DOCUMENT RESUME

ED 390 848 SP 036 411

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TITLE Helping Preservice Teachers Confront Their Conceptions about Mathematics and Mathematics Teaching and Learning.

PUB DATE Apr 94

NOTE 37p.; Paper presented at the Annual Meeting of the American Educational Research Association (New Orleans, LA, April 1994).

PUB TYPE Speeches/Conference Papers (150) -- Reports - Research/Technical (143)

EDRS PRICE MF01/PC02 Plus Postage.

DESCRIPTORS *Attitude Change; *Constructivism (Learning); Elementary Education; Higher Education; Instructional Effectiveness; *Mathematics Education; *Methods Courses; Preservice Teacher Education; Qualitative Research; *Student Attitudes; *Teaching Methods

IDENTIFIERS Pedagogical Content Knowledge; *Preservice Teachers

ABSTRACT

This study explored how implementing a constructivist approach in a mathematics methods class might change the prospective teachers' conceptions about mathematics and mathematics teaching and learning. The study used qualitative measures: interviews, classroom observation, review of written assignments, classroom interaction, and journal of student responses for 5 randomly selected students from the class of 19. In addition, the study administered the Mathematics Beliefs Scales (MBS) at the beginning and end of the course. The course's major components were mathematical inquiry and investigation through problem solving in cooperative groups and whole-class discussions, reading assignments, problem assignments, student assessment interviews, constructivist teaching plans, creating alternate algorithms, final exam, and math logs. Qualitative data results indicated that cooperative groups and use of manipulatives contributed significantly to challenging the preservice teachers' conceptions. By the end of the course nearly all students had begun to talk differently about their own learning of mathematics. For the first time they understood the meanings of rules and procedures, were willing to take risks and defend their own solutions to problems, and they had a different image of teaching mathematics. The MBS results supported this finding. (Contains 24 references.) (JB)

Helping Preservice Teachers Confront Their Conceptions about Mathematics and Mathematics Teaching and Learning

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Paper presented at the Annual Meeting of the American Educational Research Association, New Orleans, April, 1994.

Draft of an article submitted for publication

Introduction

In order for students to become productive citizens in the twenty-first century, they must learn to communicate and reason mathematically. Historically, mathematics instructors have emphasized procedures. This type of learning has been characterized as drill and practice where learners take passive roles. Educators involved in the current reform movement in mathematics education recommend that students be actively involved in constructing their own knowledge and developing powerful mathematical concepts as they explain, explore, reason, and justify solution strategies (NCTM, 1989). For many teachers, however, this approach to mathematics teaching requires a change in their conceptions about the nature of mathematics and about what it means to learn and teach mathematics (Brown, Cooney, & Jones, 1990). If the teaching of mathematics is to change, then teachers' conceptions about mathematics and mathematics teaching and learning must change first.

During the past few years, I have taught inservice and preservice elementary teachers in college courses. I communicate to them the importance of teaching mathematics with approaches that promote the idea that students should understand mathematics. Understanding mathematics as explained by Skemp (1978) is "relational understanding." Skemp (1978) defines relational understanding as knowledge that enables the student to construct several plans for solving mathematical tasks. The student conceptually understands the mathematical rules and the reasons for using them. While teaching my students to teach for relational understanding, I encourage them to make sense of mathematics for themselves. The process often proves difficult due to students' previous conceptions of mathematics. My difficulties correspond with those of other mathematics educators who have stated that substantive changes in the teaching of mathematics such as those suggested by The Curriculum and Evaluation Standards for School Mathematics (NCTM, 1989) will be difficult to achieve because of what teachers believe about the nature of mathematics and mathematics teaching and learning (Brown et al., 1990).

Because of their conceptions, many elementary teachers are unsure of how to teach mathematics in ways that help students construct relational understanding in mathematics. Teachers' conceptions are not unchangeable. Teachers develop and refine their conceptions while teaching. Their conceptions continuously change because new experiences cause them to reconsider old ones (Thompson, 1992). Teacher's conceptions about mathematics and mathematics teaching affect their thinking processes and behavior in the mathematics classroom (Ernest, 1989; Thompson, 1984).

Conceptions are composed of two components--beliefs and knowledge. Beliefs are both affective and cognitive. Beliefs are personal views, assumptions, and values (Ernest, 1989). Teachers have particular beliefs about mathematics as a subject, about how students learn mathematics, and about how to teach mathematics. Teachers may believe mathematics to be a rule-oriented subject to be transmitted. They may believe mathematics to be full of meaningful patterns and logic that must be explained so that students can discover its theory and beauty. Or teachers may believe mathematics is a problem-solving process in which mathematics is created by students as they discuss and interact in the classroom (Ernest, 1989).

Teacher knowledge, the second component of conceptions, includes both content knowledge and knowledge about teaching. Content knowledge has two components, knowledge of the basic concepts and principles of mathematics rules and facts and knowledge of the ways in which mathematical truth is established (Shulman, 1986). The content knowledge or what teachers know about mathematics affects how they teach mathematics. Knowledge about teaching is what Shulman (1986) identifies as pedagogical content knowledge. This pedagogical content knowledge enables the teacher to choose the tasks, problems, representations, and explanations that help students understand mathematics. Part of pedagogical content knowledge is the knowledge of how children learn specific content. Teacher knowledge affects teachers' instructional decisions and classroom practice.

In the spring of 1993, I had the opportunity to teach an elementary mathematics methods course, Teaching Mathematics in the Elementary School, for prospective elementary teachers. The goal of the course, as outlined in the college catalog, is to direct experiences emphasizing instructional strategies, selecting instructional materials, sequencing student activities, and diagnosing student progress. My purposes went further than this goal. I wanted these prospective teachers to learn to think mathematically and understand the nature of mathematics. I wanted to provide them with much the same problem-solving experiences that their own elementary school students would experience. Through this experience, I wanted them to adapt and alter their present knowledge of mathematics in order to construct new knowledge. The position that the learner constructs new knowledge through posing questions, investigating, predicting, and manipulating objects is a constructivist approach to learning (Fosnot, 1989). Through constructivist learning and my modeling constructivist teaching, I wanted these students to first learn mathematics by actively constructing their knowledge of mathematical concepts and then to learn to teach mathematics using the constructivist approach. To do this, I created a classroom environment where students explored, questioned, reasoned, and justified mathematics as they studied mathematics content.

I knew from the beginning that helping students develop alternative images for teaching would be difficult because they lacked experience apart from the "teaching is telling" paradigm. My previous experiences with teaching prospective teachers had impressed upon me that most of the students would have experienced learning mathematics through the behaviorist approach of teaching where teachers transmit the knowledge to be learned primarily through drill and practice and memorization. I knew that if I wanted my students to develop a constructivist understanding of learning and teaching mathematics to replace this transmission approach, I would have to show them other teaching and learning approaches. The framework which I used to design the instructional program for my course was a combination of Cognitively Guided Instruction developed at the University of Wisconsin (Carpenter, T. P., & Fennema, E., 1991) and Professional Standards for Teaching Mathematics (NCTM, 1991). The most important goal within CGI is helping teachers understand children's thinking about mathematics. The Standards emphasize the teacher's role of creating a classroom environment in which students learn to think mathematically.

