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ABSTRACT

Problem-posing abilities of 54 third graders who displayed different patterns of achievement in number concepts and novel problem solving were investigated in this paper. The children were administered a problem-posing pretest followed by an instructional program (for half the sample) and a delayed posttest. Among the findings are the limited range of problems posed by all children and the difficulties they experienced in recognizing the standard addition and subtraction sentences as representing a variety of situations, even after exposure to the program. The children showed greater problem diversity within informal contexts although, overall, still favored the basic change problems. Appendices contain sample items from number and problem-solving tests, an overview of the problem-posing program, and sample student responses. Contains 65 references. (Author/MKR)

CHILDREN'S PROBLEM POSING IN COMPUTATIONAL CONTEXTS

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CHILDREN'S PROBLEM POSING WITHIN FORMAL AND INFORMAL COMPUTATIONAL CONTEXTS

Abstract

The problem-posing abilities of 54 third-grade children who displayed different patterns of achievement in number and novel problem solving were investigated, namely, children who were strong in number and weak in novel problem solving, children who displayed the reverse of this, and children who were strong in both domains. The children were administered a problem-posing pretest, followed by an instructional program (for half the sample), and a delayed posttest. This paper reports on children's problem posing within the computational component of the study and examines three main issues: firstly, whether children are able to pose a variety of problems for the standard addition and subtraction sentences (formal context), secondly, whether children generate a broader range of problems when presented with informal computational contexts (such as a photograph), and thirdly, how children within each achievement profile respond to problem-posing activities in these formal and informal contexts. Among the findings are the limited range of problems posed by all children, and the difficulties they experienced in recognizing the standard addition and subtraction sentences as representing a variety of situations, even after exposure to the program. The primitive models theory of Fischbein et al. (1985) is examined as one possible explanation for the latter finding. The children showed greater problem diversity within the informal contexts (i.e., created compare/equalize problems, as well as multistep cases), although overall, still favored the basic change problems. Children with different achievement profiles displayed different patterns of response to the informal activities of the program. These differences were reflected in the balance of structural and computational complexity shown in their problems.

CHILDREN'S PROBLEM POSING WITHIN FORMAL AND INFORMAL COMPUTATIONAL CONTEXTS

It is well recognized that problem posing is an important component of the mathematics curriculum and, indeed, lies at the heart of mathematical activity (Brown & Walter, 1983, 1993; Kilpatrick, 1987; Moses, Bjork, & Goldenberg, 1990; Silver, 1990, 1994; Silver & Mamona, 1989 a,b). The inclusion of activities in which students generate their own problems, in addition to solving pre-formulated examples, has been strongly recommended by the National Council of Teachers of Mathematics (1989, 1991). Such activities have the added benefit of providing insight into children's understanding of important mathematical concepts as well as into the nature of their school mathematical experiences (Hart, 1981; Silver, 1994; Silver & Burkett, 1993; Simon, 1993).

Despite its significance in the curriculum, problem posing has not received the attention it warrants from the mathematics education community. We know comparatively little about children's ability to create their own problems in both numerical and non-numerical contexts, and the extent to which this ability is linked to their competence in problem solving. We also have insufficient information on how children respond to programs designed to develop their problem-posing skills (Silver, 1994). Research on these issues is particularly warranted, given the well-documented evidence that young children's creativity and open-mindedness in generating and solving problems dissipates as they progress towards the higher school grades (Campione, Brown, & Connell, 1989; Kilpatrick, 1987; Lester, 1989; Resnick & Resnick, 1992).

One of the areas in need of attention is children's ability to pose their own problems when dealing with addition and subtraction situations.

While there is substantial literature on children's ability to solve such problems (e.g., Baroody & Ginsburg, 1986; Bergeron & Herscovics, 1990; Carey, 1991; Carpenter et al., 1993; Carpenter & Moser, 1984; Carpenter, Hiebert, & Moser, 1981 1983; Carpenter, Moser, & Romberg, 1982; Fuson, 1988, 1992a,b; Riley, Greeno, & Heller, 1983), there is little information on their ability to pose them. We are thus left with an incomplete account of children's computational facility. Part of this facility requires being able to assign multiple meanings to the formal symbols (+,-). Unfortunately, children's school experiences rarely provide them the opportunity to consider different meanings for these symbols; those normally given are the simple change meanings (Fuson, 1992b).

Problems involving these change meanings include the basic "change-add-to," "combine," and "change-take-from" situations where the "missing end" must be found (a full description of the different problem types can be found in Fuson, 1992a). These examples are relatively easy for children because there is a clear mapping between the given problem situation and the operation required to solve it. They are usually the first problem types introduced to children because they enable a direct modeling of a real-world problem situation with physical embodiments (Bergeron & Herscovics, 1990; Carpenter & Moser, 1984; English & Halford, 1995; Fuson, 1992a; Gray, 1991).

In contrast to these basic problems, are those in which there is not a clear mapping between the problem situation and the operation required to find the unknown quantity. Numerous studies have documented the difficulties children experience with these types (Cummins, Kintsch, Reusser, & Weimer, 1988; Lewis, 1989; Lewis & Mayer, 1987; Mayer, Lewis, & Hegarty, 1992; Stern, 1993). Such problems include comparison situations of the type:

Sally has 6 goldfish. She has 3 more goldfish than Samantha. How many goldfish does Samantha have?

Bill has 6 marbles. He has 3 fewer than Martin. How many marbles does Martin have?

Both of these problems have an unknown reference set and are difficult for children to solve because the comparison sentence cues the opposite operation. For example, the sentence, "She has 3 more goldfish than Samantha." suggests that an addition operation is needed. The less difficult comparison problems involve an unknown difference set, such as, "Sue has 7 cherries. Penny has 4 cherries. How many more cherries does Sue have than Penny?"

Equalize problems also present difficulties because their wording frequently suggests the opposite operation, as can be seen in the following example (difference unknown):

Sue has 9 marbles. Jenny has 6 marbles. How many more marbles does Jenny need to win to have as many as Sue?

When writing number sentences for these more complex problems, children tend to choose a sentence which directly models the action in the problem, rather than a sentence which represents the arithmetic solution (Carey, 1991). Carey's research showed that first-grade children would rarely select a subtraction sentence (a - b = ...) as a representation of the above equalize problem. They would choose instead, the sentence, a + ... = b. This is not surprising, given that the subtraction sentence is not supported by the semantic structure of the problem nor by the informal solution strategy children typically would use.

