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ABSTRACT

A job analysis was conducted to define the knowledge domain in which newly licensed (certified) mathematics teachers must be knowledgea le to perform competently. The results of the job analysis were to be used to develop test specifications for the Subject Assessment in Mathematics of the Praxis Series: Professional Assessments for Beginning Teachers. A draft knowledge domain of 12 knowledge categories and 175 statements was constructed at the Educational Testing Service and submitted to review by 11 subject-matter experts. The revised draft domain was reviewed by an Advisory Committee of secondary school teachers, college faculty, and an administrator, and their revisions were reviewed through a national survey of 500 teachers, 250 college faculty, and 50 school administrators, followed by review by an additional 200 new teachers. A cut point was established to identify the core of important statements. A total of 65 statements of the 193 submitted to the national sample did not meet the criterion for inclusion. The remaining statements, finally grouped into 13 knowledge areas, were to be used as the basis for test specifications. Seven appendixes, with three appendix tables, provide supplemental information about the study, including the survey questionnaire. (Contains 5 tables and 14 references.) (SLD)

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THE PRAXIS SERIES

Professional Assessments for Beginning Teachers

Job Analysis of the Knowledge Important for Newly Licensed Teachers of Mathematics

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Educational Testing Service

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We would like to thank several people who assisted us during this study. Beth Brownstein and Jane Dass of Educational Testing Service (ETS) School and Higher Education Programs did a wonderful job developing the initial draft of the mathematics content domain. They were also responsible for identifying and contacting external subject matter experts to review the draft. We would also like to thank the members of the External Review Panel and the Advisory Committee for their significant contributions to the study. Our gratitude also extends to Richard Tannenbaum of ETS for his useful feedback and Anne Reynolds, also of ETS, for her development of a framework for content-specific pedagogy. Many thanks to Cindy Hammell of ETS for producing the appendices to this report. We also thank the representatives of the National Council of Teachers of Mathematics (NCTM) for providing the mailing lists for the job analysis and promoting the study among its members. Lastly, we thank the several hundred teachers, college faculty, and administrators who took time away from their busy schedules to complete and return the survey.

Executive Summary

A job analysis was conducted to define the knowledge domain in which newly licensed (certified) mathematics teachers must be knowledgeable to perform their jobs in a competent manner. The results of the job analysis will be used to develop test specifications for the Subject Assessment in mathematics of the Praxis Series: Professionals Assessments for Beginning Teachers™.

A draft knowledge domain was constructed by Educational Testing Service (ETS) Test Development staff who have subject-matter expertise in mathematics and ETS Research staff who have expertise in job analysis. In the process of developing the draft domain, ETS subject-matter experts reviewed state licensure (certification) requirements, previous National Teacher Examination (NTE) mathematics test specifications, current test items, relevant professional literature, and standards from the National Council for Teachers of Mathematics (NCTM). The resultant draft domain consisted of 12 major knowledge categories and 175 statements. The 12 major categories were: (1) Basic Mathematics, (2) Geometry, (3) Trigonometry, (4) Functions and Their Graphs, (5) Probability and Statistics, (6) Calculus, (7) Analytic Geometry, (8) Discrete Mathematics, (9) Abstract Algebra, (10) Linear Algebra, (11) Computer Science, and (12) Pedagogy Specific to Mathematics. The 175 statements were operationalized in behavioral terms and, hence, referred to as tasks.

This draft domain was then reviewed by an External Review Panel of 11 subject-matter experts: five classroom teachers, three college faculty, two school administrators, and one person from the NCTM. The panel reviewed the draft domain for (1) the appropriateness of its overall structure and (2) the appropriateness of the specific task statements and their completeness and clarity. Revisions suggested by the panel, including additions and deletions of knowledge categories and statements, were obtained via telephone interviews conducted by ETS Research staff. Based on the interviews, wording changes were made to 66 of the 175 statements, and nine new statements were added to the draft domain.

This revised draft domain was then reviewed by an Advisory Committee. The committee was comprised of two secondary school mathematics teachers, four college faculty, and one state administrator. The committee was charged with modifying the revised draft domain so that it accurately reflected what the members believed were important knowledge areas for newly licensed (certified) mathematics teachers. This modification process occurred during a four-day meeting held at ETS. During their review, the committee suggested considerable modification of the inventory. As a result, changes to 87 of the 183 statements in the draft were made. In addition, 16 statements were deleted and 25 statements were added. Among the additions were 11 statements comprising a new category called Mathematical Reasoning and Modeling. The committee also changed the name of the Basic Mathematics category to Arithmetic and Basic Algebra. The resulting domain consisted of 193 statements.

This revised domain was then subjected to verification/refutation through a national survey of 500 teachers (10 per state), 250 college faculty (5 per state), and 50 school administrators (1 per state) for a total of 800 education professionals (16 per state). The mailing list was made up of names from the NCTM membership roster so that appropriate people could be reached.

Names from the roster were drawn at random in a way that satisfied the state participation requirements stated above.

Teachers were included in the original sample, and we later sent the survey to an additional 200 teachers. In this supplemental sample we attempted to focus on individuals who were relatively new to the teaching profession (e.g., less than five years teaching experience). We did this to increase the likelihood that a sufficient number of responses from new teachers would be available for analysis.

All survey participants were asked to rate the statements in terms of their importance for newly licensed (certified) mathematics teachers to perform their jobs in a competent manner. Respondents used a 5-point scale ranging from a low of 0 (of no importance) to a high of 4 (very important) to make their judgments. The purpose of the survey administration was to identify a core of knowledge statements that relatively large numbers of education professionals verified to be important for newly licensed (certified) mathematics teachers. The latter objective is accomplished through the analysis of the mean importance ratings provided by three groups of education professionals (i.e., teachers and college faculty in the original sample and teachers in the supplemental sample) and by appropriate subgroups of respondents (sex, race/ethnicity, geographic region, teaching experience in the two samples combined). Task statements that are judged to be important by *all* respondent groups and subgroups define the core. The core becomes the primary data base for the development of test specifications. The derivation of test specifications from those statements verified to be important by those surveyed provides a substantial evidential basis for the content validity (content relevance) of The Praxis II Subject Assessment in mathematics.

Two types of data analysis were conducted to support the development of content valid (content relevant) test specifications for the Subject Assessment in mathematics: (1) means were computed of the importance ratings for each task statement by the three groups of education professionals and by the appropriate subgroups of respondents; and (2) correlations of the profiles of these mean importance ratings were computed across the three groups of education professionals and within the appropriate subgroups of respondents.

A cut point of a mean importance rating of 2.50 (the midpoint between moderately important [scale value 2] and important [scale value 3]) was established to identify the core of important statements. Statements that were judged by the three groups of education professionals and all subgroups of respondents to be 2.50 or higher comprised the core and therefore were considered eligible for inclusion in the development of test specifications. (However, because the survey participants were not involved in the development of the knowledge domain, they may lack certain insights that the Advisory Committee members have due to their high level of involvement in the domain's development. As a consequence, if the committee believes that a task statement rated below 2.50 should be included in the specifications and the committee can provide compelling written rationales, those task statements may be reinstated for inclusion in the test specifications.)

The results of the mean analysis conducted by teachers (primary and supplemental samples) and college faculty (primary sample) showed that 64 knowledge and ability statements were rated less than 2.50. This represents 33.2% of the content domain. In the subgroup analyses, 63 (32.6%) statements were rated below 2.50. *In total, 65 of the 193 statements (33.7%) did not meet the 2.50 criterion for inclusion.* Most of the statements were in the knowledge categories representing upper-level mathematics (i.e., Calculus, Probability and Statistics, Discrete Mathematics, Abstract Algebra, and Linear Algebra).

The computation of correlation coefficients to assess agreement in terms of perceived relative importance of the task statements revealed a very high level of agreement. The coefficients for comparisons among the two sets of teachers and college faculty exceeded .90, as did the coefficients generated during the subgroup analyses. These findings indicate that *there is substantial agreement on the relative importance given to the tasks by a diverse group of education professionals.*

The 128 task statements that were verified to be important by the surveyed teachers, the college faculty, and the demographic subgroups should be used as the foundation for the development of test specifications. Test specifications that are linked to the results of a job analysis provide support for the content validity of the derived assessment measures and may be seen as part of an initial step in ensuring the fairness to subgroups of mathematics teacher candidates of the derived assessment measures. It is reasonable to assume that, due to testing and psychometric constraints (e.g., time limits, ability to measure content reliably), not all of the verified content will be included on the assessment measures. One source of information that may be used to guide the Advisory Committee in their decision as to what verified content to include on the assessment measures is the mean importance rating. Although a rank ordering of the content by mean importance rating is not implied, it is recommended that initial consideration be given to content that is well above the cut point and represents the appropriate breadth of content coverage.

Evidence was also provided in this study of the comprehensiveness of the content domain within the 13 major knowledge categories. This information serves as a check on the adequacy of the content domain definition. If the domain was well defined, then the knowledge categories should be judged to have been well covered by their accompanying task statements. The results supported the adequacy of the content domain definition.

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Introduction

Purpose of the Study

The subject assessments for The Praxis Series: Professional Assessments for Beginning Teachers™ are designed to assess a prospective teacher's content knowledge of a specific subject area and subject-specific pedagogical knowledge. The focus of such tests is based on the premise that beginning teachers should demonstrate knowledge of the subjects they intend to teach (Grossman, Wilson, & Shulman, 1989) and, perhaps, demonstrate knowledge of teaching principles, strategies, and resources specific to those subjects (Grossman, 1989; McDiarmid, Ball, & Anderson, 1989; Reynolds, 1992). The Praxis Series can be used by state agencies as one of several criteria for initial teacher licensure (certification). Included as part of the subject assessments is a licensure examination for mathematics teachers. To identify the content domain of this examination and to support the content validity (content relevance) of this examination, a job analysis was conducted of the knowledge important for newly licensed (beginning) mathematics teachers. This report will describe the job analysis study. In particular, it will provide the rationale for conducting the job analysis, present the methods used to define job-related knowledge, describe the types of statistical analysis conducted, report the results of these analyses, and specify the implications of the results for developing test specifications.

Standards for Educational and Psychological Testing

The *Standards for Educational and Psychological Testing* (1985) is a comprehensive technical guide that provides criteria for the evaluation of tests, testing practices, and the effects of test use. It was developed jointly by the American Psychological Association (APA), the American Educational Research Association (AERA), and the National Council on Measurement in Education (NCME). The guidelines presented in the *Standards* have, by professional consensus, come to define the necessary components of quality testing. As a consequence, a testing program that adheres to the *Standards* is more likely to be judged to be valid (defensible) than one that does not.

There are two categories of criteria within the *Standards*, primary and secondary. Those classified as primary "should be met by all tests . . . unless a sound professional reason is available to show why it is not necessary, or technically feasible, to do so in a particular case. Test developers and users . . . are expected to be able to explain why any primary standards have not been met" (AERA, APA, & NCME, 1985, p. 2). One of the primary standards is that the content domain of a licensure or certification test should be defined in terms of the importance of the content for competent performance in an occupation. "Job analyses provide the primary basis for defining the content domain." (p. 64)

The use of job analysis to define the content domain is a critical component in establishing the content validity of licensure and certification examinations. Content validity is the principal validation strategy used for these examinations. It refers to the extent to which the content covered by an examination overlaps with the important components (tasks, knowledge, skills, or abilities) of a job (Arvey & Faley, 1988). Demonstration of content validity is accomplished through the judgments of subject-matter experts. It is enhanced by the inclusion of large numbers of subject-matter experts who represent the relevant areas of expertise (Ghiselli, Campbell, & Zedeck, 1981). The lack of a well-designed job analysis is frequently cited by the courts as a major cause of test invalidity.

Job Analysis

Job analysis refers to procedures designed to obtain descriptive information about the tasks performed on a job and/or the knowledge, skills, and abilities thought necessary to adequately perform those tasks (Gael, 1983). The specific type of job information collected for a job analysis is determined by the purpose for which the information will be used. For purposes of developing licensure and certification examinations, a job analysis should identify the *important knowledge or abilities necessary to protect the public*--interpreted as the importance of the content for competent performance in an occupation (AERA, APA, & NCME, 1985). In addition, a well-designed job analysis should include the participation of various subject-matter experts (Mehrens, 1987); and the data collected should be representative of the diversity within the job. Diversity refers to regional or job context factors and to subject-matter-expert factors such as race/ethnicity, experience, and sex (Kuehn, Stallings, & Holland, 1990). The job analysis conducted for mathematics teachers was designed to follow guidelines presented in the *Standards for Educational and Psychological Testing* and to adhere to accepted professional practice.

Objectives of the Job Analysis Study

The objectives of this study were: (1) to construct a comprehensive domain of knowledge that is important for newly licensed (certified) mathematics teachers and then (2) to obtain, using survey methodology, the independent judgments of a national sample of mathematics education professionals (teachers, college faculty, and state administrators) to verify or refute the importance of the domain of knowledge. The verification/refutation component plays a critical part in ensuring that the domain (in whole or in part) is judged to be relevant to the job of a newly licensed (certified) mathematics teacher by a wide array of education professionals. The components of the domain that are verified will be used in the development of test specifications for The Praxis II Subject Assessment in mathematics.

Methods

The job analysis study described in this report involved a multi-method approach that included subject-matter experts and a national survey. First, groups of subject-matter experts defined a knowledge domain important for newly licensed/certified mathematics teachers. A description of this knowledge domain was then sent out to education professionals through a large-scale national survey. The purpose of the survey administration was to: (1) obtain verification and/or refutation from large numbers of education professionals that the previous groups of subject-matter experts had defined a domain of knowledge important for newly licensed mathematics teachers. Through this process we can identify a core of important knowledge that is related to the job of the newly licensed mathematics teacher. The survey functions as a "check and balance" on the judgments of the subject-matter experts and reduces the likelihood that unimportant knowledge areas will be included in the development of the test specifications. The use of a job analysis survey is also an efficient and cost-effective method of obtaining input from large numbers of subject-matter experts and makes it possible for ratings to be analyzed separately by appropriate subgroups.

The survey participants were mathematics teachers, administrators, and college faculty whose names were obtained from the membership of the National Council of Teachers of Mathematics (NCTM). An additional list of mathematics teachers was obtained from Market Data Retrieval, an education mailing list company. The participants were asked to rate the job analysis task statements in terms of their *importance* for newly licensed/certified mathematics teachers to perform their jobs in a competent manner. The specific steps in the job analysis process are described below.

Definition of the Knowledge Domain

Development of a draft knowledge domain. The first step in the process of conducting the job analysis was to construct a preliminary knowledge domain. The domain was constructed by Educational Testing Service (ETS) Test Development staff who have subject-matter expertise in mathematics and ETS Research staff who have expertise in job analysis methodology. In the process of developing the draft, the ETS subject-matter experts reviewed state licensure (certification) requirements, previous National Teacher Examination (NTE) mathematics test specifications, current test items, relevant professional literature, and standards from NCTM (i.e., *Curriculum and Evaluation Standards for School Mathematics*, 1989).

The resultant draft domain consisted of 12 major knowledge categories: (1) Basic Mathematics, (2) Geometry, (3) Trigonometry, (4) Functions and their Graphs, (5) Probability and Statistics, (6) Calculus, (7) Analytic Geometry, (8) Discrete Mathematics, (9) Abstract Algebra, (10) Linear Algebra, (11) Computer Science, and (12) Pedagogy Specific to Mathematics. Within each category were several statements mapping the important aspects of the category. These statements were presented in behavioral terms (e.g., *order any finite set of real numbers and recognize equivalent forms of a number [e.g., 1/10, 0.1, 10^{-1}]*; *discuss informally what it means for a function to have a limit at a point*). Because the statements were presented in behavioral terms, they were referred to as tasks. This draft, included a total of 175 task statements for the 12 categories.

Evaluation of draft domain by External Review Panel. Consistent with a content validity framework, the job analysis study was designed to obtain input from many subject-matter experts at several critical points in the domain definition process. To this end, an External Review Panel of 11 mathematics education professionals was formed to review the draft domain. This group consisted of five classroom teachers, three college faculty, two school administrators, and one individual from the NCTM. Individuals were considered for membership through a process of peer recommendation. All of the review panelists have experience either teaching mathematics or supervising teachers of mathematics. Generally, they are prominent and active in professional associations and/or teacher licensure. In addition to their subject-matter expertise, the panel was formed so as to have representation by sex, ethnicity, and geographic location. Members of the panel are listed in Appendix A.

The panelists were instructed to review the draft and to make the modifications they felt were necessary to adequately cover the important aspects of teaching mathematics. They were further instructed that these modifications could include the addition of important task statements, deletion of unimportant statements, elaboration of statements with relevant examples, and revision of statements so that the language would be clear and appropriate for individuals in mathematics education. The panelists were interviewed via telephone by ETS Research staff to obtain their suggestions for modifications.

Information from the interviews was compiled, discussed with ETS Test Development Staff, and, subsequently, used to revise the draft. Of the 175 statements in the first draft, 66 were modified based on these interviews. Also, nine new statements were incorporated.

Advisory Committee meeting. The next step in the job analysis process was a meeting held September 22-25, 1989, in Princeton, New Jersey, with an Advisory Committee of seven subject-matter specialists. The committee was charged with developing a final version of the Task Analysis Inventory and with developing the specifications for the new test. Like the external review panelists, members of the Advisory Committee have documented knowledge of the subject matter. The committee comprises two classroom teachers, four college faculty members, and one school administrator and has representation by sex, ethnicity, and geographic location. Members of the committee are listed in Appendix B.

The meeting was led jointly by ETS Test Development and Research staff. Prior to the meeting, committee members were mailed a copy of the draft domain to review. They were informed about the purpose of the meeting and asked to come prepared to discuss their review. Because they will use the results obtained from a survey administration of the content domain, it is critical that committee members have a clear understanding of each statement. The group interaction during the meeting fostered discussions that generated suggestions not made during the individual interviews with the External Review Panelists. The committee members attempted to be inclusive (i.e., cover all important aspects of teaching the subject matter) rather than exclusive in defining the content domain.

During their review, the committee suggested considerable modification of the inventory. As a result, changes to 87 of the 183 statements in the draft were made. In addition, 16 statements were deleted and 25 statements were added. Among the additions were 11 statements comprising a new category called Mathematical Reasoning and Modeling.

During the meeting, the Advisory Committee also reviewed and approved the proposed rating scale for the inventory. The rating scale required respondents to make judgments regarding *importance* to the newly licensed teacher. The importance scale, which is shown below, is in compliance with professional standards (cf. AERA, APA, & NCME, 1985).

How important is it that a newly licensed (certified) mathematics teacher be able to perform this task in a competent manner?

(0) Of no importance
(1) Of little importance
(2) Moderately important
(3) Important
(4) Very important

The committee also reviewed and approved items concerning demographic and background information (e.g., sex, teaching experience, geographic location). Such items were included so that we could describe the composition of the survey respondent group and conduct analyses of the survey responses by various subgroups of respondents (e.g., males and females).

Pilot test of the Task Analysis Inventory. After the meeting, a revised Task Analysis Inventory was mailed to the committee members for final approval. Once approval was obtained, the inventory was pilot tested on a group of nine classroom teachers. The pilot participants were asked to review the survey for clarity of wording, ease of use, and comprehensiveness of content coverage. The pilot test indicated that no one had difficulty completing the inventory and that no additional changes were necessary.

