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ABSTRACT
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exploiting collateral variables for respondents. The results are
illustrated with data from the Armed Services Vocational Aptitude
Battery reported in the survey Profile of American Youth. An appendix
presents a heuristic argument in support of a large sample result in
the paper. (Contains 7 tables and 13 references.) (Author/SLD)

THE INFORMATION MATRIX IN LATENT-VARIABLE MODELS

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Princeton, New Jersey

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The Information Matrix in Latent-Variable Models[1]

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Educational Testing Service

April 1988

The Information Matrix in Latent-Variable Models

Abstract

The information matrix for the parameters in a latent-variable model is bounded from above by the information that would obtain if the values of the latent variables could also be observed. The difference is the "missing information." This paper discusses the structure of the information matrix, and characterizes the degree to which missing information can be recovered by exploiting collateral variables for respondents. The results are illustrated with data from the Armed Services Vocational Aptitude Battery.

KEY WORDS: collateral information, item response theory, latent variables, missing information principle.

1. Introduction

Latent variable models are used in the social sciences to provide parsimonious descriptions of the associations among observable variables in terms of theoretically derived constructs. Let $\mathbf{x} = (x_1, \ldots, x_n)'$ denote observable variables, θ denote latent variables, and β_1 denote parameters of the regressions of the x_js on θ through the known functions $f_j(x_j | \theta, \beta_1)$. Under the assumption of conditional independence,

$$f(\mathbf{x} | \theta, \beta_1) = \prod_j f_j(x_j | \theta, \beta_1).$$

If $g(\theta | \beta_2)$ is the density function of θ in a population of interest, then the density of the observed variables is given by the mixture

$$h(\mathbf{x} | \beta) = \int f(\mathbf{x} | \theta, \beta_1) \, g(\theta | \beta_2) \, d\theta \ ,$$

where $\beta = (\beta_1, \beta_2)$. For notational convenience, we shall suppress dependence on β_1 and β_2 in f, g, h, and elsewhere.

Some examples of latent-variable models follow:

o   An item response theory (IRT) model for an n-item educational test gives the probability that an examinee will respond correctly to item j--$x_j = 1$ rather than 0--as a function of (i) an unobservable scalar θ characterizing the proficiency of the examinee and (ii) a possibly vector-valued parameter β_{1j}

characterizing the regression of x_j on θ (Lord 1980). In this case, there is a distinct subparameter β_{1j} for each item, so that $\beta_1 = (\beta_{11}, \ldots, \beta_{1n})$.

o   In factor analysis models (Thurstone 1948), the "factor loadings" β_{1j} of the observed variable x_j are coefficients of its linear regression on unobservable trait values θ. It is often assumed that g is a standardized multivariate normal density.

o   In latent class models (Lazarsfeld 1950), β_1 implies response probabilities for items from members of classes 1 through K, but respondents' class memberships θ are not observed. Their distribution g is multinomial, with parameters β_2.

This paper concerns the structure of information matrices that arise in the estimation of β. If values of latent variables are construed as missing data (as in Dempster, Laird, and Rubin 1977), the expected information matrix associated with estimating β from values of x is bounded from above by the expected information that would obtain if values of θ were observed as well (Orchard and Woodbury 1972). The difference is "missing information." We aim to characterize this loss, and to demonstrate how covariates, or collateral variables y for respondents, can be used to recover some of the missing information for β_1. The following section gives background and notation for latent variables models. Subsequent sections discuss expected information in maximum likelihood (ML) estimation of β

from **x**, then in estimation from **x** and **y**. Results are then
presented for observed information matrices. Finally, a numerical
illustration with data from the Profile of American Youth (U.S.
Department of Defense 1982) is given.

