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ABSTRACT

In an effort to examine the impact of the changes being made at San Jose State University (California) in the calculus curriculum, multiple measures were collected and analyzed. This study focuses on the relationship between performance on a pretest and the class grade. Through written responses on the pretest, a belief and knowledge profile for each student was constructed. Students were grouped according to their answers on an item which asked them to graph 2, x, x squared, and 2 to the x power. Profiles of student perceptions and knowledge were consistent within groups and varied across groups. Results showed that the concept of multiplication was not well understood and was closely related to success in first semester calculus. Multiplication was itself still a process, and in some cases, this process produced multiple concept images within cognitive neighborhoods. (Author)

Relationships Between Understandings of Operations and Success in Beginning Calculus

Barbara J. Pence

Paper presented at the Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education

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RELATIONSHIPS BETWEEN UNDERSTANDINGS OF OPERATIONS AND SUCCESS IN BEGINNING CALCULUS

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In an effort to examine the impact of the changes being made at San José State University in the calculus curriculum, multiple measures were collected and analyzed. This study focuses on the relationship between performance on a pretest and the class grade. Through written responses on the pretest, a belief and knowledge profile for each student was constructed. Students were grouped according to their answers on an item which asked them to graph 2, x, x^2, and 2^x. Profiles of student perceptions and knowledge were consistent within groups and varied across groups. Results showed that the concept of multiplication was not well understood, and closely related to success in first semester calculus. Multiplication was itself still a process and in some cases, this process produced multiple concept images within cognitive neighborhoods.

San José State University (SJSU) is in the process of making changes in the calculus curriculum. In an attempt to trace the impact of these changes, several assessment efforts are in progress. This paper examines data from one of these studies for the purpose of investigating the relationships between understandings of operations and understandings of concepts studied during first semester calculus.

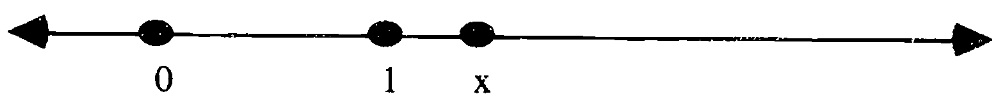
Background

Key concepts in beginning calculus involve the study of processes on functions. The road from seeing functions as processes to thinking about them as an object and finally using functions in other processes is difficult. In order to work with functions found in first semester calculus there exists a need for the encapsulation (Dubinsky, 1992) of many operations. The idea of *cognitive root* described by Tall (1992, p. 497) as "*concepts that have the dual role of being familiar to students and providing the basis for later mathematical development*" seems to apply to the role of understandings of operations relative to work with functions. At the stage when each function is still a process [take a point on the x-axis, trace a vertical line and then a horizontal line to find the value of $y = f(x)$], one basis for mathematical development includes operations such as multiplication, powers and exponents. Operations are familiar to the students; they have been using multiplication and powers in variable expressions for years. If, however, operations are not yet at the object level, then students must overcome additional obstacles in order to encapsulate the process into a single concept. This paper will investigate the linkages between the concept images of operations and understandings of processes on functions.

Methodology

Multiple measures were collected during the fall semester of 1994 for nine sections of beginning calculus including a pretest, a mid-semester test, a final, the course grade and the course grade from the second semester calculus class. The pretest and the mid-semester survey items elicited information about student cognitive knowledge, perceptions and beliefs while the other measures focused on student achievement. Complete pretest data existed for four classes, one class using the Harvard Consortium materials and three classes using Stewart. This study will concentrate on understandings as seen through the lens of the pretest and the course grades. The pretest was developed to gain insight into the students' entry level perceptions, attitudes and understandings of operations and functions. Of the eight written pretest items, the first six items elicited student comments regarding the anticipated difficulty level of the course, the grades expected, the key concepts of calculus, the perceived difficulty of representational forms, the expected applications of the content and the role of technology in the course. Content knowledge was examined through two questions. One of the content questions asked students to examine three graphs and in each case tell whether each graph was or was not a function and why. Graphs used in this question came from the research by Dreyfus and Vinner (1989). The second content question was motivated by faculty concerns that student understandings of powers and exponents are weak. This item is shown below:

> On the following number line, you will see the points representing 0, 1, and x indicated. Approximate the location of the points corresponding to 2, 2x, x^2 and 2^x.