Given the difficulties children experience in solving these more complex problems, the question is raised as to whether they also have trouble generating such examples. If children tend to choose number sentences that map directly onto a problem's semantic structure, then conversely, the problems they pose for a given number sentence might be limited to those which correspond to its semantic structure. In other words, the standard sentences, a + b = c, and a - b = c, might elicit a narrow range of problems, specifically the elementary change problems, suggesting that children fail to recognize this formal symbolism as representing a variety of situations (Baroody & Standifer, 1993). On the other hand, if children are presented with less formal situations, where the focus is away from symbolic representations (e.g., a photograph or a piece of literature), they might be more inclined to generate a greater range of problem types, including the more difficult cases we have cited. The merits of these less formal contexts in children's mathematical learning have been well documented (e.g., Jaberg, 1995; Whitin, 1995; Whitin & Wilde, 1992). The impact of these formal and informal contexts on children's problem posing were among the issues explored in the present study.

There are, of course, other factors that might have a significant bearing on children's ability to generate different computational problems. Two factors of interest here are children's facility with number and their general problem-solving skills. Unfortunately, the literature does not provide much direction on these issues. Silver and Cai (1993) found a strong positive relationship between middle school students' problem-solving and problem-posing abilities, while Silver and Mamona (1989b) found no clear link between these components in their work with middle school mathematics teachers. Earlier studies (cited in Silver, 1994) linked problem-posing skill with creativity where fluency, flexibility, and originality of response were cited as key factors. Fluency refers to the number of problems generated, flexibility to the number of different categories of problems posed, and originality to how novel a response is, compared to all other responses. While there appears some correlation between problem posing and creativity, the nature of their relationship remains unclear (Silver, 1994, Haylock, 1987). On the other hand, there seems a clearer link between mathematical competence and problem posing, where students with a strong mathematical knowledge are better able to generate problems (Ellerton, 1986; Leung, 1993).

In an effort to shed some light on the factors influencing children's computational problem posing, it was decided to investigate the relative effects of children's facility with number and their general problem-solving competence. In defining "number" for our study, we drew upon the components of number sense cited by the National Council of Teachers of Mathematics (1989) and by Sowder (1988, 1992). These include understanding cardinal number and the relative magnitudes of number, representing number relationships with manipulatives, recognising unreasonable results for calculations, and being able to recognise and use a variety of computational situations and problem structures. Our general problem-solving factor referred to children's ability to solve novel, non-computational problems for which they do not have a readily available method of solution (Baroody, 1993; Charles & Lester, 1982). In the absence of a known solution procedure, children have to apply a good deal of "original thought or deep reasoning" (Silver & Kilpatrick, 1989, p. 179).

In examining the possible effects of facility with number and competence in novel problem solving, we targeted children who displayed different profiles of achievement in these two domains. As Lesh and Lamon (1993) noted, students who are competent in mathematics "often have exceedingly different profiles of strengths and weaknesses," with their learning progressing along a variety of paths and dimensions (p. 7). When presented with problem-posing activities, children who are strong in the number domain but weak in novel problem solving, for example, might display different patterns of response to children who display the reverse profile (i.e., weak in number but strong in novel problem solving). Children who are competent in both domains might show other patterns of response and perhaps display superior problem-posing skills.

To investigate this, we worked with third-grade children (8 year-olds) who exhibited these different profiles of achievement, as measured by tests we designed in number and novel problem solving. As indicated in the following sections, 54 children were initially administered a problem-posing pretest. Half of them subsequently participated in a two-month problem-posing program, which was followed by a posttest administered to all children. The program incorporated activities set within addition and subtraction contexts, as well as activities dealing with novel, noncomputational problem situations. We included this range of activities in order to provide a rich and interesting program of experiences for the children, as well as to compare their responses across the different problem situations. For this paper however, we confine our discussion to the computational component of the study and address three main issues:

1. Are children able to pose a variety of problems for the standard addition and subtraction sentences, that is, do they recognize the formal symbolism as representing a range of problem situations?

2. Do children generate a broader range of problem types for informal computational situations, that is, where there is an absence of formal symbolism?
3. How do children with different profiles of achievement in number and novel problem solving respond to problem-posing activities within these two contexts?

In addressing these issues, we firstly describe how our subjects were chosen. We then review their responses to the computational component of the problem-posing pretest. A description of the problem-posing program and how the children responded to it are then presented. Finally, we review the children's performance on the posttest and consider the implications of our findings.

Selection of Subjects: Initial Assessment of Number and Novel Problem Solving

Six classes of eight year-olds (N = 154) from one state and one non-state school were given the initial testing instruments. The schools were situated in neighboring suburbs of predominantly middle-class families in a major city in south-eastern Queensland, Australia. Fifty-four children were chosen on the basis of their responses to the tests of number sense and novel problem solving, as described shortly. All children were in their third year of elementary school and had a mean age of 8.1 years in the middle of the school year. While the children had experienced a range of computational problems in their regular class activities, they had not been exposed to problem-posing tasks of the types included in the study.

The tests of number and novel problem solving were each administered in two sessions on a whole class basis. All problems were read to the children and re-read when necessary. The children were instructed to show all of their working on the sheets provided.

We did not use commercially available tests of number as it was difficult to obtain ones which met our criteria, namely, that the items be in line with the children's school curriculum and that they provide insight into children's number sense, as we have defined it. Our aim was to assess children's conceptual understanding of number and application of number, rather than their knowledge of specific facts. To this end, we did not use multiple choice items (cf. Ginsburg et al.'s 1993 argument re "dumb tests," p. 286) and included questions which asked for justification and explanation. A reliability analysis of the number test yielded a Cronbach alpha of .77. Appendix A contains a sample of items from the test.

Finding suitable commercial tests of novel problem solving that met our criteria proved equally difficult. Firstly, our items were to represent novel, yet meaningful situations for the children. Secondly, the items were to address reasoning processes that were considered important in children's mathematical development and that were sufficiently varied to provide a reasonable assessment of the children's ability to solve novel problems. The test we developed comprised problems that entailed deductive, inductive, and spatial reasoning, these being recognized as significant processes in the mathematics curriculum (Baroody, 1993; English & Halford, 1995; NCTM, 1989). The items included combinatorial examples, deductive reasoning problems, noncomputational patterns, and spatial puzzles, with the puzzles being taken from Rowe (1986). The combinatorial and deductive problems had been used extensively in prior research with children of this age (e.g., English, 1993; English & Halford, 1995). The Cronbach alpha reliability for the problem-solving test was .67. Sample items appear in Appendix B.

