DOCUMENT RESUME

ED 389 618 SE 057 263

AUTHOR	Dann, Emily; And Others
TITLE	"Unconsciously Learning Something:" A Focus on Teacher Questioning.
SPONS AGENCY	National Science Foundation, Washington, D.C.
PUB DATE	Oct 95
CONTRACT	MDR-9053597
NOTE	11p.; Paper presented at the Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education (17th, Columbus, OH, October 21-24, 1995). For entire conference proceedings, see SE 057 177.
PUB TYPE	Reports - Research/Technical (143) -- Speeches/Conference Papers (150)
EDRS PRICE	MF01/PC01 Plus Postage.
DESCRIPTORS	*Cognitive Processes; Grade 7; Junior High Schools; *Junior High School Students; Longitudinal Studies; Mathematics Instruction; *Questioning Techniques
IDENTIFIERS	Combinatorics; *Representations (Mathematics)

ABSTRACT

The mathematical behaviors of a group of seventh grade students were observed as part of a longitudinal study of how children build mathematical ideas. The children, having built representations of their solutions to a combinatorics task, were challenged by their teacher to explain and discuss their ideas, and to extend them to similar situations. This report focuses on how teacher questioning facilitated students as they: (1) justified their ideas; (2) extended ideas to problems with similar structure; (3) made connections to previous tasks; and (4) generalized their conjectures in the context of isomorphic problems. (Author/MKR)

"Unconsciously Learning Something:" A Focus on Teacher Questioning

Emily Dann, Ralph S. Pantozzi and Elena Steencken

Paper presented at the Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education

(17th PME-NA, Columbus, OH, October 21-24, 1995)

"UNCONSCIOUSLY LEARNING SOMETHING:" A FOCUS ON TEACHER QUESTIONING

Emily Dann, Rutgers University
Ralph S. Pantozzi, Rutgers University
Elena Steencken, Rutgers University

The mathematical behaviors of a group of seventh grade students have been observed as part of a longitudinal study of how children build mathematical ideas. The children, having built representations of their solutions to a combinatorics task, are challenged by their teacher to explain and discuss their ideas, and to extend them to similar situations. This report focuses on how teacher questioning facilitates students as they 1) justify their ideas; 2) extend ideas to problems with similar structure; 3) make connections to previous tasks; and 4) generalize their conjectures in the context of isomorphic problems.

Several children have been observed over the course of a longitudinal study[1] of the development of mathematical ideas. During this time, they have been exposed to a constructivist classroom setting, where students are encouraged to build concrete representations and justify solutions to mathematical tasks. After two days of investigation into a combinatorics problem, students were asked how they felt about this activity. One student, Jeff, responded:

> I don't know, it feels, like, I know I'm, I'm, it's like I'm unconsciously learning something, like I know I'm doing something to figure something out, it's just that...yeah, like cause in math we'll go over a subject, and in science we'll say, well, we're learning about "Jane Adams" and we'll study her, but in this it's sort of like you just learn it over, sort of, while you're in the mid[dle]... [when you're] doing something.

It seems that Jeff has indicated an awareness that his learning was significantly different and unlike that which he has experienced in the past. Notice that he indicates that he has "unconsciously learned something." Although Jeff may not have been able to fully articulate his ideas, he did indicate that his learning occurred in the process of *doing* mathematics. The problem-solving activities in which he and his classmates were engaged had been designed to prompt students to search for meaning and build connections between previous, relevant experiences. It could be useful to analyze the details of the learning experience that prompted Jeff's response. This paper will detail a sequence of episodes in which teacher-student interaction and student-student conversations contributed to this process. We focus on the role of teacher interventions, through questioning and probing, while students are actively constructing ideas. We will present episodes

[1] This research is supported in part by a grant from the National Science Foundation #MDR-9053597 to Rutgers, the State University of New Jersey, Robert B Davis, and Carolyn A. Maher, directors. Any opinions, findings, and conclusions or recommendations expressed in this publication are those of the authors and do not necessarily reflect the views of the National Science Foundation.

that illustrate how teacher questioning plays a part in students' efforts to justify and extend those ideas.

