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ABSTRACT

This study investigated whether the van Hiele model accurately describes the geometric thinking of gifted students in the 6th through 8th grades prior to a formal course in geometry and made comparisons with what has been found with other populations. The results from 120 students who completed a 25-item multiple choice paper-and-pencil test, developed by the Cognitive Development and Achievement in Secondary School Geometry Project, and 64 students who participated in 30-45 minute individual interviews were analyzed. Although the responses of the students on the multiple-choice test did form a hierarchy overall, 35.8% of the gifted students tested skipped levels in the van Hiele model. Analysis of the clinical interviews confirmed that individuals do not demonstrate the same level of thinking in all areas of geometry. Many of the students lacked correct basic definitions of terms such as congruent and similar, but they attempted to deduce the definitions from contextual clues. Once they established a definition, correct or incorrect, most students reasoned consistently from it. Although reasoning was a strength of most of the subjects, they did not know how to construct an acceptable formal geometric proof. (Author)

Geometric Understanding in Gifted Students Prior to a Formal Course in Geometry

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Abstract

This study investigated whether the van Hiele model accurately described the geometric thinking of gifted students in the sixth through eighth grades prior to a formal course in geometry and made comparisons with what has been found with other populations. The results from 120 students who completed a 25-item multiple choice paper-and-pencil test, developed by the Cognitive Development and Achievement in Secondary School Geometry Project, and 64 students who participated in 30-45 minute individual interviews were analyzed. Although the responses of the students on the multiple-choice test did form a hierarchy overall, 35.8% of the gifted students tested skipped levels in the van Hiele model. Analysis of the clinical interviews confirmed that individuals do not demonstrate the same level of thinking in all areas of geometry. Many of the students lacked correct basic definitions of terms such as congruent and similar, but they would attempt to deduce the definitions from contextual clues. Once they established a definition, correct or incorrect, most students reasoned consistently from it. Although reasoning is a strength of most of the subjects, they did not know how to construct an acceptable formal geometric proof. Despite their younger age, these gifted students demonstrated higher overall van Hiele levels than the usual student entering a high school geometry course. However, gifted students, particularly the 35.8% that skipped levels and do not fit the model, need Level 2 and Level 3 experiences in order to provide a foundation for their reasoning. Provided with this additional foundation, gifted middle school students should be capable of a proof-oriented geometry course.

The van Hiele Model of Geometric Understanding and Gifted Students

Dutch educators P. M. van Hiele and Dina van Hiele-Geldof proposed a linearly-ordered model of geometric understanding which asserts that five hierarchical levels of geometric thinking exist and that a successful learner passes through each in order. This study examines whether the van Hiele model accurately describes the geometric thinking of gifted students prior to a formal course in geometry and makes comparisons with what has been found with other populations.

The van Hiele Model

Levels of Geometric Thought

According to the van Hiele model of geometric understanding (van Hiele, 1959/1985; van Hiele, 1986; van Hiele-Geldof, 1984), students progress through five levels of thought as their understanding of geometry develops. The levels, which are sequential and hierarchical, have been described by Clements and Battista (1992) as:

Level 1 (Visualization): Figures are recognized by appearance alone. A figure is perceived as a whole, recognizable by its visible form, but the properties of a figure are not perceived. Visual prototypes are often used in identifying figures. For example, a student might identify a rectangle by comparing it to a door, which he knows is a rectangle. At this level, a student should recognize and name figures and distinguish a given figure from others that look somewhat the same. Decisions are based on perception, not reasoning.

Level 2 (Analysis): Here, properties are perceived, but they are isolated and unrelated. Since each property is seen separately, no relationship between properties is noticed and relationships between different figures are not perceived. For example, a student at this level would know that a triangle had three sides and three angles, but would not realize that as an angle gets bigger, the side opposite it also gets bigger. Figures are seen as collections of properties instead of visual images. A student at this level should recognize and name properties of geometric figures.

Level 3 (Abstraction): At this level, definitions are meaningful, with relationships being perceived between properties and between figures. Students at this level can give informal arguments to justify their reasoning. Logical implications and class inclusions are understood. The role and significance of formal deduction, however, is not understood.

Level 4 (Deduction): At this level deduction is meaningful. The student can construct proofs, understand the role of axioms and definitions, and know the meaning of necessary and sufficient conditions. Students at this level should be able to reason formally using axioms, theorems, and definitions. They can construct proofs such as those typically done in a high school geometry class.

