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ABSTRACT

This study examined effects of cultural and curricular variables on algebraic reasoning in early and middle adolescence. Four algebra curricula in England and Russia were included in the design. Two age groups were included in the samples: 10- to 14-year-olds, and 14- to 16-year-olds. Algebraic reasoning processes were examined using a written test and interviews. Profound cross-cultural and cross-curricular differences were found in students' algebraic deductive reasoning for both age groups. (Author)

Development of Algebraic Reasoning in Children and Adolescents: A Cross-Cultural and Cross-Curricular Perspective

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DEVELOPMENT OF ALGEBRAIC REASONING IN CHILDREN AND ADOLESCENTS: A CROSS–CULTURAL AND CROSS–CURRICULAR PERSPECTIVE

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This study examined effects of cultural and curricular variables on algebraic reasoning in early and middle adolescence. Four algebra curricula in England and Russia were included in the design. Two age groups were included in the samples: 10 to 14 years, and 14 to 16 years. Algebraic reasoning processes were examined using a written test and interviews. Profound cross–cultural and cross–curricular differences were found in students' algebraic deductive reasoning for both age groups.

Comparative and developmental studies consistently reveal cross–cultural and cross–sectional differences in mathematical reasoning (e.g., Crosswhite et al., 1985; Inhelder & Piaget, 1958). While cross–cultural variability has been established, specific cultural and social variables accountable for the variation remain unclear. Candidate factors include educational variables; family processes; cultural belief systems and practices; semiotic systems; social factors; and interactions between these variables. Sources of within–cultural, cross–sectional variability in mathematical reasoning also remain unclear. Age has been firmly established as an important contributing factor, and cross–sectional differences have been primarily attributed to cognitive developmental stages (e.g., Küchemann, 1981; Piaget, 1983), or to an interaction between developmental and socio–cultural factors (e.g., curriculum, cognitive tools, SES) (e.g., Davydov, 1975).

There are inherent difficulties in identifying explanatory variables, and establishing mechanisms via which these factors affect reasoning. First, sources of variation are usually examined in isolation from one another. Consequently, critical sources of group differences, and interrelationships among contributing variables remain unclear (see, e.g., Reyes & Stanic, 1988). For example, within–cultural cross–sectional studies have attributed variation in algebraic reasoning to cognitive developmental factors without an accompanying analysis of the effects of socio–cultural and curricular variables (see, e.g., Küchemann, 1981). To develop a cohesive model of reasoning, multiple sources of variation must be examined within a single design (Stedman, 1994). Second, to establish relationships between specific socio–cultural variables and specific cognitive outcomes and processes involved in mathematical reasoning, sufficient variability has to be obtained in both sets of variables.

This study attempted to identify explanatory variables affecting mathematical reasoning (to detect and measure effects, and to point to likely candidate factors), and to establish underlying mechanisms by (1) examining multiple sources of variation in a single design; and (2) obtaining sufficient variability in socio–cultural contexts. The latter was viewed as a preliminary move toward establishing links between variability in specific socio–cultural factors and variability in reasoning.

To address the problems, two curricular settings were identified that incorporated profoundly different models for developing algebraic reasoning. The first curriculum, *National Mathematics Project* in England (Harper, Küchemann, et al., 1987), has a strong concrete–to–abstract orientation. The curriculum tends to replicate a natural progression in development—the progression from more concrete to more abstract concepts (e.g., in developing algebraic letter concepts and concepts of mathematical structure) (e.g., Piaget, 1983). Thus, *NMP* emphasizes inductive, case–based reasoning and learning—the investigation of a number of particular cases to formulate and to assess the validity of algebraic generalizations, emphasizing the importance of empirical checks. With respect to developing children's ability to create and operate on abstract algebraic objects, to recognize and use structure, and to perform algebraic transformations, *NMP* uses a procedural–to–structural curricular model (Kieran, 1992). Emphasis on less abstract numerical input–output interpretations ("procedural interpretations") of algebraic constructs precedes emphasis on structural interpretations, and on algebraic transformations.

