DOCUMENT RESUME

ED 339 561 SE 057 204

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TITLE Radical Constructivism as a Basis for Mathematics Reform.
PUB DATE Oct 95
NOTE 8p.; Paper presented at the Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education (17th, Columbus, OH, October 21-24, 1995). For entire conference proceedings, see SE 057 177.
PUB TYPE Reports - Research/Technical (143) -- Speeches/Conference Papers (150)

EDRS PRICE MF01/PC01 Plus Postage.
DESCRIPTORS Affective Behavior; Cognitive Development; *College Mathematics; *Constructivism (Learning); *Geometry; Higher Education; *Mathematics Instruction; *Problem Solving
IDENTIFIERS *Connections (Mathematics); *Reform Efforts

ABSTRACT

This paper describes the use of radical constructivism as a basis for curriculum reform in university mathematics courses and reports on research conducted on two of the courses developed, one in geometry and one in problem solving. The theoretical underpinnings of the project are described along with the implications for course design and instruction. Finally, results from qualitative research conducted on the courses are presented. The courses were found to foster intellectual autonomy, challenge students to rethink mathematics from a conceptual rather than procedural perspective, promote confidence in their mathematics knowledge, become more positive mathematics learners, and make connections among algebra, geometry, and calculus concepts. (Author)

Radical Constructivism as a Basis for Mathematics Reform

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Paper presented at the Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education

(17th PME-NA, Columbus, OH, October 21-24, 1995)

RADICAL CONSTRUCTIVISM AS A BASIS FOR MATHEMATICS REFORM

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This paper describes the use of radical constructivism as a basis for curriculum reform in university mathematics courses and reports on research conducted on two of the courses developed. The theoretical underpinning of the project is described along with the implications for course design and instruction. Finally, results from qualitative research conducted on two of the courses is presented. The courses were found to foster intellectual autonomy, challenge students to rethink mathematics from a conceptual rather than procedural perspective, promote confidence in their mathematics knowledge, become more positive mathematics learners and make connections among algebra, geometry, and calculus concepts.

It is often said that we teach as we are taught. Undoubtedly, the nature of instruction in mathematics courses taken in college greatly influence the teaching styles and practices of teachers. "Very few teachers have had the experience of constructing for themselves any of the mathematics that they are asked to teach. " (National Research Council, 1989). In designing a middle school mathematics teacher education program, we recognized the importance of having mathematics taught in a manner compatible with the goals of their pedagogical courses. We have much experience with methods courses following the recommendations of the National Council of Teachers of Mathematics and the National Research Council being offered to students who take mathematics courses based on logical positivism and behaviorism - mathematics courses which are lecture based and emphasize practicing taught procedures. This conflict has not been lost on prospective teachers. They struggle with the question, "Why am I being asked to teach in a way I have never experienced in a mathematics class?" Beginning teachers will instinctively use teaching methods like those experienced in the many mathematics courses taken in high school and college. We have not always been able to overcome the impact of many hours listening to lectures and practicing procedures which are then marked right or wrong. Thus, we recognized the importance of mathematics courses for prospective teachers which emphasize sense making, encourage collaboration and promote intellectual autonomy.

The purpose of this paper is to describe a theoretical basis for mathematics instruction and report findings from analyses of courses based on the theory. As one component of our four-year teacher education project funded by the National Science Foundation, four mathematics courses for prospective middle school mathematics teachers were designed; Number Theory, Algebra, Geometry, and Problem Solving. Each course was created by a course development team (one for each course) composed of two or more mathematicians, a mathematics educator and several mathematics education graduate students. We were most fortunate in having a mathematician who understood the reform movement and believed in opportunities for students to construct their own knowledge as a member of each team. A team met for a year planning the geometry course and a semester for each of the other courses. Our thinking was influenced by the NCTM Professional Standards (1991) and a constructivist epistemology (von Glasersfeld, 1995a). Examples of the mathematics and instruction will be drawn from the geometry and the problem solving courses; analyses of the other courses are in progress.

Epistemology

In designing courses and planning lessons, it is useful to have a clearly defined epistemological theory. For this project, radical constructivism as described by von Glasersfeld (1995a, 1995b) served as the theoretical orientation. In this theory of knowing, which has been used for other mathematics educational reforms, it is assumed that knowledge cannot be transmitted but must be constructed by the learner. Students have only their personal experiences upon which to rely in this constructive process and each person has unique experiences. Of course a person's experiences include other persons and thus it is not a 'lonely voyage.' Thus activities in which students are encouraged to work together in solving a problem, to listen, explain and challenge peers provide rich potential learning opportunities.

A second principle of radical constructivism has to do with the nature of knowledge. For the logical positivist, knowledge is out there, out there for the behaviorist to observe. For the radical constructivist, knowledge is an individual construction which results from attempting to make sense of our experiences. Knowledge is not true or false but viable or not viable. As von Glasersfeld states,

> [We must] Give up the requirement that knowledge represent an independent world, and admit instead that knowledge represents something that is far more important to us, namely what we can *do* in our *experiential* world, the successful ways of dealing with the objects we call physical and the successful ways of thinking with abstract concepts. (pp. 6-7)

A radical constructivist epistemology places importance on constructing models of student's thinking. As von Glasersfeld states,

> In the endeavor to arrive at a viable model of the student's thinking, it is important to consider that whatever a student does or says in the context of solving a problem is what, at the moment, makes sense to the student. It may seem to make no sense to a teacher, but unless the teacher can elicit an explanation or generate a hypothesis as to how the student has arrived at the answer, the chances of modifying the student's conceptual structures are minimal. (p. 15)

Certain classroom practices are suggested by radical constructivism. First, we must negotiate a set of social norms in which emphasis is on making sense rather than following procedures specified by an instructor. The goal is for each individual to develop a rich network of schemes which are viable. The social norms might include

- A task requires time and investigation; we should not be expected to know how to do a task but instead develop our own procedures for accomplishing the task. An exploratory mind-set is essential.
- Students are expected to explain their reasoning to peers; viability is established by convincing others. An assertion (proof) which stands the test of time is said to be viable.
- Collaboration is an accepted environment for learning.