Level 5 (Rigor): The student at this level understands the formal aspects of deduction, establishing and comparing mathematical systems. Symbols without referents can be manipulated according to the laws of formal logic. A student at this level should understand the role and necessity of indirect proof and proof by contrapositive and be able to function in non-Euclidean systems.

Clements and Battista (1992) also proposed the existence of a Level 0 which they call pre-recognition. Subjects at this level attend to only a subset of the visual characteristics of a shape, resulting in an inability to distinguish between many shapes. For example, they may distinguish between triangles and quadrilaterals by focusing on the number of sides the polygons have, but not be able to distinguish between any of the quadrilaterals.

Other Aspects of the Theory

According to the van Hieles' model, the learner cannot achieve one level without mastering the previous levels. While a teacher can reduce content to a lower level and it may appear to be mastered because the student has rotely memorized the material, a student cannot skip a level and still achieve understanding (Clements & Battista, 1992). Progress from one level to the next is more dependent on educational experiences than on age or maturation, and certain types of experiences can facilitate (or impede) progress within a level and to a higher level. Language is critical in progressing through levels. The same word can mean different things to people operating at different levels. For example, a square is a shape that looks like first base to a person functioning at Level 1 and is a polygon with four congruent sides and four right angles to someone at Level 2. Two people operating at different levels might not even be able to understand each other.

Previous Research

Previous research tends to support the van Hieles' description of the model. Burger and Shaughnessy (1986) found mainly Level 1 thinking for subjects in grades K-8. They described the levels as dynamic rather than static and more continuous than discrete. Fuys, Geddes, and Tischler (1988) utilized instructional modules in geometry in six to eight 45-minute individual sessions with 16 sixth graders and 16 ninth graders. They found entry levels of 1 and 2, but several students, especially those deemed above average in mathematics ability prior to instruction, exhibited Level 3 behavior by the completion of the six hours of clinical interviews and instruction.

Usiskin (1982) found a hierarchy of levels existed in the 2,699 students enrolled in 99 high school geometry classes that he examined in 13 schools in 5 states. He found that he could assign a van Hiele level to 88% of these students by using a paper-and-pencil test developed by the Cognitive Development and Achievement in Secondary School Geometry Project (CDASSGP) (Usiskin, 1982), but some students were in transition between levels and therefore difficult to classify. Almost 40% of the students finishing high school geometry were below Level 3.

Using Guttman's scalogram analysis, Mayberry (1983) found sufficient evidence among 19 undergraduate preservice elementary teachers to support the hierarchical aspect of the theory, but she rejected the hypothesis that an individual demonstrated the same level of thinking in all areas of geometry included in school programs. These results were replicated with preservice teachers in Spain for Levels 1 through 4 (Gutiérrez & Jaime, 1987; Gutiérrez, Jaime, & Fortuny, 1991).

After synthesizing existing research, Clements and Battista (1992) concluded that van Hiele theory appears related to Piaget's theories. As an example, they cite Denis' findings based on research with secondary students that "the van Hiele levels appear to be hierarchical across concrete and formal operational Piagetian stages" and there exists "a significant difference in van Hiele level between students at the concrete and formal operational stages" (Clements & Battista, 1992, p. 437).

Research indicates that gifted, average, and retarded children all follow the same pattern of progression through the Piagetian stages (Carter & Ormrod, 1982; Roeper, 1978; Weisz & Zigler, 1979). Gifted students showed superiority on Piagetian tasks over students of normal intelligence at every age level tested. Piaget proposed that the transition to the formal operational stage occurs at ages 11 to 12. Carter and Ormrod (Carter & Ormrod, 1982) found that the majority of subjects of average intelligence were still transitional to formal operations even as late as age 15. They also found that the gifted subjects entered formal operations successfully by 12 - 13 years of age (p. 114).

Since there is a significant difference in van Hiele level between students at the concrete and formal operational stages and since gifted subjects, although they progress through the same sequence of Piagetian stages, enter the formal operational stage earlier than subjects of average intelligence, the question of whether the van Hiele model applies to gifted students as well as it does for non-gifted students arises. This study examines whether the van Hiele model accurately describes the geometric thinking of academically gifted students prior to a formal course in geometry and makes comparisons with what has been found with other populations.

