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AUTHOR De Corte, Erik; And Others
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ABSTRACT

Two related studies are reported about the (lack of) activation of real-world knowledge in elementary school students' understanding and solution of school arithmetic word problems. In the first study a set of word problems was collectively administered to 75 fifth-graders during a typical mathematics lesson. Half of the problems were standard items that could be unambiguously solved by applying the most obvious arithmetic operations with the given numbers, while the other half were problematic items for which the appropriate mathematical model was less obvious and indisputable. In the second study a similar problem set was individually administered to 30 fifth-graders in an attempt to collect more fine-grained data about the strength and awareness of conceptions and beliefs concerning the role of real-world knowledge in understanding and solving word problems. The results of both studies support the hypothesis that students tend to exclude common-sense knowledge and realistic considerations during the solution of arithmetic word problems. Furthermore, this tendency seems to be rooted in implicit but resistant beliefs about the role of real-world knowledge in mathematical modeling of school word problems. Contains 20 references. (Author/MKR)

WORD PROBLEMS: GAME OR REALITY? STUDIES OF CHILDREN'S BELIEFS ABOUT THE ROLE OF REAL-WORLD KNOWLEDGE IN MATHEMATICAL MODELING

Erik De Corte, Lieven Verschaffel* and Sabien Lasure
Center for Instructional Psychology and Technology (CIP&T)
University of Leuven, Belgium

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* L. Verschaffel is a senior research associate of the National Fund for Scientific Research, Belgium.

Address for correspondence: Erik De Corte, Center for Instructional Psychology and Technology, University of Leuven, Vesaliusstraat 2, B-3000 Leuven, Belgium.

Abstract

Two related studies about the (lack of) activation of real-world knowledge during elementary school pupils' understanding and solution of school arithmetic word problems, are reported. In the first study a set of word problems was collectively administered to 75 fifth-graders during a typical mathematics lesson. Half of the problems were standard items that can be unambiguously solved by applying the most obvious arithmetic operation(s) with the given numbers, while the other half were problematic items for which the appropriate mathematical model is less obvious and indisputable, at least if one seriously takes into account the realities of the context evoked by the problem statement. In the second study a similar problem set was individually administered to 30 fifth-graders in an attempt to collect more fine-grained data about the strength and the awareness of the (mis)conceptions and (mis)beliefs concerning the role of real-world knowledge in pupils' understanding and solution of school word problems. The results of both studies support the hypothesis that pupils tend to exclude commonsense knowledge and realistic considerations during their solution processes of school arithmetic word problems. Furthermore this tendency seems to be rooted in implicit but resistant beliefs about the role of real-world knowledge in mathematical modeling of school word problems.

Introduction

Arithmetic word problems constitute an important part of the mathematics program at the elementary school. The most important reason for using this type of problems in schools is to train pupils in applying their formal mathematical knowledge and skills in real-worldlike situations. However, for several years it has been argued that the practice of word problems in school mathematics does not foster in pupils a tendency to include commonsense knowledge and realistic considerations in the different stages of their solution processes, i.e. the initial understanding of the problem, the construction of a mathematical model, the actual computational activities, and the interpretation and evaluation of the outcome of these computations. Rather than functioning as realistic contexts inviting or even forcing pupils to use their commonsense knowledge and experience about the real world, school arithmetic word problems are perceived as artificial, puzzle-like tasks that are unrelated to the real world. This tendency among students to ignore real-world knowledge - and even to accept premises about the problem context that are empirically false - when solving arithm[illegible] problems, is generally attributed to the following major characteristics of current instructional practice:

(1) the impoverished and stereotyped diet of standard word problems which can be unambiguously modeled and solved through the most obvious arithmetic operation(s) with the numbers given in the problem, and

(2) the focus at teaching pupils to solve these problems by identifying and executing <u>the</u> correct arithmetic operation, rather than using word problems as exercises in realistic modeling, focusing on the proper consideration of the assumptions and the appropriateness of the model underlying any proposed solution (De Corte & Verschaffel, 1985; Freudenthal, 1991; Kilpatrick, 1987; Nesher, 1980; Reusser, 1986; Säljö, 1991; Schoenfeld, 1991; Treffers, 1987; Verschaffel & De Corte, in press).

