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ABSTRACT

Based on a synthesis of the literature and observations of young children over 2 years, a framework for assessing probabilistic thinking was formulated, refined, and validated. For each of four major constructs incorporated into this framework--sample space, probability of an event, probability comparisons, and conditional probability--four different levels of thinking were established that reflect a continuum from subjective to numerical reasoning. The framework was validated through data obtained from 24 children in grades 1 through 3 who served as case studies. Results suggest that, while the framework produces a unified picture of children's thinking in probability, there is "static" in the system which generates inconsistencies among construct levels. The framework has implications for curriculum development and assessment. Contains 18 references. (Author)

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A FRAMEWORK FOR ASSESSING YOUNG CHILDREN'S THINKING IN PROBABILITY

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Based on a synthesis of the literature and observations of young children over two years, a framework for assessing probabilistic thinking was formulated, refined and validated. For each of four major constructs incorporated into this framework—sample space, probability of an event, probability comparisons, and conditional probability—four different levels of thinking were established which reflected a continuum from subjective to numerical reasoning. The framework was validated through data obtained from 24 children of grades 1 through 3 who served as case studies. Results suggest that while the framework produces a unified picture of children's thinking in probability, there is "static" in the system which generates inconsistencies among construct levels. The framework has implications for curriculum development and assessment.

Although there has been considerable research into young children's thinking and misconceptions in probability (Fischbein, 1975; Fischbein, Nello, & Marino, 1991; Garfield & Alhgren, 1988; Piaget & Inhelder, 1975, Tversky & Kahneman, 1982; Shaughnessy, 1992), none of this research has generated a framework for systematically assessing young children's thinking in probability. Given the call for including probability in the elementary school curriculum (National Council of Teachers of Mathematics, 1989) and the inclusion of probability in state and national assessments (e.g., Illinois Goal Assessment Program, 1993; Mullis, Dossey, Owens, & Phillips, 1993), there is a need to describe children's probabilistic thinking as a basis for generating appropriate curriculum and assessment programs.

Aims of the Research

Based on a synthesis of the research literature related to children's thinking about probability (e.g., Fischbein, Nello, & Marino, 1991; Piaget & Inhelder, 1975; Shaughnessy, 1992) and related neo-Piagetian research that postulates the existence of different levels of complexity in children's thinking (e.g., Biggs & Collis, 1991; Case, 1985), this study attempted to:

- develop and refine a framework for describing and predicting how young children think in probabilistic situations; and
- use the framework to generate assessment protocols to validate the framework.

Theoretical Considerations

The thesis of this study maintains that for children to exhibit probabilistic thinking, there is a need for them to understand probability concepts that are multifaceted and develop over time. In order to capture the manifold nature of probabilistic thinking, our Framework (Figure 1) incorporates four key constructs: sample space, probability of an event, probability comparisons, and conditional probability. In this study *sample space* refers to listing or identifying the complete set of outcomes of a one- or two-stage probability experiment. *Probability of an event* involves identifying and justifying which of two or three events are most likely to occur. *Probability comparisons* entail determining and justifying: (a) which probability situation is more likely to generate the target event in a random draw; or (b) whether the two probability situations offer the same chance for the target event. *Conditional probability* refers to recognizing and justifying why the probability of an event may or may not be changed by the occurrence of another event.

The first three of these constructs have been investigated by several researchers (Acredolo, O'Connor, Banks & Horobin, 1989; English, 1993; Fischbein, Nello & Marino, 1991; Piaget & Inhelder, 1975). Few studies on the fourth construct, conditional probability, have been directed at young children. However, interpretations have been made from data on tasks involving elements of conditional probability (Borovcnik & Bentz, 1991; Falk, 1988; Konold, 1989; Shaughnessy, 1992). Notwithstanding the extent of research into children's probabilistic thinking, it has seldom investigated the four constructs in combination, and has not produced universal agreement on the scope of children's thinking in probability (Shaughnessy, 1992).

