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ABSTRACT

This case study was carried out with a student who was 9 years old and in the fourth grade when the study began. She was selected after the administration of an exploratory questionnaire to 66 pupils in primary school. The main features of the case were the absence of a common referent and a corresponding unit to tne free generation of references related to the addition of fractions. The girl was interviewed twice with the same instrument. The interviews consisted of 10 tasks. The link generated by this student between different classes of objects (referents) through a sum affected only fraction references (not natural number references). Likewise, the child was unable to establish a unit when facing the requirement to construct the additive situation by her own means while she could assign sense, recognize and select an adequate unit in simple tasks, and reconstruct whole processes. Contains 15 references. (Author)

Preservation of the Common Referent in the Addition of Fractions: A Case Study

Marta Elena Valdemoros

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(17th PME-NA, Columbus, OH, October 21-24, 1995)

PRESERVATION OF THE COMMON REFERENT IN THE ADDITION OF FRACTIONS: A CASE STUDY

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This case study was carried out with a student who was 9 years old and in the fourth grade when the the study began. She was selected after the application of an exploratory questionnaire to 66 pupils in primary school. The main features of the case were the absence of a common referent and a corresponding unit to the free generation of references related to the addition of fractions. The girl was interviewed twice with the same instrument. The interviews consisted of 10 tasks. We confirmed that the link generated by this student between different classes of objects (referents) through a sum affected only fraction references (not natural number references). Likewise, the child was unable to establish a unit when facing the requirement to construct the additive situation by her own means while she could assign sense, recognize and select an adequate unit in simple tasks, and reconstruction of whole processes.

The problem considered herein centers on the difficulties experienced by the child in the elementary school, when he or she must assign a common referent to the addition of fractions. Are such difficulties of a general nature and do they arise in the context of addition of natural numbers? Or, rather, do they come out in the field of fractional numbers, connected with cognitive processes of greater complexity?

Evidence obtained in some previous studies (Valdemoros, 1993a, 1994a) offers direct support to the outline of this problem, since they allow one to recognize that most of the students included in that research (attending third and fourth grades of primary school, whose teaching of such numbers is undertaken in these grades in Mexico) relate different concrete referents with certain sums of fractions; that is, they refer a specific additive situation to various kinds of objects. Likewise, the aforesaid study established that the difficulty to construct compatible references with the addition of fractions has always been accompanied by the absence of a unit of measure to which each fraction involved in the sum is referred, in the field of "problem invention" by those students.

Supported by the weight that linguists and semiologists (Ducrot & Todorov, 1981; Eco, 1991 among others) assign to *references*, at the level of meaning formation, we grant here great attention to those, in the concrete framework of addition of fractions. The correlative concept of elaborations built around such references is constituted by the *unit*, which has been widely recognized as a fundamental cognitive component, both for the construction of fractional number ideas (Piaget et al., 1966; Kieren, 1983, 1984, 1988; Bergeron & Herscovics, 1987; Hiebert & Behr, 1988) as well as for the resultant integration of their relations and operations.

Method

In order to make an in-depth inquiry, we designed a case study for third and fourth grades of primary school. The institution chosen was a public school with a good performance within the local educational system.

So as to select the subjects that would be interviewed, we administered to 66 students a questionnaire comprised of 13 adaptations of tasks previously pilot-tested and submitted to the analysis of several specialists in the area. We assigned the questionnaire an eminently selective character, since it was the starting point for the case studies. The questionnaire included 5 problems involving the identification of fractions (with the meanings of the part-whole relation, measure, indicated quotient, and ratio); 4 tasks centered on the equivalence relation between fractions, 2 problems referred to the pictorial resolution of a sum (in the presence of a given figure), and 2 tasks requiring the "invention of a problem" by the pupils (each one related to a sum and a subtraction of fractions and without any given figure).

Seven children were chosen to carry out individual, videotaped interviews. The design thereof was specific for each case taking into account its particular profile (in spite that they all showed difficulties with the construction of references, the intrinsic details thereof differed). All cases were controlled by means of a triangulation scheme consisting of the comparison of results obtained during the interviews with responses to the questionnaire and with the notes of an observer. The results were submitted to a qualitative analysis (Valdemoros, 1993b, Valdemoros & Orendain, 1994, Valdemoros & Campa, 1994).