Large-Scale Survey

Survey instrument. The finalized survey consisted of three parts. Part I included 13 major knowledge categories: Arithmetic and Basic Algebra, Geometry, Trigonometry, Functions and Their Graphs, Probability and Statistics, Analytic Geometry, Calculus, Discrete Mathematics, Abstract Algebra, Linear Algebra, Computer Science, Mathematical Reasoning and Modeling, and Content-Specific Pedagogy. Under these categories were 193 specific task statements. Survey respondents were asked to rate the statements using the importance scale shown above.

For each major knowledge category, there was also a content coverage question in Part I. Survey participants were asked to indicate how well each major knowledge category was covered by its task statements. Respondents made their judgments using a 5-point rating scale (1=Poorly, 2=Somewhat, 3=Adequately. 4=Well, 5=Very well). The participants also had an opportunity to identify and write in task statements that they believed should be added to the domain.

In Part II of the survey, participants were asked to indicate the weight (emphasis) that each of the major knowledge categories should receive on the assessment. This was accomplished by distributing 100 total points across the major areas. These point distributions were converted into percentages, representing the percent of items that the survey respondents believed should be devoted to each area.

In Part III, participants were asked for demographic and background information. As previously noted, these items are used to describe the respondents and to perform subgroup analyses. A copy of the final survey is provided in Appendix C.

Survey participants. The primary sample for this study consisted of 500 teachers (10 per state), 250 college faculty (5 per state), and 50 school administrators (1 per state) for a total of 800 (16 per state). The mailing list was made up of names from the NCTM membership roster so that appropriate people could be reached. Names from the roster were drawn at random in such a way as to satisfy the state participation requirements stated above.

Teachers were included in the primary sample, and we also sent the survey to an extra 200 teachers. In this supplemental sample, we attempted to focus on individuals who were relatively new to the teaching profession (e.g., less than five years' teaching experience). We did this to increase the likelihood that a sufficient number of responses from new teachers would be available for analysis. It is important to survey new teachers because they are most like the population that will eventually take the assessment. Names were obtained from Market Data Retrieval (MDR), an education mailing list company. For our study, MDR was unable to specifically identify new teachers, but, as a surrogate strategy, was able to identify teachers who were new to their schools.

Survey administration. The surveys were administered to the primary sample in December 1989. Surveys were administered to the supplemental sample in May 1991. Each survey was accompanied by a letter of invitation to participate and a postage-paid envelope for return of the completed survey. A reminder postcard was mailed approximately one week after the survey mailing. The cover letters for the primary and supplemental samples and the follow-up postcard are provided in Appendix D.

The purpose of the survey administration was to identify a core of task statements that relatively large numbers of education professionals judged to be relevant (verified as important) to newly licensed (certified) mathematics teachers. The latter objective is accomplished through an analysis of the mean importance ratings provided by the three groups of education professionals and by the relevant subgroups of respondents. Task statements that are judged to be important by *all* respondent groups and subgroups define the core. The core becomes the primary data base for the development of test specifications. The derivation of test specifications from those knowledge and ability statements verified as important by the surveyed education professionals provides a substantial evidential basis for the content validity (content relevance) of the Subject Assessment in mathematics.

Results

Survey Respondents: Primary Sample

Response rate. Of the 800 inventories mailed to the primary sample, 16 were returned incomplete for a variety of reasons (e.g., wrong address, individual was retired and declined to participate). Of the remaining 784, 462 (58.9%) were completed and returned.

Demographic characteristics. Results of the analysis of the responses to the demographic questions are summarized in Appendix E. The typical respondent was over 35 years old, White, had at least a master's degree, and had more than 20 years' experience in teaching mathematics. More of the respondents were male than were female (63.0% to 34.6%). The respondents who taught tended to do so in high school or in college.

Survey Respondents: Supplemental Sample

Response rate. Of the 200 inventories mailed to the supplemental sample, 11 were returned incomplete. Of the remaining 189, 82 (43.4%) were completed and returned.

Demographic characteristics. Demographic distributions for this sample are also provided in Appendix E. As intended, most of the respondents were teachers (76/82 = 92.7%). Of the respondents, 43.9% (36/82) had 5 or fewer years of teaching experience. Hence, the sampling strategy was partially successful in identifying relatively new teachers. In addition to teaching experience, this sample was different from the primary sample on other demographic variables. For example, the supplemental sample tended to be younger, had a higher percentage of women and minority respondents, and had fewer respondents with a master's degree than the primary sample.

Analysis of Importance Ratings

Two types of data analysis were conducted to support the development of content valid (content relevant) test specifications for the Subject Assessment in mathematics: (1) Means of the importance ratings were computed for each task statement by the three groups of education professionals and by the appropriate subgroups of respondents, and (2) correlations of the profiles of these mean importance ratings were computed across the three groups of education professionals and the appropriate subgroups of respondents.

Means. The mean analysis is used to determine the level (absolute value) of importance attributed to the task statements. Means were computed for teachers and college faculty in the primary sample and teachers in the supplemental sample. Means were also computed for appropriate subgroups of respondents in the two samples combined (sex, race/ethnicity, geographic region, teaching experience). An analysis of importance ratings by geographic region is consistent with the recent legal emphasis on addressing regional job variability when conducting job analyses for content domain specification purposes (Kuehn et al., 1990). We used the regional categorizations established by the National Association of State Directors of Teacher Education and Certification (NASDTEC) in our analysis. Sex and race/ethnicity subgroups were included because they represent protected "classes" under Title VII of the Civil Rights Act of 1964. (Responses from all racial and/or ethnic minority participants were aggregated because of the insufficient number of respondents, i.e., < 30, from any single racial/ethnic group.) We used a dichotomous breakdown of teaching experience at the 5-year point was chosen so that the judgments of less experienced teachers and more experienced teachers could be represented separately.

A respondent category was required to have at least 30 respondents to be included in the mean analysis (e.g., ≥ 30 college faculty, ≥ 30 females). This is a necessary condition to ensure that the mean value based upon the sample of respondents is an accurate estimate of the corresponding population mean value (Walpole, 1974). Consequently, there were insufficient numbers of state administrators to analyze their responses separately. Task statements that meet or exceed a mean importance value of 2.50 (to be discussed in a later section) by *all three groups of education professionals* (teachers and college faculty in the primary sample and teachers in the supplemental sample) and by *all subgroups of respondents* may be included in the development of the test specifications. In addition, mean ratings were computed for the responses to the content coverage section and the recommendation for test content section of the survey. These analyses were computed for the three groups of education professionals and for the total sample.

Correlations. The correlational analysis is used in this study to determine the extent of agreement among the three groups of education professionals and among the demographic subgroups of respondents about the relative importance of the task statements. Relative importance refers to the similarity of the pattern of mean ratings generated by the different respondent groups. For example, the profile of 193 mean ratings for teachers in the primary sample is correlated with the profile of 193 mean ratings for teachers in the supplemental sample. If these two profiles are similar (the shapes of the profiles are complementary), the value of the correlation coefficient will be close to 1.00.

Criterion for Interpretation of Mean Importance Ratings

As the purpose of a job analysis is to ensure that only the most important task statements are included in the development of test specifications, a criterion (cut point) for inclusion needs to be established. A criterion that was used in a similar study (i.e., Rosenfeld & Tannenbaum, 1991) is a mean importance rating that represents the midpoint between moderately important and the next higher scale value. For the importance rating scale used in the present job analysis, the value of this criterion is 2.50 (midpoint between moderately important and important). It is believed that this criterion is consistent with the intent of content validity, which is to measure only important knowledge in the assessment measure. Therefore, task statements that receive a mean importance rating of 2.50 or more may be considered eligible for inclusion in the development of test specifications; task statements that receive a mean rating of less than 2.50 may not be considered for inclusion. (Because survey participants were not involved in the development of the content domain, however, they may lack certain insights that the Advisory Committee members have because of their high level of involvement in the domain's development. Consequently, if the committee believes that a task statement rated below 2.50 should be included in the specifications and the committee can provide *compelling written rationales*, those task statements may be reinstated for inclusion in the test specifications.)

Mean Importance Ratings

Education professionals. Means and standard deviations were computed for teachers and college faculty in the primary sample and for teachers in the supplemental sample. Because of their length, these results are provided in Appendix F.

Those task statements rated less than 2.50 by either the teachers or college faculty in the primary sample or by the teachers in the supplemental sample are provided in Table 1. Of the 193 individual task statements, 64 (33.1%) were rated below 2.50 by one or more of the three groups. The following dimensions yielded the highest percentages of statements with low ratings: Probability and Statistics (10 of 19 statements), Discrete Mathematics (17 of 21), Abstract Algebra (7 of 7), and Linear Algebra (10 of 12).

Teachers gave more of the lower ratings. Both teacher samples gave average ratings below 2.50 to 57 task statements, 48 of which were common to both. In contrast, the average college faculty rating was less than 2.50 on only 28 statements. Of the 28, all were given average ratings below 2.50 by the primary sample of teachers, while 27 were rated below the criterion by the supplemental sample of teachers.

Demographic subgroups. Mean ratings were computed for demographic subgroups based on sex, race/ethnicity, geographic region, and teaching experience (for teachers only). These data are presented in a table in Appendix G.

Table 1
Statements Rated Below 2.50 by Teachers and College Faculty

	Primary Sample		Supplemental Sample
	Teachers (N = 180)	College Faculty (N = 159)	Teachers (N = 77)
ARITHMETIC AND BASIC ALGEBRA			
9 Find positive integral powers and roots of perfect powers	2.36	2.28	
GEOMETRY			
44 Know and use basic facts about non-Euclidean geometries	2.26		2.24
TRIGONOMETRY			
56 Find trigonometric form of complex numbers and apply DeMoivre's Theorem			2.34
PROBABILITY AND STATISTICS			
73 Calculate the expected value of a function of a discrete random variable	2.09	2.41	2.36
74 Model an applied problem using mathematical expectation	2.46		
76 Explain the consequences of the Central Limit Theorem	2.08		2.26
77 Use the Central Limit Theorem to calculate probabilities	2.00		2.16
78 Solve problems using the uniform and chi-square distributions	1.87	2.18	2.08
79 Solve continuous probability problems with random variables, etc.	1.77	2.11	2.11
80 Solve continuous probability problems with joint and conditional probability	1.94	2.10	2.15
81 Solve expected value problems for continuous random variables	1.81	2.01	2.05
82 Develop test to accept or reject a given null hypothesis	1.81	2.43	1.96
83 Discuss sample size, significance level, power, type I, II error relationships	1.71	2.18	1.88
CALCULUS			
91 Prove via epsilon-delta that the limit of a function equals the calculated value	2.40	2.35	2.42
102 Approximate the roots of a function	2.44		
111 Evaluate improper integrals	2.47		
114 Interchange the order of integration in double integrals	1.94	2.28	2.20
115 Find volumes of solids using double integrals	1.99	2.41	2.25
116 Determine limits of sequences and simple infinite series			2.42
117 Use tests to show convergence or divergence of series	2.36		2.22
118 Determine the interval of convergence of a power series	2.19	2.41	2.15
119 Determine Taylor series of functions like sin x, cos x, e^x, and ln x	2.18	2.46	2.15

Table 1 (cont.)

	Primary Sample		Supplemental Sample
	Teachers (N = 180)	College Faculty (N = 159)	Teachers (N = 77)
DISCRETE MATHEMATICS			
123 Use Laws of Algebra of Propositions to evaluate equivalence	2.36		2.49
125 Use Euclidean Algorithm to find greatest common divisor of two numbers			2.49
126 Work with numbers expressed in bases other than base ten	2.34		
127 Solve modular congruences	2.05		2.08
128 Prove theorems using modular systems	1.67	2.11	1.92
129 Find values of functions defined recursively	2.34		2.03
130 "Translate" between recursive and closed form expressions for a function	2.02	2.45	1.90
131 Determine if a binary relation is reflexive, symmetric, antisymmetric, transitive	2.21		2.13
132 Determine if a binary relation is an equivalence relation	2.26		2.11
133 Determine if set is ordered with respect to a binary relation	2.10		2.03
134 Prove countability of rational numbers and non-countability of real numbers	1.84	2.18	1.96
135 Use basic terminology in graph theory	2.15		2.15
136 Identify conditions under which a graph can be traversed	1.81	2.29	1.81
137 Know and use the three basic ways of traversing a binary tree	1.77	2.03	1.83
138 Solve simple linear programming problems			2.37
139 Find and use finite differences of a function	2.00	2.42	2.06
140 Interpolate using Newton's forward and backward difference formulas	1.66	1.85	1.77
ABSTRACT ALGEBRA			
141 Determine if a set together with an operation is a group	2.21		1.93
142 Use definition of a group to deduce elementary properties of a group	2.13		1.90
143 Determine if a set together with two operations is a ring	1.94		1.85
144 Determine if a set together with two operations is a field	2.20		1.85
145 Use definition of a field to deduce elementary properties	2.11		1.83
146 Determine whether mathematical systems are groups, rings, or fields	2.09		1.86
147 Determine if subset of a group is subgroup or normal subgroup	1.74	2.20	1.81
LINEAR ALGEBRA			
In a finite dimensional real vector space:			
148 determine if a subset is a subspace	1.72	2.47	1.89
149 determine if a set of vectors is linearly independent	1.87		2.04
150 determine if a set of vectors is basis for vector space	1.89		1.90

Table 1 (cont.)

		Primary Sample		Supplemental Sample
		Teachers (N = 180)	College Faculty (N = 159)	Teachers (N = 77)
151	determine the dimension of the span of a set of vectors	1.77	2.34	1.83
152	determine if vectors are orthogonal using the dot product	1.93		1.99
153	determine effects of linear transformation on a vector space	1.79	2.30	1.92
154	determine image and kernel of a linear transformation	1.54	2.06	1.79
155	Add, subtract, and scalar multiply vectors using geometric interpretations			2.38
157	Demonstrate understanding and use basic properties of inverses			2.41
158	Determine and apply matrix representation of a linear transformation	2.27		2.29
COMPUTER SCIENCE				
164	Trace and debug existing computer algorithms	2.23		2.36
165	Program in two computer languages, one of which is a structured language	1.95	2.30	2.16
MATHEMATICAL REASONING AND MODELING				
170	Estimate actual and relative error in numerical answer	2.46		
173	Determine whether an isomorphism exists between two mathematical systems	2.13	2.46	2.29
174	Determine if one mathematical model will describe two different situations	2.37		
175	Understand the different levels of mathematical impossibility			2.41
176	Use the axiomatic method in modeling and problem solving	2.43		2.19
CONTENT-SPECIFIC PEDAGOGY				
191	Evaluate impact of learning theorists on mathematics education	2.04	2.37	2.30

Those task statements rated less than 2.50 by any of the demographic subgroups mentioned above are provided in Table 2. Of the 193 individual task statements, 63 (32.6%) were rated below 2.50 by one or more of the subgroups. Similar to the results of the analysis of the education professionals, the following dimensions yielded the highest percentages of statements with low ratings: Probability and Statistics (10 of 19 statements), Discrete Mathematics (16 of 21), Abstract Algebra (7 of 7), and Linear Algebra (9 of 12). Of the 63 statements rated below the criterion in the demographic analyses, 62 also fell below the criterion for the education professionals. *In total, 65 of the 193 statements (33.7%) did not meet the 2.50 criterion for inclusion by either the education professionals or the demographic subgroups. This leaves 128 task statements that can be used (without written rationale) as the foundation for test specifications.*

Correlations of the Profiles of Mean Importance Ratings

Education professionals. Correlations were computed among arrays of means for the teachers and college faculty in the primary sample and the teachers in the supplemental sample. The obtained correlations were: Teachers (primary) and Teachers (supplemental), r=.97; Teachers (primary) and College Faculty (primary), r=.96; and College faculty (primary) and Teachers (supplemental), r=.91.

Demographic subgroups. Correlations were computed among arrays of means for the selected subgroups of respondents (e.g., males and females). This is done as a way of evaluating agreement among subgroups. The correlations between the various subgroups are all in the upper .90s (see Table 3, page 17). This, combined with the results for the education professionals, indicates a high level of agreement among the respondent groups with respect to the relative importance of the statements. This is consistent with other findings in the job analysis literature (e.g., Schmitt and Cohen, 1989).

Mean Ratings of Content Coverage

The survey participants were asked to indicate, using a 5-point rating scale, how well the statements within each of the 13 major knowledge categories covered the important aspects of the category. Responses to this provide an indication of the adequacy (comprehensiveness) of the content domain. The scale values were 1=Poorly, 2=Somewhat, 3=Adequately, 4=Well, 5=Very well. The mean ratings for the teachers and college faculty in the primary sample, the teachers in the supplemental sample, and all respondents in the total sample are presented in Table 4 (page 17). The overall mean ratings (i.e., for all employment categories in the total sample) meet or exceed 4.00 on all categories except Computer Science, which has a moderately high rating of 3.87. This supports the notion that the major knowledge categories were reasonably well covered and that the overall content domain was comprehensive.

Table 2
Statements Rated Below 2.50 by Demographic Subgroups

		Sex		Race/ Ethnicity		Geographic Region				Teaching Experience (teachers only)	
		F	M	POC	W	NE	C	S	FW	0-5	6+
ARITHMETIC AND BASIC ALGEBRA											
9	Find positive integral powers and roots of perfect powers		2.31		2.38	2.31	2.29		2.46		2.37
GEOMETRY											
44	Know and use basic facts about non-Euclidean geometries		2.31		2.39	2.34	2.33		2.40	2.18	2.26
TRIGONOMETRY											
56	Find trigonometric form of complex numbers and apply DeMoivre's Theorem			2.44						2.42	
PROBABILITY AND STATISTICS											
73	Calculate the expected value of a function of a discrete random variable	2.29	2.29	2.34	2.28	2.32	2.31	2.28	2.25	2.39	2.14
74	Model an applied problem using mathematical expectation										2.47
76	Explain the consequences of the Central Limit Theorem	2.35	2.36	2.31	2.34	2.44	2.43	2.28	2.27	2.23	2.12
77	Use the Central Limit Theorem to calculate probabilities	2.25	2.26	2.17	2.25	2.28	2.39	2.19	2.16	2.10	2.04
78	Solve problems using the uniform and chi-square distributions	2.09	2.03	2.02	2.04	2.15	2.1	1.92	2.06	2.13	1.90
79	Solve continuous probability problems with random variables, etc.	2.04	1.97	2.07	1.98	2.07	2.04	1.93	1.93	2.16	1.83
80	Solve continuous probability problems with joint and conditional probability	2.10	2.04	2.15	2.05	2.11	2.12	2.00	2.03	2.25	1.97
81	Solve expected value problems for continuous random variables	1.97	1.93	1.95	1.93	1.96	1.98	1.91	1.94	2.09	1.85
82	Develop test to accept or reject a given null hypothesis	2.04	2.13	2.05	2.08	2.17	2.13	2.02	2.05	2.13	1.81
83	Discuss sample size, significance level, power, type I, II error relationships	2.02	1.89	1.95	1.93	1.95	2.02	1.87	1.91	1.94	1.73
CALCULUS											
91	Prove via epsilon-del. that the limit of a function equals the calculated value		2.34	2.44	2.41	2.40	2.42	2.41		2.31	2.42
102	Approximate the roots of a function							2.45		2.48	2.47
104	Solve problems using the Mean Value Theorem of differential calculus									2.43	

Note: F = Female (N=203); M = Male (N=328); POC = People of Color (N=43); W = White (N=483); NE = Northeast (N=124); C = Central (N=138); S = South (N=141); FW = Far West (N=129); 0-5 = 0 to 5 years' teaching experience (N=34); 6+ = 6 or more years' teaching experience (N=223).

