DOCUMENT RESUME

ED 389 557 SE 057 200

AUTHOR	Li, Bey-bey; Adams, Verna M.
TITLE	Teacher Guidance in an Exploratory Mathematics Class.
SPONS AGENCY	National Science Foundation, Washington, D.C.
PUB DATE	Oct 95
CONTRACT	RED-9254922
NOTE	8p.; Paper presented at the Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education (17th, Columbus, OH, October 21-24, 1995). For entire conference proceedings, see SE 057 177.
PUB TYPE	Reports - Research/Technical (143) -- Speeches/Conference Papers (150)
EDRS PRICE	MF01/PC01 Plus Postage.
DESCRIPTORS	*Classroom Communication; *Concept Formation; Grade 7; Grade 8; Junior High Schools; Junior High School Students; *Mathematics Instruction; Middle Schools; *Probability; Teaching Methods
IDENTIFIERS	*Combinatorics; *Permutations (Mathematics); Scaffolding

ABSTRACT

This paper is concerned with understanding how a scaffolding process is utilized in the natural setting of a middle-school mathematics class. Wood, Bruner, & Ross (1976) characterize scaffolding as a learning process of a novice which is assisted and dominated by the adult. Rogoff and Gardner (1984) also point out that "to make messages sufficiently redundant" is one way to provide scaffolding. This study examined classroom discourse when a new topic was introduced to the class. The teacher connected a new topic (combinations) to old content (permutations). He used abundant, similar, but slightly changed examples as referents to help the students attach meaning to symbols for permutations and then began to turn over the discourse to the students to support their development of that new content. (Author)

Teacher Guidance in an Exploratory Mathematics Class

Bey-bey Li and Verna M. Adams

Paper presented at the Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education

(17th PME-NA, Columbus, OH, October 21-24, 1995)

TEACHER GUIDANCE IN AN EXPLORATORY MATHEMATICS CLASS

Bey-bey Li, Washington State University
Verna M. Adams, Washington State University

This paper is concerned with understanding how a scaffolding process is utilized in a natural setting of a middle-school mathematics class. Wood, Bruner, & Ross (1976) characterize scaffolding as a learning process of a novice which is assisted and dominated by the adult. Rogoff and Gardner (1984) also point out that "to make messages sufficiently redundant" (p.109) is one way to provide scaffolding. In this study, we examine the classroom discourse when a new topic is introduced to the class. The teacher connects a new topic (combinations) to old content (permutations). He uses abundant, similar, but condition changed slightly, examples as referents to help the students attach meaning to symbols for permutations and then begins to turn over the discourse to the students to support students' development of that new content.

To identify scaffolding, we draw on Wood, Bruner, & Ross's (1976) description of scaffolding as "a process that enables a child or novice to solve a problem, carry out a task or to achieve a goal which would be beyond her/his unassisted efforts. This scaffolding consists essentially of the adult 'controlling' those elements of the task that are initially beyond the learner's capacity" (p.90). Rogoff and Gardner's (1984) use of data from a study of mothers preparing their children for a memory test illustrate how an adult's instruction serves as a scaffold for the learner. They emphasize that "to make messages sufficiently redundant" (p.109) is one way to provide scaffolding. They also conclude that the adult assists children with new challenging problems by guiding children to make connections to more familiar contexts. We view scaffolding as a process that transfers responsibility back and forth between the teacher and students until it is completely turned over to the students in problem-solving situations. Errors and uncertain responses of students in the classroom dialogue function for the teacher as signals that students are in or beyond their zones of proximal development (Vygotsky, 1978, 1986; Wertsch, 1985). If students are functioning in their zones of proximal development, then teacher intervention may help them function successfully. Greenfield (1984) pointed out that "Errors, either anticipated or actual, are used as a signal to upgrade the scaffold, transferring responsibility from the learner to the teacher" (p.136). The challenge for the teacher is to communicate in a way that helps the teacher identify the students' thinking and that allows the students to participate and redefine the task.

We also draw on Hiebert's (1988) theory of cognitive processes involved in increasing students' competence with written mathematics symbols. Five sequential types of processes are distinguished: "(1) *connecting* individual symbols with

Preparation of this paper was supported in part by National Science Foundation, Grant No. RED-9254922. Any opinions, conclusions, or recommendations are those of the authors and do not necessarily reflect the views of the National Science Foundation.

referents; (2) *developing* symbol manipulation procedures; (3a) *elaborating* procedures for symbols; (3b) *routinizing* the procedures for manipulating symbols; and (4) using the symbols and rules as referents for *building* more abstract symbol systems" (p.335). The first two processes include building referents on students' previous experiences and manipulating referents, observing the result, and translating referents to symbol world. Their purpose is to provide symbols with meaning. The subsequent cognitive processes shift from a heavy dependence on referents to a mediation of the symbols and rules themselves.

This study examines how a teacher uses scaffolding to support the above first two cognitive processes by using abundant, similar, but condition changed slightly, examples as referents. As part of the scaffolding process, he turns over the discourse to the students.

