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ABSTRACT

This study sought to determine: (1) what children understand about "chance" when they begin secondary school?; and (2) how common and how influential the use of informal heuristics, approaches, and biases is in their thinking about probability in the school context. Children's understanding of chance, attributions of events to influences, beliefs, and intuitions were explored. Interviews and questionnaires were conducted with 11- and 12-year-old children, eliciting their stories of chance, and focusing on the language of chance, including attributions and experience. Results were examined in terms of availability, representativeness, equiprobability, and C. Konold's "outcome approach." Religious beliefs, superstition, and language were shown to have significant influence on the children's thinking. Results support the need to facilitate the increased recognition and understanding of the heuristics and approaches commonly applied by children. (An appendix includes questionnaires which also list responses interpreted as "equiprobability" and as "representatives." Contains 18 references.) (BGC)

11-12 year old children's informal knowledge and its influence on their formal probabilistic reasoning

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Using interviews (n=38) and a questionnaire (N=236), probability items based on previous work (Green, 1982, Kahnemann et al, 1982, Lecoutre, 1992, Konold, 1989,1991) and some new instruments were developed and validated with 11-12 year old children. The children's understanding of 'chance', attributions of events to influences, beliefs and intuitions were explored.

Many of the children's responses are found to be consistent with the application of 'availability', 'representativeness', and 'equiprobability' heuristics, or with an 'outcome approach'. Their understanding of the meaning of the term 'chance' is explored in relation to their experience, language and beliefs. Religious beliefs, superstition and language have some significant influence on the children's thinking.

Reliable equiprobability and representativeness scales were built from errors in the probability items, but the outcome approach interpretation proved unreliable, in contrast to Konold's result. Overlapping interpretation, effect of context and methodological concerns are discussed.

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INTRODUCTION

> "READER (M) When and why did the absolutely potty idea of quoting odds on weather conditions begin in Britain? I've heard this from the United States on short wave, where the "30% chance of rain" was regarded as a joke. Perhaps you could ask one of the experts what a "25% chance of rain" is going to produce? Does it mean showers at four hourly intervals, or six hours of rain in twenty-four? In my view the terminology is useless.
>
> DUNKLEY (BBC) I can remember listeners writing to FEEDBACK urging the BBC to **adopt** this usage which is so widely used in North America, not, incidentally, as a joke at all. Andrew Lane, Assistant Producer at the BBC Weather Centre says they started this system eighteen months ago and that the aim is to give a more precise indication of likelihood than you used to get from vague phrases such as "isolated showers". He reckons that everyone understands a 100% possibility of rain, or a zero possibility, and he suspects nobody has much trouble with a 50-50 likelihood. 10% means there's very little chance, 90% a very big chance, and everything else is pro rata." (BBC, Radio 4, 'FEEDBACK', 18.3.94, 09:45)

This dialogue illustrates the problem probabilistic thinking can cause even adults in a practical setting. Yet the topic is generally regarded to be of increasing importance in society, and this is reflected in its growing recognition in the curriculum all over the world, (Kapadia and Borovnic, 1991). The probability curriculum presents a new kind of thinking to the child, in contrast to the causal, deterministic explanations common to most disciplines, and most applications of mathematics. Is the child ready for this challenge in secondary school? What obstacles may the child, and her teacher be faced with ? We adopt the point of view that the child arrives with knowledge, derived from the experience, beliefs and language encountered in their everyday social interactions, i.e. as part of their culture. This knowledge may provide support and obstacles to the growth of a normative probabilistic understanding, and we need to understand this informal knowledge and how it affects or influences the probabilistic thinking expected in the school curriculum. Anticipating a little, we pose the simple question: is it reasonable to assume that children understand the common term 'chance' correctly, and do they believe that the outcome of the throw of a die or coin is a random event ?

LITERATURE

Early research into probabilistic thinking emphasised development, either through Piagetian stages (Piaget & Inhelder, 1975, original French version: 1951), or through subject-specific hierarchical levels (Green, 1982), or through particular strategies in comparison of odds (Falk et al., 1980). The survey of Green is particularly relevant to this paper. Green's analysis parallelled that of [illegible] MS studies of the 1970s which assigned children's responses to academic prob[illegible]ems into a hierarchy of levels of difficulty. There was some diagnosis of errors in these studies, though little in the case of the probability work, i.e. in Green. Thus, whereas inferences can be made about the roots of the child's construction of the concept of decimals, fraction and ratio, and the logic of the misconceptions caused as being intelligent extensions of correct conceptions of whole number, the same cannot be said of the errors children make in Green's items. What is the source of the child's probabilistic thinking, and why do they make these errors?

Recent research has targeted informal probabilistic thinking: applying heuristics (Kahnemann et al., 1982; Lecoutre, 1992); using intuitions (Fischbein, 1975, 1987); operating with the gist of numbers (Reyna & Brainerd, in press). Finally, Konold (1989, 1991) and others investigated the individual's goals when approaching a problem, which could be decision orientated, leading to the 'outcome approach', rather than frequentist, leading to a probabilistic approach. Briefly, (we will expand on the terms below, since our research uses many items drawn from this literature) the literature points to notions of everyday cognition being structured around certain intuitions, mental models and heuristics which may or may not be conscious. These are presumably constructed by the individual through experience of everyday problems involving uncertainty which require a decision to be made, and the individual forms intuitive ideas of likelihood to cope with this. Of course, as with our intuitive construction of frameworks and models of everyday motion, such as the Aristotelian conception of force, the models we have of stochastic situations are at variance with the standard, normative or academic theory of probability. The genesis of this cognition would make a fascinating ethnomathematical study beyond our scope, though some of the interview results in this paper point to the significance of events such as games of chance in structuring the cognition.

The terms 'intuition', 'heuristic', 'approach', 'bias', etc. have a close, sometimes overlapping, meaning in their use within research. We will be using the term 'heuristics' for 'rules of thumb' used as part of 'informal thinking' (i.e. not school taught), which sometimes lead to correct answers, but sometimes lead to 'biases'. 'Outcome approach' (Konold, 1989) versus 'frequentist approach' suggests a choice of framework before proceeding with the solving. All these reasoning processes can be (but don't have to be) characterised by "Self-evidence... Intrinsic certainty... Perseverance... Globality... Implicitness... " (Fischbein, 1987), i.e. be 'intuitive'.

The literature on the heuristics of availability, representativeness and equiprobability , and the work on the outcome approach formed a starting point for our research, the questions used in interviews and the items devised for the questionnaire in multiple choice format. It is significant that most previous work took place with older students, that they were based on generally small numbers in interview situations. (Large numbers in Lecoutre et al's work being counterbalanced by a small number of items.) The move to a questionnaire format posed the problem of interpretation.

Communication between researcher and subject has become an important issue in probability. Borovcnik & Bentz, (1991) show how interpretations by Piaget & Inhelder, (1975), or Green (1982) of results with children can go wrong. Results from research with adults, and especially the items and methodology employed (c.f. Konold, 1989; Kahnemann et al., 1982) must certainly be used in research with children cautiously.

It is characteristic of the research in this field to be atomistic: each researcher being concerned with the identification of a particular pattern of thinking in a certain situation. It is an open question therefore whether there is an overlap of the heuristics so far identified, whether they are indeed even commonly used by school children and how far they account for misconceptions or errors in thinking in relevant academic settings and contexts.

The research question here therefore can be defined in terms of the problems:

1. What do children understand about 'chance' when they begin secondary school?

2. How common and how influential is the use of informal heuristics, approaches and biases in their thinking about probability in the school context?