For this item students are asked to connect symbolic and visual representations and to link units, variables, and operations on the number line. Although the original problem was conceptualized for use on a computer using a dynamic geometry system such as Cabri Geometry, this became impossible due to the lack of availability of computers. Thus, the problem became static with the variable x located so that it was less than 1.5. Actually it was placed at the point corresponding to the square root of two. Each of the four functions corresponds to an operation. Locating the point corresponding to 2 required repeated addition with the input being that of the location of the unit. As with 2, the function of 2x could be processed by repeated addition. On the other hand, the image of x^2 and 2^x required both the location of the unit and x. For each of the four functions students were required to process an operation.

Analysis

Data collected was examined both quantitatively and qualitatively. Results from 76 completed pretests and class grades for each of these students was examined collectively. The first step of the analysis was to sort the pretests into levels of understanding on the operation item. The sort produced six categories which were:

(1) students who were not able to locate any of the four functions correctly;

(2) students who located 2 but were not able to locate 2x;

(3) students who located 2 and 2x but were unable to locate x^2 or 2^x;

(4) students who located 2, 2x and x^2 but had trouble locating 2^x;

(5) students who located 2, 2x and 2^x but had trouble locating x^2; and

(6) students who located all four functions 2, 2x, x^2 and 2^x.

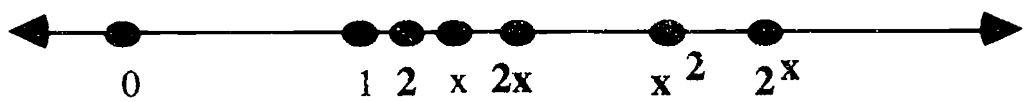
The six categories are hierarchical with categories 4 and 5 conceptualized as parallel. In the table, the categories form the structure for examining relationships between understandings and average anticipated grade, average actual course grade, failure rate, average score on the function item, and the representation reported to cause the most difficulty when solving a problem. Patterns exist both between and

Category #	# of Students	# Repeatg	Ave Anticip grade A=4	Ave Actual grade A=4	% of students who failed	Ave score on funct item (n=3)	Most difficult representation (graph, table, equation)
1	5	4	3.0	0.5	60%	2.8	(3, 1, 1)
2	34	17	3.6	1.4	53%	2.5	(16, 5, 5)
3	19	11	3.5	0.8	63%	2.7	(3, 2, 11)
4	2	1	3.5	0.0	100%	2.0	(0, 0, 2)
5	2	1	3.5	3.5	0%	2.0	(0, 0, 2)
6	14	8	3.4	2.5	36%	2.4	(5, 2, 3)

within categories. First, students in categories 1 - 3 were more likely to be repeating the course of calculus and the difference between the anticipated and actual grade was larger. Second, the score on the function item did not seem to be related to understandings of operations. Since the vertical line test was the major justification for answers to this question, the lack of connection with the understandings of operations is not surprising. Third and one of the most interesting pattern was that of self monitoring. Students in categories 1 - 4 (79%) expected grades of at least a B+ but earned grades of D or lower or dropped out. Students in category 6 were better able to monitor their progress or at least their progress against standards set by the instructor. Could it be that the category 1 - 4 students are progressing in the development of their own understandings but are not to the stage of contrasting the result of these understandings across conflicting concepts in their own cognitive structures much less in comparing their structures with those being set forth by their instructors?

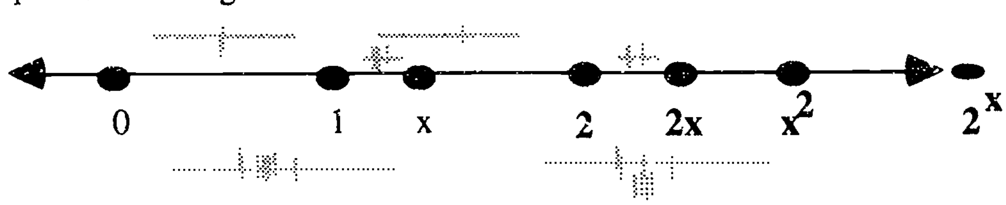
Although the tables describe both within and across category patterns, examination of sample student work helped in the exploration of student understandings of operations. Due to space restrictions, student work will be shared for categories 1, 2, 3 and 6 only.

Category 1. The five students in this category had trouble locating 2.