Children's responses on each of the tests were scored on an interval scale, with maximum scores of 30 for the number test and 20 for the problem solving. The scores ranged from 8 to 29 on the number test (mean of 18.4, SD = 4.3) and from 4 to 20 on the problem-solving test (mean of 10.7, SD = 3.6). We classified children as strong in number sense if they scored at least 24 points out of 30, and weak if they scored 16 or fewer. Those considered competent in novel problem solving scored at least 16 points out of 20, with those weak in the domain scoring 10 points or fewer. As previously indicated, we were interested in children who demonstrated different profiles of achievement in the two domains. We initially identified children who fell into the four categories:

1. strong in number sense but weak in novel problem solving (N=20)
2. weak in number sense but strong in novel problem solving (N=14)
3. strong in both domains (N=20)
4. weak in both domains (N=13)

We interviewed each of the children in these categories to probe further their number sense and problem-solving skills. This was prior to administering a problem-posing pretest for the instructional program. As a result of these interviews, we chose not to include children in the fourth category (i.e., weak in both domains). These children were on special remediation programs in their school and had difficulties not only in mathematics but also in other areas including literacy. Given that they were lacking in basic skills needed for participation in our program and also required concentrated, individual guidance in their learning, we felt our program could not effectively meet their needs. Hence our subsequent discussion focuses on children in the first three categories, hereafter abbreviated to SN/WP (category 1), WN/SP (category 2), and SN/SP (category 3). As noted above, there were only 14 children who fell into the second category, using our given cut-off points. We tried to obtain equal sized groups by adjusting these points, but this did not give us the demarcation we required between each category. We thus had to remain with the present numbers.

Pretest

All 54 children were administered the pretest on an individual basis, with each child asked for a verbal response only. All responses were videotaped. The computational component of the pretest comprised four tasks, as shown in Table 1.

INSERT TABLE 1 ABOUT HERE

In contrast to the formal context involving a standard subtraction sentence, the items in the informal context (photograph and non-goal specific statement) were free of symbolic representations (except for the numbers of toys given). The merits of these open-ended contexts for encouraging effective problem posing and solving have been well documented (e.g., Charles & Lester, 1982; Silver, 1990; Baroody, 1993). Silver (1990) recommends the inclusion of non-goal specific situations in the curriculum as they provide good opportunities for students to learn useful problem-solving knowledge and skills, as well as to engage in generative aspects of mathematical thinking such as problem posing and conjecturing. We also included such activities in our problem-posing program, as indicated later.

Children's Responses to the Pretest

Children's responses to the four pretest items, as well as to the program and posttest items, were classified according to the type of problem they posed and also, whether the problem was multistep. The children offered a limited range of problems, these being confined to three main types: change-add-to/combine (missing end), change-take-from/separate (missing end), and equalize/compare (unknown difference set). There was also an "others" category, which included no response or an inappropriate response. Typical responses in each of the problem categories on the pretest were as follows:

Change-Add-to or Combine Problems, with Missing end

Anne had 8 yogurt cups and Kelly gave her 3 more. How many has Anne now? (a change-add-to problem generated from the photograph by Barbara, a SN/SP student)

Catherine had three yogurt containers and Stephanie had eight. How many did they have altogether? (a combine problem generated from the photograph by Claire, a SN/WP student)

Change-take-from or separate problems, with missing end

There were 12 boats sailing. Eight of the boats sank. How many were left? (a change-take-from problem created for the number sentence by Lauren, a WN/SP student)

There were 12 kings and eight stopped ruling their part of the land. How many kings are still ruling, are still in charge of places? (a separate problem offered by Nicholas, a SN/WP student)

Since there was only one "missing change" (Fuson, 1992) subtraction problem offered, it was incorporated within the missing-end cases.

Equalize and compare problems, with unknown difference set

Amy has four glass jars and Belinda has two glass jars. How many more glass jars does Amy have than Belinda? (a compare problem generated from the photograph by Reilly, a SN/SP student)

How many more toy cars does she need if she wants the same number of dolls as cars? (an equalize problem created for the non-goal specific statement by Natalie, a WN/SP student).

Others

Problems assigned to this category included cases in which problem data were ignored, where a problem could not be created, or where an inappropriate problem was offered, such as:

There were 12 people and there were eight pies. How many pies did they each get?" (offered for the number sentence by Ian, a WN/SP student)

Multistep Problems

A problem was considered multistep if it entailed two or more operations, either a repeated application of one operation or a combination of different operations (Baroody, 1993). In the case of the latter, the problem type was ascertained from the nature of the question posed. For example, Ian (a WN/SP student) created the following multistep compare problem for the photograph.

Leanne has eight yogurt containers, five bottle tops, and seven butter lids. And Belinda has one yogurt container, seven bottle tops, and three lids. How many more things does Leanne have than Belinda?

Table 2 displays the distribution of problem types created by the children on the pretest. Children across all achievement categories clearly favored the basic change problems for the formal context. There were no significant differences among the achievement categories in their choice of problem type. All children offered a significantly greater number of basic change problems than any other type for their first number sentence attempt, with 72% of all responses being of this type, $X^2(1) = 9.76$, $p < 0.01$. This decreased to 57% of all responses in the basic change category for the second number sentence and was accompanied by an increased percentage of "other" responses (39% of all responses). The increase in this latter category was due to children in the SN/WP and SN/SP categories who had difficulty in creating a second problem for the given number sentence. The children clearly found it difficult to conceive of the standard subtraction sentence as representing anything other than a basic change situation.

The informal context on the other hand (viz., the photograph and non-goal specific statement) was more conducive to the generation of compare and equalize problems. A significantly greater number of these problems was produced for the informal situations than for the formal number sentence attempts, $X^2(1) = 24.75$, $p < 0.001$. The children did not differ significantly in their creation of compare/equalize problems for the photograph, with approximately 40% of children in each achievement category generating these. The non-goal specific statement did not elicit as many compare/equalize problems, although 35% of the SN/SP children did offer these (in contrast to 15% and 21% of the SN/WP and WN/SP children respectively). The children also found it more difficult to create an appropriate problem for the non-goal specific statement than for the photograph, especially the SN/WP children where 25% of them could not do so.

As can be discerned from Table 2, there were no significant differences amongst the achievement categories in their production of change-add-to/combine problems for the informal context. The children favored these addition problems over the subtraction change problems, with only one child (SN/WP) creating a change-take-from problem for the photograph. The creation of multistep problems was also rare, with only three children (two SN/WP and one SN/SP) generating such a problem.

