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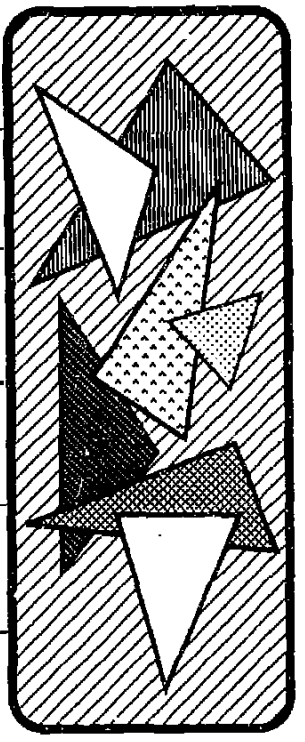
ABSTRACT

This document is designed to assist teachers and other school personnel in the planning and teaching of the eighth grade mathematics course. Contents include: (1) Overview of Grade 8 Mathematics (mission statement, purpose and philosophy, goals, National Council of Teachers of Mathematics' Professional Standards for Teaching Mathematics, and uses of technology and manipulatives); (2) Essential Elements of Instruction and Learning Objectives; (3) Texas Assessment of Academic Skills (TAAS) (focus, domains, objectives, targets, and instructional strategies); (4) Sample Lessons for Teaching Grade 8 Mathematics; and (5) Evaluation (philosophy, types of evaluation, samples, test-taking strategies, grading, and homework). TAAS features three domains: concepts, operations, and problem solving. The Essential Elements are: problem solving; patterns, relations, and functions; number and numeration concepts; operations and computation; measurement; geometry; and probability, statistics, and graphing. "Math Portfolios: A New Form of Assessment," a paper by Jean Kerr Stenmark, is appended. Contains 10 references. (MKR)

GUIDELINES FOR TEACHING GRADE 8 MATHEMATICS



Texas Education Agency
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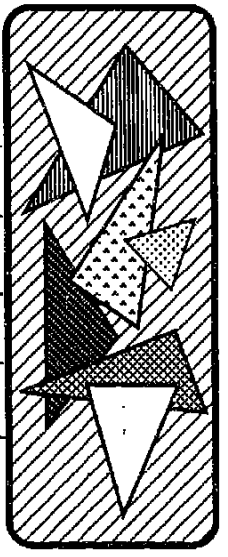

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FOREWORD

Guidelines for Teaching Grade 8 Mathematics is designed to help teachers and other school district personnel plan and teach eighth grade mathematics. The publication presents the philosophy and intent of the course and discusses the required essential elements, TAAS instructional targets, instructional strategies, and the use of technology and manipulatives. Also included are sample objectives and activities to illustrate how the essential elements for eighth grade mathematics can be taught. School district personnel may want to use these suggestions to develop their own curriculum documents for the course.

We hope these guidelines will be useful in planning and teaching mathematics in Grade 8 and in equipping the mathematics classroom.

Lionel R. Meno
Commissioner of Education

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CONTENTS

Overview of Grade 8 Mathematics

Mission Statement

Guidelines for Teaching Grade 8 Mathematics is designed to assist teachers and other school personnel in the planning and teaching of the eighth grade mathematics course. This document discusses the philosophy and intent of the course, the goals, essential elements, and sample objectives, activities, and lesson plans. Also included is a discussion of the uses of technology and manipulatives, as well as Texas Assessment of Academic Skills (TAAS) domains, objectives, and targets; evaluation techniques and samples; and suggested resources and references. School personnel may want to use these suggestions to develop their own curriculum documents for eighth grade mathematics. These guidelines will be useful to district personnel in planning curriculum, teaching mathematics, and equipping classrooms.

When students are allowed to experience the numerous perspectives of mathematics, they will be more successful. This document has been developed with that in mind. A teacher's understanding of the content matter taught is most important. A teacher's understanding of how a student can achieve mastery of concepts; how the mistakes made by students and a teacher's readiness to reduce or eliminate these occurrences; and how a teacher uses books, resources, and materials sets the stage for successful experiences in learning and teaching mathematics.

The eighth grade essential elements and the eighth grade TAAS objectives were used as a basis for the lesson activities in these guidelines. Some of the activities may be challenging for students. Challenging or advanced topics should be taught because they demonstrate to students what they are capable of learning, understanding, and accomplishing in mathematics. This document is designed to inspire teachers to inspire students to want to learn mathematics. Only when educators begin to expand their horizons in this way will a significant impact be made upon students.

Purpose and Philosophy

Mathematics is useful, exciting, and creative and can be enjoyed by all middle school students. Problem-solving skills and logical reasoning are developed while students explore and make sense of their world through the study of mathematics. Unfortunately, mathematics has been viewed by many students as boring, irrelevant, and routine and as externally dictated by a rigid system of rules governed by standards of speed, accuracy, and memory. In the past, computational facility has been emphasized instead of a broad, integrated view of mathematics. While computational skills are important, learner characteristics and the vitality of mathematics itself cannot be overlooked. Mathematics in Grade 8 should be broad based and concept driven and should reflect relevant mathematics and its interrelationships with technology.

Middle school students are in a transitional period, forming lifelong values and skills. Decisions about what students will study and how they will learn can dramatically affect their futures, and failure to study mathematics can result in a loss of opportunities. Attitudes that affect the decisions are often formed during middle school years, so the curriculum must be useful, interesting, and relevant, and it must foster a positive disposition toward mathematics.

The middle school mathematics curriculum should expand students' knowledge of numbers, computation, estimation, measurement, geometry, statistics, probability, patterns and functions, and the fundamental concepts of algebra. Traditionally, textbooks have offered the same topics, approach, and presentation grade after grade, with little change from Grade 5 through Grade 8. Chapters on new material have been included in the last half of textbooks in sections not covered because of lack of time. Thus the result has been a reexamination of materials students have already seen, which promotes a negative image of mathematics and an inadequate background for secondary school mathematics.

A broad-based and flexible view of the middle school curriculum is important for several reasons. First of all, basic skills for the 1990s and beyond mean far more than computational facility. With calculators readily available, the need for tedious paper and pencil proficiency is obsolete, and topics such as geometry, probability, and statistics have become more important through technology. Secondly, if students have been unable to master basic computational skills in elementary school, they are likely to be unsuccessful with the same techniques during the middle school years. Thirdly, many mathematical topics currently omitted actually can assist students in arithmetic concepts and skills through a fresh approach. The eighth grade curriculum should include the following:

- problem situations that establish the need for new ideas and motivate students
- communications with and about mathematics
- mathematical reasoning
- a broad range of topics, including number concepts, computation, estimation, functions, algebra, statistics, probability, geometry, and measurement
- topics taught as an integrated whole
- technology, including calculators, computers, and videos

Goals

According to the *Curriculum and Evaluation Standards for School Mathematics* (the *Standards*) developed by the National Council of Teachers of Mathematics (NCTM), the five overall curriculum goals for students are:

- learning to value mathematics
- becoming confident in their ability to do mathematics
- becoming mathematical problem solvers
- learning to communicate mathematically
- learning to reason mathematically

Moreover, the educational system of tod v demands new societal goals for education:

- mathematically literate workers
- lifelong learning
- opportunity for all
- an informed electorate

Specifically, teaching the mathematics curriculum to middle school students must be related to the characteristics of the learners and their needs today and in the future. *Everybody Counts* (National Research Council, 1989) posits that "self-confidence built on success is the most important objective of the mathematics curriculum" (page 45). Individuals must be able to cope with mathematics in their later lives—as employees, parents, and citizens. Ability to do so depends on attitudes toward mathematics conveyed in school. The mathematics curricula must not leave a legacy of misunderstandings and apprehension.

National Council of Teachers of Mathematics: Professional Standards for Teaching Mathematics

The *Professional Standards for Teaching Mathematics (NCTM, 1991)* are based on four assumptions about the practice of teaching. These assumptions are abbreviated versions of the more extensive ones found in the original document (NCTM, 1991, pages 21-22).

(1) The goal of teaching mathematics is to help all students develop mathematical power. Teachers must help every student develop conceptual and procedural understandings of number, operations, geometry, measurement, statistics, probability, functions, and algebra and the connections among ideas. They must engage all students in formulating and solving a wide variety of problems, making conjectures and constructing arguments, validating solutions, and evaluating the reasonableness of mathematical claims.

(2) What students learn is fundamentally connected with how they learn it. Students' opportunities to learn mathematics are a function of the setting and the kinds of tasks and discourse in which they participate.

(3) All students can learn to think mathematically. The goals such as learning to make conjectures, to argue about mathematics using mathematical evidence, to formulate and solve problems, and to make sense of mathematical ideas are not just for some group thought to be "bright" or "mathematically able."

(4) Teaching is a complex practice and hence not reducible to recipes or prescriptions. First of all, teaching mathematics draws on knowledge from several domains: knowledge of mathematics, of diverse learners, of how students learn mathematics, of the contexts of the classroom, school, and society. Good teaching depends on a host of considerations and understandings. Good teaching demands that teachers reason about pedagogy in professionally defensible ways within particular contexts of their own work.

The *Professional Standards for Teaching Mathematics* identifies a particular set of instructional standards for the effective teaching of mathematics. These standards describe the nature of the tasks, patterns of communication and the learning environment. More specifically, five of these standards focus on instructional strategies. They are:

STANDARD 1: WORTHWHILE MATHEMATICAL TASKS

The teacher of mathematics should pose tasks that are based on:

- sound and significant mathematics;
- knowledge of students' understandings, interests, and experiences;
- knowledge of the range of ways that diverse students learn mathematics;

and that

- engage students' interests;
- develop students' mathematical understandings and skills;
- stimulate students to make connections and develop a coherent framework for mathematical ideas;
- call for problem formulation, problem solving, and mathematical reasoning;
- promote communication about mathematics;
- represent mathematics as an ongoing human activity;
- display sensitivity to, and draw on, students' diverse background experiences and dispositions;
- promote the development of all students' dispositions to do mathematics.

STANDARD 2: THE TEACHER'S ROLE IN DISCOURSE

The teacher of mathematics should orchestrate discourse by:

- posing questions and tasks that elicit, engage, and challenge each student's thinking ability;
- listening carefully to students' ideas;
- asking students to clarify and justify their ideas orally and in writing;
- deciding what to pursue in depth from among the ideas that students bring up during a discussion;
- deciding when and how to attach mathematical notation and language to students' ideas;

- deciding when to provide information, when to clarify an issue, when to model, when to lead, and when to let a student struggle with a difficulty;
- monitoring students' participation in discussions and deciding when and how to encourage each student to participate.

STANDARD 3: STUDENTS' ROLE IN DISCOURSE

The teacher of mathematics should promote classroom discourse in which students:

- listen to, respond to, and question the teacher and one another;
- use a variety of tools to reason, make connections, solve problems, and communicate;
- initiate problems and questions;
- make conjectures and present solutions;
- explore examples and counterexamples to investigate a conjecture;
- try to convince themselves and one another of the validity of particular representations, solutions, conjectures, and answers;
- rely on mathematical evidence and argument to determine validity.