In this article, I describe the major course activities and represent by examples the methods and problems I used to challenge prospective teachers' conceptions about mathematics. I discuss the data through written essay responses and oral answers to individual interviews. I also present the quantitative results from an analysis of variance on a Mathematics Beliefs Scales (Fennema, E., Carpenter, T. P., & Peterson, P. L., 1987) questionnaire.

The main questions that guided my research are, Could I through modeling the constructivist teaching affect a change in these students' conceptions about mathematics? At what point during this course did their conceptions about mathematics and mathematics learning and teaching begin to change? What are the conceptual changes?

Methodology

My perspective was one of "practitioner researcher" (Liston & Zeichner, 1991). These authors refer to practitioner research as "inquiries that are conducted into one's own practice in teaching or teacher education" (p. 147). I wanted to implement a teaching instructional approach that would give prospective teachers the opportunity to learn how to teach mathematics that focuses on how students construct knowledge. I wanted to find out whether exposing these students to alternative models of teaching could change their conceptions about mathematics and mathematics teaching and learning. Liston and Zeichner (1991) state that there are growing numbers of self-studies by teacher educators. Schools like Michigan State University and University of Florida have made "efforts to use their teacher education programs as laboratories for the study of teacher education" (Liston & Zeichner, 1991, p. 152). Ross (1987) states that by conducting this type of research a teacher acquires additional knowledge about content.

My main research approach was the ethnographic approach of participant observation, interviewing, and collecting artifacts. Since validity and reliability are of central concerns in research, some of the steps I took to ensure the validity and reliability of my research were collecting data over a long period of time, using several methods of data collection (both qualitative and quantitative), discussing findings with classroom participants, and keeping bias under control. The quantitative data that I collected is a result of pre and posttest administrations of the Mathematics Beliefs Scales (Fennema et al., 1987) questionnaire. I analyzed this data using the analysis of variance procedure on the pre and posttest means.

During their first class, I asked all students to complete the Mathematics Beliefs Scales (Fennema et al., 1987) questionnaire. The questionnaire focused around two main points: 1) how do you believe children learn mathematics, and 2) how should teachers assess mathematics understanding? In an effort to allow each student the opportunity to express personal ideas about how one should teach mathematics, I also asked my students to write on the question "What does it mean to teach mathematics in the elementary schools?"

Before the class began, I had randomly selected five students from the class roster that I would follow closely. I would interview them at the beginning and the end of the course, as well as observe them closely during class. I paid close attention to their written assignments and classroom interaction. In addition, I kept a journal about their responses to classroom activities.

Before I interviewed them I verified that the students were in field placements in schools. I wanted to be sure that they had opportunities to implement some of the ideas they would be learning in the course. My interviews with these five students covered not only their responses on the questionnaire, but other questions such as, What is mathematics? What does it mean to know mathematics? How is mathematics learned?

Thirteen weeks later, at the end of the course, I asked students to write in their math logs answers to the following questions: "How have your ideas about teaching changed since the course began?" "What was the turning point for you toward a change in your conceptions about mathematics and mathematics teaching and learning?" "What are your goals for your students in learning mathematics?" "What is the role that you will play in the mathematics classroom?" During the last class, I again asked students to respond to the Mathematics Beliefs Scales (Fennema et al., 1987) questionnaire in order to evaluate quantitatively if there was a significant change in their beliefs about mathematics and mathematics teaching and learning.

Each night students worked in cooperative groups exploring solutions to problems. One student from each group kept a record of the group's discourse and interaction. I collected these notes. In addition, I walked around from group to group taking my own notes. Later, I used both sets of notes to assess the students and to help me in my research in understanding and changing prospective teachers' conceptions about mathematics teaching and learning. Other sources of data that I collected were written work completed by the students (i.e. assignments, quizzes, tests) and audio-tapes of interviews.

I analyzed the data following the procedures described by Spradley (1980) as the Developmental Research Sequence. This ethnographic research model is a cyclic process of questioning, collecting data, recording data, and analyzing data. While I conducted my research I repeated the sequence of questioning, collecting, recording, comparing, contrasting, and analyzing. Data analysis was an integral part of the research cycle.

Overview of the Course

A total of 19 prospective teachers completed this particular section of Teaching Mathematics in the Elementary School at the University of Florida. The students consisted of prospective elementary and middle-school teachers with specializations in special education, language arts, social studies, computer technology, and mathematics. Each class met weekly for one 4-hour period.

The course's major components were mathematical inquiry and investigation through problem solving in cooperative groups and whole-class discussions, reading assignments, problem assignments, student assessment interview, constructivist teaching plan (actually taught and assessed by the individual student), creating alternate algorithms different from conventional procedures (such as invert and multiply for the division of fraction algorithm), final exam, and math logs. Occasionally, we would view videotapes of constructivist teaching in elementary classrooms. I chose activities that I thought would help convince them that teaching for mathematical understanding is possible and justifiable.

Each class session began with one student giving an oral presentation of the research reading for that week. This student also provided a personal reaction to the reading. All students were urged to react to or reflect on the reading assignment, the previous class, and/or the homework problems. We worked with manipulatives while reviewing concepts which the students had encountered during their homework assignments. We discussed how to choose the best manipulatives to represent and to promote students' understanding of the various mathematics concepts. This discussion usually lasted about three-fourths of an hour. The balance of the class time was spent on mathematical problem solving. During the last half hour, the groups shared their discoveries and solutions with the whole class. Homework assignments usually included problems that related to that night's activities, reading assignments, and writing in their math logs.

I knew that I could not conceivably cover in one course every concept of mathematics that these students would need to understand in order for them to teach the elementary or middle school curriculum. Therefore, I chose topics that would cause them to think about the mathematics and were representative of the curriculum with which they would be working. I selected topics for investigation from those that they would be teaching in elementary and middle school: place value, number theory, exponents, fractions, and geometry. The reading assignments addressed constructivist learning (Carpenter, T. P., & Fennema, E., 1991; Kamii & Lewis, 1990; NCTM, 1989; Yackel, Cobb, Wood, Wheatley, & Merkel, 1990). constructivist teaching (Ball, 1988; Lampert, 1990, 1989; NCTM, 1991; van de Walle, 1990), mathematics beliefs (Ernest, 1989). I selected assignments that would help them understand the research behind the reform movement in mathematics that is advocated by many educators and teachers. The students wrote in math logs about what they had learned, about the ideas and concepts that were difficult for them, and about emotions and reactions to the problem-solving activities and assignments. I collected the logs and responded to them in writing or by meeting with the students individually.

Researcher's Focus

The three questions that I wanted answered were--Could I through modeling constructivist teaching affect a change in these students' conceptions about mathematics? At what point during this course did their conceptions about mathematics and mathematics learning and teaching begin to change? What were the conceptual changes? I was further interested in knowing what components of my instructional program had influenced a change in their conceptions of mathematics and mathematics teaching and learning.

It is my view that prospective teachers should become involved in the process of learning mathematics while learning to teach the content. While students explain their solutions to problems and make curriculum decisions about teaching certain concepts, students can learn to make worthwhile use of their mathematical content knowledge. It is the responsibility of teacher educators to help prospective teachers understand the mathematics content while also learning how to teach it. Many prospective teachers will actually relearn the content by observing, inventing, verifying, and reconsidering hypotheses during class discussions. Some of the preservice teachers in my class found that they were relearning the content. This is illustrated by their comments below:

"I am excited about my new found mathematics. . . ."

