DOCUMENT RESUME

ED 389 554 SE 057 197

AUTHOR	Zang, Richard A.
TITLE	Inferring Internal Strategic Problem Representation and Its Development: A Two-Year Case Study with Marcia.
SPONS AGENCY	National Science Foundation, Washington, D.C.
PUB DATE	Oct 95
NOTE	8p.; Paper presented at the Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education (17th, Columbus, OH, October 21-24, 1995). For entire conference proceedings, see SE 057 177.
PUB TYPE	Reports - Research/Technical (143) -- Speeches/Conference Papers (150)
EDRS PRICE	MF01/PC01 Plus Postage.
DESCRIPTORS	Case Studies; *Cognitive Processes; *Concept Formation; *Elementary School Students; Grade 4; Intermediate Grades; Interviews; Mathematics Education; *Problem Solving
IDENTIFIERS	*Representations (Mathematics)

ABSTRACT

This study is part of a longitudinal study of 22 elementary school children observed over a 3-year time span in the classroom and in individual task-based interviews. Through the examination of videotaped structured clinical interviews of Marcia, the first in 1992 when she was 9 years old (4th-grade), this study seeks to draw inferences about her internal strategic problem representations while engaged in problem-solving activity, and to analyze how these representations developed over a 2-year time span. An analysis of Marcia's task-based interviews shows that strategies are accessible to her, and even more importantly, that they do not necessarily need to be "taught." The research reported here shows that Marcia, when left to her own devices, not only invented strategies and representations to aid her in finding a solution to the problem, but she often did so in noncanonical ways. (Author)

Inferring Internal Strategic Problem Representation and Its Development: A Two-Year Case Study With Marcia

Richard A. Zang

A Paper Presented at the Seventeenth Annual Meeting for the Psychology of Mathematics Education (North American Chapter)

October 21-24, 1995

INFERRING INTERNAL STRATEGIC PROBLEM REPRESENTATION AND ITS DEVELOPMENT: A TWO-YEAR CASE STUDY WITH MARCIA*

Richard A. Zang, University of New Hampshire at Manchester

This study was part of a longitudinal study which entailed observing 22 elementary-school children over a 3-year time span in the classroom and in individual task-based interviews. Through the examination of videotaped structured clinical interviews of Marcia, the first in 1992 when she was 9 years old (4th-grade), this study sought to elucidate inferences of her internal strategic problem representations while engaged in problem-solving activity, and to analyze how these representations developed over a two-year time span. An analysis of Marcia's task-based interviews shows that strategies are accessible to her, and more importantly, that they do not necessarily need to be "taught". The research reported here shows that Marcia, when left to her own devices, not only invented strategies and representations to aid her in finding a solution to the problem, but often she did so in noncanonical ways.

If one views learning as the acquisition of competencies, and further, internal representations as descriptors for such competencies, it would stand to reason that children with rich systems of internal representation developed from exposure to appropriately chosen task-based situations, would be in a better position to assimilate, and thus learn, new competencies when exposed to new problem solving situations. It follows then, that we need to better understand these internal systems of representation to foster learning. Moreover, we need to develop ways to assess internal representation acquisition of individual children.

By designing task-based interviews with a homomorphic nature to them (sequences of geometrically presented figurate numbers), and by conducting these interviews two years apart, cognitive representations were able to be compared over that time span. Moreover, as cognitive representations describe competencies, various types of competencies (e.g., reversibility of strategy) could be compared. The richness of possible behaviors allowed for by the protocols, and subsequently observed, enabled Marcia's cognitive development to be charted.