In sum, the pretest results highlighted firstly, the overall limited range of problems the children were able to generate for both the formal and informal contexts. Secondly, the results pointed out children's difficulty in recognizing a formal subtraction sentence in terms of a range of problem situations, their interpretation being confined to the basic change types. Thirdly, it appeared that the informal situations were more conducive to children's generation of a broader range of problems, viz., the inclusion of compare and equalize problems. Finally, there were no significant differences in the responses of the children in the three achievement categories, this being partly due to our small cell sizes. We now give consideration to the problem-posing program and the children's responses to the computational component.

The Problem-Posing Program

Half of the children from each achievement category participated in the problem-posing program, while the remaining 27 children continued with their normal class lessons which did not address problem-posing activities of the type included in the study. The children were randomly assigned to the program and nonprogram groups. The program was conducted in a vacant room in each of the two schools from which the children were drawn. It was implemented by a research assistant with specialist teaching qualifications in the early school years.

The program comprised 16, 45-minute sessions, with two sessions conducted per week during the third term of the school year. The constraints of the school timetable prevented us from conducting our desired three sessions per week. As stated previously, the program incorporated activities set within addition and subtraction contexts, as well as activities dealing with novel, noncomputational problem situations. As indicated in Table 3, the latter activities took place during the first nine sessions of the program, while the computational examples took place in the next five sessions. Because the novel problem situations proved to be time consuming, we did not spend as much time on the computational examples as we had orignially planned. The remaining two sessions of the program were devoted to an open-ended, applied problem (planning a class activity), which we considered important to retain in order to integrate the children's problem-posing skills in a purposeful fashion (Baroody, 1993).

In all sessions, we attempted to create an environment which was both constructive and interactive (Cobb, Yackel, & Wood, 1992; Jones, Thornton, & Putt, 1994). We placed a strong emphasis on children's interactions with their peers and the teacher, in the hope that this collaboration would help children internalize their learning (Vygotsky, 1978). We attempted to capture some of these interactions on tape, however the noise levels of surrounding rooms prevented us from doing so effectively. Of interest to the present discussion are the sessions devoted to the computational situations. These included the formal context, in which the children created problems for standard addition and subtraction sentences, and the informal context comprising non-goal specific situations, a stimulus picture, and a piece of literature. We describe these in the remainder of this section and, for uniformity, begin with the formal context.

INSERT TABLE 3 ABOUT HERE

Creating Problems for Standard Number Sentences

Whole group discussion

A set of approximately 30 standard addition and subtraction sentences were written on individual cards and displayed before the whole group. The sentences only involved number facts to 18, all of which the children had learnt in class. After discussing the nature of the sentences, one example was selected (e.g., 8 - 5 = 3) and the children invited to create a verbal story problem that could be solved by the sentence. To encourage problem diversity (following children's limited offering of basic change problems), the teacher reviewed a child's problem and posed a question such as, "What if Martha began her problem like this: I had 8 toys and Sally had 5 toys. How might she complete her problem then?" This was followed by small group work.

Small group activity

Working in pairs, the children selected two of the number sentence cards, one of each operation, and formulated a story problem for each. They were invited to tape record their problems prior to recording them in their journals, which would enable them to subsequently share and publish their problems (Silverman, Winograd, & Strohauer, 1992). Throughout this time, the teacher moved amongst the children, questioning them on the nature of their problems and encouraging them to consider alternative situations for the one sentence. For example, if a child wrote the same basic change problems for the sentence, 6 + 4 = 10 (e.g., "I had 6 marbles and I won 4 more. How many did I have altogether?"), the teacher might say, "What other question might you ask about your marbles?" If necessary, the teacher would offer more explicit guidance and encourage the children to model alternate problems (e.g., "What if Martha had 10 marbles and you only had 4 marbles?").

Whole group discussion

The children subsequently shared their problems with the larger group, with discussion focusing on the similarities and differences in the structure of their problems. As before, the children were encouraged to consider other questions that might be asked about the information given in their problems. A short period was devoted to the children solving one of their friends' problems.

Generating Problems from Two Non-Goal Specific Situations

Whole group discussion

The following stories about Rufus, the dog, (Baroody, 1993) were displayed on large sheets and read to the children.

1. Rufus managed to get into the Bradley house one afternoon. He chewed up four of Amy's shoes, three of her toys, and six of her socks. He also chewed up five of Brad's shoes, seven of his toys, and two of his socks.
2. Mrs Smith baked two dozen biscuits. Rufus made off with twelve biscuits. He buried eight of them before Mrs Smith discovered him.

The children discussed the situations and responded to questions about the information given. As the children were unfamiliar with problem situations of this type, it was explained that these examples gave information only and that the children's task was to think of some problems about the damage Rufus caused, that is, some questions they could ask about the given information. Initial responses were discussed within the whole group situation, with children encouraged to raise a number of different questions. This was followed by small group work.

Small group activity

The children chose a partner and worked collaboratively on posing problems for either one or both of the Rufus stories. They were encouraged to discuss their problems prior to recording them in their individual journals. The teacher roamed among the groups, questioning the children on their created problems, and where appropriate, encouraging them to think of different problem types (e.g., "What other question could you ask about Rufus and the toys?" "Could you make up a problem where we would have to think about Amy's toys as well as Brad's toys?").

Whole group discussion

Following the small group activity, the children came together to share their problems. This not only provided a meaningful source of problem-solving activities, but also enabled children to reflect on the structure of their problems as they compared and contrasted the different examples posed by their peers.

Generating Problems from a Stimulus Picture

Whole group discussion

Initial discussion focused on the array of items displayed in an illustration of a toy shop window, with the children subsequently invited to create a problem based on these items. As before, the children's problems served as vehicles for further problem posing. For example, if a child offered the problem, "Jane bought 5 dolls and Sue bought 3 dolls. How many dolls did they buy altogether?" the teacher would ask the children to think of other problems about Jane's and Sue's dolls. If the children had difficulty in creating a range of problems, the teacher would provide further encouragement (e.g., "What if Sue was unhappy because she only had 3 dolls and wanted 5 dolls . . . ") .

Small and large group discussions

These discussions followed similar formats to the previous activities, with each group of children given a copy of the toy shop picture and encouraged to think about different problem situations that might be generated. As before, the children were invited to consider other questions that might arise from the information given in their problems. They were also provided the opportunity to solve one or two of each other's problems.

