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ABSTRACT

Research in mathematical problem solving has produced significant results in trying to understand what people do to solve problems. An important part of the solution process is the presence of both cognitive and metacognitive strategies. This paper documents the extent to which 13 ninth grade students are able to recognize the basic structure of a problem given in three different contexts. In the analysis, it was important to identify a set of distinctions that the students coordinate during the process of solution. This set of distinctions involves the use of some kind of representation of the problem, the search for connections with other ideas, flexibility in approaching the solutions, and confidence in the results. These ingredients become essential in evaluating the quality of students' work. (Author/MKR)

Students' Recognition of Structural Features in Mathematical Problem Solving Instruction

Manuel Santos T

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STUDENTS' RECOGNITION OF STRUCTURAL FEATURES IN MATHEMATICAL PROBLEM SOLVING INSTRUCTION

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Research in mathematical problem solving has produced significant results in trying to understand what people do to solve problems. An important part of the solution process is the presence of both cognitive and metacognitive strategies. This paper documents the extent to which students are able to recognize the basic structure of a problem given in three different contexts. In the analysis, it was important to distinguish a set of distinctions that the students coordinate during the process of solution. This set of distinctions involves the use some kind of representation of the problem, the search for connections with other ideas, the flexibility in approaching the solutions, and confidence of the results. These ingredients become essential to evaluate qualities of the students' work.

Problem solving has been identified as an important component of mathematical instruction (NCTM, 1989; Schoenfeld, 1994). As a consequence, teachers encourage their students to engage in problem solving activities during the development of their courses. However, what types of problems and to what extent students should discuss these problems during instruction are issues that teachers need to discuss on a regular basis. It is common to hear that it is difficult to find or design good problems for the class discussion, and teachers often continue working with routine problems that they have been using regularly in their classes. Thus, if we accept that problem solving is a way of thinking that should be present not only in mathematics instruction, but in the process of interacting with problems in other contexts, then it becomes important to explore how other contexts could play an important role in the selection of problem solving activities for the classroom. This paper analyzes the work done by tenth grade students who were asked to work on three problems that share similar structure. Thus, it was important to document what type of strategies and difficulties were shown by the students who noticed connections among the problems. The discussion of the students' approaches play an important role not only in understanding the processes shown while working on the problems but also in evaluating the potential of some activities associated with problem solving instruction.

Background to the Study

Research in mathematical problem solving has suggested that it is important to provide learning experiences for the students in which they have opportunity to get engaged in actual mathematical experiences. Schoenfeld (1992) found that the process of doing mathematics includes the use of resources or basic mathematical knowledge (facts, procedures, algorithms), the use of heuristic strategies, the pres-

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ence of metacognitive activities (monitoring and control), and an understanding of the nature of the mathematical practice (conception of the discipline). As a consequence, it is necessary to investigate to what extent the students' problem solving behaviors could be improved when the instruction they receive takes into account learning activities related to those dimensions. Santos (1995) pointed out that in order to develop the students' mathematical disposition to learn mathematics is important to provide a class environment in which students consistently are asked to a) work on tasks that offer diverse challenges; b) discuss the importance of using diverse types of strategies including the metacognitive strategies; c) participate in small and whole group discussions; d) reflect on feedback and challenges that emerge from interactions with the instructor and other students; e) communicate their ideas in written and oral forms; and f) search for connections and extensions of the problems. These learning activities play a crucial role in helping students see mathematics as a dynamic discipline in which they have the opportunity to engage in mathematical discussions and thus value the practice of doing mathematics.

The need to document how the students approach different types of tasks is based on the great influence that problem solving has shown in the learning of mathematics. The number of research studies in this area has been significant in the last 25 years (Schoenfeld, 1994; Charles & Silver, 1988, Lester, 1994). One important direction in problem solving has been to categorize the way students solve problems. Several frames of analysis or theoretical models emerged from that research direction and have contributed to the understanding of the process used by the problem solver. The role of qualitative tasks or nonroutine problems has been important during the process of gathering information of the students' work. As a consequence, some research results in problem solving have challenged or transformed the teaching of mathematics. Here, it becomes important to study the potential of diverse tasks or problems that involve different contexts as a means to use them in mathematical problem instruction. The analysis of the students' approaches while working on problems with similar structure will help us understand what aspects of problem solving appear as important when students actually recognize the structure of the problems during the solution process.

