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ABSTRACT

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The Influence of Innovative Instructional Processes On Mathematical Belief Systems

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THE INFLUENCE OF INNOVATIVE INSTRUCTIONAL PROCESSES ON MATHEMATICAL BELIEF SYSTEMS[1]

by

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Abstract

This paper describes a study of the effects of an innovative mathematics content course on the beliefs of elementary education majors at Indiana University (Bloomington). The primary focus of the study was on how students' belief systems might have been affected as a result of taking this course. The sample used in the study were eight undergraduate students enrolled in the new course (T104) during the first semester it was taught (Spring 1990). Data for the study were collected in several ways: non-participant observations, students' interviews, and document analysis. Findings indicate that the three basic components of T104 instructional approach (problem solving, cooperative learning, and written reflections) have potential to challenge students to question their mathematical belief systems regarding what it means to know, do, learn, and teach mathematics.

Introduction

At present, society and the media identify poor quality teaching as one of the primary causes of American students' low mathematics achievement in relation to other countries. Our nation is demanding better mathematics teachers, yet too effort has been made to understand what teachers or prospective teachers think about knowing, learning, and teaching mathematics. Many studies offer prescriptive answers for teacher preparation, but fail to acknowledge the human aspect inherent in the teaching profession. Formal education involves complex interactions between teachers and students who are influencing and being influenced by each other in both cognitive and noncognitive factors. A noncognitive factor inherent in the human endeavor of teaching, that current research about teachers and teacher education have not fully taken into account, is teachers' beliefs about mathematics and mathematics pedagogy.

Beliefs, or belief systems, are becoming increasingly explored elements within current educational research, and an interest in the area of teachers' thought processes is emerging. Clark & Peterson (1986), for example, note the importance of teachers' implicit theories and beliefs about education. Beliefs and belief systems have been defined and studied in different ways by mathematics educators. Schoenfeld (1987), who broadly defines mathematical belief systems as one's mathematics world view, has pointed to

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[2]The authors are members of a team directed by Frank K. Lester, Jr. working on the T104 course described herein. Other members of the team include Diana L. Kroll, Carol J. Fry, Francisco Egger Moellwald, and Sue Mau.

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beliefs as one component of students' metacognition that can be better understood and perhaps influenced by thought-provoking instruction on the part of a metacognitively-aware instructor.

Other mathematics educators (Lester, 1987; Lester, Garofalo, & Kroll, 1989) define mathematical belief systems more specifically, claiming that mathematical belief systems comprise one's subjective knowledge about self, mathematics, the environment, and mathematical tasks. In a study of metacognitive behaviors of seventh graders involved in problem solving, Lester, Garofalo, and Kroll (1989) found beliefs to be one of several influential noncognitive factors that influence students' problem-solving abilities.

In our study, we conceive belief systems in terms of beliefs about self, epistemological beliefs, ontological beliefs, and ethical beliefs. Incorporating the two definitions of mathematical belief systems provided above into our own conception, we contend that each individual has his/her own unique belief system which internalizes his/her experiences through a filtering process that is unique to the culture in which the individual lives. Thus, individuals encode mental ideas and representations of their experiences in the world according to their belief systems.

Beliefs expose our fundamental ideas about our life experiences, whether we consciously acknowledge those beliefs or not, and they directly affect our actions (Fishbein & Ajzen, 1975). Research has indicated that the study of teacher preparation should include not only an examination of content-knowledge, but also of thinking processes, attitudes, and beliefs held by prospective elementary teachers (Peterson, Fennema, Carpenter, & Loef, 1989; Thompson, 1989). The beliefs a future elementary teacher has about what mathematics is and what it means to know and do mathematics are driving forces in that prospective teacher's learning, applying, and teaching of mathematical ideas (Carpenter, 1989; Kloosterman & Stage, 1989; Lester, Garofalo, & Kroll, 1989; Peterson, 1988a, 1988b; Shulman, 1986).

Teaching actions are directly influenced by teachers' beliefs, and in turn those teacher actions have a tremendous impact on students' belief systems. Teachers, when they were students, often experienced mathematics classes that consisted of a predictable pattern of lectures followed by seat work. Their quizzes and tests required regurgitation of exactly what their teachers had spelled out in class. Not surprisingly, their mathematical beliefs are based on these experiences. They are likely to teach in the same manner, perpetuating the long-standing chain of beliefs about mathematics as being: (1) mechanical in nature, (2) a fixed body of procedures that can be performed without thinking, (3) activities that must be done independently, and, (4) difficult except for those people who happen to be lucky enough to be good at it. Individuals internalize such beliefs through continuous exposure to situations at school and at home that reinforce the notion that this is the true nature of mathematics.

If educators envision a mathematics curriculum in which students at all levels acquire a different way of perceiving mathematics and hope to develop in students positive beliefs and attitudes toward mathematics, then schools must consider the extent to which teachers' beliefs play a role in students' motivation, performance, and achievement in mathematics. Everybody Counts (National Research

Council [NRC], 1989) is an interesting report that identifies three problem areas: teaching mathematics, learning mathematics and mathematics teacher preparation. This report also offers some insights into how to overcome them. According to this report, a crucial factor in revitalizing curricular practice is the preparation of teachers, and

> Teachers themselves need experience in doing mathematics--in exploring, guessing, testing, arguing, and proving--in order to develop confidence that they can respond constructively to unexpected conjectures that emerge as students follow their own paths in approaching mathematical problems. (NRC, 1989, p. 65)

In line with this report, the National Council of Teachers of Mathematics (NCTM) published the Curriculum and Evaluation Standards for School Mathematics (NCTM Standards, 1989a) and the Professional Standards for Teaching Mathematics (NCTM Professional Standards, 1989b). The NCTM reports present a different conception of what it means to know, learn, and teach mathematics, which reflects the call for mathematics education reform voiced in Everybody Counts (NRC, 1989). Underlying the need for curricula change that is outlined in these documents is an accompanying need to rethink mathematics teacher preparation, and to begin exploring ways in which future teachers might be prepared to teach mathematics in the spirit of these reports.