METHODOLOGY

In studying the children's informal understanding of the term 'chance', the interviews sought children's stories of chance events; explanations and elaborations were encouraged and these interviews then provided a collection of case studies. The importance of beliefs, experiences and informal use of language emerged strongly from some of these interviews, and encouraged us to devise some new instruments for the questionnaire stage to attempt to measure these:

1. The attribution instrument used a 5 point Likert scale to assess children's strength of attribution of the outcome of an event (there were 8 in all, selected form the literature, e.g. Konold, 1991 on random-attributions by experts and novices, and from children's interview responses) to chance, to luck, to the act of God or to some causal influence (such as skill).

2. The language precision instrument, which measures the ability of a child to associate a probabilistic term to a scale of chance from 0 through 6, 50:50 being half way up the scale, i.e 3, and others being determined according to the normative use by 24 adult graduate mathematicians, (assignments by more than 10% of the graduates were regarded as normative, and a score was calculated as the average numerical distance of the child's assignments from a normative value.)

3. Children were asked to rate how often they experienced certain activities, from games to gambling.

The research reported here attempted to explore the extent to which the above mentioned heuristics. (equiprobability, representativeness and availability) and 'outcome approach' are applicable to the informal reasoning of 11-12 year old children. Items from previous research investigating the application of heuristics, and of the 'outcome approach' by older students and adults were used or adapted, as well as items from Green's survey (1982) of children's probability concepts. Results of Green's items were analysed, re-interpreting certain responses as revealing a bias or an 'outcome approach'. Thus we sought to check whether the heuristics could really account for errors in the Green's items which in some sense represent the academic curriculum for probability. The interview data allowed us to check the interpretations and devise the multiple choice format, while the questionnaire allowed us to explore statistically the robustness of the interpretation for large numbers, construct possible diagnostic scales and look for relationships and patterns in the children's background and probabilistic scores.

The reason for the choice of age group, i.e. 11-12 year old children was that at this age the child's reasoning, language and experience are already rich; the children can fill in a questionnaire, or communicate with an adult in conversation. (In fact we chose to read out the questions to whole classes, thus minimising reading difficulties and making sure all questions were likely to be completed.) Also, there has usually been little formal academic learning of the subject of probability in school, thus making it easier to induce children to express their informal ideas in conversation.

The questionnaire sample had a mean 'probability concepts level' as defined by Green (1982) of 1.11. This is similar to Green's sample, for the same age group which had a mean of 1.14 (N=640). Differences in probabilistic thinking scores by ethnicity and gender, although sometimes reaching significance, were not large. This suggests that the results of this sample may not be untypical of the wider U. K. population. even though its ability was relatively low (verbal ability: 89.2, numerical ability: 90.0, non verbal ability: 94.2; 100 being the U.K. mean).

RESULTS

Language of chance, attributions and experience

The use of the term 'chance' occurred in a number of ways:

(a) Something happens by chance, when it is unplanned, unintended or unpredictable. Often, the event is unexpected, and hence may be connected with rare events, as Konold et al reported, 1991:

> T: *Example of chance? You want the phone to ring and you go upstairs and it starts ringing by chance... You expect something, but it doesn't happen....You're walking.. .. and something bumps into you.... Like- chance is something that happens, but kind of you're not expecting it, but it happens.* (Interview no 27.)

> P: *100 tosses of a coin ? It would be chance to get 100 out of 100, but possible to get 55 on one and 45 on the other.* (Interview no. 35)

(b) 'There is a chance' or 'have a chance' meaning to have an opportunity or possibility of something happening, e.g. when you leave school you have a chance to go to university, a chance to win (interview no. 24), a chance to get the answer right or wrong (interview no 23). Another close meaning was to have a choice "to go there or you don't have to go, its up to you" (interview no 18).

(c) 'More chance, good chance, no chance', meaning an expression of the likelihood of an outcome or event. (Interviews 5, 10 and 20).

(d) 'To take a chance' as in taking a risk. This was rare:

> R: *If you're using some chemicals- its a risk using it, so you take a chance* (Interview no 19)

The children's understanding of luck was usually in the context of the degree of good or bad fortune in the outcome, rather than the conditions of the event, as described by Wagenaar and Keren, 1988. But there are two meanings which could influence the child's thinking about chance: first, the child can interpret chance and luck as the same thing, luck has the same properties of unexpectedness, unplanned outcomes. Second: the child sees luck to be associated strongly with superstition, fortune or fate. Thus some people are luckier than others, so their chances with a dice may be higher than yours or mine:

> H: *If you roll it and get a 4, it is just luck, it's chance......* (interview 32).

The child was also asked to rate their agreement with the various definitions of chance and luck identified in the interviews. The average rating, (the scale from 1 to 5) for the meaning of luck declined from " something good" (4.2) to "something to do with dice, coins, spinners etc." (2.5). With the concept of chance there was a decline from "something uncertain" (3.7) through "gives an opportunity" (3.5) " no pattern" (3.2) and "nothing is chance" (3.2) down to "something to do with dice, coins, spinners etc." (2.5) See Figs.1, 2, below.

Fig.1.
Graph of mean agreement with explanations of 'something lucky' in the questionr aire sample

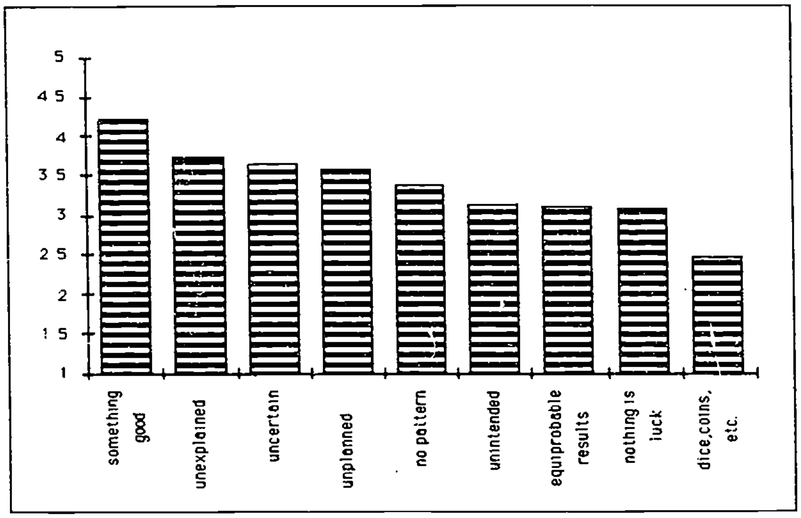
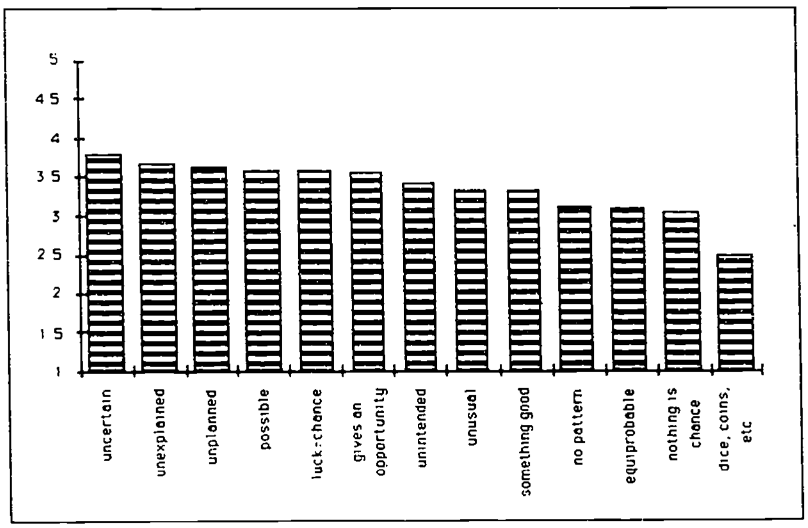


Fig.2.
Graph of mean agreement with explanations of 'something that happens by chance' in the questionnaire sample .