STANDARD 4: TOOLS FOR ENHANCING DISCOURSE

The teacher of mathematics in order to enhance discourse, should encourage and accept the use of:

- computers, calculators, and other technology;
- concrete materials used as models;
- pictures, diagrams, tables, and graphs;
- invented and conventional terms and symbols;
- metaphors, analogies, and stories;
- written hypotheses, explanations, and arguments;
- oral presentations and dramatizations.

STANDARD 5: LEARNING ENVIRONMENT

The teacher of mathematics should create a learning environment that fosters the development of each student's mathematical power by:

- providing and structuring the time necessary to explore sound mathematics and grapple with significant ideas and problems;
- using the physical space and materials in ways that facilitate students' learning of mathematics;
- providing a context that encourages the development of mathematical skill and proficiency;
- respecting and valuing students' ideas, ways of thinking, and mathematical dispositions;

and by consistently expecting and encouraging students to:

- work independently or collaboratively to make sense of mathematics;
- take intellectual risks by raising questions and formulating conjectures;
- display a sense of mathematical competence by validating and supporting ideas with mathematical argument.

Uses of Technology and Manipulatives

Calculators and computers are tapped for important roles in mathematics education at all levels and across topics. Changes in technology and the broadening of the areas in which mathematics is applied have resulted in growth and changes in the discipline of mathematics itself. The new technology has altered the very nature of the problems important to mathematics and the methods mathematicians use to investigate them.

The *Standards* call for the following regarding technology in the classroom:

- appropriate calculators for all students at all times
- a computer for every classroom for demonstration
- access to a computer for individual and group work
- students learning to use the computer as a tool for processing information and performing calculations to solve problems

Calculators and computers offer teachers and students a rich learning aid. Their potential is great and as yet untapped both in developing concepts and in developing positive attitudes and persistence in problem solving.

Computers can be utilized in a variety of ways in the mathematics classroom, and the appropriateness of a particular approach depends on the goals. Three qualitatively different methods suggested by R. Taylor in *The Computer in the School : Tutor, Tool, Tutee* are:

- as a sophisticated teaching machine
- to be programmed (or taught) by the student
- as a mode for applications in research and development through software that displays graphs, manipulates symbols, analyzes data, and performs mathematical procedures. Applications such as spreadsheets, word processing, data bases, and communication packages have the appeal of matching the classroom's use of technology with that of society's.

Calculator use is not for the purpose of replacing paper-and-pencil computations but to reinforce them. According to N. Kober in *What We Know About Mathematics Teaching and Learning*, calculator use is apt to sustain independent thought, not replace it. For example, students can be challenged to invent calculator algorithms to replace procedures taught in textbooks. The students explain why their procedures work and debate the advantages and disadvantages of their procedures over others. Calculators are programmable, produce graphics, and work in fractional and algebraic notation. Teachers need to be innovative, experiment, and share ideas.

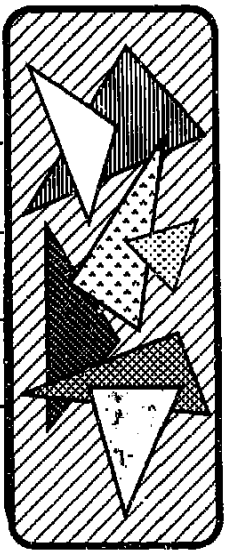
Furthermore, manipulatives offer an excellent way to enable students to connect between mathematical ideas. Learning is enhanced when students are exposed to a concept in a variety of manipulative contexts. As an example, fractions represented with colored cubes, pattern blocks, fraction bars, fraction circles, and Cuisenaire rods help students understand the concept of fractions independent of the physical representation. In addition to using manipulatives for new concepts, activities should be oriented to help students connect between concrete, pictorial, and abstract representations of ideas.

The primary goal in teaching mathematics is to develop conceptual understanding of mathematical ideas. Technology and manipulatives are tools to achieve this goal if implemented in classroom activities.

I hear and I forget. I see and I remember. I do and I understand.

Chinese proverb

Essential Elements of Instruction and Learning Objectives



Essential Elements of Grade 8 Mathematics

The State Board of Education in 1989 revised the essential elements of instruction for mathematics, Grades 1-8. These revised essential elements follow closely the recommendations made by the National Council of Teachers of Mathematics in its nationally recognized *Curriculum and Evaluation Standards for School Mathematics*. According to the Texas Education Agency (1989), "The mathematics curriculum review committee and the Agency [TEA] have tried to be sensitive to a balance between changes expected of teachers and improvements necessary to help students learn mathematics more effectively." Some of these major changes include:

- narrowing the spiral of the curriculum –beginning some topics later and finishing some topics sooner in the curriculum to eliminate some of the redundance
- revising the role of review in the curriculum so that the majority of each grade level is new material and so that review is placed in relevant contexts
- emphasizing the development of problem-solving skills in relevant and interesting situations
- incorporating calculators and computers throughout all grades as problem-solving tools
- adding an essential element on patterns, relations, and functions
- separating the teaching of operations and computation so that all students learn the meaning of the operations
- strengthening the areas of probability, statistics, and geometry
- emphasizing the importance of communication in mathematics
- building on a sound foundation of concepts rather than on rote procedures

- putting mathematics into meaningful contexts

The revised essential elements for Grade 8 are as follows:

(1) **Problem Solving.** Experience in solving problems designed to systematically develop students' problem-solving abilities through a variety of strategies and approaches. The student shall be provided opportunities to engage in the following types of activities:

(A) develop an organized approach to solving application and nonroutine problems appropriate for Grade 8;

(B) analyze problems by identifying relationships, discriminating relevant from irrelevant information, sequencing, observing patterns, prioritizing, and questioning;

(C) communicate an understanding of a problem by describing and discussing the problem and recording the relevant information;

(D) select appropriate strategies from a variety of approaches;

(E) select appropriate materials and methods for solutions; and

(F) generate and extend problems.

(2) **Patterns, Relations, and Functions.** Use of models and patterns to develop the algebraic concepts of relations and functions. The student shall be provided opportunities to:

(A) use patterns to develop the concept of negative exponents;

(B) extend function notation from concrete models to graphic representation;

(C) generate ordered pairs with and without a calculator to graph linear equation;

(D) investigate compound statements as they apply to simple reasoning situations (and, or, if, if not); and

(E) extend the investigation of number patterns such as those found in Pascal's triangle and the Fibonacci sequence.

(3) **Number and Numeration Concepts.** Concepts and skills associated with the understanding of numbers and the place value system. The student shall be provided opportunities to:

(A) extend scientific notation to numbers with a wide range of values using a calculator when appropriate;

(B) compare and order rational numbers;

(C) apply the concepts of significant digits to solving problems with a calculator;

(D) investigate irrational numbers and their representations on a calculator as they arise from problem situations;

(E) describe the properties of terminating, repeating, and non-repeating decimals and convert between fractions and decimals; and

(F) extend basic number concepts and properties to algebraic applications.

(4) **Operations and Computation.** Use of manipulatives to develop the concepts of basic operations on numbers and to apply these concepts to the computational algorithms. The student shall be provided opportunities to:

(A) select an appropriate operation and/or strategy to solve a problem and justify the selection;

(B) estimate and solve application problems involving percent;

(C) add, substract, multiply, and divdie rational numbers in problem situations;

(D) estimate and solve application problems by writing and solving simple linear equations; and

(E) solve linear equations and inequalities with integer, fraction, and decimal solutions.

(5) **Measurement.** Concepts and skills using metric and customary units. The student shall be provided opportunities to:

(A) estimate and solve application and nonroutine problems involving surface area and volume;

(B) solve right triangle problems using the Pythagorean Theorem, indirect measurement, and the properties of 30-60-90 and 45-45-90 triangles;

(C) use precision and relative error, given the degree of accuracy required and the nature of a particular problem situation;

(D) use the concept of volume for prism/cylinders as the product of the area of the base and the height; and

(E) use the concept of volume for cone/pyramids as one-third the product of the base and the height.

(6) **Geometry.** Properties and relationships of geometric shapes and their applications. The student shall be provided opportunities to:

(A) find the missing parts of similar figures;

(B) investigate the relationships between angles formed when parallel lines are cut by a transversal using graphing technology when appropriate;

(C) use a variety of methods to perform basic constructions;

(D) construct a geometric figure congruent to a given figure;

(E) draw three-dimensional figures from different perspectives;

(F) graph similar figures, reflections, translations, linear equations, and linear inequalities on a coordinate plane; and

(G) use geometry to solve problems in such areas as art, architecture, construction, etc.

(7) **Probability, Statistics, and Graphing.** Use of probability and statistics to collect and interpret data. The student shall be provided opportunities to:

(A) understand and apply reasoning with proportions in problem situations;

(B) select an appropriate format for presenting collected data;

(C) evaluate arguments based on data analysis;

(D) find the probability of simple and compound events;

(E) use mathematical probabilities and experimental results for making predictions and decisions; and

(F) investigate bias to determine validation of an inference made from a set of data.

Sample Learning Objectives

The following are sample objectives that mathematics educators could use to teach the essential elements for Grade 8.

ESSENTIAL ELEMENT 1: Problem Solving

(A) compare and contrast several approaches to problem solving.

(B) ask questions that lead to solutions of problems.

(C) make predictions about the solutions to problems.

(D) given a list of strategies, choose the ones appropriate for solving specific problems.

(E) use materials other than paper and pencil to solve problems.

(F) record a list of possible questions that may be answered, given a set of data.

ESSENTIAL ELEMENT 2: Patterns, Relations, and Functions

(A) The student will evaluate a numeric expression involving negative exponents, using calculators and computers where appropriate.

(B) Given examples of the concrete model of a function, the student will graph a function on a coordinate grid.

(C) The student will generate a table of ordered pairs for a linear function, with or without a calculator.

(D) The student will determine the truth of a conjunction (and) or disjunction (or) of two statements.

(E) The student will investigate patterns in the terms of figurative sequences, such as those for triangular, square, and pentagonal numbers.

ESSENTIAL ELEMENT 3: Number and Numeration Concepts

(A) The student will identify numbers from science that are generally expressed in scientific notation.

(B) The student will order a set of rational numbers by graphing the numbers on a number line.

(C) The student will explore the limits of the calculator for displaying a computational result with a large number of significant digits.

(D) The student will explore the limits of the calculator display for representing an irrational number as a decimal.

(E) The student will use a calculator to explore patterns for converting fractions with 9, 99, or 999 as a denominator of a decimal.

(F) The student will use algebra manipulatives to simplify an algebraic expression.

ESSENTIAL ELEMENT 4: Operations and Computation

(A) determine whether an approximate or exact answer is more appropriate for a given problem.

(B) estimate solutions to problems involving percent.

(C) use operations on rational numbers to solve application problems, using a calculator when appropriate.

(D) formulate and solve equations from problem situations.