Methods, Procedures, and Frame of Analysis

Thirteen grade nine students, all volunteers, participated in the study. They worked on the problems for about 45 minutes. Each student worked the problems individually and was asked to think aloud while solving the problem. It is important to mention that the teacher of this group of students has been implementing problem solving activities during the last three years of his teaching. An interviewer took notes during the whole process and was available to provide clarification questions when required by the students. Three problems were used as means to gather information.

1. A carpenter makes \$800 for the first week of work and then \$860 for the next two weeks. What were his total earnings for that period, and what was his average salary?
2. A tank is filled to a depth of 80 centimeters and two identical tanks are filled to a depth of 86 centimeters. What is the average depth of the water in the tanks?
3. Peter travels 80 km per hour for one hour, then at 86 km per hour for two hours. How far did Peter travel, and what was his average speed?

The work shown by the students was analyzed by considering the type of resources and strategies that the students used to solve or make progress while working on the tasks. It is important to mention that during the analysis aspects of the mathematical practice which helped students identify similarities among the problems were explored. During this process, three levels were identified as a means to characterize the students' work. The high level appears when a student shows the important mathematical ideas associated with the task in his or her solution and he or she provides a consistent argument that supports such a solution. A medium level is identified when a student shows significant progress to the solution but misses to consider some cases. Finally, a low approach involves the student showing little understanding of the key issues of the task and addresses only superficial parts of the problem solution.

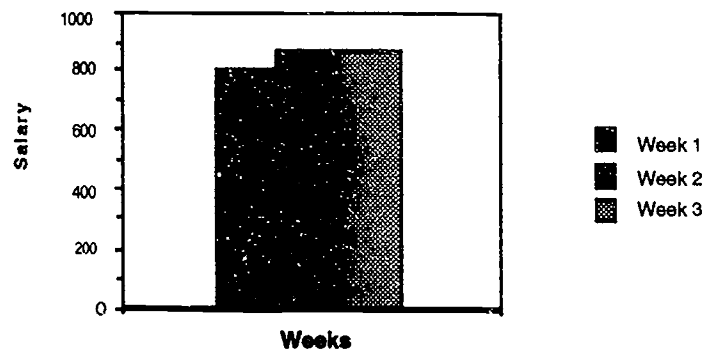
Students' Approaches to the Problems

Eighty percent of the students showed significant progress toward solving the problems. Although the most popular approach was to focus on operations, it was important to observe that various students used graphical representations. For example, seven students relied on a table and figure to solve the first problem, and these students noticed that to solve the second and third problems they were going to use a similar approach. That is, they were able to identify the common structure of the problems. It seems that using a representation helped them make the connections. Some students who relied only on calculations did not make explicit statements about the relationships among the problems. For example, four students were able to solve the first two problems, and they wrote that they did not recall the formula working in the third problem. Only one student graphed the three problems together by presenting the data in accumulative form and explained relationships among the representations. A set of distinctions that students showed during the solution process helped categorize the quality of the responses. These distinctions include: (a) The use of representation as a means to work the data (table, list) and to show the result, (b) Connections in which some students linked the common features among the problems, (c) Flexibility in trying to graph and explain extensions of the problems (accumulative graph), and (d) Confidence shown by some students when they compared the responses to the problems. To illustrate differences among the students' responses, an example taken from the students' work is used to illustrate the quality of the responses for high, medium and low levels.

Students who decided to represent the data graphically showed tables and, in some cases, bar diagrams. For example, some students utilized the following representations:

For the problem that involves finding the total earnings, seven students arranged the data of the problem on a table and showed a bar diagram. It was interesting to observe that these students also represented the second problem similarly and immediately (while working on the representation) noticed that the three problems could be approached in the same way.