The questionnaire presented terms describing degrees of uncertainty, and pupils had to give their quantitative interpretation of how close the term is to impossible (0) or certain (6). The distance between each response and the normative answer was calculated, and the mean of these distances referred to as the PROBABILITY LANGUAGE PRECISION (PLP) score. The overall mean for the pupils was 1.29, with a standard deviation of 0.76. The biggest mean distances (i.e. wrong answers) were for 'sure to happen' and 'always happens'. The highest number of missing answers, probably because of not knowing the meaning of the word 'seldom', was for the term 'seldom happens'. The "best" results were for 'probable' and for 'even chance'. There are perhaps some lessons here for the probabilistic terms that teachers can expect the children to understand appropriately and the terms which should be taught or avoided.

The child's understanding of chance (or luck) can further be probed by asking them to consider the influences on events, such as the throw of a die, car accidents, the weather etc.

T: *They happen by chance. You don't take care, you just walk and it happens. ...you're going to get hit.*

Int: So if you don't take care, and it happens: it's by chance?

T: *Yes.*

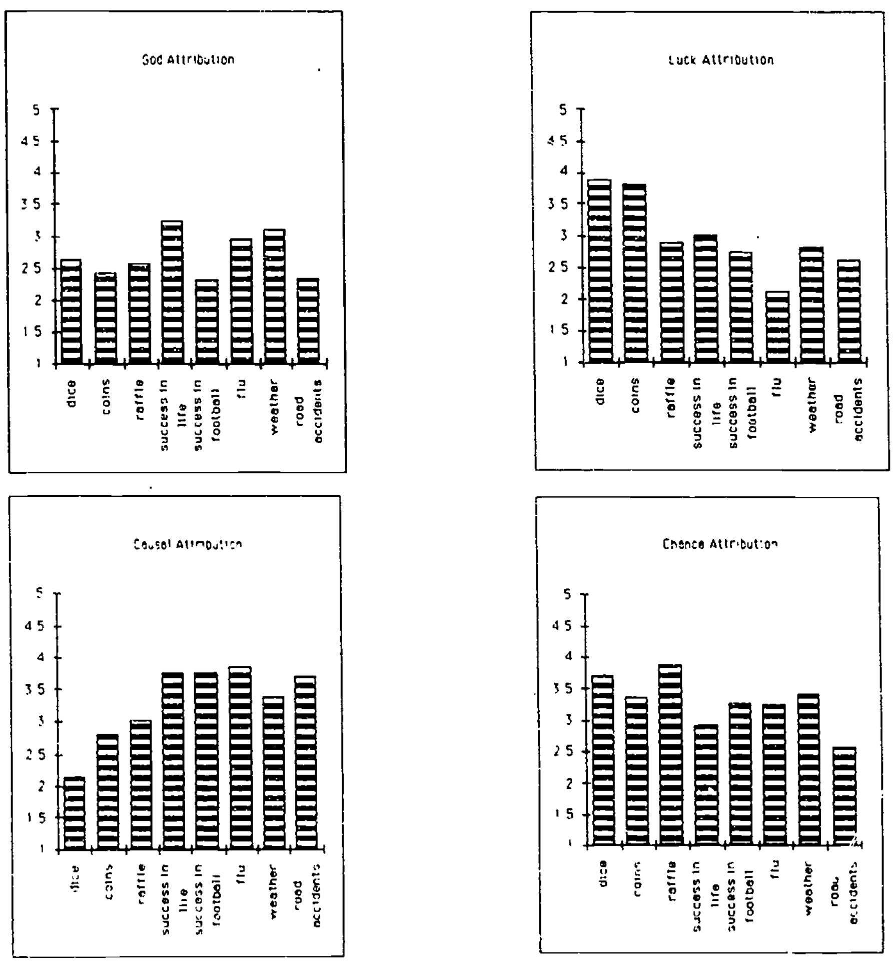
But several interviews saw the intervention of other factors, such as God as the important influence:

Int.: ... a road accident, was it meant by God to happen?

R: *Yes. Even though it seems bad, in some way it might be for the best. Something might come out that is good, the driver was doing something wrong, it'll teach him to put that right, so he won't have an accident.* (Interview 31.)

In other interviews children rejected the role of chance and came up with causal explanations such as lack of skill or care of the driver. This motivated the construction of an attribution instrument, drawing on techniques from attribution theory, and some items used in the literature to identify the strength of religious belief and superstition of the child.

Fig.3.
Graphs of mean agreement with the attribution statements in the questionnaire sample



Finally, it should be noted that the high attribution of non-chance influences to the standard normative models for introducing stochastics to children, i.e. dice, coins etc. are related to some extent to the religious and superstitious background of the children. A factor analysis of the background attitudinal items and the attribution items taken together led to two clear factors emerging: the first includes all the attribution to God and strength of religious background items, and the second includes most of the superstition-belief items with some of the attribution to luck items:

Factor I from the factor analysis of the beliefs and attributions questions: GOD ATTRIBUTION AND RELIGIOUSNESS

	Item	*Loading*
Part 2		
6.	If it will rain or not in Manchester a week from today depends on God's will.	0.74
9.	Getting a '6' on a normal dice does **not** depend on God. *	0.54
12.	Your success in life depends on what God decided that your life will be like.	0.77
14.	Winning a raffle depends on God's will.	0.73
15.	When a road accident happens, it was meant to happen by God, for some reason that we cannot understand.	0.75
19.	Getting 'Tails' when tossing a coin depends on God.	0.78
30.	If you get the flu next month does **not** depend on God. *	0.55
31.	Winning in a football game depends on God.	0.75
Part 4		
2.	Frequency of visits to church / mosque / synagogue / etc.	0.73
3.	Certainty in the existence of God.	0.71
4.	Belief that God knows everything.	0.77
5.	Belief that God controls everything.	0.82

* Due to the negative wording of this item its results were reversed.

Factor 2 from the factor analysis of beliefs and attributions questions: LUCK ATTRIBUTION AND SUPERSTITIOUSNESS

	Item	*Loading*
Part 2		
3.	Your success in life depends on luck.	0.55
17.	Getting 'Tails' when tossing a coin depends on how lucky you are.	0.45
Part 4		
6.	It is unlucky to walk under a ladder.	0.76
7.	It is unlucky to break a mirror.	0.76
8.	Do you have a lucky number?	0.47
9.	Do you have an unlucky number?	0.60
10b.	Are the predictions of the horoscope correct?	0.47
Part 5		
10.	Some people are luckier than others in raffles, etc. *	0.59

* Due to the negative wording of this item its results were reversed.

Availability

The 'availability' heuristic (see Kahnemann et al., 1982) bases probability judgments on relevant instances or occurrences that can be brought to mind or recalled from memory. This method is often useful and correct, but can also lead to biases. For example, given a list of relatively more famous men and less famous women (and vice versa), their subjects judged erroneously that there were more men in the list. Again, people judge erroneously that there are more words that start with an 'r' than words that have 'r' as their third letter. This is because it is easier to recall words according to their first letter than to do so according to their third letter.

