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AUTHOR Watanabe, Tad

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ABSTRACT

Fifth grade children's (n=16) understandings of simple fractions such as 1/2, 1/3, and 1/4 were investigated. The study identified three alternative conceptions of fractions: (1) 1/"N" is one of "N equal" parts, (2) 1/N is one of "N equal" parts, and (3) parts must fit together to form the whole. In addition, many participants believed that the perimeter measured the area. This conception of area measurement also influenced the participants' problem-solving activities significantly. (Author/MKR)

Incongruity and Complexity of Young Children's Understanding of Simple Fractions

Tad Watanabe

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INCONGRUITY AND COMPLEXITY OF YOUNG CHILDREN'S UNDERSTANDING OF SIMPLE FRACTIONS

Tad Watanabe, Towson State University

Fifth grade children's understandings of simple fractions such as 1/2, 1/3 and 1/4 were investigated. The study identified three alternative conceptions of fractions: 1/*N* is one of *N* equal parts, 1/N is one of ***N equal*** parts, and parts must fit together to form the whole. In addition, many participants believed that the perimeter measured the area. This conception of area measurement also influenced the participants' problems solving activities significantly.

In an earlier work (Watanabe, in press, 1991), it was reported that second grade children held and used different, and often inconsistent, meanings of the fraction one-half in different contexts. The findings from the research raised a number of questions concerning children's understanding of fractions. Do older children hold inconsistent meanings for simple fractions such as 1/2, 1/3, and 1/4? If so, what are some of the meanings that are commonly held by the older students? How do they develop these meanings? What are the factors that influence children's construction of a variety of fraction meanings? Why is it possible for some children to hold inconsistent meanings of simple fractions without perturbation? And, finally, what are the influences of formal instruction and how can teachers cause perturbation in children so that they may construct more consistent meanings of fractions? To answer these and other questions, a series of investigations has been conducted. In this report, I will report findings from a study with fifth grade students on their understanding of simple fractions. The primary focus of this report is to identify and describe the alternative conceptions of these participants.

Theoretical Framework

Dykstra, Boyle & Monarch (1992) pointed out that the phrase, "alternative conception" has been used to describe a variety of meanings. I will use the phrase to mean, "the fundamental beliefs students have about how the world works, which they apply to a variety of different situations" (Dykstra et al., 1992, p.621). In this case, the "world" really refers to the participants' mathematical world. I agree with Dykstra et al. that calling students' alternative conceptions as "misconceptions" is inappropriate. Children's conceptions are not at random, but they are results of rational processes. We must understand how children formulate these conceptions so that we can provide appropriate learning opportunities for the students.

This research was conducted within a framework consistent with the constructivist epistemology. The aim of investigation was, therefore, not to evaluate children's fraction understanding measured against some pre-set standards. Rather, the goal of the study was to understand how children were making sense of mathematical ideas - in this case, simple fractions. Furthermore, this study was conducted with the belief that, if we do take constructivism seriously, the first step in teaching children mathematics must be understanding *children's* understanding, i.e., we must pay close attention to children's prior concepts (Steffe, 1988). In addition, it was also assumed that children's imagery (Wheatley & Reynolds, 1993) and their metaphors (Lakoff & Johnson, 1980) play central roles in their construction of mathematical meanings. Therefore, the analysis of children's concepts will include their imagery and metaphors.

Methodology

Sixteen fifth graders, 7 boys and 9 girls, from a single classroom participated in the research. These children were interviewed individually. The interviews were semi-structured in that a set of common tasks was prepared in advance, and each interview started with the same question. However, the interviewer made changes based on his on-the-spot analysis.

A variety of tasks were used during the interviews; however, the analysis reported in this paper is based on the following four tasks:

One-Half Task: In this task, students were shown 16 partially shaded figures (see Figure 1), and they were asked to identify those that were half-shaded. After the participant had selected those figures s/he believed were half-shaded, the interviewer asked her/him to justify how s/he knew those figures were half-shaded.

Cookie Question: For this question, three figures shown in Figure 2 were used. These figures were obtained from congruent squares by partitioning them into two congruent parts in different ways. The interviewer first showed two copies of each shape and demonstrated that they were identical by placing one on top of the other. He then arranged the two into the square and placed it in front of the participant. This was done with all three figures. After this demonstration, one copy of each shape was given to the participant, and s/he was asked to pretend they were their favorite kind of cookies. Then, the question was posed: You are really hungry, but you can have only one piece. Which one would you choose? After the participant selected one, s/he was asked to justify their selection. If the participant picked one shape as the largest, s/he was reminded of the initial demonstration and asked if that would help them make her/his decision.