Table 2 (cont.)

		Sex		Race/ Ethnicity		Geographic Region				Teaching Experience (teachers only)	
		F	M	POC	W	NE	C	S	FW	0-5	6+
111	Evaluate improper integrals									2.45	
114	Interchange the order of integration in double integrals	2.19.	2.12	2.24	2.13	2.18	2.22	2.01	2.19	2.07	2.00
115	Find volumes of solids using double integrals	2.24	2.20	2.31	2.19	2.15	2.25	2.09	2.37	2.20	2.05
116	Determine limits of sequences and simple infinite series									2.43	
117	Use tests to show convergence or divergence of series		2.45		2.47		2.46	2.42		2.17	2.34
118	Determine the interval of convergence of a power series	2.39	2.25	2.42	2.28	2.33	2.29	2.18	2.42	2.10	2.19
119	Determine Taylor series of functions like sin x, cos x, e^x, and ln x	2.28	2.29		2.26	2.33	2.36	2.14	2.35	2.10	2.18
DISCRETE MATHEMATICS											
123	Use Laws of Algebra of Propositions to evaluate equivalence										2.38
126	Work with numbers expressed in bases other than base ten		2.49								2.36
127	Solve modular congruences	2.38	2.23	2.31	2.28	2.31	2.28	2.23	2.35	2.16	2.05
128	Prove theorems using modular systems	2.01	1.87	2.02	1.90	1.91	1.90	1.85	2.05	1.94	1.71
129	Find values of functions defined recursively	2.45			2.49	2.49		2.40		2.03	2.28
130	"Translate" between recursive and closed form expressions for a function	2.21	2.22	2.24	2.20	2.20	2.19	2.10	2.37	1.93	2.00
131	Determine if a binary relation is reflexive, symmetric, antisymmetric, transitive		2.42	2.48	2.45		2.41	2.48	2.45	2.17	2.19
132	Determine if a binary relation is an equivalence relation						2.48			2.14	2.23
133	Determine if set is ordered with respect to a binary relation	2.38	2.33	2.42	2.34	2.34	2.31	2.39	2.35	1.9	2.11
134	Prove countability of rational numbers and non-countability of real numbers	2.16	1.95	2.05	2.01	2.19	1.88	1.88	2.19	1.83	1.88
135	Use basic terminology in graph theory	2.43	2.31		2.33	2.40	2.27	2.33	2.44	2.10	2.16
136	Identify conditions under which a graph can be traversed	2.06	1.99	2.07	1.99	2.08	1.99	1.91	2.08	1.64	1.83
137	Know and use the three basic ways of traversing a binary tree	1.94	1.86	2.07	1.86	1.92	1.87	1.81	1.97	1.62	1.81
138	Solve simple linear programming problems									2.34	2.48
139	Find and use finite differences of a function	2.18	2.22	2.31	2.18	2.27	2.11	2.13	2.31	1.86	2.04
140	Interpolate using Newton's forward and backward difference formulas	1.78	1.77	2.00	1.74	1.78	1.77	1.63	1.93	1.64	1.70

Table 2 (cont.)

		Sex		Race/ Ethnicity		Geographic Region				Teaching Experience (teachers only)	
		F	M	POC	W	NE	C	S	FW	0-5	6+
ABSTRACT ALGEBRA											
141	Determine if a set together with an operation is a group	2.43	2.47	2.40	2.45	2.41	2.44	2.46		1.87	2.17
142	Use definition of a group to deduce elementary properties of a group	2.38	2.35	2.36	2.35	2.31	2.38	2.33	2.42	1.84	2.10
143	Determine if a set together with two operations is a ring	2.23	2.20	2.21	2.20	2.11	2.23	2.15	2.34	1.77	1.94
144	Determine if a set together with two operations is a field	2.37	2.41	2.33	2.37	2.36	2.34	2.39	2.47	1.77	2.14
145	Use definition of a field to deduce elementary properties	2.33	2.29	2.36	2.28	2.28	2.30	2.28	2.37	1.74	2.07
146	Determine whether mathematical systems are groups, rings, or fields	2.29	2.32	2.29	2.30	2.25	2.31	2.26	2.42	1.81	2.05
147	Determine if subset of a group is subgroup or normal subgroup	2.	1.90	2.17	1.90	1.98	1.94	1.80	2.10	1.74	1.76
LINEAR ALGEBRA											
	In a finite dimensional real vector space:										
148	determine if a subset is a subspace	2.07	2.05	2.19	2.03	2.02	2.10	1.99	2.11	1.80	1.76
149	determine if a set of vectors is linearly independent	2.19	2.22	2.36	2.18	2.21	2.20	2.13	2.30	1.97	1.91
150	determine if a set of vectors is basis for vector space	2.16	2.15	2.28	2.13	2.15	2.16	2.06	2.26	1.73	1.92
151	determine the dimension of the span of a set of vectors	2.02	1.99	2.07	1.98	1.97	2.04	1.93	2.06	1.60	1.81
152	determine if vectors are orthogonal using the dot product	2.19	2.22	2.21	2.20	2.25	2.18	2.15	2.27	2.03	1.93
153	determine effects of linear transformation on a vector space	2.08	1.99	2.17	2.00	2.07	1.96	1.93	2.14	1.80	1.83
154	determine image and kernel of a linear transformation	1.87	1.76	1.98	1.77	1.80	1.82	1.73	1.86	1.67	1.61
155	Add, subtract, and scalar multiply vectors using geometric interpretations									2.43	
158	Determine and apply matrix representation of a linear transformation		2.40		2.43		2.39	2.37		2.33	2.27
COMPUTER SCIENCE											
164	Trace and debug existing computer algorithms	2.42	2.33	2.29	2.38	2.34	2.47	2.33	2.34	2.44	2.25
165	Program in two computer languages, one of which is a structured language	2.06	2.15	2.14	2.11	2.06	2.22	2.03	2.15	2.13	2.00

Table 2 (cont.)

		Sex		Race/ Ethnicity		Geographic Region				Teaching Experience (teachers only)	
		F	M	POC	W	NE	C	S	FW	0-5	6+
MATHEMATICAL REASONING AND MODELING											
170	Estimate actual and relative error in numerical answer										2.48
173	Determine whether an isomorphism exists between two mathematical systems	2.45	2.23	2.41	2.29	2.29	2.35	2.23	2.40	2.19	2.17
174	Determine if one mathematical model will describe two different situations							2.45			2.41
175	Understand the different levels of mathematical impossibility			2.45				2.49		2.48	
176	Use the axiomatic method in modeling and problem solving		2.48	2.41				2.34		2.13	2.39
CONTENT-SPECIFIC PEDAGOGY											
191	Evaluate impact of learning theorists on mathematics education	2.38	2.12	2.30	2.20	2.15	2.31	2.08	2.33	2.34	2.09

Table 3
Correlations of the Mean Importance Ratings Among Demographic Subgroups

	1	2	3	4
Sex				
1. Female (N=203)	1.00			
2. Male (N=328)	.98	1.00		
Racial/Ethnic Background				
1. Minority (N=43)	1.00			
2. Majority (N=483)	.98	1.00		
Geographic Region				
1. Northeast (N=124)	1.00			
2. Central (N=138)	.99	1.00		
3. South (N=141)	.99	.99	1.00	
4. Far West (N=129)	.98	.99	.99	1.00
Teaching Experience (Teachers only)				
1. 1 - 5 years (N=34)	1.00			
2. Greater than 5 years (N=223)	.96	1.00		

Table 4
Mean Ratings of Content Coverage

	Primary Sample		Supplemental Sample	Total Sample
Knowledge Category	Teachers (N=180)	College Faculty (N=159)	Teachers (N=77)	All Employment Categories (N=544)
Arithmetic and Basic Algebra	4.39	4.31	4.49	4.38
Geometry	4.27	4.04	4.36	4.21
Trigonometry	4.34	4.19	4.42	4.30
Functions and Their Graphs	4.24	4.12	4.42	4.24
Probability and Statistics	4.17	4.06	4.35	4.14
Analytic Geometry	4.06	3.91	4.32	4.02
Calculus	4.31	4.25	4.30	4.32
Discrete Mathematics	4.08	3.96	4.26	4.06
Abstract Algebra	4.05	3.86	4.16	4.02
Linear Algebra	4.05	3.86	4.30	4.00
Computer Science	3.96	3.70	4.08	3.87
Mathematical Reasoning and Modeling	4.22	4.03	4.28	4.17
Content-Specific Pedagogy	4.24	4.08	4.23	4.17

Mean Percentage Weights for Test Content Emphasis: Recommendations for Test Content

In Part III of the survey, Recommendations for Test Content, participants are asked to indicate how many test questions (out of 100) should be included from each of the knowledge categories. This information may be used by the Advisory Committee to assist them in making decisions about how much emphasis the knowledge categories should receive in the test specifications. The mean weights for the teachers and college faculty in the primary sample, the teachers in the supplemental sample, and all respondents in the total sample are presented in Table 5. Arithmetic and basic Algebra and Geometry received the highest average ratings, while Abstract Algebra, Linear Algebra, and Discrete Mathematics received the lowest.

Table 5
Mean Percentage Weights for Test Content Emphasis

	Primary Sample		Supplemental Sample	Total Sample
Knowledge Category	Teachers (N=180)	College Faculty (N=159)	Teachers (N=77)	All Employment Categories (N=544)
Arithmetic and Basic Algebra	17.59	14.78	20.53	17.02
Geometry	12.83	10.97	13.40	12.29
Trigonometry	9.18	8.16	9.87	8.96
Functions and Their Graphs	9.56	9.12	7.76	9.03
Probability and Statistics	5.98	6.80	6.79	6.46
Analytic Geometry	7.09	6.39	5.95	6.64
Calculus	6.89	8.25	5.75	7.21
Discrete Mathematics	4.49	5.53	3.88	4.77
Abstract Algebra	3.21	4.18	2.78	3.53
Linear Algebra	3.75	4.63	4.03	4.14
Computer Science	5.39	5.21	5.50	5.32
Mathematical Reasoning and Modeling	6.58	7.31	5.73	6.65
Content-Specific Pedagogy	7.46	8.67	8.03	7.97

Summary and Conclusions

A job analysis was conducted to define a knowledge domain in which newly licensed (certified) mathematics teachers must be knowledgeable to perform their jobs in a competent manner. A draft domain of important knowledge statements was constructed by ETS Test Development staff with expertise in mathematics and ETS Research staff with expertise in job analysis. This draft domain was reviewed by an External Review Panel of subject-matter experts and revised as they felt necessary. The revised draft was then reviewed, modified, and approved by an external Advisory Committee. The revised knowledge domain was then subjected to verification/refutation through the use of a national survey of mathematics teachers, teacher educators, and state administrators. The survey participants were asked to rate specific task statements of the domain using a 5-point importance scale. A cut point of 2.50 (midpoint between moderately important and important) was established to designate task statements as eligible (≥ 2.50) or ineligible (< 2.50) for inclusion in the development of test specifications.

The results of the mean analysis conducted by teachers and teacher educators indicated that 64 of 193 task statements were rated less than 2.50. This represents 33.2% of the content domain. When the same analysis was conducted for demographic subgroups, very similar results were obtained. *In total, 65 of the 193 statements (33.7%) did not meet the 2.50 criterion for inclusion in these two analyses.*

The 128 task statements that were verified to be important by those surveyed should be used as the foundation for the development of test specifications. Test specifications that are linked to the results of a job analysis provide support for the content validity of the derived assessment measures and may be considered part of an initial step in ensuring the fairness to subgroups of mathematics teacher candidates of the derived assessment measures. It is reasonable to assume that because of testing and psychometric constraints (e.g., time limits, ability to measure content reliably) not all of the verified content will be included in the new assessment measure. One source of information that may be used to guide the Advisory Committee in their decision as to what verified content to include is the mean importance rating. Although a rank ordering of the content by mean importance rating is not implied, it is recommended that initial consideration be given to content that is well above the criterion and represents the appropriate breadth of content coverage as stipulated in the test specifications.

The computation of correlation coefficients to assess relative agreement in terms of perceived importance of the task statements revealed a very high level of agreement. All coefficients exceeded .90. These findings indicate that *there is substantial agreement in the importance ratings given by a diverse group of education professionals.*

Evidence was also provided in this study of the comprehensiveness of the content domain within each of the 13 major knowledge categories. The results indicated that the survey respondents thought the categories were reasonably well covered by their task statements.

Finally, we collected data in the Recommendations for Test Content section of the survey regarding the emphasis that should be given in the test to each of the 13 categories. This information will be used by the Advisory Committee in their decisions about the appropriate weighting of the test.

In summary, this study took a multi-method approach to identify a content domain that is related to the job of the newly licensed mathematics teacher. The job analysis process allowed for input from many practicing professionals in mathematics education. The results of the study will be used to develop specifications for the mathematics test that will be included as part of the subject assessments of The Praxis Series: Professional Assessments for Beginning Teachers™.

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Appendix A

External Review Panel

Deborah Ball
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Ms. Ball received her Ph.D. in Educational Policy from Michigan State University. She has taught elementary school for 13 years and college for 2 years. Currently, she teaches Preservice Mathematics, Introduction to Education, and a Doctoral Seminar on Disciplinary Knowledge. She has received an Excellence in Teaching Award and the Outstanding Dissertation Award while at Michigan State.

Steven Conrad
Roslyn High School
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Mr. Conrad has an M.S. degree from Yeshiva University. He has been a high school mathematics teacher for 23 years. The courses he typically teaches are Geometry, Algebra II and Trigonometry, Pre-Calculus, Calculus, and AP Calculus. He is a member of the American Federation of Teachers (AFT) and has received the Presidential Award for his teaching.

Ms. Gloria Donaldson
Oklahoma School of Science and Math
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Ms. Donaldson received an M.A.T. degree from the University of Alabama. She is a Charter Mathematics Instructor with 24 years of teaching experience. The courses she has taught range from seventh grade math to Calculus. She was awarded the Andalusia Chamber of Commerce Teacher of the Year award and the Richover Faculty Award and was a nominee for the Presidential Award. She has twice been the President of Alabama Teachers of Mathematics and has served on the Johns Hopkins Center for the Advancement of Academically Talented Youth.

John Dossey
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Mr. Dossey received his Ph.D. from the University of Illinois. He has taught courses such as Problem Solving, Number Theory, and History of Mathematics for 24 years. He is a Past President of the National Council of Teachers of Mathematics. He has served on the Board of Governors for the Mathematics Association of America and the Executive Committee of the Mathematics/Science Board of the National Research Council.

Shirley Frye
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National Council of Teachers of Mathematics
Reston, VA

Mr. Gates received a Ph.D. from George Washington University. In addition to being the Executive Director of the National Council of Teachers of Mathematics, he has 20 years of teaching experience at the junior high, senior high, and college levels. He has been given the National Science Foundation Institute Award, the General Electric Institute Award, and the General Motors Award for his contributions.

Linda Hackett
Foxcroft School
Middleburg, VA

Ms. Hackett has been a secondary school mathematics instructor for 10 years. She typically teaches AP Calculus, Pre-Calculus, and Algebra II. She received her M.Ed. from Edinborough University.

Michael Lambe
Grossmont Union High School
Spring Valley, CA

Mr. Lambe received his M.S. from the State University of New York at Buffalo. He has taught secondary school mathematics for 23 years. In addition to being the Department Head of Mathematics at Grossmont Union, Mr. Lambe typically teaches Intermediate Algebra, Pre-Calculus, Analysis, and Trigonometry. He has served on the Commission on Teacher Credentialing and the California Mathematics Council.

James R. Leitzel
Associate Professor and Vice Chairman
Department of Mathematics
The Ohio State University
Columbus, OH

Mr. Leitzel received his Ph.D. from Indiana University. He has been teaching mathematics for 27 years. The kinds of courses he teaches are collegiate-level mathematics and graduate mathematics courses for teachers. He also leads inservice workshops for teachers of mathematics.

Frank Reardon
Division of Teacher Education for Mathematics
Department of Education
Harrisburg, PA

Mr. Reardon is the Mathematics Advisor for the Division of Teacher Education for Mathematics in Pennsylvania. He received an M.A. degree from Temple University.

J. T. Sutcliffe
St. Marks School of Texas
Dallas, TX

Ms. Sutcliffe received an M.A.T. degree from Northwestern University. She is currently the Mathematics Department Chair at her school. She has taught primarily Algebra II, Pre-Calculus, and Calculus for 22 years. In recognition of her teaching ability, she has received the Presidential Award and the Ross Perot Award.

Appendix B

Advisory Committee

Russell Daniel
District Supervisor for K-12 Mathematics
Philadelphia School District
Philadelphia, PA

Mr. Daniel holds an M.A. degree in Mathematics Education from Temple University and has served as a consultant to the College Board on several occasions. In addition to his full-time job as District Supervisor for K-12 Mathematics, he teaches College Algebra and Pre-Calculus on a part-time basis at Lincoln University, Villanova University, and Swarthmore College.

Loretta Fischer
Lindbergh School District
St. Louis, MO

Ms. Fischer holds a B.A. degree from the University of Colorado an M.A.T. from Webster University. She has had 19 years of experience teaching Algebra, Applied Mathematics, FORTRAN, BASIC, and AP Computer Science in grades 9-12. She is a member of the American Federation of Teachers and of the Missouri Advisory Council for the Certification of Educators.

Judith Jacobs
California Polytechnic Institute
Pomona, CA

Ms. Jacobs, who specializes in pedagogy, was recommended by the Department of Education in California. She serves on the Mathematics Advisory Committee for the California test.

Guy R. Mauldin
Science Hill High School
Johnson City, TN

Mr. Mauldin holds a B.A. and an M.S. degree from Mississippi State University and has done course work for a Ph.D. from the University of Kentucky. He is a high school mathematics teacher with 17 years of teaching experience in college and 14 years in high school. He has taught Geometry, Algebra II, Advanced Mathematics (pre-calculus), and AP Calculus. He is a member of the National Education Association and has served as a reader for AP Calculus.

Billy E. Rhoades
Professor of Mathematics
Indiana University
Bloomington, IN

Mr. Rhoades has an A.B. degree from Ohio Northern University, an M.S. from Rutgers University, and a Ph.D. from Lehigh University. He has 36 years of mathematics teaching experience at the college and graduate levels. He was the recipient of the Jones Award for superior teaching at Lafayette College and the Distinguished Alumni Award from Ohio Northern University. He has served as a reader for AP Mathematics.

Betty P. Scarborough
Instructor in Mathematics
Mississippi State University
Starkville, MS

Ms. Scarborough holds a B.S. from Mississippi State College for Women and an M.A.T. from Tulane University. She has also done some graduate work at Mississippi State. She has had 21 years of teaching experience at both the high school and college levels. The courses taught include Algebra, Trigonometry, Calculus I-IV, Differential Equations, Linear Algebra, Business Calculus, Finite Mathematics, and Informal Measurement and Geometry. She is a member of the American Mathematical Society, Mathematics Association of America, and Mississippi Teachers of College Mathematics. She has served as a reader for AP Mathematics.