2. Background and Notation

In the terminology of Dempster, Laird, and Rubin (1977),
estimating β from a sample of N independent observations of $(\theta, \mathbf{x})$
is a "complete-data" problem. The loglikelihood for β is

$$\lambda_{\theta X} = \sum_{i=1}^{N} \ln [f(\mathbf{x}_i | \theta_i, \beta_1) \ g(\theta_i | \beta_2)] . \tag{2.1}$$

Assume that both f and g are twice-differentiable, and define the
gradient vector $s(\theta, \mathbf{x})$ for β for the loglikelihood of a single
observation by

$$s(\theta, \mathbf{x}) = [s^1(\theta, \mathbf{x}), s^2(\theta, \mathbf{x})]$$

$$= [\frac{\partial}{\partial \beta_1} \ln \ f(\mathbf{x} | \theta, \beta_1), \ \frac{\partial}{\partial \beta_2} \ln \ g(\theta | \beta_2)]. \tag{2.2}$$

Under regularity conditions, the MLE $\hat{\beta}$ solves the likelihood
equation

$$0 = \sum_{i=1}^{N} s(\theta_i, \mathbf{x}_i)$$

and for large N is approximately multivariate normal in repeated
samples, with mean β and covariance given by the inverse of the

expected information matrix N $I_{\theta X}$. Using the fact that $E_{X\theta}(s)=0$, we further our purpose by writing $I_{\theta X}$ in the following manner:

$$I_{\theta X} = \int\int s(\theta,x)\ s'(\theta,x)\ p(\theta|x,\beta)\ d\theta\ h(x|\beta)\ dx$$

[where $p(\theta|x,\beta)$ is obtained by Bayes theorem as $f(x|\theta)g(\theta)/h(x)$]

$$= E_X[E_\theta(ss'|x)]$$

$$= Var_{X\theta}(s)\ .$$

Let $I^1_{\theta X} = Var_{X\theta}(s^1)$ denote the block of the information matrix that pertains to β_1, and note that (2.2) implies that the off-diagonal block of $I_{\theta X}$ for elements of β_1 and those of β_2 is zero.

Suppose that collateral variables y such as educational or demographic status could also be observed for respondents, and let $p(y|\gamma)$ denote their density in a population of interest. In an extension of conditional independence, it is desirable to posit that

$$f(x|\theta,\beta_1,y) = f(x|\theta,\beta_1)\ ; \qquad\qquad (2.3)$$

that is, θ also explains the associations among observed responses x and collateral variables. (See Thissen, Steinberg, and Wainer 1987 on how this assumption can be tested in the context of IRT.) When (2.3) holds, the joint density of (x,θ,y) is

4          $1\hat{0}$

$$p(x,\theta,y|\beta_1,\beta_2,\gamma) = f(x|\theta,\beta_1)\, g(\theta|y,\beta_2)\, p(y|\gamma).$$

(Note the slight change in the meaning of g; in a similar manner, h becomes $h(x|y,\beta)$ when collateral variables are present.) As long as β_1, β_2, and γ are distinct, the loglikelihood induced by observing y along with θ and x is equal to (2.1) plus a constant insofar as β is concerned. Moreover,

$$I_{\theta XY}^1 = I_{\theta X}^1 , \qquad\qquad (2.4)$$

where

$$I_{\theta XY}^1 = E_Y E_X E_\theta (s^1 s^{1\prime}|x,y) = \text{Var}_{YX\theta}(s^1) .$$

Since the off-diagonal blocks of $I_{\theta XY}$ for elements of β_1 with those of both β_2 and γ are zero, it can be concluded that observing y provides no additional information about β_1 if both θ and x are also observed.