"It occurred to me that I really never knew why I 'borrowed' when subtracting."

"I knew the algorithm forward and backwards, but the algorithm didn't mean anything to me [until now]."

Mathematical problem solving was the true focus of my instructional program. I chose problems that were mathematically rich enough for the students to be challenged and yet basic enough for them to apply in their own teaching in elementary schools. I chose problems that would create enough cognitive conflict to motivate the students to find solutions. The problems that I chose provided an environment for them to learn to model their own thinking through using manipulatives and drawing pictures and diagrams. More importantly, I chose problems that would help them to reflect on their conceptions about the nature of mathematics and develop an understanding of mathematics equivalent to Skemp's (1978) "relational understanding" With relational understanding the students would be able to link the concepts with the mathematical rules and procedures. I hoped that during the problem solving the students would discover patterns and form connections among mathematical ideas. Within the cooperative groups and, afterwards, during whole-group discussion, students were asked to speculate about solutions, agree or disagree about proofs, and communicate with each other to understand the mathematical situations. With this process of negotiation, we reasoned together to reach agreement on important concepts and the connection to procedures. Through negotiating, we created knowledge in ways that mathematical truths are created in the discipline of mathematics (Lampert, 1990). I gave up my role as the teacher who determines when answers are correct, so that students could engage in a mathematical discourse that would allow them to determine mathematical truth for themselves. As a guide, I led them to conjecture, agree, and disagree about evidence for proof of problem solutions. Each student was constructing personal knowledge while sharing in the class discourse. Often we spent an hour discussing one mathematics problem. By focusing on one problem we could look in detail at where the concept might have originated and where it could go in the future. We pulled together the alternative solutions and integrated the process of learning.

Mathematical Thinking in Progress

On the first day of class I made it clear that this mathematics methods course would not follow the traditional format. I saw my role as one of "guide." I wanted to guide the students in their journey in thinking new ways about mathematics. I used representations that were familiar to form a foundation for them to construct knowledge upon present understandings. I worked with the students to help them see and overcome potential difficulties in their abilities to construct new understanding. I wanted them to become dissatisfied with their present conceptions of mathematics and mathematics teaching and learning. However, before I could help them construct the conceptual knowledge that I knew they lacked and persuade them to try alternative approaches, they needed to encounter conflict and uncertainty in their conceptions. I wanted them to probe their understandings to help them find the limits of their knowledge and to discover that they needed to change their conceptions. Immediately, I began to guide them in confronting their conceptions.

The first day I gave them a division of fraction problem to solve (Ball 1991).

> When you will be teaching you may often need to devise different real-world situations and applications of content to help your students develop different concepts. You will find that this strategy can be challenging. An example of this type of situation is the following problem. Can you tell me a good situation or story for using 1 3/4 divided by 1/2?

I gave them manipulatives, pattern blocks, fraction squares, fraction circles, and paper to use, if they desired. They worked in cooperative groups where they had to agree on a group solution that would later be shared with the entire class.

One purpose of this problem situation was to help the preservice teachers become aware of different problem-solving strategies that students use. I wanted them to see that children, like they, could have different thinking patterns. I also wanted them to become aware of their lack of knowledge in a basic concept (division of fractions). I hoped to challenge them to begin to view mathematics in a new and different way. While trying to understand how to represent this problem, they had to focus their attention on the mathematical content. If they finished solving the problem and became satisfied with their answer, they were to discuss as a whole group how children might begin to solve this problem.

Every group, in the beginning, used the traditional algorithm to verify the correct answer first: change the first fraction to an improper fraction, invert the second fraction, multiply the numerators and denominators together, and then simplify.

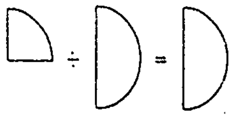
$$1\ 3/4 \div 1/2 = 7/4 \div 1/2 = 7/4 \times 2/1 = 14/4 = 7/2 = 3\ 1/2$$

However, I reminded them that the task was to draw a representation or develop a real-life situation that illustrated the concept of dividing by 1/2. During the next hour every group approached the problem by dividing by 2 instead of 1/2.

$$1\ 3/4 \div 2 = 7/4 \div 2 = 7/4 \times 1/2 = 7/8$$

When I reminded each group to compare this answer, 7/8, to the answer they had obtained from the algorithm, they began to question what it meant to divide by 1/2. All groups tried using different manipulatives. As the students worked, I visited with each group to hear their discourse. To facilitate their thinking in new directions I asked questions, such as Tell me what you were thinking when you did this? and How many times will that half fit into your diagram? Finally, every group was satisfied with their approach. We came back to the whole group and discussed their solutions. Before they gave their answers, one student from each group told of the process they had followed from the beginning to the point when they had a satisfactory answer.

One group explained that they had begun with a simpler problem, $1/4 \div 1/2 = 1/4$ which they illustrated with \$.25 divided by \$.50 = \$.50. They used money because they found it difficult to work with fractions. However, the answer didn't make sense to them and didn't agree with the answer from the algorithm. They tried 1/4 of a circle divided by 1/2 of a circle.



They weren't satisfied with this illustration either. Finally, in an attempt to explain the problem using money, they developed an elaborate story:

> Johnny has $.50; Susie has $.25. The odds are 2 : 1 on a football game. Susie says, "Cowboys will win." She and Johnny bet $.25 each because she only has a quarter. However, Johnny has only a $.50 piece. Cowboys won. So Susie says, "We'll divide my quarter by your $.50 piece. I'll give you the quarter and you give me the $.50 piece." Therefore, Susie has $.50 and Johnny has only $.25.

Even though the above story may seem confusing to us, the development of this story was a step in the students process of making sense of the problem. Through this process, they began to see the relationship between the conceptual and procedural knowledge of division of fractions. This process helped them to develop their final story which did provide the answer of 3 1/2.

Another group's approach included comparing 1/2 to 30 minutes. They drew a diagram like this:

30 ------ 1/4) 1/2
30 ------ 1/4

30 ------ 1/4) 1/2
30 ------ 1/4 　　　3 1/2 of the halves

30 ------ 1/4) 1/2
30 ------ 1/4

30 ------ 1/4

Their story problem became

> It takes 30 minutes to eat 1/4 of a piece of pie. How long would it take to eat 1/2 of a pie?

They checked their answer and got 1 hour. This didn't make sense and they tried again several times until they came up with the final solution in the form of a question.

> It takes me 1 hour to eat 1/2 of a pie or 30 minutes to eat 1/4 of a pie. How long will it take to eat 1 3/4 pies?

Another group's final answer was the following situation.

> One and three fourths pizza is left after a party. If I eat 1/2 each day, how many days will it take to eat the leftover pizza?

Their diagram looked like this:

Monday-----1/2
Tuesday----1/2
Wednesday-----1/2
Thursday---------?