The case presented herein is that of Belen (a 9-year-old fourth-grade student) who exhibited good performance on the questionnaire, where (at the "problem inventing level") the corresponding task required her to "Invent a problem which contains 1/5 + 1/10," and she wrote:

> Two ladies are going to make a cake and they need 1/5 of mixture and 1/10 of lard. How much did the two of them get together? 1/5 + 1/10 = 15/50 (a text without a common referent for addition and lacking of a unit of measure—a common occurrence among these children).

This obstacle was observed for 23 children of the described group (that is, 23/66).

Belen was interviewed twice. The first interview took place some weeks after the application of the questionnaire. The second interview was developed eleven months later.

Belen's Interviews

For Belen's case study we designed ten different tasks (Valdemoros & Campa, 1994):

- The "re-invention" of an additive problem with fractions from the questionnaire (see Fig. 1).

TASK 1
Invent a problem which contains 1/5 + 1/10

Figure 1

- Two tasks referred to the "invention of problems" with natural number addition and subtraction (see Fig. 2).

TASK 2	TASK 3
Invent a problem which contains: 12+6	Invent a problem which contains: 19-9

Figure 2

- Three identification tasks of certain fractions with the aid of concrete materials and in the presence of two continuous wholes and a discrete whole (see Fig. 3).

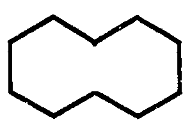
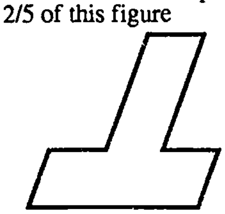
TASK 4	TASK 5	TASK 10
Use the blocks to represent 3/4 of this figure 	Use the blocks to represent 2/5 of this figure 	Identify 1/5 in the following set: 

Figure 3

- Two tasks for the reconstruction of the continuous whole from the part (see Fig. 4).

TASK 6	TASK 7
If △ is 1/3, draw 1	If ◇ is 1/4, draw 1.

Figure 4

- Finally, two addition and subtraction tasks with fractions, in the presence of a certain figure and by means of the manipulation of concrete materials (Zullie's geometrical blocks, 1975). See Fig. 5.

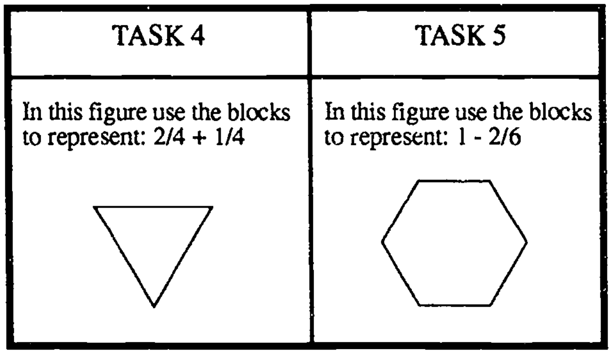


Figure 5. This was the common design for both interviews maintained with Belen.

The Main Results and Their Interpretation

During the first interview, when faced again with the task of fraction addition from the questionnaire, Belen "invented" a text of similar in nature to the above, with identical difficulties and semantic distortions (that is, she generated a text without a common referent for the sum and lacking of a unit of measure). She "invented" and adequately solved the natural number addition and subtraction problems. Belen expressed correct solutions with respect to the elementary activities of fraction recognition and reconstruction of the whole from the part. The girl, with more caution and effort, did also adequately solve the fraction addition and subtraction tasks, in the presence of an already identified figure and using concrete materials (geometrical blocks of different shape and size).

In the second interview, Belen exhibited results similar to those on the previous instrument. When we asked her what part of Task 1 could be changed, Belen wrote that the text was adequate and she wouldn't change anything in it. The girl easily solved the other tasks included in the second interview (specifically, the elementary activities of fraction recognition, the reconstruction of the whole from the part, and the natural number addition and subtraction problems). With more difficulty, she also correctly solved the fraction addition and subtraction tasks in the presence of an already identified figure and using concrete materials (Zullie's blocks).

Confronting both interviews, we confirmed that the main progress evidenced by Belen was the final development of a more efficient algorithm (during the second interview she wrote: 1/5 + 1/10 = 3/10). However, the central feature of this case—the generation of a text without a common referent for the sum and lacking of a unit of measure—didn't change.