Generating Problems from Literature

This activity proceeded along similar lines to the previous activities. The children read and discussed the book, Blue Gum Ark (Clapman, 1988), an Australian version of Noah's Ark. The various native animals were identified and the number of each animal appearing in the story was recorded on a large sheet for later reference. The children posed and shared problems about the creatures in the story and illustrated their work, if they wished. As before, the teacher fostered problem diversity through guided questioning. The children subsequently solved one or two of their friends' problems.

Children's Responses to the Problem-Posing Program

As previously noted, the children maintained individual journals of the problems they created during each of the activities. These journals provided the data for the present analysis of their responses. The children were reluctant to record their problems on tape, requesting that they write their problems first. This proved somewhat of a handicap as there were some children who were able to offer more diverse problems in their discussions with the teacher and their peers, but when it came time to write their problem, switched to a basic change example. Their regular class teachers commented that the children appeared concerned with writing a problem they thought would be "correct" and thus opted to record the familiar basic change problems. The difficulties some children experienced with written expression might also have hindered their recording of more divergent problems.

Table 4 shows the numbers of children who recorded change problems only, compare/equalize problems only, or both change and compare/equalize problems for each of the problem situations.

INSERT TABLE 4 ABOUT HERE

As on the pretest, all children conceived of the number sentences in terms of basic change situations. Even with encouragement to consider other examples, the children remained with the change-add-to/combine and change-take-from/separate problems and were reluctant to record a problem that did not map directly onto the semantic structure of a sentence. Creating a "different" problem was achieved largely by changing the context (e.g., eating cookies to eating apples or losing toys). On the other hand, as can be seen in Table 5, the children were productive in their creation of problems for the number sentences, with all of the WN/SP children generating three or more problems.

INSERT TABLE 5 ABOUT HERE

The children also showed a preference for the basic change problems on the informal activities of the program (see Table 4), especially for the non-goal specific situations and the stimulus picture. "Change only" problems were favored significantly over the other types for both the non-goal specific situations X^2 (1) = 12.0, $p < 0.001$, and for the stimulus picture, X^2 (1) = 9.48, $p < 0.01$.

As indicated in Table 4, the children had difficulty in posing compare/equalize problems for the non-goal specific situations, with only three children (two WN/SP and one SN/SP) doing so. All of the SN/WP children wrote change problems only. This lack of problem diversity may have been due partly to the children's unfamiliarity with these non-goal specific situations and to the nature of the particular examples we chose; a broader context might have elicited more diverse problems. Some children had initial difficulty with the open-ended nature of these non-goal specific situations and, rather than pose a problem which required working with the data, they simply asked a question about the data that were already given, such as, "How many biscuits did Rufus bury?" With the exception of one child (SN/SP), children who wrote basic change problems remained with this type, despite the teacher's encouragement to consider other questions about Rufus and the items he stole. Nevertheless, as shown in Table 5, several children in each achievement category were able to generate three or more problems for these situations, with the SN/WP children being the most productive.

As Table 4 shows, more children were able to create compare/equalize problems for the toy shop stimulus picture and the piece of literature than for the non-goal specific situations. The literature, in particular, encouraged the generation of these problems. There was a significantly greater number of children creating compare/equalize problems for the literature than there were for the non-goal specific situations, X^2 (1) = 5.88, $p < 0.05$. There was no significant difference in the number of children creating these problems for the literature and the stimulus picture.

Although the numbers are small, it is interesting to compare the proportions of children from each achievement category who were able to generate compare/equalize problems for each of the three informal situations. The least creative children in each case were those in the SN/WP category. As previously noted, none of these children created a compare/equalize problem for the non-goal specific situations and only one child did so for the stimulus picture; 30% could do so however, for the literature. In contrast, 29% of the WN/SP children could create compare/equalize problems for the non-goal specific situations, 43% could do so for the stimulus picture, and 57% for the literature. These WN/SP children outperformed the others on the first two of these situations. It was only on the literature that the SN/SP children performed best, with 70% of them posing compare/equalize problems.

It is also interesting to look at the children's creation of multistep problems (see Table 5). Although these were not emphasized in the program, there was a noticeable increase in their creation between the pretest and the program. On the pretest, only three children (one SN/SP and two SN/WP children) created such problems, whereas children from each category generated multistep examples for the informal situations of the program. The WN/SP children however, showed the least preference for multistep cases during the program. While these WN/SP children produced problems that lacked computational complexity (i.e., incorporated only one operation), they nevertheless were able to create problems that displayed greater structural complexity (i.e., represented the more complex compare/equalize situations), as we have indicated. In contrast, the SN/WP children created problems with little structural complexity (i.e., represented basic change situations) but with greater computational complexity (i.e., incorporated more than one operation). The SN/SP children demonstrated both of these patterns. That is, on the piece of literature, 70% of them recorded at least one compare/equalize problem but only 30% created multistep problems. On the stimulus picture, where only one SN/SP child created a compare/equalize problem, 70% of these children recorded multistep problems.

A final point worth mentioning is the observed difference in the written expression of the children in each of the categories. The WN/SP children frequently had difficulty here, as can be seen in Jaimie's response in Fig. 1. To ensure this did not hamper the children's recording of their creations, written assistance was provided where necessary. In contrast, the SN/SP children were competent in written expression, as can be seen in Reilly's case (Fig. 1). The SN/WP children showed considerable variation here, but generally were more adept than their WN/SP counterparts.

INSERT FIGURE 1 ABOUT HERE

In summary, the children's responses to the problem-posing program again highlighted their limited interpretation of the standard number sentences. While the children were able to generate several change problems for a given sentence, there was little variation in the structure of their problems. The informal activities did encourage greater diversity in problem type, although the basic change problems were still favored overall. Of the informal situations, the literature was the most effective in eliciting compare and equalize problems. Whereas the non-goal specific situations were the least conducive to this, they nevertheless yielded the greatest productivity in terms of the number of problems generated. The informal situations also encouraged the creation of multistep problems, which rarely appeared on the pretest.

Although significant differences among the achievement categories were not evident, potentially important patterns of response emerged in the children's problem creations for the informal situations. The SN/WP children were the least creative in the types of problems they generated. In contrast, the WN/SP children showed the greatest diversity, except on the literary item where the SN/WP children performed best. Children in each category showed an increased generation of multistep problems on the program. Interesting contrasts in the children's use of multistep problems and in their problem diversity were evident. Whereas the SN/WP children's creations lacked diversity and had limited structural complexity, many displayed computational complexity. The WN/SP children on the other hand, demonstrated greater structural complexity in their problems but less computational complexity. The SN/SP children displayed botn patterns of response.