Then they decided they needed a more clear explanation

Monday
  breakfast 1/4
  dinner 1/4
Tuesday
  breakfast 1/4
  dinner 1/4
Wednesday
  breakfast 1/4
  dinner 1/4
Thursday
  breakfast 1/4

Other solutions were in the form of questions and stories:

Sarah and Denise are making a cake for Mrs. Steele's birthday. The recipe calls for 1 3/4 cups of sugar. But the only measuring cup they have measures 1/2 cup. How many times will they have to fill up the 1/2 cup to measure out 1 3/4 cups of sugar?

We went to a pizza restaurant and ordered 2 pizzas. The waiter who was hungry ate 1/4 of one of the pizzas. There were only 3 of us so we divided what was left. Two of us each got 1/2 of the first pizza. The third person got the half that hadn't been touched by the waiter from the second pizza. And since the waiter was so hungry, we gave him the remaining 1/4 pizza.

Your mother has bought 1 3/4 feet of ribbon to make Christmas bows for the tree. Each bow requires 1/2 feet of ribbon. How many bows can she make?

Activities like this led the preservice teachers to a growing awareness of the need to attach meaning to such symbols as fractions and such operations as division of fractions. One student whose specialization was mathematics struggled with her own lack of understanding during the experience of dividing two fractions and wrote in her math log:

> It was great beginning the course this way. It's hard to put into words how I feel now my eyes have been opened. 'I was blind but now i see.' I never thought about how to do that type of problem other than the way I was taught in school (flip the 1/2 and multiply). I am excited about my new found mathematics, but I am also severely depressed. I feel as if all my years of school were worthless. I wish I could go back and do it over. Maybe I would then really learn something--and understand it, too!

As an assignment for the next class, the students had to develop an alternative algorithm different from the invert and multiply one used for solving the division of fraction problem : 1 3/4 ÷ 1/2. Again, the groups spent close to an hour negotiating how to complete this assignment. One group treated the fraction like a measurement division problem. They said:

> You want to know how many halves go into the dividend so we found a common denominator and then did repeated subtraction until we couldn't subtract any more.
>
> 1 3/4 = 7/4
> 1/2 = 2/4
>
> 7/4 - 2/4 = 5/4
> 5/4 - 2/4 = 3/4
> 3/4 - 2/4 = 1/4 Stop.
>
> We can't subtract 2/4 anymore. So, we then counted the number of times we had subtracted. We got 3 times. To get our fraction we put the remainder over the original divisor; 1/2 ÷ 1/4 = 2/4 = 1/2. We combined the whole number of 3 and the fraction of 1/2 and got 3 1/2.

Clearly, this algorithm is much longer and involved and harder to learn, but it is based on linking the concepts of subtraction and division. The students were making connections. Seeing relationships will make it easier for them to remember with meaning and form a basis for higher order thinking.

Another group's algorithm was similar:

1/2) 1 3/4

3 1/2
2/4) 7/4
-6/4
1/4

First, we have 1 3/4, we want to see how many halves are in it. It is 3 halves and a half leftover.

As the preservice teachers reflected further in their math logs, they discussed the connection between developing one's own algorithm for working problems that connected the conceptual knowledge with the procedural knowledge that they already possessed. One student wrote:

> Even though the class was hard, I liked being challenged and believe me, I sure was challenged! While we were developing the algorithm, I began to understand how difficult it can be for some students to understand math problems. This is the first time I ever approached a math problem this way. It makes a lot more sense and was much more meaningful. By connecting the procedure to the concept of dividing fractions, I can see a practical use for this problem that I never saw before. I will remember it.

During the course of the semester, we worked other problems that were conceptually rich and encouraged higher order thinking. Some of the problems we solved and discussed were

1) Explain and represent the concept of 7/0.
2) If there were 22 people in the class and everyone shook hands one time with each person, how many hand shakes will there be (Billstein, Libeskind, & Lott, 1989)?
3) 49 X 49. We looked at factors of these numbers, developed such problems as 7^2 X 7^2. Find patterns of squares from 1^2 to 17^2 to develop the exponent rules--power rule, product rule, and quotient rules for exponents (Lampert, 1990).
4) Permutations (Ball, 1988)
5) How do we interpret and explain that a minus times a minus is a plus (example: (-3 X -5))?
6) What concrete representation could we create (example: -8 - (-2))?

These problems forced students to think about their present concepts of this mathematical content. Through this problem-solving process, they came to know what they understood about this content. Working through these mathematical problems helped them to confirm or redefine their conceptual knowledge. This process of proving mathematics created the necessary cognitive conflict needed for them to reconsider their conceptions. Thus, they relearned the mathematics content through the process of solving problems and became more open to alternative ways of learning and teaching. I used these activities to facilitate the students' awareness of different representations, patterns, diagrams, and manipulatives that can be used to solve problems. I wanted to guide students in attaching meaning to symbols and operations.

Fractions remained a rich medium through which students learned to teach mathematics. Looking at a record of one group's discourse during the following problem clarifies how they negotiated to make sense in finding alternatives to traditional algorithms and constructed knowledge through direct modeling:

> We have 36 donuts. Five of us are going to share. How many dozens or how much of a dozen will each person get? (I got the idea for this problem from a conversation with Deborah Ball.)

Record of one group's discourse:

> Violet counts our manipulatives 1 through 36. "Okay, 36 donuts." Puts them into 3 dozen.
>
> Nancy begins to count to 36 also.
>
> Steven chooses paper and draws five people.
>
> Violet puts circles into 5 rows.
> Violet: We each get half a dozen and one donut.
>
> Nancy: Why don't we just go--one for you, one for you . . . into five categories until they are all gone?
>
> Violet: Okay.
>
> Steven does the algorithm. $5\overline{)36}$
>
> Nancy: So everyone gets 1/2 dozen plus one and one left over. One half dozen + 1 donut + 1/5 of a donut. Isn't 1/5 of a donut the same as 1/12 of a dozen?
>
> Violet: Each person gets 7 1/5 donuts. We need to find out what fraction 7 1/5 is of a dozen. Is there 12 on the geoboard? What could we use? A clock. A clock has 12.

Nancy: If we are on a clock, how would you have 7 1/5? Why are we on a clock?

Steven: Let's get out our fraction manipulatives.

Nancy: I know what to do! Find an easier problem. What if there were 6 people? Everyone would get a half dozen.

Steven: Six equals 1/2. Ah!

Violet sighs.

Nancy: What if there were 4 people or 3 people? See how it comes out even every time.

Violet: Hey! It's 3. Three dozen to begin with. See the pattern--36 donuts--
6 people, 3/6. Each person gets 1/2 dozen.
4 people, 3/4. Each person gets 3/4 dozen.
3 people, 3/3. Each person gets 1 dozen.
So, 5 people, 3/5. Each person gets 3/5 dozen.

Meanwhile, Steven has been working the algorithm: $\frac{7\ 1/5}{12} = 3/5$

As the prospective teachers began to attach meaning to familiar symbols and operations, they came to see mathematics as a connected web. The rules could be demonstrated through exploring the meanings of the procedures. The students began to connect formal mathematics to their informal experiences. They began to make discoveries for themselves, to interact, to communicate, and verify their solutions.