3.   Estimating β from x

Of course it is never possible to observe θ. But, since θ is missing for all respondents regardless of the values of θ and x, it can be considered missing data that is "missing completely at random," and appropriate likelihood and sampling distribution inferences follow by marginalizing over θ in the complete-data likelihood (Little and Rubin 1987). It is common practice to

estimate β from values of $\mathbf{x}$ alone--ignoring y even if it is available--by maximizing the "incomplete-data" loglikelihood

$$\lambda_X = \sum_i \ln h(\mathbf{x}_i|\beta) - \ln [E_\theta(\exp \lambda_{\theta X}|\mathbf{x}_1,\ldots,\mathbf{x}_N;\beta)] \ .$$

(In the context of IRT, Bock and Aitkin 1981 refer to this procedure as "marginal maximum likelihood" estimation.) Again under regularity conditions, the MLE solves what is now an "incomplete-data" likelihood equation $0=\partial\lambda_X/\partial\beta$. Provided differentials can be passed through the integral,

$$\frac{\partial\lambda_X}{\partial\beta} = \sum_i \frac{\partial}{\partial\beta} \ln h(\mathbf{x}_i|\beta)$$

$$= \sum_i \int \frac{\partial}{\partial\beta} [f(\mathbf{x}_i|\theta,\beta_1)g(\theta|\beta_2)] \ h^{-1}(\mathbf{x}_i|\beta) \ d\theta$$

$$= \sum_i \int \frac{\partial}{\partial\beta} [\ln f(\mathbf{x}_i|\theta,\beta_1)g(\theta|\beta_2)] \ f(\mathbf{x}_i|\theta) \ g(\theta) \ h^{-1}(\mathbf{x}_i) \ d\theta$$

$$= \sum_i \int s(\theta,\mathbf{x}_i) \ p(\theta|\mathbf{x}_i,\beta) \ d\theta$$

$$= \sum_i E_\theta(s|\mathbf{x}_i) \ .$$

Accordingly, the incomplete-data information matrix is $N \ I_X$, where

$$I_X = E_X[E_\theta(s|\mathbf{x}) \ E_\theta(s'|\mathbf{x})] = \text{Var}_X[E_\theta(s|\mathbf{x})] \ ,$$

the final equality using $E_X[E_\theta(s|x)]=0$. I_X is related to $I_{\theta X}$ through a decomposition of variance--an instance of Orchard and Woodbury's (1972) "missing information principle:"

$$I_{\theta X} = Var_X[E_\theta(s|x)] + E_X[Var_\theta(s|x)]$$

$$= I_X + I_{\theta|X} \quad . \tag{3.1}$$

$I_{\theta|X}$, the missing information, is the average variance of the complete-data gradient vector given x but not θ; that is, variation in s over possible values of θ that could give rise to observed data x, averaged over x. If the variance of $p(\theta|x,\beta)$ were zero for all x--loosely speaking, if x determined θ with complete accuracy--then $Var_\theta(s|x) = 0$ for all x, and no information would be lost as a result of not observing θ. If values of θ are not completely determined by x, this variance increases and information about β is decreased. The proportional decrease, from diagonal elements of $I_{\theta X}$ to those of I_X, need not. be the same for all elements of β.

4.   Estimating β from x and y

When collateral variables y are available for respondents, the extended incomplete-data loglikelihood is

$$\lambda_{XY} = \sum_i \ln [h(x_i|\beta) \, p(y_i|\gamma)]$$

$$= \ln [E_\theta(\exp \lambda_{\theta X}|x_1,y_1,\ldots,x_N,y_N;\beta)] + \sum_i \ln p(y_i|\gamma) \quad .$$

As long as β is distinct from γ, the shape of the likelihood surface with respect to β involves only the first term--the conditional distribution of the xs, given the observed values of the ys. In particular, the likelihood equation for β is $0 = \partial\lambda_{XY}/\partial\beta$, where

$$\frac{\partial\lambda_{XY}}{\partial\beta} = \sum_i \int s(\theta,x_i)\, p(\theta|x_i,y_i,\beta)\, d\theta$$

$$= \sum E_\theta(s|x_i,y_i) \; ,$$

with $p(\theta|x,y,\beta) = f(x|\theta,\beta_1)\, g(\theta|y,\beta_2) / h(x|y,\beta)$. The asymptotic distribution of the MLE under repeated samples of N (x,y) pairs does involve the distribution of y, however. The block of the information matrix for (β,γ) that pertains to β is N I_{XY}, where

$$I_{XY} = E_Y E_X[E_\theta(s|x,y)\, E_\theta(s'|x,y)] = Var_{YX}[E_\theta(s|x,y)] \; .$$

Large sample ML inferences about β under repeated sampling of (x,y) can be based on this block alone, since the off-diagonal block pertaining to the crossing of β and γ is zero. (Section 5 concerns repeated samples of N (x,y) values with the ys fixed at prespecified values.)

