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ABSTRACT

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The Emergence of the Splitting Metaphor in a Fourth Grade Classroom

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THE EMERGENCE OF THE SPLITTING METAPHOR IN A FOURTH GRADE CLASSROOM

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In this classroom a child introduced the word **split** when he was asked to describe his mental activity in performing the addition of two numbers; the teacher initiated a spatial representation using a broken line. Subsequently, the children used their own pictorial representations. The paper presents an analysis of the numerical diagrams used by these fourth graders to represent their **splitting notion**. Splitting evolved into a metaphor that fostered the emergence of these children's numerical meanings.

Introduction

Metaphors according to Lakoff (1987, 1994), Lakoff and Johnson (1980), and Johnson (1987) are not just a matter of language but of thought and reason. Metaphors, for them, are mapping "motivated by structures inhering in everyday experience" (Lakoff, 1987, p. 287). Metaphors, they contend, are mappings that put into correspondence two different domains of experience preserving their basic logic—an image schemata domain that structure our experience preconceptually and a conceptual abstract domain. In this fourth grade classroom, splitting became more than a peculiar way of speaking; it became a metaphor establishing a correspondence between the physical experiential domain of breaking and dividing into parts and the conceptual domain of unfolding or deunitizing a numerical unit into smaller subunits.

During the first months of the school year, the ***splitting metaphor*** and its spatial representations became, for the children, a thinking tool to conceptualize place value and solve word problems. Children used the ***splitting metaphorical expression*** to describe their mental actions of deunitizing a composite unit into simpler units to fit their particular goal when operating with natural numbers. These children's diagrams indicate the enactment of their mental actions to operate with natural numbers in different contextual situations. The ***splitting metaphor*** was represented through numerical diagrams that acquired different degrees of complexity and they were additive (decomposition of a unit into simpler units of different magnitude) or multiplicative (decomposition of a unit into simpler units of the same magnitude) in nature. Initially, the splitting metaphor emerged, as a linguistic device in a classroom discussion, from the numerical conceptual evolution of one child; the teacher legitimized it by initiating a spatial representation of it and incorporating it in the flow of the classroom conversation. The splitting notion seems to be a natural notion to children since they immediately started to

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use it to make diagrams or spatial representations of their own mental ways of operating with numbers.

The Study

The teaching experiment. During the school year, the fourth-grade arithmetic class was team taught by the researcher and the collaborating teacher. All instructional tasks for the arithmetic class were generated to make emphasis on natural numbers as composite units (Steffe, von Glasersfeld, Richards, and Cobb, 1983; Steffe and Cobb 1988) that could be themselves iterated to form a larger unit or decomposed into smaller subunits. Besides the instruction during the arithmetic class, these fourth graders were interviewed weekly in small groups of at most three students. These groups were constituted taking into account the inferred numerical understanding of each child and his or her willingness to work with the other members of the group. Every class and interview was videotaped and transcribed, field notes were taken, and the task pages and children's scrap paper were collected.

Organization of the teaching activity. Typically, children were given tasks on paper or verbally to solve individually while encouraged to consult and discuss with other classroom members. After the task appeared to be solved by the majority of the students, a whole class discussion took place. Children presented their solutions using the overhead projector or the board. A sequence of teacher's questions and children's explanations of their solutions characterized the interaction. Learning from Cobb (1989, 1990), and Cobb, Yackel, and Wood (1992), since the beginning of the school year there was an explicit mutual agreement between the teachers and the students about their responsibility to listen carefully to the solution of others and to express their agreement or disagreement by giving a reason for it. This agreement was consolidated throughout the school year. After the first three months of the school year, children's mental engagement in the classroom activity was manifested by collective applause when a child presented a solution that they considered to be novel or they perceived the presenter appeared to have difficulties but successfully completed the task.

The Emergence of Splitting as a Metaphor

These children's conceptualizations of units of ten and their understanding of the place-value structure of the Hindu-Arabic notation of numbers was minimal and their operations with numbers was strictly procedural and dependent on conventional algorithms. Our first concern was to help students to find their own meaningful ways of operating with numbers. To do this, we emphasized counting and mental computation. Most of the time, numbers were presented verbally to make relevant not only the units of ten but the relationship between number words and number symbols. To facilitate mental computation, at most three numbers were given to the children at once. At the beginning when numbers were presented in a written form, we used a rectangular 2-by-2 array of squares to locate the numbers in three of them and the fourth was empty for the children to write the sum.