(E) solve linear equations involving integers, decimal, and fractions.

ESSENTIAL ELEMENT 5: Measurement

(A) The student will estimate the amount of material needed to build a model, draw plans for the model, and find the surface area and volume of the model.

(B) The student will develop the Pythagorean Theorem using square tiles/graph paper.

(C) The student will determine the best unit of measure to use for the least amount of error for measuring a given set of objects.

(D) The student will discover what happens to the total surface area and volume of a rectangular prism when the length of each edge is multiplied/increased by the same number.

(E) The student will discover what happens to the total surface area and volume of a cone/pyramid when the dimensions are multiplied/increased by the same number.

ESSENTIAL ELEMENT 6: Geometry

(A) Using proportions, the student will find the height of a tree when the length of the tree's shadow, and the height and shadow length of another object are known.

(B) The student will discover relationships by measuring and comparing angles formed when two lines intersect.

(C) The student will use computer software to draw perpendicular lines and parallel lines.

(D) The student will use a compass and straightedge to copy a triangle.

(E) The student will construct three-dimensional figures from cubes and draw views from the four sides as well as the top.

(F) The student will compare the figure formed from the graph of a set of ordered pairs to the graph of the figure when the domain and range values are increased or decreased by a constant.

(G) The student will use translations, rotations, and reflections to create a pattern which can be used to develop a tessellation drawing.

ESSENTIAL ELEMENT 7: Probability, Statistics, and Graphing

(A) The student will solve application problems using a proportion.

(B) The student will choose an appropriate measure of central tendency for a set of data.

(C) The students will use data analysis to determine if a relationship exists between two variables.

(D) The students will list the sample space for compound events.

(E) The students will use sampling to estimate population sizes.

(F) The students will determine the adequacy of a sample for making predictions.

Texas Assessment of Academic Skills

Focus

The Texas Education Agency implemented the Texas Assessment of Academic Skills (TAAS) testing program in 1990. The program is in effect for the 1990-1995 period. The purpose of the assessment program is to provide Texas schools with an accurate measure of student achievement. The scope of content of the TAAS includes more of the instructional targets delineated in the essential elements than previous state assessments. Every section of the TAAS test contains a certain number of broad objectives. These objectives remain constant from grade to grade because they represent the core concepts that form the basis for a sound instructional progression from Grade 1 through Grade 12. What will differ from grade to grade are the instructional targets—or essential elements that comprise each objective. A portion of this extended set of instructional targets is selected for assessment annually, but not every target is tested every year.

The broadened scope of the TAAS assessment program allows for a different focus, one that addresses the academic requirements of the 1990s. Skill areas that demand little more than rote memorization are de-emphasized, while areas that improve a student's abilty to think independently, read critically, write clearly, and solve problems logically receive increased emphasis. This emphasis is in keeping with current national trends in education, which stress the importance and necessity of teaching students higher order thinking skills.

Domains, Objectives, and Targets

The TAAS features three domains—concepts, operations, and problem solving. Each domain contains objectives that are derived from the essential elements. For every objective, there are instructional targets that describe the kinds of mathematical experiences that will reflect that objective. Each instructional target was taken for the most part directly from the essential elements as delineated in the *State Board of Education Rules for Curriculum*. Each target is defined in behavioral terms appropriate for pencil-and-paper testing.

DOMAIN: Concepts

Objective 1: The student will demonstrate an understanding of number concepts.

(a) Use scientific notation
(b) Use exponential notation with integral exponents
(c) Compare and order rational numbers
(d) Round whole numbers and decimals
(e) Determine relationships between and among fractions, decimals, and percents

Objective 2: The student will demonstrate an understanding of mathematical relations, functions, and other algebraic concepts.

(a) Recognize and use rational number properties and inverse operations
(b) Determine missing elements in patterns
(c) Identify ordered pairs and solution sets in one and two dimensions
(d) Apply ratio and proportion
(e) Use exponents and properties of exponents
(f) Evaluate variables and expressions (formulas)
(g) Solve simple equations involving integers, decimals, and fractions

Objective 3: The student will demonstrate an understanding of geometric properties and relationships.

(a) Identify lines, rays, angles, and planes
(b) Recognize properties of two- and three-dimensional figures
(c) Use right-triangle geometry with Pythagorean property, similarity, indirect measurement, and ratios
(d) Use similarity, congruence, and symmetry
(e) Recognize basic geometric constructions

Objective 4: The student will demonstrate an understanding of measurement concepts using metric and customary units.

(a) Use metric and customary units
(b) Convert within the metric system
(c) Convert within the customary system
(d) Determine perimeter and circumference
(e) Find area, surface area, and volume
(f) Recognize precision

Objective 5: The student will demonstrate an understanding of probability and statistics.

(a) Use counting methods
(b) Find the probability of simple and compound events
(c) Determine the mean, median, and the mode
(d) Use frequency distributions

DOMAIN: Operations

Objective 6: The student will use the operation of addition to solve problems.

(a) Add rational numbers (fractions, decimals, integers, and percents)

Objective 7: The student will use the operation of subtraction to solve problems.

(a) Subtract rational numbers (fractions, decimals, integers, and percents)

Objective 8: The student will use the operation of multiplication to solve problems.

(a) Multiply rational numbers (fractions, decimals, integers, and percents)

Objective 9: The student will use the operation of division to solve problems.

(a) Divide rational numbers (fractions, decimals, integers, and percents)

DOMAIN: Problem Solving

Objective 10: The student will estimate solutions to a problem situation.

(a) Estimate solutions

Objective 11: The student will determine solution strategies and will analyze or solve problems.

(a) Identify strategies for solving or solve proportion problems
(b) Determine methods for finding or find percent and percentage
(c) Determine methods for solving or solve measurement problems
(d) Formulate or solve problems using geometric concepts
(e) Analyze or solve probability and statistics problems
(f) Make predictions

Objective 12: The student will express or solve problems using mathematical representation.

(a) Formulate equations/inequalities
(b) Analyze or interpret graphs, charts, tables, maps, or diagrams and use the information derived to solve problems

Objective 13: The student will evaluate the reasonableness of a solution to a problem situation.

(a) Evaluate reasonableness

Instructional Strategies

A 50-Minute Lesson Format

1. Eye Opener Activity/Magic Tricks—3 minutes
2. Vocabulary and Facts—10 minutes
3. Main Lesson—30 minutes
4. Mental Math—5 minutes
5. Closure/Review—2 minutes

Eye Opener:
In the school library are magic books. Finding a trick that may or may not be math related will enhance your students' interests. Students will look forward to the eye opener. The eye opener may also be some type of activity that is quick and to the point. This may be teaching the students how to multiply by nine using their fingers.

Vocabulary and Facts:
This is essential for the students. Ask students at the beginning what is the product of 2 and 3? Most will probably not know if they have not previously studied math vocabulary. Note however that most all the students recognize the number form of $2 \times 3 =$.

Main Lesson:
Having done the above, students are ready for the main lesson.

Mental Math:
End the math class by teaching students a mental math skill. For example, how to multiply by 25 mentally.

Closure/Review:
Summarize the lesson's activities and concepts.

Following Directions

Administer this test to the students at the beginning of the school year to stress the importance of following directions. Tell students this is a timed test. They have one minute to work on it. Place the test face down on their desks. Say "start" for students to begin. Observe how many students did not read directions. You may be surprised!

DIRECTIONS:

- This is a timed test.
- Work quickly and carefully.
- Write the answers in the blanks provided.
- Work only problems three and ten.

1. 11 + 4 =
2. 6 x 3 =
3. Your name is
4. Raise your right hand!
5. 13 - 3 =
6. Stand up, then sit down!
7. 20 x 0 =
8. Walk quickly to the wall closest to you, touch the wall, then go back to your seat and sit down!
9. 3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 =
10. Come to the front and turn in your paper!

Twelve Suggestions that Work

1. The eye opener activity at the beginning of every session helps to ensure that students are attentive and focused prior to the presentations of mathematics materials.

2. The facts and vocabulary activity provides students an opportunity to be successful (during reviews); to strengthen their knowledge of mathematics (facts/vocabulary); to respond openly in a group setting.

3. The quick analysis that follows each session improves presentations that follow.

4. Providing instructions/expectations prior to each activity improves the student's opportunities for success.

5. Rewarding students with praise or small gifts after exemplary performances or active participation reinforces the importance of the learning process.

6. Activities that involve each individual in the group make use of peer pressure which improves participation by students who tend to be passive learners (chanting, spelling vocabulary words, walking among students, and insisting on participation by reluctant participants proves to be motivational).

7. Treating the taking of notes as a following instructions activity helps in getting students to focus in activities.

8. Mental games (Number Sense and quick answers to problems involving a series of operations) prove challenging and a change of pace to the regular classroom routine.

9. The rapid move from the eye opener activity to the facts vocabulary activity to the main concept activity to the mental math activity to the review activity has students constantly engaged.

10. The daily handouts that are provided for student use serve to reinforce facts, vocabulary, and concepts introduced during the sessions.

11. Constantly reminding students of the importance of listening, following directions, being observant, and taking an active role in the session's different activities reinforces expectations set forth at the beginning.

12. The diagnostic test administered at the beginning of the program helps to determine group needs.

Fifty Two-Card Deck: A Teaching Tool for Basic Facts

Consider using a standard deck of playing cards to introduce or review certain facts with your students. Prior to introducing the use of playing cards, familiarize students with the following information (if possible, use an actual deck of cards).

1. Each deck consists of four suits:
   - A) black cards
     1. spades
     2. clubs
   - B) red cards
     1. hearts
     2. diamonds

   Tell students that the four suits can represent the four seasons in a year (spring, summer, fall, and winter).

2. Each suit contains 13 cards: Ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, Jack, Queen, King. The Jack, Queen, and King are called *face cards*. If you let the Ace = 1, Jack = 11, Queen = 12, and King = 13, the sum of all the playing cards is 364. Since the deck contains 2 Jokers, you can make the following observation: The 52 playing cards and Joker (Joker = 1), when added, equals 365 (the number of days in a year). The extra Joker (Joker = 1), when added, give 366 (the number of days in a leap year).

   The standard deck has 12 face cards (NOTE: Each suit has 3 face cards. Since there are 4 suits, there are 4 x 3 or 12 face cards). The 12 face cards can represent the number of months in one year.

3. Each deck has 52 regular playing cards and 2 Jokers. The 52 regular playing cards can represent the number of weeks in a year.

4. A year measures the time it takes the Earth to revolve once around the sun. Since it takes the Earth 365 1/4 days to revolve once around the sun, then one year = 365 1/4 days. For convenience, it was agreed to let 1 year = 365 days with the understanding that every four years, the accumulated 1/4's (1/4 + 1/4 + 1/4 + 1/4 = 1) would require a leap year consisting of 366 days.

Resource
McAllen ISD's *The Magic of Math* handbook for teachers.