While the above-mentioned criticism against school word problems has been raised several times, the empirical evidence supporting it is rather scarce and anecdotal, and therefore not very compelling. We will report two related studies in which we collected empirical data about the (lack of) activation of real-world knowledge during elementary school pupils' understanding and solution of school arithmetic word problems. The methodology used in both studies consists of confronting pupils with a set of arithmetic word problems half of which are standard items (S-items) that can be solved unambiguously by applying the most obvious arithmetic operation(s) with the given numbers, while the other half are problematic items (P-items) for which the appropriate mathematical model is less obvious and indisputable, at least if one seriously takes into account the realities of the context evoked by the problem statement. A similar approach has recently been used by Greer (1993). While his problem set involved only items relating to multiplication and division, we broadened the scope of the research somewhat by considering problems involving various aspects of addition and subtraction too.

Study 1

Method

The subjects were 75 pupils (10-11 year-old boys and girls) from three fifth-grade classes of three schools in which word problem solving was taught in the typical way. This implies that the pupils had been frequently confronted with traditional school word problems rather than with authentic problem situations, and that realistic modeling had not been systematically addressed in teaching (see also De Corte & Verschaffel, 1989).

A paper-and-pencil test was constructed consisting of ten matched pairs of items. Each pair consisted of

- a standard item (S-item) that can be solved unambiguously by applying the most obvious arithmetic operation(s) with the given numbers (e.g., "Steve has bought 5 planks of 2 meters each. How many planks of 1 meter can he saw out of these planks?"), and
- a parallel problematic item (P-item) for which the appropriate mathematical model is less obvious and indisputable, at least if one seriously takes into account the realities of the context evoked by the problem statement (e.g., "Steve has bought 4 planks of 2.5 meters each. How many planks of 1 meter can he saw out of these planks?").

The ten P-items are listed in Table 1. The problematic modeling assumptions of these ten problems are illustrated in Table 2.

Insert Table 1 about here

The ten pairs of problems were administered on the same day in two series, each containing the P-variant of five problem pairs and the S-variant of the five other pairs. The problems in each series were presented in two different orders, and in each class one half of the children started with one series while the other half was given the other series first. The administration of the problems was done by the class teacher as part of a normal mathematics lesson.

The problems were presented on A4-sheets. With respect to each problem, pupils were asked to write down not only their answer, but also how they arrived at this answer (e.g., by mentioning the calculations), and possible other comments they might have (e.g., explaining their stumbling block when they were not able to solve the problem, supplementing their numerical answer with some comments, criticizing the problem statement, etc.).

Analysis

Children's reactions to the problems were analyzed in two ways for evidence of the activation and use of real-world knowledge and realistic considerations about the problem context:

(1) by distinguishing in their answers between realistic answers and non-realistic ones;

(2) by distinguishing in their computations and additional comments between realistic comments and no realistic comments.

When a child gave an answer to the problem that was scored as realistic or produced a non-realistic answer that was accompanied by a realistic comment, his(her) overall reaction to that particular problem was scored as a "realistic reaction" (RR). Take, for example, the "planks" item mentioned above. A RR score was not only given to a child who produced the realistic answer "8 planks", but also to a child who responded with "10 planks" but who added the comment that "Steve would have a hard time putting together the remaining pieces of 0.5 meter". The code NR ("non-realistic reaction") was given for children who answered a problem in a non-realistic manner and did not give any further realistic comment. Table 2 gives examples of non-realistic reactions (NR) and realistic reactions (RR) for all ten P-items from Table 1.

Insert Table 2 about here

Hypotheses

The overall hypothesis of the first study was that due to their extensive experience with an impoverished diet of standard word problems, and to the lack of systematic attention at the mathematical modeling perspective in their mathematics lessons, pupils will demonstrate a strong tendency to exclude real-world knowledge and context-bound considerations from their problem-solving endeavours of the P-items, and - consequently - will solve them as if they were not at all problematic.

Results and discussion

Table 3 gives the number of pupils who reacted in a realistic (RR) and in a non-realistic (NR) way for each of the ten P-items from Table 1 separately. As said before, the number of RRs refers to the sum of the pupils who responded in a realistic way, and those who wrote down an answer that was scored as non-realistic, but that was accompanied by any realistic consideration.

Insert Table 3 about here

The data in Table 3 strongly support the hypothesis. As predicted, the pupils demonstrated a very strong overall tendency to exclude real-world knowledge and realistic considerations when confronted with the problematic versions of the problems. In total, only

128 out of the 750 reactions to the P-items (= 17 %) could be considered as realistic (RR), either because the pupil wrote a realistic answer or made an additional realistic comment.

Table 3 also shows that for two out of the ten P-items a considerable number of realistic answers or comments were observed: the "buses" item (P4) and the "balloons" item (P7). The question arises why these two problems elicited considerably more RR than the other P-items. A plausible explanation is that in P4 and P7 the modeling difficulty is restricted to the last phase of the solution process, wherein the pupil has to make sense of the result of the arithmetic operation, namely a quotient with a remainder (Silver, Shapiro & Deutsch, 1993). In all other P-items the underlying realistic modeling difficulty requires context-based adaptations at the beginning of the solution process, namely the construction of the mathematical model, rather than at the end of it (Verschaffel, De Corte & Lasure, 1994).