The implications for the instruction flowing from this theory are:

1. The mathematics to be studied must be analyzed to determine the major concepts and relationships.
2. It is important to build models of students thinking.
3. Based on these first two practices, tasks are designed which have potential learning opportunities.
4. All activities must be potentially meaningful to the students.
5. Meaning must be negotiated; it cannot be transmitted or legislated.
6. A major responsibility of the teacher is to facilitate classroom discourse.
7. This entire process is recursive.

Procedure

Each session of the two courses were video recorded for the full semester. Field notes were collected and each instructor reflected on lessons after each class session. The geometry course was taught by a mathematics education doctoral student and the problem solving course was taught by a mathematics education professor. The principles on which these courses were designed were:

1. The courses should focus on central ideas in mathematics and promote progressive schematization rather than specified procedures.
2. Activities must be interesting and potentially meaningful to the students. In each case an effort was made to approach the subject from a different perspective than they had seen previously. For example, many of the properties of plane geometry were developed from a study of spherical geometry, e.g., straightness.
3. Students are to be encouraged to become intellectually autonomous rather than simply doing what the instructor said whether it made sense to them or not.

4. A problem-centered instructional model (Wheatley, 1991) was adopted.

5. Collaboration was encouraged.

6. Students were required to justify the viability of their solutions. Rather than the teacher judging responses as right or wrong, students presented their solutions to the class and the class had to be convinced of the validity of the solution.

7. Technology was to be used whenever feasible. For example, two computer microworlds were developed for the geometry course and spreadsheets were used extensively in the number theory course.

8. Assessment was based, in large part, on informed professional judgment and portfolios.

In teaching the courses, considerable attention was devoted to negotiating social norms conducive to inquiry and intellectual autonomy. Students often entered the courses with a belief that mathematics is a set of facts and procedures to be explained by the teacher and remembered by them. The courses were designed to foster the view that mathematics is the *activity* of constructing patterns and relationships. Students were encouraged to take responsibility for their knowledge construction in conjunction with other members of the class.

Because the instructor rejected the role of mathematical authority, the students began to assume responsibility for justifying their actions. These justifications took the form of students presenting their solutions to problems they had solved and responding to questions raised by their peers or the instructor. At times students who disagreed or had an alternative solution went to the board and began explaining their point of view without any action by the teacher; none was required. In both courses the mode of instruction utilized was problem centered learning as described by Wheatley (1991).

Approximately one-fourth of the geometry course was devoted to a study of spherical geometry. The decision to study spherical geometry in a course for prospective middle school students was based on our belief that interesting and significant questions could be raised which would deepen the meaning given to plane geometry concepts such as straightness, angle, and quadrilateral. The topic was also potentially meaningful and interesting since we do not live on a flat surface and NASA activities have raised our consciousness of the earth as a sphere. The statement, "The sum of the angles of a triangle is 180 degrees" takes on a richer meaning once triangles have been drawn on a beach ball with marking pens and the sum of the angles determined. In addition to spherical geometry, topics in plane geometry and measurement were studied through problem solving.

The students became quite interested in the study of spherical geometry as evidenced by observations, their journal entries, interviews and written evaluations at the end of the course. Students were thrown into a state of disequilibrium by some of their findings as they engaged in these activities. Of particular value were the two computer microworlds in which students could explore paths on a sphere. Not only were the students motivated to study spherical geometry but they made significant geometric (mental) constructions.

In the problem solving course, the nature of solutions became increasingly more organized and sophisticated. Initially, students attempted to identify formulas and substitute numbers but soon realized this approach would not work on the nonroutine problems they faced. But as they participated in the negotiation of a different way of doing mathematics, they became more thoughtful about their activity. For example, in week six of the course students presented a variety of solutions to the following problem.

> A column of soldiers 25 miles long marches 25 miles a day. One morning, just as the day's march began, a messenger started at the rear of the column with a message for the man at the front of the column. During the day he marched forward, delivered the message to the first man in the column and returned to his position just as the day's march ended. How far did the messenger walk?

This problem required rather sophisticated problem solving strategies and considerable power in thinking in terms of rates. Additional information about the problem solving course can be found in Trowell (1994).

Analysis of the courses indicated that 1) students were challenged to rethink mathematics concepts previously studied but not understood; 2) students developed confidence in their mathematics knowledge; 3) students became more positive as mathematics learners; and 4) students increased their competence and made connections among algebra, geometry, and calculus concepts.

Summary

In a *Call for Change: Recommendations for the Preparation of Teachers of Mathematics*, the Mathematics Association of America states that, ". . . collegiate mathematics classrooms must become a place where students actively do mathematics rather than simply learn about it" (p. 2). This statement could be interpreted as embracing a constructivist approach to mathematics teaching. In this study, evidence for the power of university mathematics courses based on radical constructivism was obtained. While certainly not the only viable theoretical orientation for successful mathematics teaching, radical constructivism, growing out of Piagetian theory, provides a sound basis for facilitating mathematics learning.

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