As mathematics educators interested in preparing teachers for their future career, and who are curious about the extent to which future teachers are aware of the many facets of what it means to be a teacher (in particular, a teacher of mathematics), we sought a context where we could find prospective teachers willing to share their reflections on this issue with us. At Indiana University (Bloomington), a unique mathematics content course (T104) has been developed for preservice elementary teachers. The goals of the course mirror many of the goals discussed in the Curriculum Standards (1989a) and The Professional Standards (1989b). Developers of this course have tried to incorporate a new vision of teaching mathematics via problem solving by challenging students to construct and/or reconstruct their understanding of mathematics. Because one of the objectives of this course was to get students to question their beliefs about knowing, doing, learning, and teaching mathematics, it was an ideal setting in which to investigate the effects of innovative instruction on prospective teachers' mathematics belief systems.

This paper describes our study of how T104 affected the beliefs of preservice elementary teachers. We hoped to gain an understanding of prospective teachers' thinking about themselves as knowers and doers of mathematics, and about the dynamic process involved in their rethinking of their beliefs about learning and teaching mathematics. As background for why T104 would be expected to affect students' belief systems, the paper begins with a description of how the philosophy and instructional process in T104 differ from those of other mathematics courses.

A Description of T104

A typical mathematics classroom is described by the following scenario. The teacher begins by checking the homework from the previous day. If there were difficulties on any of the problems, the

teacher generally works them out on the board. Once this is done, the topic for the day is presented in about 15 or 20 minutes, usually by the teacher giving students definitions and algorithms that they need for that day's homework. This is followed by a few examples on the board, and from there the students proceed to work a list of problems from that lesson. This type of instruction implies that there is specific mathematical knowledge that the teacher has and by imitating what he/she does in the form of homework problems, and through passive absorption, students will gain this "knowledge."

This perception of mathematics education, predominant in contemporary American society, concentrates on the pursuit of outcomes, and also presents facts unconditionally, encouraging mindless absorption. As Langer (1989) noted, "If something is presented as an accepted truth, alternative ways of thinking do not even come up for consideration. Such a single-minded way of viewing the world can generalize to virtually everything we do. By teaching absolutes we pass our culture from one generation to the next. It brings stability. But the cost may be high" (p. 35).

One increasingly explored philosophical position among mathematics educators today is that of constructivism, a particular philosophical position that assumes a subjective perspective of epistemology. Others suggest that constructivism is a post-epistemological perspective (Noddings, 1990), that is, more of a pedagogical view. We contend that constructivism is an epistemological philosophy that holds particular implications for pedagogy, and it is this philosophy upon which the curriculum and instruction of T104 is founded.

The course, *Mathematics for Elementary Teachers Via Problem Solving*, known to us and Indiana University students as T104, is a required mathematics content course designed specifically for prospective elementary teachers. The emphasis in this course is on learning mathematical concepts via problem solving in a cooperative learning situation. Students work in groups throughout the semester and change groups periodically. In agreement with research on cooperative learning (Sharan, 1990), we believe that cooperative work offers the following advantages: (1) group problem solving is often broader, more creative, and more insightful than individual efforts, (2) interaction with other students stimulates additional problems, insights and discoveries, (3) students motivate one another to excel and to accept more challenging problems, (4) motivation to persevere with a problem is increased, (5) socialization skills are developed and practiced, (6) students are exposed to a variety of thinking and problem-solving styles different from their own, (7) students learn to depend upon themselves and each other (rather than the teacher) for problem solutions, and (8) conceptual understanding is deeper and longer-lasting when ideas are shared and discussed. Group problem solving in mathematics encourages students to construct their own mathematical understanding, while sharing and negotiating meaning, which is an important aim of T104.

The goals of T104 include helping students to: (a) develop an adult-level perspective and insight into the nature of mathematics taught in the elementary school, (b) improve their ability to engage in mathematical thinking and reasoning, (c) increase their ability to use mathematical knowledge to solve

problems, and (d) learn mathematics through problem solving. The course is organized around key, unifying mathematical ideas, which we call "Big Ideas." Some of these big ideas are conjecturing, verification, decomposition, and multiple representations. These key mathematical ideas are taught through problem-solving activities and are woven throughout the mathematics content areas of numeration, operations, geometry, measurement, number theory and rational numbers. The big ideas help connect the content areas and offer the students an integrated view of mathematics.

In addition to a problem-solving approach and cooperative work in learning mathematics, T104 emphasizes reflective thinking and writing. Through initial problems, extension problems, questions about the problems and other activities, students discuss and reflect as a group on how to solve a problem, why a solution makes sense, why algorithms work, what big ideas are present in an activity, how concepts are connected, etc. In addition to the in-class reflection, students are assigned written reflections. Some examples of reflection assignments are as follows: "Write an algorithm for adding two numbers that are written in a base other then ten and explain why it works," "Make up and explain a divisibility test for 6 in base six," "Do you believe mathematics was invented or discovered? Explain your thinking." These questions challenge students to reflect on both the cognitive and metacognitive aspects of their thinking about mathematical tasks.

Student assessment in T104 is conducted through several means. Students keep a notebook of their class activities, homework, and reflections. Several times during the semester, these notebooks are graded based on completeness, accuracy, and insight. Students also develop and present group projects that are evaluated in terms of thoroughness, creativity, mathematical justification of the project, and clarity of presentation.

Probably the most interesting aspect of student assessment involves the tests. Students complete two mid-term examinations and one final examination which are composed of three parts: a group problem-solving activity, an individual follow-up to the group problem, and an individual part that is not directly related to the group problem. Students express a variety of reactions to this assessment process.

As one might expect, a course where alternative assessment practices, problem solving, cooperative work and student reflection are emphasized has a much different role for the teacher than does a course not having these emphases. The teacher's role in T104 is to facilitate discussion, ask questions of students or small groups to prompt thinking, challenge students' misconceptions and serve as a model for these prospective teachers. This brief description of T104 sets the stage for the specifics of our study.

A Description of the Study

The questions addressed in the study are: (1) How do students perceive the instructional process implemented in T104 as compared to other mathematics content courses?, (2) What are students' reactions to specific components of T104 (e.g., problem solving, cooperative work, and written reflections)?, (3) How have students' awareness of changes in their beliefs been articulated?, and (4) How has the

instructional process used in T104 affected students' beliefs about doing, knowing, learning, and teaching mathematics? A historical account of how we pursued these questions is described below.