In this research 'availability' was used to interpret responses to the following item, from Green's survey (1982):

> "When an ordinary 6 sided dice is thrown which number or numbers is it hardest to throw, or are they all the same?" Source: Green (1982)

Several of the interviewed children remembered from their experience with board games waiting a long time for a '6' on the dice, often needed when getting started with a game. This made them conclude that '6' is harder to get than other scores (16 pupils, out of 38 with whom dice were discussed in the interviews). For example:

Note: 'I' stands for Interviewer, other letters are the initials of the interviewed children.

C.: *6' is harder. 'Cause every time when you throw a dice, it always runs on a '2', or a '1', or a '3', or a '5'. When you want it to land on a '6' it will never land, but when you don't want it, it will come.*

I.: You remember that from games?

C.: *Yes, snakes and ladders, monopoly. Every time when you want a '6' to get out, it never does it to you. But with the other people, they always get '6'-es. But when it comes to my turn, I never get them.* (INTERVIEW NO 11)

Sometimes this view does not emerge immediately, but only after some probing. Although an alternative interpretation exists, that the children see '6' as less likely because they are comparing 'getting 6' with 'not getting 6', this did not arise in the interviews.

In the questionnaires the same question was used, and results of this sample were compared with the same age group in Green's sample:

Table 1
Distribution of responses to QUESTION 1 in the questionnaire sample and in Green's sample

	This sample (N=236)	Green's sample (N=640)
No answer	2 (1%)	
All numbers are the same (correct answer)	151 (64%)	67%
6 is hardest to get *	45 (19%)	23%
6 and other numbers are hardest to get *	14 (6%)	2%
A number other than 6 is hardest to get	10 (4%)	1%
Numbers not including 6 are hardest to get	3 (1%)	
Other answer	11 (5%)	

*responses interpreted as revealing 'availability'

As can be observed, a similar percentage of pupils in Green's sample and in this sample (25%) thought that when throwing a dice '6' is harder to get. The interviews support the interpretation of these responses as reflecting the use of the 'availability' heuristic, children basing their generalisation on memories of long waiting for '6' in board games.

Other examples of availability influencing cognitive beliefs include:

(a) a belief that some people (e.g. elder brothers, sister, parents) win at games of chance because they are better at throwing coins or dice, and

(b) the belief by one boy that he is a skilled coin-flicker, justifying his place in the team as a consequence.

Representativeness

Kahnemann et al.'s (1982) explanation of 'representativeness' is that if A resembles B, then the probability that A originates from B is considered high. With this heuristic adults tend to assess the likelihood of a sample result by the similarity of this result to the corresponding population, without taking into account sample size. They tend to expect that a randomly generated sequence of events will represent the essential characteristics of that process even when the sequence is short. For example, when tossing a coin for heads or tails they will regard the sequence HTHTTH as more likely than (a) HHHTTT, or (b) HHHHTH, (a) being too orderly, and (b) including too many Heads.

'Representativeness' also accounts for the well known 'gambler's fallacy': believing in chance as a self-correcting process. After a long run of reds on the roulette wheel, for example, many people believe that black is now due. These kinds of misconceptions of chance are widespread among adults, and even highly educated adults (ibid).

In this research responses consistent with use of the 'representativeness' heuristic appeared mainly in the context of coins. For example, the following question is from Green (1982):

QUESTION 2

'An ordinary coin is tossed <u>five times</u> and 'heads' appears <u>every time</u>. Tick the correct sentence below:

__(A) Next time the coin is more likely to turn up 'heads' again.
__(B) Next time the coin is more likely to turn up 'tails'.
__(C) Next time 'heads' is as likely as 'tails'.
__(D) Don't know.'

In the interviews several of the children thought 'tails' is more likely, explaining, for example:

I.: An ordinary coin is tossed five times and heads appears every time. Next time you toss the coin is it more likely to turn up heads again, or tails, or just as likely?

K.: *It's much more likely to land tails now. Because it's been landing so many times on there.* (INTERVIEW NO. 7)

I.: An ordinary coin is tossed five times and heads appears every time. Next time you toss the coin is it more likely to turn up heads again, or tails, or just as likely?

W.: *Tails*

I.: Why?

W.: *Probably it will be heads, tails, heads, tails, heads, tails, and keep going like that...* (INTERVIEW NO 9)

This question was also used in the questionnaire.

Table 2
Distribution of responses to QUESTION 2 in the questionnaire sample and in Green's sample

	This sample	Green's sample
Next time the coin is more likely to turn up 'heads' again	23 (10%)	14%
Next time the coin is more likely to turn up 'tails'. †	19 (8%)	14%
Next time 'heads' is as likely as 'tails'. *	194 (82%)	67%
Total	236 (100%)	95%**

* Response considered correct
† Response considered using 'representativeness'
** The rest of the respondents answered "I don't know".

Another question from Green's survey used in this research was:

QUESTION 3

A 1p and a 10p coin are tossed together.	1p coin	10p coin
One possible result, Heads on the 1p coin	H	T
and Tails on the 10p coin, has already been		
put in the table: H for heads and T for tails.		
Write in all the other possible results.		

Source: Green (1982)

In the interviews several pupils found only one other possibility, 'tails' on the 1p and 'heads' on the 10p. For example:

I.: A 1p coin and a 10p coin are tossed together. One possible result is heads on the 1p and tails on the 10p. What are the other possibilities?

M.: *Could fall in the opposite sides: 'tails' on the 1p, 'heads' on the 10p.*

I.: Any other possibilities?

M.: *... No.*

This tendency to see only possibilities in which one coin shows 'heads' and the other 'tails' is interpreted as use of 'representativeness'. Another type of answer was: finding all combinations, but thinking their probabilities are not equal:

I.: A 1p coin and a 10p coin are tossed together. One possible result is heads on the 1p and tails on the 10p. What are the other possibilities?

W.: *TT, HH, TH.*

I.: Which results are more likely, or are they all just as likely?

W.: *HT, TH are more probable. Probably, if one's going to land on H the other will land on tails. HH will happen about 1 in 10; TT about 1 in 10.* (INTERVIEW NO 9)

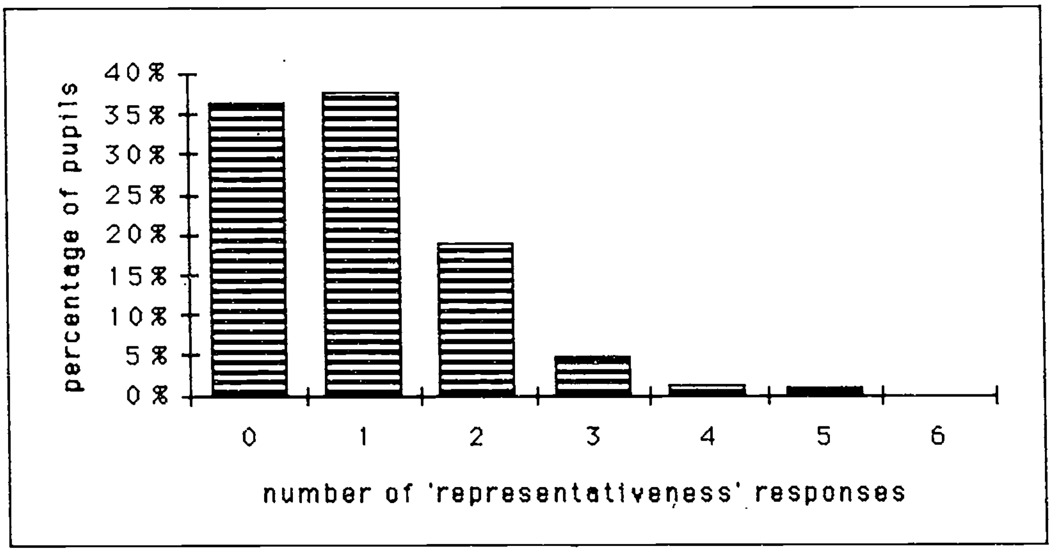
The original question was also used in the questionnaires. The results are as follows:

Table 3
Distribution of responses to QUESTION 3 in the questionnaire sample and in Green's sample

	This sample	Green's sample
No answer	2 (1%)	
Correct	125 (53%)	49%
One possibility missing	9 (4%)	}42%
Only 'HT' and 'TH'	95 (41%)	}
Other	5 (2%)	
Total	236 (100%)	

The analysis of responses to this question was more detailed in this research than in Green's: here we classified the kind of mistake. Almost half the pupils mentioned only two possibilities, of 'tails' on one coin and 'heads' on the other, probably applying the 'representativeness' heuristic, as evidenced in the interviews. Altogether 6 error-responses in the questionnaire were analysed as revealing 'representativeness', and the number of these were referred to as the REPRESENTATIVENESS score (range 0-6). A reliability test of this score (the Guttman split-half procedure, SPSS, 1990) resulted in a coefficient of 0.43. The following graph shows the distribution of this score in the questionnaire sample:

Figure 4
Graph of percentages of the frequencies of the REPRESENTATIVENESS score in the questionnaire sample



Mean= 0.99 Standard deviation=1

As can be observed in the graph, representativeness was applied in at least two cases (1/3 of the questions) by over a quarter (25.8%) of the pupils.

Equiprobability bias

Lecoutre (1992) added to the list of heuristics identified in probabilistic thinking what he called the 'equiprobability bias': the tendency to treat chance outcomes as equiprobable by nature. He interviewed adults with problems such as comparing the likelihood of getting a 5 and a 6 versus two 6-es, when simultaneously throwing two dice (ibid, p. 557). He tested students with various backgrounds in probability theory, and about 50% of them answered incorrectly that both possibilities were equally likely. The most frequent line of argument found was: "The two results to compare are equiprobable because it's a matter of chance" (ibid, p. 561) This model accounted for more than 65% of the equiprobability responses. According to this model "random events should be equiprobable by nature." (ibid, p. 561) This is referred to as the 'equiprobability bias'.

Some children in this research's interviews seemed to be using this heuristic. An example from the interviews:

I.: What do we mean when we speak of 'things happening by chance' in our lives?

K.: *You've got a chance of getting it and a chance of not getting it. Chances of heads and tails on a coin: you've got even chance to get heads and even chance to get tails.*

I.: Do road accidents happen by chance?

K.: *You've got a 50-50 chance and not chance of it, because there's the chances that somebody will crash into you, and you've got the chance that you'll crash into them, half-half.*

Or in another interview:

I.: What is the chance of getting a '4' on a 10-sided die?

J.: *Probable.*

I.: Could you give me the chances in a number form?

J.: *50-50. Even chance of getting a '6' or a '4' or a '3'.* *

I.: What is the chance of getting a number bigger than '6'?

J.: *Probable. 50-50.*

I.: And of getting an even number?

J.: *Probable. 50-50.*

J. has a fixed answer about chance, 'probable', which might mean 'possible', and '50-50', which seems to be a natural property of chance: the asterisked explanation shows that J. has an image of chance strongly related to several equiprobable possibilities, and the term 50-50 is used to express this.

Equiprobability was also common in the following item from Green (1982):

QUESTION 4

A mathematics class has 13 boys and 16 girls in it. The teacher does a raffle. Each pupil's name is written on a slip of paper. All the slips are put in a hat. The teacher picks out one slip without looking. Tick the correct sentence :

__(A) The name is more likely to be a boy than a girl.
__(B) The name is more likely to be a girl than a boy.
__(C) It is just as likely to be a girl as a boy.
__(D) Don't know.

Source: Green (1982)

In the interviews 10 out of 23 pupils answered that to pick a boy was just as likely as a girl. Some of the explanations seemed to reflect a 'natural' implication that chance is normally equiprobable, although this was mostly not explicit. For example:

P.: *It is just as likely to be a girl as a boy.*

I.: Why?

P.: *Because it's just... if you just put your hand in it and you pull one out, it doesn't matter if there's more girls than boys.* (INTERVIEW 21)

This question was used in the questionnaire:

Table 4
Distribution of responses to QUESTION 4 in the questionnaire sample and in Green's sample

Response	This sample	Green's sample
No answer	1 (.4%)	
(A) The name is more likely to be a boy than a girl.	10 (4%)	5%
(B) The name is more likely to be a girl than a boy.*	115 (49%)	38%
(C) It is just as likely to be a girl as a boy. †	110 (47%)	53%
Total 236 (100%)		96% **

* Answer considered correct
† Answer considered using 'equiprobability'
** The rest of the respondents answered 'Don't know'.

Answer (C) could be the result of applying the 'equiprobability bias', i.e. thinking that chance by nature gives equal results. But it could also be the result of an 'outcome approach', i.e., thinking that as the situation is unclear no 'decision' can be made about the result. A second part of the question was added to Green's question, in order to check how consistent is the pupil's tendency to choose the 'equal' answer:

Table 5
Distribution of responses to QUESTION 5 in the questionnaire sample:

QUESTION 5
What if the mathematics class had 15 girls and 5 boys:

Response		Frequency	Percentage
(A)	The name is more likely to be a boy than a girl.	11	4.7%
(B)	The name is more likely to be a girl than a boy *	199	84.3%
(C)	It is just as likely to be a girl as a boy. †	26	11.0%
Total		236	100.0%

* Answer considered correct
† Answer considered using 'equiprobability'

A cross tabulation of responses to the two parts of the question showed the number of pupils that are consistently equiprobable:

Table 6
Cross tabulation of responses to QUESTIONS 4 and 5 in the questionnaire sample

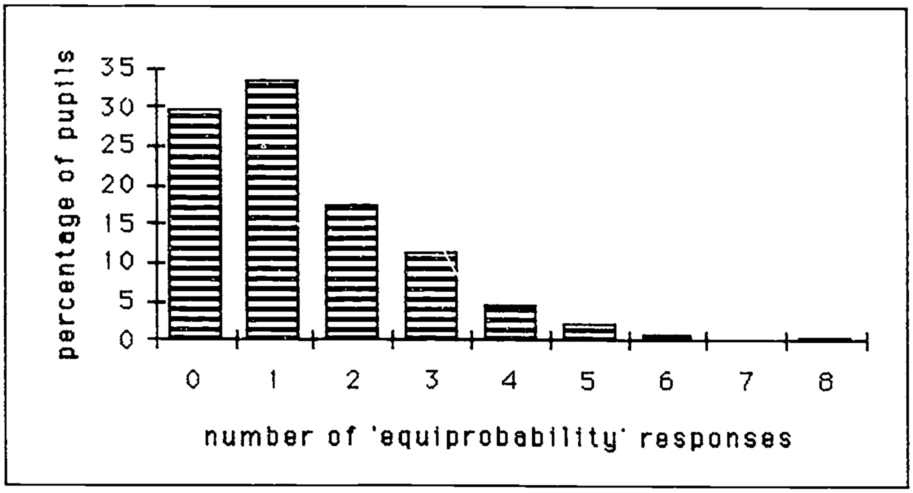
	5	(A)	(B)	(C)
4	(A)	1	8	1
	(B)	2	110	3
	(C)	8	80	22

The biggest number of pupils got both questions right. But a considerable number (80, 33.9%) moved from 'equal chances' in 4 to 'a girl' in 5. This can be explained by the 'outcome approach' (a decision about the outcome being more clear cut in 5 than in 4).