Thinking Mathematically: Extra Time Fillers

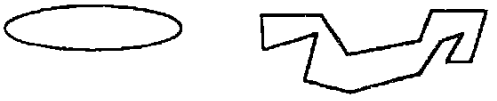
Using Number Concepts

1. Two to four players, two sets of number cards numbered 0 to 9, paper and pencil. Draw a game board, one per player, that contains four rectangles. Shuffle the cards and place them face down. The first player takes the top card and writes the digit in any of his or her rectangles. The next player draws the next card and writes the digit in any of his or her rectangles. Continue until each player has all four digits filled. The player with the largest number wins. The game may also be played so that the winner is the player with the smallest number. Guide the students into realizing that there is a method of placing the numbers in the rectangles.

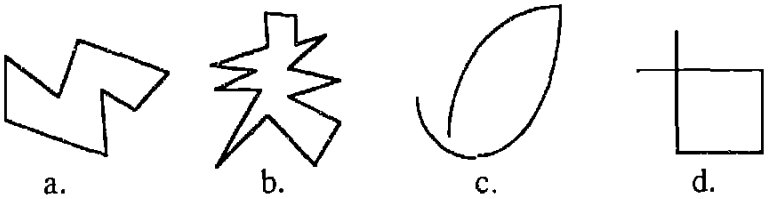
2. Draw a grid of squares, 3 by 3. Fill in the squares with the numbers 2 through 10 so that each row, column, and diagonal adds up to 18.

Visual Reasoning

3. Think of shapes being stretched and bent. Imagine that they are made of rubber bands. These figures are both closed loops.



Which of these figures are equivalent to those above? (a and b) (Both are closed figures.)



a. b. c. d.

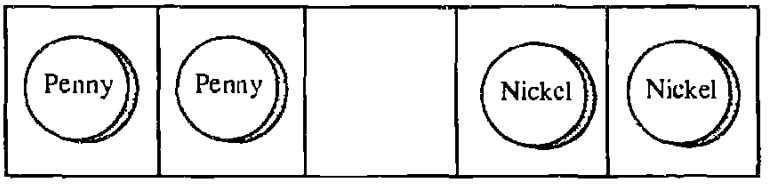
Look at this figure.

Which of the figures below could be made from it? (b and c) (Both are made up of three line segments with a common connecting point.)

a. b. c. d.

Investigating Patterns

4. Draw a five square board. Place two pennies on the left and two nickels on the right. The object of the game is to switch the order of the coins. Coins may move one square into an open square or jump over one coin. Pennies can move only right and nickels only may move to the left. The first move is always with a penny.



Resource

Mathematics in Action. Macmillan/McGraw-Hill, 1991, New York, NY.

Mathematics Shortcuts

Here are a few quick mathematics shortcuts which students enjoy learning and using:

When multiplying by 25, divide the other number by 4. If there are no remainders, add two 0's to the answer. If there is a remainder of 3, write a 75 at the end. If there is a remainder of 2, write a 50 at the end. (3/4 = 75%, 2/4 = 1/2 = 50%)

When multiplying by 50, divide the other number by 2. If there is no remainder, write two 0's at the end of the answer. If there is a remainder of one, then write 50 at the end instead of the two 0's.

When multiplying a two-digit number by 101, take the two-digit number and write it twice: 23 x 101 = 2323.

When multiplying a number by 100, take the decimal point of the other number and move it two times to the right.

When multiplying a number by 1000, take the decimal point of the other number and move it three times to the right.

When multiplying a number by 10000, take the decimal point of the other number and move it four times to the right.

When dividing a number by 10, take the decimal point of the other number and move it once to the left.

When dividing a number by 100, take the decimal point of the other number and move it to the left two times.

When dividing a number by 1000, take the decimal point of the other number and move it three times to the left.

Squaring two-digit numbers ending in 5:

Always write 25 for the last part of your answer. Add one to the other digit. Take this answer and multiply it to that first digit:

$\underline{35}^2$

$5^2 = \underline{25}$ Always write 25 for the last part of your answer.

$4 \times 3 = \underline{12}$ Add one more to the 3, you get 4. Multiply 3 x 4 and you get 12 which is the first part of the answer.

Therefore: $\underline{35}^2 = \underline{1225}$

Always write a 25 for the last part of your answer. To get the first part, always add one more to the tens place digit and multiply these two numbers.

Sample Lessons for Teaching Grade 8 Mathematics

The following lessons represent the kind of mathematical experiences recommended for students in eighth grade mathematics classes. Several different manipulatives are included in these activities. It is important that students use these manipulatives as they work through the activities. Manipulatives and concrete objects enable middle school students to better understand the mathematical problems and concepts they so often struggle to learn. Students' experiences with manipulatives are recommended in the essential elements, TAAS's instructional targets, and NCTM's *Curriculum and Evaluation Standards for School Mathematics*.

Many activities in this section also recommend that students work together in pairs or small groups. Working together in cooperative groupings promotes communication, mathematical confidence, and students' problem-solving abilities.

EE: 3E *Related EE: 1B* *TAAS Objectives: 1D, 1E*

Objective The student will use a calculator to explore patterns for converting fractions with 9, 99, or 999 as a denominator to a decimal.

Activity Repeating decimal patterns

Materials Calculators

Procedure

Distribute calculators to the students. Have the them convert the first four fractions in the table below to decimals using their calculators.

fraction	decimal
1/9	
2/9	
3/9	
4/9	
5/9	
6/9	
7/9	
8/9	
9/9	

Ask the students if they can predict the decimal representations for the other fractions in the list. Have them check their predictions using their calculators.

Evaluation Have the students write a description of the procedure for converting fractions with 9, 99, or 999 to a decimal. Encourage them to include examples.

Extension

The pattern in the table suggests that 9/9 can be expressed as $.\overline{9}$. This suggests that $.\overline{9} = 1$ since 9/9 = 1. Challenge students to show/explain whether $.\overline{9} = 1$ is a true statement.

Have students use a similar method, to explore fractions with 99 and 999 as denominators.

EE: 3F *Related EE: 1B* *TAAS Objectives: 2*

Objective The student will use the distributive property to solve problems.

Activity Broken calculator

Materials Calculators

Procedure

Provide the students with calculators. Tell them to pretend that the 6 key on the calculator is broken. Write 527 X 6 on the chalkboard and ask the students how they can solve the problem using their broken calculators. If the students need a hint, remind them that multiplication is the same as repeated addition. A student may suggest adding 527 + 527 + 527 + 527 + 527 + 527, which is one way to solve the problem. Challenge the students to think of another method. Guide the students to see that they can break 527 x 6 into parts. For example, (527 x 4) + (527 x 2). Encourage them to use the calculator's memory to accumulate parts of the problem. Use the following keystrokes to solve the example: 5, 2, 7, X, 4, =, M+, 5, 2, 7, X, 2, M+, MR. Ask the students to think of all the ways they can break down 527 x 6 into two pieces and list them on the chalkboard.

Evaluation

Have the students solve the following problems. Ask them to write down at least three ways to solve each.

1. 43 x 58 when the 5 key does not work.
2. 328 x 49 when the 9 key does not work.
3. 687 x 123 when the 3 key does not work.

EE: 3B *TAAS Objective: 1C*

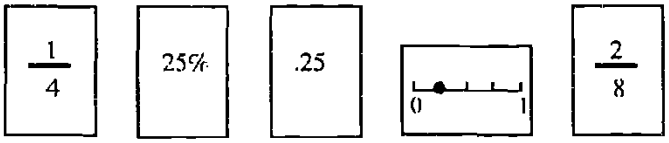
Objective The student will compare rational numbers.

Activity Equivalents card game

Materials Index cards

Procedure

Create a set of cards containing several families of 4 - 7 cards each. Each family should contain some combination of a fraction, percent, decimal, and number line representation for the same value. You will need enough cards to accommodate the class size.



Five representations of the same value.

Randomly distribute the cards to the students. Tell the students to find all of the other students in the class holding cards that belong to the same family as their card. The students will need to move around the class and make many comparisons to form their family groups. Do not tell them the number of cards contained in a particular family. Let them find out for themselves.

Evaluation Have each group show their cards to the rest of the class. Have the other members of the class verify that each card belongs to the family.

EE: 3B *TAAS Objective: 1*

Objective The student will compare and order rational numbers.

Activity Rational number trains

Materials Index cards

Procedure

Create a set of rational number cards using index cards (one card for each student in your class). Each card in the set should contain a different value.

Divide the class into three large groups (approximately 8-10 students) of equal size. Randomly distribute the rational number cards to the students. When you give the signal, have each group attempt to form a line in descending order. Give the students three minutes to work. Depending on the ability level of your students, however, you may want to allow more or less time. Each student must hold his or her card at all times.

When time has expired, award a point to the group with the longest ordered train of students starting with the first student in line. The first six numbers in the following line, for example, are in correct order.

9/10, 7/8, 3/4, 3/5, 1/2, 1/3, 2/5, -2/9, -1/6, -1/10

After a few rounds, have the students discuss possible strategies they can use to work more efficiently as a large group. For example, each student could compare his or her card to zero to be placed in a high or low subgroup which could work together more efficiently. Once the subgroups are placed in order, they could be combined.

After several rounds, the group with the most points wins.

Evaluation Teacher observation of students working in groups.

EE: 4B *Related EE: 1D* *TAAS Objectives: 10A, 11B*

Objective The student will estimate and solve application problems involving percents.

Activity Estimating tips

Materials None

Procedure

Hold a discussion with the class about tipping waiters and waitresses when eating at a restaurant. Explain that it is common to provide a tip of approximately 15% of the total cost of the meal. Demonstrate how a 15% tip can be estimated using the following steps.

1. Find 10% of the cost mentally by moving the decimal to the left one place; round so that half of it can be found mentally.

2. The sum of the amount from Step 1 and half of the amount from Step 1 approximates 15% of the cost of the meal.

Example:

Cost of meal: $34.39
Step 1: 10% of $34.39 = $3.439, which rounds to $3.40.
Step 2: 1/2 of $3.40 = $1.70 and $3.40 + $1.70 = $5.10

Have students practice estimating a tip for various meal costs.

Evaluation Have the students verbalize their thoughts as they work through problems mentally.

Extension Divide the check among several people and decide on each person's amount.

EE: 4D *Related EE: 1C* *TAAS Objectives: 2G, 12A*

Objective The student will formulate equations from problem situations

Activity Interpreting equations

Materials None

Procedure

Write the equation "$m + 9 = 16$" on the chalkboard and have students make up several problem situations it could be used to represent. You may need to provide students with an example to get started.

> Together, Martin and Evan had 16 airplane models. If 9 models belong to Evan, how many belong to Martin?

Discuss equations involving two operations as well. Then provide the students with the following set of equations to write problems for independently.

1. $5m + 5 = 30$
2. $100/n = 25$
3. $m - 23 = 120$
4. $9c = 27$
5. $15 + 2x = 35$
6. $45/j = 9$
7. $3a + 10 = 22$

Evaluation Divide the class into pairs. Tell each student to write a word problem that an equation could be used to represent. Challenge the students to formulate the equation for their partner's problem.