The second, Davydov's elementary mathematics curriculum in Russia, assumes the natural progression in concept development is not the most efficient. Davydov draws on Vygotsky's distinction between development of spontaneous and scientific concepts. While spontaneous concepts develop as an abstraction of properties of concrete instances, scientific concepts develop in the opposite direction: from formal definitions of properties, to an ability to identify those properties in concrete instances. Formal education is viewed as the environment that can, and must, foster development of scientific concepts. Thus, Davydov's curriculum emphasizes abstract deductive, law–based reasoning—the logical derivation of particular (e.g., numerical) cases from general mathematical principles and relationships where those principles and relationships are first expressed algebraically. Davydov's curriculum can be characterized as abstract–to–concrete and structural–to–procedural. Concepts of "relation or structure" are developed prior to numerical work, and prior to emphasis on algebraic transformations.

Curricular variables across cultures were confounded with other potentially important variables such as language, family and cultural beliefs and practices, etc. To control potential confounds, and to investigate alternative sources of variation, two "non–experimental schools" were selected in the same countries. Schools were in the same geographical area, and had comparable student and teacher populations; however, they did not have curricula that were designed to develop specific kinds of algebraic reasoning.

This paper specifically examines curricular effects on components of algebraic deductive reasoning, including letter interpretation, formulation of equations, and children's understanding of the logical necessity of deductive conclusions derived from algebraic proof.

Method

For purposes of comparison, four groups were included: (1) students and graduates of Davydov's curriculum, implemented in Moscow School #91 in Moscow, Russia (n=120); (2) students in a non–experimental school in Moscow (n=89); (3) students in an upper school in England that had implemented *NMP* for seven years (n=120); and (4) students in an upper school in England with a "non–experimental" curriculum (n=120).

Outcome variables were measured through written open–ended problems and follow–up interviews. Students were tested and interviewed within their schools. The following task from the *CSMS* study (Küchemann, 1981) examined ability to interpret letters as variables:

Which is larger, $2n$ or $n+2$? Explain.

The following task, adapted from Clement, Lochhead, and Monk (1981), measured ability to formulate algebraic equations that represented verbally described quantitative relationships:

> Write an equation using the letters S and T to represent the following statement:
>
> "There are six times as many students as teachers at this school." Use S for the number of students and T for the number of teachers.

The following problem from Lee and Wheeler (1989) examined students' tendency and ability to formulate algebraic deductive arguments:

> A girl multiplies a number by 5 and then adds 12. She then subtracts the original number and divides the result by 4. She notices that the answer she gets is 3 more than the number she started with. She says, "I think that would happen, whatever number I started with." Is she right? Explain carefully why *your* answer is right.

Data were aggregated by age and culture–curriculum composites ("Groups"). Thus, four groups were used in the analyses: Russian non–experimental curriculum ("R–NEX"), Russian experimental curriculum ("DV"), English non–experimental curriculum ("E–NEX"), and English experimental curriculum ("NMP"). Subjects within each of the curricular groups were divided into two age groups: 10–14 and 14–16 years. Since the data were categorical, and frequencies of categories of responses were aggregated across groups and ages, log–linear analysis was deemed an appropriate approach to data analysis. Log–linear models were used to discern cultural, curricular, and age–related effects on algebraic reasoning.

Results

For Küchemann's task, Figure 1 compares the percentages of correct conditional responses (e.g., $2n$, when $n>2$) for the various curricular groups and age groups.