The Study

The class described in this paper was an elective class for 7th graders (5 students) and 8th graders (13 students). Its purpose was to explore mathematical topics that were not provided in the students' "regular" math class. The teacher was a fifth-year teacher with a major in mathematics in his teacher preparation program. He has excellent rapport with middle school students and great enthusiasm for mathematics. The teacher felt that he had less time pressure and greater flexibility to design and run this class than other math classes. The classroom had a relaxed atmosphere, yet the teacher had a firm control over student misbehavior. He enthusiastically guided students to deal with numbers, especially large numbers, and made connections to real world experiences. A variety of interactions were used in the classroom: the teacher led whole-class discussions, the students worked individually and in small groups, and groups gave problem-solving presentations.

Data collection consisted of daily videotaping, field notes, collection of materials used by the students, interviews of selected students and the teacher, and an initial and a final whole-class survey of students' attitudes and perceptions of the class. The mathematical content during the 8-week period of data collection was a unit on methods of counting, including permutations and combinations. This paper focuses on the introduction of a new topic and symbol. Analysis of the discourse revealed scaffolding patterns of discourse. From that observation, a more serious focus on scaffolding evolved. In the paper, we attempt to show how the teacher introduced new content, developed students' understanding through the use of sufficiently redundant examples as referents, and then turned over the learning process to the students.

Building New Content on Existing Knowledge

Prior to this episode, the teacher introduced real-life examples of permutations related to the Rose Bowl game, phone numbers of a town, and credit card issues. The whole-class discussions were followed by two sets of exercises worked in small groups and one done individually. Toward the end of that topic, students were using symbols for permutations to present problems and calculate numerical answers. When the teacher introduced combinations into the class dialogue, he emphasized that it was important to note the difference between combinations and permutations: "If you don't understand the difference, it is going to be very, very confusing for you." He started by building the new content (combinations) on familiar content (permutations), pointing out differences:

Teacher: Who can explain to me what a permutation is—in their own words? Yeah, TN?

TN: A permutation is [pause] a, but something, like a number, like ${}_3P_1$, and P stands for permutations.

For TN, a permutation was two numbers associated by a symbol *P*. The teacher accepted TN's response, but then tried to help her to re-organize her understanding and connect the symbol to vivid referents. He narrowed his question to focus on the concept.

Teacher: What does the word mean by itself? Disregard the number—[speaking slowly with emphasis] permutation?

TN: Oh, the arrangement.

Again, the teacher did not reject TN's incomplete explanation of permutations. Instead, he decided to give the class a concrete example. Below, we show how the teacher used redundancy to connect the numerical component of the situation to concept development.

Use of Sufficiently Redundant Examples as Referents

The redundancy occurs in both the type and number of examples. The teacher builds an introduction to combinations based on the students' understandings of permutations:

Teacher: All right, now, the word permutation—one key element in that idea is a very specific arrangement of things. If I were to, if we're to elect two people out of the eighteen votes, the president and the vice president in this, uh, classroom—if I elect LP as the president and LN as the vice-president, that arrangement is very important. It's totally different than if I elected LN as the president and LP as vice-president. It's the same two people, but I changed the arrangement, and I get a different situation. So, when, in a permutation, you change the arrangement and get something different, that's, that's called a permutation. You change the arrangement, you get something different. Yeah?

TN: So, would it be, would it be like 18 times 17?

Although TN selected the right numbers to fit this example, her questioning response indicated that she was still struggling to attach numbers to the example. The teacher not only agreed with her, but added more verbal explanation of the meaning of those numbers. Then, he slightly changed his example situation to move to combinations:

Teacher: That is exactly like that ${}_{18}P_2$ [writes "${}_{18}P_2$" on the chalkboard] which is how many ways I can take and arrange 2 people from 18, would be ${}_{18}P_2$. Now, that's a different problem—what if I say we're in a magazine sale and we win, uh, say candy or ice cream—and say I just want to send two people down to the office to pick up ice cream. That is not a permutation. Just say I picked LN and LP to go down to the office. If I picked LN and LP to go down to tne office, that's the same thing as if I picked LP and LN to go down to the office. It doesn't matter which one I picked first, I just picked those two people. So if I switched the order, I picked LN first and then LP, that's the same as I picked LP and LN. That doesn't matter: I just sent two people down to the office to pick up ice cream for us. Who cares what order they're in, so that's not a permutation because the order is not important. That's called a combination [Writes "combination" on the chalkboard].

In this introduction, the teacher utilized numbers provided by TN, connecting the new concept to a familiar concept by example. On the left side of the chalkboard, he listed a pattern of 15 permutations using notations such as ${}_1P_1$, ${}_2P_1$, ${}_2P_2$, ${}_3P_1$, continuing to ${}_5P_5$. These notations were familiar to students. Before attaching each notation with meaningful referents, he asked the students to calculate the numerical answers for each permutation. Then they reviewed and wrote the general formula "nPr = n!/(n-r)!" on the chalkboard.