Children's Responses to the Postest

The problem-posing program finished just before the children's two-week mid-semester break towards the end of September. A delayed posttest was administered to all 53 children during the first two weeks of their return (one child had left the school during the vacation period). The posttest paralleled the pretest, with the pretest photograph reused. The posttest was administered on an individual basis, with all responses videotaped. Table 6 displays the distribution of the program and nonprogram children's problem types for the formal context (two problem-posing attempts for the number sentence, 9 - 6 = 3) and the informal context (the photograph and the non-goal specific statement, "Sally has three toy animals on one shelf in her room and and seven racing cars on another"). The two items within each context have been collapsed for ease of analysis.

INSERT FIGURE 6 ABOUT HERE

As was expected, the program did not alter children's limited interpretation of the standard subtraction sentence (see Table 6). All of the program children's problems (except one) were of a change-take-from/separate type. The nonprogram children performed in a similar manner, although one child (WN/SP) did offer a compare/equalize problem.

While the children showed greater diversity on the informal context, they still favored the basic change types. A significantly greater number of responses were of this type than any other, X^2 (1) = 39.68, $p < .001$. There was a noticeable difference however, between the pretest and the posttest in children's generation of these change types for the informal situations. Only one response on the pretest was of a subtraction change type (see Table 2), in contrast to 25% of all responses of this type on the posttest. One explanation for this could be the increase in children's creation of multistep problems between the pretest and posttest. Twenty-five percent of all responses on the posttest were multistep and half of these were subtraction change problems. The increase in multistep problems on the informal activities of both the program and the posttest is interesting, given that these were not targeted in the program and that the nonprogram children also increased their use of these (namely, from one response on the pretest to 24% of their responses on the posttest). This appears due to children's mathematical development in the intervening period and may reflect their emerging belief that "good story problems" contain lots of interesting information, some pertinent and some extraneous to the problem question (Silverman et al., 1992).

No significant differences in the children's generation of the different problems types were noted on the posttest, neither between the program and nonprogram children, nor between the children in each achievement category. Both the program and nonprogram children created few compare/equalize problems for the informal context. Program children in the SN/SP category created the greatest number of compare/equalize problems, with 35% of their responses being of this type. On the other hand, these children created few multistep problems (15% of their responses to the informal context). Their SN/SP counterparts in the nonprogram group showed the reverse pattern (only one compare/equalize problem was offered but 30% of their responses were multistep). A similar pattern was displayed by the SN/WP children in the program group, where only 17% of their responses were of the compare/equalize type but 33% were multistep (reflecting their pattern of response on the program). The WN/SP children in both the program and nonprogram groups generated few compare/equalize problems (14% of their responses), but did generate some multistep problems (29% of their responses).

DISCUSSION

The present study explored three main issues: firstly, whether children are able to pose a variety of problems for the standard addition and subtraction sentences, secondly, whether children generate a broader range of problems when presented with informal computational contexts, and thirdly, how children with different profiles of achievement in number and novel problem solving respond to problem-posing activities in these formal and informal contexts. Fifty-four 8-year-olds were administered a problem-posing pretest. This was followed by an instructional program, in which half the children participated, and then a delayed posttest, administered to all.

The findings from all three components of the study revealed an overall limited range of problems the children were able to pose for both the formal and informal contexts. The children posed three main problem types: change-add-to/combine (missing end), change-take-from/separate (missing end), and compare/equalize problems (unknown difference set). Multistep problems were also generated, mainly on the program and posttest. In all of the formal activities, the children had difficulty in recognizing the standard addition and subtraction sentences as representing a variety of problem situations (cf. findings of Huinker, 1992). Rather, they interepreted the formal symbolism in terms of the basic change situations and were reluctant to pose problems that did not correspond to the semantic structure of a sentence. The children were able to create several of these change problems on the problem-posing program, largely by altering the context of their problem. The activities of the program were clearly inadequate in broadening the children's interpretation of these formal statements. The program needed to include several more sessions in which children could both solve and pose computational problems that extended beyond the basic change types.

The children showed greater diversity in problem type within the informal contexts of the study (non-goal specific situations, stimulus picture, photograph, literature), where they generated some compare and equalize problems, as well as multistep cases. Although the children were not prolific in their creation of compare/equalize problems, there were nevertheless some potentially important trends emerging. Firstly, the non-goal specific situations were not as conducive to children's creation of these problems as were the other informal activities. This was due partly to children's lack of familiarity with these open-ended situations and perhaps to the context in which these were couched. The children responded more favorably to the photograph and literature, especially the latter.

Secondly, there was a noticeable increase in children's generation of multistep problems for the informal activities between the pretest and the program; this trend continued on the posttest, even though such problems were not specifically targeted on the program. Half of the children's multistep problems on the posttest were basic subtraction change problems, accounting for the increase in these problem types on the posttest. This increase was accompanied by a decline in children's creation of compare/equalize problems. In other words, it appeared that the children opted for computational complexity, rather than structural complexity, in posing their problems on the posttest. Both forms of complexity have a role to play in children's computational problem posing, as we have shown. Of particular interest is the interaction of the two in the development of children's problem-posing skill, an issue that warrants further investigation (cf. Nesher & Hershkovitz's, 1994, argument on how this interaction accounts for variance in children's solving of two-step problems).

The third trend pertains to the ways in which children in each of the achievement categories incorporated these complexities within their problem posing during the program. Caution is exercised in drawing any generalizations however; rather, the emerging patterns of response should be taken as avenues for further exploration. An analysis of the children's recorded problems for the informal activities indicated that children classified as strong in number sense but weak in novel problem solving were the least divergent in their problem posing (i.e., favored the basic change types). Whereas these children showed a lack of structural complexity in their problems, they did display some computational complexity, with nearly half of them opting for multistep problems on one or more of the informal activities. Children with the reverse profile of achievement (WN/SP) tended to display the reverse pattern of response. That is, they showed greater structural complexity in their problems than the SN/WP children but less computational complexity. Children who were strong in both number and novel problem solving displayed both patterns of response.

Our data are not sufficiently strong to clearly identify the relative effects of facility with number and competence in novel problem solving on children's computational problem posing. However, it is tempting to suggest that competence in novel problem solving, rather than facility with number, might contribute to the generation of more diverse and more structurally complex problems. Facility with number on the other hand, might facilitate the creation of more computationally complex, rather than more structurally complex, problems. It follows that the intersection of these complexities (i.e., problems that are both structurally and computationally complex) would be seen in the problems of children who are competent in both number and novel problem solving. Our data, albeit limited, do not support this hypothesis, however. Clearly, these conjectures warrant futher testing with a larger sample of children on a more comprehensive program of computational problem-posing activities.