In addressing this need, our framework enables young children's probabilistic thinking to be described and predicted across four levels for each of the four constructs. These levels have evolved from our observations of young children's probabilistic thinking over a two-year period. Moreover, the notion of levels of thinking within specific knowledge domains is also in concert with cognitive research that recognizes developmental stages (Piaget & Inhelder, 1975) and, more particularly, with neo-Piagetian theories that postulate the existence of sub stages or levels that recycle during stages (Biggs & Collis, 1991; Case, 1985).

As is highlighted in Figure 1, Level 1 is associated with *subjective* thinking, Level 2 is seen to be *transitional* between subjective and naive quantitative thinking, Level 3 involves the use of *informal quantitative* thinking and Level 4 incorporates *numerical reasoning*. Further it is claimed that a child's probabilistic thinking at a given time is stable across all four constructs.

Methodology

Subjects

The population for the study comprised children in grades one through three at a University laboratory school. Eight children, randomly sampled from each of these grades, served as case studies. None of these children had been exposed to prior probability instruction.

The Validation Process

To validate the framework we sought to: a) ascertain whether children's thinking at a particular level was stable across all four constructs; and b) confirm and refine the characteristics of each level within the framework. Cochran's Q test (Siegel & Castellan, 1998) was used to assess the stability of framework levels and qualitative analysis was used to address the rest of the validation.

Data Collection and Instrumentation

The framework and the validation process guided the design of the data collection instruments and procedures. A structured interview assessment based on the framework comprised 22 tasks—six tasks associated with *sample space*, four with *probability of an event*, seven with *probability comparisons*, and five with *conditional probability*. This interview, audiotaped for subsequent analysis, was administered by members of the research team to each of the case study students.

Each question in the interview assessment was scored according to a three-part rubric: 1) fully met, 2) partially met, and 3) didn't meet the framework criteria. Children's thinking on all questions was analyzed and coded by level for each construct of the framework using the double coding procedure described by Miles & Huberman (1984). As a result of this analysis, children's dominant level of thinking with respect to each construct of the framework was determined.

Validating the Framework: Results and Discussion

In validating the framework a major concern was to examine stability of children's thinking across the constructs of sample space, probability of an event, probability comparisons, and conditional probability. The results of Cochran's Q test indicated that there were no significant differences among the thinking levels generated by the four probability constructs. That is, each of the four constructs were generally in harmony in identifying a child's probabilistic thinking level.

Notwithstanding the results of these analyses, there were not more than five children for whom the thinking levels were in complete agreement across the four constructs. Our observations and interpretations suggest that while the framework produces a unified picture of children's thinking in probability, there is 'static' in the system which generates inconsistencies among the levels based on each of the constructs. Moreover, it is our contention that this static results from children's tendencies to unexpectedly regress back to subjective judgments, even when their probabilistic thinking is more indicative of "transitional" or "informal quantitative" reasoning.

A second area of interest in the validation process was the refinement of descriptors of children's probabilistic thinking at each level and across all four constructs. The analysis of children's thinking revealed that children exhibiting level 1 thinking were narrowly and consistently bound to subjective judgments. They did not provide a complete listing of the outcomes in a one-stage experiment and they almost always used subjective judgments rather than quantitative ones in situ-