Stephen S. Willoughby
Professor of Mathematics
University of Arizona
Tucson, AZ

Mr. Willoughby has taught all levels from first grade through graduate school and has served as professor of mathematics at the University of Wisconsin (Madison) and at New York University, where he was also Chairman of the Department of Mathematics, Science, and Statistics Education. He is now professor of Mathematics at the University of Arizona. He received bachelor's and master's degrees from Harvard University and a doctorate from Columbia University. He was President of the National Council of Teachers of Mathematics from 1982 to 1984 and Chairman of the Council of Scientific Society Presidents in 1988. He is a member of the National Board of Advisors for SQUARE ONE TV, the Children's Television Workshop mathematics program, and a member of several other advisory boards for projects on mathematics education. He is now directing a four-year National Science Foundation sponsored project to develop and test K-6 supplementary mathematics materials for a technological society. He has published more than 170 articles and books on mathematics and mathematics education and is senior author of the innovative K-8 mathematics series Real Math, published by Open Court.

Appendix C

Task Analysis Inventory

TASK ANALYSIS INVENTORY

FOR MATHEMATICS TEACHERS

By

Educational Testing Service
Princeton, New Jersey

INTRODUCTION

Educational Testing Service (ETS) is developing a new generation of assessments for the purpose of licensing teachers. The inventory that follows is part of our development effort and is designed to gather information concerning the job of the secondary-school mathematics teacher. It was developed by high school mathematics teachers, college mathematics professors, mathematics teacher educators, and state department of education officials working with ETS staff.

The educators who collaborated on the construction of this questionnaire recognize that mathematics teachers in secondary school may be required to teach a variety of courses ranging from remedial arithmetic to calculus, and they will have to teach students with varying levels of ability and background. For these reasons, the collaborators believe that mathematics teachers should have a broad and deep understanding of many different areas of mathematics in order to integrate the subject matter and to teach it in a meaningful way. Part I of the inventory asks you to react to a list of tasks and to rate each task with respect to its importance for a newly-licensed (certified) mathematics teacher. Try not to relate each task to your own job, but rather to what you believe a new teacher should know and be able to do in order to be a competent mathematics teacher at any level of secondary school. Part II of the inventory asks that you make recommendations concerning the content of the new test. Finally, Part III asks you for some background data. The information you provide will ultimately be used to guide the development of the successor to the National Teacher's Examination for Mathematics. It is expected that the new test will differ from the current examination in both content and design. In addition to the development of a new test, this study represents an important contribution in its own right to our understanding of mathematics teaching as a profession. We expect the results of the study to be widely disseminated and to have ramifications for teacher education and development.

The inventory has been mailed to a carefully selected sample of 800 professionals like you. The value of a survey like this one is directly related to the number of individuals who complete and return their questionnaires. Because you represent a large number of professionals, your responses are very important to us. Please take the time to complete the inventory.

PART I - INVENTORY OF TASKS

The purpose of Part I is to ascertain what you believe a new teacher should know and be able to do in order to be a competent mathematics teacher. On the following pages you will find thirteen broad categories (e.g., arithmetic and basic algebra, geometry, calculus, mathematical reasoning and modeling, etc.). Beneath each category is a list of task statements related to that category. The categories are based on mathematical topics rather than typical courses. Because of this, some tasks that are taught in a particular course may appear in a different category from the name of that course.

For each statement in this section, you will be asked to make the following judgment:

IMPORTANCE: How important is it that a newly-licensed (certified) mathematics teacher be able to perform this task in a competent manner?

0 Of no importance
1 Of little importance
2 Moderately important
3 Important
4 Very important

To familiarize yourself with the categories and task statements you may wish to briefly glance through Part I before you make your rating judgments.

Using the scale below, please circle the number that best describes your judgment as to how important it is that a newly-licensed (certified) mathematics teacher be able to perform the task in a competent manner.

IMPORTANCE

0 Of no importance
1 Of little importance
2 Moderately important
3 Important
4 Very important

A. ARITHMETIC AND BASIC ALGEBRA	IMPORTANCE
1. Order any finite set of real numbers and recognize equivalent forms of a number (e.g., 1/10, 0.1, 10^{-1})	0 1 2 3 4
2. Use numbers in a way that is most appropriate in the context of a problem (e.g., appropriately rounded numbers, numbers written in scientific notation, using 5(100 - 1) for 5(99), etc.)	0 1 2 3 4
3. Demonstrate an understanding of the properties of counting numbers (e.g., prime or composite, even or odd, factors, multiples)	0 1 2 3 4
4. Add, subtract, multiply, and divide rational numbers expressed in any form	0 1 2 3 4
5. Apply the order of operations to problems involving addition, subtraction, multiplication, division, roots, and powers, with and without grouping symbols	0 1 2 3 4
6. Interpret and apply the concepts of ratio, proportion, and percent in appropriate situations	0 1 2 3 4
7. Solve problems that involve measurement in the metric system	0 1 2 3 4
8. Solve problems that involve measurement in the traditional system	0 1 2 3 4
9. Find positive integral powers and roots of perfect powers (e.g., what is the cube root of 9261)	0 1 2 3 4
10. Estimate values of expressions involving decimals, exponents, and radicals (e.g., what is a good estimate for the square root of 21)	0 1 2 3 4
11. Solve problems involving average, including arithmetic mean and weighted average	0 1 2 3 4
12. Evaluate algebraic expressions and formulas (e.g., If $x = 5$ and $y = 6$, what is the value of $x^2 + 5y$?)	0 1 2 3 4
13. Solve for a selected variable in a literal equation or formula (e.g., If $A = bh/2$, then $h = 2A/b$)	0 1 2 3 4
14. Classify a number as a rational, irrational, real, and/or complex	0 1 2 3 4
15. Identify the properties (e.g., closure, commutativity, associativity, distributivity) of the basic operations (i.e., addition, subtraction, multiplication, division) on the standard number systems	0 1 2 3 4

Using the scale below, please circle the number that best describes your judgment as to how important it is that a newly-licensed (certified) mathematics teacher be able to perform the task in a competent manner.

IMPORTANCE

0 Of no importance
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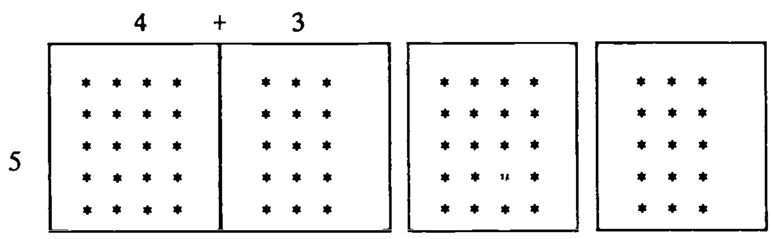
A. ARITHMETIC AND BASIC ALGEBRA (continued)	IMPORTANCE
16. Given newly defined operations on a number system, determine whether the closure, commutative, associative, or distributive properties hold	0 1 2 3 4
17. Identify the additive and multiplicative inverses of a number	0 1 2 3 4
18. Add, subtract, and multiply polynomials	0 1 2 3 4
19. Divide polynomials	0 1 2 3 4
20. Add, subtract, multiply, and divide algebraic fractions	0 1 2 3 4
21. Translate verbal expressions and relationships into algebraic expressions or equations	0 1 2 3 4
22. Solve and graph linear equations and inequalities in one or two variables	0 1 2 3 4
23. Solve and graph systems of linear equations and graph inequalities in two variables	0 1 2 3 4
24. Solve and graph nonlinear algebraic equations and graph inequalities	0 1 2 3 4
25. Perform standard algebraic operations involving complex numbers, radicals, and exponents, including fractional and negative exponents	0 1 2 3 4
26. Determine any term of a binomial expansion using Pascal's triangle or some other method (e.g., What is the fourth term of the binomial expansion of $(a + b)^5$?)	0 1 2 3 4
27. Solve equations and inequalities involving absolute values (e.g., $\|x + 2\| = 6$, $\|x + 2\| + 3x = 6$)	0 1 2 3 4
28. Present geometric interpretations of algebraic principles (e.g., the triangle inequality and the distributive principles). See example	0 1 2 3 4



5(4+3) = (5 x 4) + (5 x 3)

Using the scale below, please circle the number that best describes your judgment as to how important it is that a newly-licensed (certified) mathematics teacher be able to perform the task in a competent manner.

IMPORTANCE

0 Of no importance
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4 Very important

B. GEOMETRY	IMPORTANCE
29. Use relationships (e.g., congruency, similarity) among two-dimensional geometric figures to solve problems	0 1 2 3 4
30. Use relationships (e.g., congruency, similarity) among three-dimensional geometric figures to solve problems	0 1 2 3 4
31. Solve problems involving the properties of parallel and perpendicular lines	0 1 2 3 4
32. Solve problems using special triangles, such as isosceles and equilateral	0 1 2 3 4
33. Solve problems using the relationships of the parts of triangles, such as sides, angles, medians, midpoints, and altitudes	0 1 2 3 4
34. Apply the Pythagorean Theorem to solve problems	0 1 2 3 4
35. Solve problems using the properties of special quadrilaterals, such as the square, rectangle, parallelogram, rhombus, and trapezoid	0 1 2 3 4
36. Describe relationships among sets of special quadrilaterals, such as the square, rectangle, parallelogram, rhombus, and trapezoid	0 1 2 3 4
37. Solve problems using the properties (e.g., angles, sum of angles, number of diagonals, and vertices) of polygons with more than four sides	0 1 2 3 4
38. Solve problems using the properties of circles, including those involving inscribed angles, central angles, chords, radii, tangents, secants, arcs, and sectors	0 1 2 3 4
39. Compute the perimeter and area of triangles, quadrilaterals, and circles, and regions that are combinations of these figures	0 1 2 3 4
40. Compute the surface area and volume of right prisms, pyramids, cones, cylinders, and spheres, and solids that are combinations of these figures	0 1 2 3 4
41. Solve problems involving reflections, rotations, and translations of points, lines, or polygons in the plane	0 1 2 3 4
42. Execute geometric constructions using straight-edge and compass (e.g., bisect an angle, erect a perpendicular)	0 1 2 3 4

Using the scale below, please circle the number that best describes your judgment as to how important it is that a newly-licensed (certified) mathematics teacher be able to perform the task in a competent manner.

IMPORTANCE

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B. GEOMETRY (continued)	IMPORTANCE
43. Prove that a given geometric construction yields the desired result	0 1 2 3 4
44. Know and use basic facts about non-Euclidean geometries and the ramifications of the three postulates pertaining to the existence of parallel lines	0 1 2 3 4
C. TRIGONOMETRY	
45. Define and use the six basic trigonometric relations on the right triangle ...	0 1 2 3 4
46. Solve problems involving right triangles which have angles of 30° or 45° without tables or calculators	0 1 2 3 4
47. Identify the relationship between radian measures and degree measures of angles	0 1 2 3 4
48. Apply the law of sines and the law of cosines in the solution of problems ..	0 1 2 3 4
49. Define and use the six basic trigonometric functions as defined on the unit circle using radian measure	0 1 2 3 4
50. Solve problems involving trigonometric functions evaluated at such numbers as π, $\pi/6$, $2\pi/3$, $9\pi/4$, and $-\pi/3$	0 1 2 3 4
51. Recognize the graphs of the six basic trigonometric functions and identify their period, amplitude, phase displacement or shift, and asymptotes	0 1 2 3 4
52. Apply the formulas for the trigonometric functions of $x/2$, $2x$, $x + y$, and $x - y$ in terms of the trigonometric functions of x and y	0 1 2 3 4
53. Prove identities using the basic trigonometric identities	0 1 2 3 4
54. Solve trigonometric equations and inequalities	0 1 2 3 4
55. Given a point in the rectangular coordinate system, identify the corresponding point in the polar coordinate system	0 1 2 3 4
56. Find the trigonometric form of complex numbers and apply DeMoivre's Theorem	0 1 2 3 4

Using the scale below, please circle the number that best describes your judgment as to how important it is that a newly-licensed (certified) mathematics teacher be able to perform the task in a competent manner.

IMPORTANCE

0 Of no importance
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D. FUNCTIONS AND THEIR GRAPHS	IMPORTANCE
57. Understand function notation for functions of one variable and be able to work with the algebraic definition of a function (i.e., for every x there is at most one y) and be able to identify whether a graph in the plane is the graph of a function	0 1 2 3 4
58. Given a graph, select from a list the most appropriate equation for the graph	0 1 2 3 4
59. Given an equation, graph it	0 1 2 3 4
60. Use the definition of a function as a mapping and be able to work with functions given in this way (e.g., $f: (x, y) \to (x^2 + y^2, x^2 - y^2)$)	0 1 2 3 4
61. Find the domain and/or range of a function	0 1 2 3 4
62. Use the properties of algebraic, trigonometric, logarithmic, and exponential functions to solve problems (e.g., finding composite functions and inverse functions)	0 1 2 3 4
63. Find the inverse of a one-to-one function in simple cases and know why one-to-one functions have inverses	0 1 2 3 4
64. Determine the graphical properties and sketch a graph of a linear, step, absolute value, or quadratic function (e.g., slope, intercepts, intervals of increase or decrease, axis of symmetry)	0 1 2 3 4
E. PROBABILITY AND STATISTICS	
65. Organize data into a presentation that is appropriate for solving a problem (e.g., construct a histogram and use it in the calculation of probabilities)	0 1 2 3 4
66. Solve probability problems involving finite sample spaces by actually counting outcomes appropriately	0 1 2 3 4
67. Solve probability problems using counting techniques (e.g., If 3 cards are drawn from a standard deck of 52 cards, what is the probability that all 3 are aces?)	0 1 2 3 4
68. Solve probability problems involving independent trials (e.g., If a coin is tossed 5 times, what is the probability that at least 3 heads occur?)	0 1 2 3 4

Using the scale below, please circle the number that best describes your judgment as to how important it is that a newly-licensed (certified) mathematics teacher be able to perform the task in a competent manner.

IMPORTANCE

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E. PROBABILITY AND STATISTICS (CONTINUED)	IMPORTANCE
69. Solve problems using the binomial distribution and be able to determine when the use of the binomial distribution is appropriate	0 1 2 3 4
70. Solve problems involving joint probability	0 1 2 3 4
71. Find and interpret common measures of central tendency (population mean, sample mean, median, mode) and know which is the most meaningful to use in a given situation	0 1 2 3 4
72. Find and interpret common measures of dispersion (range, population standard deviation, sample standard deviation, population variance, sample variance)	0 1 2 3 4
73. Calculate the expected value of a function of a discrete random variable given the probability distribution function of the random variable or its expected value (e.g., If $E(X) = 2$, what is $E(2X + 2)$?)	0 1 2 3 4
74. Model an applied problem by using the mathematical expectation of an appropriate discrete random variable (e.g., fair coins, expected winnings, expected profit)	0 1 2 3 4
75. Solve problems using the normal distribution	0 1 2 3 4
76. Explain the consequences of the Central Limit Theorem and why it establishes the importance of the normal distribution in the study of statistics	0 1 2 3 4
77. Use the Central Limit Theorem to calculate probabilities (e.g., A random sample of 100 observations is selected from a population with $\mu = 30$ and $\sigma = 16$. Approximate the probability that $\bar{X} \leq 28$.)	0 1 2 3 4
78. Solve problems using the uniform and chi-square distributions	0 1 2 3 4
79. Solve problems involving continuous probability using such concepts as random variables, probability density function, and cumulative distribution function	0 1 2 3 4
80. Solve problems involving continuous probability using such concepts as joint and conditional probability	0 1 2 3 4
81. Solve expected value problems for continuous random variables	0 1 2 3 4
82. Develop a test to determine whether to accept or reject a given null hypothesis, H_0	0 1 2 3 4

Using the scale below, please circle the number that best describes your judgment as to how important it is that a newly-licensed (certified) mathematics teacher be able to perform the task in a competent manner.

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E. PROBABILITY AND STATISTICS (CONTINUED)	IMPORTANCE
83. Discuss the concepts of and relationships between sample size, level of significance, power, and type I (α) and type II (β) errors	0 1 2 3 4
F. ANALYTIC GEOMETRY	
84. Determine the equations of lines and planes given appropriate information	0 1 2 3 4
85. Make calculations in 2-space or 3-space (e.g., distance between two points, the coordinates of a midpoint of a line segment, distance between a point and a plane)	0 1 2 3 4
86. Given a geometric definition of a conic section, derive the equation for the conic section (e.g., given that a parabola is the set of points that are equidistant from a given point and a given line, derive its equation)	0 1 2 3 4
87. Determine which conic section is represented by a given equation if the axis of symmetry is parallel to one of the coordinate axes	0 1 2 3 4
88. Determine which conic section is represented by a given equation if no restrictions are placed on its location in the plane	0 1 2 3 4
G. CALCULUS	
89. Discuss informally what it means for a function to have a limit at a point	0 1 2 3 4
90. Calculate limits of functions or determine that the limit does not exist	0 1 2 3 4
91. Prove via an epsilon-delta proof that the limit of a function is actually equal to the calculated value	0 1 2 3 4
92. Solve problems using the properties of limits (e.g., $\lim_{x \to c} f(x) + \lim_{x \to c} g(x) = \lim_{x \to c} (f(x) + g(x))$)	0 1 2 3 4
93. Use limits to show that a particular function is continuous	0 1 2 3 4
94. Use L'Hopital's rule, where applicable, to calculate limits of functions	0 1 2 3 4
95. Relate the derivative of the function to a limit or to the slope of a curve	0 1 2 3 4

Using the scale below, please circle the number that best describes your judgment as to how important it is that a newly-licensed (certified) mathematics teacher be able to perform the task in a competent manner.

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G. CALCULUS (continued)	IMPORTANCE
96. Explain conditions under which a continuous function does not have a derivative	0 1 2 3 4
97. Differentiate algebraic expressions, trigonometric functions, and exponential and logarithmic functions using the sum, product, quotient, and chain rules	0 1 2 3 4
98. Use implicit differentiation	0 1 2 3 4
99. Make numerical approximations of derivatives and integrals	0 1 2 3 4
100. Use differential calculus to analyze the behavior of a function (e.g., find relative maxima and minima, concavity)	0 1 2 3 4
101. Use differential calculus to solve problems involving related rates and rates of change	0 1 2 3 4
102. Approximate the roots of a function (e.g., using Newton's method with derivatives)	0 1 2 3 4
103. Use differential calculus to solve applied minima-maxima problems	0 1 2 3 4
104. Solve problems using the Mean Value Theorem of differential calculus	0 1 2 3 4
105. Explain the significance of, and solve problems using, the Fundamental Theorem of Calculus	0 1 2 3 4
106. Demonstrate an intuitive understanding of the process of integration as finding areas of regions in the plane through a limiting process	0 1 2 3 4
107. Integrate functions using algebraic substitutions	0 1 2 3 4
108. Integrate functions using "integration by parts"	0 1 2 3 4
109. Integrate functions using trigonometric substitutions	0 1 2 3 4
110. Integrate logarithmic and exponential functions	0 1 2 3 4
111. Evaluate improper integrals	0 1 2 3 4
112. Use integral calculus to calculate the area of regions in the plane	0 1 2 3 4
113. Calculate the volume of solids formed by rotating plane figures about a line	0 1 2 3 4

Using the scale below, please circle the number that best describes your judgment as to how important it is that a newly-licensed (certified) mathematics teacher be able to perform the task in a competent manner.