EE: 4D *Related EE: 1B* *TAAS Objectives: 2B, 2C*

Objective The student will formulate equations from problem situations.

Activity Name the rule

Materials Chalkboard

Procedure

Display the following information on the chalkboard or overhead projector.

[7] ->> {15}
[5] ->> {11}
[10] ->> {21}

[11] ->> { ? }
[4] ->> { ? }

Tell the class to use the first three lines of information to develop a rule for finding the number in the braces and to write a variable expression illustrating the rule. [2m + 1] Have them use the rule to find the number that goes in braces in the last two lines. [23, 9]

Have the students try these. For each problem, have students determine a rule and use it to find the numbers that go in the braces in the last two lines. Also, have them write a variable expression illustrating the rule.

1.
[5] --> (11)
[9] --> (19)
[21] --> (43)
[15] --> (?)
[4] --> (?)

2.
[3] --> (9)
[9] --> (81)
[10] --> (100)
[12] --> (?)
[7] --> (?)

3.
[8] --> (10)
[3] --> (0)
[5] --> (4)
[6] --> (?)
[11] --> (?)

4.
[31] --> (26)
[5] --> (0)
[19] --> (14)
[21] --> (?)
[14] --> (?)

Evaluation Have students make up their own problems. You may want to have students exchange their problems and challenge other students with the problems they create.

Objective The student will estimate and solve application problems involving percents.

Activity Using the calculator percent key

Materials Newspapers, calculators

Procedure

Show students how to use the percent key on the calculator to add a sales tax to a dollar amount or to subtract a discount from a dollar amount. Have students assume they are buying an item which costs $27.86.

To add a 8.25% sales tax to $27.86, have students enter 27.86 into the display and then press +, 8.25, %, = . When working the example, students will notice that 2.29845 is displayed after pressing the percent key and before pressing equal. This number represents the tax. Pressing equal gives the total cost of the item including tax. (30.15845)

Discuss that 30.15845 rounds to 30.16 and 30.15101 rounds to 30.16 because you are paying in a store.

Subtracting a discount is done similarly. To take a 15% discount from $27.86, have the students enter 27.86 into the display and press -, 15, %, = . The number in the display (4.179) prior to pressing equal represents the discount. Pressing equal gives the reduced price. (23.681)

Provide students with copies of a newspaper and have them find advertisements. Have them list five items along with their prices. Have students assume the prices are being discounted by 25% and tell them to compute the total cost of the items including the discount and tax.

Evaluation Check work done on problems described above.

Extension Have students investigate whether or not it makes a difference in the total cost of an item to add the tax before taking a discount versus taking a discount and then adding tax.

EE: 5B *TAAS Objective: 3C*

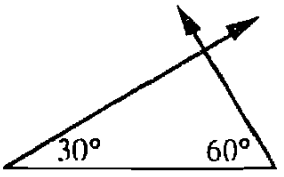
Objective The student will use properties of 30-60-90 and 45-45-90 triangles to solve problems.

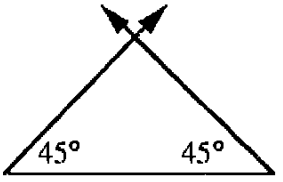
Activity Exploring 30-60-90 and 45-45-90 triangles

Materials Rulers, protractors, grid paper

Procedure

Have the students measure and draw lines of various lengths. (These could be prepared by the teacher.) Instruct them to construct a 30° angle at one endpoint and a 60° angle at the other. Tell them to extend the segments until they intersect to form a triangle. Direct them to repeat the steps with 45° angles.





Have the students measure the new angles and discuss the type of angle formed. Instruct them to measure the legs and compare the lengths.

Evaluation Continue with other examples and have the students turn in written descriptions of their results.

Extension Discuss that 45^0-45^0-90^0 triangle is an isosceles triangle.

EE: 5A *Related EE: 1E* *TAAS Objective: 4E*

Objective The student will build a three-dimensional model to scale and find the surface area and volume.

Activity Exploring volume and surface area

Materials Cubes

Procedure

Divide students into groups of four and provide each group with 64 cubes. Have students construct as many different rectangular prisms as possible. The dimensions, surface area, and volume should be recorded for each prism constructed. You may also want to have the groups draw a picture of each prism using dot paper.

Evaluation Have the students repeat the activity using eight cubes. Have them identify the prism with the least surface area and the greatest surface area.

Extension Discuss formulas for volume.

EE: 5B *Related EE: 3D* *TAAS Objective: 3C*

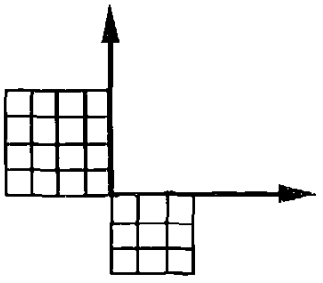
Objective The student will develop the Pythagorean Theorem using square tiles/graph paper.

Activity Pythagorean Theorem

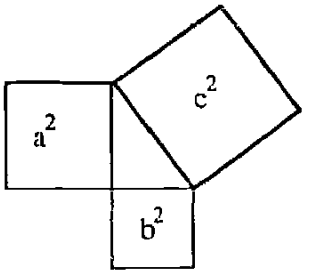
Materials Square tiles, right triangle handout

Procedure

Have students work in groups of four. Provide each group with 50 square tiles and a handout containing a large right triangle. Instruct the students to construct a square using 9 tiles and another square using 16 tiles. Tell the students to place the two squares corner to corner along the legs of the right triangle on the handout as shown below.



Challenge students to find how many tiles it will take to construct a third square in order to complete the triangle. The square root (or a side of the third triangle) is the length of the missing side.



$a^2 + b^2 = c^2$
$3^2 + 4^2 = 5^2$
$\sqrt{25} = \sqrt{25}$
$5 = 5$

Have students repeat the steps for other Pythagorean Triples.

Evaluation Have students find the hypotenuse of right triangles when the two legs are known. Discuss and record results.

Extension Find *a* or *b* with the other and *c* is known.

EE: 7E *Related EEs: 1B, 7A, 7F* *TAAS Objectives: 5D*

Objective The student will use sampling to estimate population sizes.

Activity Estimating population size

Materials 150 blue marbles, 150 red marbles, paper bag

Procedure

Introduction

To help students understand the importance of sampling techniques, hold a discussion with the students about estimating the size of various animal populations.

Questions to ask:

- How would we go about determining the number of catfish in a lake or the number of deer in Big Bend National Park?
- Would it be necessary to count them all?
- Is it possible to use smaller samples to determine the size of entire populations?

Preparation

Prepare a bag with 200 red marbles. If marbles are not available, other objects such as plastic chips, beans, or even slips of paper may be used instead.

Inform the class that you have a collection of red marbles that represents a population of deer. Tell the class that to study the population of deer, we must capture some deer, mark them, release them back into the population, and then see how many of them show up in new samples. To simulate capturing and marking the deer, count out 50 red marbles and replace them with 50 blue marbles which will represent the marked deer. Create a table on the chalkboard for collecting data. Table headings should include sample size, number of red marbles, number of blue marbles, and percent of blue marbles.

Data Collection

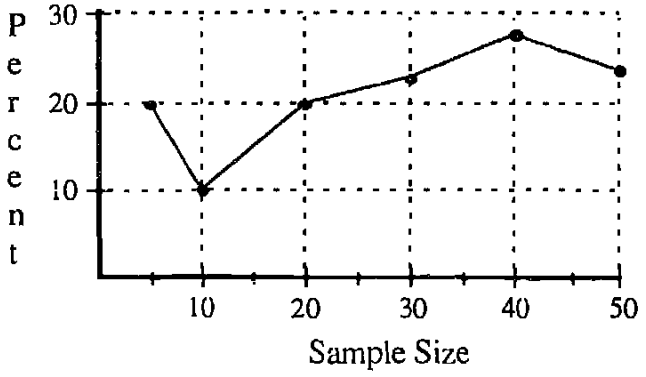
Ask a student to draw five marbles from the bag and record the number of red and blue marbles included in the sample in the table on the chalkboard. Have the class calculate the percent of blue marbles (marked deer) in the sample and record this information as well.

Have another student draw five more marbles from the bag. Add them to the five marbles previously drawn to increase the sample to ten. Record the total number of blue and red marbles in the table on the chalkboard. Again, have the class calculate the percent of blue marbles and record this information as well.

Continue the same procedure, having students draw ten marbles at a time to make samples of 20, 30, 40, and 50 marbles. With each increase in sample size, record the data in the table on the chalkboard.

Have students create the following: a line graph illustrating the relationship between sample size and percent of blue marbles. (See the following sample table and graph.)

Sample Size	Red Marbles	Blue Marbles	% Blue
5	4	1	20%
10	9	1	10%
20	16	4	20%
30	23	7	23%
40	29	11	28%
50	38	12	24%



Questions to ask:
- Why does the line graph fluctuate as the sample size changes?
- What happens to the line as the sample size increases?
- Which sample size do you think provides the most accurate estimate for the percent of blue marbles?
- How can the percent of blue marbles (marked deer) in a sample be used to estimate the total number of marbles (deer) in the bag?

Conclusions
Using the data on percent of blue marbles in a sample, have students estimate the total number of marbles in the bag. For example, the data from the sample table above indicates that 24% of the total number of marbles in the bag are blue. Since it is known that there are 50 blue marbles, then 24% of N (total marbles) = 50. Since 50 / .24 = 208.33, there are approximately 208 deer in the population.

Finally, allow students to check their estimate of the total deer population by counting the total number of marbles in the bag.

Questions to ask:
- Was the estimate reasonable?
- Would a larger sample size provide us with more accurate data?
- Would a smaller sample size provide us with an acceptable estimate? If so, would this be true in general or for this experiment only?

Evaluation

- Teacher observation of individuals or groups of students working on similar problems.
- Clearly written inferences from the data.
- Correctly created graph to match data.

Extensions Given the total number of marbles in the bag, students will estimate the number of marbles of each color in the bag. Also, they can try using three different colors instead of two.

EE: 7E *Related EEs: 7A, 7D* *TAAS Objectives: 2D, 12B, 13*

Objective The student will collect data to determine if a relationship exists between two variables.

Activity Exploring pulse rates and respiration

Materials Small bucket, balloons (30), large pan, large beaker, meter stick, scale (preferably metric).

Procedure

Introduction

Hold a class discussion about the circulatory system and the respiratory system and the relationship between the two. Below are some examples of the kinds of questions to explore.

1. Is there a relationship between an individual's mass and lung capacity?
2. Is there a relationship between an individual's height and lung capacity?
3. Is there a relationship between an individual's mass and pulse rate?
4. Is there a relationship between an individual's height and pulse rate?
5. Is there a relationship between lung capacity and pulse rate?
6. Is there a relationship between lung capacity and an individuals breathing rate?
7. Do females have different pulse rates than males?
8. Do males have different lung capacities than females?

Before attempting to answer the questions listed above through experimentation, have students predict the types of relationships, if any, that may exist between the variables.