The study took place at Indiana University (Bloomington) during the Spring semester of 1990, the first time that T104 was taught. Two classes of about twenty-four students each were the focus of the study. The methods of data collection were: (a) non-participant observation, (b) interviews, and (c) document analysis. Throughout the semester, one of the primary researchers (Santos) observed (as a non-participant) one of the two sections once a week (5-6 times total in each section) in order to develop some intuition about students' reactions to T104, to assess the implementation of classroom activities, and to consider possible ways to modify the course (two other members of the mathematics education department project team also observed on a periodic basis).

Observations and brief conversations with students revealed students' desire to share their opinions about their experience in T104. Upon sharing these accounts with the other primary researcher (Raymond) in the middle of February of 1990, the two began to implement a series of studies about students' perceptions of T104, both in terms of their reactions to the innovative instruction, as well as their reflections about what mathematics is all about.

For this first study, Raymond and Santos approached students from the two classes, during the last two weeks of the semester (the end of April of 1990), and asked for volunteers to be interviewed about their experience in T104. Four volunteers from each of the two sections agreed to participate. The eight volunteers were all females between the ages of 20 and 22, with the exception of one student who was in her late thirties. The interviews were intended to be loosely-structured conversations of about one hour in length, and to be jointly conducted by the two primary researchers.

In an effort to maintain a certain level of consistency among the interviews, Raymond and Santos planned some guiding questions about the students' mathematics background, favorite or least favorite mathematics experiences, expectations prior to T104, opinions on the various components of T104 (cooperative learning, problem solving, written reflections, and alternative assessment), and the different content areas covered in T104. The questions also explored students' opinions regarding the role of the T104 instructor, their role as students in T104, and students' self-assessment regarding their confidence about themselves as "doers of mathematics." Conversations with some students extended beyond this initial set of guiding questions.

We also received permission from these respondents to read their notebooks containing written reflections, class activities, quizzes, and tests. After conducting the interviews, we compared common themes that we each noticed emerging from the interviews, and we synthesized students' responses, and developed an initial list of positive and negative reactions to T104. Our next step was to explore the extent to which the opinions expressed in the interviews would be corroborated by students' entries in their previously written reflections in the notebooks.

The analysis phase took place immediately following the interviews, during the months of May and June of 1990. We met on a daily basis to review and discuss students' interviews and reflections. To give us better insight into the students' experiences in T104 and also to get an overall picture of each student, we examined their classroom activities, homeworks, quizzes, and tests. This was done in an effort to help us make sense of the messages and meanings they conveyed in the interviews and reflections.

At the beginning of our analysis we had some doubts about the sincerity of some reflections because they almost sounded too good to be true. We were concerned that the students might be writing what they thought the teachers wanted to hear. However, as the semester progressed the students' reflections began to take on a more personal tone in that they started to be more conversational in nature. We became convinced that students really were reflecting upon their struggles throughout T104 and their beliefs about learning and doing mathematics. The interviews served as a testimony to the sincerity of the change processes that students experienced in T104.

The written reflections differed between the two sections due to the guidelines given by the instructors (co-author Masingila was one of the T104 instructors). One instructor encouraged students to keep a daily reflective journal in addition to reflecting on an assigned mathematics question of the day. The other instructor did not ask the students to keep daily reflections, but instead requested periodic reflections on mathematics topics and on their beliefs about mathematics. Toward the middle of the semester, the assigned periodic reflections of both sections were similar due to discussions between the two teachers and the coordinating team of T104.

We read through each students' reflective journal, making notes on each entry. Initially, our notes were brief summaries of the content of the entry, with occasional references to students' acknowledgement of their feelings and thinking about mathematics content. In the process of analyzing students' entries our approach started to be more and more focused. We began to focus on beliefs about mathematics that students were conveying, on their thinking about themselves as doers and learners of mathematics, on the respective roles of the teacher and the students in T104, on instances where students demonstrated that emotional and cognitive dissonance were taking place as a result of T104, on students' self-assessment regarding their confidence and performance in mathematics, and on situations where they reflected on how group work and problem solving affected their perception of mathematics.

During all the analysis time we continually reflected separately and jointly about what we were unraveling about prospective teachers' thinking about mathematics and their beliefs as a result of this distinctive mathematics course. As a final step of our data collection, we implemented interviews with the two instructors of T104. We talked with them during the first week of August 1990 about their experiences of planning, teaching, and assessing the first run of T104, about their personal expectations, and how they interpreted the actual implementation of T104. We also asked them to share their thoughts on how they believe they have influenced students' belief systems through their teaching actions, class

conversations, assessments, and personal dialogues with students via their notebooks. The conversations with the two instructors helped to corroborate students' comments from interviews and reflections.

Once the writing phase began we decided what would better illustrate our understanding and sense-making would be to simultaneously present the episodes from interviews and reflective journals in their own words, accompanied by our interpretations. The format of this next section corresponds to the broad categories within the students' mathematical belief systems that emerged from our analysis of respondents' notebooks and interviews. The categories include: (1) beliefs about self as a doer of mathematics, (2) beliefs about knowing, learning, and teaching mathematics, and (3) the disequilibration process. What follows are our impressions and interpretations of the eight respondents' stories regarding each of the aforementioned categories.

Emerging Themes

In an effort to make clear to the reader the type of problems that students were engaged in throughout T104, we offer the following example of a typical class problem solving activity.

At the beginning of the study of Number Theory, students were given the Locker Problem to solve:

> *Students at an elementary school decided to try an experiment. When recess is over, each student will walk into the school one at a time. The first student will open all of the first 100 locker doors. The second student will close all of the locker doors with even numbers. The third student will change all the locker doors with numbers that are multiples of three. (Change means closing lockers that are open and opening lockers that are closed.) The fourth student will change the position of all locker doors numbered with multiples of four; the fifth student will change the position of the lockers that are multiples of five, and so on. After 100 students have entered the school, which locker doors will be open?*

This locker problem provides students with a problem-solving situation from which are to emerge fundamental number theory concepts. Students are given time to explore and grasp the essence of the problem and its solution before any introduction to the Number Theory unit. This type of challenge presents students with the opportunity to negotiate mathematical meanings, to clearly articulate their ideas, and to experience the struggles often necessary in seeking solutions to problems. They also have a chance to put some of the problem-solving strategies they have learned (e.g., organizing a list, finding patterns, and/or solving a simpler problem) into practice while trying to make and test conjectures and find a possible generalization.