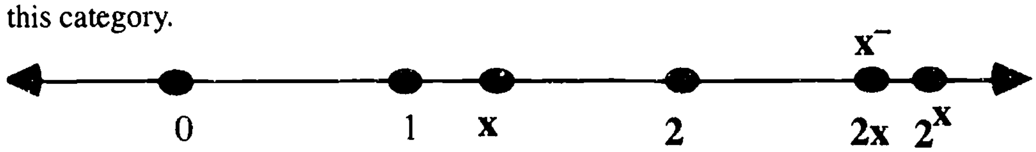
This student is repeating calculus but expects to get a B this time. She finds graphs to be the most difficult representational form to work with. Her graph is similar to the other four students in this category. There is an attempt to process most of the functions and there seems to be a belief that the constant 2 must come before the variables. The conflict between the location of 1 and 2 is not resolved. In fact, this pattern between 1 and 2 seems to be carried over to the relationship between x and 2x. To carry this analysis any further when the role of the unit is in doubt makes little sense. This woman continued in the class through the mid-semester exam. On the mid-semester exam, she was able to produce only a little work on one out of four problems, the one symbolic problem, and eventually withdrew from the class.

Category 2. In the second category, the students were able to locate 2 but were unable to find the point corresponding to 2x. Although students rarely provided any more than the diagram, this student actually gave sufficient work to help explain his thinking.



To construct the location of 2 he replicated the interval between 0 and 1. This logic of repeated addition was continued through his work with both 2x and x^2. That is, 2x was 2 plus x and x^2 was found by taking the interval from 0 to x and marking it off from x ($x^2 = x + x$). The location of 2^x seemed to be something beyond the others. Even though this student entered calculus class with high expectations, he was forced to drop the course before the final.

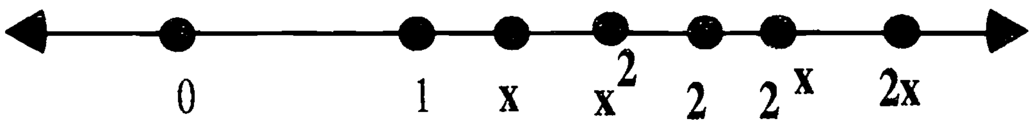
Category 3. Students in the third category correctly identified 2 and 2x. They either stopped at this point or went on to mappings which incorrectly represented both x^2 and 2^x. Many interesting linkages can be found in the work of students in this category.



The actual relationship that this student, who is repeating calculus after taking it in high school, wanted to communicate between x^2, 2x and 2^x is unclear. But, it seems as though each concept is closely related to 2x while being unrelated to 2. That is as a neighborhood is drawn closer and closer to 2x, it would always include x^2 but not 2. At what point in the shrinking of the neighborhood the location of 2^x

would be separated from 2x and x^2 it is difficult to say. Since this clustering of the concepts of 2x and x^2 appeared in more than 10 papers it is an example of a cognitive neighborhood, a construct introduced by Ervynck (1994).

Category 6. On the opposite end of the spectrum, the group of 11 students who were able to locate all four functions passed calculus with a grade of C or better. In fact, there were three A^+ grades given in the fall semester, with all three of them appearing in this category. This group included 7 students who were repeating the course, 4 of whom took the course in high school. The graph of an A^+ student is found below. This student was repeating the class and report that he



found the three representational forms equally easy to work. The location of x^2 and 2^x are not exact but the relative positions are close thus it was counted as correct. He also did well on the mid-semester exam, getting 3 out of 4 of the problems correct but did not feel confident with his results.

Discussion

Although the pretest was a written task, the results identified some interesting relationships which need further exploration. Work from students who either dropped out or failed first semester calculus showed patterns of incomplete understandings of the operation of multiplication. Their image of multiplication reflected difficulty in extending the models of multiplication beyond repeated addition with constants. Multiplication was itself still a process and in some cases, this process produced multiple concept images within cognitive neighborhoods.

This study supports the cognitive root conjecture. The idea of classification of functions by operations may be a step in the development from functions as process to function as object. Operations are familiar, that is, students have used the symbolic representations and they form the basis for later mathematical development. Thus, they may be a candidate for a cognitive root for advanced mathematics.

Trends of repeated failure among these students who have passed all of the necessary prerequisites to enter college calculus is perplexing. Why are 76% of these students unable to move beyond processing the functions of 2 and 2x? Why were these advanced students not monitoring and resolving conflict between function processes? What role does the belief system play in the cognitive image and conflict resolution? Does the multiple representation in this static task mask the potential for identification of conflict? Would a dynamic task encourage students for whom multiplication was still a process to reduce the multiple concepts contained in cognitive neighborhoods and support their movement from seeing functions as processes to thinking about them as objects and even using functions in other processes as required in their study of calculus?

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