Data Collection and Organization

Have the class create a table to record information to be gathered for each student. Table headings should include **name, height, mass, pulse rate, breathing rate,** and **lung capacity**. The following procedures can be used to collect the necessary data. Be sure to have the class record the data for each student after each procedure.

- Have students use a meter stick to measure each other's height to the nearest centimeter.
- A metric scale can be used to measure each student's mass to the nearest kilogram.
- Have students work in pairs to measure pulse rates. One student (the subject) needs to be sitting. Have the other student (the observer) place two fingertips just under the jaw of the subject. Have the observer count the beats that occur from the time you say "start" to the time you say "end." Using a watch or clock with a second hand, have the observer count beats for 15 seconds. Multiplying by 4 will convert the rate to beats per minute. Have students in each pair exchange roles and repeat the process again. Record the data for each student in the table.
- Working in pairs, have each student count the number of breaths his orher partner takes in 30 seconds. Multiplying by 2 will convert the rate to breaths per minute. For c[illegible]tency, he sure the subject is sitting.
- Lung capacity can be measured by having the student inhale deeply and then exhale completely while blowing into a balloon. Tie the balloon so that no air is lost.

Place a bucket inside a large, empty pan and fill the bucket to the brim with water. As the balloon is pushed into the bucket, water will overflow into the pan. Remove the bucket and pour the displaced water into a beaker to measure lung capacity in milliliters (cubic centimeters).

Data Analysis

With data from the table, demonstrate how the relationship between two variables can be investigated using a scatter plot. Plot values for height and mass and draw a trend line if possible. See **Figure 1** for a sample. As illustrated, a positive correlation exists between height and mass. Have students create scatter plots to investigate other possible relationships in the data.

Measures of central tendency can be used to explore possible differences between males and females.

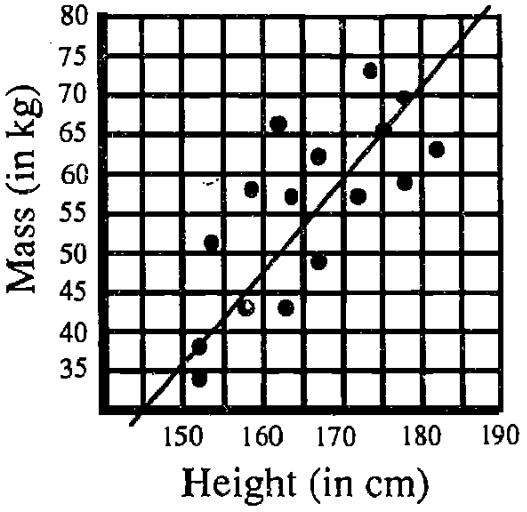


Figure 1

Conclusions

Based on their analysis of the data, students are to check the accuracy of their predictions.

Evaluation

- Accuracy of scatter plot drawings and measures of central tendency.
- Accuracy of conclusions based on scatter plot drawings and measures of central tendency.

Extensions

1. Have students investigate pulse rate differences when sitting, standing, and lying down.
2. Have the student run in place for one minute. Then, measure the pulse rate immediately after exercise. Continue to monitor pulse rate every two minutes for 10 minutes. Use these data to investigate, for example, the relationship between lung capacity and the time it takes for an individual's pulse rate to return to normal after exercise.

EE: 7D *Related EEs: 1A, 7A, 7C, 7E* *TAAS Objective: 5C*

Objective The student will find the probability of compound events.

Activity Probability experiments with a microcomputer

Materials One microcomputer equipped with BASIC programming language

Procedure

Management

This activity is written for use with a single microcomputer in the classroom. If a lab is available, however, the activity can easily be adapted to allow students to work individually or in small groups. The program may be entered in advance and saved on diskette, however, it is short and can be entered just prior to use.

The program in Listing 1 simulates rolling two number cubes 50 times. The output consists of a list of sums.

Introduction

Ask the class what sums are possible when rolling a pair of number cubes. They should be able to provide 11 possible sums: 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12. Have students list these sums on their paper.

Tell the students you are going to have the computer simulate rolling a pair of number cubes 200 times. Ask them to predict the number of times each sum will appear. Have them write down their guesses next to each possible sum on their paper. Remind them that the number of rolls they predict to occur for the 11 possible sums combined must total to 200.

Experiment

Run the program in Listing 1 to generate data for 50 rolls. Have a student create a frequency table on the chalkboard to tally occurrences of each sum. Have another student read the data from the computer screen while the other student records the data with tally marks in the frequency table.

Run the program three more times to generate data for another 150 rolls. Each time the program is executed, have a student read the data from the computer screen while another student records the data with tally marks in the frequency table.

Analysis of Data

Have the students total the tally marks in the frequency table and compare the results to the predictions on their papers. Ask the class if there are any surprises in the data collected. Quite often, students perceive rolling a pair of number cubes as a *simple* event rather than a *compound* event and predict that the sums will be evenly distributed. Ask individual students to share their predictions with the class and possible explanations for any discrepancies with the simulated data.

Guide students through the process of generating an organized list of possible outcomes for rolling a pair of number cubes. Use the organized list to determine the theoretical probability for each possible sum when two number cubes are rolled.

Based on the theoretical probabilities, ask students to compute the number of occurrences expected of each sum when the number cubes are rolled 200 times. Have students compare data with the data collected from the computer simulation. Ask students to explain any

discrepancies. Of course, students may wish to repeat the computer simulation to make sure that the computer is simulating the experiment properly.

LISTING 1

```
10  FOR I = 1 TO 50
20  LET R1 = INT (RND (1) * 6) + 1
30  LET R2 = INT (RND (1) * 6) + 1
40  PRINT R1 + R2,
50  NEXT I
```

Evaluation Have students create an organized list for other similar compound events and compute theoretical probabilities for each outcome.

Examples

(1) Assume the 6 on each cube is a 7.

(2) Divide the larger number by the smaller number.

Extensions

1. Have the class consider the event of finding the difference of the numbers on the two cubes. Simply change the operation symbol in line 40 to subtraction (-) and ignore the negative signs in the output. Be sure to have the students predict the distribution before running the simulation.

2. Also, the class could consider the event of finding the product of the numbers on the two cubes. Simply change the operation symbol in line 40 to multiplication (*).

EE: 1F *Related EEs: 1B, 1C, 4B* *TAAS Objectives: 11A, 11E, 11F*

Objective The student will record a list of questions that may be answered from a given set of data.

Activity The bottom line

Materials Activity Sheet: Form A; Activity Sheet: Form B

Procedure

Management

The students will need to work with partners. To prepare for this activity, locate several pages of problems (old texts or sample texts are excellent sources). Make a blackline copy of each page. Select and cut out four or eight problems. Entitle a plain sheet of paper "The Bottom Line—Form A" and another sheet "The Bottom Line—Form B." Doing one problem at a time, separate the data from the question. Tape the data to Sheet A and the question to Sheet B. Tape the data of the second problem to Sheet B and the question to Sheet A. Continue until all four or eight problems are cut and taped down separating the questions into two groups to make two rounds. (See sample pages). Go back and number 1-4 or 1-8 on each paper. These sheets are ready to be typed.

Introduction

Explain to the students that they will have a set of problems that have been divided into two rounds. In the first round, they will see the information part of the problem but the question will be missing. They must try to figure out the question that has been removed and generate other questions that may be answered from the given data. In the second round, they will see the question but no data. They must ask questions that will lead to the data needed. Only questions that can be answered with one word or one number may be asked. For example: "How long is the string?" "How many degrees are in the angle?" or "Does this problem involve percent?"

Place a sample problem on the overhead projector, covering the question part. Ask the class, "What questions can you answer using this information?" List all of the valid questions and discuss why any invalid questions cannot be answered with the data given. When the students come up with the intended question, underline it and say, "Yes, that is the question intended. Can you think of other questions?"

Now cover the data part of another problem and say, "You now have the question but you need some information to answer the question. You may ask me any questions that I can answer in one word or one number." List the solicited data until the students feel that they have enough information to solve the problem.

Pass out the activity sheets, giving one partner Form A and the other partner Form B. Repeat instructions for round one and instruct students to take turns according to their sheets.

Walk around to make sure everyone is following the correct procedure.

Closure

Point out that reading the question part of a problem first (the bottom line), gives purpose to reading the information part.

Evaluation Observation of student participation and verbal and nonverbal reactions should be employed in this activity. Points may be assigned for each problem.

Name _____

Partner _____

The Bottom Line - Form A

Directions: Do not let your partner see your paper. You have each other's missing questions. Your turn is indicated by the "Δ." Your partner will write what you say on his or her paper for you.

ROUND 1—Generating Questions

Δ1. An astronaut weighing 200 pounds on Earth would weigh 76 pounds on Mars.

2. What is the length of the ramp?

_____

_____

_____

ROUND 2—Gathering the Facts

The facts needed by your partner are underlined.

3. A three-minute telephone call between Milwaukee and Minneapolis costs $.90.

Δ4. How much does it cost per day to feed an elephant in captivity?

Name _____

Partner _____

The Bottom Line - Form B

Directions: Do not let your partner see your paper. You have each other's missing questions/data. Your turn is indicated by the "Δ." Your partner will write what you say on his or her paper for you.

ROUND 1—Generating Questions

1. The astronaut's weight on Mars is what percent of the weight on Earth?

_____

_____

_____

Δ2. The side view of a wheelchair ramp is shaped like a right triangle. The ramp is 5 feet high, and the base is 12 feet long.

ROUND 2—Gathering the Facts

The facts needed by your partner are underlined.

Δ3. What is the price per second?

4. Adult elephants in captivity eat 150 pounds of hay a day and 50 pounds of grain. Hay cost $0.40 per pound, and grain cost $1.05.

EE: 1D *Related EEs: 1B, 1C, 1E, 1F, 4A* *TAAS Objectives: 11A, 11B*

Objective The student will apply appropriate strategies to solve problems.

Activity Strategic drama

Materials Activity sheets, markers, tag board, overhead projector, and transparencies

Procedure

This activity presents an opportunity to work with the English teacher in developing writing skills.

Management

The class should be divided into six groups.

Introduction

Place Transparency 1 (follows activity) on the overhead projector then move to the chalk board. Ask the students to compare and contrast items A and B. List the responses in two columns on the chalkboard headed A and B which you will replace with PROBLEM and EXERCISE when appropriate. You may need to prompt students by asking questions such as the following:

a.) Which would take longer to answer?
b.) What do you need to know to do A? B?
c.) What do you call A? B?
d.) What is an algorithm? Which one is NOT an algorithm?
e.) How would a calculator be helpful for each?
f.) Is it possible that a problem for Matt may be an exercise for Robert?

Continue to solicit data and discuss until the students have a clear understanding of problems and exercises. Tell the students that knowing a variety of strategies helps to make successful problem solvers. Say: "Sometimes we forget to use the things that we know; therefore, the activity that we are going to do will help to you to remember to use problem-solving strategies."

