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ABSTRACT

This study examined how engaging calculus students in Writing to Learn Mathematics affected the types of conceptual and procedural errors that the students made on their examinations. Students in two sections of an introductory college calculus course in Fall 1994 were the respondents in this study. A classification system was developed that categorized students' errors as procedural, conceptual, or indeterminate. Procedural errors involved either syntactical or algorithmic errors. Conceptual errors involved use of inappropriate procedures, acceptance of unreasonable answers, translation mistakes, misuse of symbols, incorrect interpretation of symbols, invalid inferences, statements without justification, or contradictions of nonprocedural principles, definitions, or theorems. (Author/SW)

The Effects of Writing to Learn Mathematics on the Types of Errors Students Make in a College Calculus Class

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THE EFFECTS OF WRITING TO LEARN MATHEMATICS ON THE TYPES OF ERRORS STUDENTS MAKE IN A COLLEGE CALCULUS CLASS

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This study examined how engaging calculus students in Writing to Learn Mathematics affected the types of conceptual and procedural errors that the students made on their examinations. Students in two sections of an introductory college calculus course in Fall 1994 were the respondents in this study. We used Hiebert and Lefevre's (1986) characterization of conceptual knowledge as a framework to guide our examination of students' conceptual knowledge. To analyze the errors the students made, we developed a classification system and used some of the ideas and methods of Movshovitz-Hadar, Zaslavsky and Inbar (1987).

Many college students experience difficulty with doing mathematics (Kolata, 1988). It is not unusual to find students that use mathematical procedures with little or no understanding of the concepts behind these procedures (Hiebert & Lefevre, 1986; Schoenfeld, 1985). Some research (e.g., Oaks, 1988) has suggested that a student's difficulty in mathematics can be related to that student's beliefs that mathematics consists only of meaningless symbols and operations. Such students do not realize that there are concepts behind their procedures. They have a rote conception of mathematics that encourages them to learn only by memorizing, which ultimately prevents them from succeeding in mathematics (Oaks, 1990).

Some mathematics educators have suggested that students may be encouraged to change their conceptions of mathematics through the use of Writing to Learn Mathematics (WTLM) (e.g., Oaks, 1988), and that WTLM may benefit students' development of conceptual understanding (e.g., Gopen & Smith, 1990; Rose, 1989). However, no comparative research has been done to determine whether WTLM's proposed benefit to conceptual understanding is an actual benefit. Two comparative studies (Guckin, 1992; Youngberg, 1990) have investigated WTLM's proposed benefit to procedural ability; both of these studies focused on students in an algebra course.

Aim of the Study and Guiding Frameworks

The purpose of this study was to examine the effect of WTLM on the conceptual understanding and procedural ability of students in an introductory college calculus course. Hiebert and Lefevre (1986) characterized conceptual knowledge as that which is part of a network comprised of individual pieces of information and the relationships between these pieces of information. We are using Hiebert and Lefevre's characterization as a framework to guide our work in examining students' conceptual knowledge. To determine if WTLM helps students improve their conceptual understanding and affects their procedural ability, we are developing and will use an error classification system, based on the work of Movshovitz-

Hadar, Zaslavsky, and Inbar (1987), whose work serves as a guiding framework for our data analysis.

Methods and Data Sources

Students in two sections of an introductory college calculus course in Fall 1994 were the respondents in this study. Both classes were taught by the same instructor (the first author) with an emphasis on the mathematical concepts relevant to the course. One class, the WTLM group, participated in writing activities both inside and outside the class. The other class, the comparison group, were not assigned any writing activities. However, whenever the WTLM group was given a writing activity, the comparison group was given an activity that involved the same concepts as the WTLM's activity. Activities from both classes were discussed in class and assessed by the instructor.

Students in the WTLM group participated in a variety of writing activities. Occasionally, the students were given impromptu writing prompts during class time, to which they were asked to respond in writing. However, because class time is limited, the students were also given writing activities that were completed outside of class. The WTLM students were asked to write about topics related to course concepts and procedures. In these writing activities, the students were asked to explain course ideas in their own words, to discuss the relationship between course concepts, and to think, on paper, about concepts and procedures of the course. Students were also asked to reflect, in writing, on their study habits and performance in the course, and about the beliefs they hold about mathematics.