In general, Belen did not exhibit difficulties in recognizing the unit in simple fractional contexts (identification of the fraction tasks). The girl could also configure the unit from the part. She also did not produce errors when she added and subtracted fractions in more complex situations and using concrete material (Tasks 8 and 9), because the respective unit of each task was already established. But, the most complex requirement—the selection of a unit for the sum of Task 1—couldn't be adequately solved by Belen when she produced the corresponding text. Due to this outcome, we infer that the unit couldn't be "re-signified" by the girl (that is, endowed with new meanings) at the last elaboration level where there were some evidences of incomplete semantic processes.

With regard to the loss of the common referent for addition and the construction of unsuitable additive referents when Belen chose a referent without taking into consideration the need of its preservation in the frame of addition, we confirmed that it had its origin in the terrain of fractions. The second interview showed us that the problem we detected during the first interview didn't disappear with posterior teaching. Maybe it was not considered an important cognitive obstacle for the student.

Conclusions

We confirmed that the link generated by Belen between different classes of objects (referents) through a sum only affected fraction references (not natural number references). She was able to assign sense, recognize and select an adequate unit in simple tasks, and complete reconstruction of whole processes.

The "re-signification" of unit in additive contexts (that is, the production of new meanings for unit in more complex frames) was possible when we presented her a certain figure related to the respective task. Facing the requirement to construct the additive situation by her own means, Belen was unable to establish an unit.

Perspective of this Case

We are now carrying out other studies of the difficulties related to the preservation of a common referent for addition of fractions among students in diverse public schools. The conclusions stated in the Belen's case allow us to establish the hypotheses of the new studies. The design and results will be communicated in future reports.

References

Bergeron, J. & Herscovics, N. (1987). Unit fractions of a continuous whole. *Proceedings of the Eleventh International Conference for the Psychology of Mathematics Education*. 357- 365.

Ducrot, O. & Todorov, T. (1981). *Diccionario enciclopédico de las ciencias del lenguaje*. Mexico: Siglo XXI.

Eco, U. (1991). *Tratado de semiótica general*. Barcelona: Lumen.

Hiebert, J. & Behr, M. (1988). Capturing the major themes. In J. Hiebert & M. Behr (Eds.), *Number Concepts and Operations in the Middle Grades*. Reston, VA: National Council of Teachers of Mathematics. 1-18.

Kieren, T. E. (1983). Partitioning, equivalence and the construction of rational number ideas. *Proceedings of the Fourth International Congress on Mathematical Education*. 506-108.

Kieren, T. E. (1984). Mathematical knowledge building: The mathematics teacher as consulting architect. *Proceedings of the Fifth International Congress on Mathematical Education*. 186-194.

Kieren, T. E. (1988). Personal knowledge of rational numbers: Its intuitive and formal development. In J. Hiebert & M. Behr (Eds.), *Number Concepts and Operations in the Middle Grades*, 2. 162- 181.

Piaget, J., Inhelder, B. & Szeminska, A. (1966). *The child's conception of geometry*. London: Routledge and Keagan Paul.

Valdemoros, M. (1993a). *La construcción del lenguaje de las fracciones y de los conceptos involucrados en él*. Doctoral Dissertation. Matemática Educativa-CINVESTAV, Mexico.

Valdemoros, M. (1993b). *Fracciones y referencias: El caso de los cambios arbitrarios de unidad en situaciones aditivas*. Matemática Educativa-CINVESTAV, Mexico.

Valdemoros, M. (1994a). Various representations of the fraction through a case study. *Proceedings of the Eighteenth International Conference for the Psychology of Mathematics Education*, II. 16-23.

Valdemoros, M (1994b). Fracciones, referentes concretos y vínculos referenciales. *Memorias de la Octava Reunión Centroamericana y del Caribe sobre Formación de Profesores e Investigación en Matemática Educativa*. 21-30.

Valdemoros, M. & Orendain, M. (1994c). *Fracciones y referencias: El caso de la ausencia inicial de referentes concretos*. Matemática Educativa-CINVESTAV, Mexico.

Valdemoros, M. & Campa, A. L. (1994d). *Fracciones y referencias: El caso de la relación entre distintas clases de objetos a través de la suma*. Matemática Educativa-CINVESTAV, Mexico.

Zullie, M. E. (1975). *Fractions with pattern blocks*. Palo Alto: Creative Publications.