Finally, to have an indication of the interindividual differences in the disposition towards (non-)realistic modeling, we counted the total number of RRs on the ten P-items for each pupil separately. Fifty-nine out of the 75 pupils (= 78 %) reacted in a realistic way to less than three of the ten P-items. Almost all RRs of these pupils were found on the "buses" and the "balloons" problem. Only 16 pupils (= 22 %) provided RRs on at least three P-items, and only two of them did so on more than half of the ten P-items (see Verschaffel et al., 1994, for more details).

While Study 1 provides empirical evidence for pupils' strong tendency to exclude real-world knowledge and realistic considerations from their understanding and solving of word problems, its findings need to be put into perspective because of an important methodological limitation of the data-gathering technique used, namely a collective paper-and-pencil test. Indeed, one could argue that during their (private) solutions of the P-items some pupils may have activated real-world knowledge which was not reflected in their written answers, simply because they finally decided to react in a "conformist" rather than a "realist" way in line with their beliefs and conceptions about "the prevailing rules and conventions of the game of school arithmetic word problems" (De Corte & Verschaffel, 1985). This may have led to a significant underestimation of the number of realistic considerations in this investigation. Therefore, we set up a second study in which we investigated more directly children's awareness of the beliefs and strategies underlying their non-realistic solutions of arithmetic word problems. In addition we assessed the amount of scaffolding needed to transform these non-realistic solutions into realistic ones. The individual interview was used as the major data-gathering technique.

Study 2

Method

Study 2 consisted of two stages. In the first stage, seven matched pairs of word problems used in Study 1 were collectively administered to a group of 64 fifth-graders from three different schools. The seven P-items selected were: P1 (the "birthday-party" item), P2 (the "planks" item), P4 (the "buses" item), P5 (the "running" item), P6 (the "school-distance" item), P9 (the "rope item"), and P10 (the "flask" item) (see Table 1). The administration of the test and the scoring of the answers were done in the same way as in Study 1. Based on the results on this paper-and-pencil test, the five most "realistic" and the five most "non-realistic" problem solvers from each of the three fifth-grade classes were selected to participate in the second stage of the investigation. During this second stage, which took place one or two days later, these 15 realistic and 15 non-realistic problem solvers were individually administered the same seven problem pairs once again.

To assess the strength of children's tendency towards non-realistic mathematical modeling as well as the awareness of the beliefs and strategies underlying their non-realistic solutions, the following interviewing procedure was followed with respect to each P-item solved with a NR on the paper-and-pencil test.

First, the pupil was asked to read aloud the problem followed by his own NR written down on the answer sheet (e.g., "4 x 2,5 = 10 planks" for the "planks" item).

Then the pupil was confronted with the written notes of a fictitious classmate who had responded in a realistic manner. For instance, with respect to the "planks" item the interviewer said: "As you can see on this sheet, one of your classmates responded: 4 x 2 = 8 planks. What is the best answer? Why?". If the pupil sticked to the initial NR (i.e., "10 planks") after this first and weak form of scaffolding, a second and stronger scaffold in the direction of realistic modeling was provided by stimulating en helping the pupil to realize the concrete problem situation. For example, with respect to the "planks" item the interviewer said: "Can you draw the planks? Can you also draw what happened with these planks according to the problem statement? Can you see on your drawing how many planks of 1 meter Steve can saw out of these 4 planks?". In Table 4 we present the weak and the strong scaffold for each of the seven P-items separately.

Insert Table 4 about here

In order to control for possible disturbing effects of the interviewing technique, the same procedure (involving similar kinds of provocation and scaffolding) was applied with respect to the P-items that were solved with a RR (e.g., "8 planks" for the "planks" item) during the paper-and-pencil test. In these cases, the pupil was confronted with the fictitious written reaction of a non-realistic responder (e.g., "4 x 2,5 = 10 planks"). The same interviewing procedure was also applied with respect to the seven "unproblematic" S-items: pupils who had given a correct answer to a S-item from the paper-and-pencil test were confronted with an incorrect response resulting from the application of a wrong operation with the given numbers (e.g., a multiplication instead of a division), while those who had given an incorrect answer were confronted with the correct one.