The teacher asked students to compare ${}_1C_1$ with ${}_1P_1$, using an example of choosing one person from a class with one student. He gradually increased the number of students in the imaginary class to two, three...five. He repeated similar, but condition changed slightly, examples in order to link permutations and combinations. There was a sense formed in the class that the president and vice president issue was related to permutation problems and sending students to pick up ice cream was associated with combinations. The teacher wrote combination notations and amounts on the right side of the chalkboard to leave a visual record in front of the students. As the development continued, he frequently pointed out the slightly changed conditions for the new content. They completed their list of 15 permutations and 15 combinations and students looked at the groups of symbols and numbers on the chalkboard to figure out the relationship between patterns of permutations and combinations. TN noticed that permutations and combinations could be connected by division: "So, we could do it like a permutation, but divide it by two?"

Turn Over of Dialogue to the Student

The teacher encouraged more students to participate, changing his role from a speaker who did most of the talking to one who supported and helped students to organize their own thoughts. Dialogues in the classroom turned from lecturing by the teacher to students' observation and participation.

Teacher: Check it out. For this problem, TN is saying, I can do this just like a permutation but divide it by two and I get, cause ${}_3P_2$ is 6, divide it by 2. I get 3. That's my answer. Let's see if it works for this [pointing to ${}_3C_3$] ?

Students used *picking up ice cream* to decide that ${}_3C_3$ was equal to one. TN's rule was not satisfied in this situation. The teacher pointed out that students held different pieces of information and he encouraged all students to participate. Many students raised their hands to show that they wanted to contribute. At this moment, students did not understand the general relationship between combinations and permutations yet, but they enthusiastically participated in the discussion as they searched for pattern. Compared to the beginning of the class, the dialogues were very different. One student, FT, presented his thoughts in fragmented phrases. The teacher patiently gave short responses to FT's comments to help him complete the statement of his discovery. Then, he shifted to recording FT's idea on the chalkboard:

Teacher: Okay, let's explain it again, so...

FT: You kind of, like on the last one, it would be 3 x 2 x 1.

Teacher: Go ahead. [FT speaks but cannot be heard.] Okay, so that equals [writing ${}_3P_3$ on the chalkboard]...

FT: Oh, Yeah.

Teacher: Okay, you're saying: change this to a permutation and then do what? [writes "$= {}_3P_3$" after ${}_3C_3 = 1$ on the chalkboard].

FT: Then, put this number, second, factorial.

Helped by the teacher, FT completed the numerical relationship between combinations and permutations. The teacher went one step further to give meaning, comparison, and explanation to combinations:

Teacher: And divide this by the second number factorial. [writing "3!" under ${}_3P_3$] That's [pause] correct. And that's, how you figure out combinations. Now, I'll explain to you why that works. When you have 3 choose 3, there is only one way to do it. Just send these 3 people down. But when you compare that to the permutations, how many different ways can I mix up these same 3 people in permutations? Yeah?

JI: Six.

Teacher: Six, so I have to divide. If I get a permutation of six, but all six are the same combination, so what I do is just like we did in the last section. You have to divide by the number of repeats. [pointing to 3! on the chalkboard] So, there is 6 repeats, or 3 factorial repeats for this.

As the teacher and students elaborate on the relationship between permutations and combinations, we notice that the teacher attempts to help students become involved in the discussion and connect meanings to each of written symbols to help students to develop understandings of the new topic.

Discussion

In this classroom, cognitive processes of connecting symbols to referents and developing symbol manipulation procedures were enthusiastically supported by the teacher. Rather than teaching in a way that directly introduced mathematical written symbols and merely computing numerical answers, which often occurred in student talk, the teacher called attention to ways of talking about concepts and ways of writing and giving meaning to symbols. He built the new content (combinations) on a familiar one (permutations) by providing redundant examples when students started this novel task. Redundancy occurred when he utilized examples as referents to attach meaning to each symbol. Similar, but condition changed slightly, examples played a significant role of making symbols meaningful. The teacher also switched smoothly from lecturing to involving students in the discussion. In this transition process, students had opportunities to organize their thoughts, gain insights about the meanings of those symbols and develop the ability to manipulate them.

References

Greenfield, P.M. (1984). A theory of the teacher in the learning activities of everyday life. In B. Rogoff, & J. Lave (Eds.), *Everyday cognition: Its development in social context* (pp. 117-138). Cambridge, MA: Harvard University Press.

Hiebert, J. (1988). A theory of developing competence with written mathematical symbols. *Educational Studies in Mathematics*, *19*, 333-355.

Rogoff, B., & Gardner, W. (1984). Adult guidance of cognitive development. In B. Rogoff, & J. Lave (Eds.), *Everyday cognition: Its development in social context* (pp. 95-116). Cambridge, MA: Harvard University Press.

Vygotsky, L. (1986). *Thought and language*. Cambridge MA: MIT Press. (Translated and edited by Alex Kozulin)

Vygotsky, L. (1978). *Mind in society: The development of higher psychological processes*. Cambridge, MA: Harvard University Press. (Edited by Michael Cole, Vera John-Steiner, Sylvia Scribner, & Ellen Souberman)

Wertsch, J. V. (1985). *Vygotsky and the social formation of mind*. Cambridge, MA: Harvard University Press.

Wood, D., Bruner, J. S., and Ross, G. (1976). The role of tutoring in problem solving. *Journal of Child Psychology and Psychiatry*, *17*, 89-100.