By the middle of the third week of classes, the students were asked to add the numbers 80, 5, and 15. Numbers were located in the rectangular array. After children were given time to arrive at their result mentally, the teacher asked the children for their answers and explanations and she displayed them on the overhead. In this and the following dialogues, T stands for teacher and any two-letter set of an upper case letter followed by a lower case letter stands for the abbreviation of student's name and it will appear on italics in the body of the paper to avoid confusion.

1 Am: 80 plus 5 is 85 (counting on her fingers by one), 85 plus 15 is 100.

2 T: (writes 80 + 5 = 85, 85 + 15 = 100 as Am describes her solution). How did you add these two numbers (Pointing at 85+15)?.

3 Am: 85 plus 5, plus 5, plus 5. That's 15 (showing 3 fingers).

4 Ra: You split the 15 into 5, 5, and 5.

5 T: (simultaneously writes 85 + 5 + 5 + 5 = 100)

6 St: 5 and 15 is 20. 80 and 20 is 100.

7 T: (simultaneously writes 5+15=20, 80+20=100)

8 Pr: 80 and 15 is 95. 95 and 5 is 100.

9 T: (simultaneously writes 80+15=95, 95+5=100)

10 T: OK Ra, what did you do?

11 Ra: 80 plus 15 equals 95. I split the 15 into 5, 5, and 5. Then 95 plus 5 equals 100.

12 T: (simultaneously writes
80 + 15 = 95 , 95 + 5 = 100)
5 5 5

The interaction between the teacher and the students illustrates a simultaneous event in which children's verbalizations are transformed or mapped into conventional numerical equalities through conventional symbols that children have used in their prior school years. The most significant part of the dialogue is the *split interpretation* that *Ra* made of *Am*'s way of acting on the numbers (line 4) and the description of his solution using the splitting notion coupled with the teacher's attempt to symbolize it (lines 11 and 12). It is worth noticing that, in the above solutions, children associated the numbers in the ways that were easy to operate for them. In general, the way children associated the numbers on the rectangular array were not prompted by their position given that there is not a particular direction in which the reading of numbers must be done in the matrix-type numerical arrangement, but instead children associated them according to their emerging or predetermined strategies to add the numbers. In the course of this and the following lessons, *Ra* was given full credit for the introduction of this splitting notion in the classroom mathematical discourse. By doing this, the teacher not only respected the intellectual contributions of one of the students but let the group know that their collaboration in the elaboration of common mathematical understanding was an ongoing process in which the students' partnership was needed and welcome.

Several questions emerge from the reflections on the above dialogue: Did the students pursue their own diagrammatic representations in their efforts to explain their answers? Did these representations coevolve with their numerical meaning-making process? Did the diagrams acquire some degree of sophistication? The analysis of children's solutions of mathematical tasks indicates that children autonomously generated more sophisticated diagrams and generated other ways of talking about the decomposition or deunitization of numbers into subunits.

The Splitting Metaphor and Operations with Natural Numbers

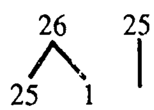
Once the teacher encouraged the splitting metaphorical expression as a way of speaking mathematically, the children considered it as a legitimate way of talking about numbers and they, on their own initiative, used it to communicate their mental actions on numbers. Sometimes, children used the term "splitting" explicitly; others, substituted it for expressions like "*take away*" or "*break into*"; still other children used their diagrams to communicate their solutions. In the following dialogue, we can observe how children, in the absence of paper and pencil, were able to conceptualize units of ten and use the splitting metaphor to find the sum of 26 and 25:

1 T: If I ask you to add, in your mind, 25 and 26, what do you get? (Several hands go up)

2 Mi: 6 and 4 is 10 ones, you can make a ten. 2 tens and 2 tens is 4 tens, so is 5 tens; that is 50. 50 and 1 is 51.

3 Pr: I split 20 into 2 tens and 20 into 2 tens; 6 and 5 is 11. 5 tens is 50 and 1 is 51.

4 Ra: 25 and 25 is 50 and 1 is 51. Let me show you ...(he goes to the board and makes the following diagram)



5 Li: 5 plus 6 is 11. 2 tens and 2 tens is 4 tens, another 10 is 50. One more is 51.

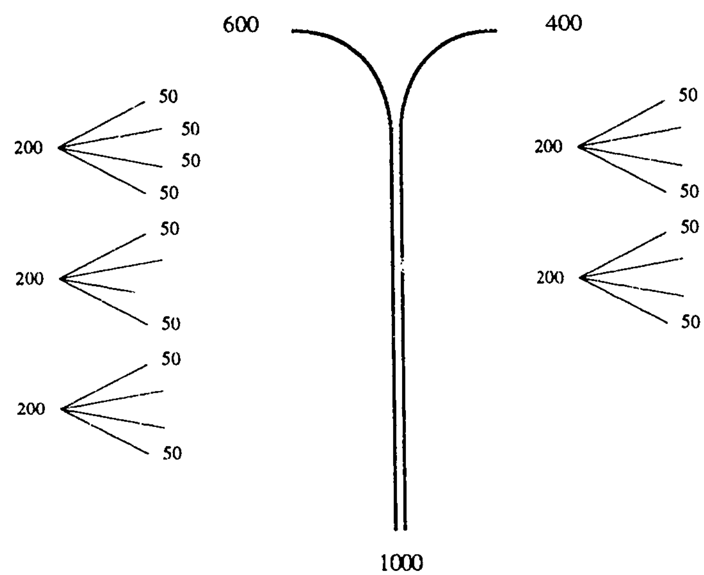
6 St: I took 6 away from 26 and 5 away from 25. 20 and 20 is 40.

6 and 4 is ten. One more is 51.

In line 2, *Mi*'s answer indicates that she conceptualized 26 as 2 tens and 6 ones, and 25 as 2 tens and 5 ones. It seems as if she had continued the splitting process for 5 as 4 and 1. *Mi* not only kept in mind the decomposition of 25 and 26 into units of ten and one but also was able to operate without confounding them. That is, 5 tens and 1 one were for her 51 and neither 6 tens nor 6 ones; *Mi*'s sophisticated reasoning seems to have been supported by some type of mental image. In line 3, splitting seems to be the mental purposive action that allowed *Pr* to deunitize 26 and 25 into units of ten and one. In line 4, *Ra* implicitly deunitized 26 into 25 and 1, added the two units of 25 and then added to it the unit of one. His verbalization was immediately followed by a diagram indicating that either he had a mental image of it to support his actions on the numbers or he generated it in the midst of verbalizing his explanations as a way of communicating with his peers. In line 6, *St*'s solution is essentially similar to that of *Mi* but she expressed the splitting action as *taking away from* as she took 5 and 6 *away from* 25 and 26 respectively.

The following task was posed by one of the students. He took a fake 1000-dollar bill from one of the banks (a bank was a plastic box with fake dollar bills of all the denominations which were kept classified) and asked his classmates if they could find the number of 50-dollar bills for which this bill could be exchanged. *Ri* offered this diagram as his solution.

Ri's diagram:



Ri's diagram is additive or a hybrid between additive and multiplicative. Additive because at level 1 (first split) the thousand unit was not deunitized into units of the same size. At level 2 (second split) the deunitization was done into units of 200, and at the level 3 (third split) the deunitization was done into units of 50. To make the diagram, *Ri* had to anticipate the consecutive unfolding of each of the units at each splitting level of the diagram. The diagram represents *Ri*'s ways of expressing his mental actions on numbers in terms of his physical experience of splitting. *Ri*'s cognitive behavior seems to be in accordance to Lakoff and Johnson's contention about "the essence of metaphor in understanding and experiencing one thing in terms of another" (p. 5, 1980).

Discussion

The splitting notion and its diagrammatic representation (whatever shape it took) became a collective way of speaking to express one's numerical reasoning and to understand the numerical reasoning of others. In the long run, splitting diagrams that were individually or collectively generated became a conceptualizing tool that fostered these children's numerical reasoning. That is, **splitting** became more than a way of speaking, it became a way of describing the mental action of decomposing numbers into subunits as conceptually perceived by the children according to their needs to operate with them. One question that would be of importance to consider is whether or not the physical notion of splitting projected into a numerical context, was used by children only in isolated instances at the beginning of the school year or if it occurred frequently and evolved throughout the school year to support children's conceptualization of fractions. An extended version of this paper will present evidence that supports this question in the positive. The role that metaphorical elaboration plays on children's numerical sense making, the conditions under which these metaphors emerge from children's efforts to operate with numbers, the influence of the social interaction on children's way of thinking, and the interdependent nature between mathematical thought and speech seem to be of interest for the teaching of arithmetic in the classroom.

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