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ABSTRACT

This booklet is a designed to introduce adult basic education/literacy instructors to a spiral sequencing approach that may be used to teach whole number skills. The booklet begins with a brief discussion of integration of numeracy into literacy instruction. Discussed next are the benefits of using a spiral sequence for numeracy facilitation and basic differences between the proposed spiral approach and the conventional linear approach found in many mathematics textbooks. The booklet's scope and structure are explained, and suggestions for using it are provided. A two-part spiral numeracy sequence, consisting of a 16-step preoperation sequence and a 23-step sequence for introducing whole number operations, is presented in detail. Concluding the booklet are linear sequences for addition, subtraction, multiplication, and division. (MN)



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A Sequencing Guide for Numeracy

Kathleen Lucas Betty Dondertman Tom Ciancone

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Whole numbers

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A Sequencing Guide for Numeracy: Whole Numbers

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About Numeracy

This book is designed as a numeracy sequencing guide to help literacy instructors introduce whole-number skills to their classes. Many literacy instructors recognize that numeracy skills equip learners to cope with important areas of their lives, such as getting a job, assessing information, shopping, and managing money. Even so, a division between mathematics and literacy does exist in many instructors' minds. Because of their own anxiety about math, some literacy instructors are unsure of how to approach math in their classes. When numeracy is integrated into literacy practice, hesitation toward mathematics may be overcome. It may help to think of numeracy as the area where literacy and mathematics overlap. The focus of numeracy is not simply getting the right answers to math questions, but seeing how and when and where it is possible to use math skills in everyday life.

Literacy instructors are ideally suited to facilitate numeracy activities. They know how to integrate real life situations into the content of their teaching, and they have experience in working with students of varying backgrounds and skill levels. The attitudes of the instructor as well as the atmosphere of the classroom can encourage students to address their numeracy needs in the context of their literacy needs. Through participatory activities, instructors and students can become comfortable with numbers. In fact, the experience in facilitating literacy that instructors bring to their numeracy practice is vital in bridging the gap between literacy and mathematics.



About the Spiral Approach

The purpose of this booklet is to provide instructors with an overview of a spiral sequence for numeracy facilitation.

The ordering of math concepts and skills found in adult math books today is linear. The four operations (addition, subtraction, multiplication, division) are separated; each operation is presented on its own. Learners progress through increasingly complex variations of one operation, culminating in the application of this one skill to word problems. Each operation is covered completely before moving on to the next. We have found that this approach has several disadvantages when working with adult literacy students. Proceeding in a linear manner through the four arithmetic operations often results in a limited understanding of mathematical applications. Learners sometimes develop mechanical skills, but lack the ability to recognize the signs of operation, name what they are doing, estimate answers, decide which operation is needed, show their method, or apply the skills to their daily lives. Isolating the operations, without building the connecting links to other operations and applications, permits many gaps in understanding. Even though students may get the answer, they often lack insight into what they have done.

Perceiving these limitations, some facilitators take a "hopping around" approach to mathematics, responding to learners' requests for specific information and math skills to deal with a particular life skill. This thematic or functional approach avoids some of the boredom and futility of working through the operations linearly; however, it has some pitfalls of its own. An instructor can end up in a blind alley. How do you answer the question, "Is a metropass a good buy for you?" without knowing how to multiply first? Mathematical skills acquisition is to a great extent cumulative. The risk with "hopping around" is that gaps in mathematical understanding are easily missed in the absence of a structure. Math skills need to be identified, sequenced, reviewed and applied, if long-term understanding is to be achieved. We propose a spiral approach which provides a strong foundation in math and which also addresses students' more immediate needs.

Although the spiral approach is a departure from the linear approach found in many math text books, the proposed sequence is actually a rearrangement of the usual order of presentation. By incorporating a spiral order to whole-number operations and making each spiral relatively small, the student has the opportunity to develop a basic understanding of all operations before going on to more difficult concepts such as carrying and borrowing. This approach reduces the problem of students



doing operations mechanically without having a fundamental idea of their use. Patterns in numbers and operations are more easily perceived because the students are encouraged to contrast and apply the operations continuously.