Analysis

For all 30 pupils and for all seven P-items the following kinds of data were used for analysis:

(1) the initial reaction to the problem during the paper-and-pencil test; this reaction was scored either as RR or NR;

(2) the reaction to the confrontation with the RR of a fictitious classmate (= weak scaffold) in case of an initial NR during the paper-and-pencil test; this reaction was also scored either as RR or NR;

(3) eventually, the reaction to the second and strongest form of scaffolding; this reaction was again scored either as RR or NR.

While the pupils' total number of RRs to the seven P-items of the paper-and-pencil test provide a measure of their actual level of realistic modeling - i.e. the level at which they can perform independently -, their sensitivity to scaffolds towards realistic modeling of the P-items that were initially answered with a NR reflect their zone of proximal development (Vygotky, 1978) or their learning potential (Hamers, Sijtsma & Ruijssenaars, 1993), i.e., those behaviors which they cannot yet perform independently but can with help from others. To measure this zone of proximal development or learning potential, a score of 2, 1 or 0 was given for each P-item not yet solved in a realistic manner during the paper-and-pencil test:

- 2 = the pupil produced a NR during the paper-and-pencil test, but replaced it by a RR after being confronted with the realistic alternative during the first, weak form of scaffolding;
- 1 = the pupil produced a NR during the paper-and-pencil test, did not replace it by a RR after being confronted with the realistic alternative, but produced a RR after receiving the second, strong scaffold;
- 0 = the pupil produced a NR during the paper-and-pencil test, and this reaction remained unchanged even after the second and strongest scaffold.

The mean of these scores (= their sum divided by the number of P-items not yet answered with a RR during the paper-and-pencil test) was considered as a measure of a pupil's learning potential, i.e. a score indicating how much a pupil gained from increasingly stronger forms of scaffolds towards realistic modeling on those P-items he could not answer in a realistic manner himself.

Hypotheses

First, it was hypothesized that - as in Study 1 - the overall number of RRs generated on the seven P-items of the paper-and-pencil test would be alarmingly low. Therefore, we predicted that the overall percentage of RRs on that test would not differ significantly from the percentage of RRs found in Study 1, which was 17 %.

Second, a positive effect of the scaffolds on the number of RRs was anticipated. Therefore, we predicted that the percentage of RRs of the 30 pupils at the end of the individual interview would be significantly higher than on the paper-and-pencil test.

However, we also expected that this overall percentage of RRs at the end of the individual interviews would still be quite low. This prediction was based on the assumption that pupils' tendency towards routine-based and non-realistic modeling would be so strong, that the confrontation with the RR (= scaffold 1) and even the subsequent hint towards the realistic reasoning underlying this RR (= scaffold 2) would often be insufficient to make them change their initial NR into a RR.

Third, we hypothesized that the so-called realistic problem solvers of the P-items from the paper-and-pencil test would benefit more from the two forms of scaffolding than the pupils with little or no RRs on that test (= the so-called non-realistic problem solvers). Their greater tendency towards realistic modeling - as expressed in their better performance on the P-items during the paper-and-pencil test - would make them also more sensitive to the two scaffolds provided during the individual interview. Therefore. we predicted a significantly greater mean learning potential score for the 15 realistic than for the 15 non-realistic problem solvers on the P-items not yet solved with a RR during the paper-and-pencil test.

Results

In line with our first hypothesis and with the findings of Study 1, the results on the collective test revealed a very strong tendency among the pupils to exclude real-world knowledge and context-based considerations from their solutions of school arithmetic word problems: only 16 % of all the reactions of the 64 pupils to the seven P-items of the paper-and-pencil test were classified as realistic. This percentage is almost exactly the same as the overall percentage in Study 1 (i.e. 17 %). As in Study 1, the "buses" problem elicited again the largest number of RRs, namely 64 %. The percentages of RRs for the six other P-items used in Study 2 varied between 17 % and 1 %.

As mentioned before, from each of the three participating classes the five pupils with the highest number of RRs and the five pupils with the lowest number of RRs were selected to participate in the second part of the study. The percentage of RRs of those 15 realistic and 15 non-realistic problem solvers was 39 % (i.e. 41 RRs out of a total of 105 responses) and 8 % (i.e. 8 RRs out of 105 responses), respectively. A one-tailed t-test showed that the difference between the two groups was significant, $t(28)=8.58$, $p < .01$.

All 15 non-realistic problem solvers produced either one or no RR on the paper-and-pencil test, and all RRs from this group occurred on the "buses" item. On the other hand, the 15 realistic problem solvers produced either two RRs (seven pupils), three RRs (six pupils), four RRs (one pupil) or five RRs (one pupil) on the paper-and-pencil test. These findings show that the 15 so-called realistic problem solvers could certainly not be considered as genuine "experts" in realistic mathematical modeling. Characterizing them as pupils with a weaker tendency towards non-realistic modeling, seemed more correct.