Students participated in this type of problem-solving activity daily, and their reflections were sometimes based upon their experiences during class. Other times their reflections were more open-ended questions related to specific mathematics content or to some metacognitive aspect of their learning. In the sections that follow, we provide examples of students' reflections, and excerpts from the interviews, that are a subset of the data which led to the emerging themes that we discuss. Excerpts from interviews, and important phrases from students' reflections are presented in bold in order to emphasize the specific items

on which we offer an interpretation. Also, the students reflections are set apart and italicized for easier reading.

Beliefs About Self As A Doer Of Mathematics

When talking with students and looking at their reflections, we observed that students addressed three issues: (1) their view of the importance of mathematics to them, (2) their feelings toward mathematics (liking or disliking), and (3) self-assessment of their ability to do mathematics.

Respondents offered their opinions on how they perceive mathematics and why mathematics is important or useful. Sometimes their opinions were explicitly stated in the journals, and in other instances, we inferred their opinions from general comments.

Katia, when reflecting on the usefulness of mathematics after about three weeks of T104, said the following:

Katia - 1/26/90

*In high school I didn't really like algebra. **I often asked how it could possibly apply to anything I had to face in the future. The only good reason I could find for taking algebra was to help me later on in college.** I didn't mind taking geometry because it wasn't near as difficult. Now that I'm in college, I have a little different idea about math. **I never used to care why a problem was done the way it was just as long as I could get the right answer. Understanding the why a problem is done the way it is becomes more of a necessity.** Being able to understand lets your mind think in a different way. I have learned a different way in which to apply my thinking. **Math has a different use than when I was in high school.***

It seemed to us that Katia was reflecting and tracing back through the evolution of her ideas about the importance of mathematics. She seemed to have gained new perspectives about mathematics since her experiences in high school, and possibly now in T104 she was at a point where she could articulate her growing views about different aspects of mathematics.

A reflection from Helena on the same topic, approached the same issue from a broader perspective.

Helena - 1/26/90

Mathematics is life. Everything that we are connected with in some way deals with math.** Even some of our descriptive words come from math terms - for example - the top of a table is called a plane, the tip of a pen is called a point, etc. It is a tragedy that all of this is taken for granted. I think a big part of the problem starts with children when first learning "math", in the formal sense in school. **A child enjoys doing various math-things (such as counting marbles) that aren't called math. Once the child is taught concepts of math, math becomes a terrible subject which he hopes to avoid. This is such a shame - because math is fun . . . . Once I realized how much I had already been using math** - for cooking, grocery shopping, crafts, etc., I really enjoyed it. **It became a challenge to figure out why things occurred as they did using math to solve my problems.

Helena saw mathematics in most everyday occurrences, but she also perceived the mathematics that is taught in schools to be distinct from mathematics that she naturally encountered in her everyday life. We inferred from the way she put her thoughts together that she was coming to see mathematics differently

than she did before, and that she understood why she, as a child, perceived mathematics outside of school and mathematics inside school as disconnected entities.

As Marta reflected on this same question, she first tried to define mathematics as a dictionary might, but then added her own interpretation of what mathematics is and its value to her in light of her experience up to this point in T104.

Marta - 1/27/90

*You've asked us to write what I think math is. **I can honestly say that I have never sat down and actually thought of a definition of math.** I think I'm going to look up the word in the dictionary in order to get the correct definition. **Then I will write what I feel math is compared to the dictionary definition.** My dictionary says that mathematics is the science that deals with quantities, expressed in numbers or other symbols, and the relations between them. I must say that I agree with that but **I have never thought of math as a science. All my life math has been a subject that I have taken and enjoyed. I think because there are so many different types of math (geometry, algebra, trigonometry, probability, etc.) I've never thought of an actual definition.***

*Over time my view has changed because each year I've taken math and every time it gets harder and we study new and different things. Some topics such as trigonometry never made complete sense to me. I found myself struggling in that class more than any others. **Since I've taken this class my views have changed even more. This is the first time I've ever had to sit down and completely understand why I use the equations that I was always told to use.** I find this class interesting and the time goes by quickly because we do lots of activities. The format is different from any other math class I've taken.*

Each of the three respondents above came from the same class, and were responding to the same issues. The instructor of this section tried to maintain a personal dialogue with the students throughout the course via their journal entries, and it is interesting to observe how uniquely each student kept the conversation going.

Another direction from which the preservice teachers voiced their attitudes toward mathematics was regarding their like or dislike of mathematics, especially when talking about previous mathematics experiences. In our interviews with the eight preservice teachers (PSTs), we asked them about mathematics courses that they had taken prior to T104, and what they liked or disliked about them. All of the respondents had taken the finite mathematics course that was a prerequisite for T104, but not all of their high school mathematics backgrounds were the same. Melinda said that in high school she studied geometry, algebra, and advanced algebra. When talking about her feelings towards those classes, she was emphatic about the fact that she **hated mathematics**! She did not like just working with numbers without applying them to real life. Her thoughts about mathematics at the end of T104 included, "**Now I don't hate math any more. This is the first math class I've ever enjoyed. It's the first time I understand math.**"

Helena told us that she had two years of algebra and one year of geometry in high school. Mathematics was her favorite subject in high school, and she intended to make mathematics her

specialization in elementary education. However, Helena did not always like mathematics, as we inferred from the following reflection.

Helena - 1/26/90

I used to <u>hate</u> math until I got into high school. The teachers were just worried about trying to pound the facts into our heads. They did not make it fun or show us how it fit in with our lives and we could not see it for ourselves. Once we were in high school, however, the teachers emphasized the daily aspects of math. Math became fun and a challenge! I think it is so much better to learn math in a relaxed environment that encourages thought-provoking ideas.