Students were excited about learning the "power" of thinking mathematically. However, the cognitive conflict from learning in new ways created frustration, as illustrated by the following excerpt from a student's math log:

> I like working math with this 'hands on' approach. The manipulatives are great! Having us actually work out the problems has made me realize just how much I do not know about basic mathematics. I felt confident before, but now I realize I don't even understand division. It is very frustrating!

Conceptual Change about the Nature of Mathematics

When students become mathematical thinkers, they undergo a change in their conceptions about the nature of mathematics. In traditional classrooms students spend time memorizing rules and procedures that the teacher transmits through lectures and working examples. Teachers treat mathematics as isolated topics and skills. Students in these classrooms come to believe that mathematics is an authoritarian, rule-oriented subject that has no meaning and cannot be understood. Students in these traditional classrooms do not get the opportunities to understand the conceptual knowledge that fits in a connected web of mathematical understanding.

During the first interview, I gave students the opportunity to talk about their experiences with mathematics and what it meant to know mathematics.

> Vanessa: I see mathematics as being all about numbers and manipulation of numbers. In arithmetic, you learn the basic skills by computing; when you get to higher mathematics then you have trained yourself to think mathematically practicing the rules over and over. Math is all about making sense of all the numbers.

> Tom: [In] math there are certain definite ways to do math--whether it is addition or subtraction, or any of those computations or problem solving. And the child must learn to do it that way. You shouldn't show them all the different ways to do it because it might confuse them. Math is a separate thing. It's not related to anything.

> Mary: I think math is a right answer. And you have to come up with only one answer.

As you can see, all of these students' responses reflect a traditional view of mathematics. They see math as being numbers, right answers, and one correct way of solving problems.

By contrast, at the end of this course, the majority of students were beginning to question the traditional view of what it means to know mathematics.

> Vanessa: Now, I see it as a problem waiting to be solved, not like a word problem, but a situation that's waiting to be solved. Before it was an impossible situation for me. [I learned] from my personal experience that there are ways to figure out problems that you can understand. It's not just numbers out there--like magic. It makes sense.

> Tom: You can just about do math with anything. It's fitting numbers to things around you. In math everything has to build. There's no going forward until they [students] understand. You can't just go back and try to reteach a concept. Math is not just procedures; it's learning to work the problems your own way.

Mary: It's not manipulation of numbers. It's not just abstract rules anymore. It's not just numbers being out there. It's something more tangible. Now, I see there's something conceptual behind the manipulations.

Note that at the end of the course, the students are beginning to see mathematics with new eyes. Mathematics has become for them a part of their everyday environment. There is no longer a "right" way to do math; there is "just the way one does math." They are seeing that mathematics permeates their everyday lives.

Conceptual Change about Mathematics Teaching and Learning

At the beginning of the course, I also gave students the opportunity to talk about their experiences with learning mathematics. I asked them to talk about the teaching in their previous classes and how they saw themselves as teachers of mathematics.

Dawn: I had two types of teachers. Some teachers gave us examples and showed us different ways of solving problems. Then we practiced the problems while we worked the homework during the rest of class. The next day we came back and asked questions. Or some teachers left us to figure it out on our own. We learned because we had to think harder. They said, 'Learning math takes awhile and takes a lot of time. It's not real easy. It's something that you have to work at to understand.'

Vanessa: Worksheets, workbooks, teacher- direction, things on the chalkboard, work to do yourself. The teacher shows you some examples of how to do the problems. Then you do it all yourself, usually at your own pace.

Joan: As a teacher, I see myself at the chalkboard showing them how to work problems--the basic skills--addition, subtraction, multiplication, and all that. I think they need to get the skills down--drill and practice, drill and practice. I don't have a problem with that. Whether they need to 'understand' and all that, I don't know. They should just be skillful in the four operations. If I do flash cards, I like to see them go like this [snaps her fingers--snap, snap, snap], not have them count on their fingers, 5, 6, 7. . . . My dad is an accountant, and he can add high figures just like that. When we are playing a game, he can add the score in a second. I'd like my students to be like that.

Terry: I think the students should understand the key words like *altogether* and *in all.* These are important before they can work the word problems. These words need to be explained first. I don't see how they can do a word problem without knowing how to add, subtract, etc. I think children need to be shown how to solve word problems. Sometimes in my math classes, I didn't even know how to set the problem up. It can be very frustrating. So, students should be shown.

These students' responses reflect passive learning rather than active construction of knowledge. Prospective elementary teachers have spent hundreds of hours in mathematics classes where "teaching is telling", and the teacher is the transmitter of knowledge. The experiences in these traditional classes have influenced the prospective teachers' conceptions that mathematics is learning procedures through drill and practice. The image they have formed of teaching mathematics is that the teacher organizes and presents the information and then provides examples for the students to duplicate.

During the students' responses about their experiences with learning mathematics and what it means to teach mathematics, the topic of word problems kept emerging. Since one element of learning and teaching mathematics is word problems, I asked students in the entire class whether children should spend time solving word problems before they practiced the skills needed to compute the answers. Some of their responses were

> Dawn: By practicing computations you learn. Eventually, you will get to a problem that you haven't done and so by practicing you will teach yourself how you did it. This way you can understand it. You will not only have memorized how to work the procedures but you will begin to understand on a higher level of meaning. I think when children are learning new concepts, they should know how to write out every step. Then, the teacher can tell exactly where you make a mistake. Personally, I think it's always important to write out the steps to keep from making stupid mistakes.

> Terry: Some students already come into the middle school or junior high not liking math and not interested. To have the teacher show them and say, 'It's not all that bad. You can do this.' Students like that. They don't want to put any effort into it. This is being lazy, but they like having the teacher do all the thinking for them.

As the course proceeded through the semester, the prospective teachers began to recognize their own abilities as mathematical thinkers, and their conceptions about mathematics learning and teaching began to change. Most of them had never experienced alternative ways in learning mathematics, and therefore, had only traditional models of teaching mathematics. The students in this course say that they expected the same from this course with the added activity of making games--something they had heard was included in other elementary mathematics methods courses. They instead had to explore mathematics through solving problems that had no easy answers. They worked together in small groups and with the whole group to discuss their thinking and ideas. By the end of the semester, many expressed a new view of what it meant to teach and learn mathematics.

> Before I would have taught them the rules, but now, to me, so much more of knowing mathematics is modeling. I could never go back to teaching without having the children model and discuss their work. Do the modeling first and then they can discover the rules for themselves.
>
> Overall, I think my role is to guide students so that they will value mathematics as they apply math concepts to real-life situations. I want to be a facilitator of knowledge rather than a 'body' delivering correct answers. By providing a problem-solving atmosphere and one that is secure enough to allow students to take risks, children will better learn to construct their knowledge. I want my students to take a more active role in their learning. Students will learn from one another as they practice self-reflection and verbalize their thoughts. I want to help my students feel confident about their math abilities. I want to integrate math with other areas of the curriculum. My essential role as teacher will be to provide a learning environment that will spark thought, new ideas, and help students make connections.
>
> As a teacher, I have a vision of my role as 'tour guide.' My part would be to lead the students to a specific math concept I want them to obtain and let them explore and discover on their own that math conceptual territory. As tour guide, I would lead them to the appropriate place to begin their exploration, then I would change roles and become that of a monitor. I will watch, listen, and keep everyone, as close as possible, in the correct relational understanding.
>
> As a teacher, it will be my job to make it interesting and challenging, to ask probing questions, and to allow my students the time to discover. I will have to take a back seat and sacrifice at times: time mainly. I want them to investigate why things are what we say they are (why is 2 + 2 = 4?). I want to provide an atmosphere where children can discuss ideas, make changes to them, and feel like they've accomplished something. By teaching this way, I hope my students will develop problem solving skills, as well as having the confidence to take risks. I want to ask probing questions to get the students to think. I want my students to be aware of the problem-solving process and to learn from others. I will try to create open-ended problem situations with several possible solutions that can be discusses. I want to help my students to see the patterns, investigate, and evaluate their thinking. I want to be like a coach. I want to be able to say. 'We have this problem--What are some ways we can approach this to solve it? I want my 'team' to get in there, wrestle with it, explore in's and out's and come up with a solution that demonstrates depth of understanding and connections.