In line with the second hypothesis, a significant effect of the two forms of scaffolding was found. Altogether, the two scaffolds resulted in an increase in the cumulative percentage of RRs from 23 % (i.e. 49 RRs out of a total 210 responses) during the paper-and-pencil test to 57 % (i.e. 120 RRs out of a total of 210 responses) at the end of the individual interviews (see Table 5). A one-tailed t-test revealed that this increase was significant, $t(29)=12.97$, $p < .001$. Additional one-tailed t-tests revealed that both forms of scaffolding contributed equally to this increase: the first, weak scaffold resulted in an increase of the cumulative percentage of RRs from 23 % (i.e. 49 RRs out of 210 responses) to 40 % (i.e. 83 RRs out of 210 responses), $t(29)=5.61$, $p < .001$, and the second and stronger scaffold produced an additional significant increase from 40 % (i.e. 83 RRs out of 210 responses) to 57 % RRs (i.e. 120 RRs out of 210 responses), $t(29)=-6.95$, $p < .001$.

Although the scaffolds led to a significant increase in the number of RRs, the cumulative percentage of RRs at the end of the individual interviews was still alarmingly low. Indeed, in 43 % of all cases (i.e. 90 out of a total of 210) the pupils still reacted in an unrealistic way even after the second and strongest scaffold towards realistic modeling. This percentage is higher than that of all cases in which scaffolding resulted in a shift from a NR to a RR (see Table 5).

Insert Table 5 about here

Moreover, a qualitative analysis of the pupils' reactions to the two forms of scaffolding indicated that they did not stick to their NRs because they were aware that in the traditional culture of school arithmetic a routine-based, non-realistic answer is more suitable than a solution that seriously takes into account the reality of the problem situation. On the contrary, if pupils frequently sticked to their NR after the first and the second form of scaffolding, it was because they did not seem to grasp the line of reasoning underlying the RR proposed by the interviewer (= scaffold 1), even not after the interviewer's suggestion to realize the problem situation (= scaffold 2). This is illustrated in the following typical protocol (I = interviewer; P = pupil).

I: Can you read aloud this problem as well as what you have written in response to it on your answer sheet?

P: Steve has bought 4 planks of 2.5 meter each. How many planks of 1 meter can he get out of these planks? Answer: 2.5 x 4 = 10; Steve can saw 10 planks of 1 meter.

I: One of your friends responded in this way: "4 X 2 = 8; Steve can saw 8 planks". Who is right? (= scaffold 1)

P: I am right. That other pupil thought the problem was about "4 planks of 2 meter each" instead of "4 planks of 2.5 meter each".

I: Can you draw what happened with these planks according to the story and show on this drawing how many planks of 1 meter Steve can saw out of these 4 planks? (= scaffold 2)

P: (The pupil draws one long and small rectangle consisting of four planks that are joined together tightly).

I: How many planks of 1 meter can Steven saw out of these 4 planks?

P: 10.

I: Can you draw one plank of 2.5 meter.

P: (The pupil draws one small rectangle with a length of 1/4 of the long rectangle).

I: How many planks of 1 meter can Steve saw out of this plank?

P: 2.

I: And how many planks of 1 meter can he saw out of 4 such planks?

P: 10, because he ends up with 4 planks of 0.5 meter and... two halves make one whole.

The quantitative and the qualitative analysis of the pupils' reactions to the two scaffolds during the individual interview provide additional support for pupils' tendency to exclude commonsense knowledge and realistic considerations from their understanding and solution of school arithmetic word problems. Moreover, this analysis suggests that this tendency is rooted in implicit but strong and resistant beliefs about the role of real-world knowledge in mathematical modeling of school word problems.

Third, the results confirmed also the hypothesis that the realistic problem solvers would benefit more from the two scaffolds than the non-realistic ones. This was tested by comparing the mean learning potential scores of the 15 realistic and the 15 non-realistic problem solvers. We remind that a pupil's learning potential score refers to the sensitivity to the two forms of scaffolding with respect to those P-items answer in a non-realistic manner during the paper-and-pencil test. A pupil received two points for every NR changed into a RR after the first scaffold, one point for every NR changed into a RR after the second scaffold, and zero points if he sticked to his initial NR until the end of the interview; the sum of all these points divided by the number of P-items solved in a non-realistic manner during the paper-and-pencil test, resulted in the learning potential score (maximum = 2; minimum = 0).

The mean learning potential score was .88 and .52 for the realistic and the non-realistic group, respectively. A one-tailed t-test revealed that this difference was significant, $t(28)=2.55$, $p < .01$.