Unlike Helena's high school experience, Teresa's was often negative. She had advanced algebra, geometry, and trigonometry in high school. She liked her college finite mathematics class, but she did not like high school trigonometry and advanced algebra because the mathematics started becoming more abstract. **"I was not very strong in math."**

Lauren took algebra and geometry in high school. She enjoyed all her mathematics classes, but **"people thought I was crazy to like math."** Juliette's high school mathematics included algebra I and II, geometry, trigonometry, calculus, probability and statistics. She said, **"I liked most, but I hated calculus. I could do it, but sometimes it was challenging, but eventually it clicked."**

Marta had nothing special to say about her likes or dislikes of her mathematics classes. She did tell us that she had taken algebra I and II, geometry, trigonometry, and analytic geometry, but she offered no insights into her feelings about those courses. Katia had two years of high school algebra, geometry, trigonometry, and college algebra. She liked all of them but algebra. **"It didn't come to me as easily. I'm not good at word problems. Other courses, like geometry were more defined."**

Talita reported that she had taken algebra and calculus. She didn't like her professors lecturing and assigning problems in high school, and she did not enjoy her finite mathematics class in college, which was taught in a similar manner.

Several of the students' comments above indicate that their liking or disliking of mathematics was related to their perception of how well they performed in mathematics classes. In our interviews and in some of the reflections the students talked about their opinions of their ability to do mathematics.

When asked to locate where she sees herself on a mathematics competence continuum, Katia said, **"It depends on the course. Basic algebra and geometry I would say I was strong. Detailed algebra I'd say neutral."** On her confidence in mathematics, she recognized the influence of T104 and expressed that **"My confidence in problem solving is high now after experiencing T104."** She went further to discuss how her views about her mathematical problem solving abilities changed, **"I used to stop when I got stuck on a problem. Now when I have dead ends, I don't stop so easily. Having to explain things in T104 has helped."**

Talita rated herself as somewhere between neutral and strong in her mathematics abilities, but she added that **"I feel a lot more confident with fractions now than I did before."** Reflecting upon her abilities as a problem solver, she said, **"Now I know you have to gradually work a problem. There are not quick simple answers. You have to see different parts."**

Two other respondents from the same class, Helena and Marta, both characterized themselves as in the range from neutral to strong, closer to strong. Neither one acknowledged much change in their competence or confidence in mathematics after experiencing T104.

Lauren was a returning student, coming back after being away from school for more than ten years. She rated herself as medium to strong in terms of competence and confidence now, but cautioned that if we had asked her this question before T104 her answer would have been much different: **"Having been away from school for some time, I did not feel so confident in my math abilities."** However, she recognized how her views of mathematics had been broadened after T104 as she talked about herself as a mathematics problem solver: **" If you have an idea of how to approach something and you demonstrate knowing how to do it, it is more important than only getting the right answer at the end. Now I realize that math has gray areas, not just right or wrong."**

The other three students from Lauren's class, Juliette, Teresa, and Melinda, all perceive themselves as better problem solvers after experiencing T104, and feel more confident and competent now. The three voiced that they tended to be scared or panicked at the beginning of T104 when they had to solve problems. When looking at their journals, our impressions of their self-assessment, regarding their problem-solving ability and mathematics in general, were similar to what they voiced in their interviews.

The students' views about their evolving abilities in mathematics not only gave insight into their attitudes toward mathematics and themselves as doers of mathematics, but also revealed their understanding of what it means to know, to learn and to teach mathematics. These views, along with other questions addressed in the interviews and journals, led us to a better understanding of the students' perceptions of knowing, learning and teaching mathematics.

Beliefs About Knowing, Learning, And Teaching Mathematics

Throughout the course, there was a great amount of frustration on the part of students because the students wanted to be given some absolute answer to the problems and wanted to be told what they needed to know. We hypothesize that part of this frustration was a result of the fact that they were still operating under their long-held assumptions about mathematics as a rigid body of knowledge governed by strict rules and procedures. The relative impact of T104 on students beliefs about mathematics has to be understood in light of the twelve or so years of mathematics schooling that these students had already experienced prior to T104.

We realize that one semester of such innovative instruction cannot completely overcome long-held beliefs that students have about mathematics. However, our study shows us that the innovations of T104

have the potential to shake up students' belief systems, and to stimulate their rethinking of what it means to know, learn, and teach mathematics. What follows is a synopsis of the respondents' positive and negative reactions to the course regarding these issues as expressed in the interviews at the end of the semester, as well as some excerpts from their on-going verbal and written reflections during T104.

We begin with some positive reactions. Most students felt that the style of teaching of T104 was the best way for them to learn mathematics, but they were not sure that this was the way that was best for everyone. They also contended that their experiences in T104 continually prompted them to think about the "why" of mathematics and to look for the reasons behind what they do when using mathematics. This seemed important to the respondents regarding their future role as teachers.

By the end of the course, most of the students really liked working in groups. However, the interviews revealed that students did not like the idea of groups or group tests at the beginning. As they started to experience group work, they began to appreciate this aspect of the course. But some of the respondents did not enjoy having to change groups periodically, especially when regrouping came prior to an exam. We believe this was a natural reaction due to their prior beliefs about the importance of grades in a mathematics class. One negative reaction to T104 voiced by most respondents was that they did not think it was fair to have a group grading component in the overall assessment process.

As a result of working in groups, T104 students began to see that there is often more than one way to approach a mathematics problem. Some students enjoyed the fact that they changed groups periodically because this allowed them to meet most of their classmates on a more personal level, and exposed them to a lot of different styles of learning.

One student, Helena, expressed a little unease about working in groups at the beginning because she did not feel comfortable with people her own age. She had been raised in a situation where she was mostly around people who were much older than herself and she felt that she could not relate well with her own peers. She recognized value in the concept of group work, but was also aware of some of the difficulties associated with it.