22 pupils (9.3%) remained with the answer 'equal chances' in both questions. We interpret this as revealing an 'equiprobability bias'.

With the limitations in the interpretations and analysis in mind, incorrect equiprobability answers, or explanations that seem to be revealing the 'equiprobability bias' were counted, forming the EQUIPROBABILITY score, with a range from 0 to 8, and a reliability coefficient of 0.39. The following graph summarises these results for the questionnaire sample:

Figure 5
Graph of percentages of frequencies of the EQUIPROBABILITY score in the questionnaire sample



Mean = 1.40 Standard deviation =1.40

19.4% of the pupils answered 3 or more of the relevant questions (i.e. over 1/3 of the questions) in this way.

The 'outcome approach'

Konold (1989) suggests a third framework of probabilistic reasoning, in addition to the 'formal' framework and the 'intuitive' framework: errors in reasoning under uncertainty arise also from a different understanding of the goal of this reasoning. Following observations and interviews Konold formulated a model referred to as the 'outcome approach', which some of the participants adopted in different degrees. According to this model, the goal in dealing with uncertainty is 'operative', i.e. to predict the outcome of a single next trial. The tendency when using this mode of reasoning is to evaluate probabilistic estimates as correct or incorrect after one trial. They often base their predictions on causal analysis of the situation. Typically, assigned probabilities serve as modifiers of the yes-no prediction, with 50% meaning that no sensible prediction can be made. The outcome-oriented individual undervalues frequency information.

In this research Konold's 'weather problem' and 'painted die problem' (1989) were adopted. The 'outcome approach' was identified to some extent in most of the interviews. Of the 16 children that were asked about the 'weather problem', dealing with the interpretation of percentages in weather forecasts, 3 gave answers all consistent with the 'outcome approach', 11 gave some answers consistent with this approach, and only 2 gave answers all of which were not consistent with this approach. An example of an answer analysed as fully consistent:

I.: If a weather forecaster says that tomorrow there is a 70% chance of rain, what does he mean?

N.: *It's gonna rain. I think the weather people are good. I watch T.V. for football. They say "it's gonna be rain, windy.", and it happens. ...*

I.: How do they get that number?

N.: *Satellite... Clouds.*

I.: Suppose the forecaster said there was a 70% chance of rain tomorrow and, in fact, it didn't rain. What would you say about the statement that there was a 70% chance of rain?

N.: *They might have got something wrong. Clouds moved somewhere else.*

I.: Suppose there were 10 days for which they said 70% chance of rain. On 3 of those 10 days there was no rain. What would you say about this forecaster?

N.: *I think they're good. We're not all perfect.*

I.: If the forecaster had been perfectly accurate, what should have happened?

N.: *All 10 days would be raining.* (INTERVIEW NO. 23)

The same interviewee reveals the 'outcome approach' also in the 'painted die problem':

I.: Now, let's look at this fair dice. I painted five of the surfaces black and one white. If I roll it 6 times, would I be more likely to get 6 blacks or 5 blacks and 1 white?

N.: *If the painting is there ... sort of like a little knot, I would expect it to fall on black every time. But if it's like normal painting... same chance of getting black and same chance of getting white.*

In this answer N. first considers the possibility that colour has added weight. The 'outcome oriented' individual, according to Konold, typically searches for causality, and in this case expects, as in this interview, six rolls of black. If the dice is normally painted, N. speaks about 'same chance'. This could still be interpreted following Konold as meaning not a quantitative estimate, but an indication that black is possible, white is possible, thus an 'outcome decision' is not possible. N. seems to think that when he is comparing the probability of A versus B there are three possibilities: either A, or B, or 'even chance' ,which means both are possible, and so nothing can be said.

Another example of equating 'not knowing' with 'equal chances', typical of the 'outcome approach', is in interview No.13:

I.: 12 coins are tossed up in the air together, and land on the table. If this is repeated a lot of times, which of the following will happen most often: 2 heads and 10 tails; 5 heads and 7 tails; 6 heads and 6 tails; 7 heads and 5 tails; all have the same chance.

P. : *All have the same chance.*

I.: Why?

P.: *Because you can't tell.*

The 'weather problem' and the 'painted die' problem were also used in the questionnaires. Here they were changed into 6 multi-choice items, where some of the answers for choice were typical 'outcome approach' responses taken from Konold's interviews. For example:

Table 7
Distribution of responses to QUESTION 6a in the questionnaire sample

QUESTION 6a
Suppose the forecaster said there was a 70% chance of rain tomorrow and, in fact, tomorrow came and it didn't rain. What would you say about his forecast? Tick the answer you most agree with:

Response		Frequency	Percentage
(A)	He was wrong. †	44	18.6%
(B)	There was a mistake in the information he got from the satellite. †	64	27.1%
(C)	His forecast could still be O. K., because he did not say definitely it will rain tomorrow. *	128	54.2%
Total		236	100.0%

* Answer considered normative
† Answer considered revealing the 'outcome approach'

Answers (A) and (B) are considered as revealing an 'outcome approach'. So, in this item 108 pupils (46%) revealed this approach.

Table 8
Distribution of responses to QUESTION 6b in the questionnaire sample

QUESTION 6b

If the forecaster had been perfectly accurate, what should have happened? Tick the one answer you most agree with:

Response		Frequency	Percentage
(A)	Rain on 7 days, no rain on 3 days. *	89	37.7%
(B)	Rain on all 10 days.†	47	19.9%
(C)	Rain on almost all of the days(8 or 9 days). †	28	11.9%
(D)	Rain on half the days. **	72	30.5%
Total		236	100.0%

* Answer considered normative
† Answers considered revealing the 'outcome approach'
** Answer considered revealing 'equiprobability'

In this question (B) is a more extreme 'outcome approach' answer, (C) is a less extreme 'outcome approach' answer. (D) is neither, and might be reflecting an 'equiprobability bias', in that uncertainty is equated with equal chances of the possible outcomes (this response was added to the EQUIPROBABILITY score). In percentages, 32% of pupils answered according to the 'outcome approach', 30% according to the 'equiprobability bias', and 38% normatively.

This 'weather problem' was used by Konold in 16 interviews with adults, whereas in this research it was used as multi choice items with children; so comparison of results must be treated with caution. In his analysis of responses to the weather problem (see Konold, 1989, table 2), 4 students (25%) expected that it will rain, compared to 50% in this sample; 6 students (37%) thought after it did not rain that the prediction was wrong, compared with 46% in this sample; the rest is not directly comparable with Konold's analysis of responses. It can be summarised that a large part of the children in this research revealed the 'outcome approach' in the weather context, 45% being the mean of the 4 parts.