One of the first points that I noted in these responses was that these students were now perceiving their roles as teachers in elementary schools to be that of "guide." The role of guide was the way I had perceived my role from the beginning of this course, but had not articulated to my students. They are beginning to recognize that it is important to create a risk-free environment or atmosphere in the classroom so that students can verbalize their solutions to problems as they investigate and interact with mathematics. They now see word problems as conceptual explorations of mathematics. These prospective teachers are now identifying mathematics learning and teaching as an investigation and interaction in the classroom. As Lampert (1990) discusses in her research, the content of the lessons have become the arguments about proving solution strategies.

A final question that I wanted answered was At what point in the semester did these students begin to change their conceptions about mathematics and mathematics teaching and learning? In an effort to answer this question, I interviewed, at the end of the course, the five originally selected students. In the following responses, they discuss the experience that was for them the turning point when they were willing to consider alternative approaches to teaching mathematics:

> Mary: Everything combined. I did the manipulating to clarify what I wanted to articulate to my group and the class. But having to articulate it made me do the manipulating which clarified everything. But I wouldn't have had to figure out with the manipulatives what I understood if you hadn't ask us to explain 'why' we had worked the problem a certain way. Before I thought about math as one right answer. You had to come up with only one answer, but having to articulate it made me try to find with the manipulatives a way to explain so that others could understand. When I heard others tell how they solved the problems, I saw, hey, you can work it this way and this way.

> Dawn: Clearly, the problems that you gave us in class. Like the simple donut problem (How many dozen or how much of a dozen does each person get if you have 36 donuts and 5 children?). I said to myself you should know how to work this. At the beginning of class, I remember I was frustrated. You would give us problems to work, like fractions. We would have to figure them out. It was difficult and frustrating because by now we should know this 'stuff' and be able to figure them out.

> Vanessa: It was a gradual change. By myself I might not have been able to become confident, but it helped to be in a group. I know I couldn't have come up with some of the 'stuff.' Seeing the way others in my group were thinking sometimes changed the way I was thinking. I said, 'Maybe there's something there I should be looking at.' The manipulatives helped a lot because when I can't look at something I don't understand, especially in math. With the manipulatives I could

see the connections to the algorithms. But the discourse helped also. For example, when someone would talk about something so simple as how they add large numbers mentally, I saw something I'd never seen before. I never used to do that. And now I can add that way, too.

Terry: A definite turning point for me was when the class worked out several place- value problems with the base-ten blocks. It occurred to me that I really never knew why I 'borrowed' when subtracting. I don't even think that I knew that the number 32 is really three tens and 2 one. And--I got A's. I can't believe how little I understand about mathematics.

Tom: The day we worked on the problem 1 3/4 ÷ 1/2 was the day that opened my eyes. I knew the algorithm forward and backwards, but the algorithm didn't mean anything to me. I didn't know what was really going on. I gained a tremendous amount of insight into a world of math that I did not know existed.

From the interview responses and the students' final writings in their math logs, I identified several factors that had contributed to challenging the preservice teachers' conceptions about what it meant to know mathematics and how it should be learned and taught. One factor was learning in cooperative groups. Learning in cooperative groups gave them an opportunity to hear how other students were thinking, to verbalize their own thinking, and to clarify for themselves their own approaches to problems. Manipulatives were another important factor in helping teachers model their thinking. Using the manipulatives to demonstrate the concepts behind the procedures forced them to confront their own knowledge about the content. It was through the use of manipulatives that they began to understand the algorithms. The most powerful component for conceptual change within the students' thinking was the problem situations themselves. These problems were the medium that created the environment for the discourse and the use of manipulatives. It was this environment created by the problems which challenged the preservice teachers' current conceptual knowledge creating the cognitive conflict necessary to either confirm what they knew or relearn the concepts. As learners of mathematics, they were beginning to develop a conceptual understanding of familiar mathematics procedures.

At the beginning of the course, practically all the students talked about mathematics as "the basics"--adding, subtracting, multiplying, and dividing. Knowing mathematics meant being able to memorize facts and manipulate numbers. At the end of the course, they talked differently about their own learning of mathematics and what they had begun to conceive about mathematics. Many of the students came for the first time to understand the meanings of rules and procedures and why they actually worked. They became willing to take risks and defend their own solutions to challenging problems. They modeled their thinking with manipulatives and diagrams. Along with a change in conceptions about mathematics came a new image of what it is to teach mathematics. In the following excerpts from their math logs, they write:

> We keep hearing in our education courses, '. . . be a constructivist teacher; teach for conceptual understanding.' However, we are never taught that way. In this class we learned how to learn math constructivistly by not only working with the manipulatives but by being taught that way. We were allowed to discuss how we approached our problems and to hear others. It has been a great eye-opener for me. I admit that I have "traditional-style bones' in my body, but I now see why that style or method of teaching is not always effective, especially for my new ideas of how to teach math. I liked most being shown how to use the manipulatives to understand how children think. I know more about what it means to 'know mathematics.'

> I found this course to be intellectually challenging. Instead of constantly making math games, I have had the opportunity to expand my personal conceptual knowledge of math and feel more confident about teaching it. This class has shown me what math is and how to teach it so that students will understand it.

> This experience of being taught in the ways we are told to teach has helped me most. Being in a constructivist class helped me to understand the readings on constructivist teaching. My teaching habits and philosophies have changed a lot during the course. I'm glad I had the opportunity to re-examine my mathematics beliefs.

> This way of learning to teach mathematics has been good for me, but frustrating at times. It allowed me to view mathematics concepts--in a way completely different from what I've experienced in my math classes. Unfortunately, it showed me just how much of math I don't 'understand' but have been able to do procedurally. The problem solving in our groups was the most important part in helping me grasp a better understanding. I liked the format of putting theory into practice--of working in cooperative groups and sharin" our thought processes.