Helena - 1/24/90

Cooperative learning is a wonderful way to learn. So much can be accomplished. It helps because not everyone thinks the same way. This allows for new or different insights on a problem. *It is a lot easier to remember concepts when you are able to figure them out yourself or with the help of your peers. Sometimes a teacher just can't communicate a concept so that it is understandable.* ***There is a problem with team-learning however. The key to this is the team.. It only works efficiently if everyone works together and communicates with each other.*** *My group did great in the beginning. We were able to get a lot done, and we had fun. Now, we don't get along very well. It takes a long time to get anything done and often we don't finish everything we need to. It has become a real chore to work together.* ***I think the problems started when we had to assume a role in the problem-solving*** *(i.e., calculator person, captain, and questioner). We started developing a distinct difference of opinions. As long as we were able to just work we were fine. However,* ***because each of us have a different personality, when we had to assume a specific role, we started having problems.***

Helena's perspective on group work evolved quite a bit throughout the semester. A later reflection exemplifies this.

Helena - 2/14/90

*Today was test day. **I was very anxious about the group part of the test.** This part went rather well - I should have been more worried about the individual part . . .*

*I feel like I have come a long way in the class. Even though my scores have been well so far in the class, **I feel as if it is not the most important thing. I have learned different ways of dealing with people - I have improved greatly.** This is what is most important for me. I have been able to get a new insight on myself. I really enjoy math and spend a great deal of time at it so I do well. **However, I have realized that I get overbearing - I have learned that class participation is not just giving answers and explaining them, but rather it includes actively allowing others to work out problems and giving suggestions along the way.***

Helena pointed out how cooperative learning can be beneficial for herself and others as learners of mathematics and future teachers. At the end of T104, she and others expressed that changing groups periodically was valuable to them as future teachers in that it gave them practice at dealing with a variety of personalities, and forced them to explain themselves in more than one way in order to make sure everyone understood (as a teacher must do).

In one of Melinda's reflections, we saw a connection between her reactions to group work with her changing views of what it means to learn and to teach mathematics.

Melinda - 3/28/90

*Before having this class, I was not confident at all in my ability to solve problems. In fact, in rereading my early journal entries, I am amazed at how much my views have changed. **By working problems with my group, I have become much more confident in my own ability to do problems. I think this is because before coming to this class, I really didn't know how to approach a problem. The teacher always told us exactly how to solve a problem. However, in this class, we have been forced to solve the problem on our own with a group. I realized that I don't have to have a teacher tell me everything. I can figure out problems.** Also working in the groups has taught me to use several different approaches to solve a problem rather than just the way I would do it. **Because we have been forced to figure out problems ourselves and because we have been exposed to many different methods of solving problems, my own confidence in my ability to solved problems has increased.***

By the end of the semester, Melinda had the following things to say about her beliefs of learning and teaching mathematics, and also about how group work could influence learning and be beneficial for teaching mathematics.

Melinda - 4/24/90

*When I look back on this semester. I'm amazed at how much I've learned. At first, I thought the group problem solving concept was a waste of time. However, what I've learned has proved me wrong. **To begin, I've learned that I can solve problems on my own; I don't have to have a teacher guiding me every step of the way. I realized that this is a good thing because I am going to be the teacher!** I think this has also made me more aware that students will be able to work things out without me always being there Besides gaining more confidence, I have learned to work with others better also. **Our groups really worked as a team to accomplish a common goal. Many times we had to "teach" each other because we had to rely on one another. It's***

important that teachers learn how to teach by communicating clearly, and we certainly got a lot of experience doing that. I have learned so much about math this semester that I can't even begin to name everything. However, the thing that stands out in my mind the most is the way I look at math problems. Before, I simply did the step-by-step procedures. Now I honestly understand what is really happening. I also don't give up on math problems so easily - I've learned to look at them in several different ways. My problem solving techniques have definitely improved. I could go on and on about what I've learned in this class, but I couldn't do it justice with words. I just know that I was very leery of the teaching techniques used in the class; now that I have experienced them, I believe I will be a better teacher because of them.

Upon reviewing all of the respondents' interviews and reflective journals, we came to an understanding of some of the views of learning, and teaching mathematics that had been challenged by T104. The students' perspectives about the teacher's role in a mathematics class had changed in some cases, although we are not sure that they've concluded that this is a better way to teach. Most of the students saw their T104 instructor as a coach when they needed guidance, rather than a dispenser of knowledge, while they saw their own role in the learning process as more of an active participant, and felt more responsible for their own learning.

According to the respondents, the three components of T104 that seemed to affect their beliefs most were cooperative learning (mostly regarding the sharing and communicating of ideas), learning via problem-solving activities, and reflective writing. Marta expressed her thoughts this way.

Marta - 5/1/90

*Now that the semester is over, I have a chance to think back over my classes and what I've learned. T104 was a class that I really had to work at to get a decent grade. As time went on, I started to get the hang of how things worked in the class. **I enjoyed working in groups and being able to talk things over with other people. This enabled me to get experience with explaining my thoughts and knowledge and also being able to get help from others when I needed it. It was also nice to get away from the traditional teaching style.** However, I feel that it could've been used a little bit more perhaps to go over what was assigned in the text book.*

Similar sentiments were not only voiced through written reflections, but also expressed in the interviews of several respondents. Teresa said that

"working in groups was the best component of T104. It helps with understanding, and you don't have to be afraid of asking, and in the groups they are more patient and have more time to talk with you. It helps to explain to people on a small scale (if you are timid). It's good to work with others and to learn differences in personality." Reflecting about the value of changing groups after some weeks, Teresa added, **" It was good to change groups because it helped us to learn how to explain things for different persons. This will be very similar to teaching different kids. However, some students need to know how to work in groups because some people say, this is right, and they try to dominate the whole group."** Her written reflections included:

"They helped to organize my thoughts . . . but I did not like to have my reflections graded, especially on insight."

Katia also commented on the written reflections, saying, **" It was not easy to write them. Sometimes I'd write and go back to see if I explained well. I was not sure what the teacher wanted."** This instance depicted how a student can believe that the important thing in the class is to try to figure out what the teacher has in mind, and to find the correct answer for the teacher. Even though the T104 instructors had continually emphasized the idea that there was not always one right answer, and that the students needed to find their own way to understand a problem, there was still evidence of lingering beliefs that the important idea in mathematics was to get the answer and to please the teacher.

