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ABSTRACT

This paper reports on a teaching experiment involving decimal instruction conducted with a class of elementary students (n=20) in a public school of a small city. After doing extensive work with multiplication, division, ratio, and fractions through an innovative mathematics curriculum, fifth grade students were introduced to decimal numbers. To develop their understanding of decimal notation, students worked through three open-ended, contextual problems that encouraged them to make connections between decimals and previously encountered mathematical constructs such as ratio and fraction. After instruction, students' performance on decimal tasks indicated that students developed a robust understanding of decimal concepts. Based on these positive results, the authors assert that building decimal instruction upon students' ratio reasoning and fraction sense is a key component to helping students develop meaningful strategies for understanding and working with decimal numbers. Contains nine references. (Author/MKR)

Introducing Fifth Graders to Decimal Notation Through Ratio and Proportion

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INTRODUCING FIFTH GRADERS TO DECIMAL NOTATION THROUGH RATIO AND PROPORTION

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This paper reports on a teaching experiment involving decimal instruction. After doing extensive work with multiplication, division, ratio and fractions through an innovative mathematics curriculum, fifth grade students were introduced to decimal numbers. To develop their understanding of decimal notation, students worked through three open-ended, contextual problems which encouraged them to make connections between decimals and previously encountered mathematical constructs such as ratio and fraction. After instruction, students' performance on decimal tasks indicate that students developed a robust understanding of decimal concepts. Based on these positive results, the authors assert that building decimal instruction upon students' ratio reasoning and fraction sense is a key component to helping students develop meaningful strategies for understanding and working with decimal numbers.

The various difficulties elementary school students have as they begin to work with decimal fractions have been well documented by mathematics education researchers (Carpenter, Corbitt, Kepner, Lindquist, & Reys, 1981; Resnick, Nesher, Leonard, Magone, Omanson, & Peled, 1989; Wearne & Hiebert, 1988, 1989). Hiebert and Wearne (1985) have hypothesized that children's struggles with decimals stem from the fact that what students learn about decimals is largely syntactic. In many schools in the United States, students are taught the rules governing decimal operations but are not given sufficient time or opportunity to develop a deep understanding of the notation itself. Without developing a meaning for the symbols from which decimals are constructed, students struggle to conceptually understand and successfully compute with decimals.

In the past, several teaching experiments involving decimal instruction have been conducted in an attempt to understand what types of classroom activities might help students construct a meaningful understanding of decimal numbers (Hiebert, Wearne, & Taber, 1991; Wearne & Hiebert, 1988, 1989). However, these teaching experiments have typically treated the teaching of decimals as a distinct and separate instructional unit (Ibid.). Little attention is given to the elementary school mathematics curriculum in which the decimal instruction is embedded and where in that curriculum such instruction belongs. Consequently, minimal effort in the research of decimal instruction has been given to how the understanding of decimals is connected to the understanding of other mathematical constructs presented earlier in the elementary school mathematics curriculum.

In this paper, we will report on the results from a teaching experiment involving instruction of decimal fractions embedded in a unique curriculum. Students who participated in this curriculum appeared to use their prior mathematical experiences with ratio and fractions to develop a strong conceptual understanding of decimal notation.

Research Context

The novel curriculum in which this decimal instruction took place is built around the construct of splitting (Confrey, 1994). Splitting actions, which include sharing, folding, and magnifying, are believed to stem from primitive notions which occur intuitively in children. Since these notions can lead directly to multiplication, and concurrently division and ratio, Confrey (1994) has argued that in order to support the intuitive splitting actions of children, students should be introduced to the constructs of multiplication, division, and ratio as a trio, early in their schooling. Consequently, Confrey developed a curriculum in which students are first introduced to all these multiplicative constructs simultaneously in the third grade.

Over the past three years, Confrey has piloted her curriculum with a class of elementary school students (n = 20) in a public school of a small city. When these students were in third grade, they were introduced to multiplication, division, and ratio as a trio of mathematical ideas. When this same class was in fourth grade, instruction focused on strengthening the construct of ratio and introducing fractions as a subset of ratio (Confrey & Scarano, 1995). As fifth graders, having developed a rich network of mathematical ideas, students were then introduced to decimal fractions. This paper focuses only on the aspects of this curriculum related to decimal instruction.

Decimal Instruction

Students' introduction to decimals was done over a six-week period and was built around three open-ended, contextual problems which students worked on in small groups. The first of these problems gave students the opportunity to review and further develop the ratio concepts they encountered earlier in the curriculum and to begin to connect these concepts to decimal notation. In the second and third contextual problems, students worked directly with decimal notation and computation involving decimals.

In addition to the contextual problems, numerous whole class discussions were held. The whole class discussions were used to help students with decimal concepts and operations which they would need to work on the contextual problems and on the homework. Students were typically assigned homework four nights per week and worked on these assignments individually. The homework gave students the opportunity to further practice and develop problem-solving and computational skills with decimals.

Assessment Tools

Students' understanding of decimal concepts was assessed through a series of written and interview tasks. All twenty students were tested prior to the start of decimal instruction and were given a similar test at the end of decimal instruction. The items on the pre and posttests were taken largely from previous research studies on decimal instruction (Hiebert & Wearne, 1985; Hiebert, Wearne, & Taber, 1991; Wearne & Hiebert, 1988, 1989; Resnick et al., 1989) and included a diversity of decimal tasks. In addition to the written test, four students worked on decimal tasks in an interview setting two weeks after decimal instruction ended. Tasks given in interviews were similar to the tasks on the written tests.