IMPORTANCE

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4 Very important

G. CALCULUS (continued)	IMPORTANCE
114. Interchange the order of integration in double integrals	0 1 2 3 4
115. Find volumes of solids using double integrals	0 1 2 3 4
116. Determine the limits of sequences and simple infinite series	0 1 2 3 4
117. Use standard tests to show convergence (either conditional or absolute) or divergence of series (e.g., comparison, ratio, etc.)	0 1 2 3 4
118. Determine the interval of convergence of a power series	0 1 2 3 4
119. Determine the Taylor series of functions such as sin x, cos x, e^x, and ln x	0 1 2 3 4
H. DISCRETE MATHEMATICS	
120. Use the basic terminology and, given the definitions, use the symbols of logic	0 1 2 3 4
121. Solve problems involving the union and intersection of sets, subsets and disjoint sets	0 1 2 3 4
122. Use truth tables to verify statements	0 1 2 3 4
123. Use Laws of Algebra of Propositions to evaluate equivalence of complex logical expressions (e.g., De Morgan's Laws)	0 1 2 3 4
124. Solve basic problems involving permutations and combinations	0 1 2 3 4
125. Use the Euclidean Algorithm to find the greatest common divisor of two numbers	0 1 2 3 4
126. Work with numbers expressed in bases other than base ten	0 1 2 3 4
127. Solve modular congruences	0 1 2 3 4
128. Prove theorems using modular systems	0 1 2 3 4
129. Find values of functions defined recursively	0 1 2 3 4
130. "Translate" between recursive and closed form expressions for a function	0 1 2 3 4

Using the scale below, please circle the number that best describes your judgment as to how important it is that a newly-licensed (certified) mathematics teacher be able to perform the task in a competent manner.

IMPORTANCE

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H. DISCRETE MATHEMATICS (continued)	IMPORTANCE
131. Determine if a binary relation on a set is reflexive, symmetric, antisymmetric, or transitive	0 1 2 3 4
132. Determine if a binary relation on a set is an equivalence relation	0 1 2 3 4
133. Determine if a set is ordered with respect to a binary relation	0 1 2 3 4
134. Prove the countability of the rational numbers and non-countability of the real numbers	0 1 2 3 4
135. Use basic terminology in graph theory (e.g., graphs, trees, binary trees)	0 1 2 3 4
136. Identify the conditions under which a graph can be traversed in such a way that each edge is traversed exactly once and the conditions under which each vertex is visited exactly once	0 1 2 3 4
137. Know and use the three basic ways of traversing a binary tree, and use a binary tree to represent an algebraic expression	0 1 2 3 4
138. Solve simple linear programming problems	0 1 2 3 4
139. Find and use finite differences of a function	0 1 2 3 4
140. Interpolate using Newton's forward (advancing) or backward difference formulas	0 1 2 3 4
I. ABSTRACT ALGEBRA	
141. Determine if a particular set together with a given operation is a group	0 1 2 3 4
142. Use the definition of a group to deduce elementary properties of a group	0 1 2 3 4
143. Determine if a particular set together with two operations is a ring	0 1 2 3 4
144. Determine if a particular set together with two operations is a field	0 1 2 3 4
145. Use the definition of a field to deduce elementary properties of a field	0 1 2 3 4
146. Determine whether various mathematical systems (e.g., rational numbers, matrices, motions of an equilateral triangle) are groups, rings, or fields	0 1 2 3 4
147. Determine if a particular subset of a group is a subgroup or a normal subgroup	0 1 2 3 4

Using the scale below, please circle the number that best describes your judgment as to how important it is that a newly-licensed (certified) mathematics teacher be able to perform the task in a competent manner.

IMPORTANCE

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J. LINEAR ALGEBRA	IMPORTANCE
In a finite dimensional real vector space:	
148. determine if a subset is a subspace	0 1 2 3 4
149. determine if a set of vectors is linearly independent	0 1 2 3 4
150. determine if a set of vectors is a basis for the vector space	0 1 2 3 4
151. determine the dimension of the span of a set of vectors	0 1 2 3 4
152. determine if vectors are orthogonal using the dot product	0 1 2 3 4
153. determine the effects of a linear transformation on a vector space	0 1 2 3 4
154. determine the image and kernel of a linear transformation	0 1 2 3 4
155. Add, subtract, and scalar multiply vectors using geometric interpretations of these operations and use in real world applications	0 1 2 3 4
156. Scalar multiply, add, subtract, and multiply matrices	0 1 2 3 4
157. Demonstrate an understanding and use the basic properties of inverses of matrices	0 1 2 3 4
158. Determine and apply the matrix representation of a linear transformation	0 1 2 3 4
159. Use matrix techniques to solve systems of linear equations	0 1 2 3 4

K. COMPUTER SCIENCE	
160. Demonstrate an understanding of the roles of the hardware and software components of a computer system (e.g., output devices, CPU, disks, operating systems, secondary storage devices)	0 1 2 3 4
161. Know basic computer terminology (e.g., files, I/O, records)	0 1 2 3 4
162. Use "user friendly" software (e.g., classroom instruction packages, graphics software, spreadsheets)	0 1 2 3 4
163. Develop computer algorithms to solve mathematical problems	0 1 2 3 4

Using the scale below, please circle the number that best describes your judgment as to how important it is that a newly-licensed (certified) mathematics teacher be able to perform the task in a competent manner.

IMPORTANCE

0 Of no importance
1 Of little importance
2 Moderately important
3 Important
4 Very important

K. COMPUTER SCIENCE (continued)	IMPORTANCE
164. Trace and debug existing computer algorithms	0 1 2 3 4
165. Program in two computer languages, at least one of which is a structured language (e.g., FORTRAN, PASCAL)	0 1 2 3 4

L. MATHEMATICAL REASONING AND MODELING

166. Demonstrate an understanding of a physical situation or a verbal description of a situation and develop a mathematical model of it	0 1 2 3 4
167. Determine appropriate mathematical strategies to solve a problem. These strategies might include conjectures, counterexamples, inductive reasoning, deductive reasoning (mathematical induction, proof by contradiction, direct proof, and other types of proof), and deciding which tools are appropriate (e.g., discussion with others, mental math, pencil and paper, calculator, computer, trees and graphs, fingers)	0 1 2 3 4
168. Recognize the reasonableness of results given the context of a problem	0 1 2 3 4
169. Using estimation, test the reasonableness of results	0 1 2 3 4
170. Estimate the actual and relative error in the numerical answer to a problem by analyzing the effects of roundoff and truncation errors introduced in the course of solving a problem	0 1 2 3 4
171. Having solved a problem, reconsider the strategies used. Are there other appropriate strategies? Which strategies are the most efficient? Can these strategies be used to solve other problems? Can these strategies be used to prove a more general result?	0 1 2 3 4
172. Communicate results in an appropriate form (e.g., correct English sentences, tables, charts, graphs)	0 1 2 3 4
173. Determine whether an isomorphism exists between two mathematical systems	0 1 2 3 4
174. Determine whether one mathematical model will describe two apparently different situations	0 1 2 3 4

Using the scale below, please circle the number that best describes your judgment as to how important it is that a newly-licensed (certified) mathematics teacher be able to perform the task in a competent manner.

IMPORTANCE

0 Of no importance
1 Of little importance
2 Moderately important
3 Important
4 Very important

L. MATHEMATICAL REASONING AND MODELING (continued)	IMPORTANCE
175. Demonstrate an understanding of the different levels of mathematical impossibility, such as: I lack the mathematical skills to do it; No one has been able to do it as yet (e.g., prove Goldbach's conjecture or Fermat's last theorem); No one will ever be able to do it (e.g., trisect a general angle with straight edge and compass)	0 1 2 3 4
176. Use the axiomatic method in modeling and problem solving	0 1 2 3 4

M. CONTENT-SPECIFIC PEDAGOGY

In addition to content, mathematics teachers also need to know something about pedagogy specific to mathematics. Please note in the following statements that the phrase " a particular group of students" is not meant to exclude activities and tasks aimed at teaching an individual student.

177. Evaluate a given scope and sequence of mathematics topics	0 1 2 3 4
178. Develop a scope and sequence of a mathematics topic for a particular level and justify it	0 1 2 3 4
179. Given an example of a student's work that contains an error arising from a misconception, identify the misconception and suggest methods for correcting it	0 1 2 3 4
180. Identify the prerequisite knowledge and skills that students ought to have before being taught a particular topic	0 1 2 3 4
181. Develop questions that ask students to display their current level of understanding of a particular topic	0 1 2 3 4
182. Given a particular problem, identify several problem-solving strategies (e.g., guess and check, reduce to a simpler problem, draw a diagram, work backwards) that might assist students to solve the problem	0 1 2 3 4
183. Use appropriate forms of representation (e.g., analogies, drawings, examples, symbols, manipulatives) for mathematics subject matter for a particular group of students that help make mathematics understandable and interesting	0 1 2 3 4
184. Use a variety of teaching strategies (e.g., laboratory work, supervised practice, group work, lecture) in mathematics appropriate for a particular group of students and a particular topic	0 1 2 3 4
185. Integrate concepts to show relationships among topics	0 1 2 3 4

Using the scale below, please circle the number that best describes your judgment as to how important it is that a newly-licensed (certified) mathematics teacher be able to perform the task in a competent manner.

IMPORTANCE

0 Of no importance
1 Of little importance
2 Moderately important
3 Important
4 Very important

M. CONTENT-SPECIFIC PEDAGOGY (continued)	IMPORTANCE
186. Relate mathematical concepts and ideas to real-world situations	0 1 2 3 4
187. Identify, evaluate, and use curricular materials and resources for mathematics (e.g., textbooks and other printed materials, computer software, base 10 blocks, geoboards, egg cartons, etc.) in ways appropriate for a particular group of students and a particular topic	0 1 2 3 4
188. Know procedures for controlling the social atmosphere of a classroom without restricting divergent mathematical thought	0 1 2 3 4
189. Know how society is affected by its general level of mathematical knowledge and know how societal influences differentially affect the mathematics education of gender, racial, ethnic, and socio-economic groups	0 1 2 3 4
190. Know how to use information about different gender, racial, ethnic, and socio-economic groups to enhance these groups' learning of mathematics ..	0 1 2 3 4
191. Evaluate the impact of learning theorists (e.g., Dewey, Piaget, Pestalozzi, the Van Hieles, Montessori, Thorndike) on mathematics education	0 1 2 3 4
192. Identify, evaluate, and use appropriate evaluation strategies (e.g., observations, interviews, oral discussions, written tests) to assess student progress in mathematics	0 1 2 3 4
193. Write specific evaluation items to test for specific mathematical skill	0 1 2 3 4

CONTENT COVERAGE

The purpose of this section is to obtain your judgment of how well this inventory covered the important tasks a newly-licensed (certified) mathematics teacher should be able to perform. We are interested in obtaining separate judgments for each of the thirteen categories included in the inventory.

Please circle the number on the rating scale that best represents your judgment.

How well did the tasks cover the important aspects of this category?

1 Poorly
2 Somewhat
3 Adequately
4 Well
5 Very well

194.	ARITHMETIC AND BASIC ALGEBRA	1	2	3	4	5
195.	GEOMETRY	1	2	3	4	5
196.	TRIGONOMETRY	1	2	3	4	5
197.	FUNCTIONS AND THEIR GRAPHS	1	2	3	4	5
198.	PROBABILITY AND STATISTICS	1	2	3	4	5
199.	ANALYTIC GEOMETRY	1	2	3	4	5
200.	CALCULUS	1	2	3	4	5
201.	DISCRETE MATHEMATICS	1	2	3	4	5
202.	ABSTRACT ALGEBRA	1	2	3	4	5
203.	LINEAR ALGEBRA	1	2	3	4	5
204.	COMPUTER SCIENCE	1	2	3	4	5
205.	MATHEMATICAL REASONING AND MODELING	1	2	3	4	5
206.	CONTENT-SPECIFIC PEDAGOGY	1	2	3	4	5

207. Please use this space to list any important categories that you feel were not included in the inventory (see previous page for a listing of the thirteen categories)

208. Please use this space to list any important task statements that you feel were not included within the thirteen categories. For each statement please indicate the category to which it belongs.

PART II - RECOMMENDATIONS FOR TEST CONTENT

Here are the thirteen topics covered in Part I of this inventory. If a licensing examination for mathematics teachers contained 100 questions, how many questions should be included from each topic? If you feel a topic should not be included in the exam, put 0 in the space provided. Make sure your responses sum to 100.

GENERAL TOPICS	NUMBER OF TEST QUESTIONS (out of 100)
209. ARITHMETIC AND BASIC ALGEBRA	________
210. GEOMETRY	________
211. TRIGONOMETRY	________
212. FUNCTIONS AND THEIR GRAPHS	________
213. PROBABILITY AND STATISTICS	________
214. ANALYTIC GEOMETRY	________
215. CALCULUS	________
216. DISCRETE MATHEMATICS	________
217. ABSTRACT ALGEBRA	________
218. LINEAR ALGEBRA	________
219. COMPUTER SCIENCE	________
220. MATHEMATICAL REASONING AND MODELING	________
221. CONTENT-SPECIFIC PEDAGOGY	________
TOTAL	100

PART III - BACKGROUND INFORMATION

The information which you provide in answering the questions in this section is completely confidential and will be used for research purposes only. Your responses will be grouped statistically with those of other individuals who are participating in this survey. A vital part of the statistical analysis consists of grouping people with varying experience and varying backgrounds. To do this, we need your answers to the following questions.

222. In which state do you work?

1. Alabama
2. Alaska
3. Arizona
4. Arkansas
5. California
6. Colorado
7. Connecticut
8. Delaware
9. District of Columbia
10. Florida
11. Georgia
12. Hawaii
13. Idaho
14. Illinois
15. Indiana
16. Iowa
17. Kansas
18. Kentucky
19. Louisiana
20. Maine
21. Maryland
22. Massachusetts
23. Michigan
24. Minnesota
25. Mississippi
26. Missouri
27. Montana
28. Nebraska
29. Nevada
30. New Hampshire
31. New Jersey
32. New Mexico
33. New York
34. North Carolina
35. North Dakota
36. Ohio
37. Oklahoma
38. Oregon
39. Pennsylvania
40. Rhode Island
41. South Carolina
42. South Dakota
43. Tennessee
44. Texas
45. Utah
46. Vermont
47. Virginia
48. Washington
49. West Virginia
50. Wisconsin
51. Wyoming

223. What is your age?

1. Under 25
2. 25-34
3. 35-44
4. 45-54
5. 55-64
6. Over 65

224. What is your sex?

1. Female
2. Male

225. How do you describe yourself?

1. American Indian, Eskimo, or Aleut
2. Black or African American
3. Mexican American or Chicano
4. Oriental or Asian American
5. Puerto Rican
6. Other Hispanic or Latin American
7. White
8. Other

226. Which of the following <u>best</u> describes your highest educational attainment?

1. Less than a Bachelor's degree
2. Bachelor's degree
3. Bachelor's degree + additional credits
4. Master's degree
5. Master's degree + additional credits
6. Doctorate

227. Which of the following <u>best</u> describes your current employment status?

1. Temporary substitute (assigned on a daily basis)
2. Permanent substitute (assigned on a longer term basis)
3. Regular teacher (not a substitute)
4. Principal or Assistant Principal
5. School Administrator
6. Curricular Supervisor
7. State Administrator
8. College Faculty
9. Other (please specify)________________________________

228. How many years have you taught mathematics?

1. Less than a year
2. 1 - 2 years
3. 3 - 5 years
4. 6 - 10 years
5. 11 - 15 years
6. 16 - 20 years
7. 21 or more years
8. Never taught mathematics

229. What grade level(s) are you currently teaching? (Circle <u>all</u> that apply)

1. Preschool/Kindergarten
2. Grades 1-4
3. Grades 5-8
4. Grades 9-12
5. College
6. Do not currently teach
7. Other (please specify)________________________________

230. Which of the following best describes your current teaching assignment? (Circle all that apply)

1. Arithmetic and Basic Algebra (e.g., General Mathematics, Consumer Mathematics, Pre-Algebra, Algebra I)
2. Pre-Calculus Mathematics (e.g., Geometry, Algebra II, Trigonometry, Pre-Calculus)
3. College-Level Mathematics (e.g., Calculus, Linear and Abstract Algebra, Probability and Statistics, Analysis, Discrete Mathematics)
4. Computer Science
5. Do not currently teach
6. Other (please specify)____________________________

Thank you for completing this inventory.
Please return it in the enclosed envelope or send it to:

Educational Testing Service, 11-R
Princeton, NJ 08541

Appendix D

Cover Letters and Follow-Up Postcard

Cover Letter for Primary Sample

NATIONAL COUNCIL OF
Teachers of Mathematics

NCTM

1906 Association Drive, Reston, Virginia 22091 ■ (703) 620-9840

December 1989

Dear Colleague:

I am writing to ask your cooperation in a project that should be of importance to members of the National Council of Teachers of Mathematics.

Educational Testing Service (ETS) is in the process of developing a new generation of licensing tests for the National Teachers Examinations (NTE). One of these new tests is the NTE Mathematics test. ETS has asked for our help in the design and content of this examination.

As part of the developmental process, ETS has worked closely with teachers, college faculty, and school administrators to identify potentially important tasks. The enclosed questionnaire has been developed as a way to obtain your judgments on the importance of these tasks for beginning teachers. The data obtained from these questionnaires will be used to define the content of the new examination.

I urge you to please take the time to complete the enclosed questionnaire. A preliminary study has indicated that the survey takes approximately 30 minutes to complete.

Please return your completed questionnaire within 10 days in the enclosed postage-paid envelope.

Your cooperation and participation in this important project is appreciated.

Sincerely,

James D. Gates

James D. Gates
Executive Director

JDG/mf
Enclosures

BEST COPY AVAILABLE

Cover Letter for Supplemental Sample

EDUCATIONAL TESTING SERVICE PRINCETON, N.J. 08541

609-921-9000
609-734-1090 (Fax)
CABLE-EDUCTESTSVC

DIVISION OF APPLIED
MEASUREMENT RESEARCH

May 1991

Dear Colleague:

I am writing to ask your cooperation in a project that should be of importance to teachers, college faculty, administrators, and other professionals in your field. Educational Testing Service (ETS) is in the process of developing a new generation of assessments for the purpose of licensing teachers. One type of assessment will be created to measure the prospective teacher's subject-matter or specialty-area knowledge and will likely be administered upon completion of the undergraduate teacher education program. One such assessment is a new version of the NTE Mathematics test. I am asking for your help as we develop this examination.

As part of the developmental process, ETS has worked closely with an advisory committee of classroom teachers, college faculty, and school administrators to identify potentially important knowledge and skill areas in mathematics instruction. The enclosed inventory has been constructed as a way to obtain your judgments on the importance of these areas for the newly licensed (certified) mathematics teacher. Your responses and those of other professionals to this inventory will guide the development of the new examination.

You will notice that the inventory asks for some background information about you; this is solely for purposes of describing respondents. Your answers will be treated in strict confidence.

A postage-paid envelope is enclosed for the return of your completed questionnaire. Thank you for your participation in this important project.

Sincerely,

Scott Wesley

Scott Wesley, Ph.D.
Associate Research Scientist

Enc. (2)

Follow-Up Postcard for Primary and Supplemental Samples

TASK ANALYSIS INVENTORY FOR MATHEMATICS TEACHERS

Dear Colleague:

I recently sent you an inventory to obtain your opinions of what a newly-licensed mathematics teacher should know and be able to do. If you have not already done so, please complete the inventory and return it in the postage-paid envelope to:

Educational Testing Service
Mail Stop 11-P
Princeton, NJ 08541

If you have already returned the inventory, please accept my thanks for your help in this important project.