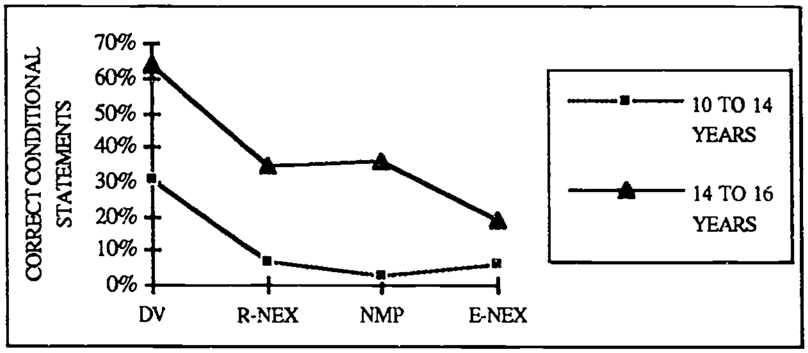

Figure 1. Percentages giving correct conditional responses

For interpretation of letters as variables, Group and Age had independent effects (goodness of fit $c^2(3) = 2.26$; p=0.52). The effect size of Group was large (0.53), while the effect size of Age was moderate (0.30). Data analyses suggested curricular effects. Davydov's group gave correct responses more often than other groups (p<.0001), and the English non–experimental group gave correct responses less often than other groups (p<.0001).

Figure 2 shows percentages formulating a correct algebraic equation in response to Clement et al.'s (1981) task. For formulation of a correct equation, Group and Age had independent effects (goodness of fit $c^2(3) = 7.49$; p=0.06). The effect size of Group was moderate (0.42), while the effect size of Age was small (0.14). Analyses revealed curricular effects. Davydov's group wrote correct equations more often than other groups (p<.0001). Experimental groups wrote correct equations more often than non–experimental groups (p<.0001).

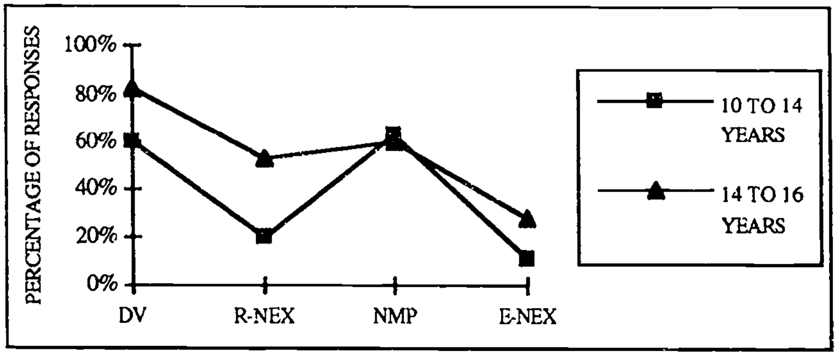

Figure 2. Percentages formulating a correct algebraic equation (S=6T)

Figure 3 compares the percentages of students independently formulating an algebraic deductive argument in response to Lee and Wheeler's task. For use of algebraic deductive reasoning, Group and Age had independent effects (goodness of fit $c^2(3)=0.85$; p=0.8375). Effect sizes of Group (0.5) and Age (0.67) were large. Analyses suggested cultural effects, and a combination of cultural and curricular effects. Russian groups formulated proofs more often than English groups

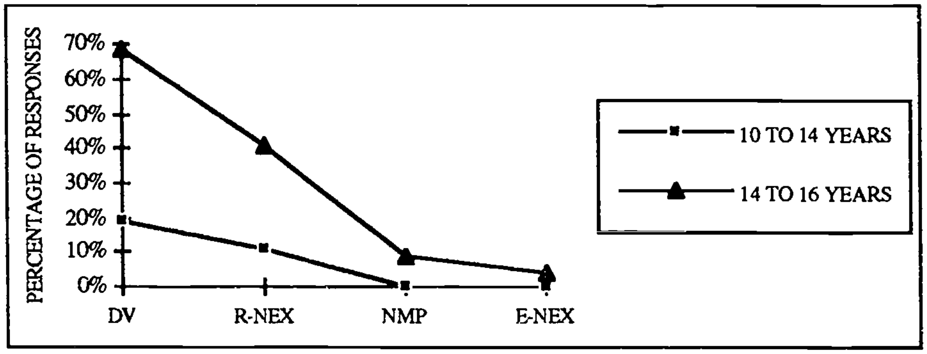

Figure 3. Percentages formulating an algebraic proof during the written test

(p<.0001), and Davydov's group formulated proofs more often than other groups (p<.0001).