Thus, while the results of the second study strongly confirm elementary school pupils' disposition towards non-realistic modeling of school arithmetic word problems, they provide at the same time evidence for the existence of interindividual differences in this disposition, by showing that pupils of different levels of actual performance with respect to realistic modeling, differ also in terms of their zone of proximal development, in the sense that they are differently sensitive to scaffolds towards realistic modeling on problems they could not solve properly themselves initially.

Conclusions and discussion

The results of the two studies reported support the general hypothesis that elementary school pupils have a strong tendency to exclude real-world knowledge and realistic considerations from their solutions of school arithmetic word problems. Indeed, the results on the P-items from the paper-and-pencil tests used in Study 1 and 2 show that pupils' answers only rarely reflect adjustments for context-bound constraints based on realistic considerations. Moreover, the quantitative and qualitative data from the individual interviews in Study 2 suggest that this tendency is very strong and resistant: while confronting the pupil with the correct answer (= scaffold 1) and with the underlying reasoning (= scaffold 2) led to a significant increase in the number of RRs, almost half of the P-items were still answered in an non-realistic way at the end of the interview. The individual interview data suggest also that pupils did not persist in giving NRs because of a metacognitive awareness of their tendency towards non-realistic modeling taking into account the instructional context of school arithmetic. To the contrary, their overall persistence to give NRs seemed to result rather from a routine-based approach to the problems in which there was little or no place for real-world knowledge and context-based considerations about the concrete problem situation. However, two additional results need to be mentioned.

First, certain P-items elicited considerably more RRs than others. In this respect we refer to the relatively large number of RRs on division problems involving a remainder, such as the "balloons" item from Study 1 and the "buses" item used in both studies. A common characteristic which differentiates these two items from the other P-items, is that the modeling difficulty arises in the last phase of the solution process wherein the pupil has to consider the outcome of the calculation within the context of the problem situation. To the contrary, in the other P-items the realistic modeling difficulty occurs in the beginning of the solution process, namely in the construction of a proper mathematical model.

Second, we also found considerable interindividual differences in pupils' tendency towards non-realistic modeling. Whereas most pupils produced little or no RRs during the paper-and-pencil tests of Study 1 and 2, a few others responded more than half of the P-items in a realistic way. In Study 2 we found a significant and positive relationship between pupils' actual level of performance with respect to realistic modeling and their zone of proximal development.

Starting from these findings - and from the results of a similar study by Greer (1993) -, several suggestions for further research derive (see also Greer, 1993; Verschaffel et al., 1994).

First, there is a need for a more systematic description and classification of the distinct types of word problems from the perspective of the difficulties involved in realistic modeling (e.g., division problems involving remainders, addition and subtraction problems about sets with joint elements, multiplication and division problems about direct proportional reasoning...). In addition, the relative difficulty of these distinct categories of "problematic" word problems has to be empirically investigated.

Second, we need to get a deeper understanding of the instructional factors that are responsible for the development of the strong tendency among pupils to solve word problems in a mindless and non-realistic way. As mentioned in the first section of this article, this tendency is generally attributed to two major characteristics of the culture of the current mathematics classroom, namely (1) the impoverished and stereotyped diet of standard word problems which can be unambiguously modeled and solved through the most obvious arithmetic operation(s) with the given numbers, and (2) the teaching methods which do not stimulate proper consideration and discussion of the appropriateness of the mathematical model underlying the proposed solutions. However, there is hardly any evidence supporting the assumptions concerning the influence of these aspects of the current classroom culture on the development of the tendency towards non-realistic modeling in school children. Therefore, future research should address this important issue. One useful line of research in this respect consists in the careful and critical analysis of current textbooks and worksheets focusing on the nature and variety of word problems. Another promising approach is to analyze teachers' conceptions and beliefs about realistic modeling, and their impact on the actual teaching behavior in the mathematics lessons.