Method

Subjects

The present study focuses on the levels of geometric understanding among students in the sixth through eighth grades who have been identified by their school districts as academically gifted. The subjects had mathematics percentile ranks of 97 or above on the Iowa Test of Basic Skills or the Stanford Achievement Test and teacher recommendations indicating other distinguishing characteristics relevant to mathematics achievement. The population consists of 120 students, drawn from over 50 different school districts, who participated in a National Science Foundation sponsored Young Scholars Program targeted for gifted youth from rural areas during 1990-94. None of the students included in this study had taken a formal course in geometry.

Procedure and Instruments

Two methods of examining the geometric thinking of these gifted students were employed. To enable comparisons with a large general population of students enrolled in high school geometry classes, the 25-item multiple choice paper-and-pencil test developed by the Cognitive Development and Achievement in Secondary School Geometry Project (CDASSGP) (Usiskin, 1982) was administered to all 120 gifted students prior to their participation in the program. In addition, in order to more deeply probe their reasoning, the research conducted 30-45 minute individual interviews with sixty-four randomly selected gifted students.

Paper-and-Pencil Tests. The 25-item multiple choice paper-and-pencil test developed by the Cognitive Development and Achievement in Secondary School Geometry Project (CDASSGP)

(Usiskin, 1982), with 5 proposed answers per item and 5 items per level, was originally developed to test the van Hiele theory by asking:

> (a) Is the theory *descriptive*, in the sense that a unique level can be assigned to each student; and if so (b) is the theory *predictive*, in the sense that the student's van Hiele level can be utilized to predict his or her performance in the traditional tenth-grade geometry course. (Usiskin & Senk, 1990, p.242)

This test was considered as five 5-item tests for purposes of reliability. The K-R formula 20 reliabilities for the five parts reported by Usiskin (1982, p. 29) are .31, .44, .49, .13, and .10. He notes that one reason for the low reliabilities is the small number of items at each level and reports that similar tests with 25 items at each level would have reliabilities of .74, .82, .88, .43, and .38. Thorough evaluations of this instrument by Wilson and Crowley as well as a response from Usiskin and Senk have been published (Wilson, 1990; Crowley, 1990; Usiskin & Senk, 1990).

Answering 4 of 5 questions correctly at a level in this test indicated mastery of that level, a criterion used by CDASSGP to minimize Type I error. If a student met the criterion for mastery of each level up to and including level *n* and failed to meet the criterion for mastery of all the levels above level *n*, the student was assigned to level *n*. If the student could not be assigned to a level in this manner, the student was said to "not fit." Based on an analysis of the results of this test with the 2,699 high school students examined by CDASSGP, Usiskin (1982) concluded that level 5, as described by the van Hieles, either does not exist or is not testable. In this study, the entire instrument was administered to all 120 students, and the results were analyzed both with and without the level 5 items being included.

The Interviews. Sixty-four randomly selected subjects participated in a 30 - 45 minute individual interview, conducted by the researcher prior to their participation in the program. The questions used as a starting point in the interview were a subset of the instrument developed and validated by Mayberry (1981). Guided by Usiskin's conclusion (1982) that level 5 either does not exist or is not testable and by the findings of Burger and Shaughnessy (1986) and Fuys, Geddes, and Tischler (1988) of the levels that students of these grades might be expected to attain, no Level 5 questions were administered. Level 4 questions were administered in one content strand only. The square strand was chosen for the Level 4 questions because it was felt that students would be more familiar with the content of this strand than the other content strands available. In addition to all of the questions from Levels 1-4 of the square strand, questions of interest from the right triangle strand and the isosceles triangle strand were utilized.

Results and Discussion

Paper-and-Pencil Tests

The distribution of the CDASSGP test scores in this study appear in Table 1.

Insert Table 1 about here

Despite their younger age, these gifted students demonstrated higher overall van Hiele levels than the usual student entering a high school geometry course. For example, Senk (1989) using this same CDASSGP instrument with students beginning a high school geometry course, found that of the 241 students in 11 schools in 5 states who "fit the model" according to the test results, 27% had not mastered Level 1, 51% mastered Level 1, 15% mastered Level 2, 7% mastered Level 3, only one student (.4%) mastered Level 4, and no students had mastered Level 5. Of the 77 gifted students who "fit the model" in the current study, only 5% had not mastered Level 1 and 17% were classified as having attained van Hiele Levels 4 or 5. In Senk's study, only 22% were above Level 2. 49% of the gifted students in the current study were above Level 2.