Quantitative Analysis

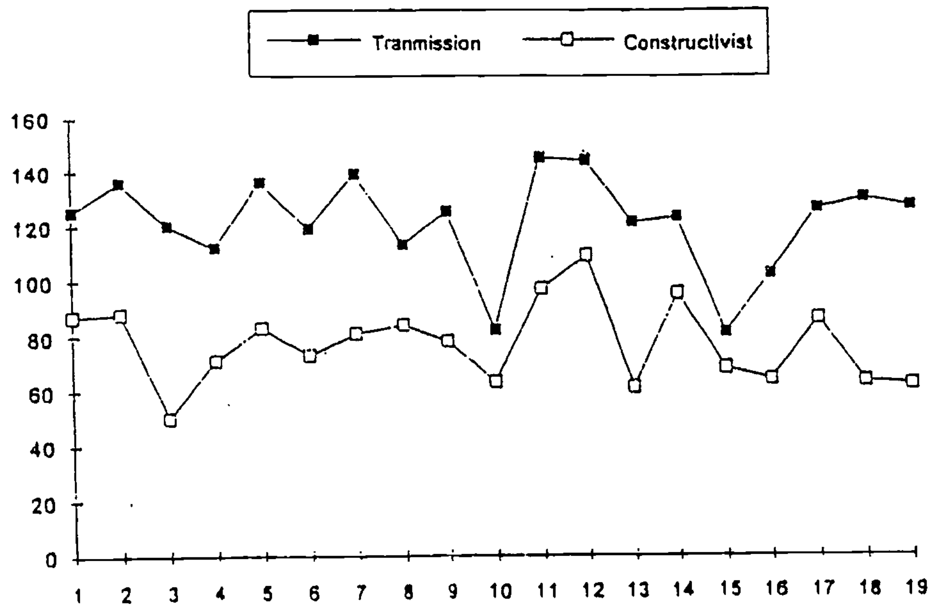
In an effort to further support the qualitative portion of my research, I decided in the planning of the course to use the Mathematics Beliefs Scales (Fennema et al., 1987) questionnaire as a pretest and a posttest. This questionnaire elicits two types of information. Some items address how teachers believe children learn mathematics. Other items cover how one believes teachers should teach mathematics. Fennema et al. (1987) established the validity of the questionnaire in their research on changing teacher conceptions about how children learn mathematics. They provided workshops in which they presented and described how children learn specific mathematics content. Through this research they found that if teachers are provided with information on how children learn specific content, teachers will change their conceptions, thereby changing their instructional methods. Similar to Fennema et al. workshops, I provided preservice teachers with information about how children learn specific mathematics content. Therefore, I decided that by using the Mathematics Beliefs Scales as my instrument, I would be successful in finding out if changes did occur in preservice teachers' conceptions about mathematics teaching and learning as a result. I asked them to complete the questionnaire during the first class and the last class. I told my students that their responses on the questionnaire would in no way affect their grade or success in this course. In the beginning of the course. I explained that my purpose in giving the questionnaire was to help me better understand their conceptions of mathematics teaching and learning. At the end of the course, I explained that I was interested in understanding their present beliefs about teaching and learning mathematics .

Following are some sample questions and student responses. As can be seen from this sampling of the data, there is a difference in the students' responses at the beginning of the course and at the end. This information is consistent with my findings from the interviews, observations, students' work, and their math logs.

Table 1
Sample questions from mathematics beliefs scales

	Pretest	Posttest
24. Most children cannot figure math out for themselves and must be explicitly taught.		
Strongly Agree	0	1
Agree	1	0
Undecided	3	0
Disagree	10	0
Strongly Disagree	5	18
28. Children should be allowed to invent ways to solve simple word problems before the teacher demonstrates how to solve them.		
Strongly Agree	5	16
Agree	3	3
Undecided	7	0
Disagree	4	0
Strongly Disagree	0	0
35. Frequent drills on the basic facts are essential in order for children to learn them.		
Strongly Agree	3	0
Agree	5	0
Undecided	7	1
Disagree	3	13
Strongly Disagree	1	5

The graph below displays the students' pre and posttest results. The pretest represents a transmission approach to learning and teaching mathematics, and the posttest represents a constructivist approach to learning and teaching mathematics.

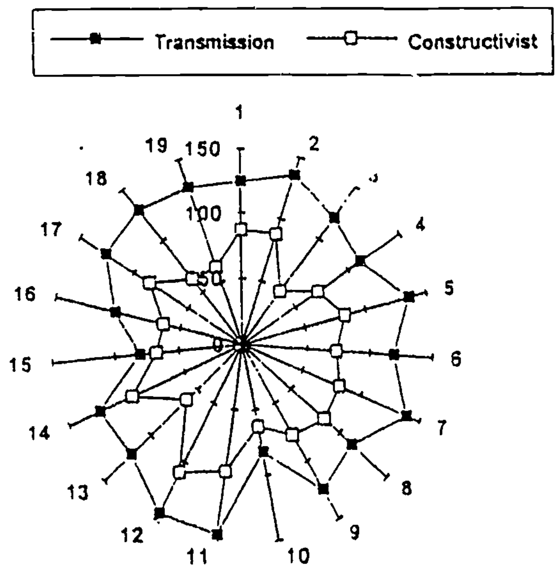


Graph 1. Line graph of pre and posttest scores

Lower scores indicate a movement toward a more constructivist approach to teaching and learning mathematics. As you can see, at the beginning of the course the students viewed mathematics learning and teaching with a more transmission approach. Preservice teachers entered the course believing that children should be presented with number facts that they drilled and practiced rather than constructing the relationships between the concepts and the procedures. They believed that teachers should assess children's mathematical understanding through paper and pencil computation rather than through classroom discussion that involved solving problems. They thought that the focus of the classroom should be on teacher presentation and demonstration of solving problems rather than on the children working together to solve the problems.

At the end of the course, the students saw mathematics as a personal construction of knowledge. The role of the teacher, for them, became a facilitator whose responsibility is to create an environment where children could think and verbalize their ideas.

Another meaningful way of viewing the difference on how preservice teachers' conceptions of learning and teaching mathematics changed from the beginning of the course to the end of the course is displayed in the following graphic illustration.



Graph 2. Web graph comparing transmission and constructivist approaches

The zero point represents the most constructivist score possible on this questionnaire. The outer scores represent the pretest scores obtained from the first administration of the questionnaire. The inner scores represent the posttest scores obtained from the last administration of the questionnaire. Note that the students' posttest scores are converging on the zero point. This convergence indicates that at the end of the course the students' beliefs regarding the learning and teaching of mathematics were approaching a constructivist viewpoint.

Table 2 below contains the descriptive statistics for the pre and posttest.

Table 2
Descriptive Statistics

	pretest	posttest
Range	64	59
Mean	121.37	77
Median	125	78
Mode	125	63
Standard Deviation	17.77	15.01
Variance	315.91	225.33

The range of preservice teacher beliefs is presented in the first row of Table 2 for both the pre and posttest. Notice that the range is smaller in the posttest than in the pretest. The pretest range is greater because two students entered the course with stronger constructivist views of learning and teaching. Their scores are the very low scores (the lower the score, the more constructivist the view) seen in the top line of Graph 1. The lower range in the posttest indicates that the preservice teachers' beliefs about learning and teaching mathematics were more closely aligned with one another by the end of the course.

Observations about the quantitative data are particularly interesting when correlated with the qualitative data. The belief that a teacher educator can change the conceptions about mathematics teaching and learning of preservice teachers is confirmed. The mean of the pretest is higher than the mean of the posttest. The higher pretest score indicates that as a whole, the preservice teachers entered the course with beliefs about learning and teaching mathematics that reflected more strongly the transmission approach. The 51 point difference in the means strongly shows that by the end of the course, the students beliefs about teaching and learning mathematics had changed toward a more constructivist approach to teaching.