Place Transparencies 2a and 2b, the sample miniplay, on the overhead projector and read aloud (with expression) to the class.

After reading the miniplay, solicit the answer to Mrs. Sloe's last sequence. Then place the Follow-up problem on the overhead projector.

Ask a student to stand and read the problem twice--slowly the second time. After discussing and finding the solution to the problem, pass out a set of assignment sheets for each group (see the sample assignment sheet that follows). Say: "Each group is responsible for writing a miniplay that does not exceed 15 minutes. You will find your strategy at the top of your sheet. You may choose to write a mystery, a horror story, or a comedy. You will have the remainder of the period to work with your group. I will take questions only after group discussions.

Provide the students with a bank of sample problems. As the students present their plays, arrange the signs on the wall to make a problem-solving strategies display.

Solutions
Mrs. Sloe's problem: 13 (an ordered list of prime numbers)

The rice problem: Start both timers. Wait and start the rice when the seven-minute timer runs out which will leave four minutes on the 11-minute timer. You only need to flip the eleven minute timer to get a total of 15 minutes.

The Cold Springs thefts: The thefts were occurring every two nights. The number of books taken were decreasing squares. The street names were in alpha order, and the houses were in counting order.

Evaluation Look for depth and individual input in each presentation.

Variations and Extensions

1. Decrease the number of groups and strategies to narrow the focus.
2. Increase the number of groups to include more strategies
3. Include an additional group to address an organized approach to problem solving such as Polya's four steps to problem-solving.

Make a transparency of this sheet then use masking tape and a large index card to make a cover for the follow-up section which will be used later in the lesson.

TRANSPARENCY 1

INTRODUCTORY EXERCISE

A

A famous chef who always steamed his rice exactly 15 minutes. He had two sand glass timers: One for 7 minutes and one for 11 minutes. How did he use the two timers to cook his rice for exactly 15 minutes?

B

$$\frac{7305 + 480}{15}$$

FOLLOW-UP PROBLEM

In the small town of Cold Springs, a rash of odd thefts occurred. The first one occurred on Sunday night in the first house on Allen street, and 49 books were taken. The second occurred on Tuesday morning in the second house on Broad street, and 36 books were taken. The third one occurred Thursday night in the third house on Cooper street, and 25 books were taken. The thief was caught at the fourth house by a detective who decided to think like the thief. He figured out when, where, and what the thief would take. How?

Transparency 2a

Jack's Last-Minute Scramble

Characters

Jack B. Sloe Mrs. Sloe Narrator

Narrator: You are in the home of Jack B. Sloe. It is 8:15 p.m. Thursday night, and Jack is watching a good movie on HBO. Enters Mrs. Sloe.

Mrs. Sloe: Jack, have you done your homework?

Jack: I just have to study for a quiz.

Mrs. Sloe: You have a quiz and you're looking at TV!?! Turn it off now and do not go to your room but to the kitchen table and get started. You are not going to bed until you are ready for your quiz. I'm setting the timer on the stove for 30 minutes. I'll be back to check.

Jack: Oh man, just when the movie was getting to the best part.

Narrator: Jack's quiz is in math. He has to be able to choose strategies and solve problems. His teacher has told the class that the quiz will consist of five problems and no two problems are to be solved using the same strategy. His class has studied seven different strategies, however, Jack was out for an orthodontist appointment when the class was working on two of the strategies. Jack is now at the kitchen table.

Jack: (Looking at his notes, talking to himself.) Let's see. I know Guess and Check . . . working backwards, no problem . . . draw a picture or diagram . . . I can do that . . . make a list, ... Logical reasoning. . . hmm, that's the one where we had to write what we were thinking. That's funny. She said five out of seven. That's only one, two, three, four, five. Five! I'm missing two (flipping pages). O man, I know she will pick the ones I'm missing. There goes my weekend.

Narrator: Mrs. Sloe is passing by the table and picks up on Jack's distress.

Mrs. Sloe: Having trouble? Need some help?

Jack: My test tomorrow is over problem-solving, and I'm missing two of the strategies that we have to use.

Mrs. Sloe: Did you get your make-up work for when you went to get your braces tightened?

Transparency 2b

Jack: O man, I forgot.

Mrs. Sloe: Why don't you call Jermaine. Isn't he in your math class?

Jack: Yeah, that's a good idea.

Narrator: Mrs. Sloe leaves, and Jack makes the call.

Jack: (On the phone), Hey, man. What's up? . . . Yeah she was looking good in those Daisy Dukes . . .

Mrs. Sloe: (From the next room) Jack, get your assignment and get off the phone now!

Jack: Hey look, Jermaine, I can't talk. I just need to get two problem-solving strategies that I'm missing . . . (Pause) . . . If I knew which ones I was missing I wouldn't be calling you. (Pause) Okay, I have that one . . . yeah . . . yeah . . . yeah . . . Wait a minute, I don't have Find a Pattern . . . (Writing it down) Go 'head, I got it . . . Wait. Simpler . . . similar. (Writing) What's that? (Pause) Oh well, call me back when you finish. (Hangs up the phone.)

Mrs. Sloe: You get it?

Jack: Yeah, but I don't really understand them.

Mrs. Sloe: Maybe I can help. Let's see (sitting down). Find a Pattern . . . If I said 2, 4, 6, 8, what would come next?

Jack: That's easy, 10!

Mrs. Sloe: Okay try this--100, 95, 90, 85.

Jack: Eighty, that's easy too!

Mrs. Sloe: Okay, 2, 3, 5, 7, 11. What's next?

Jack: I don't get that one. First you were counting by two's or adding two. Then you were subtracting fives. You're not multiplying and you're not dividing: I don't get it.

Narrator: Jack is stuck because he has limited his thinking to the operations. That's a good way to start looking for a pattern, but you have to be willing to expand your thinking when that doesn't work. Mrs. Sloe and Jack worked on the two strategies until he felt ready for his quiz. Jack made 90 on the quiz because he missed part of the problem on working backward. By the way. Do you know the next number in Mrs. Sloe's pattern?

Sample Assignment Sheet

DRAW A PICTURE

Some problems are better understood and more quickly solved when you draw a diagram or picture. Your group will send that message to the class in your play. Each group member should know what everyone in the group is doing. The exchange of phone numbers is not a requirement but a good idea. You are to use your tag board to make a sign that identifies your strategy. The sign should be an attention getter. Each member must be responsible for something. The group must collect and staple all assignment sheets together to be turned in prior to the presentation of the play.

Group member _____ Responsibility _____ Phone number

1. _____

2. _____

3. _____

4. _____

5. _____

Give a detailed description of any problems that you/your group encountered and how you resolved them while preparing for your group presentation.

_____

_____

_____

_____

Name _____

Date of Presentation _____ Score _____

Objective The student will draw three-dimensional figures from different perspectives.

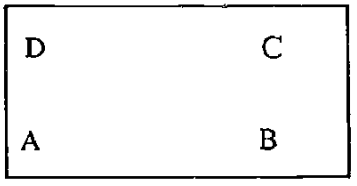
Activity A different perspective

Materials centimeter cubes, isometric dot paper, 3" x 5" index cards

Procedure

Demonstrate how to draw a cube on isometric dot paper on the overhead projector or chalkboard. Have students practice drawing cubes with volumes of $1cm^3$, $8cm^3$, and $27cm^3$.

Have students label the corners of an index card A, B, C, and D as shown below.



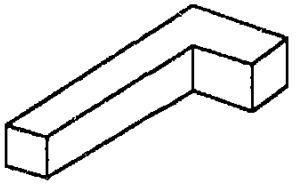
Direct the students to build the following three-dimensional figures with centimeter cubes on the index card. The students will build the prisms from the front view. (Side AB of the index card is parallel to the front of the desk.)

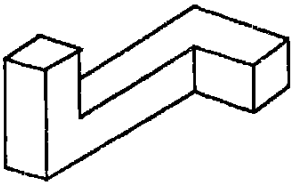
1) L = 1cm, W = 1cm, H = 2cm
2) L = 1cm, W = 2cm, H = 1cm
3) L = 2cm, W = 1cm, H = 1cm
4) L = 1cm, W = 1cm, H = 3cm
5) L = 1cm, W = 2cm, H = 3cm
6) L = 2cm, W = 3cm, H = 3cm

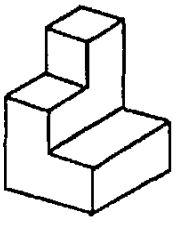
Have students practice dra ing the rectangular prisms from a corner A view.

Students will build any figure then draw two different views (corners A, B, C, D) of the figure on isometric dot paper.

Sample figures for students to build:

1. 

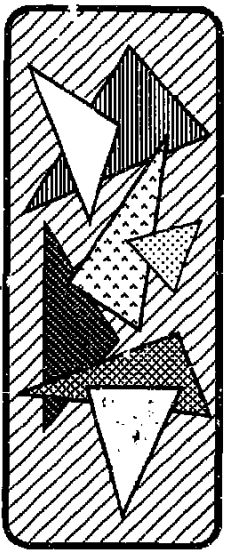
2. 

3. 

Evaluation Accuracy of student drawings.

Extension Have each student design a figure, draw the figure from the top view, and exchange the view with another student who will then reproduce the figure with centimeter cubes.

Evaluation



Philosophy

NCTM's *Professional Standards for Teaching Mathematics* and *Curriculum and Evaluation Standards for School Mathematics* (the *Standards*) emphasize the connection between assessment of students and analysis of instruction. In other words, mathematics teachers should monitor students' learning (both formatively and summatively) in order to assess and adjust teaching. Teachers must observe and listen in order to tailor teaching strategies. Information about what students are understanding should be used to revise and adapt short- and long-range plans, and students' understandings should guide teachers in shaping the learning environment. Also, teachers are responsible for describing students' learning to administrators, parents, and students themselves.

Students' mathematical power depends on various understandings, skills, and dispositions. The development of students' abilities to reason mathematically—to conjecture, justify, and revise based on evidence and to analyze and solve problems must be assessed. Students' dispositions toward mathematics (confidence, interest, perseverance, etc.) are also a key dimension that teachers should monitor.

The importance of using assessment to improve instruction is crucial. Information should be gathered from multiple sources using numerous assessment techniques and modes that are aligned with the curriculum. Assessment techniques must reflect the diversity of instructional methods implied in the *Standards* and the various ways students learn and process information. Instructional decisions should be based on this convergence of information from different sources.

Types of Evaluation

While paper and pencil tests are one useful medium for judging aspects of students' mathematical knowledge, teachers need information gathered in a variety of ways and using a range of sources. Observing, interviewing, and closely watching and listening to students are all important means of assessment. While monitoring students, teachers can evaluate the learning environment, tasks, and discourse that have been taking place. Using a variety of strategies, teachers should assess students' capacities and inclinations to analyze situations, frame and solve problems, and make sense of concepts and procedures. Such information should be used to assess how students are doing, as well as how well the tasks, discourse, and environment are fostering students' mathematical power and then to adapt instruction in response.