Insert Table 2 about here

However, as seen in Table 1, over 35% of the gifted subjects tested did not fit the model. This is in contrast to the Cognitive

Development and Achievement in Secondary School Geometry Project study in which only 12% of the over 2,600 students about to take high school geometry did not fit the model.

In comparison to students closer to their own age, Fuys, Geddes, and Tischler (1988) found no one functioning above Level 2 in interviewing sixth and ninth grade average and "above average" subjects while Burger and Shaughnessy (1986) found mainly Level 1 thinking in grades K-8. Table 3 shows the lowest van Hiele level not mastered on the CDASSGP test by the gifted students in the current study.

Insert Table 3 about here

Proof Readiness. In order to examine the predictive power of a student's van Hiele level, Senk (Usiskin & Senk, 1990) compared the van Hiele levels indicated by the CDASSGP test prior to a high school geometry course with their performance in proof writing as measured by the CDASSGP Proof Test at the end of the course in the spring. Applying her findings to the current study, only 5% of the gifted students, all 6th and 7th graders, have not mastered Level 1 and so have a probability of success in proof writing of less than .35. 25% of the students have mastered Level 1 and have a probability of successful proof writing between .35 and .60. The remaining 70% of these gifted students have van Hiele levels 2 or greater and have probability of proof writing success greater than .75.

Interviews

The percentage of subjects at each van Hiele level as determined by the interviews is given in Table 4 for the Square and Right Triangle Strands.

Insert Table 4 about here

Analysis of the clinical interviews also confirmed the hypothesis that the van Hiele levels are hierarchical in gifted subjects. Table 5 summarizes the coefficients of reproducibility from the Guttman scalogram analysis of the four levels in the square strand and the three levels in the right triangle strand.

Insert Table 5 about here

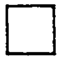
Analysis of the clinical interviews confirmed Mayberry's rejection of the hypothesis that an individual demonstrates the same level of thinking in all areas of geometry included in the school program (Mayberry, 1983). Excluding the 15 subjects who exhibited mastery of the highest levels of both the square and right triangle strands administered, only 8 of the remaining 49 subjects were deemed to be thinking at the same level in the two content areas. As might be anticipated, the subjects were thinking at higher levels when considering squares than when considering right triangles or isosceles triangles. In particular, only 1.6% of the subjects were below Level 2 thinking on the Square Strand, compared to 26.5% below Level 2 on the Right Triangle Strand. This does not imply that the students were not capable of thinking in terms of relationships with right triangles. However, they were less familar with the properties of right triangles and the relationships that exist between the properties and between the such figures than in the square domain. For example, all the students knew that a square had four equal sides without really having to deduce it. Fifty-four of the 64 knew that a right triangle always has a longest side, but only 27 of those 54 appeared to know it as a fact rather than having to deduce it from by several drawings. Their greater familarity with the properties and relationships in the Square Strand makes these properties and relationships more accessible and provides a greater comfort level, which, in turn, makes the students more likely to think in terms of properties and relationships. This is consistent with the findings of Burger and Shaughnessy who described the reasoning of post-geometry 10th - 12th graders:

> Flashes of Level 2 reasoning would occur but usually only as a result of probing. Such students, left to their own devices, seemed to prefer the relative safety of Level 2 reasoning and tended to avoid deduction, even though they knew it was available. (1986, p. 45)

In addition, these gifted students appeared to function at the minimum level they perceived a particular task or the interviewer required. For example, when identifying various quadrilaterals as squares or rectangles, 12 of the 64 students identified a non-square rhombus, , as a square.

Insert Table 6 about here

Later in their interviews, 11 of the 12 said that squares must have 4 or all right angles. The twelfth students, a sixth grader, specified that all the angles in a square must be equal. When shown the 3 rhombi again, all 12 students now identified only the two squares as squares. When asked why they had previously identified the other shape, , as a square, all twelve said because "it looks like a square" or "it has the shape of a square", answers indicative of Level 1 reasoning, even though all 12 students knew that a square had all right or equal angles and demonstrated that they could apply that part of the definition in identifying squares and other tasks.

Vinner and Hershkovitz (1980) and Fuys, Geddes, and Tischler(1988) noted that some students can know a correct verbal description of a concept but have a certain visual image associated with that concept so strongly that the student has difficulty applying the verbal description correctly. The identification of a non-square rhombus as a square doesn't appear to fall into this category, however, since the students all drew a figure like  when asked to draw a square and also later readily used the fact that a square has four equal angles in comparing a square to other figures.