Third, research should address the question whether it is possible to develop in pupils a disposition towards realistic mathematical modeling through instruction. This would not only be relevant from a practical point of view, but would provide an additional important way to test the impact of the above-mentioned instructional factors on the development of the strong tendency towards non-realistic modeling among pupils. In our view, the design of such an appropriate instructional program should be guided by the following principles. First, the impoverished diet of standard word problems offered in traditional mathematics classrooms must be replaced by more authentic problem situations that stimulate pupils to pay explicit attention at the complexities involved in realistic mathematical modeling. Second, the way in which mathematical modeling is taught should reflect a constructive, active and collaborative view of learning. This implies the use of a variety of instructional techniques, such as modeling of skilled problem-solving behavior by the teacher, coaching and scaffolding during small group and individual assignments, and whole-class discussions between the teacher and pupils as well as among pupils. Such an approach to teaching is necessary to disclose in the classroom the difficulties and complexities of realistic modeling. Third, special attention should be given to the establishment of a new mathematics classroom culture by explicitly negotiating new social norms about what counts as a good mathematics problem, a good solution procedure, a good response, and by renegotiating the role of the teacher and the students in a mathematics class. Recently, we set up an exploratory teaching experiment in which a program based on these three design principles was implemented in a fifth-grade class (Verschaffel & De Corte, 1995). The results are promising and provide initial support for the hypothesis that it is possible to develop in pupils a disposition towards realistic mathematical modeling. This is achieved by immersing them into a classroom culture in which word problems are not anymore conceived as training tasks for routine solutions, but as exercices in realistic modeling focussing on proper consideration of the assumptions and the appropriateness of the model underlying any proposed solution.

Finally, we should investigate the influence on children's problem solving processes of the broader socio-cultural context in which their mathematical modeling skills are analyzed. For example, is the tendency towards non-realistic mathematical modeling restricted to problems presented in the setting of a mathematics class or a mathematics test, or does it also spread out to other problem-solving activities in school (e.g., physics, social sciences...) and even to out-of-school contexts (see, e.g., Nunes, Schliemann & Carraher, 1993; Säljö, 1991)?

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Table 1. Ten P-items involved in Study 1

P1 Carl has 5 friends and Georges has 6 friends. Carl and Georges decide to give a party together. They invite all their friends. All friends are present. How many friends are there at the party? (= the "birthday" item)

P2 Steve has bought 4 planks of 2.5 meter each. How many planks of 1 meter can he get out of these planks? (= the "planks" item)

P3 What will be the temperature of water in a container if you pour 1 liter of water at 80° and 1 liter of water of 40° into it? (= the "water" item)

P4 450 soldiers must be bused to the their training site. Each army bus can hold 36 soldiers. How many buses are needed? (= the "buses" item)

P5 John's best time to run 100 meters is 17 seconds. How long will it take to travel 1 kilometer? (= the "runner" item)

P6 Bruce and Alice go to the same school. Bruce lives at a distance of 17 kilometers from the school and Alice at 8 kilometers. How far do Bruce and Alice live from each other? (= the "school distance" item)

P7 Grandfather gives his 4 grandchildren a box containing 18 balloons, which they share equally. How many balloons does each grandchild get? (= the "balloons" item)

P8 Rob was born in 1978. Now it's 1993. How old is he? (= the "age" item)

P9 A man wants to have a rope long enough to stretch between two poles 12 meters apart, but he has only pieces of rope 1.5 meters long. How many of these pieces would he need to tie together to stretch between the poles? (= the "rope" item)

P10 This flask is being filled from a tap at a constant rate. If the depth of the water is 4 cm after 10 seconds, how deep will it be after 30 seconds? (This problem was accompanied by a drawing of a cone-shaped flask) (= the "flask" item)

Tabel 2. Examples of reactions scored as NR (non-realistic reaction) and as RR (realistic reaction) for the ten P-items from Study 1

P-item 1: the "birthday party" item

NR: 5 + 6 = 11; there will be 11 friends at the party.

RR: You cannot know how many friends there will be, because some of the invited friends can belong to the friends of Carl and at the same time to the friends of Georges.

P-item 2: the "planks" item

NR: 4 x 2.5 = 10; he can saw 10 planks of 1 meter.

RR: He can saw 8 planks of 1 meter; it is impossible to get planks of 1 meter out of the four remaining planks of 0.5 meter each.

P-item 3: the "water" item

NR: 8O + 40 = 120; the temperature of the water will be 120°.

RR: (80 + 40) : 2 = 60; the temperature of the water will be 60°.

P-item 4: the "buses" item

NR: (1) 450 : 36 = 12.5; 12.5 buses will be needed.

(2) 450 : 36 = 12.5; they will need 12 buses.

RR: (1) 450 : 36 = 12.5; they will need 13 buses.

(2) 450 : 36 = 12.5; they will need 12 buses and some additional cars.

P-item 5: the "runner" item

NR: 10 x 17 = 170; John will run 1 kilometer in 170 sec.

RR: 10 x 17 = 170, but his real time will be a bit more than 170 sec, because John will certainly get tired after a while.

P-item 6: the "school distance" item

NR: (1) 17 + 8 = 25; Bruce and Alice live at a distance of 25 km from each other.
(2) 17 - 8 = 9; Bruce and Alice live at a distance of 9 km from each other.