Perhaps the main issue in need of attention is why children have difficulty in posing a diverse range of problems for both formal and informal situations. One explanation is children's lack of exposure to varied problem situations within the classroom, as has been indicated by analyses of commonly used school texts (e.g., Fuson, 1992; Stigler, Fuson, Ham, & Kim, 1986). While children's real-world experiences include many situations that extend beyond the basic change types, it seems they have difficulty in transferring this knowledge to informal computational activities within the classroom setting (Hiebert & Carpenter, 1992; Resnick, 1987; Resnick & Resnick, 1992).

A second explanation which could account for children's limited problem-posing skill may be found in the primitive models theory of Fischbein, Deri, Nello, and Marino (1985). We confine their ideas to the formal context, as further development of their theory would be needed to address children's responses to the informal activities. Fischbein et al. argue that "each fundamental operation of arithmetic generally remains linked to an implicit, unconscious, and primitive intuitive model." These models comprise the original meanings that were assigned to the operations and are thus behavioral in nature and meaningful to children. The models are assumed to exert an unconscious influence on children's problem-solving efforts and to be largely responsible for the difficulties children encounter when dealing with computational situations. The constraints these models place on children's performance appear to remain long after a concept has been formalized. Hence if children's concepts of addition and subtraction are intuitively tied to the basic change situations, then these mental models are likely to place constraints on children's ability to entertain other problem cases. This would apply especially to their interpretation of the formal addition and subtraction sentences whose standard structure maps onto these primitive and enactively meaningful change notions. Children would thus have difficulty in posing problems that did not conform to these intuitive models of addition and subtraction.

The constraints imposed by these models on children's problem posing would likely be strengthened by classroom activities that strongly favor the basic change situations over all other types. We need to broaden children's problem-solving and problem-posing experiences to include a variety of problem situations (Baroody & Standifer, 1993; NCTM, 1989). We also need to present activities which encourage children to assign multiple meanings to the formal symbols and to associate the standard number sentences with problems that do not directly match their semantic structure. Given that children have difficulty in recognizing the standard forms as efficient representations of sentences such as, a + ... = b and .... - a = b (Carey, 1991), it is important that children's exploration of number relationships includes activities to develop this understanding. If children develop flexibility in their use of language describing quantitative comparison, which includes understanding the symmetric nature of the relations "x fewer than y" and "y more than x" (Stern, 1993), they might be in a better position to create more diverse problems, both within formal and informal contexts. Finally, we need to make greater use of informal situations, including children's everyday experiences, as vehicles for fostering children's problem-posing abilities.

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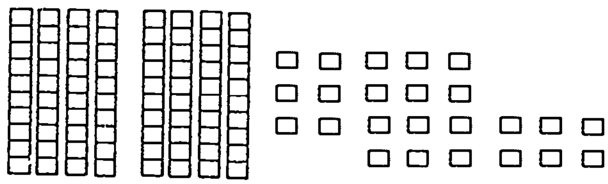
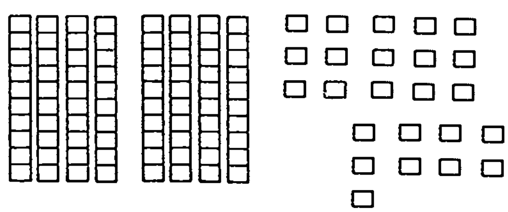
Appendix A

Sample Items from the Number Test

1. Sam wants to show the number 57 with his blocks. Color the blocks he could use to show his number.



2. Is there another way Sam could show the number 57 with his blocks? If you think there is, show this by coloring some of the blocks below.



3. Which is larger: 82 or 79? ....

   Tell how you know..

   ....

4. How is the number 24 different from the number 42? ....

   ....

5. Mary and Sue counted seventeen butterflies. Mary wrote 17 in her book. Sue wrote 71 in her book. Who is correct and why?

   ....

   ....

6. Write in the missing numbers in this number pattern.

   22, 32, 42, ..., 62, 72, ..., 92

How did you work out which numbers to write? ....

....

7. Sally worked the following examples and made a mistake in two of them. Find her mistakes and show what she should have done.

35	46	54
+ 42	+ 24	- 20
77	60	30

8. Write a number sentence that will give you the answer to this problem: Kelly won 5 medals. This is 3 fewer than the number of medals that Kim won. How many medals did Kim win?

Appendix B

Sample Items from the Problem-Solving Test

1. John and Sue are making greeting cards. They have yellow paper, blue paper, and pink paper. They can also use silver labels, gold labels, and green labels. How many different cards can they make if each card has coloured paper and a label?

2. Kelly built a tower with some coloured blocks. Use the clues to work out which block was the bottom block.

   Clues:

   * The red block was just below the yellow block.
   * The blue block was just above the white block and just below the red block.
   * The yellow block was somewhere between the green block and the white block.

3. To help them clean their home, Mr and Mrs Brown use the following plan:

   Day 1: Clean the floors Day 2: Wash the clothes

   Day 3: Dust the furniture Day 4: Clean the bathroom

   Day 5: Clean the floors Day 6: Wash the clothes

   Day 7: Dust the furniture

   What do Mr and Mrs Brown do on Day 8?

   What do Mr and Mrs Brown do on Day 10?

   Can you find a fast way of working out what Mr and Mrs Brown will do on Day 17? Describe your fast way below.

4. Look at the shape in the single box on the left (near the star). Two of the shapes in the boxes on the right go together to make that shape. Put a ring around these two shapes.

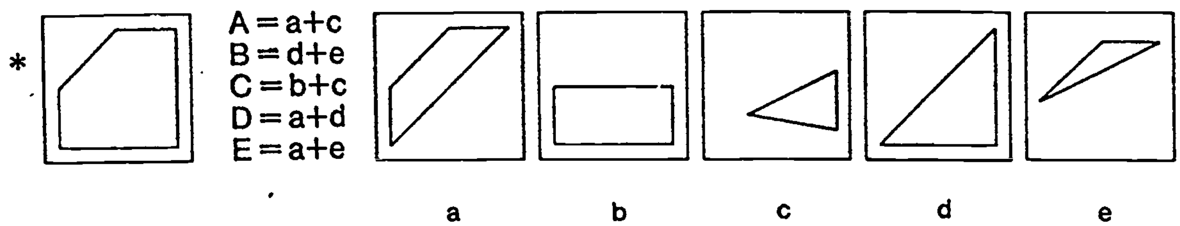


Table 1

Pretest Computational Items

Formal Context

1. The child was to pose a problem that could be solved by the number sentence, 12 - 8 = 4.
2. The child was then asked to pose a different problem that could be solved by the same sentence.