We now focus on the effect of including y upon information about β_1. Expressions for the I_X^1 block suffice, since the nullity of the off-diagonal block in $I_{\theta X}$ implies the nullity of the corresponding block in I_X (see Appendix). Using (2.4) and applying the missing information principle,

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$$I^1_{\theta X} = I^1_{XY} + I^1_{\theta|XY} \; ,$$

where

$$I^1_{\theta|XY} = E_X E_Y [Var_\theta(s^1|x,y)] \; .$$

As in (3.1), the missing information corresponds to a loss in the precision with which β_1 can be estimated. The loss is expressed as expected variation of s^1 over possible values of the latent variable ϑ, conditional now on values of y as well as those of x.

Intuitively, less information should be lost if y is observed along with x. An expression for how much of the missing information has been recovered begins with another decomposition of the total variation in s^1. Since

$$I^1_{\theta X} = I^1_{\theta XY} = Var_{XY\theta}(s^1) \; ,$$

it follows that

$$I^1_{\theta X} = Var_X[E_Y E_\theta(s^1)] + E_X\{Var_Y[E_\theta(s^1)]\} + E_X E_Y[Var_\theta(s^1)]$$

$$= I^1_X + I^1_{Y|X} + I^1_{\theta|XY} \; .$$

I^1_X is the variance in expected values of s^1 over x, averaging over y and θ. $I^1_{Y|X}$ is the expected variance of the average values of. s^1 with respect to θ as y varies. It represents variation in

$E_\theta(s^1)$ explained by y beyond that explained by x. $I^1_{\theta|XY}$ is the expected variation in s^1 remaining unexplained after both x and y have been accounted for.

The portion of missing information about β_1 that is recovered by using y, then, is $I^1_{XY} - I^1_X = I^1_{Y|X}$--another application of the missing information principle, with (x,y) treated as the complete data and x as the incomplete data. When y and θ are independent, this term is zero because for each x, $E_\theta(s^1)$ takes the same value at all values of y. No information about β_1 is lost by ignoring y in this case. When y and θ are not independent, the degree to which information about β_1 increases depends not simply upon the strength of their relationship, but on the strength of their relationship conditional on x. There is less to be gained by using collateral information when θ is already well determined by x alone.

These results indicate that greater benefit accrues from using collateral information as it relates more strongly to the latent variable, and as less information is available from the observed responses x. Mislevy's (1987) analyses in the context of item response theory indicate that in typical applications of educational and psychological testing, readily available collateral variables such as educational and demographic data can often account for a third of the population variance, and increase the precision of β_1 roughly as much as two to six additional test items. This gain is substantial in applications such as educational assessment or attitudinal surveys, where a subject

might be administered only five or ten items; it is potentially useful in adaptive testing, where he might receive fifteen well-chosen items. The proportional gain is not impressive with individual achievement tests, where test lengths of 60 to 100 items are common.

5. "Conditional Expected" Information

The preceding sections concern information matrices that require marginalization over the sample spaces of both x and y. They reflect the point of view one has before observing either x or y values. This section presents results for expected information conditional on given values of y.