Similarly, 3 students identified the non-square rectangles as squares even though they later said that all a square's sides had to be the same length. Two students later corrected themselves when they were asked how squares and rectangles are alike. One of the seventh graders explained that she said the non-square rectangle was a square because she remembered her teacher telling her that "All rectangles are squares, but not all rectangles are squares." She said that it didn't seem right to her, but that was what her teacher said, so it must be that way.

Concepts and Logical Reasoning

Burger and Shaughnessy (1986) have characterized the van Hiele levels as very complex structures that involve the development of both concepts and reasoning processes. This dual nature of geometric understanding in gifted students is particularly evident in the portions of the interviews dealing with inclusion relationships.

Only 31 of the 64 students identified the squares shown then as being rectangles as well as squares.

Insert Table 7 about here

Sixteen of the other 33 students later said that all squares are also rectangles and were able to give correct arguments such as offered by one sixth grader: "A rectangle is a polygon with four sides and four right angles and all squares have these features." Fourteen of the remaining 17 believed that squares were not rectangles because, as one sixth grader said, "Squares have four congruent sides. Rectangles have 2 small and 2 long." The remaining three maintained that squares just look different than rectangles, a Level 1 response.

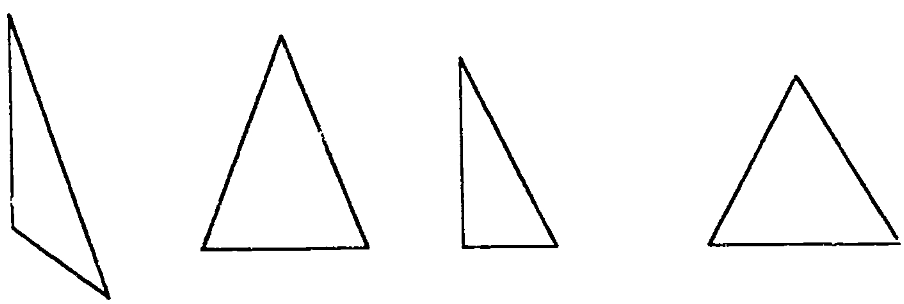
Even though isosceles and equilateral triangles are a standard part of the school mathematics curriculum prior to the end of sixth grade, the subjects provided a wide range of definitions for the term "isosceles triangle" as shown in Table 8.

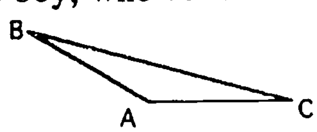
Insert Table 8 about here

Fifty-one of the 63 subjects knew that an isosceles triangle had two sides and/or angles the same, but only 32 consistently interpreted this to mean "at least two." In fact, four 8th graders and one 7th grader specifically stated that the third side of the triangle had to be longer or shorter than the other two sides and an additional 8 students, while not specifically stating it as part of their definition, interpreted "two = sides" as "exactly two = sides" as indicated by explanations later in their interviews. Two 7th graders and two 6th graders said that two sides had to be equal, but were inconsistent in whether they interpreted this to mean "exactly two" or "at least two." Only 15 of the 63 subjects included a reference to "two equal angles" or "at least two equal angles" in their definition of isosceles triangle.

When questioned, 14 subjects of the sixty-three admitted that they were unsure of the correct definition of isosceles triangle, but 12 of these students provided definitions such as no sides congruent, all sides congruent, all angles less than 90°, or containing one angle greater than 90°. Only two subjects answered "I don't know" when asked for a definition of an isosceles triangle. Many of these definitions appear to be deductions based on the structure of previous questions. For example, one of the Level 1 questions encountered prior to the request for the definitions was:

Which of these figures are isosceles triangles?



One seventh grade boy, who defined an isosceles triangle by making a drawing such as 

and saying "If the three points were ABC, A is obtuse, B & C are acute.", explained his reasoning in the following manner:

> In that other question, I wasn't sure what isosceles meant. So I looked at the pictures. I knew the third one was a right triangle, so that wasn't the answer. The second and fourth ones looked like regular old triangles. I said the one that looked really different, the first one. Then when I needed a definition, I figured out what made it what it was - the one obtuse angle and the two acutes.