There are other advantages to the spiral approach. Students are less likely to feel bored and frustrated with focusing too long on one operation, because they are soon working on a variety of operations. In fact, many adults have some familiarity with different operations. The problems that students encounter daily involve all four operations. In a numeracy/literacy class, continually moving among the four operations reflects reality and makes the mind more alert. The work requires thought; it cannot be done mechanically. This approach leads to an overall understanding rather than simply an answer. The emphasis shifts to the process of problem solving, "the process of applying previously acquired knowledge to new and unfamiliar situations."(from a position paper on basic mathematical skills – National Council of Supervisors of Mathematics)

The spiral sequence allows for continuous assessment of a learner's understanding. For example, if students can find answers to addition and subtraction questions but cannot decide which operation to use to solve a word problem, they do not fully understand adding and subtracting. This is when clarification and practice can be facilitated. The spiral sequence builds in a process of review; in order to take the next step in division, the student must recall the previously learned skills. It is natural to combine a needed review with the next skill step.

Often, attendance in literacy classes is interrupted or sporadic. If a person is able to attend classes for only a short time, the spiral sequence would give exposure to a greater variety of skills. With short-term attendance, it is even more essential that learners are not deprived of making connections as early as possible. Using a spiral sequence, it is possible to group or pair students because most will soon have some basic knowledge of all four operations.

By alternating regularly among the four operations, linking them as they are applied, the learner is better able to construct a clear and complete picture. The spiral sequence gives learners the opportunity to compare, contrast, make decisions, ask questions, and bring to consciousness their problem-solving methods. This interaction facilitates a naming of the functions as well as a meaningful dialogue about the learning process. In conclusion, this approach is one that brings meaning to working with numbers while at the same time contributing to literacy.



About This Sequencing Guide

Once numeracy activities begin, many facilitators soon discover that it is very difficult to find material. Usually, only skill-based math texts are available. Most basic-level math material is either too juvenile or assumes more literacy skills than most participants in literacy classes have achieved. It is in response to this need for math materials written for adult literacy learners that the Adult Basic Education Unit of the Toronto Board of Education has begun to create numeracy materials. The greatest need for numeracy materials and guidance is at the basic level, for learners who have almost no background with numbers. We have focused on this need and hope that facilitators can see, within this approach, ways to adapt the materials.

The numeracy sequence proposed in this book is in two parts. The first part of the sequence deals with pre-operation concepts and skills. Based on a learner's pace of learning and previous experience with numbers, the order presented here may be rearranged. It is essential, however, that a learner understand the meaning of numbers and be able to count. Taking time for pre-operation skills sets a strong foundation for numeracy. The participatory approach taken by the instructor at this stage will set the tone for working comfortably with numbers in many aspects of daily life. The second part of the sequence deals with whole-number operations. These are presented in an order that continuously builds on previously learned skills.

There are two essential features to the spiral sequence proposed in this book. One is that the spirals are small. This means that we introduce each operation at a basic level, then return to each operation to introduce new skills one at a time. The other feature is that the spiral includes skills called "Making Connections". These skills involve integrating two or more operations. The spiral sequence itself presents the operational skills in a repeating pattern of steps starting with considering a problem, going on to introduce the new concept and arithmetic skill, and then applying the skill in a variety of ways. As well as integrating operations, this approach involves integrating relevant material throughout.

A fuller description of each of the steps follows. Please note that in this guide we have put these steps as part of the operations skills, and not the pre-operation skills.



Considering a Problem

This step makes use of discussion to establish a context for the next math skill. The facilitator may discuss situations where this new skill can be used to solve a problem. There may be a discussion about various ways to solve the problem mathematically. This is also an opportunity for the learners to relate their experiences. These discussions may reveal strategies that individual learners use to cope with these situations in their lives.

Concept, Sign Recognition and Vocabulary

This step needs to be covered before the actual arithmetic skill is introduced. Students need language to talk about what they are doing and mathematical notation to show what they are doing.

Facts

This step refers to arithmetic knowledge that must be practised and learned by heart until it becomes automatic. Multiplication tables are one example. Number combinations that add up to 10 are another example.

Skills

This step is different from "Facts" in that skills are not known by heart, as facts are, but are applied. Long division is an example. We don't know the answer to a long division problem just by looking at it, but we apply what we know in order to work it out.

Applications

This step includes a variety of activities. Discussions, word problems, and estimating are examples of applications. Word problems require students to make their methods conscious, use notation, and combine literacy and numeracy skills. It is crucial in this part of the sequence, to adapt to the particular needs of the learners. Working with grocery fiyers, estimating how much rice to cook, calculating overtime pay, and looking at bills are examples of relevant applications.