To test whether the mean differences between the pre and posttest scores were statistically significant, I conducted a single-factor analysis of variance. I am aware that I have conducted a pre-experimental study, more specifically, known as a one-group pretest-posttest design. I do not consider, however, the following factors as threats to internal validity: history, maturation, testing, and instrumentation. Even though these students were not randomly assigned, the selection of students taking the course did not affect the pre-posttest means since selection is not a factor threatening internal validity.

The results of the analysis of variance are presented in Table 3.

Table 3
Source Table

Source	Sums of Squares	df	Mean Squares	F Observed	F Critical
Between Groups	18701.29	1	18701.29	69.11	4.11
Within Groups	9742.42	36	270.62		

$\rho = 0.05$

With the F observed 65 point higher than the F critical, there was a significant difference in the means. The null hypothesis that my instructional program made no difference in changing the beliefs of preservice teacher regarding teaching and learning mathematics was rejected. Therefore, the alternative hypothesis that my instructional program did change the beliefs of preservice teachers regarding teaching and learning mathematics was accepted.

Implications for Teacher Education

A major goal in any mathematics teacher preparation course, as it was in mine, is to assist the preservice teachers in becoming effective teachers of mathematics. I believe that for preservice teachers to be effective teachers of mathematics they must first develop an understanding of the nature of mathematics. In my course, prospective teachers developed this understanding as they interacted with one another during the problem solving and the classroom discussions about their solutions. "Understanding is situated or context bound because social interactions are fundamental and inseparably bound to the development of the tools for thinking and understanding of how to use them" (Cochran, DeRuiter, & King, 1993, p. 266). Therefore, teacher educators must develop teacher preparation programs that provide preservice teachers with experiences that parallel those that they will encounter in teaching. Teacher educators should create "context bound" environments in their courses. These environments should engage preservice teachers in activity which involves them in learning about how learning occurs.

By asking them questions about *how* they were thinking and about *what* they were thinking, I acknowledged their ideas. Through recognizing their thinking, I provided a model for them which demonstrated that teachers should continuously elicit from students how they are thinking in order to assess what they are learning. My acknowledgement of their thoughts and understandings helped to make prospective teachers more aware of how their own students would think and understand mathematics.

The experiences of learning to teach mathematics in teacher preparation courses should go beyond providing preservice teachers with activities that they can later utilize. Teacher preparation programs for mathematics should help preservice teachers develop their pedagogical content knowledge by focusing on how mathematics is learned. Teacher education programs should put students in the situations of learning that assist them in answering their frequently asked question How will I know when students have learned what I have taught? When teachers understand how their students think and understand, they can plan instructional programs that better meet their students' individual learning needs.

All learners must have opportunities to explore their ideas in order to find limitations and strengths in their own conceptions. These opportunities for cognitive conflict are the catalysts for conceptual change. However, students must experience sufficient cognitive conflict to motivate them to extend their thinking. Cognitive conflict causes students to confront their conceptions--knowledge and beliefs--about the nature of mathematics and how it is learned. For preservice teachers, this confrontation opens them to a new belief that mathematics knowledge is personally created. Accepting the new belief that mathematics is constructed by learners leads the prospective teachers to consider, reflect on, and try out new approaches to teaching. Teacher preparation courses should be created so that the students enrolled in them can experience sufficient cognitive conflict with their conceptions to open them to new approaches of teaching and learning.

Mathematics teacher educators need to challenge preservice teachers to become mathematical thinkers. When preservice teachers strengthen their own understanding of mathematics through the process of becoming mathematical thinkers, they better understand how children learn mathematics. Preservice teachers will use this information about how children learn mathematics when planning, implementing, and assessing their instruction.

Conclusions

The basic goal of any teacher education course for mathematics is to produce teachers who will help their students to learn to communicate and reason mathematically. With this purpose in mind, educators produced the Curriculum and Evaluation Standards for School Mathematics (NCTM, 1989). The basic goal of the Standards document is "the development of mathematical power for all students" (p.1). I wanted to help the preservice teachers in my class to develop their own mathematical power so that they would be better prepared to encourage their students in constructing mathematical power. Therefore, I used the Curriculum and Evaluation Standards for School Mathematics (NCTM, 1989) and the Professional Standards for teaching Mathematics (NCTM, 1991) as the framework for my course. I also incorporated principles from the Cognitively Guided Instruction program.

"The goal of CGI is to prepare teachers who can foster children's active mental involvement in worthwhile mathematical tasks" (Zeichner & Liston, 1990, p. 6).

I had come to recognize through previous teaching experience with preservice teachers that their conceptions about teaching mathematics were formed from the ways that they had been taught. In other words, their previous teachers had been the models for how they thought teaching should be. I believed that if I wanted to affect a change in their conceptions about what it is to know mathematics and how mathematics should be taught, the most effective method would be to provide them with a new model for learning and teaching mathematics.

I wanted my students to question what they knew about mathematics and to integrate the new knowledge that they were constructing with the old knowledge. Both the qualitative and quantitative data support the conclusion that I met my goal of changing my students' conceptions about mathematics and mathematics learning and teaching. The following quote from one of the students in my class explains how my course impacted his conceptions about mathematics learning and teaching:

> I feel like a kid who has had his eyes opened to a new type of candy. Looking back I am amazed at the difference MAE 4310 has made in my view of mathematics and mathematics instruction. As a teacher, I want my students to understand how math concepts relate to one another and may be integrated with other concepts. Furthermore, I intend to encourage the children to verbalize their thought process and strategies for solving problems. I have a lot more confidence that I can teach mathematics.

I believe that providing experiences for my students that are similar to those that they would be providing for their students placed the preservice teachers in the role of learners. By creating an environment where learning was the focus, I helped increase these prospective teachers' understanding about mathematics and how it is learned. The learning environment that consisted of productive mathematical tasks, cooperative groups, modeling with manipulatives, and inquiring discourse created the environment for conceptual change. The most powerful component for conceptual change within the students' thinking was the problem situations themselves. These problems were the medium that created the environment for the discourse and the use of manipulatives. It was this environment created by the problems which challenged the preservice teachers' current conceptual knowledge creating the cognitive conflict necessary to either confirm what they knew or relearn the concepts. As learners of mathematics they were beginning to develop a conceptual understanding of familiar mathematics procedures.

Preservice teachers indicate that education courses provide them with information about theory, but seldom give concrete examples of how to put this theory into practice. By modeling how constructivist teachers teach, I provided my preservice teachers with an example of "theory in practice." They could adapt and incorporate this new model of learning and teaching mathematics into their own pedagogy. Through modeling the constructivist approach, I became a part of the preservice teachers' learning environment. I demonstrated the role of the constructivist teacher, a guide who leads students into the construction of their own knowledge. This teacher creates an environment in which students feel free to engage in learning without risks, selects the tools and materials which serve to aid the students in their learning, and elicits dialog from the students that encourages them to share their understandings and thinking with others. In the words of one preservice teacher in my class, the role of the teacher is "to guide students so that they value mathematics as they apply math concepts to real-life situations."

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