Once they gave a definition, most students reasoned consistently from it. For example, one eighth grade girl who defined an isosceles triangle as "The sides are all different sizes", answered the question "Are some right triangles isosceles triangles?" by saying "Yes. A triangle could have a 90° angle and have two different angles for the rest." A seventh grade male, who defined an isosceles triangle as having "no congruent angles or sides", answered "Triangle DEF has three congruent sides. Is it an isosceles triangle?" by saying "no, isosceles has no congruent sides." A seventh grade female, who defined an isosceles triangle as "a triangle with an angle over 90°.", answered that same question, "No. In order for it to be an isosceles triangle at least one of the sides must be larger." When asked "Are some right triangles isosceles triangles?", this same seventh grade girl replied, "Isosceles triangles must have an angle over 90° and having one angle at 90° would not leave enough degrees for two more angles." Only four students were inconsistent is applying their stated definitions.

As illustrated above, the reasoning ability of these gifted subjects, as demonstrated during the interviews, was far beyond what may have been anticipated, given their lack of knowledge of basic definitions and concepts. In most cases, the students built valid logic structures based upon their conjectured definitions. This type of thinking is indicative of Level 3, but has been accomplished without knowledge of the specific definition or geometric content.

Of the 120 students, 76 demonstrated mastery of Level 3, but 29 of the 76 (38%) failed to demonstrate mastery of Level 1 or Level 2 questions. Five of the 120 students (4%) exhibited mastery of the Level 5 questions on the CDASSGP test although they had failed to do so for at least two of the previous levels. Four students (3%) did not master even Level 1 and exhibited thinking characteristic of Clements' and Battista's hypothesized Level 0, Pre-recognition, supporting the existence of such a level.

Conclusions and Recommendations

This study investigated whether the van Hiele model accurately described the geometric thinking of academically gifted subjects prior to a formal course in geometry by analyzing the results from a 25-item multiple choice paper-and-pencil test, developed by the Cognitive Development and Achievement in Secondary School

Geometry Project, administered to 120 gifted students and the results of 30-45 minute individual interviews, based on a subset of Mayberry's test, administered to 64 gifted students.

While the responses of the students did form a hierarchy overall, 35.8% of the gifted students tested skipped levels in the van Hiele model. Analysis of the clinical interviews confirmed that individuals do not demonstrate the same level of thinking in all areas of geometry included in the school program.

Many of these gifted subjects had not been exposed to or did not remember what the critical defining attributes of various figures were. However, they tended to look for similarities and differences in figures (a characteristic of subjects who have attained at least Piaget's Pre-Operational Stage) and deduce what the defining attributes might be. Many of the students lacked correct basic definitions of terms such as congruent and similar, but they would attempt to deduce the definitions from contextual clues. Once they established a definition, correct or incorrect, most students reasoned consistently from it.

Generally, these students were capable of handling inclusion relationships if they had suitable definitions of the elements involved, a characteristic of Piaget's Concrete Operational Stage as well as van Hiele's Level 3. But an equilateral triangle can not be an isosceles triangle if you think that an isosceles triangle has exactly two congruent sides.

Deduction is a strength of most of the subjects. However, they have not been exposed to the "rules of the game" and so do not know how to construct an acceptable formal geometric proof. In addition, they do not know the role of axioms and definitions and the meaning of necessary and sufficient conditions. It should be noted that deductive reasoning is a skill which can be developed outside the context of geometry, as it apparently has with many of these subjects.

Despite their younger age, these gifted students demonstrated higher overall van Hiele levels than the usual student entering a high school geometry course. Using probabilities developed in the CDASSGP study (Usiskin & Senk, 1990), 70% of the students, who were able to have a level assigned, have a probability of proof writing success greater than .75. However, gifted students, particularly the 35.8% that have skipped levels and do not fit the model, need Level 2 and Level 3 experiences in order to provide a foundation for their reasoning. Provided with this additional foundation, gifted middle school students should be capable of a proof oriented geometry course.