The focus of this study is Marcia's *choice of representation* of the task, and her *strategic decisions* and methods of solution. In each structured clinical interview,

* The research reported in this study was partially funded by a grant from the National Science Foundation (NSF), "A Three-Year Longitudinal Study of Children's Development of Mathematical Knowledge", directed by Robert B. Davis and Carolyn A. Maher of Rutgers University. The opinions expressed and conclusions reached do not necessarily represent those of the NSF.

an analysis of Marcia's external representations (what the child constructed and verbalized) has been conducted, and inferences of internal strategic representations have been made consistent with Marcia's observed behavior (Zang, 1995). Among the features of strategic representations looked for were: 1) spontaneous use of formal symbolic representation in place of concrete manipulatives; 2) heuristic processes; 3) ability to generalize; and 4) reversibility of reasoning.

This is a qualitative case study, the purpose of which is purely exploratory and descriptive. There is necessarily a certain subjective nature to the reporting of the results of such a study. No attempt was made to conduct independent analyses by more than one researcher to achieve intercoder reliability; i.e., this technique was not used. *Ergo*, no assertion is made about the reliability of the inferences drawn.

However, the research team involved has allowed for an essential feature of scientific progress, by carefully preparing scripted protocols for the structured task-based interviews (Goldin, DeBellis, DeWindt-King, Passantino, & Zang, 1993; Zang, 1995). This had the dual effect of not only preparing the clinicians involved to mitigate their individual interview styles and adhere to a planned sequence of questions and contingencies, but also to permit a degree of comparability and reproducibility so that these results can be further extended and compared with those of other researchers.

The Tasks

The domain of the problem solving activities revolve around 5 problem contexts involving number sequences with attending figure/geometric representations. Materials (index cards, red and black chips, markers of different colors, paper, and pencil) were placed ahead of time on the table in front of the child for each of the interviews. The first task-based interview consisted of laying 3 cards, one at a time, in front of the child as illustrated in figure 1. The following series of questions was then asked: 1) "What card do you think would follow that one?" [then asked again with reference to the 4th card]; 2) "Do you think this pattern keeps going?"; 3) "How would you figure out what the 10th card would look like?"; 4) "Here's a card

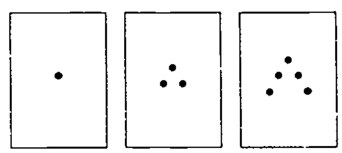


Figure 1. Task #1

(showing one with 17 dots in the shape of a chevron) ... can you make the card that comes *before* it?"; and 5) "How many dots would be on the 50th card?"

For each question discussed above, the following stages exist: 1) Posing of the question (free problem solving); 2) Heuristic suggestion (if not spontaneously evident); e.g., "Can you show me using some of these materials?"; 3) Guided use of heuristic suggestion; e.g., "Do you see a pattern in the cards?"; and 4) Exploratory (metacognitive) questions; e.g., "Do you think you could explain how you thought about the problem?" The clinician always sought to elicit a complete, coherent verbal reason and a coherent external representation before proceeding to the next question. It is important to note that the canonical card did not have to be drawn or the canonical pattern described, in order for a response to be considered a complete and coherent reason and a complete coherent external representation.