Sincerely,

Scott Wesley, Ph.D.
Associate Research Scientist
Educational Testing Service

Appendix E

Demographic Distributions

	Primary Sample (N = 462)		Supplemental Sample (N = 82)	
	Number	Percent	Number	Percent
AGE (years)				
Under 25	1	0.2	1	1.2
25-34	2	0.4	25	30.5
35-44	49	10.6	33	40.2
45-54	202	43.7	17	20.7
55-64	153	33.1	3	3.7
65 and over	45	9.7	1	1.2
No response	10	2.2	2	2.4
SEX				
Female	160	34.6	43	52.4
Male	291	63.0	37	45.1
No response	11	2.4	2	2.4
RACE/ETHNICITY				
American Indian or Alaskan Native	2	0.4	1	1.2
Black or African American	13	2.8	5	6.1
Mexican American or Chicano	2	0.4	2	2.4
Oriental or Asian American	6	1.3	1	1.2
Puerto Rican	0	0.0	1	1.2
Other Hispanic or Latin American	1	0.2	3	3.7
White	419	90.7	64	78.0
Other	3	0.6	3	3.7
No response	16	3.5	2	2.4
HIGHEST EDUCATIONAL ATTAINMENT				
Less than Bachelor's	0	0.0	0	0.0
Bachelor's	0	0.0	7	8.5
Bachelor's + Credits	29	6.3	35	42.7
Master's	27	5.8	9	11.0
Master's + Credits	257	55.6	29	35.4
Doctorate	139	30.1	1	1.2
No response	10	2.2	1	1.2

	Primary Sample (N = 462)		Supplemental Sample (N = 82)	
	Number	Percent	Number	Percent
CURRENT EMPLOYMENT STATUS				
Temporary Substitute	1	0.2	0	0.0
Permanent Substitute	0	0.0	1	1.2
Regular Teacher (not a substitute)	180	39.0	76	92.7
Principal or Assistant Principal	6	1.3	0	0.0
School Administrator	2	0.4	0	0.0
Curricular Supervisor	11	2.4	3	3.7
State Administrator	9	1.9	0	0.0
College or University Faculty	159	34.4	0	0.0
Other	81	17.5	1	1.2
No response	13	2.8	1	1.2
TEACHING EXPERIENCE				
Less than 1 year	0	0.0	2	2.4
1-2 years	1	0.2	13	15.9
3-5 years	2	0.4	21	25.6
6-10 years	14	3.0	14	17.1
11-15 years	24	5.2	11	13.4
16-20 years	39	8.4	10	12.2
21 or more years	371	80.3	10	12.2
Never taught mathematics	1	0.2	0	0.0
No response	10	2.2	1	1.2
GRADES CURRENTLY TEACHING [1]				
Preschool/Kindergarten	4	0.9	0	0.0
Grades 1 - 4	5	1.1	0	0.0
Grades 5 - 8	38	8.2	6	7.3
Grades 9 - 12	192	41.6	78	95.1
College	197	42.6	2	2.4
Do not currently teach	53	11.5	1	1.2
Other	32	6.9	3	3.7
No response	0	0.0	1	1.2

[1] Multiple responses were allowed. Total will not add up to 462 in the main sample and 82 in the supplementary sample.

	Primary Sample (N = 462)		Supplemental Sample (N = 82)	
	Number	Percent	Number	Percent
CURRENT TEACHING ASSIGNMENT [1]				
Arithmetic and Basic Algebra	135	29.2	65	79.3
Pre-Calculus Mathematics	215	46.5	49	59.8
College-Level Mathematics	184	39.8	10	12.2
Computer Science	52	11.3	10	12.2
Do not currently teach	59	12.8	1	1.2
Other	78	16.9	7	8.5
No response	0	0.0	1	1.2
GEOGRAPHIC REGION				
Northeast	100	21.6	24	29.3
Central	120	26.0	18	22.0
South	118	25.5	23	28.0
Far West	113	24.5	16	19.5
No response	11	2.4	1	1.2

1 Multiple responses were allowed. Total will not add up to 462 in the main sample and 82 in the supplementary sample.

Appendix F

Importance Ratings for Primary and Supplemental Samples

74

		Main Sample				Supplemental Sample	
		Teachers (N = 180)		College Faculty (N = 159)		Teachers (N = 77)	
		Mean	SD	Mean	SD	Mean	SD
ARITHMETIC AND BASIC ALGEBRA							
1	Order finite set of real numbers and recognize equivalent forms	3.79	0.45	3.77	0.50	3.66	0.65
2	Use numbers in a way that is most appropriate in the context of a problem	3.32	0.81	3.35	0.80	3.39	0.79
3	Understand the properties of counting numbers	3.72	0.53	3.70	0.55	3.74	0.53
4	Add, subtract, mulitply, and divide rational numbers expressed in any form	3.84	0.44	3.82	0.49	3.91	0.34
5	Apply the order of operations to problems, with and without grouping symbols	3.86	0.39	3.78	0.55	3.92	0.27
6	Interpret and apply the concepts of ratio, proportion, and percent	3.78	0.43	3.72	0.52	3.82	0.38
7	Solve problems that involve measurement in the metric system	3.03	0.87	3.19	0.83	3.09	0.81
8	Solve problems that involve measurement in the traditional system	3.32	0.75	3.26	0.92	3.33	0.64
9	Find positive integral powers and roots of perfect powers	2.36	1.09	2.28	1.15	2.60	0.99
10	Estimate values of expressions involving decimals, exponents, and radicals	3.35	0.71	3.37	0.79	3.21	0.86
11	Solve problems involving average, including arithmetic mean and weighted average	3.24	0.79	3.29	0.79	3.43	0.66
12	Evaluate algebraic expressions and formulas	3.80	0.44	3.72	0.55	3.83	0.42
13	Solve for a selected variable in a literal equation or formula	3.71	0.56	3.74	0.56	3.59	0.66
14	Classify a number as rational, irrational, real and/or complex	3.46	0.71	3.46	0.83	3.27	0.93
15	Identify the properties of the basic operations on the standard number system	3.29	0.85	3.34	0.77	3.27	0.88
16	For newly defined operation, determine whether properties hold	2.73	0.85	2.96	0.92	2.78	0.99
17	Identify additive and multiplicative inverses	3.56	0.65	3.50	0.71	3.47	0.70
18	Add, subtract, and multiply polynomials	3.72	0.53	3.62	0.64	3.72	0.51
19	Divide polynomials	3.42	0.71	3.33	0.91	3.51	0.79
20	Add, subtract, multiply, and divide algebraic fractions	3.59	0.68	3.56	0.74	3.72	0.53
21	Translate verbal expressions and relationships into algebraic expressions or equations	3.85	0.40	3.87	0.33	3.78	0.51
22	Solve and graph linear equations and inequalities in one or two variables	3.74	0.51	3.75	0.50	3.71	0.54
23	Solve and graph systems of linear equations and graph inequalities in two variables	3.64	0.57	3.64	0.59	3.53	0.72

Note: Mean ratings that are less than 2.50 are shaded.

		Main Sample				Supplemental Sample	
		Teachers (N = 180)		College Faculty (N = 159)		Teachers (N = 77)	
		Mean	SD	Mean	SD	Mean	SD
24	Solve and graph nonlinear algebraic equations and graph inequalities	3.38	0.70	3.32	0.82	3.37	0.78
25	Perform standard algebraic operations with complex numbers, radicals, exponents	3.42	0.71	3.47	0.75	3.61	0.57
26	Determine any term of a binomial expansion using Pascal's triangle or other method	2.67	0.86	2.71	1.03	2.64	0.95
27	Solve equations and inequalities involving absolute values	3.22	0.71	3.21	0.86	3.21	0.88
28	Present geometric interpretations of algebraic principles	2.61	0.99	3.01	0.94	2.72	1.10
GEOMETRY							
29	Use relationships among two-dimensional geometric figures to solve problems	3.69	0.54	3.66	0.58	3.61	0.61
30	Use relationships among three-dimensional geometric figures to solve problems	2.89	0.79	3.03	0.84	3.07	0.85
31	Solve problems using properties of parallel and perpendicular lines	3.70	0.57	3.62	0.62	3.68	0.55
32	Solve problems using special triangles, such as isosceles and equilateral	3.72	0.51	3.65	0.61	3.79	0.44
33	Solve problems using the relationships of parts of triangles, such as side, angles	3.54	0.62	3.40	0.75	3.59	0.59
34	Apply the Pythagorean Theorem to solve problems	3.91	0.35	3.83	0.43	3.89	0.31
35	Solve problems using properties of special quadrilaterals	3.48	0.62	3.41	0.76	3.57	0.66
36	Describe relationships among sets of special quadrilaterals	3.47	0.68	3.35	0.79	3.36	0.76
37	Solve problems using properties of polygons with more than four sides	3.14	0.80	2.92	0.90	3.22	0.78
38	Solve problems using properties of circles	3.36	0.74	3.22	0.86	3.46	0.66
39	Compute perimeter and area of triangles, quadrilaterals, circles	3.68	0.57	3.68	0.64	3.70	0.59
40	Compute surface area and volume of prisms, pyramids, cones, cylinders, and spheres	3.10	0.78	3.13	0.88	3.25	0.73
41	Solve problems involving reflection, rotation, translation of points, lines, polygons	2.66	0.87	2.92	0.89	2.70	0.91
42	Execute geometric constructions with straightedge and compass	3.06	0.93	3.10	0.99	3.04	0.90
43	Prove that a geometric construction yields the desired result	2.61	0.91	2.85	1.00	2.67	1.03
44	Know and use basic facts about non-Euclidean geometries	2.26	0.89	2.52	0.98	2.24	0.99
TRIGONOMETRY							
45	Define and use the six trigonometric relations on the right triangle	3.80	0.45	3.77	0.53	3.77	0.45
46	Solve problems of right triangles with common angles without tables/calculators	3.52	0.76	3.42	0.84	3.60	0.55

		Main Sample				Supplemental Sample	
		Teachers (N = 180)		College Faculty (N = 159)		Teachers (N = 77)	
		Mean	SD	Mean	SD	Mean	SD
47	Identify relationship between radian measures and degree measures of angles	3.48	0.73	3.51	0.76	3.46	0.69
48	Apply law of sines and law of cosines in solving problems	3.39	0.76	3.26	0.87	3.41	0.84
49	Define and use six basic trigonometric functions defined on unit circle	3.48	0.73	3.49	0.79	3.40	0.81
50	Solve problems involving trigonometric functions of π values	3.29	0.82	3.39	0.78	3.31	0.84
51	Recognize graphs of the six trigonometric functions	3.25	0.84	3.36	0.81	3.05	0.93
52	Apply formulas for trigonometric functions involving x and y	2.87	0.95	2.89	0.92	2.82	0.93
53	Prove identities using basic trigonometric identities	2.99	0.94	3.09	1.06	3.05	0.97
54	Solve trigonometric equations and inequalities	3.19	0.83	3.29	0.82	3.07	0.95
55	Identify point in polar coordinate system from rectangular coordinate system	2.86	0.91	2.97	0.90	2.64	1.03
56	Find trigonometric form of complex numbers and apply DeMoivre's Theorem	2.59	1.02	2.60	0.94	2.34	1.09
FUNCTIONS AND THEIR GRAPHS							
57	Understand function notation and work with algebraic definition of a function	3.72	0.55	3.75	0.57	3.56	0.66
58	Given a graph, select the most appropriate equation for the graph	3.46	0.61	3.54	0.67	3.43	0.74
59	Given an equation, graph it	3.72	0.52	3.73	0.52	3.76	0.52
60	Use the definition of a function as a mapping	2.90	0.87	2.97	0.82	2.89	0.92
61	Find the domain and/or range of a function	3.58	0.65	3.59	0.77	3.45	0.78
62	Use properties of algebraic, trigonometric, logarithmic, and exponential functions	3.15	0.82	3.30	0.77	3.00	0.85
63	Find inverse of a one-to-one function in simple cases	3.16	0.86	3.37	0.81	2.95	0.90
64	Sketch graph of a linear, step, absolute value, or quadratic function	3.30	0.83	3.47	0.73	3.23	0.88
PROBABILITY AND STATISTICS							
65	Organize data into a presentation that is appropriate for solving a problem	3.23	0.83	3.53	0.66	3.33	0.88
66	Solve probability problems involving finite sample spaces	3.16	0.93	3.42	0.74	3.19	0.93
67	Solve probability problems using counting techniques	3.18	0.83	3.35	0.75	3.28	0.85
68	Solve probability problems involving independent trials	3.08	0.82	3.37	0.68	3.35	0.83
69	Solve problems using the binomial distribution	2.65	0.91	2.95	0.87	2.60	0.94

		Main Sample				Supplemental Sample	
		Teachers (N = 180)		College Faculty (N = 159)		Teachers (N = 77)	
		Mean	SD	Mean	SD	Mean	SD
70	Solve problems involving joint probability	2.55	0.92	2.71	0.86	2.76	0.98
71	Find and interpret common measures of central tendency	3.24	0.91	3.56	0.68	3.23	0.91
72	Find and interpret common measures of dispersion	2.67	1.00	3.34	0.73	2.85	0.91
73	Calculate the expected value of a function of a discrete random variable	2.09	0.97	2.41	0.99	2.36	0.97
74	Model an applied problem using mathematical expectation	2.46	0.98	2.79	0.92	2.59	0.97
75	Solve problems using the normal distribution	2.58	0.97	3.12	0.84	2.88	0.93
76	Explain the consequences of the Central Limit Theorem	2.08	1.02	2.68	0.99	2.26	0.99
77	Use the Central Limit Theorem to calculate probabilities	2.00	1.02	2.57	0.97	2.16	1.01
78	Solve problems using the uniform and chi-square distributions	1.87	1.02	2.18	0.95	2.08	0.99
79	Solve continuous probability problems with random variables, etc.	1.77	0.96	2.11	0.97	2.11	0.97
80	Solve continuous probability problems with joint and conditional probability	1.94	1.01	2.10	0.93	2.15	1.06
81	Solve expected value problems for continuous random variables	1.81	0.94	2.01	0.94	2.05	0.99
82	Develop test to accept or reject a given null hypothesis	1.81	1.03	2.43	1.05	1.96	1.05
83	Discuss sample size, significance level, power, type I, II error relationships	1.71	0.99	2.18	1.11	1.88	1.06
ANALYTIC GEOMETRY							
84	Determine the equations of lines and planes given appropriate information	3.44	0.74	3.56	0.65	3.37	0.83
85	Make calculations in 2-space or 3-space	3.36	0.82	3.41	0.71	3.33	0.87
86	Given a geometric definition of a conic section, derive the equation	2.90	0.89	2.94	0.90	2.92	0.94
87	Determine which conic section is represented by a given equation with restrictions	3.06	0.90	3.08	0.92	2.95	0.90
88	Determine which conic section is represented by a given equation without restrictions	2.56	1.00	2.53	1.04	2.69	1.09
CALCULUS							
89	Discuss informally what it means for a function to have a limit at a point	3.36	0.84	3.66	0.57	3.25	0.89
90	Calculate limits of functions or determine that the limit does not exist	3.24	0.89	3.48	0.69	3.19	0.92
91	Prove via epsilon-delta that the limit of a function equals the calculated value	2.40	1.14	2.35	1.13	2.42	1.03
92	Solve problems using the properties of limits	2.89	1.01	3.09	0.88	2.80	1.06

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		Main Sample				Supplemental Sample	
		Teachers (N = 180)		College Faculty (N = 159)		Teachers (N = 77)	
		Mean	SD	Mean	SD	Mean	SD
93	Use limits to show that a particular function is continuous	2.84	1.03	3.05	0.89	2.78	1.04
94	Use L'Hopital's rule, where applicable, to calculate limits of functions	2.64	1.13	2.83	0.98	2.59	0.98
95	Relate the derivative of a function to a limit or to the slope of a curve	3.30	0.92	3.61	0.62	3.08	1.11
96	Explain conditions under which a continuous function does not have a derivative	2.84	0.96	3.05	0.89	2.70	1.05
97	Differentiate expressions and functions using sum, product, quotient, and chain rules	3.14	1.00	3.40	0.79	3.03	1.03
98	Use implicit differentiation	2.89	1.03	3.08	0.98	2.79	1.07
99	Make numerical approximations of derivatives and integrals	2.67	0.99	2.94	0.99	2.68	1.09
100	Use differential calculus to analyze the behavior of a function	3.14	1.00	3.45	0.82	3.01	1.02
101	Use differential calculus to solve problems involving related rates and rates of change	3.01	0.99	3.34	0.79	2.96	1.03
102	Approximate the roots of a function	2.44	1.07	2.65	0.95	2.53	0.99
103	Use differential calculus to solve applied minima-maxima problems	3.07	1.01	3.33	0.74	2.93	0.98
104	Solve problems using the Mean Value Theorem of differential calculus	2.64	1.02	2.68	0.96	2.63	1.04
105	Explain significance of, and solve problems using Fundamental Theorem of Calculus	2.99	0.99	3.23	0.89	2.87	1.07
106	Demonstrate understanding of integration as finding areas of regions through limits	3.08	0.91	3.51	0.70	2.79	1.11
107	Integrate functions using algebraic substitutions	2.80	1.08	3.03	0.98	2.83	1.14
108	Integrate functions using "integration by parts"	2.67	1.09	2.96	1.00	2.71	1.12
109	Integrate functions using trigonometric substitutions	2.56	1.10	2.66	1.07	2.73	1.15
110	Integrate logarithmic and exponential functions	2.73	1.16	3.03	0.98	2.75	1.13
111	Evaluate improper integrals	2.47	1.05	2.73	1.05	2.56	1.18
112	Use integral calculus to calculate area of regions in the plane	2.98	1.04	3.38	0.83	2.90	1.09
113	Calculate volume of solids formed by rotating plane figures about a line	2.81	1.07	3.01	1.02	2.74	1.16
114	Interchange the order of Integration in double integrals	1.94	1.05	2.28	1.08	2.20	1.15
115	Find volumes of solids using double integrals	1.99	1.09	2.41	1.11	2.25	1.19
116	Determine limits of sequences and simple Infinite series	2.66	1.04	2.96	0.91	2.42	1.23
117	Use tests to show convergence or divergence of series	2.36	1.05	2.65	1.08	2.22	1.08