Theory

Students, when investigating solutions to mathematical problems, often attempt to generalize their solutions based upon recognition of patterns. In addition, they gain confidence in their extensions as more evidence accumulates in support of their proposed pattern. This confidence may be confused with deeper understanding. For example, in a research study involving ten year old Stephanie's recognition of a doubling pattern after having built towers from plastic cubes selecting from two colors, a conjecture was made that 1024 unique towers could be built that are 10 cubes tall. (Maher & Martino, in press) One might have reasonably inferred from Stephanie's recognition of a doubling pattern that she connected the generation of towers from the pattern. Subsequent teacher probing, however, indicated that Stephanie could not match her generation of actual towers with her pattern recognition. In fact, in tracing the development of the idea, one finds that Stephanie took over one and one half years to build this understanding (Maher & Martino, in press). Vinner (1994) has reported similar findings; he cautions us that what may appear to be a meaningful generalization on the part of the learner may actually be "pseudo-conceptual" behavior. By this he means student-teacher and student-student discussion is based only on teacher cues and student guesses.

Research on teacher questioning (Martino and Maher, 1994) suggests that appropriate teacher intervention can facilitate students' building of justifications of problem solutions, particularly applied at points in time when students are cognitively ready to revisit their ideas. Questions which stimulate students to justify and generalize their ideas may give teachers a greater insight into children's thinking. Questioning used in this way may also provide an alternative model for students to consider as they engage in discussions about their own work. In fact, a long-term case study of one student, Jeff, who has been engaged in thoughtful mathematical problem solving since grade 1, has indicated qualitative differences in his ability to question other students and listen to their ideas (Maher, Martino, and Pantozzi, 1995).

The purpose of this research is to analyze the problem solving activity in which Jeff and his classmates were engaged that prompted him to claim that he had "unconsciously learned something." Specifically, we will focus on three episodes of student conversation that was triggered by teacher intervention in the classroom community that seeks to foster students' construction of mathematical ideas and creation of generalizations.

Background

The students in this study come from a working class school district in New Jersey which has been the site of an on-going longitudinal study[2] in classrooms centered around problem solving. Since grade 1, the students have participated in

[2] The study is now in its seventh year.

problem-solving sessions under the guidance of a teacher/researcher intermittently during the school year. The children were seventh graders at the time of this study.

Methods and Procedures

Thirteen children were seated around tables in two groups of four and one group of five. A camera at each table videotaped the activity. The classroom exploration took place over three days, consisting of two 80-minute sessions and a third session of 40 minutes. Videotapes were transcribed and analyzed by a research team, and the transcripts along with other records were used to produce a video portfolio[3] to trace the development, among individuals and groups of students, of ideas relating to the fairness of the games. The transcripts of the classroom sessions and follow-up student interviews, along with students' written work and assessments, researcher notes, and interpretations of students' work constitute the data for the study.

Design

As one strand of a longitudinal study, the children have worked on combinatorics activities. For this research, we report on the students' investigations of games of chance. The activity required that they determine the "fairness" of a game of chance involving rolling sets of dice and called for their determination of a suitable sample space. After speculating about the possible outcomes when rolling three 6-sided dice, the teacher/researcher introduced tetrahedral dice so that the students could more easily support the conjectures that they had made regarding the number of elements in the sample space. The episodes presented here refer to the students work with the tetrahedral dice.

Episodes

Three episodes from the videotape transcripts provide data for this study.

Episode 1: October 27, 1994. The students had shared their ideas about the number of possibilities when rolling three tetrahedral dice. They were then asked to consider two other cases, where three dice were rolled and when two dice were rolled. After some discussion, the students decided that there were four outcomes for one die and 16 outcomes for two dice.

1	Teacher:	Now, what if I'm rolling three of them [the dice]?
2	Bobby:	I've got it.
3	Magda:	Sixty-four.
4	Michelle:	No! Wait..It's more.