Informal Context

3. The child was to create a problem from a large photograph of children playing with various sets of discrete items (containers, lids, bottle tops, etc.).
4. The child was to transform the following non-goal specific statement into a problem: Sarah has 5 dolls on one shelf in her room and 4 toy cars on another shelf.

Table 2

Proportion (and Frequency) of Responses of Each Problem Type on the Pretest by Problem Situation and Achievement Category

Problem Situation	Achievement Category	Problem Type: Change-add-to/ Combine	Change-take-from/ Separate	Compare/Equalize	Other
Formal Context					
Number sentence: First attempt	SN/WP (N=20)		.8 (16)	.05(1)	.15 (3)
	WN/SP (N=14)		.57 (8)		.43 (6)
	SN/SP (N=20)		.75(15)	.1 (2)	.15 (3)
Number sentence: Second attempt	SN/WP		.55 (11)	.05(1)	.4 (8)
	WN/SP		.5 (7)	.07(1)	.43(6)
	SN/SP		.65 (13)		.35(7)
Informal Context					
Photograph	SN/WP	.45 (9)	.05 (1)	.4 (8)	.1 (2)
	WN/SP	.5 (7)		.43(6)	.07(1)
	SN/SP	.6 (12)		.4 (8)	
Non-goal specific situation	SN/WP	.6 (12)		.15(3)	.25 (5)
	WN/SP	.65 (9)		.21(3)	.14 (2)
	SN/SP	.5 (10)		.35(7)	.15 (3)

Generation of Multistep Problems

Photograph	SN/WP (N=20)	.1 (2)
Non-goal specific situation	SN/SP (N=20)	.05(1)

Table 3.

Overview of Problem-Posing Program

SESSION	ACTIVITY
1 - 5	Solving and creating combinatorial and deductive reasoning problems using hands-on materials
6 - 9	Patterning activities involving both number and non-number examples, with children completing and generating their own patterns; use of hands-on materials and activity cards
10	Generating problems from two non-goal specific situations involving the adventures of the dog, Rufus
11 - 12a	Creating problems for standard addition and subtraction number sentences
12b - 13	Generating problems from a stimulus picture of a toy shop
14	Generating problems from a piece of literature, namely, Blue Gum Ark (Clapman, 1988)
15 - 16	Posing and solving problems within an applied problem situation, namely, planning a class stall for the school fair

Table 4

Proportion (and Frequency) of Children who Created Change Only, Compare/Equalize Only, and Change & Compare/Equalize Problems

		Problem Type			
Problem Situation	Achievement Category	Change Only	Compare/ Equalize Only	Change and Compare/Equalize	No Response
Formal Context					
Number	SN/WP (N=10)	1(10)			
Sentences	WN/SP (N=7)	1 (7)			
	SN/SP (N=10)	1(10)			
Informal Context					
Non-goal	SN/WP	1 (10)			
Specific	WN/SP	.57(4)	.29(2)		.14(1)
Situations	SN/SP	.9 (9)		.1(1)	
Stimulus	SN/WP	.9 (9)		.1 (1)	
Picture	WN/SP	.57(4)	.29(2)	.14(1)	
	SN/SP	.9 (9)	.1 (1)		
Literature	SN/WP	.7 (7)	.2 (2)	.1 (1)	
	WN/SP	.42(3)	.29(2)	.29(2)	
	SN/SP	.3 (3)	.3 (3)	.4 (4)	

Table 5

Proportion (and Frequency) of Children on the Program who Generated (a) Three or More Problems and (b) Multistep Problems

Problem Situation	Achievement Category	Generated Three or More Problems	Generated Multistep Problems
Number	SN/WP (N=10)	.5 (5)	
Sentences	WN/SP (N=7)	1.0 (7)	
	SN/SP (N=10)	.4 (4)	
Non-goal	SN/WP	.5 (5)	.4 (4)
specific	WN/SP	.43 (3)	.14(1)
situations	SN/SP	.4 (4)	.1 (1)
Stimulus	SN/WP	.1 (1)	.4 (4)
Picture	WN/SP	.14 (7)	.3 (2)
	SN/SP		.7 (7)
Literature	SN/WP	.1 (1)	.3 (3)
	WN/SP	.14(1)	.14 (1)
	SN/SP	.1 (1)	.3 (3)

Table 6

Proportion (and Frequency) of Responses of Each Problem Type on the Posttest by Problem Situation, Achievement Category, and Condition

Problem Situation	Achievement Category	Change-add-to/ Combine P**	Change-add-to/ Combine NP	Change-take-from/ Separate P	Change-take-from/ Separate NP	Compare/Equalize P	Compare/Equalize NP	Other P	Other NP
Formal Context									
Two number	SN/WP (N*=38)			1.0(18)	.95(19)				.05 (1)
sentence	WN/SP (N =28)			1.0(14)	.79(11)		.07(1)		.14 (2)
attempts	SN/SP (N =40)			.95(19)	.85(17)			.05(1)	.15 (3)
Informal Context									
Photograph &	SN/WP (N=38)	.61(11)	.7(14)	.11(2)	.2 (4)	.17(3)	.1 (2)	.11(2)	
Non-goal	WN/SP (N=28)	.64 (9)	.29(4)	.22(3)	.43(6)	.14(2)	.14(2)		.14(2)
specific	SN/SP (N=40)	.5 (10)	.45(9)	.15(3)	.45(9)	.35(7)	.5 (1)		.5 (1)
situation									

*N (Total number of responses) **P = Program children NP = Nonprogram children

Generation of Multistep Problems for the Informal Context

	P	NP
SN/WP	.33(6)	.15(3)
WN/SP	.29(4)	.29(4)
SN/SP	.15(3)	.3 (6)

Caption for Figure 1

Samples of Children's Recorded Problems

Jane (SN/WP)

There were ten bunyups
and Three dingos. How many
alltogether

10 +3 =13

Jaimie (WN/SP)

TOm bort 1 packet of tinnes balls
James bort 3 packet of Tinnes balls
how many more packets of balls
James got than tom

Reilly (SN/SP)

Lachlan bought a skateboard and Reilly bought
three little balls and two balloons.
How many more things did Reilly buy than Lachlan?