CONSTRUCT	Level 1 (Subjective)	Level 2 (Transitional)	Level 3 (Informal Quantitative)	Level 4 (Numerical)
SAMPLE SPACE	• lists an incomplete set of outcomes for a one-stage experiment	• lists a complete set of outcomes for a one-stage sample space, and • lists the outcomes of a two-stage experiment in a limited and unsystematic way	• adopts and partially applies a generative strategy to make a complete listing of outcomes for a two-stage case	• adopts and applies a generative strategy which enables a complete listing of the outcomes for a two- and three- stage case
PROBABILITY OF AN EVENT	• predicts most/ least likely event based on subjective judgments • distinguishes "certain," "impossible," and "possible" events in a limited way	• predicts most/ least likely event based on quantitative judgments but may revert to subjective judgments • distinguishes "certain," "impossible," and "possible" events within reasonable parameters	• predicts most/ least likely events based on quantitative judgments including situations involving non-contiguous outcomes) • uses numbers informally to compare probabilities • distinguishes "certain," "impossible," and "possible" events, and justifies choice quantitatively	• predicts most/ least likely events for single stage experiments • assigns a numerical probability to an event (it may be a real probability or a form of odds.)
PROBABILITY COMPARISONS	• compares the probability of an event in two different sample spaces, usually based on various subjective or numeric judgments • cannot distinguish "fair" probability situations from "unfair" ones	• makes probability comparisons based on quantitative judgments (may not quantify correctly and may have limitations where non-contiguous events are involved) • begins to distinguish "fair" probability questions from "unfair" ones	• makes probability comparisons based on consistent quantitative judgments • justifies with valid quantitative reasoning, but may have limitations where non-contiguous events are involved • distinguishes "fair" and "unfair" probability generations based on valid numerical reasoning	• assigns a numerical probability measure and compares • incorporates non-contiguous and contiguous outcomes in determining probabilities • assigns equal numerical probabilities to equally likely events
CONDITIONAL PROBABILITY	• following a particular outcome, predicts consistently that it will occur next time, or alternatively that it will not occur again (over-generalizes)	• begins to recognize that the probability of an event changes in a non-replacement situation • can recognize when certain and impossible events will arise in non-replacement situations	• can determine changing probability measures in a non-replacement situation • recognizes that the probability of all events change in a non-replacement situation	• assigns numerical probabilities in replacement and non-replacement situations • distinguishes dependent and independent events

Figure 1. Probabilistic thinking framework

ations involving probability. Children reflecting level 2 thinking could list the complete set of outcomes in a one-stage experiment. However, they didn't always use these outcomes when responding to probabilities, especially in tasks involving conditional probability. Level 2 is a period of transition where probability constructs are not always coordinated.

While acknowledging the subjective 'static' discussed above, children exhibiting level 3 thinking characteristically used quantitative judgments when dealing with tasks based on probability constructs. They revealed a consistent predisposition to use numbers in describing and comparing probabilities, albeit not always expressed as correct probability measures or odds. This predisposition to use numbers carried across into conditional probability situations, where children were able to recognize that the probabilities of all events changed in a non-replacement experiment. Children typifying this level of thinking, also tended to use more generative strategies in listing outcomes of two-stage experiments. Moreover, our analysis of children's probabilistic thinking revealed that level 3 thinkers had begun to coordinate thinking in sample space and thinking in probability in a more systematic manner.

The move from level 3 thinking to level 4 thinking needs further investigation, as none of the children in our study exhibited level 4 thinking across all four constructs. There was, however, evidence in this study that some children had begun to use more precise measures of probability and listings of multi-stage sample spaces. Our observations suggest that lack of knowledge of fractions inhibited the thinking of children who were otherwise predisposed to more precise probability measures.

In validating the framework, we have described children's probabilistic thinking at each of the four levels in content-specific terms. That is, we have related the children's probabilistic thinking across the four constructs to a continuum of four levels of quantitative reasoning. Moreover, the notion of levels of probabilistic thinking appears to be in concert with the theoretical position of cognitive researchers such Biggs & Collis, 1991; Case, 1985. They claim the existence of more general cognitive structures which incorporate sub stages or levels of cognitive functioning that recycle across broader stages of development. Their theoretical position adds further support to the existence of distinct levels of probabilistic thinking among children found in our study.

The framework generated by this study enables children's probabilistic thinking to be described and predicted in a unified and systematic manner. It does have limitations in that the levels of children's thinking on the four constructs were not completely stable and appeared to be subject to "static" as children unexpectedly regressed to subjective reasoning. Future research may reveal more stable patterns if children whose thinking has generally progressed beyond level 1 probabilistic thinking, are assessed on the basis of their dominant level when they occasionally revert to subjective judgments. The framework has implications for curriculum development and assessment in relation to probability programs for children in the primary grades.

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