Helena expressed in the interviews that when it came to written reflections,

"at first I despised it. I'm not a good writer, but it helped to put thoughts on paper. I saw value in needing to communicate with people. It helped me to communicate and helped with tests when I had to write it out [the solution of problems and explaining why they worked]. **I really put effort into writing, and the grading and the dialogue with the instructor** [the written comments on the student's journal entries] **helped make me see if I was on the right track."**

By looking at the attitudes respondents expressed towards their views of knowing, learning and teaching mathematics it appeared that the students' change in perception on these issues resulted from experiences of emotional and cognitive struggles throughout the semester. Students' attitudes toward mathematics as learners and future teachers, and their views of what mathematics is, had been shaken up by this course. We made this inference after analyzing each respondent's complete journal and noting episodes of disequilibration and the struggles they had to face when their prior views of mathematics and learning mathematics were challenged in this course.

The Disequilibration Process

We label this section the disequilibration process in order to depict, in the Piagetian sense, the moments when students' mathematics belief systems were thrown into a state of unbalance. Throughout the semester, the students' feelings and attitudes toward mathematics and learning mathematics and toward the course, itself, seemed to oscillate. We came to this interpretation by following the students written reflections throughout the semester. In some cases the students also voiced the inner turmoil they experienced in T104 when they talked with us in the interviews.

By working cooperatively to solve problems and to make sense of mathematical concepts, students learned to reflect, communicate, and articulate their thoughts in such a way that they sometimes were able to resolve their disequilibrium in verbal and written forms. The way one student viewed mathematical understanding obviously changed as shown by the following remark:

"This class never ceases to amaze me... I never thought I would be excited about triangles, but when you actually discover their properties instead of simply being told, it makes the whole idea much more meaningful."

In brief, students' beliefs about solving problems and what it means to know, learn, and teach mathematics began to be questioned by the nature of T104. It was as a result of this questioning that students began to build beliefs and attitudes which were appropriate for continued personal growth in mathematics.

The first excerpt we share exhibits a student's cognitive conflict regarding class expectations at the beginning of the semester. She particularly comments on how she is unsure of the different components of this mathematics class.

Juliette - 1/12/90

In class we stop after every activity & write down what we think. What's the purpose! Well here's what I wrote. In the last 20 minutes of class I've seen other ways to look at patterns. ***Gee, I never thought of that, the phrase pops up often. I just have to realize that different ways of accomplishing the same thing are possible.*** *Sometimes I over look that fact. The pattern for Monday is gonna be hard!*

When we solve problems in groups we need to devise a plan of attack with the groups. Work with others in the group they have ways of thinking that we don't. ***Talk things out visualize & then once a solution is obtained, question it. It is really correct is this the only way? Are there other possibilities. We're always writing down what we think in here! Good or Bad? I'm not sure!*** *We'll see how the semester goes.*

Juliette was questioning the value of writing in mathematics, but she displayed surprise at what she discovered and learned from what she wrote. She also questions other steps in the learning process of T104, that of group work, communication, and the questioning of the reasonableness of results, and she was still not sure if this was a good way to approach mathematics or not.

Teresa also expressed doubts about the class at the beginning of the semester, but her reflections illustrated more emotional dissonance combined with cognitive questioning.

Teresa - 1/8/90

I am not too sure about this class. It really scares me because I have a hard time thinking quickly & suddenly. I would rather have homework problems I could take home and practice on. I just hope I can make it through this class. I try really hard but if you don't get what's going on sometimes it just won't soak through. *I do think the instructor tries to make things interesting by giving us challenges & creating games for problems. These teaching methods will come in handy for teaching small children. I wish there were other things for us to do besides deriving formulas. I would like to maybe do a little algebra or geometry.* ***I really don't understand why we need to take this hard of a class when the mathematics we will be teaching is so simple. I probably won't teach math at all because the children usually have separate rooms for reading & math but it will give me more opportunities.***
I am really excited about teaching school. I wish it was time to get a job now. I am definitely ready to work rather than go to school and study.

Teresa started reflecting about her concerns and fears about T104. She seemed to be more comfortable in the traditional style of mathematics class, and she wondered why she has to take a mathematics class that she perceived to be more difficult. It was interesting to us that what seemed to bother her was that this class presented a challenge for her because it did not follow the normal patterns of the mathematics classroom experiences that she was used to in the past. She was also not sure why she had to take a mathematics course of this type since, in her perspective, it would not be relevant to her career as a teacher.

Teresa had some misconceptions about elementary school teaching, and her instructor, Mary Kate, engaged in a dialogue with her throughout the semester to talk about these perceptions. In her interview with us, Teresa was able to look back over the semester conversations and she expressed a clearer understanding of her future teaching role, and how mathematics would play a part in it.

Teresa continued to compare her traditional mathematics learning experiences to those that she had in T104. She still felt uncomfortable with the idea that the teacher's role in mathematics was not so directive.

Teresa - 1/17/90

I do not like the way we have to guess on problems. I do not feel like there is any skill involved in this procedure. I wish there was some way I could use a process to or steps to figure things out. I wonder if the things I learned in Finite (M118) will even help me in this course. I just hope it all goes OK. I really want a decent grade out of this class, but right now it seems like it will be really hard. I wish there were more directions like when a question is asked I wish that what the instructor wants you to do was more clarified. I am not asking for a procedure or an answer. I would just like to know what it is that the person wants you to do. There are many ways of looking at things, but there is only one way they want you to do it.

We noticed through the reflections that, as the semester progressed, other respondents expressed their struggles between the fact that knowing mathematics means to show correct results versus the fact that knowing mathematics means to display understanding and to be able to explain the reasons behind the mathematics.

Lauren - 2/7/90

We spent lots of time working on operations in bases other than 10. Unfortunately, the way I did the problems as homework has not been ruled as an acceptable method of doing this. We must show that we truly understand the concept behind regrouping by performing the operation in the given base. I can do this - but I hope nobody "upstairs" in the math department phrases a question on the exam next week asking me "how would I do" or "can you" because I might be tempted to press my luck and tell them!

The main problem I have in this stuff is that I have functioned within base 10 for regrouping and recognizing place value for so long. I've spent more time trying to prove to myself I really know how and why this works in the past 6 days than in the past 30 years! Can it be that I have been doing these operations without knowing what I've been doing? Yes! I don't think so, I've spent too much time helping third graders learn regrouping. I've just never tried to explain this from scratch before.