Some examples of writing activities are as follows:

- Explain to a friend, in writing, what a function is.
- What is a derivative?
- Why would someone want to find a derivative?
- How are Rolle's Theorem and the Mean Value Theorem related?
- Explain the First Derivative Test. Why does it work?
- What is the best way to study for a mathematics class? Why?
- Discuss your reaction to your performance on the test. Discuss ways in which you could improve your preparation for and performance on the next test.

The comparison group did not participate in the writing activities. However, whenever the WTLM group was given a writing activity (generally twice a week), the comparison group was given an assignment or quiz (graded or not, depending on whether the WTLM group's activity was graded) that will involve problems of the same content as the WTLM group's writing activity. For example, when the WTLM group was asked to describe, in writing, all of their thoughts and actions as they attempted to solve a certain homework problem, the comparison group was asked to solve the same problem and be prepared to discuss their thoughts and actions. Both groups received feedback on their work through written comments and discussion in class.

The data for the study consist of student responses from both classes on three in-class examinations and one final examination. All examinations were identical for both classes. The examinations included both routine exercises and nonroutine problems. We used ideas from the error classification system developed by Movshovitz-Hadar et al. (1987) as a basis for our data analysis. Movshovitz-Hadar et al. developed a classification system for errors in secondary mathematics (not including calculus). They classified student errors according to the following six categories: (a) Misused Data, (b) Misinterpreted Language, (c) Logically Invalid Inference, (d) Distorted Theorem or Definition, (e) Unverified Solution, and (f) Technical Error (Movshovitz-Hadar et al., 1987). We used this model, and Hiebert and Lefevre's (1986) framework of conceptual understanding, as a starting point, and described categories that emerged from our data for classifying students' errors in calculus, which has not been previously done. We analyzed the students' errors in a qualitative manner that Movshovitz-Hadar et al. called *constructive analysis*.

Findings

Discussion of the Categories that Emerged

By analyzing three midterm examinations and the final examination at the end of the semester, we collected 1,241 errors that we considered for this study. We noted 636 other errors but did not categorize these since we were concentrating on students' conceptual and procedural understanding of calculus ideas and these errors were not specific to calculus and involved mathematics content the students were taught in previous courses. We classified the 1,241 errors into the following categories: (a) Procedural, (b) Conceptual, and (c) Indeterminate. We will describe each category and give its characteristic elements. In order for an error to fit in a certain category, it must meet the criteria for at least one characteristic element.

The *Procedural Error* category consists of errors involving procedural knowledge, as defined by Hiebert and Lafevre (1986); procedural knowledge is composed of two parts: (a) the symbols and syntax of mathematics, and (b) the rules, algorithms, and procedures for performing mathematical tasks. The two parts of procedural knowledge are incorporated in the three characteristic elements of the Procedural Error category. The first characteristic element is that the error violates one or more of the syntactic rules for writing mathematical symbols in an acceptable way. The second characteristic element involves writing a symbol incompletely or improperly. Note that this does not include valid mathematical terms or symbols that are *used* improperly but written correctly. An example of this is from a student who wrote "$\lim_{x \to c} =$" without using a function, f. The third characteristic element is that there is an error in the statement of or use of a rule, procedure, or algorithm used for completing mathematical tasks in a step-by-step, linear fashion. Note that this does not include errors in selecting an appropriate procedure or in evaluating the outcome of a procedure. An example illustrating this element is a student who used a distorted version of the quotient rule in calculating a derivative.

The *Conceptual Error* category consists of errors involving conceptual knowledge, as defined by Hiebert and Lefevre (1986); conceptual knowledge is that which is part of a network comprised of individual pieces of information together with the relationships between these pieces of information. We have determined eight characteristic elements for this category.

The first characteristic element of this type of error is that a procedure that is inappropriate for the problem at hand has been selected. For example, part of the solution for a problem involved finding the derivative of a function but a student found the limit of the function instead. The second characteristic element is a failure to reject an answer that is unreasonable or whose incorrectness could have been discovered by checking. An example of this is the student who determined that a particular circle had a radius of -4. We developed these two characteristic elements based on Hiebert and Lefevre's (1986) discussion of errors that involve conceptual knowledge that is associated with a procedure: "Conceptual knowledge, if linked with a procedure, can monitor its selection and use and can evaluate the reasonableness of the procedural outcome" (p. 12).