Expected information conditional on y is pertinent to the problem of experimental design, for example. If it is possible to stratify on y when gathering data, expected information for various combinations of y values can be compared to choose an optimal sampling scheme. This requires expectations over x conditional on fixed values of y. Let $y = (y_1, \ldots, y_N)$ be a vector of N specified values of y. Define the mixture density

$$P_y(\theta|\beta_2) = N^{-1} \sum_i^N g(\theta|y_i, \beta_2) \quad ,$$

which represents the marginal density of θ in samples drawn in accordance with y. The complete-data expected information matrix

that corresponds to this density is defined analogously to $I_{\theta X}$ in Section 3, as

$$I_{\theta X(y)} = \iint s(\theta,x)\, s'(\theta,x)\, p_y(\theta|x,\beta)\, d\theta\, h_y(x|\beta)\, dx \,, \qquad (5.1)$$

where

$$h_y(x|\beta) = \int f(x|\theta,\beta_1)\, p_y(\theta|\beta_2)\, d\theta$$

and

$$p_y(\theta|x,\beta) = f(x|\theta,\beta_1)\, p_y(\theta|\beta_2)\, /\, h_y(x|\beta) \,.$$

$I_{\theta X(y)}$ is the expected information about β corresponding to repeated observations of (x,θ) sampled in accordance with y. When, as in practice, observations consist of (x,y), one can calculate expected information when estimation ignores y values and when estimation uses y values. These are, respectively,

$$I_{X(y)} = \int \left[\int s\, p_y(\theta|x,\beta)d\theta\right]\, \left[\int s'\, p_y(\theta|x,\beta)d\theta\right]\, h_y(x|\beta)\, dx$$

$$= E_{X(y)}[Var_{\theta(y)}(s|x)] \qquad (5.2)$$

and

$$I_{Xy} = N^{-1} \sum_{i}^{N} \int [E_\theta(s|x,y_i)\, E_\theta(s'|x,y_i)]\, h(x|y_i,\beta)\, dx$$

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$$- E_y E_{x|y} [\text{Var}_\theta(s|x,y)] \ . \tag{5.3}$$

By the missing information principle, the gain in information about β_1 expected when exploiting y for this particular value of y, or $I^1_{Xy} - I^1_{X(y)}$, is at least positive semidefinite.

6. Observed Information

The expected matrices discussed in the preceding sections are functions of the true values of β. In practice, they ar sometimes approximated by substituting maximum likelihood estimates $\hat\beta$ for β. The resulting "estimated expected information matrices" are consistent estimates of the desired values. They are to be distinguished, however, from "observed information" matrices.

An observed information matrix is the negative inverse of the second derivative matrix of the loglikelihood, calculated with maximizing value $\hat\beta$, the observed responses $(x_1,\ldots,x_N)$, and, if required, $(y_1,\ldots,y_N)$. Observed information reflects the point of view after N sampled values of both x and y have been observed, and indicates the precision with which β has been estimated from the realized sample (Efron and Hinkley 1978). In a large-sample normal approximation to the posterior distribution of β under Bayesian inference, the posterior variance is the negative inverse of the observed information.

Define the complete-data second derivative $d(\theta,x)$ as $\partial s(\theta,x)/\partial\beta'$. Using Louis' (1982) expressions for observed information in missing data problems,

$$N \; I_x = \sum_i^N \; [\int (-d) \; p(\theta | x_i, \hat{\beta}) \; d\theta - \int ss' \; p(\theta | x_i, \hat{\beta}) \; d\theta]$$

and

$$N \; \ddot{I}_{xy} = \sum_i^N \; [\int (-d) \; p(\theta | x_i, y_i, \hat{\beta}) \; d\theta - \int ss' \; p(\theta | x_i, y_i, \hat{\beta}) \; d\theta] \quad .$$

In contrast to results with expected information, the off-diagonal blocks of I_x and I_{xy} for the crossing of β_1 and β_2 need not be zero. Moreover, the appealing decomposition of expected information into variance components of s does not carry over to observed information; it depended on the fact that $E(ss') = -E(d)$. For unfortuitous combinations of x and y values, I_x can exceed I_{xy} in some or even all diagonal elements. Inasmuch as observed information is quite generally a consistent estimate of expected information, however, the results of Section 4 suggest that one would expect to find the greater diagonal entries in I_{xy} more often than not.