When teachers have access to reflective writing, such as Lauren's entry above, they may get a glimpse at the kinds of struggles the students are facing. In the case of Lauren, it was beneficial for herself to put an effort into her reflections, and to keep a record of all her personal cognitive and emotional dissonance, upon which she could reflect and learn from throughout this semester and beyond. This opened a clear channel of communication between herself and the instructor.

As we analyzed the reflections toward the end of the semester, we discovered students' writings that showed more maturity and responsibility toward their own learning in mathematics. However, the students were still questioning themselves and the value of this mathematics experience, which leads us to believe that they had learned to be more critical of their learning experiences and themselves as learners and were able to live with and accept episodes of disequilibration.

Juliette - 4/25/90

It's hard to write down what I've gained from this class. ***At times I hated it, and wondered why we were wasting our time with this stupid class. But sometimes I saw the light & realized but it didn't always click. What I did learn was how to work in groups, I see*** *how it's useful but with kids? I have to know so that I can teach them, but? Will I really teach them this? I don't know?* ***As a freshman I guess I have I lot to learn & realize . . .***

The style of class was kinda hard to get used to just because it was so different . I think it should have been easier to adjust to if Mary Kate was more organized. Sometimes I felt like we missed things just because Mary Kate wasn't organized. Next year will be better for the students because we were the test group but what about us? Hopefully things will improve.

The respondents in this study were all well aware that this was the pilot run of T104, and many of them expressed the feeling that they were guinea pigs. This feeling was one of the motivations they shared with us for wanting to participate in this study. They wanted to offer their views on the course to someone who was interested in prospective teachers' thinking about mathematics, and possibly offer insight from the student's perspective that could help in the revision of T104 for future students.

The power that reflections have in inspiring dialogue between students and the teacher is demonstrated by this last emotional entry from one of the respondents' journals. Here Melinda directly addressed her instructor regarding all the the feelings she had about her experiences and what she learned in T104.

Melinda - 4/26/90

Mary Kate I'm getting ready to walk into our last T104 class. I'm a little sorry that this will be our last class together. This past semester there were times when I hated T104, and yes, I even got frustrated with you!! However, something very special has happened to me this semester. All my life I have hated math with a passion. Now that I've had this class I can't believe that I once felt that way. You have taught me that math has meaning behind it *- it is not just doing step -by- step procedures with numbers.* ***I am so glad that I took this class before I started teaching. I would never have wanted my dislike of math to reflect on my class. Now math is exciting, and I will try my best to keep more "***

Melindas" who don't like math from walking out of my classroom feeling that way. I've already said that I can't begin to name the many things I've learned in this class, especially about being a good teacher. T104 (as a course) isn't perfect yet, but Mary Kate I could tell that you gave it your all. I just wanted to say thank you for teaching me that math doesn't have to be hated. I will never forget this class.

In the interviews, the respondents were able to think back on the whole T104 experience and remembered some episodes that challenged their emotions and their ways of thinking about mathematics. Some of them recognized that the written reflections serve as a testimony for them to observe now and in the future the ways in which they evolved. When respondents agreed to let us review their written reflections, and to look over their entire notebook, they wanted to make sure that we would return their notebooks and journals so that they would have them for the future.

The three emergent themes of attitudes toward mathematics; knowing, learning, and teaching mathematics; and disequilibration that were explored above were all linked to the main thrust of T104, which urged students to be reflective about their thinking while doing mathematics and to be aware of their mathematical belief systems. From our perspective, the most important thing that might be gained from the experience of T104 is not that students necessarily display a positive attitude about issues surrounding mathematics teaching and learning, but that they begin to be more critical about how they know and learn mathematics and to reflect on their beliefs and how their beliefs might influence their ways of knowing, learning, and , possibly, teaching mathematics.

Conclusion

We learned that T104 students' belief systems were called into question as a result of experiencing a mathematics course taught from a new perspective. Long-held mathematics beliefs can be challenged when students are given the opportunity to take control of their own learning and to construct an understanding of mathematics via group problem solving.

It is evident that students' beliefs about the nature of mathematics and about themselves as "doers of mathematics" can fluctuate when they are continually asked to reflect on questions regarding these and related issues. It is possible that T104 students will continue to be more critical of their thinking and more aware of their mathematics beliefs due to T104's emphasis on the dynamic nature of knowing, learning, and teaching mathematics.

We hesitate to suggest that T104 actually altered students' beliefs about mathematics in the short period of four months, but we do believe that the innovative instructional practices implemented in T104 have the potential to challenge students' beliefs by bringing awareness of them to the surface.

By forcing students to find their own ways of making sense of mathematics, and by encouraging them to reflect on the challenges presented in T104, prospective teachers might carry this process of questioning and rethinking beliefs throughout their teaching practice. However, a more longitudinal approach to studying the impact that T104 might have in reshaping preservice teachers' beliefs and attitudes toward mathematics teaching is necessary. This study served as a springboard for a series of

related studies investigating how T104 might really affect students' belief systems, which might, in turn, have an impact on their future teaching actions.

We are skeptical of how consistent the information we gathered in this study might be with that which may be gathered in future semesters because the instructors of the first two sections of T104 were also involved in the planning, revising and evaluating of T104. We believe that these instructors probably put more heart into their instruction than future teachers might invest. As mathematics educators interested in the constructivist philosophy, these two instructors probably had added insight into their teaching of T104 that a more typical mathematics instructor might possess. It was this concern which inspired us to conduct interviews with the two instructors and also to include a more extensive teacher component in our next study.

In concluding, we feel we need to acknowledge some of the more influential components of T104, in light of the eight respondents' opinions toward how their beliefs were called into question. The students' reaction to the teacher's role, and their corresponding new roles as students responsible for their own learning and understanding, combined with the continuous reflection and articulation required in the group problem-solving component of T104, seemed to be most influential on students' belief systems.

We believe that more research is needed to investigate future teachers' belief systems about themselves as learners and doers of mathematics, and as future elementary teachers who will be responsible for teaching mathematics. We discovered that innovations in mathematics instruction have the potential to shake up students' belief systems, but more careful and systematic studies are necessary to discover the extent to which future teachers' belief systems can be truly influenced in more positive and lasting ways.

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