Making Connections

This step refers to word problems and other activities (such as decoding and inverse operations) that use more than one operation. Students are always encouraged to interpret and assess information, make decisions, and exercise judgement, thereby developing confidence. Learning math skills by way of this literacy approach, which integrates the skills into the everyday life of the learner, is the challenge of numeracy.

Clearly, this approach must be adapted to suit various situations. Learners' needs, abilities, personalities, and backgrounds with numbers vary greatly. This sequence can suit the needs of the learners and the demands of the learning situation. For example, a larger spiral may be more appropriate for learners who have some previous background with math. Numeracy facilitation is something new to most instructors. We recognize that there is a need for training to reinforce the facilitation skills that are required to use this material more fully.

This sequencing guide is a natural companion to the book <u>Numbers in our Lives - Numeracy Methods and Materials</u>, which introduces basic numeracy concepts and whole-number skills. To that end, we have included page number references from Numbers in our Lives beside the descriptions of each step. There is enough space to add page numbers from other reference materials as well. Facilitators can refer to <u>Numeracy Books for Adult Learners</u>, an annotated bibliography also published by the Adult Basic Education Unit. For those people who prefer to develop materials themselves however, this sequencing guide is just as valuable on its own. We welcome feedback from anyone using this sequencing guide.

Kathy Lucas, Betty Dondertman, Tom Ciancone



A Spiral Numeracy Sequence Part One: Pre-Operation

1. Recognizing and Drawing Shapes

Page References

2. One-to-One Correspondence

Make pairs





3. Recognizing and Drawing Numerals

1234567890

Numbers in our Lives pages 7-12



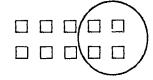
4. Reading and Writing Numbers Using Numerals

What is your telephone number?

It is 393–1995

5. Counting 1 - 10

Circle 4 squares.



6. Counting to 50

How many days did you work in April?

Count them on the calendar.

7. Ordering and Comparing Numbers

Rewrite these numbers in order. 23 27 25 26 24

8. Counting to 100

Which number follows 79'

Page References

NL pages 13-15

NL pages 19-23

NL pages 24-28

NL pages 29-33

NL pages 34-38



9.	Cou	nting	to	1000
----	-----	-------	----	------

Page References

How many days are in a year? 365

NL pages 39-41

10. Place Value: 1 - 999

NL pages 42, 45

If you have 2 hundred-dollar bills, 1 ten-dollar bill, and 3 loonies, you will have \$213.

11. Ordering and Comparing Numbers to 1000

NL pages 46-48.

Which is larger? 405 or 410

12. Counting by 2's, 5's, and 10's

These are even numbers: 2, 4, 6, 8, 10, 12 . .

NL pages 49-51

13. Rounding

Is 93 closer to 90 or 100?

NL pages 53-54



14. Estimating

Approximately how much money do you have with you today?

Page References NL page 52

NL pages 55-58

15. Reading and Writing Numbers in Words

1 one

2 two

20 twenty

21 twenty-one

16. Reading and Writing Ordinal Numbers

1st first

2nd second

3rd third

'NL page 59



A Spiral Numeracy Sequence Part Two: Whole Number Operations

1. Adding: one-digit numbers, answers to 10

Considering a Problem

Discussion to establish context

Addition: Concept, Sign Recognition, Vocabulary

two plus three is five

Addition Facts: O - 10

Applications

Estimating, word problems, discussions . . .

Page References

Numbers in our Lives page 101

NL pages 102 - 108

NL pages 109 - 117

NL pages 118 - 120

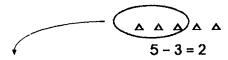


2 Subtracting: one-digit numbers

Considering a Problem

Discussion to establish context

Subtraction: Concept, Sign Recognition, Vocabulary



five minus three is two

Subtraction Facts: O - 10

Applications

Estimating, word problems, discussions . . .

Page References

NL page 121

NL pages 122 - 127

NL pages 128 - 134

NL pages 135 - 136



Page References

3. Making Connections: adding and subtracting

Decoding

Inverse Operations

Applications

Estimating, word problems, discussions . . .