The third characteristic element is "translating an expression from natural language into a mathematical term or equation that represents a relation different from the one described verbally" (Movshovitz-Hadar, Zaslavsky & Inbar, 1987, p. 10) or vice versa. The fourth characteristic element is "designating a mathematical concept by a symbol traditionally designating another concept" (Movshovitz-Hadar, Zaslavsky & Inbar, 1987, p. 10) or referring to a mathematical concept using language traditionally used in reference to a different concept; for example, using $f'(2x-4)$ to mean $\frac{d}{dx}(2x-4)$. The fifth characteristic element is "incorrectly interpreting graphical symbols as mathematical terms or vice versa" (Movshovitz-Hadar, Zaslavsky & Inbar, 1987, p. 10) or incorrectly interpreting mathematical symbols. An example illustrating this element is a student who interpreted $f'(3) = -8$ as the point (3, -8). The third, fourth and fifth characteristic elements are based on Movshovitz-Hadar, Zaslavsky and Inbar's (1987) Misinterpreted Language category. They describe their error category in the following way: "This category includes those mathematical errors that deal with an incorrect translation of mathematical facts described in one (possibly symbolic) language to another (possibly symbolic)" (p. 10).

The sixth characteristic element of this type of error is making a logically or conceptually invalid inference. That is, invalidly drawing new information from information that was previously given or inferred (Movshovitz-Hadar, Zaslavsky & Inbar, 1987). For example, a student who was given the statement of the Intermediate Value Theorem—"if f is continuous on the closed interval [a, b], then f takes on all f(x) values between f(a) and f(b)"—was then asked "Can the theorem be used to show that f is continuous?" The student responded that this statement of the theorem could be used to show that f is continuous. Another example of this type of error occurred in a problem that required the student to find the absolute maximum value of a function. The student found a critical value (x) for the function and claimed that the function reached a maximum at this x-value without actually determining where f increased and decreased (or any other evidence). The seventh characteristic element is making a statement without providing sufficient motivation for it or explanation of the reasons why the statement is true. Our sixth and seventh characteristic elements are based on Movshovitz-Hadar, Zaslavsky and Inbar's (1987) Logically Invalid Inference category: "In general, this category includes those errors that deal with fallacious reasoning and not with specific content" (p. 10).

The eighth characteristic element of this type of error is making a statement or giving an answer that contradicts or neglects a nonprocedural (in the sense of Hiebert & Lefevre, 1986) principle, definition, or theorem. For example, a student did not list certain x-values as points of discontinuity even though they were points of discontinuity. Another example of this type of error is a student who stated that the limit of a function existed even though the right-hand limit did not equal the left-hand limit. Our eighth characteristic element is related to the Distorted Theorem or Definition error category of Movshovitz-Hadar, Zaslavsky and Inbar (1987) that contains errors concerning "the distortion of a specific and identifiable principle, rule, theorem, or definition" (p. 11).

The *Indeterminate Error* category consists of errors that involved (a) both procedural and conceptual knowledge and where it was not possible for us to categorize the error as predominantly procedural or predominantly conceptual, or (b) neither procedural nor conceptual knowledge. This error category, and examples of error we classified as indeterminate, will be discussed more fully during the presentation.

Connection Between WTLM and the Error Categorization

Rose (1989, 1990) has identified a variety of perceived benefits of writing in mathematics. The benefits she categorized as beneficial to students as writers included, among others, that writing can (a) promote understanding, (b) facilitate reasoning and problem solving, (c) help generate meaning, (d) reveal what was misunderstood, (e) stimulate the posing of questions, (f) promote independent learning, and (g) help retention of content. It was our intent in this study to examine whether some of these perceived benefits are actual benefits. Thus, we explored whether students who were engaged regularly in WTLM over the course of a semester would make fewer and/or a different type of conceptual and procedural errors.

At the present, we have categorized the 1,241 errors into the three categorized that emerged from the data and were supported by the frameworks guiding our study. Up until now, all the data has been anonymous. We are now beginning to examine the connection between WTLM and the type and frequency of errors. This will be discussed in detail during the presentation.

Significance

This study adds to the growing body of research on WTLM in an important way by addressing the lack of comparative research on the proposed benefits of WTLM. This study also yields information that is valuable to educators who seek ways to improve students' conceptual understanding.

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