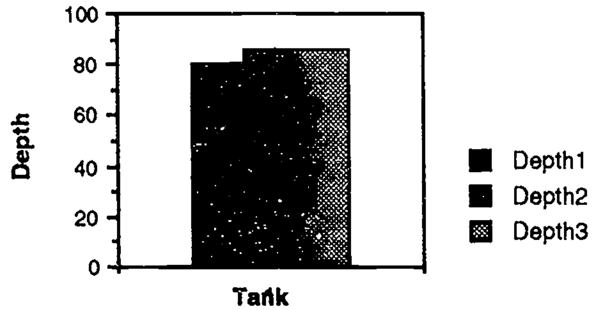
Data from "The Carpenter's Salary"



Week	1	2	3	Average
Salary	800	860	860	840
Tank	1	2	3	Average
Depth	80	86	86	84
Hour	1	2	3	Average
Spped	80	86	86	84

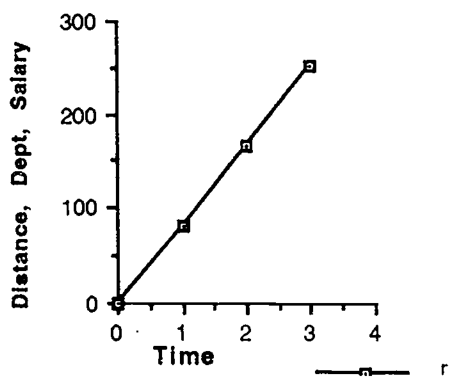
It was clear that students who represented the problem graphically were able to identify similar properties among the problems. For example, two students who had used bar graphs to represent the first problems, immediately noticed that the shapes of the graphs were the same. These students mentioned that all three problems could be solved in the same way. They also mentioned that the context of the problem did not influence the form of solution. The responses given by these students were categorized as the high level type. When the students used only a table or paid attention only to the numerical results used to determine the relationships among the problems, then the responses were categorized as the medium level type. For example, three students spotted similarities among the problems

Data from "Tanks"



based on a list of what happened in each situation individually. That is, they focused on the average number asked in each problem to support their responses. An interesting contrast with the students who used graphs was that these students worked on the three problems completely before realizing that they shared similar

Accumulative Data



structures; while the students who used graphs did not need to complete the three problems before noticing such similarities.

One student showed the relation between the bar representation and the linear graphs by showing an accumulative representation. He noticed that the information given in the three problems could be easily read from this representation.

The accumulative representation shows exactly how many km had been traveled or how much money had been earned by a given time. The students who failed to solve the problems or make progress toward the solution experienced difficulties in trying to understand the conditions and what they were asked to do. For example, one student asked for the speed formula to approach problem three.

Discussion of Results and Instructional Implications

The results show that it is possible to identify a set of characteristics that distinguishes various approaches in the students' work. On one side there were students who spent significant amount of time analyzing the conditions of the problems and worked on a well structured plan. These students showed the use of different representations as a means to approach the problems. The fact that the students explicitly searched for various representations helped them interpret the information and observe some connections. On the other side, other students tended to approach the problems by using numerical representation, and it was difficult for them to visualize that the problems shared a similar structure. Although the students were asked only to work on the problems, it is interesting to note that those who used more than one representation were able to see the problems in a wider perspective compared with the students who used only one representation. That is, the use of several representations played an important role in the transfer of the students' ideas.

It is also evident that the first group of students (who spent more time understanding the conditions) showed more of a disposition to work on these tasks, and they showed some kind of flexibility in using more than one approach, including graphical representation. It seems that being flexible while representing the information given in the problem allowed students to observe features that were not evident under the numerical representation. An important implication here is that it is important to encourage students to use more than one representation to deal with the information. In addition, it is important that students consistently are asked to identify similarities and differences among methods of solution and structural properties of problems that involve different contexts.

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