4. Multiplying: to 5 x 10

Considering a Problem

Discussion to establish context

NL page 137

NL page 138

NL page 139

NL page 143



Multiplication:	Concept,	Sign	Recognition,
	Vocabulo	ıry	

4 x 3 = 12

four times three is twelve

Multiplication Facts: O to 5 as multiplier

 $9 \times 3 = 27$

Applications

Estimating, word problems, discussions . . .

5. Dividing: divisors up to 5, dividends up to 50

Considering a Problem

Discussion to establish context

Division: Concept, Sign Recognition, Vocabulary

14 + 7 = 2

fourteen divided by seven is two

Page References

NL pages 144 - 152

NL pages 153 - 161

NL pages 162 - 163

NL page 164

NL pages 165 - 172



Division Facts: divisors 1 to 5

$$30 + 5 = 6$$

Applications

Estimating, word problems, discussions . . .

6. Making Connections: multiplying and dividing

Sign Recognition

$$15 + 5 = 3$$

$$0 \times 7 = 0$$

Inverse Operations

$$4 \times 7 = 28$$

$$28 + 7 = 4$$

Applications

Estimating, word problems, discussions . . .

Page References

NL page 181

NL pages 182-183

NL page 184



7. Adding: one-digit numbers, answers to 19

Considering a Problem

Discussion to establish context

Addition Facts: answers to 19

Applications

Estimating, word problems, discussions . . .

8. Subtracting: from two-digit numbers to 19

Considering a Problem

Discussion to establish context

Page References

NL page 187 – 188

NL page 189, and 193 - 211

NL pages 212 - 215 (mixed)

NL page 187 - 188



Subtraction Facts: numbers to 19

17 - 9 = 8

Applications

Estimating, word problems, discussions . . .

9. Making Connections: adding and subtracting

Applications

Estimating, word problems, discussions ...

10. Multiplying: to 10 x 10

Considering a Problem

Discussion to establish context

Multiplication Facts: up to 10 times table

 $7 \times 6 = 42$

Page References

NL pages 190 - 209

NL pages 212 - 215 (mixed)

> NL pages 212 - 215

NL pages 219 - 220

NL pages 221 - 233



Page References

Applications

Estimating, word problems, discussions . . .

NL pages 237 - 240 (mixed)

11. Dividing: one-digit divisor, dividends up to 100

Considering a Problem

Discussion to establish context

NL pages 219 - 220

Division Facts: one–digit divisor, one–digit answer

NL pages 221 - 230

$$8)$$
 $\frac{7}{56}$ $63 + 9 = 7$

Applications

Estimating, word problems, discussions . . .

NL pages 237 - 240 (mixed)



Page References

12. Making Connections: multiplying and dividing

Decoding

$$10 + 2 = 5$$

Inverse Operations

Applications

Estimating, word problems, discussions . . .

NL pages 234 - 236

NL pages 237 - 240

13. Adding: two- and three-digit numbers

Considering a Problem

Discussion to establish context

Addition: two digits, no carrying

NL pages 243 - 244

NL pages 245 - 247



Addition: with carrying

Page References

NL pages 248 - 252

Applications

Estimating, word problems, discussions

NL page 253

14. Subtracting: two– and three–digit numbers

Considering a Problem

Discussion to establish context.

NL pages 243 - 244

Subtraction: two and three digits, no borrowing

NL pages 254 - 256



Subtraction: with borrowing

Subtraction: borrowing with zeroes

Applications

Estimating, word problems, discussions . . .

15. Making Connections: adding and subtracting

Applications

Estimating, word problems, discussions . . .

Page References

NL pages 257 - 259

NL page 260

NL pages 284 - 291



Page References

16. Multiplying: one-digit multiplier,two or three digits on top

Considering a Problem

Discussion to establish context.

Multiplication: one-digit multiplier.

no carrying

34

x 2

Multiplication: one-digit multiplier, with carrying

45

x 3 135

Applications

Estimating, word problems, discussions . . .

NL pages 243 - 244

NL pages 261 - 266

NL pages 267 - 273

NL page 274



17. Dividing: long division, one-digit divisor

Considering a Problem

Discussion to establish context

Division: long division, one-digit divisor

Division: with zero as place holder

Applications

Estimating, word problems, discussions . . .

Page References

NL pages 243 - 244

NL pages 275 - 283

NL pages 284 - 291 (mixed)



Page References

18. Making Connections: multiplying and dividing

Applications

Estimating, word problems, discussions . . .