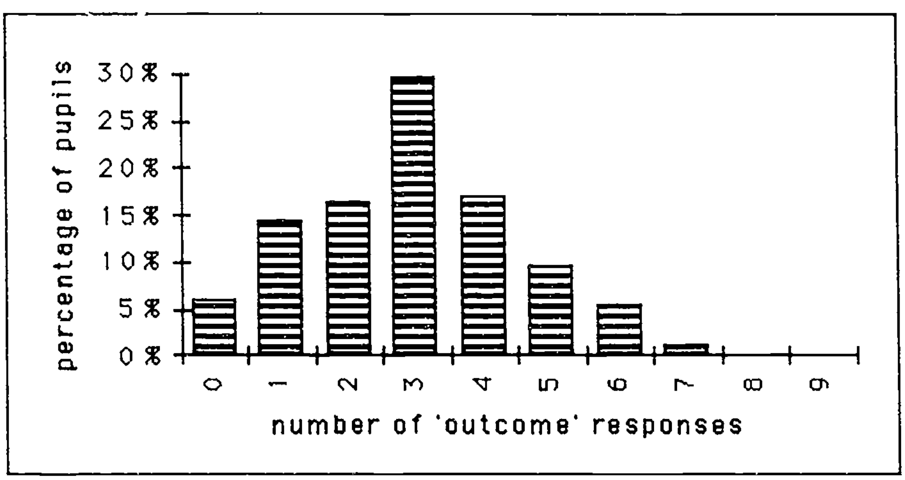
In question 5, dealing with a 'painted die', a lower percentage of pupils, 23-4% revealed this approach. A possible reason for this mean being considerably lower than that in the 'weather problem' is that the weather problem might be seen as much more realistic, and perhaps therefore evokes a decision type, non-frequentist answer, compared with the 'painted dice problem'. This could be an example of the influence of **context** on the extent to which the 'outcome approach' is applied.

Apart from Konold's 'weather problem' and 'painted die problem', three other responses for questions from Green's survey were also analysed as reflecting the 'outcome approach. The total number of such responses will be referred to as the OUTCOME score, having possible values of between 0 and 9.

Testing the OUTCOME score for reliability gave the low result of -0.22. This result in fact indicates that part of the items in this score are negatively correlated. Even when examining separately only the 6 items that are based on Konold's questions (part 8, questions 1 and 2) the resultant reliability coefficient was -.03. These results indicate that there are problems in the attempt to measure the degree of application of the 'outcome approach' in the way it was done in this research. Perhaps this problem is a result of the transition from interview setting to multiple-choice questionnaire setting. Another explanation could be that the 'outcome approach' is applied inconsistently because of the different contexts, as was suggested earlier. Perhaps the use of the 'outcome approach' is triggered more by context and wording than by individual orientation. Or perhaps children are more fickle, i.e. are more likely to be influenced by context than adults. All this demands further research.

Keeping the limitation of the low reliability found for this score in mind, the following graph shows the distribution of the OUTCOME score in the questionnaire sample:

Figure 6
Graph of percentages of the frequencies of the OUTCOME score in the questionnaire sample



Mean = 2.95 Standard deviation =1.6.

As can be observed in the graph, the 'outcome approach' was applied to a wide extent. 63.1% of the pupils gave at least 3 such responses (1/3 of the questions).

CONCLUSIONS

1. Children have a variety of understandings of the term chance, which to some extent overlaps with luck. Their everyday use of the term is sometimes at variance with randomness. Their use of language of probabilistic terms is sometimes low, and some terms are better understood than others.

2. Children's beliefs, both religious and superstitious, affect their understanding of chance, and stochastic situations such as dice, coins etc., e.g. 72% of the children believed that some people are luckier than others in games of chance. 43% of the children thought that their preference for Heads (or tails) really made a difference. 35% believed the outcome of the dice throw depends on how you throw it and 20% by God's will.

3. We found the use of common heuristics such as representativeness, (e.g. the gambler's fallacy), equiprobability and the outcome approach (e.g. in the weather problem), and explained errors in Green's items from the same perspective: there were two reasonably reliable scales for the equiprobability and representativeness, but the outcome approach responses were unreliable. Even the subset of items related directly to Konold's previous findings gave unreliability.

   The interpretation of some responses, i.e. where the child affirms the chances are equal because there are two possible outcomes/ events, could be either 'equiprobability' or 'outcome approach'. The theoretical distinction between the two appears to overlap in this case.

DISCUSSION

The children's use of language, understanding of various probability terms and beliefs about the influence of chance in the world reveal some significant confusions.

'Chance' is a concept with varied meaning, only one of which contains associations of randomness. The attribution of chance to some events is strongly conditional on other influences; teachers must especially note the situation with dice and coins! The conflicting and overlapping understanding of chance and luck may be important and related to children's belief in the possibility of personal or divine intervention in stochastic processes.

Heuristics seemed to be helpful in interpreting responses both in the interviews and in the questionnaires, although in the latter case interpretation was more difficult, some responses leading to more than one possible interpretation.

25% of the questionnaire sample believed that '6' is harder to get than other numbers when throwing a die. Responses in several interviews supported the interpretation that children were generalising from memories they had of waiting for a '6' in games they played. This is a clear cultural link: the child's experience with board games using dice in which '6' has a special role leads to a certain view of the likelihoods of the numbers on the dice. In societies with different kinds of games different results might be expected, and the influence of teaching could hold out hopes of modifying this belief.

Some of the interviewed pupils' responses were consistent with an application of 'representativeness'. This sometimes meant expecting even small samples to reflect all possibilities of the parent population, for example in expecting that when two coins are tossed it is likely that one will show 'heads' and the other 'tails'. Another type of 'representativeness' is expecting a sample not to be too orderly, for example in thinking that when having 4 children the most likely is to have 3 boys and 1 girl, and vice versa. In this case 'representativeness' has led to the correct answer, but formal calculation of probabilities is clearly beyond the children's knowledge in this case. The interviews gave support to the interpretation of this type of response as 'representativeness'. 26% of the questionnaire sample revealed this bias in at least one third of the relevant questions.

Some responses in the interviews were consistent with the 'equiprobability bias', in their tendency to regard chance as intrinsically associated with equal outcomes. 19% of the pupils in the questionnaire sample answered at least one third of the relevant questions in this way. The 'equiprobability bias' might also be culturally influenced: most games and random devices common in the west are based on equal chances for the possible outcomes. It would be interesting to check how common is the 'equiprobability bias' in a society with different types of games, for example where 'cowrie shells' are used for games of chance in Africa, that have unequal chances of falling on either side (see Zaslavsky, 1973).

Using Konold's 'outcome approach' (1989) as a new perspective for analysing children's understanding of probabilistic situations, the interviews and the questionnaire results showed a frequent application of this approach. The year group in focus seem to use this approach quite frequently, although not necessarily systematically across all problems (e.g. the tendency to use it in the weather problem was considerably higher than the tendency to use it in a problem with dice). Context seemed to be relevant to the degree of application of the 'outcome approach', resulting in a negative reliability coefficient for the outcome score.

The emphasis on the outcome of an event seems to be related to causation, and yet our inability to construct a reliable scale for outcome approach <u>or</u> for causality of attribution suggest that both may be strongly influenced by context. And our understanding supports such a theoretical view: that the situation presented largely determines a normative classification of the problem (social, scientific or academic), that this determines also the kind of goal to be adopted and so the approach to the task.

Teachers and researchers might learn to recognise the heuristics and approaches that are commonly applied by children: 'representativeness', 'availability' and 'equiprobability', and the 'outcome approach'. This could be a constructive tool in the analysis of children's responses, and might form a basis for diagnosing and correcting misconceptions. It was sometimes hard to distinguish between the 'outcome approach' and the 'equiprobability bias' when interpreting responses. The relationship between the situation, the chosen approach to a problem and the heuristics adopted is not yet clear, and certainly seems worthy of further study.

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APPENDIX: LIST OF RESPONSES INTERPRETED AS 'REPRESENTATIVENESS' AND USED IN THE SCORE

Responses interpreted as 'representativeness' are either marked by an asterisk or specified.