Results

In general, the students in this study performed exceptionally well on all written and interview tasks. Individually, only three students out of twenty did not show significant improvement (p-value < .05) between pre and posttests. As a group, the pretest average of 15.5% correct responses rose to 80.8% correct responses on the posttest (Table 1). A paired t-test conducted on this data revealed the group's improvement was highly significant (p < .001).

The items on the written tests were grouped into four different content scales: tasks relating to the meaning of decimal notation, ordering tasks, fraction to decimal tasks, and computation with decimal tasks. Group performance on each scale was computed to get pre and posttest averages (see Table 1 for scale averages). On all four scales, student improvement from pre to posttest was highly significant (p < .001).

Table 1

Class pre and posttest averages on the items in each content scale and on overall written test.

Scale	Pretest	Postest:
Meaning of Decimal Notation	41.7	80.6
Ordering Tasks	43.5	84.0
Fraction to Decimal Tasks	38.8	88.8
Computation with Decimals	18.3	67.8
Overall	**15.5**	**80.8**

Because many of the items on the written tests were taken from other studies done on decimal instruction, it was possible to compare the performance of students in this study to the performance of students in other studies. In their investigation of the invented rules students use to order decimals, both Resnick et al.(1989) and Sackur-Grisvard and Leonard (1989) classify students according to the type of rule they used to complete an ordering task. They also report on what percentage of their sample consistently ordered decimal numbers correctly, and thus could be classified as experts.

For the purposes of comparison, we reviewed the individual student's performances on decimal ordering tasks in our study. Fourteen students out of twenty got all ten of the ordering tasks on the written tasks correct and could thus be classified as experts. As the data in Table 2 illustrates, the percentage of students in our study who could be classified as experts is substantially higher than the percentage of experts found in other groups of students reported in other studies.

In another study, Hiebert & Wearne (1985) collected data on students' performances on different computation tasks. Table 3 presents both the addition and

Table 2
Percent of students classified as experts in completing decimal ordering tasks across three studies

Study Group	#students	%experts
THIS STUDY (1995)		
U.S. Fifth Graders	20	70.0
Resnick et. al. (1989)		
U.S. Fifth Graders	17	18.0
Israeli Sixth Graders	21	19.0
French Fifth Graders	38	53.0
Sackur-Grisvard & Leonard (1985)		
French Fifth Graders	49	53.1
French Sixth Graders	57	52.6

Table 3
Percent of correct student responses on decimal computation items found in the Hiebert & Wearne (1985) study and in this study.

H&W Items:	Grade 5 n=99	Grade 6 n=55	Grade 7 n=272	Grade 9 n=196	This Study's Items:	Grade 5 n=20
5.1+.46	20	62	70	79	4.5+8.6	80
6+.32	6	25	39	77	6.24+8+.873	70
.86-.3	12	35	51	81	5.42-.3	75
4.7-.24	5	42	53	69	7.6-.34	85

subtraction items and the student performance data on these items from Hiebert & Wearne's study along with similar information from our study. In comparison to students tested in Hiebert & Wearne's study, the students in our study performed substantially better than their peers in the same grade, and at least as well or better than older students.

Discussion

From the results presented above, it seems evident that students in our study developed a strong and robust conceptual understanding of decimal notation which allowed them to successfully complete a variety of decimal tasks. From our test data and our observations and interviews with these students, we believe that students' prior work with ratio and fraction was crucial to students' ability to develop a deep, conceptual understanding of decimals numbers.

For instance, because of their work with ratio and fractions, students viewed decimals as simply another form of fractions. In response to the question, "What are decimals?", three students (of the four who responded) said:

Carrie: A way to write fractions in base 10.

Max: Well, I just think it's sort of like a fancy way of writing fractions. Like if you can't use fractions in everything like the computer or calculator, you need to adapt it.

Kai: I think it's just a shorter way of writing fractions. Instead of writing 97/100, you just do point 97.

Such a vision of decimals gave students great flexibility in dealing with these numbers. When they had difficulties, they simply converted the decimal numbers to fractional form. In fact, whenever students were initially introduced to an operation (addition, subtraction, multiplication, or division) with decimals, their first reaction was usually to use fractions to complete the given computation.

In completing ordering tasks, students often used ratio reasoning. For example, in comparing .8 to .34, one student said, ".8 goes to 10 and .34 goes to 100. 8 is a lot closer to 10 than 34 is to 100, so .8 is bigger." In addition, students who used the "add a zero to the end trick" to compare two decimals of different lengths (e.g. .8 and .08) usually understood why the "trick" worked. "Since .8 is 8/10 and 8/10 is equal to 80/100, I can just write .8 as .80. It's the same thing," explained one student.

Conclusion

As the above examples illustrate, students frequently and easily connected and applied their knowledge of ratio and fraction to their work with decimals. It is our contention that the ratio curriculum and its approach to decimals supported and encouraged these connections and applications. Thus, given the meaningful, conceptual understanding of decimals that the students in this study developed, we recommend that ratio and fraction concepts be developed earlier and more broadly in the elementary school mathematics curriculum. Effective decimal instruction can then be grounded in students' understanding of these multiplicative constructs.

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