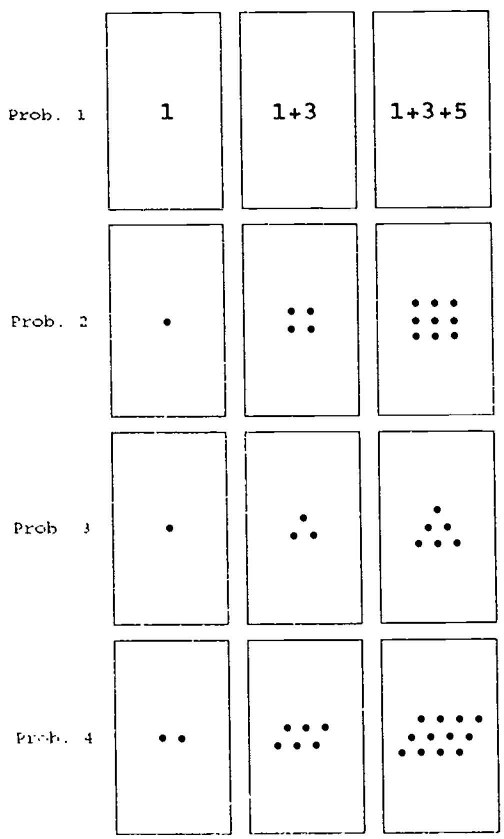


Figure 2. Task #2

The next task-based interview, as it relates to this present study, was conducted two years later (1994). Task #2 involved 4 problem contexts as illustrated in figure 2. Each of the 4 problems utilized the same basic paradigm as in the first task. In formulating/designing this task-based interview, and what task(s) it would encompass as its domain, it was decided that the richness of variety embodied in figurate number sequences and their interrelatedness (i.e., homomorphic nature) to task-based interview #1 would be the most appropriate vehicle to look at developing internal strategic problem representations. In light of and in recognition of the fact that we as researchers cannot see the cognitions being employed as problem solving occurs, and instead we must infer these cognitions (with the aid of self-reporting by the child and their accompanying external representations), it is prudent to provide a cornucopia of figurate number tasks (each one building on the other) such that externalizing the internal representations provides as much of a cognitive window as possible to viewing their internal strategic representations.

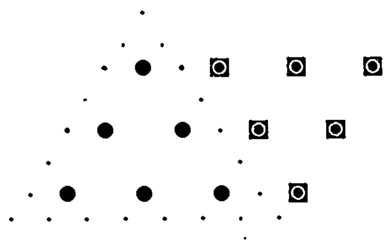
Some Highlights

During Task #1, Marcia developed an add-two strategy, but not in the canonical fashion (i.e., adding two dots to the bottom of the previous figure). Instead, she would draw each dot in contiguous order starting from the lower left portion of the chevron. The two dots she said she was adding, were always the last two dots on the bottom right side of the chevron (she gave quite a lengthy explanation of this). When asked what the 10th card would like, she appeared to discover that the number of dots on the left side of the chevron represents the number of that card (e.g., 3 for 3rd card, 4 for 4th card, *et cetera*), and that the number of dots on the right side was one less. She stopped constructing the cards *en route* to the 10th card, after the 7th card (having constructed the 4th, 5th, 6th, and 7th cards), and proceeded to give a verbal accounting of what the 10th card would look like. The inference made is that this is when the discovery took place, or at least the point that it manifested itself externally, for it was here that she was able to apparently generalize, and extend mentally, the operations necessary to form an internal representation of the 10th card. The inference made is that she then held a more imagistic internal representation, in that, for the first time, there was a direct correlation between the number of dots she was focussing on (left side of the chevron) and the geometry of the shape; all done simultaneously (tieing the numeric concept into a portion of the shape). Most of the children eventually abandoned the geometry of the problem (Zang & Goldin, 1993) whereas Marcia learned to exploit it.

When asked the "reversibility" question -- "Here's a card (showing one with 17 dots in the shape of a chevron) ... can you make the card that comes *before* it?" -- Marcia said and gestured that there were 9 dots going up on the presented card, and so there would have to be 8 dots going up (referring to the left side of the chevron) on the card before it.

Two years later when the "reversibility" question was posed during Task #2, Problem 3, she apparently realized it was unnecessary to know that number (the number of dots on the left side of the triangle); one merely had to remove the bottom row (no matter how many dots may be in it, or any other part of the figure).

Marcia was asked if she noticed any relationship between the 3rd and 4th set of cards during Task #2. She noticed that two of every card from the 3rd set is embodied in the corresponding card of the 4th set (albeit one of the two from the 3rd set is inverted). Along with a good verbal accounting, she explained as follows:



Using the 3rd presented card, subject covered the dots indicated by the ◙'s, then encircled the other dots as indicated by the dotted line.

Here, it was inferred, she utilized an internal imagistic representation(s); to "see" instances of certain problems embodied in still other instances of other problems.