		Main Sample				Supplemental Sample	
		Teachers (N = 180)		College Faculty (N = 159)		Teachers (N = 77)	
		Mean	SD	Mean	SD	Mean	SD
118	Determine the interval of convergence of a power series	2.19	1.02	2.41	1.11	2.15	1.17
119	Determine Taylor series of functions like sin x, cos x, e^x, and ln x	2.18	1.07	2.48	1.13	2.15	1.21
	DISCRETE MATHEMATICS						
120	Use basic terminology and, given the definitions, use the symbols of logic	2.98	0.92	3.22	0.88	3.04	1.01
121	Solve problems of union and intersection of sets, subsets, and disjoint sets	3.15	0.88	3.42	0.72	3.23	0.95
122	Use truth tables to verify statements	2.95	0.90	3.20	0.88	2.92	1.04
123	Use Laws of Algebra of Propositions to evaluate equivalence	2.36	0.95	2.76	1.02	2.49	1.11
124	Solve basic problems involving permutations and combinations	3.05	0.88	3.30	0.76	2.86	1.02
125	Use Euclidean Algorithm to find greatest common divisor of two numbers	2.59	0.99	2.92	1.03	2.49	1.13
126	Work with numbers expressed in bases other than base ten	2.34	1.00	2.72	1.18	2.65	1.12
127	Solve modular congruences	2.05	0.92	2.53	1.06	2.08	1.06
128	Prove theorems using modular systems	1.67	0.90	2.11	1.09	1.92	1.11
129	Find values of functions defined recursively	2.34	0.98	2.79	0.95	2.03	1.15
130	"Translate" between recursive and closed form expressions for a function	2.02	0.93	2.45	1.01	1.90	1.19
131	Determine if a binary relation is reflexive, symmetric, antisymmetric, transitive	2.21	1.01	2.76	1.04	2.13	1.14
132	Determine if a binary relation is an equivalence relation	2.26	1.03	2.85	1.02	2.11	1.14
133	Determine if set is ordered with respect to a binary relation	2.10	1.01	2.63	1.07	2.03	1.13
134	Prove countability of rational numbers and non-countability of real numbers	1.84	1.02	2.18	1.13	1.96	1.16
135	Use basic terminology in graph theory	2.15	1.08	2.53	0.96	2.15	1.17
136	Identify conditions under which a graph can be traversed	1.81	1.02	2.29	1.03	1.81	1.12
137	Know and use the three basic ways of traversing a binary tree	1.77	0.97	2.03	1.02	1.83	1.17
138	Solve simple linear programming problems	2.51	0.97	2.85	0.97	2.37	1.27
139	Find and use finite differences of a function	2.00	0.94	2.42	1.04	2.06	1.17
140	Interpolate using Newton's forward and backward difference formulas	1.66	0.93	1.85	0.98	1.77	1.09

		Main Sample				Supplemental Sample	
		Teachers (N = 180)		College Faculty (N = 159)		Teachers (N = 77)	
		Mean	SD	Mean	SD	Mean	SD
ABSTRACT ALGEBRA							
141	Determine if a set together with an operation is a group	2.21	1.15	2.94	1.07	1.93	1.12
142	Use definition of a group to deduce elementary properties of a group	2.13	1.18	2.77	1.15	1.90	1.13
143	Determine if a set together with two operations is a ring	1.94	1.12	2.63	1.15	1.85	1.15
144	Determine if a set together with two operations is a field	2.20	1.19	2.79	1.27	1.85	1.13
145	Use definition of a field to deduce elementary properties	2.11	1.18	2.66	1.25	1.83	1.11
146	Determine whether mathematical systems are groups, rings or fields	2.09	1.11	2.72	1.13	1.86	1.17
147	Determine if subset of a group is subgroup or normal subgroup	1.74	1.03	2.20	1.26	1.81	1.15
LINEAR ALGEBRA							
	In a finite dimensional real vector space:						
148	determine if a subset is a subspace	1.72	1.08	2.47	1.19	1.89	1.16
149	determine if a set of vectors is linearly independent	1.87	1.11	2.62	1.16	2.04	1.17
150	determine if a set of vectors is basis for vector space	1.89	1.14	2.52	1.21	1.90	1.21
151	determine the dimension of the span of a set of vectors	1.77	1.06	2.34	1.22	1.83	1.17
152	determine if vectors are orthogonal using the dot product	1.93	1.11	2.53	1.11	1.99	1.17
153	determine effects of linear transformation on a vector space	1.79	1.11	2.30	1.13	1.92	1.18
154	determine image and kernel of a linear transformation	1.54	1.02	2.06	1.18	1.79	1.21
155	Add, subtract, and scalar multiply vectors using geometric interpretations	2.71	1.03	3.13	0.89	2.38	1.18
156	Scalar multiply, add, subtract, and multiply matrices	2.70	1.00	3.18	0.88	2.52	1.19
157	Demonstrate understanding and use basic properties of inverses	2.53	1.08	2.97	0.95	2.41	1.13
158	Determine and apply matrix representation of a linear transformation	2.27	1.08	2.67	1.07	2.29	1.14
159	Use matrix techniques to solve systems of linear equations	2.68	1.03	3.02	0.94	2.56	1.11
COMPUTER SCIENCE							
160	Demonstrate an understanding of the roles of hardware and software components	3.19	0.93	3.27	0.99	3.14	0.97
161	Know basic computer terminology	3 10	0.95	3.25	0.98	3.18	0.93

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		Main Sample				Supplemental Sample	
		Teachers (N = 180)		College Faculty (N = 159)		Teachers (N = 77)	
		Mean	SD	Mean	SD	Mean	SD
162	Use "user-friendly" software	3.35	0.89	3.49	0.71	3.38	0.86
163	Develop computer algorithms to solve mathematical problems	2.65	1.01	2.85	0.94	2.80	1.03
164	Trace and debug existing computer algorithms	2.23	1.10	2.52	1.10	2.36	1.12
165	Program in two computer languages, one of which is a structured language	1.95	1.15	2.30	1.10	2.16	1.28
MATHEMATICAL REASONING AND MODELING							
166	Develop mathematical model of physical situation	3.09	0.93	3.46	0.70	2.99	0.95
167	Determine mathematical strategies to solve a problem	3.37	0.83	3.61	0.60	3.24	0.82
168	Recognize reasonableness of results given the context of a problem	3.58	0.75	3.71	0.56	3.28	0.90
169	Using estimation, test the reasonableness of results	3.60	0.73	3.63	0.61	3.35	0.81
170	Estimate actual and relative error in numerical answer	2.46	0.99	2.77	0.92	2.68	1.02
171	Having solved a problem, reconsider the strategies used	2.98	0.91	3.28	0.76	3.05	0.93
172	Communicate results in an appropriate form	3.43	0.81	3.64	0.66	3.50	0.82
173	Determine whether an isomorphism exists between two mathematical systems	2.13	0.99	2.46	0.99	2.29	1.15
174	Determine if one mathematical model will describe two different situations	2.37	0.96	2.80	0.99	2.53	0.96
175	Understand the different levels of mathematical impossibility	2.53	1.04	2.94	0.96	2.41	1.08
176	Use the axiomatic method in modeling and problem solving	2.43	1.01	2.65	1.01	2.19	1.10
CONTENT-SPECIFIC PEDAGOGY							
177	Evaluate a given scope and sequence of mathematics topics	3.10	0.93	3.21	0.82	3.24	0.77
178	Develop scope and sequence of a mathematics topic and justify it	2.88	0.97	3.06	0.87	3.14	0.85
179	Identify misconception in a student's work and suggest methods for correcting	3.50	0.73	3.61	0.65	3.45	0.71
180	Identify students' prerequisite knowledge and skills for a topic	3.46	0.75	3.57	0.61	3.59	0.59
181	Develop questions to ask students that display their level of understanding	3.44	0.71	3.57	0.60	3.49	0.60
182	Given a problem, identify several problem-solving strategies	3.51	0.72	3.64	0.56	3.53	0.64
183	Use appropriate forms of representation	3.44	0.77	3.59	0.60	3.59	0.55
184	Use variety of appropriate teaching strategies	3.44	0.77	3.62	0.76	3.63	0.56

		Main Sample				Supplemental Sample	
		Teachers (N = 180)		College Faculty (N = 159)		Teachers (N = 77)	
		Mean	SD	Mean	SD	Mean	SD
185	Integrate concepts to show relationships among topics	3.35	0.74	3.46	0.72	3.47	0.64
186	Relate mathematical concepts and ideas to real-world situations	3.47	0.69	3.54	0.66	3.69	0.52
187	Identify, evaluate, and use curricular materials and resources	3.23	0.76	3.39	0.79	3.49	0.62
188	Know procedures to control social atmosphere without restricting divergent thought	3.40	0.75	3.36	0.75	3.60	0.59
189	Know how society is affected by level of mathematics knowledge	2.91	0.97	3.06	0.87	3.34	0.86
190	Use information about different groups to enhance learning	2.80	0.95	3.04	0.85	3.17	0.84
191	Evaluate impact of learning theorists on mathematics education	2.04	0.97	2.37	1.09	2.30	0.93
192	Identify, evaluate, and use appropriate evaluation strategies	3.36	0.79	3.38	0.76	3.55	0.66
193	Write evaluation items to test for a specific mathematical skill	3.47	0.77	3.58	0.63	3.64	0.56

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Appendix G

Importance Ratings for Demographic Subgroups

	Sex		Race/ Ethnicity		Geographic Region				Teaching Experience (teachers only)	
	F	M	POC	W	NE	C	S	FW	0-5	6+
ARITHMETIC AND BASIC ALGEBRA										
1 Order finite set of real numbers and recognize equivalent forms	3.81	3.74	3.76	3.77	3.77	3.77	3.73	3.80	3.75	3.75
2 Use numbers in a way that is most appropriate in the context of a problem	3.52	3.31	3.36	3.39	3.37	3.36	3.45	3.39	3.48	3.32
3 Understand the properties of counting numbers	3.84	3.66	3.88	3.71	3.70	3.72	3.74	3.74	3.84	3.71
4 Add, subtract, multiply, and divide rational numbers expressed in any form	3.89	3.79	3.95	3.82	3.84	3.85	3.82	3.82	3.94	3.85
5 Apply the order of operations to problems, with and without grouping symbols	3.89	3.80	3.83	3.84	3.84	3.87	3.83	3.80	3.97	3.86
6 Interpret and apply the concepts of ratio, proportion, and percent	3.87	3.73	3.83	3.78	3.79	3.72	3.79	3.84	3.81	3.79
7 Solve problems that involve measurement in the metric system	3.26	3.07	3.27	3.12	3.07	3.05	3.20	3.23	3.15	3.04
8 Solve problems that involve measurement in the traditional system	3.43	3.28	3.52	3.33	3.36	3.25	3.43	3.30	3.52	3.30
9 Find positive integral powers and roots of perfect powers	2.54	2.31	2.55	2.38	2.31	2.29	2.52	2.46	2.82	2.37
10 Estimate values of expressions involving decimals, exponents, and radicals	3.47	3.32	3.38	3.38	3.53	3.28	3.34	3.38	3.18	3.33
11 Solve problems involving average, including arithmetic mean and weighted average	3.38	3.28	3.60	3.30	3.41	3.26	3.33	3.30	3.52	3.26
12 Evaluate algebraic expressions and formulas	3.84	3.75	3.76	3.79	3.79	3.80	3.76	3.79	3.94	3.79
13 Solve for a selected variable in a literal equation or formula	3.75	3.70	3.71	3.72	3.76	3.72	3.68	3.70	3.76	3.66
14 Clas y a number as rational, irrational, real and/or complex	3.63	3.35	3.50	3.45	3.52	3.40	3.49	3.39	3.33	3.42
15 Identify the properties of the basic operations on the standard number system	3.48	3.23	3.43	3.31	3.31	3.25	3.37	3.34	3.42	3.26
16 For newly defined operation, determine whether properties hold	3.02	2.78	3.00	2.85	2.83	2.77	2.99	2.87	2.91	2.72
17 Identify additive and multiplicative inverses	3.63	3.47	3.60	3.52	3.53	3.54	3.51	3.55	3.56	3.53
18 Add, subtract, and multiply polynomials	3.79	3.60	3.72	3.67	3.62	3.67	3.66	3.73	3.82	3.70
19 Divide polynomials	3.57	3.29	3.63	3.37	3.31	3.36	3.41	3.48	3.68	3.41
20 Add, subtract, multiply, and divide algebraic fractions	3.72	3.51	3.67	3.58	3.56	3.60	3.57	3.62	3.76	3.61
21 Translate verbal expressions and relationships into algebraic expressions or equations	3.92	3.80	3.86	3.84	3.83	3.80	3.87	3.88	3.82	3.83
22 Solve and graph linear equations and inequalities in one or two variables	3.87	3.67	3.88	3.73	3.78	3.67	3.72	[illegible].82	3.76	3.73
23 Solve and graph systems of linear equations and graph inequalities in two variables	3.73	3.55	3.72	3.61	3.66	3.53	3.60	3.[illegible]	3.56	3.62

Note: F = Female (N=203); M = Male (N=328); POC = People of Color (N=42); W = White (N=483); NE = Northeast (N=124); C = Central (N=138); S = South(N=141); FW = Far West (N=129); 0-5 = 0 to 5 years' teaching experience (N=34); 6+ = 6 or more years' teaching experience (N=223). Mean ratings that are less than 2.50 are shaded.

		Sex		Race/ Ethnicity		Geographic Region				Teaching Experience (teachers only)	
		F	M	POC	W	NE	C	S	FW	0-5	6+
24	Solve and graph nonlinear algebraic equations and graph inequalities	3.50	3.28	3.47	3.35	3.40	3.26	3.37	3.45	3.38	3.38
25	Perform standard algebraic operations with complex numbers, radicals, exponents	3.58	3.40	3.51	3.47	3.50	3.39	3.50	3.50	3.68	3.44
26	Determine any term of a binomial expansion using Pascal's triangle or other method	2.79	2.69	2.88	2.71	2.76	2.61	2.79	2.76	2.59	2.68
27	Solve equations and inequalities involving absolute values	3.35	3.15	3.33	3.21	3.24	3.16	3.29	3.20	3.26	3.21
28	Present geometric interpretations of algebraic principles	2.91	2.74	3.02	2.79	2.77	2.74	2.79	2.93	2.88	2.61
GEOMETRY											
29	Use relationships among two-dimensional geometric figures to solve problems	3.78	3.61	3.74	3.67	3.67	3.66	3.67	3.69	3.71	3.66
30	Use relationships among three-dimensional geometric figures to solve problems	3.16	2.88	3.10	2.98	2.93	2.88	3.03	3.11	3.24	2.90
31	Solve problems using properties of parallel and perpendicular lines	3.78	3.63	3.58	3.70	3.72	3.66	3.65	3.73	3.74	3.69
32	Solve problems using special triangles, such as isosceles and equilateral	3.79	3.68	3.74	3.72	3.74	3.71	3.71	3.71	3.79	3.73
33	Solve problems using the relationships of parts of triangles, such as side, angles	3.65	3.40	3.67	3.48	3.52	3.45	3.51	3.50	3.62	3.55
34	Apply the Pythagorean Theorem to solve problems	3.89	3.87	3.88	3.88	3.85	3.85	3.88	3.93	3.88	3.91
35	Solve problems using properties of special quadrilaterals	3.63	3.40	3.47	3.49	3.52	3.47	3.46	3.51	3.59	3.50
36	Describe relationships among sets of special quadrilaterals	3.59	3.31	3.47	3.41	3.39	3.42	3.39	3.46	3.35	3.45
37	Solve problems using properties of polygons with more than four sides	3.26	2.95	3.23	3.05	3.00	3.04	3.06	3.17	3.24	3.15
38	Solve problems using properties of circles	3.45	3.26	3.56	3.31	3.32	3.32	3.34	3.34	3.38	3.39
39	Compute perimeter and area of triangles, quadrilaterals, circles	3.78	3.63	3.77	3.69	3.62	3.66	3.74	3.71	3.68	3.68
40	Compute surface area and volume of prisms, pyramids, cones, cylinders, and spheres	3.31	3.04	3.19	3.14	3.02	3.04	3.18	3.32	3.35	3.11
41	Solve problems involving reflection, rotation, translation of points, lines, polygons	2.97	2.69	2.88	2.78	2.85	2.78	2.79	2.77	2.56	2.68
42	Execute geometric constructions with straight-edge and compass	3.15	3.04	3.28	3.06	2.91	3.11	3.11	3.17	3.15	3.04
43	Prove that a geometric construction yields the desired result	2.81	2.69	2.91	2.73	2.65	2.66	2.78	2.86	2.68	2.62
44	Know and use basic facts about non-Euclidean geometries	2.54	2.31	2.51	2.39	2.34	2.33	2.51	2.40	2.18	2.26
TRIGONOMETRY											
45	Define and use the six trigonometric relations on the right triangle	3.82	3.76	3.70	3.79	3.79	3.78	3.79	3.78	3.85	3.79
46	Solve problems of right triangles with common angles without tables/calculators	3.59	3.46	3.37	3.52	3.52	3.45	3.51	3.56	3.64	3.53
47	Identify relationship between radian measures and degree measures of angles	3.52	3.47	3.42	3.50	3.53	3.49	3.47	3.49	3.39	3.48

	Sex		Race/ Ethnicity		Geographic Region				Teaching Experience (teachers only)	
	F	M	POC	W	NE	C	S	FW	0-5	6+
48 Apply law of sines and law of cosines in solving problems	3.48	3.28	3.33	3.36	3.31	3.31	3.35	3.44	3.41	3.39
49 Define and use six basic trigonometric functions defined on unit circle	3.57	3.41	3.44	3.47	3.46	3.46	3.47	3.52	3.44	3.46
50 Solve problems involving trigonometric functions of π values	3.47	3.28	3.33	3.35	3.37	3.32	3.38	3.35	3.33	3.29
51 Recognize graphs of the six trigonometric functions	3.38	3.22	3.07	3.29	3.35	3.25	3.21	3.30	3.15	3.20
52 Apply formulas for trigonometric functions involving x and y	3.07	2.81	2.86	2.90	2.92	2.91	2.83	2.97	2.88	2.85
53 Prove identities using basic trigonometric identities	3.16	3.00	3.05	3.06	2.96	3.03	3.11	3.13	3.03	3.01
54 Solve trigonometric equations and inequalities	3.32	3.17	3.17	3.23	3.21	3.25	3.22	3.22	3.00	3.18
55 Identify point in polar coordinate system from rectangular coordinate system	2.90	2.90	2.81	2.89	2.88	2.92	2.81	2.98	2.58	2.83
56 Find trigonometric form of complex numbers and apply DeMoivre's Theorem	2.62	2.58	2.44	2.59	2.55	2.58	2.59	2.63	2.42	2.53
FUNCTIONS AND THEIR GRAPHS										
57 Understand function notation and work with algebraic definition of a function	3.75	3.70	3.58	3.73	3.67	3.72	3.72	3.76	3.53	3.70
58 Given a graph, select the most appropriate equation for the graph	3.63	3.40	3.47	3.49	3.48	3.47	3.43	3.60	3.47	3.44
59 Given an equation, graph it	3.80	3.68	3.74	3.72	3.68	3.75	3.67	3.81	3.82	3.71
60 Use the definition of a function as a mapping	3.08	2.88	2.93	2.95	2.98	2.93	2.96	2.97	2.82	2.91
61 Find the domain and/or range of a function	3.66	3.54	3.58	3.58	3.54	3.60	3.59	3.62	3.38	3.57
62 Use properties of algebraic, trigonometric, logarithmic, and exponential functions	3.24	3.17	3.16	3.20	3.21	3.15	3.21	3.24	2.94	3.13
63 Find inverse of a one-to-one function in simple cases	3.26	3.21	3.00	3.24	3.22	3.26	3.14	3.30	3.00	3.11
64 Sketch graph of a linear, step, absolute value, or quadratic function	3.43	3.34	3.33	3.38	3.39	3.37	3.41	3.33	3.30	3.28
PROBABILITY AND STATISTICS										
65 Organize data into a presentation that is appropriate for solving a problem	3.44	3.35	3.37	3.38	3.37	3.41	3.36	3.38	3.59	3.21
66 Solve probability problems involving finite sample spaces	3.31	3.29	3.19	3.30	3.30	3.33	3.19	3.36	3.35	3.14
67 Solve probability problems using counting techniques	3.36	3.23	3.36	3.27	3.25	3.32	3.27	3.26	3.44	3.17
68 Solve probability problems involving independent trials	3.31	3.24	3.28	3.26	3.28	3.28	3.21	3.29	3.56	3.10
69 Solve problems using the binomial distribution	2.82	2.77	2.68	2.79	2.85	2.82	2.77	2.73	2.70	2.63
70 Solve problems involving joint probability	2.72	2.63	2.78	2.64	2.67	2.74	2.61	2.63	2.94	2.56
71 Find and interpret common measures of central tendency	3.38	3.37	3.31	3.37	3.44	3.40	3.34	3.30	3.39	3.22