[3] See Maher, C.A. & Martino, A.M. (under review) Conditions contributing to a young shild's development of mathematical proof: A 4-year study. *Journal for Research in Mathematics Education.*

5 Bobby: Sixty-four.

6 Teacher: Why don't you write those out? I'd like you to show me a way of representing all those outcomes if you're rolling three.

Episode 2: October 27, 1994. Ankur suggested that rolling four dice would result in 256 possibilities. The students were again asked how it worked with fewer dice. Ankur responded to the case of rolling two dice by producing the first two columns of Figure 1. He made a tree diagram by connecting 1 to 1, 1 to 2, 1 to 3, 1 to 4, 2 to 1, etc., until there were 16 lines visible. When asked to interpret what he had done, he produced the 16 ordered pairs in Figure 2.

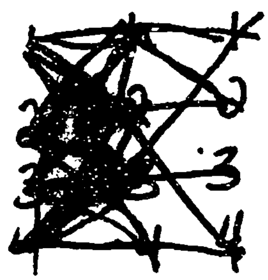

Figure 1

2	1,1	1,2	1,3	1,4
	2,1	2,2	2,3	2,4
	3,1	3,2	3,3	3,4
	4,1	4,2	4,3	4,4

Figure 2

Ankur was asked to extend the idea represented by his diagram:

7 Teacher: Okay, so you've convinced me now that there are sixteen. Okay, now how would you do it if you were rolling three now?

8 Ankur: Sixty-four.

9 Teacher: I want to be able to see them in my head, so show me how you get these sixty-four. Show me, show me how it works.

10 Ankur: Add four more numbers. [Writing the third column of 1,2,3,4 in Figure 2.] This one [pointing to the third column in Figure 1] has four here...you count these too... [the 16 possibilities indicated in columns 1 and 2]

11 Teacher: Okay, can you work together, work out a way to show me how you generate [your method?] to get to two fifty-six? Continue what you're doing and also the way you could keep doing it. Show me the way you begin to think about it.

In both episodes, the students proposed generalizations based upon their previous findings. After the students discussed these generalizations, they created various types of tree diagrams, representing the number of possibilities that they proposed. Figures 3, 4, and 5 represent a portion of the students' written work.

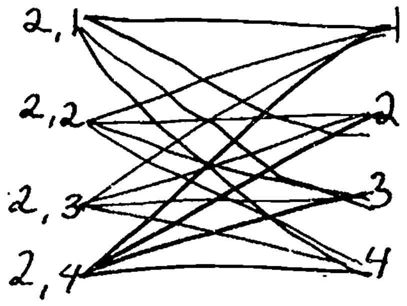

Figure 3

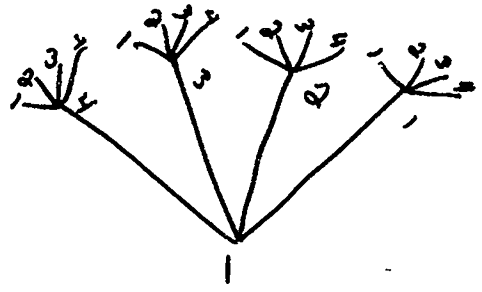

Figure 4

Episode 3: October 28, 1994. In this episode, the students discussed their written work in a small group interview. Michelle began by explaining her work in Figure 5.