7. A Numerical Illustration

This section illustrates the ideas developed above in the context of an item response theory (IRT) model for mental test data. The values of $I^1_{\theta X(y)}$, $I^1_{X(y)}$, and I^1_{Xy}--the quantities relevant to "conditional expected information--are approximated here by evaluating (5.1) through (5.3) with $\hat{\beta}$ in place of β.

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7.1 The Data

Observed responses **x** are vectors of responses to four items from the Arithmetic Reasoning test of the Armed Services Vocational Aptitude Battery (ASVAB), Form 8A, as observed in the sample of respondents from the survey Profile of American Youth (U.S. Department of Defense 1982) whose data are reported by Mislevy (1986). Response counts are shown in Table 1 for the N=776 respondents as a whole, and as broken down into the four categories of a demographic design with subsample sizes N_y of 263, 228, 140, and 145. A correct response is indicated by a 1, an incorrect response by a 0.

Table 1 about here

7.2 The Model

Using the numerical procedure described in Mislevy (1987), the 2-parameter logistic (2PL) IRT model was fit to the response counts in Table 1 by maximizing a loglikelihood of the form of (2.1). Under the usual IRT assumption of conditional independence, and with β_1 representing the item parameters $(a_1, b_1, \ldots, a_4, b_4)$, we have

$$f(\mathbf{x}|\theta,\beta_1) = \prod_{j=1}^{4} \exp[1.7a_j x_j(\theta-b_j)]/\{1+\exp[1.7a_j(\theta-b_j)]\},$$

or, equivalently,

$$= \prod_{j} P_j(\theta)^{x_j} Q_j(\theta)^{1-x_j},$$

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where $P_j(\theta) = f_j(x_j=1|\theta,a_j,b_j)$ and $Q_j(\theta) = 1-P_j(\theta)$. The parameters a_j and b_j give the (linear) regression of the logit of x_j on θ. Normal densities were assumed for $g(\theta|y,\beta_2)$, for $y = 1,\ldots,4$. The population parameters $\beta_2 = (\mu_1,\sigma_1,\ldots,\mu_4,\sigma_4)$ were constrained in order to make the model identified, by incorporating the computationally convenient constraints $\Sigma \mu_y = 0$ and $\Sigma(\sigma_y^2 + \mu_y^2)/4 = 1$. The resulting MLE's are shown in Table 2.

Table 2 about here

From this point on, the MLEs shown in Table 2 will be treated as known true values for the purpose of approximating expected information matrices. Using the results of Section 5, expected information matrices will be calculated conditional on the observed subsample proportions $p(y) = (.339, .294, .180, .187)$. The density $p_y(\theta)$ that obtains after fixing y in this manner is thus a mixture of four normal components:

$$p_y(\theta|\beta_2) = \sum_{y=1}^{4} g(\theta|\mu_y,\sigma_y)\, p(y).$$

Values of y account for about 18-percent of the variation of θ in the mixture.

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7.3 Formulae

Let u_j be an element of β_1. Let $W_j(\vartheta)$ take the value $1.7(\theta-b_j)$ if $u_j = a_j$, and the value $-1.7a_j$ if $u_j = b_j$. The element of the complete-data gradient vector corresponding to u_j is

$$s(u_j;\theta,\mathbf{x}) = [x_j - P_j(\theta)] \ W_j(\theta) \ .$$

Computing expected information requires the expected count of each response pattern $\mathbf{x}_\ell = (x_{\ell 1},\dots,x_{\ell 4})$. In subpopulation y, this value is

$$\hat{N}_{\ell y} = N_y \int f(\mathbf{x}_\ell|\theta,\beta_1) \ g(\theta|\mu_y,\sigma_y) \ d\theta \ .$$

For the undifferentiated population (given the observed values of y) as a whol , $\hat{N}_\ell = \Sigma \ \hat{N}_{\ell y}$. These values are given in Table 3.