Tangram Task: The seven tangram pieces were placed in front of the participant, arranged into a square. An identical square with outlines of each piece drawn inside was also presented on a separate sheet of paper. The participant was then asked to identify what part of the square each tangram piece was.

Identification: Different partially shaded figures were presented on grid papers. The participants were asked if the shape was 1/2 (or 1/3, or 1/4) shaded.

Findings

One of the alternative conceptions identified may be summarized in the phrase, "1/N is one of N equal parts." It is true that one of N equal parts is 1/N of the whole, and this is probably the most common way we approach fractions in lower grades. However, for many of the participants, this conception of fractions limited

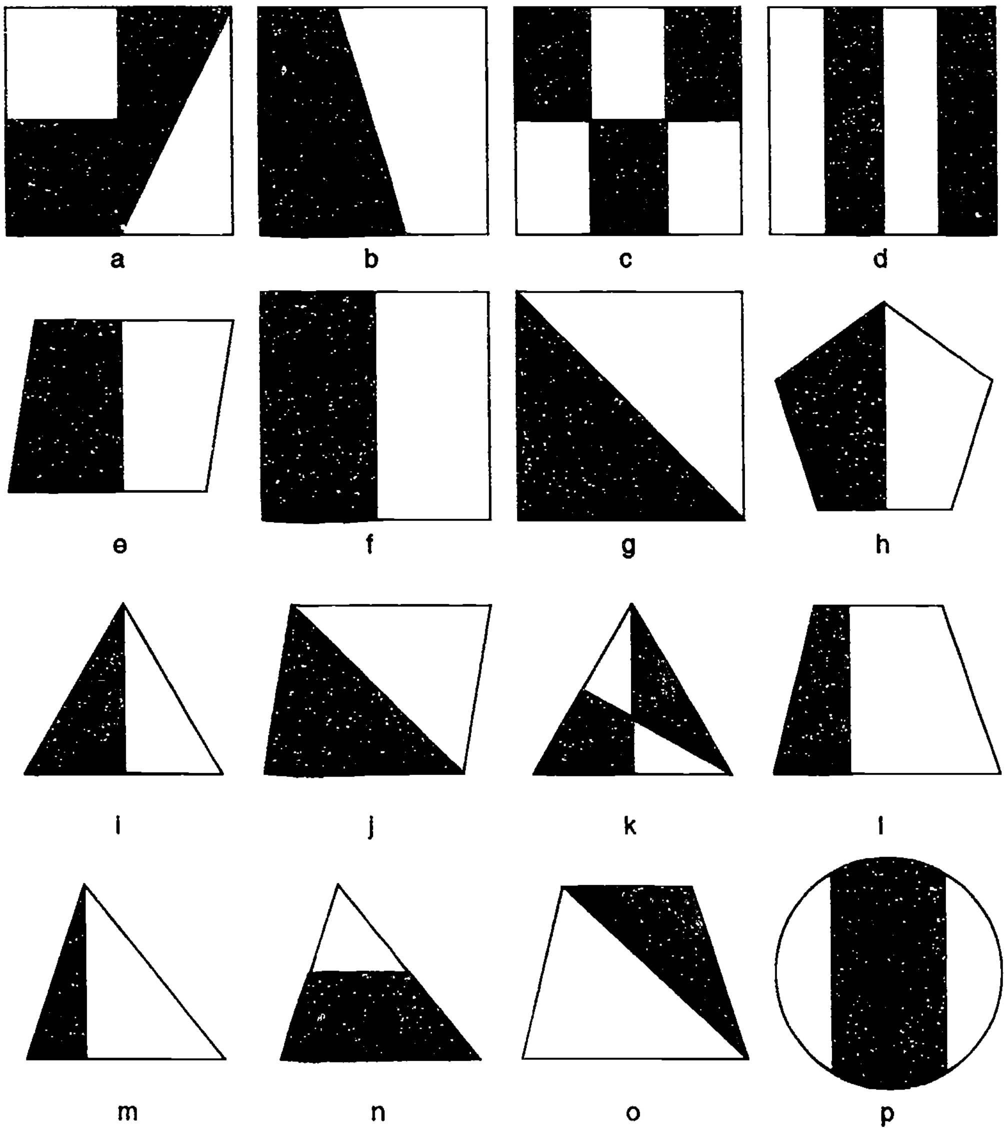

Figure 1. 16 designs used in the One-Half *task.*

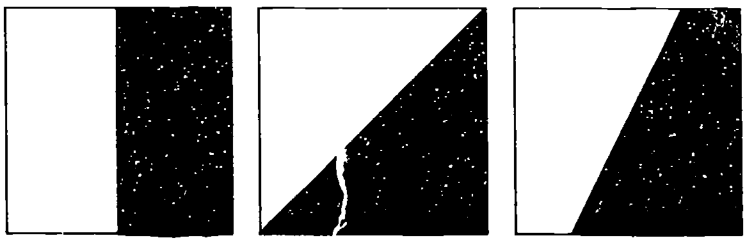

Figure 2. Three shapes used in the Cookie Question

their ability to deal with a number of tasks. There were two versions of this alternative conception.