		Sex		Race/ Ethnicity		Geographic Region				Teaching Experience (teachers only)	
		F	M	POC	W	NE	C	S	FW	0-5	6+
72	Find and interpret common measures of dispersion	2.90	2.98	2.90	2.95	3.06	2.98	2.88	2.90	3.03	2.68
73	Calculate the expected value of a function of a discrete random variable	2.29	2.29	2.34	2.28	2.32	2.31	2.28	2.25	2.39	2.14
74	Model an applied problem using mathematical expectation	2.61	2.62	2.68	2.60	2.59	2.58	2.62	2.68	2.67	2.47
75	Solve problems using the normal distribution	2.86	2.85	2.68	2.86	2.97	2.90	2.82	2.73	3.15	2.60
76	Explain the consequences of the Central Limit Theorem	2.35	2.36	2.31	2.34	2.44	2.43	2.28	2.27	2.23	2.12
77	Use the Central Limit Theorem to calculate probabilities	2.25	2.26	2.17	2.25	2.28	2.39	2.19	2.16	2.10	2.04
78	Solve problems using the uniform and chi-square distributions	2.09	2.03	2.02	2.04	2.15	2.10	1.92	2.06	2.13	1.90
79	Solve continuous probability problems with random variables, etc.	2.04	1.97	2.07	1.98	2.07	2.04	1.93	1.93	2.16	1.83
80	Solve continuous probability problems with joint and conditional probability	2.10	2.04	2.15	2.05	2.11	2.12	2.00	2.03	2.25	1.97
81	Solve expected value problems for continuous random variables	1.97	1.93	1.95	1.93	1.96	1.98	1.91	1.94	2.09	1.85
82	Develop test to accept or reject a given null hypothesis	2.04	2.13	2.05	2.08	2.17	2.13	2.02	2.05	2.13	1.81
83	Discuss sample size, significance level, power, type I, II error relationships	2.02	1.89	1.95	1.93	1.95	2.02	1.87	1.91	1.94	1.73
ANALYTIC GEOMETRY											
84	Determine the equations of lines and planes given appropriate information	3.47	3.50	3.45	3.49	3.48	3.48	3.47	3.52	3.44	3.41
85	Make calculations in 2-space or 3-space	3.44	3.39	3.43	3.40	3.42	3.40	3.40	3.40	3.38	3.34
86	Given a geometric definition of a conic section, derive the equation	3.04	2.91	3.00	2.95	2.99	2.99	2.90	2.95	2.88	2.91
87	Determine which conic section is represented by a given equation with restrictions	3.13	3.05	3.02	3.08	3.11	3.09	3.04	3.09	2.88	3.05
88	Determine which conic section is represented by a given equation without restrictions	2.76	2.54	2.74	2.60	2.52	2.66	2.58	2.71	2.64	2.60
CALCULUS											
89	Discuss informally what it means for a function to have a limit at a point	3.51	3.45	3.45	3.47	3.52	3.44	3.46	3.47	3.24	3.34
90	Calculate limits of functions or determine that the limit does not exist	3.37	3.34	3.26	3.35	3.40	3.32	3.33	3.33	3.19	3.23
91	Prove via epsilon-delta that the limit of a function equals the calculated value	2.59	2.34	2.44	2.41	2.40	2.42	2.41	2.51	2.31	2.42
92	Solve problems using the properties of limits	3.05	2.93	2.88	2.97	2.95	3.05	2.93	2.95	2.72	2.89
93	Use limits to show that a particular function is continuous	3.09	2.86	2.95	2.94	2.99	2.97	2.93	2.88	2.75	2.84
94	Use L'Hopital's rule, where applicable, to calculate limits of functions	2.79	2.67	2.64	2.72	2.75	2.70	2.73	2.67	2.53	2.64
95	Relate the derivative of a function to a limit or to the slope of a curve	3.35	3.41	3.38	3.38	3.48	3.34	3.41	3.29	2.97	3.27

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		F	M	POC	W	NE	C	S	FW	0-5	6+
96	Explain conditions under which a continuous function does not have a derivative	2.97	2.89	3.00	2.91	2.96	2.88	2.93	2.92	2.58	2.83
97	Differentiate expressions and functions using sum, product, quotient, and chain rules	3.15	3.30	3.37	3.22	3.35	3.19	3.29	3.14	2.97	3.13
98	Use implicit c rentiation	2.95	2.98	3.14	2.94	3.12	2.86	2.95	2.96	2.67	2.88
99	Make numerical approximations of derivatives and integrals	2.83	2.76	2.79	2.78	2.81	2.79	2.73	2.83	2.58	2.69
100	Use differential calculus to analyze the behavior of a function	3.15	3.30	3.37	3.22	3.39	3.16	3.22	3.22	3.03	3.11
101	Use differential calculus to solve problems involving related rates and rates of change	3.10	3.18	3.23	3.13	3.30	3.11	3.10	3.09	3.00	3.00
102	Approximate the roots of a function	2.65	2.52	2.63	2.55	2.60	2.60	2.45	2.63	2.48	2.47
103	Use differential calculus to solve applied minima-maxima problems	3.10	3.19	3.26	3.14	3.29	3.11	3.10	3.14	2.84	3.06
104	Solve problems using the Mean Value Theorem of differential calculus	2.79	2.64	2.79	2.68	2.89	2.60	2.63	2.71	2.43	2.67
105	Explain significance of, and solve problems using, Fundamental Theorem of Calculus	3.05	3.06	3.09	3.04	3.12	3.04	3.04	3.02	2.93	2.96
106	Demonstrate understanding of integration as finding areas of regions through limits	3.15	3.27	3.16	3.22	3.35	3.16	3.17	3.23	2.68	3.04
107	Integrate functions using algebraic substitutions	2.95	2.91	3.17	2.89	3.01	2.90	2.88	2.91	2.87	2.80
108	Integrate functions using "integration by parts"	2.85	2.79	3.00	2.78	2.92	2.75	2.76	2.83	2.65	2.69
109	Integrate functions using trigonometric substitutions	2.78	2.61	2.88	2.64	2.79	2.61	2.59	2.72	2.68	2.60
110	Integrate logarithmic and exponential functions	2.87	2.86	3.09	2.83	3.02	2.81	2.78	2.86	2.68	2.74
111	Evaluate improper integrals	2.64	2.59	2.79	2.58	2.71	2.56	2.55	2.61	2.45	2.50
112	Use integral calculus to calculate area of regions in the plane	3.05	3.15	3.17	3.10	3.23	3.07	3.10	3.06	2.87	2.97
113	Calculate volume of solids formed by rotating plane figures about a line	2.89	2.90	2.95	2.88	2.96	2.86	2.86	2.90	2.83	2.78
114	Interchange the order of integration in double integrals	2.19	2.12	2.24	2.13	2.18	2.22	2.01	2.19	2.07	2.00
115	Find volumes of solids using double integrals	2.24	2.20	2.31	2.19	2.15	2.25	2.09	2.37	2.20	2.06
116	Determine limits of sequences and simple infinite series	2.75	2.75	2.60	2.75	2.83	2.76	2.63	2.78	2.43	2.61
117	Use tests to show convergence or divergence of series	2.55	2.45	2.51	2.47	2.52	2.46	2.42	2.56	2.17	2.34
118	Determine the interval of convergence of a power series	2.39	2.25	2.42	2.28	2.33	2.29	2.18	2.42	2.10	2.19
119	Determine Taylor series of functions like sin x, cos x, e^x, and ln x	2.28	2.29	2.51	2.26	2.33	2.36	2.14	2.35	2.10	2.18

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		F	M	POC	W	NE	C	S	FW	0-5	6+
DISCRETE MATHEMATICS											
120	Use basic terminology and, given the definitions, use the symbols of logic	3.26	3.03	3.19	3.11	3.08	3.10	3.13	3.16	3.03	2.99
121	Solve problems of union and intersection of sets, subsets, and disjoint sets	3.37	3.22	3.26	3.28	3.28	3.30	3.27	3.26	3.23	3.16
122	Use truth tables to verify statements	3.16	2.98	3.05	3.05	3.11	3.04	2.95	3.13	3.10	2.92
123	Use Laws of Algebra of Propositions to evaluate equivalence	2.63	2.53	2.58	2.55	2.56	2.53	2.55	2.63	2.52	2.38
124	Solve basic problems involving permutations and combinations	3.13	3.16	3.07	3.15	3.27	3.13	3.07	3.14	3.00	3.00
125	Use Euclidean Algorithm to find greatest common divisor of two numbers	2.84	2.71	2.79	2.75	2.70	2.75	2.75	2.82	2.55	2.57
126	Work with numbers expressed in bases other than base ten	2.74	2.49	2.57	2.58	2.58	2.56	2.51	2.70	2.90	2.36
127	Solve modular congruences	2.38	2.23	2.31	2.28	2.31	2.28	2.23	2.35	2.16	2.05
128	Prove theorems using modular systems	2.01	1.87	2.02	1.90	1.91	1.90	.85	2.05	1.94	1.71
129	Find values of functions defined recursively	2.45	2.53	2.50	2.49	2.49	2.51	2.40	2.60	2.03	2.28
30	"Translate" between recursive and closed form expressions for a function	2.21	2.22	2.24	2.20	2.20	2.19	2.10	2.37	1.93	2.00
131	Determine if a binary relation is reflexive, symmetric, antisymmetric, transitive	2.53	2.42	2.48	2.45	2.51	2.41	2.48	2.45	2.17	2.19
132	Determine if a binary relation is an equivalence relation	2.55	2.52	2.53	2.52	2.55	2.48	2.57	2.51	2.14	2.23
133	Determine if set is ordered with respect to a binary relation	2.38	2.33	2.42	2.34	2.34	2.31	2.39	2.35	1.90	2.11
134	Prove countability of rational numbers and non-countability of real numbers	2.16	1.95	2.05	2.01	2.19	1.88	1.88	2.19	1.83	1.88
135	Use basic terminology in graph theory	2.43	2.31	2.52	2.33	2.40	2.27	2.33	2.44	2.10	2.16
136	Identify conditions under which a graph can be traversed	2.06	1.99	2.07	1.99	2.08	1.99	1.91	2.08	1.64	1.83
137	Know and use the three basic ways of traversing a binary tree	1.94	1.86	2.07	1.86	1.92	1.87	1.81	1.97	1.62	1.81
138	Solve simple linear programming problems	2.69	2.63	2.58	2.64	2.68	2.65	2.61	2.65	2.34	2.48
139	Find and use finite differences of a function	2.18	2.22	2.31	2.18	2.27	2.11	2.13	2.31	1.86	2.04
140	Interpolate using Newton's forward and backward difference formulas	1.78	1.77	2.00	1.74	1.78	1.77	1.63	1.93	1.64	1.70
ABSTRACT ALGEBRA											
141	Determine if a set together with an operation is a group	2.43	2.47	2.40	2.45	2.41	2.44	2.46	2.51	1.87	2.17
142	Use definition of a group to deduce elementary properties of a group	2.38	2.35	2.36	2.35	2.31	2.38	2.33	2.42	1.84	2.10
143	Determine if a set together with two operations is a ring	2.23	2.20	2.21	2.20	2.11	2.23	2.15	2.34	1.77	1.94

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		F	M	POC	W	NE	C	S	FW	0-5	6+
144	Determine if a set together with two operations is a field	2.37	2.41	2.33	2.37	2.36	2.34	2.39	2.47	1.77	2.14
145	Use definition of a field to deduce elementary properties	2.33	2.29	2.36	2.28	2.28	2.30	2.28	2.37	1.74	2.07
146	Determine whether mathematical systems are groups, rings, or fields	2.29	2.32	2.29	2.30	2.25	2.31	2.26	2.42	1.81	2.05
147	Determine if subset of a group is subgroup or normal subgroup	2.04	1.90	2.17	1.90	1.98	1.94	1.80	2.10	1.74	1.76
LINEAR ALGEBRA											
	In a finite dimensional real vector space:										
148	determine if a subset is a subspace	2.07	2.05	2.19	2.03	2.02	2.10	1.99	2.11	1.80	1.76
149	determine if a set of vectors is linearly independent	2.19	2.22	2.36	2.18	2.21	2.20	2.13	2.30	1.97	1.91
150	determine if a set of vectors is basis for vector space	2.16	2.15	2.28	2.13	2.15	2.16	2.06	2.26	1.73	1.92
151	determine the dimension of the span of a set of vectors	2.02	1.99	2.07	1.98	1.97	2.04	1.93	2.06	1.60	1.81
152	determine if vectors are orthogonal using the dot product	2.19	2.22	2.21	2.20	2.25	2.18	2.15	2.27	2.03	1.93
153	determine effects of linear transformation on a vector space	2.08	1.99	2.17	2.00	2.07	1.96	1.93	2.14	1.80	1.83
154	determine image and kernel of a linear transformation	1.87	1.76	1.98	1.77	1.80	1.82	1.73	1.86	1.67	1.61
155	Add, subtract, and scalar multiply vectors using geometric interpretations	2.74	2.89	2.95	2.82	2.86	2.76	2.86	2.85	2.43	2.64
156	Scalar multiply, add, subtract, and multiply matrices	2.80	2.91	2.93	2.85	2.88	2.76	2.94	2.88	2.68	2.65
157	Demonstrate understanding and use basic properties of inverses	2.68	2.70	2.67	2.69	2.68	2.63	2.72	2.74	2.52	2.49
158	Determine and apply matrix representation of a linear transformation	2.53	2.40	2.58	2.43	2.52	2.39	2.37	2.54	2.33	2.27
159	Use matrix techniques to solve systems of linear equations	2.80	2.82	2.72	2.81	2.85	2.77	2.79	2.84	2.74	2.63
COMPUTER SCIENCE											
160	Demonstrate an understanding of the roles of hardware and software components	3.33	3.17	3.05	3.24	3.24	3.29	3.20	3.18	3.22	3.16
161	Know basic computer terminology	3.37	3.09	3.12	3.20	3.23	3.23	3.19	3.14	3.25	3.10
162	Use "user-friendly" software	3.48	3.35	3.23	3.42	3.37	3.45	3.41	3.38	3.50	3.33
163	Develop computer algorithms to solve mathematical problems	2.83	2.72	2.79	2.76	2.72	2.93	2.64	2.77	2.88	2.67
164	Trace and debug existing computer algorithms	2.42	2.33	2.29	2.38	2.34	2.47	2.33	2.34	2.44	2.25
165	Program in two computer languages, one of which is a structured language	2.06	2.15	2.14	2.11	2.05	2.22	2.03	2.15	2.13	2.00

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		F	M	POC	W	NE	C	S	FW	0-5	6+
MATHEMATICAL REASONING AND MODELING											
166	Develop mathematical model of physical situation	3.28	3.11	3.26	3.16	3.12	3.09	3.11	3.41	2.97	3.07
167	Determine mathematical strategies to solve a problem	3.48	3.43	3.48	3.44	3.45	3.36	3.36	3.62	3.22	3.35
168	Recognize reasonableness of results given the context of a problem	3.67	3.58	3.40	3.63	3.62	3.57	3.55	3.70	3.34	3.51
169	Using estimation, test the reasonableness of results	3.70	3.54	3.41	3.61	3.62	3.61	3.54	3.61	3.50	3.53
170	Estimate actual and relative error in numerical answer	2.82	2.54	2.83	2.63	2.62	2.60	2.61	2.75	2.78	2.48
171	Having solved a problem, reconsider the strategies used	3.25	3.09	3.19	3.15	3.12	3.15	3.05	3.28	3.00	3.00
172	Communicate results in an appropriate form	3.63	3.49	3.55	3.55	3.64	3.45	3.54	3.58	3.69	3.42
173	Determine whether an isomorphism exists between two mathematical systems	2.45	2.23	2.41	2.29	2.29	2.35	2.23	2.40	2.19	2.17
174	Determine if one mathematical model will describe two different situations	2.68	2.51	2.62	2.56	2.60	2.51	2.45	2.75	2.52	2.41
175	Understand the different levels of mathematical impossibility	2.70	2.72	2.45	2.72	2.76	2.71	2.49	2.93	2.48	2.50
176	Use the axiomatic method in modeling and problem solving	2.63	2.48	2.41	2.53	2.51	2.58	2.34	2.70	2.13	2.39
CONTENT-SPECIFIC PEDAGOGY											
177	Evaluate a given scope and sequence of mathematics topics	3.36	3.12	3.36	3.19	3.20	3.20	3.26	3.19	3.22	3.13
178	Develop scope and sequence of a mathematics topic and justify it	3.17	2.98	3.10	3.04	3.10	3.04	3.11	2.97	3.28	2.90
179	Identify misconception in a student's work and suggest methods for correcting	3.69	3.49	3.40	3.58	3.52	3.55	3.57	3.59	3.56	3.47
180	Identify students' prerequisite knowledge and skills for a topic	3.67	3.50	3.60	3.55	3.55	3.51	3.63	3.56	3.64	3.48
181	Develop questions to ask students that display their level of understanding	3.67	3.44	3.50	3.53	3.58	3.41	3.55	3.58	3.55	3.44
182	Given a problen., identify several problem-solving strategies	3.69	3.53	3.48	3.60	3.58	3.56	3.59	3.63	3.55	3.51
183	Use appropriate forms of representation	3.65	3.50	3.50	3.56	3.48	3.56	3.57	3.62	3.70	3.45
184	Use variety of appropriate teaching strategies	3.72	3.49	3.60	3.57	3.55	3.55	3.58	3.65	3.73	3.46
185	Integrate concepts to show relationships among topics	3.58	3.34	3.46	3.43	3.46	3.38	3.43	3.48	3.58	3.36
186	Relate mathematical concepts and ideas to real-world situations	3.70	3.46	3.57	3.55	3.53	3.50	3.55	3.64	3.82	3.49
187	Identify, evaluate, and use curricular materials and resources	3.54	3.24	3.49	3.34	3.26	3.31	3.40	3.46	3.45	3.28

		Sex		Race/ Ethnicity		Geographic Region				Teaching Experience (teachers only)	
		F	M	POC	W	NE	C	S	FW	0-5	6+
188	Know procedures to control social atmosphere without restricting divergent thought	3.66	3.27	3.58	3.41	3.35	3.43	3.40	3.52	3.73	3.42
189	Know how society is affected by level of mathematics knowledge	3.30	2.90	3.26	3.03	2.88	3.00	3.19	3.13	3.31	3.00
190	Use information about different groups to enhance learning	3.18	2.83	3.14	2.95	2.97	2.92	2.99	2.99	3.15	2.87
191	Evaluate impact of learning theorists on mathematics education	2.38	2.12	2.30	2.20	2.15	2.31	2.08	2.33	2.34	2.09
192	Identify, evaluate, and use appropriate evaluation strategies	3.61	3.32	3.58	3.41	3.38	3.32	3.47	3.55	3.64	3.38
193	Write evaluation items to test for a specific mathematical skill	3.69	3.43	3.56	3.52	3.53	3.43	3.56	3.61	3.70	3.50

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