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Table 1

% of Subjects at Each van Hiele Level as Determined by the CDASSGP Test

		van Hiele Level						
Grade	n	not mastered 1	1	2	3	4	5	no-fit
8	46	0.0	15.2	10.9	26.1	2.2	10.9	34.8
7	36	2.8	8.3	8.3	25.0	5.6	2.8	47.2
6	38	7.9	23.7	23.7	7.9	7.9	2.6	26.3
Total	120	3.3	15.8	14.2	20.0	5.0	5.8	35.8

Table 2

% of Gifted Students and Students Entering
High School Geometry at Each van Hiele Level
on the CDASSGP Test Excluding "No-Fits"

		van Hiele Level					
Grade	n	not mastered 1	1	2	3	4	5
8	30	0	23	17	40	3	17
7	19	5	16	16	47	11	5
6	28	11	32	32	11	11	4
Total	77	5	25	22	31	8	9
High School*	241	27	51	15	7	0	0

* Data for students entering high school geometry was reported by Senk (1989, p. 315). Other data is from gifted students in the current study.

Table 3

Lowest van Hiele Level Not Mastered
on the CDASSGP Test in % of Subjects

		van Hiele Level					
Grade	n	1	2	3	4	5	none
8	46	6.5	26.1	17.4	37.0	2.2	10.9
7	36	8.3	22.2	11.1	50.0	5.6	2.8
6	38	18.4	28.9	26.3	15.8	7.9	2.6
Total	120	10.8	25.8	18.3	34.2	5.0	5.8

Table 4

% of Subjects at Each van Hiele Level as Determined by the Interviews in the Square and Right Triangle Strands

			van Hiele Level					
Grade	n	Strand	not mastered 1	1	2	3	4	no-fit
8	20	square	0.0	0.0	35.0	25.0	20.0	20.0
		rt. Δ	0.0	20.0	20.0	45.0	n/a	15.0
7	24	square	0.0	0.0	25.0	8.3	45.8	20.8
		rt. Δ	0.0	20.8	12.5	50.0	n/a	16.7
6	20	square	0.0	5.0	30.0	10.0	35.0	20.0
		rt. Δ	10.0	30.0	10.0	25.0	n/a	25.0
Total	64	square	0.0	1.6	29.7	14.1	34.4	20.3
		rt. Δ	3.1	23.4	14.1	40.6	n/a	18.8

n/a = not administered

Table 5

Coefficients of Reproducibility for van Hiele Response Patterns in Interview

Grade	n	Squares Errors/4 Levels	Rep	Right Triangles Errors/3 Levels	Rep
8	20	4	.95	3	.95
7	24	5	.95	4	.94
6	20	4	.95	5	.92
Total	64	13	.95	12	.94

Table 6

% of Students Identifying Various Shapes as Squares

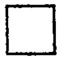
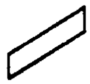
		Shape					
Grade	n						
8	20	100.0	100.0	20.0	0.0	0.0	0.0
7	24	100.0	100.0	8.3	4.2	4.2	0.0
6	20	100.0	100.0	30.0	10.0	10.0	0.0
Total	64	100.0	100.0	18.8	4.7	4.7	0.0

Table 7

% of Students Identifying Various Shapes as Rectangles

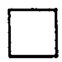
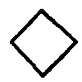
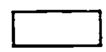
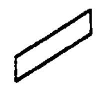
		Shape					
Grade	n						
8	20	40.0	40.0	10.0	100.0	100.0	45.0
7	24	50.0	50.0	0.0	100.0	100.0	4.2
6	20	55.0	55.0	10.0	95.0	95.0	40.0
Total	64	48.4	48.4	3.1	98.4	98.4	28.1

Table 8

Number of Students At Each Grade Level Providing Specific Definitions of "Isosceles Triangle"

	Grade			
	6	7	8	Total
At least 2 sides =	5	8 (5)[a]	3 (3)[a]	16
2 sides =, interpreted as "at least 2"	2 (1)[a]	7 (1)[a]	7(1)[a]	16
2 sides =, interpreted as "exactly 2"	3	1	4(1)[b]	8
2 sides =, inconsistent interpretation	2	2	0	4
2 sides = with 3rd different	0	1	4(2)[a]	5
No sides ≅	2	2	2	6
All sides ≅	2	0	0	2
2 = <s	2 (1)[c]	8 (8)[c]	4 (3)[c]	14
All <s < 90°	2 (1)[b]	0	0	2
One < > 90°	0	2	0	2
I don't know	1	0	1	2

Notes. [a]Number in parentheses refers to number of subjects who also referred to two angles or at least two angles being the same. [b]One subject defined an isosceles triangle as being "a 3-sided figures with all angles less than 90° and with 2 sides of the same length and 2 angles that are the same" with the exactly two interpretation. [c]Number in parentheses refers to number of subjects who also referred to two sides or at least two sides being the same.