1/*N* is one of *N* equal parts: SW selected all figures divided into two parts with one part shaded as one-half shaded, including figures (l) - (o) in Figure 1 that were not half-shaded. He had also constructed some form of understanding that 2 out of 4, 3 out of 6, etc., were equivalent to one-half. Therefore, the only shape he did not select as half-shaded was figure (f). Obviously he paid no attention to the size relationship among the parts, nor the relationship between the shaded part and the whole. Rather, for him, the important factor is the *number* of the parts and the relationship between the number of shaded parts and the number of unshaded parts. Because this alternative conception does not consider size relationship among parts of the whole or between the part and the whole, the resulting "fractions" have no quantitative significance. Therefore, SW decided that the triangular shape in the Cookie Question was the largest of the three even though, according to his conception, all three figures are one-half of the same square.

1/N is one of *N equal* parts: With this alternative conception, besides having to have N parts, all parts must be equal in size. For example, in response to an Identification question both KR and LD decided that the following figure shown was not 1/3 shaded because the three parts were not equal in size. With this alternative conception, the children were paying attention to the size relationship, but it was the size relationship among the parts, not between the part and the whole, that occupied their attention.

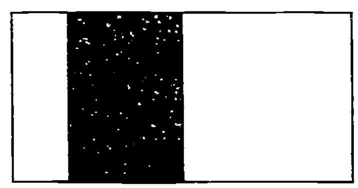

Figure 3. KR and LD said this figure was not 1/3-shaded

Another alternative conception identified was: parts must fit together to make the whole. This conception caused problems for three of the participants as they tried to decide what part of the large square the parallelogram tangram piece was. Many of them tried to cover the square using the parallelogram piece, and one even used two small triangles to make the parallelogram to assist her effort. For most of them, the fact that the parallelogram will not cover the square evenly was the major problem. KR explained why she could not find the answer by saying, "not all the sides are straight, so it won't fit evenly in the box."

Another major alternative conception influenced these participants' problem solving processes involving fractions, although it was not exactly a conception of fractions. This alternative conception was, "perimeter measures the area." In the Cookie Question, eight of the 16 participants selected one of the three figures as the largest even after they were reminded of the initial demonstration. Only two participants were able to decide that the three shapes were the same size when the problem was posed initially. The most common strategy used by the participants to justify their selections, usually the triangular piece, was to compare shapes by placing them next to each other and compare the lengths of the sides. Even when one shape was placed on top of another, many participants simply compared the lengths of "corresponding" sides, not the area. For them, comparing the lengths of sides was a legitimate way of comparing the "largeness" of the pieces. Although this alternative conception may not be directly related to children's conception of fractions, it may be the case that their work with fractions may have facilitated this conception. For example, during the Identification task, KR often counted the number of squares along each side of the both shaded and unshaded figures. This was a valid way for him to test that the parts were equal, and it works fine with most typical fraction exercises where the parts are congruent. If the measures of corresponding sides are equal, then, the parts are equal in size. Such an experience may have encouraged the formation of this alternative conception.

Discussion

Even among fifth grade students who participated in this study, the understanding of fractions was very much context bound. Thus, they were very capable of responding correctly to the One-Half tasks, yet half of the participants were unable to reason that the three pieces of the Cookie Question were the same size, a half of the square, even after they were reminded of the initial demonstration. These participants had received at least three years of formal instruction on fractions. They had studied fraction arithmetic in fourth grade and they were studying decimals. Yet, many participants have little number quantitative sense with fractions.

Furthermore, many of the participants' alternative conceptions identified in this study appeared to have grown out of the formal instruction, unlike many alternative conceptions in science such as, "motion implies force." This is both discouraging and hopeful. It is discouraging to learn that the formal instruction is contributing to the formation of these alternative conceptions. But, it is hopeful that if mathematics educators become aware of the possible problems with some of common ideas about fractions, they would be able to make appropriate adjustments sc that they can keep these conceptions from being developed. It appears that the common part-of-a-whole approach to fractions must be complemented with much more emphasis on size relationships, especially the relationship between the part and the whole.

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