English groups were more likely than Russian groups to use purely numerical reasoning on this task (p<.0001); e.g., 76% of *NMP* students used only numerical examples, with no use of algebra. "Empirical proofs" were often formulated in response to this task; i.e., children used inductive numerical arguments, concluding a generalization held for an infinite set after verifying that a generalization held for particular numerical cases. Curricular effects were evident: Davydov's group used "empirical proofs" less often than other groups (p<.0001), whereas *NMP* students used "empirical proofs" more often than other groups (p<.0001). Forty–seven percent of *NMP* students formulated an empirical proof, while 10% of Davydov's group used empirical arguments.

If a student did not use algebra on this item during the written test, he/she was asked to do so during the follow–up interview. Students' ability to use algebra as a tool for reasoning could therefore be examined, as well as their tendency to do so. When prompted to use algebra, 17% of high track ("Red Track") *NMP* students formulated an algebraic proof, while 38% of high track *NMP* students used an algebraic equation/expression only as a template to generate numerical examples.

Discussion

Davydov's group was more likely to interpret letters as variables, to formulate correct equations, and to formulate algebraic proofs. For algebraic deductive reasoning, differences between Davydov's group and other groups tended to increase with children's age—suggesting effects of instruction tend to increase with age. Though age is an important contributing factor in development of algebraic reasoning, comparison of younger and older children's responses across groups suggests socio–cultural factors can amplify development of algebraic reasoning—over–shadowing effects of age (Figure 3).

While the Russian groups and *NMP* group acquired component understandings required in algebraic deductive reasoning (variables, equations), there were profound differences in their use of algebraic deductive arguments. Differences did not appear to be due to across–group differences in children's tendency to use, or apply algebraic concepts and skills. Rather, findings suggested children in the curricular groups had acquired very different kinds of understandings of algebraic reasoning, constructs, and operations.

These results were consistent with other findings from this study (Morris, 1995). In comparison with other curricular groups, Davydov's group was more likely to use algebraic deductive arguments; to believe algebraic proof establishes "universal validity"; to use arithmetical structure; to manipulate algebraic expressions correctly; and to acquire concepts of generalized numbers, variables, and givens. In comparison with other curricular groups, the *NMP* group was more likely to use inductive, numerical arguments on proof tasks; to believe algebraic proof requires empirical support; to compute, rather than use arithmetical structure; and to use only procedural interpretations of algebraic constructs. In comparison with the English non–experimental group, the *NMP* group was more likely to acquire concepts of generalized numbers and variables; to formulate correct equations; and to manipulate algebraic expressions correctly. Thus, *while the approach developed some component understandings*, prolonged emphasis on inductive, case–based reasoning and numerical input–output interpretations seems to promote empirical, rather than theoretical reasoning (see Hatano et al., 1995). Using numerical reasoning, children attempted to establish "whether a generalization worked," rather than "why it worked."

When prompted to use algebra on proof tasks, *NMP* students tended to substitute numbers to make sense of algebraic statements, to test cases, and to generate empirical evidence. Russian groups operated at a different level—operating at the level of *relationship or structure*. This was particularly evident among Davydov's group. Approximately 70% of Davydov's graduates operated at the level of structure—writing proofs, and demonstrating an understanding of the logical necessity of deductive conclusions derived from proofs.

Findings suggest different curricular approaches tend to lead to different conceptual organizations of children's mathematical knowledge. This conceptual organization, in turn, affects how and whether children utilize and apply their algebraic knowledge and skills in the solution of particular problems.

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