An unexpected outcome during Task #2 was when Marcia employed a gnomon-like strategy while engaged in Problem 2. A gnomon to a geometric figure (A), is another geometric figure, such that when the gnomon is suitably attached to A, the resulting figure (A') is similar (in a geometric sense) to A. Because of her verbal accounting, gesturing, and drawing (a representative example of the type of behavior she e hibited is seen in figure 3), the inference made is that the dots that comprise the gnomon are the dots she is focussing attention on (cognitive dots), and thus are the dots she is using to draw a pattern from. The inference made is that the focus on these cognitive dots is her strategy, whereas the overall process of attaching (mentally) part A with a gnomon (resulting in A') is the heuristic process. This discussion serves to illustrate McClintock's (1984) suggestion that heuristic processes may be viewed from another standpoint; i.e., as inherent in and thus residing in the mathematical problems themselves.

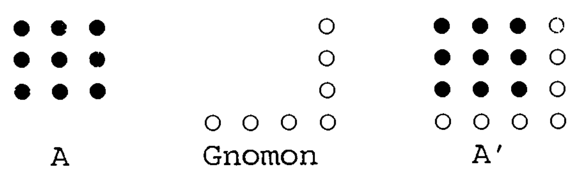


Figure 3. Cognitive Dots

Conclusion

The structured individual task-based interviews proved to be an appropriate research tool as they were able to draw out the *processes* Marcia used (e.g., strategy use), as opposed to the more traditional *product* so often emphasized in the classroom

(Goldin et al., 1993; Zang, 1995). This is an important distinction in that teachers have always manifested an overriding concern to *measure* learning. In their quest to measure, they inevitably turn to the product of the problem solving (test scores, time to solution, etc.), and have traditionally not focused on the process.

One possible implication of this research (to the extent that results are generalizable) is that we as mathematics educators should be mindful that a teacher's expectations of the canonical response can have the cognitive effect of being so overriding, that they can interfere with the teacher being receptive to really insightful ways of thinking by a child. As such, this research suggests that one possible way of teaching these strategic representations, is not to offer instruction in them *per se* (i.e., in some procedural way), but rather, to provide a rich environment wherein the children will construct them on their own, in much the same way as occurred during these task-based interviews with Marcia. Another possible implication of this research is that we can introduce geometric concepts much earlier than traditionally thought -- teaching children to exploit the visual aspects of a problem in their early years of problem solving, long before the more formal operational approach is encountered.

Acknowledgments

Special thanks are due to many people at Rutgers University where this research was conducted, especially the research team, and particularly Gerald A. Goldin, who not only directed the team but also served as the author's thesis advisor.

References

Goldin, G. A., DeBellis, V. A., DeWindt-King, A. M., Passantino, C. B., & Zang, R. A. (1993). Task-Based interviews for a longitudinal study of children's mathematical development. In I. Hirabayashi, N. Nohda, K. Shigematsu, & F. Lin (Eds.), Proceedings of the 17th International Conference of the Psychology of Mathematics Education (PME) Volume 1 (pp. 197-203). Tsukuba, Ibaraki, Japan.

McClintock, C. E. (1984). Heuristic processes as task variables. In G. A. Goldin & C. E. McClintock (Eds.), Task variables in mathematical problem solving (pp 171-234). The Franklin Institute Press.

Zang, R. A., & Goldin, G. A. (1993). Inferring children's internal strategic problem representations. In J. R. Becker & B. J. Pence (Eds.), Proceedings of the 15th Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education Volume 2 (p. 285). San Jose, California: San Jose State University, The Center for Mathematics and Computer Science Education.

Zang, R. A. (1995). Inferring Internal Strategic Problem Representation and its Development: A Two Year Case Study with Four Elementary School Children. Dissertation Abstracts International, 55 (12A), 3775. (University Microfilms International No. 9514127)