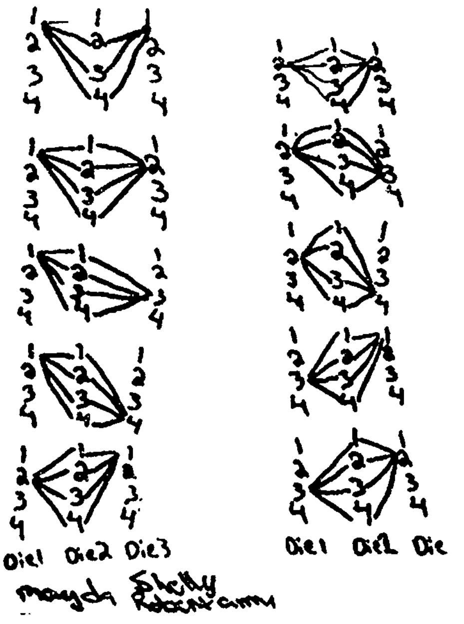

Figure 5

12 Michelle: This [Figure 5] is sort of a little like theirs [Figure 3] except theirs is separated.

13 Teacher: Can you explain it to me?

14 Michelle: If you roll the one on the first die... but then they all went to one [the third die shows 1] and then two and then three and then four [referring to the first four charts in Figure 5] and then if we did it with the twos and then threes and then fours [referring to the remaining charts - some not shown here].

15 Teacher: Oh, how neat.

16	Teacher2:	Can I ask something for clarification? These numbers [in the first column of each chart] go with the first die?
17	Michelle:	Yeah.
18	Teacher2:	And the middle ones are the second die? [the second column of each chart]Okay and how many would there be all together? How many outcomes? Possibly sixty-four? Help me to see that in this picture.
19	Michelle:	Um, it just shows like..
20	Jeff:	Sixteen for each [number of the first die rolled].
21	Ankur:	Yeah.
22	Michelle:	There are sixteen down here -yeah, it would be...
23	Ankur:	Four here, four here, four here, four here [points to each chart in Figure 5] four times four.
24	Jeff:	Four times four which got sixteen and then you multiplied.
25	Teacher:	I don't see that Michelle multiplied though. Can you see it Michelle?
26	Ankur:	These are all the combinations. [Referring to Figure 5 and additional charts, not shown.]
27	Michelle:	This [the first four charts of Figure 5] is just for the one thing [die]. There's like sixteen for each like like, like if you rolled a one for the first die number on the chart.
28	Teacher:	Huh, the question is, show us sixty-four. I guess I thought I saw it and now I'm not sure.

Later in the conversation, Michelle suggested how her diagram (Figure 5) might be extended if additional dice were rolled, while referring to her work in Figure 4.

29	Teacher:	Now if you were to do it, for rolling it four times, what would your chart look like, how would you do it?
30	Ankur:	Another four numbers on the side. [of the charts in Figure 5]
31	Teacher:	Another four numbers on the side, what would that look like?
32	Michelle:	I guess it would sort of be harder to...like...do it that way.
33	Teacher:	Well could you do it though? Is it possible?
34	Michelle:	Well it would be like this is, [Figure 4] ah...See here's our number one. I looked at their chart things. And like it's not exactly the same, but it sort of is and I remember we did um when we like did the towers we did like a tree thing. I don't know if any

of you people remember, but I remember when we did like a tree thing.

35 Teacher: What was the towers?

36 Michelle: When, I forget, I just knew when we were working on towers.

37 Jeff: Towers of three, and two different colors, how many can you make...

38 Michelle: And, yeah, then we did it in trees, so when we... I remembered that so we did it like that. And this is what you would roll on the first die [pointing to first tier of Figure 4] and this is like what you would roll on the second die [pointing to second tier]and this is what you could roll on the third die [points to third tier].

Near the end of the session, Ankur offered a general rule that he had developed.

39 Teacher: Do you want to tell us what that is? [Referring to a rule Ankur mentioned previously]

40 Ankur: Well it's the number of sides, that's a four, in this case, times, to the power of like the number of dice you have [Writes 4^4 on his paper.]

41 Teacher: Does that work, if you had a six-sided die and you were rolling it twice?

42 Ankur: That's six to the second power.

43 Teacher: Tell me, this [pointing to the base number]is the number of sides? And...

44 Ankur: This, the number here [pointing to the base number] is the number of sides. And this [pointing to the exponent] is the number of dice.