NL pages 284 – 291

19. Making Connections: review all operations

Decoding and mixed practice

$$87 + 15 = 102$$
 $56 - 27 = 29$

$$8 \times 6 = 48$$
 $55 + 11 = 5$

Applications

Estimating, word problems, discussions . . .

20. Multiplying: two-digit multiplier

Considering a Problem

Discussion to establish context

Multiplication: by 10, 100, 1000

$$53 \times 10 = 530$$

Applications

Estimating, word problems, discussions . . .

21. Dividing: two-digit divisor

Considering a Problem

Discussion to establish context

Division: two-digit divisors

Division: by 10, 100, 1000

$$2600 + 100 = 26$$

Page References

Page References

Applications

Estimating, word problems, discussions . . .

22. Making Connections: multiplying and dividing

Applications

Estimating, word problems, discussions . . .

23. Review: all operations

Applications

Estimating, word problems, discussions . . .



The Steps of the Spiral Sequence in Summary

Part One: Pre-operation

- 1. Recognizing and Drawing Shapes
- 2. One-to-One Correspondence
- 3. Recognizing and Drawing Numerals
- 4. Reading and Writing Numbers Using Numerals
- 5. Counting 1 10
- 6. Counting to 50
- 7. Ordering and Comparing Numbers
- 8. Counting to 100
- 9. Counting to 1000
- 10. Place Value: 1 999
- 11. Ordering and Comparing Numbers to 1000
- 12. Counting by 2's, 5's, and 10's
- 13. Rounding



- 14. Estimating
- 15. Reading and Writing Numbers in Words
- 16. Reading and Writing Ordinal Numbers

Part Two: Whole Number Operations

- 1. Adding: one-digit numbers, answers to 10
- 2. Subtracting: one-digit numbers
- 3. Making Connections: adding/subtracting
- 4. Multiplying: to 5 x 10
- 5. Dividing: divisors up to 5, dividends up to 50
- 6. Making Connections: multiplying/dividing
- 7. Adding: one-digit numbers, answers to 19
- 8. Subtracting: from two-digit numbers to 19
- 9. Making Connections: adding/subtracting
- 10. Multiplying: to 10 x 10
- 11. Dividing: one-digit divisor, dividends up to 100
- 12. Making Connections: multiplying/dividing



- 13. Adding: two- and three-digit numbers
- 14. Subtracting: two- and three-digit numbers
- 15. Making Connections: adding/subtracting
- 16. Multiplying: one-digit multiplier, two or three digits on top
- 17. Dividing: long division, one-digit divisor
- 18. Making Connections: multiplying/dividing
- 19. Making Connections: review all operations
- 20. Multiplying: two-digit multiplier
- 21. Dividing: two-digit divisor
- 22. Making Connections: multiplying/dividing
- 23. Review: all operations



A Linear Sequence: Addition

- 1. Concept
- 2. Addition Facts: 1 10
- 3. Addition Facts: adding one-digit numbers, answers greater than 9
- 4. Addition: two and three digits, no carrying
- 5. Addition: with carrying
- 6. Addition: mixed review
- 7. Applications



A Linear Sequence: Subtraction

- 1. Concept
- 2. Subtraction Facts: 0 10
- 3. Subtraction Facts: 0 19, no borrowing
- 4. Subtraction: two and three digits, no borrowing
- 5. Subtraction: with borrowing
- 6. Subtraction: with borrowing using zeroes
- 7. Subtraction: mixed review
- 8. Applications



A Linear Sequence: Multiplication

- 1. Concept
- 2. Multiplication Facts
- 3. Multiplication: one digit, no carrying
- 4. Multiplication: one digit, carrying
- 5. Multiplication: two digits, no carrying
- 6. Multiplication: two and three digits, carrying
- 7. Multiplication: by 10, 100, 1000
- 8. Multiplication: mixed review
- 9. Applications



A Linear Sequence: Division

- 1. Concept
- 2. Division Facts
- 3. Long Division: one-digit divisors, without and with remainder
- 4. Division: using zeroes as place holders
- 5. Division: two-digit divisors
- 6. Division by 10, 100, 1000
- 7. Division: mixed review
- 8. Applications



Other books available from the Adult Basic Education Unit:

- ☐ Numbers in Our Lives: Numeracy methods and materials
- ☐ Planning Numeracy Lessons for an ESL-Literacy Classroom: A beginning
- ☐ Numeracy Books for Adult Learners:An annotated bibliography



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