RR: I cannot solve this problem; the answer can be either 9 or 25.

P-item 7: the "balloons" item

NR: 18 : 4 = 4.5; each grandchild will get 4.5 balloons.

RR: (1) 4 X 4 = 16; so, each grandchild gets 4 balloons and there remain 2 balloons.
(2) 18 : 4 = 4.5; but I am afraid that they will have a hard time blowing up balloons cut in half.

P-item 8: the "age" item

NR: 1993 - 1978 = 15; Rob is 15 years old.

RR: (1) Rob is either 14 or 15.
(2) 1978 + 15 = 1993 but if Rob's birthday is in one of the remaining months of the year, he is still 14.

P-item 9: the "rope" item

NR: 12 : 1.5 = 8; he will need 8 pieces of rope to stretch between the poles.

RR: 12 : 1,5 = 8; but if you take into account the amount of rope needed to tie the pieces of rope to each other and to the two poles, the man certainly will need more than 8 pieces

P-item 10: the "flask" item

NR: 4 X 3 = 12; the depth of the liquid in the flask will be 12 cm.

RR: 4 X 3 = 12, but the answer will be more than 12 cm because of the cone-shaped form of the flask.

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* For the P-item for which no RRs were made in Study 1, a fictitious example of a RR was made.

Table 3. Number and percentage of non-realistic (NR) and realistic (RR) pupil reactions to the ten P-items in Study 1

P-item	NR		RR	
	N	%	N	%
1	60	80	15	20
2	65	87	10	13
3	62	83	13	17
4	38	51	37	49
5	73	97	2	3
6	73	97	2	3
7	31	41	44	59
8	73	97	2	3
9	75	100	0	0
10	72	96	3	4
Total	622	83	128	17

* RR stands for "realistic reactions", i.e. all reactions reflecting activation of context-dependent real-world knowledge in the solution of the problem or in the additional comments.

Table 4. Weak (S1) and strong (S2) scaffolds used for the seven P-items of Study 2

P1-item (the "birthday" item)

S1: One of your classmates said that it is impossible to solve this problem. Who is right? You or your classmate?

S2: Can you give me the name of a good friend in the class? Imagine you and ... are giving a party together and that you both invite your best friends. Imagine that your five best friends are present, and that the six best friends of ... are present. Are you sure that there are 11 guests at the party?

P2-item (the "planks" item)

S1: One of your classmates responded in this way: 4 X 2 = 8; Steve can saw 8 planks. Who is right?

S2: Can you draw the planks? Can you also draw what happened with these planks according to the story. Can you see on this drawing how many planks of 1 meter Steve can saw out of these 4 planks?

P4-item (the "buses" item)

S1: One of your classmates responded in this way: 450 : 36 = 12.5; they will need 13 buses. Who is right?

S2: You have answered that they will need 12.5 buses. What does that answer mean to you? 12.5 buses?

P5-item (the "school distance" item)

S1: One of your classmates said that it is impossible to solve this problem. Who is right? You or your classmate?

S2: Can you make a drawing of the situation described in the problem? Is this the only drawing you can make out of this problem statement?

P6-item (the "runner" item)

S1: One of your classmates responded that it is impossible to know precisely how long it would take John to run 1 kilometer. Who is right?

S2: Do you know your best time on the 100 meter? Imagine that it is 17 sec. If you had to run 1 kilometer, do you think that you would succeed in running every remaining 100 meter in that same best time?

P9-item (the "rope" item)

S1: One of your classmates responded in the following way: I don't know exactly, but the answer will certainly be more than 8 pieces. Who is right?

S2: Here is a picture of the man who is tying together the pieces of rope to stretch between the two poles. Do you still think that 8 pieces will be enough to stretch between the two poles?

P10-item (the "flask" item)

S1: One of your classmates said that it is impossible to give a precise answer to that question. Who is right?

S2: If this is the part of the flask that was filled after 10 sec (interviewer points at the colored part of the cone-shaped flask on the pupil's response sheet), can you indicate on this figure what the level of the water will be after 20 sec? And after 30 sec?

Table 5. Absolute and cumulative frequencies and percentages of realistic reactions of the 30 pupils to the seven P-items at the different stages of Study 2.

		IRR*	RRF	RRS	NRR
Absolute	N	49	34	37	90
	%	23%	16%	18%	43%
Cumulative	N	49	83	120	210
	%	23%	39%	57%	100%

* IRR = immediate realistic reaction during the collective test

RRF = realistic reaction after the first, weak scaffold

RRS = realistic reaction after the second, strong scaffold

NRR = still a non-realistic reaction at the end of the individual interview