45 Michelle: But that's like four times four times four times four.

46 Ankur: Yeah.

47 Michelle: Oh, okay.

48 Teacher: Okay, so you had a general rule, didn't you? With x's and y's? You were showing me?

49 Ankur: Yeah.

50 Teacher: How does that work? Show me. Why don't you say...

51 Michelle: Is that [4^4]for the sixty-four?

52 Ankur: No, it's for two hundred fifty-six. Sixty-four is four to the third. Four times four is sixteen times four.

53 Michelle: Oh, OK.

54 Teacher: Ankur, suppose I had a twelve-sided die and I was rolling it three times, what would the rule be?

55 Ankur: Twelve times twelve times twelve, twelve to the third.

56 Teacher: [to other students]What do you think?

57 Jeff: I agree.

58 Teacher: So what's the general rule you're telling me?

59 Ankur: It's like if 4 is equal to x [writes x on his paper] and this is y [writes y as an exponent].

60 Teacher: So what does the x represent? Why don't you write it out?

Conclusions & Implications

These episodes indicate instances (lines 6, 9, and 11) where a teacher/researcher has posed questions that were designed to promote the interaction of students and prompt them into explanations and justifications of their ideas. We note that these questions were directed not only to elicit a response from one student, but to invoke the participation of others in the group. Instead of confirming the students' findings, the questions focused on further elaboration of their proposed generalizations (lines 31, 41). In response, Michelle made a connection to a previous task (line 34), and Ankur suggested a general rule (line 59).

At any point where students offer generalizations, teachers may make the decision to build connecting structures for students. It is in this "territory" where we believe that teacher decisions are critical. In the episodes presented, the students were given opportunities to revisit and share their ideas with each other. We suggest that Jeff's belief in having "learned unconsciously" might have arisen through his involvement in situations where the bounds of his inquiry were not externally framed. One implication is that appropriate teacher intervention, and ample time for students to build mathematical ideas, may be crucial tools in helping students build further connections between the deep ideas underlying their work.

The teacher questioning and student discussion presented here may serve to illuminate elements of "unconscious thinking." Such thinking may include the process of students' building powerful schemata through active reconsideration of ideas, prompted by teacher questioning.

References

Davis, R.B. (1984). *Learning Mathematics: The Cognitive Science Approach to Mathematics Education*. New Jersey: Ablex Publishing.

Maher, C.A., Davis, R.B. & Alston, A. (1991) Brian's representation and development of mathematical knowledge: A four year study. *The Journal of Mathematical Behavior, 10*, 163-210.

Maher, C.A. & Martino, A.M. (under review) Conditions contributing to a young child's development of mathematical proof: A 4-year study. *Journal for Research in Mathematics Education.*

Maher, C.A. & Martino, A. M. (in press). Young Children Invent Methods of Proof: The Gang of Four. In L. Steffe, P. Cobb, G. Goldin, B. Greer & P. Nesher (Eds.), *Theories of Learning.* Hillsdale, NJ: Lawrence Erlbaum Associates.

Maher, C.A. & Martino, A. M. (in press). Conditions for Conceptual Change: From Pattern Recognition to Theory Posing. In H. Mansfield, (Ed.), *Young Children and Mathematics: Concepts and their Representations.* Curtin University of Technology, Australia: Australian Association of Mathematics Teachers.

Maher, C.A., Martino, A.M., and Pantozzi, R.S. (1995) Listening Better and Questioning Better: A Case Study. In L. Meira, D. Carraher, (Eds.), *Proceedings of the 19th International Conference for the Psychology of Mathematics Education* (Vol. 3 pp. 82 - 89). Recife, Brazil: The Program Committee of the 19th PME Conference.

Vinner, Shlomo. (1994). Traditional Mathematics Classrooms - Some Seemingly Unavoidable Features. In J. Ponte and J. Matos, (Eds.) *Proceedings of the 18th International Conference for the Psychology of Mathematics Education* (pp. 353 - 360). Lisbon, Portugal: The Program Committee of the 18th PME Conference.