Table 3 about here

Expected information matrices are now obtained as follows:

$$I^1_{\theta X(y)} = \Sigma_\ell \ \hat{N}_\ell \int s^1_\ell \ s^{1\prime}_\ell \ p_y(\theta|\mathbf{x}_\ell) \ d\theta \ ,$$

where $s^1_\ell = s^1(\theta,\mathbf{x}_\ell)$ and $p_y(\theta|\mathbf{x}_\ell) = f(\mathbf{x}_\ell|\theta,\beta_1) \ p_y(\theta|\beta_2) \ / \ h_y(\mathbf{x}|\beta)$;

$$I^1_{X(y)} = \sum_\ell \hat{N}_\ell \; [\int s^1_\ell \; p_y(\theta|x_\ell) \; d\theta][\int s^{1\prime}_\ell \; p_y(\theta|x_\ell) \; d\theta] \; ;$$

and

$$I^1_{Xy} = \sum\sum_{\ell y} \hat{N}_{\ell y} \; [\int s^1_\ell \; p(\theta|x_\ell,y) \; d\theta][\int s^{1\prime}_\ell \; p(\theta|x_\ell,y) \; d\theta] \; ,$$

where $p(\theta|x_\ell,y) = f(x_\ell|\theta,\beta_1) \; p(\theta|\beta_2,y) \; / \; h(x_\ell|y,\beta)$.

7.4 Results

Tables 4, 5, and 6 present $I^1_{\theta X(y)}$, $I^1_{X(y)}$, and I^1_{Xy}. These matrices are block-diagonal, with only off-diagonal elements for the a and b parameters of a given item taking possibly non-zero values. The proportions of effective information and partial recovery for the diagonal elements are summarized in Table 7. Compared to the information expected if (x,θ) were observed, the degree of information expected when only x is observed averages 36-percent for a parameters and 85-percent for b parameters. Using (x,y) yields corresponding values of 40-percent and 87-percent. Averaging over item-level results, the degree to which missing information is recoverable (for the observed values of y) is 7-percent for a's and 17-percent for b's.

Tables 4-7 about here

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Appendix

Section 4 uses the large-sample result that if the off-diagonal block in $I_{\theta X}$ corresponding to the crossing of β_1 and β_2 is null, so is the corresponding block in I_X. This appendix gives a heuristic argument in support of that claim. Since the observed information matrix I_x introduced in Section 6 is a consistent estimate of I_X, it suffices to show that the expectation of the off-diagonal block in I_x is null.

From Section 6,

$$N \ I_x = \sum_{i}^{N} \ [\int (-d) \ p(\theta | x_i, \hat{\beta}) \ d\theta - \int ss' \ p(\theta | x_i, \hat{\beta}) \ d\theta] \ .$$

Substituting β for $\hat{\beta}$, and taking expectation over x gives

$$E_X \ I_x = E_X[E_\theta (-d)] - E_X[E_\theta (ss')] \ . \tag{A.1}$$

It is clear from (2.2) that the off-diagonal block of d is null for all x and all θ, so the corresponding block of the first term on the right of (A.1) is null. The second term is $\text{Var}_{\theta X}(s)$, or $I_{\theta X}$, which, from Section 2, also has a null off-diagonal block. The off-diagonal block of the matrix difference between the two terms must be null too.