Part 1

3. An ordinary coin is tossed <u>five times</u> and 'Heads' appears <u>every time</u>. Tick the correct sentence below :
   ___(A) Next time the coin is more likely to turn up 'Heads' again.
   ___(B) Next time the coin is more likely to turn up 'Tails'. *
   ___(C) Next time 'Heads' is as likely as 'Tails'

10. A 2p and a 10p coin are tossed together. One possible result, Heads on the 2p coin and Tails on the 10p coin, has already been put in the table: H for heads and T for tails. Write in all the other possible results.

2p coin	10p coin
H	T
___	___
___	___
___	___

Response analysed as 'representativeness': H T or T H.

11. 4 red marbles, 4 blue marbles and 2 green marbles are put into a bag which is then shaken. Three marbles are picked out without looking - 2 red and 1 blue. Then <u>one more marble</u> is picked out. Which colour is it most likely to be?
   ___(a) Red has the best chance
   ___(b) Blue has the best chance
   ___(c) Green has the best chance*
   ___(d) All colours have the same chance

Explanation for 11 interpreted as 'representativeness': "Red and blue have already been picked, now it's the turn of green".

2. Among families with 4 children, the most common type is:
   (Tick the answer you most agree with)
   ___(A) 4 of the same sex (that is all boys or all girls).
   ___(B) 3 of one sex and 1 of the other (that is 3 boys and 1 girl, or 3 girls and 1 boy). *
   ___(C) 2 boys and 2 girls
   ___(D) all types are equally common

3. Among families with two children, the most common type is:
   (Tick the answer you most agree with)
   ___(A) 1 boy and 1 girl *
   ___(B) 2 of the same sex (2 boys or 2 girls)
   ___(C) the two types are equally common

APPENDIX: LIST OF RESPONSES INTERPRETED AS 'EQUIPROBABILITY' AND USED IN THE SCORE

5a. A mathematics class has 13 boys and 16 girls in it. The teacher does a raffle. Each pupil's name is written on a slip of paper. All the slips are put in a hat. The teacher picks out one slip without looking. Tick the correct sentence :

(A) The name is more likely to be a boy than a girl. ☐

(B) The name is more likely to be a girl than a boy. ☐

(C) It is just as likely to be a girl as a boy. ☐ *

AND

5b. What if the mathematics class had 15 girls and 5 boys:

(A) The name is more likely to be a boy than a girl. ☐

(B) The name is more likely to be a girl than a boy. ☐

(C) It is just as likely to be a girl as a boy. ☐ *

6. Two bags have each got some black counters and some white counters in them.

Bag C : 5 black and 2 white

Bag D : 5 black and 3 white

If one picks a counter out of the bag without looking - which bag (C or D) gives a better chance of picking a <u>black</u> counter or do they give the same chance?

(A) Bag C ☐

(B) Bag D ☐

(C) Same chance ☐

Why?

Response analysed as 'equiprobability': It is the same because it is chance.

8. Two other bags have black and white counters :

Bag G has 12 black counters and 4 white counters.

Bag H has 20 black counters and 10 white counters.

If one picks a counter out of the bag without looking - which bag gives a better chance of picking a <u>black</u> counter?

(A) Same chance ☐ *

(B) Bag G ☐

(C) Bag H ☐

Why?

Response analysed as 'equiprobability': The same because it is chance.

9. Mark and Steven play a dice game.

Mark wins 1 penny if the dice comes 2 or 3 or 4 or 5 or 6.
If it comes up 1 Steven wins some money.
How much should Steven win when he throws a 1 if the game is to be <u>fair</u>?
ANSWER ________ p

Response interpreted as 'equiprobability': 1p.

d. If the forecaster had been perfectly accurate, what should have happened? Tick the one answer you most agree with:
___(A) Rain on 7 days, no rain on 3 days.
___(B) Rain on all 10 days.
___(C) Rain on almost all of the days (8 or 9 days).
___(D) Rain on half the days. *

6. Here are three cards, two with a triangle and one with a square: (Lecoutre's rhombus/ house problem)

With the triangle and the square we can build a house, with two triangles we can build a diamond.

We put the three cards in a box, and draw two cards from the box without looking.
With the two cards drawn, we will be able to build either a house or a diamond .
Do you think there is:
(Tick the answer you most agree with)
___(A) an equal chance of getting a house and a diamond? *
___(B) more chance of getting a house than a diamond?
___(C) more chance of getting a diamond than a house?

APPENDIX: LIST OF RESPONSES INTERPRETED AS 'OUTCOME APPROACH' AND USED IN THE SCORE

5a. A mathematics class has 13 boys and 16 girls in it. The teacher does a raffle. Each pupil's name is written on a slip of paper. All the slips are put in a hat. The teacher picks out one slip without looking. Tick the correct sentence :
(A) The name is more likely to be a boy than a girl. ☐
(B) The name is more likely to be a girl than a boy. ☐
(C) It is just as likely to be a girl as a boy. ☐ *

AND

5b. What if the mathematics class had 15 girls and 5 boys:

(A) The name is more likely to be a boy than a girl. ☐

(B) The name is more likely to be a girl than a boy. ☐

(C) It is just as likely to be a girl as a boy. ☐ *

12. Two discs, one orange and one brown, are marked with numbers. (Green's spinner problem)

Each disc has a pointer which spins round. If you want to get a 1, is one of the discs better than the other, or do they both give the same chance?

___(a) Brown is better for getting a 1

___(b) Orange is better for getting a 1

___(c) Both discs give the same chance

___(d) No one can say*

Why did you choose this answer?

Response interpreted as 'outcome approach': Same chance because you don't know.

1a. What does it mean when a weather forecaster says that tomorrow there is a 70% chance of rain? Tick the answer you most agree with:

___(A) It is likely to rain tomorrow.

___(B) 70% of the sky is covered with clouds, so it will rain tomorrow. *

___(C) It will rain tomorrow, but not very strong rain. *

___(D) The satellite information shows clearly it will rain tomorrow. *

b. Suppose the forecaster said there was a 70% chance of rain tomorrow and, in fact, tomorrow came and it didn't rain. What would you say about his forecast? Tick the answer you most agree with:

___(A) He was wrong. *

___(B) There was a mistake in the information he got from the satellite. *

___(C) His forecast could still be O. K., because he did not say definitely it will rain tomorrow.

c. Suppose you want to find out how good a particular forecaster's predictions are. You observe what happened on 10 days for which a 70% chance of rain has been predicted. On 7 of those days it rained, and on 3 of those 10 days there was no rain. What would you say about the accuracy of this forecaster? Tick the answer you most agree with:

___(A) Not bad, we all make mistakes sometimes. *

___(B) Very good, 7 out of 10 is exactly 70%.

___(C) Medium, could be better. *

d. If the forecaster had been perfectly accurate, what should have happened? Tick the one answer you most agree with:

___(A) Rain on 7 days, no rain on 3 days.

___(B) Rain on all 10 days. *

___(C) Rain on almost all of the days (8 or 9 days). *

___(D) Rain on half the days.

5a) Suppose you have a normal, six sided dice. Five of the surfaces are painted black and one white. If you roll it 6 times, would you be more likely to get 6 blacks, or 5 blacks and 1 white? Tick the answer you most agree with.

____a) more likely to get 6 blacks. *
____b) more likely to get 5 blacks and 1 white
____c) same chance of getting 6 blacks or 5 blacks and 1 white.

b) If you roll it 60 times, how many times would you expect white to turn up?

____a) white would not turn up. *
____b) white would turn up 1 to 7 times. *
____c) white would turn up 8 to 12 times.
____d) white would turn up 13 to 20 times
____e) white would turn up over 20 times.