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Table 1

Observed Counts of Response Patterns

Response				y				
1	2	3	4	1	2	3	4	Total
0	0	0	0	23	20	27	29	99
0	0	0	1	5	8	5	8	26
0	0	1	0	12	14	15	7	48
0	0	1	1	2	2	3	3	10
0	1	0	0	16	20	16	14	66
0	1	0	1	3	5	5	5	18
0	1	1	0	6	11	4	6	27
0	1	1	1	1	7	3	0	11
1	0	0	0	22	23	15	14	74
1	0	0	1	6	8	10	10	34
1	0	1	0	7	9	8	11	35
1	0	1	1	19	6	1	2	28
1	1	0	0	21	18	7	19	65
1	1	0	1	11	15	9	5	40
1	1	1	0	23	20	10	8	61
1	1	1	1	86	42	2	4	134
Total				263	228	140	145	776

Table 2

Maximum Likelihood Estimates of β

Item	a_j	b_j	y-value	μ_y	σ_y
1	1.006	-.421	1	.485	1.164
2	.672	-.213	2	.073	.855
3	.775	.139	3	-.513	.642
4	.834	.402	4	-.502	.640

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Table 3

Expected Counts of Response Patterns

| Response | | | | y | | | | |
1	2	3	4	1	2	3	4	Total
0	0	0	0	27.17	27.85	31.70	32.34	119.06
0	0	0	1	4.88	5.81	5.37	5.54	21.60
0	0	1	0	7.55	8.97	8.46	8.73	33.71
0	0	1	1	3.16	3.56	2.31	2.41	11.44
0	1	0	0	12.76	15.04	14.69	15.13	57.62
0	1	0	1	4.80	5.49	3.77	3.92	17.98
0	1	1	0	7.07	8.15	5.77	6.00	26.99
0	1	1	1	6.00	5.55	2.35	2.46	16.36
1	0	0	0	15.72	18.75	16.23	16.79	67.48
1	0	0	1	8.38	8.95	5.08	5.31	27.72
1	0	1	0	12.04	13.04	7.68	8.01	40.77
1	0	1	1	14.77	11.66	3.80	3.99	34.22
1	1	0	0	17.86	19.80	12.37	12.90	62.93
1	1	0	1	19.51	16.26	5.76	6.05	47.58
1	1	1	0	26.58	22.79	8.46	8.88	66.71
1	1	1	1	74.75	36.33	6.20	6.55	123.83
Total				263.00	228.00	140.00	145.00	776.00

Table 4

Expected Complete-Data Information

	a_1	b_1	a_2	b_2	a_3	b_3	a_4	b_4
a	176.75		290.81		254.24		249.38	
b	-68.60	371.64	-37.80	201.01	31.42	250.70	75.89	267.98

Note: Only diagonal blocks are shown; all other entries are zero.

Table 5

Expected Incomplete-Data Information, Ignoring y

	a_1	b_1	a_2	b_2	a_3	b_3	a_4	b_4
a	51.76		114.82		90.72		95.65	
b	-55.09	298.63	-31.91	179.77	31.70	215.75	70.80	224.58

Note: Only diagonal blocks are shown; all other entries are zero.

Table 6

Expected Incomplete-Data Information, Using y

	a_1	b_1	a_2	b_2	a_3	b_3	a_4	b_4
a	53.39		119.79		105.18		116.72	
b	-46.78	308.91	-22.49	183.28	44.33	222.03	85.86	232.98

Note: Only diagonal blocks are shown; all other entries are zero.

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Table 7

Recovery of Missing Information (Diagonal Elements)

	a_1	a_2	a_3	a_4	average
$I_{X(y)}/I_{\Theta X(y)}$	.29	.39	.36	.38	.36
$I_{Xy}/I_{\Theta X(y)}$	.30	.41	.41	.47	.40
$\dfrac{I_{Xy} - I_{X(y)}}{I_{\Theta X(y)} - I_{X(y)}}$	.01	.03	.09	.14	.07

	b_1	b_2	b_3	b_4	average
$I_{X(y)}/I_{\Theta X(y)}$	.80	.89	.86	.84	.85
$I_{Xy}/I_{\Theta X(y)}$	.83	.91	.89	.87	.87
$\dfrac{I_{Xy} - I_{X(y)}}{I_{\Theta X(y)} - I_{X(y)}}$	.14	.17	.18	.19	.17

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