Principles relevant for judging assessment instruments at all levels also apply to program evaluation. Inherent in the *Standards* is an assumption that all evaluation processes should use multiple assessment techniques aligned with the curriculum and consider the purpose of assessment. Mathematics education described in the *Standards* places new demands on instruction and forces a reassessment of the manner and method by which students' progress is charted. Testing instruments must reflect the scope and intent of the instructional program to have students solve problems, reason, and communicate, and they must enable teachers to understand students' perceptions of mathematical ideas and processes and their abilities to function in mathematical contexts. Also, the testing instruments must be sensitive enough to help teachers identify individual areas of difficulty in order to improve instruction.

Available assessment techniques suggested in the *Standards* include multiple-choice, short-answer, discussion, or open-ended questions; interviews; homework; projects; journals; essays; portfolios; presentations; and dramatizations. These techniques are appropriate for students in whole-class settings, small groups, or individually, with the mode of assessment written, oral, or computer oriented.

Samples

The purpose of an assessment should dictate the questions asked, methods employed, and uses of resulting information. Methods for gathering information should be appropriate to the developmental level and maturity of the students.

The following samples are excerpted and adapted from the NCTM *Curriculum and Evaluation Standards for School Mathematics*.

Sample 1

Individuals' pulse rates vary. What is the normal pulse rate for students in your class? Consider various characteristics and conditions (such as exercise). How do they relate to pulse rate?

Evaluation should focus on the reasonableness of students' questions, representations of data, verification of results, and generalizations. This exercise is appropriate for group work and can extend over several days. A score can be given to each group, and specific segments of work can be scored separately. Calculators and computers should be employed.

Sample 2

Three of five doctors interviewed recommend Dinosaur vitamins. Write a question to go with this statement to make a problem and solve it.

This problem is appropriate for large-group discussion. Assessment should focus on the choice of questions which must be logically connected to the statement. Solving one's own problem gives an indication of ability to solve and encourages the formulation of reasonable questions.

Sample 3

Seven students have test scores of 57, 96, 73, 86, 92, 75, and 89. Find the average score. How much is the average score increased if each student's score is increased by

a. 5 points?
b. x points?

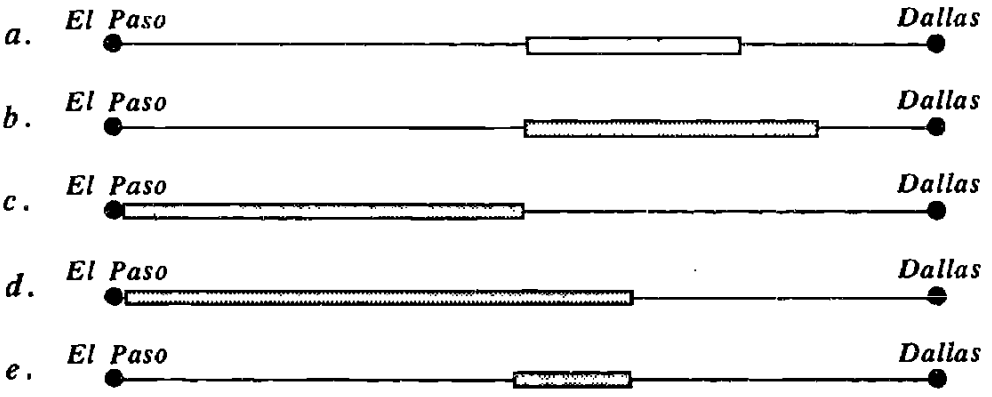
Write a statement about how much the average score is increased if each individual score is increased by x points. Convince another student that the statement is true.

Students can work in small groups. A computer is useful for investigating other situations or to consider the scores of an entire class. This sample focuses on students' ability to generalize from specific cases.

Sample 4

On a trip from El Paso to Dallas, Sara fell asleep after half the trip. When she awoke, she still had to travel half the distance she traveled while sleeping. For what part of the entire trip had she been asleep? Assuming that the shaded part in each diagram shows when Sara was asleep, which diagram best depicts the answer?

This sample assesses whether students can interpret the problem and identify the correct representation of the solution. Fractions must be considered in relation to different units. Students must think of the trip as a whole and then consider the portion of the total trip during which Sara was asleep.



Sample 5

Find the least common multiple of the following numbers:

a. 7 and 49
b. 8 and 9
c. 1 and 5
d. 5 and 6 and 20

Assessment should focus on whether students can arrive at the correct answer with facility. Note that in selection "a" one number is a multiple of the other; in selection "b" the numbers are relatively prime; in answer "c" one number is 1; and "d" might require a modification of procedures previously used.

Test-Taking Strategies

1. If the score is based on the number of questions you answer correctly, you should answer every question (even if you have to guess). Do this before going to the next problem.
2. You should put a light mark next to questions that you may be able to work but that will require more time. If time permits, you can then try to work these. Erase the light marks before turning in your answer sheet.
3. Make sure you scan all of the answer choices. Some of the choices are those that are obtained when a common error is made.
4. Sometimes the answer choices may give you an idea as to how to work a problem.
5. If you do not know how to solve a problem using standard methods you may try working it backwards--that is trying each answer and seeing which one works. As your first test, pick a middle-sized answer and plug it in. Is it to little? If so, then you need answers with bigger numbers. If this middle-sized answer is too big, then test only the answer choices with smaller numbers.
6. Estimating the result of a certain calculation can reduce the amount of time needed to obtain a solution. If the answers are spread apart from each other, estimating is easy. If they are close to each other, you may not be able to estimate effectively.
7. Sometimes answer choices can be eliminated because you can see without computation that they cannot possibly be right.
8. Make sure that the question you answer is the question that was asked.
9. When first approaching a word problem, put down your pencil, read the problem over quickly to get the general idea, identify the desired unkown, and scan quickly over the answer choices.
10. Some problems contain too much information. Weed out that which is unnecessary as you set up equations, and don't worry if you do not use all the information given.
11. Remember that an answer can be represented in more than one way. If you're fairly sure you did a problem correctly, check to see if the answer you got is the same as one of the answer choices given for the problem but given in a different form.
12. When you can eliminate one answer choice or more, try to make an educated guess and record an answer.
13. Make full use of the time alloted. If you have attempted all of the problems and have some time left, take the time to check your work.
14. If you are allowed to use scratch paper, use it. Do not recopy a problem from a test to the scratch paper if it is unnecessary.
15. Avoid spending too much time on any one question.
16. Make sure that the answer sheet is filled in correctly. Be careful when erasing; take the time necessary to ensure that answers to problems worked are placed correctly on the answer sheet.

Resource: McAllen ISD: *The Magic of Math* handbook for teachers.

Grading

Different purposes are served by assessing students' knowledge in the mathematics classroom. One of these purposes is the grading of students. Teachers are faced with a number of decisions that they must make concerning timing, form, rigorousness, and usability of assessment. Any single form of assessment is too limiting to describe fully a student's mathematical knowledge. Thus a variety of methods over time would best evaluate the different aspects of mathematical knowledge.

Standard 1 in the Evaluation Section of the *Standards* is titled "Alignment." Standard 1 calls for the methods and tasks for assessing students' learning to be aligned with the curriculum's:

1. Goals, objectives, and mathematical content;
2. Relative emphases given to various topics and processes and their relationships;
3. Instructional approaches and activities, including the use of calculators, computers, and manipulatives.

Teachers might begin changes in classroom assessment following five steps suggested by Lester and Kroll (1991).

Step 1: Start small.

Step 2: Incorporate assessment into the class routine.

Step 3: Set up an easy and efficient record-keeping system.

Step 4: Establish an assessment plan.

Step 5: Personalize the assessment plan.

In assessing students, teachers need to remember that giving a correct answer does not mean a student knows a concept. Students may give a correct solution for the wrong reason. Thus information on reasoning must come from open-ended questions and verbal interactions. Effective assessment of critical thinking depends to a great extent on how well teachers facilitate the communication of evidence of students' understandings, critical thinking, and reasoning.

Finally, portfolios have become increasingly used in mathematics assessment. The article by Jean Stenmark in the Appendix, "Math Portfolios: A New Form of Assessment," discusses the purpose and uses of portfolios to assess student learning. Included in a portfolio might be: written descriptions of the results of mathematical investigations; extended analyses of problem situations; descriptions and diagrams; statistics and graphics; reports; responses to open-ended questions or homework; group reports; copies of awards; photographs of projects; and other ideas. Dates should be placed on all entries. Portfolios can provide essential evidence of performance and ability beyond factual knowledge.

Homework

Assignments may be given for completion in class or at home. Two purposes of assignments might be to train students in meeting responsibility and to provide practice after a skill-oriented lesson. The assignments may be scored by the students themselves or by the teacher. Starting assignments during class time provides students the opportunity to enlist needed help from the teacher or peers. Homework provides an opportunity for parental involvement.

David Johnson in *Every Minute Counts* offers some possible starts to a class period that are more effective than roll call. For example, as soon as students enter the classroom, they may take out paper and pencil and solve a typical problem chosen by the teacher from the previous night's assignment. If only a few students are able to solve the exercise, reteaching is in order. Also, answers to homework assignments may be written on transparencies before students enter for checking work at the start of class. A walk around the class to view scores on each page indicates how well each student is mastering objectives. Collecting homework immediately after checking inhibits learning from mistakes. If students are not provided opportunities to see and correct errors, there is a good chance they will make the same mistakes again.

A few basic guidelines for assigning homework suggested by David Johnson are listed below.

- Know what is being assigned.
- Know what the students can do.
- Know the importance of timing. (Don't give homework too early in the class or just as the students are dismissed.)
- Make sure the students understand the assignment.
- Be careful in assigning reading in the mathematics textbook.
- Be positive about homework.
- Don't spend precious planning time correcting papers.
- Think about valuing homework scores in grading.

Appendix

Math Portfolios: A New Form of Assessment

by Jean Kerr Stenmark

From *Teaching K-8*. August/September 1991 (pages 62-68).

Some ideas on the creation of math portfolios—what should be in them, how to assess them, questions about assessment . . . plus an introduction by Shirley Frye, Immediate Past President of the NCTM.

This excellent reference by Jean Stenmark can be the guide you need for initiating the use of math portfolios in your classroom. A portfolio, as you know, is a showcase of student work that focuses on growth over time. The contents reveal the changes and consistencies of each student's attitudes and thinking processes, and provide a basis for discussion between you and the student. The portfolio also provides evidence of areas of achievement and success for the learner, the teacher and the parent.

Implementing the portfolio as a new form of assessment enables you, the teacher, to develop a comprehensive profile of each student's progress and growth in mathematical ideas. Monitoring the acquisition of skills and concepts—and the ability to combine them with the individual's interests and perspective—offers a most effective way to provide feedback to the teacher about the student's understanding of the content.

The collection of work is determined by the goals of the mathematics program for your children and the facets of interest of your students. The exhibit of important work illustrates the attainment and performance of the individual through the year and becomes the basis for pride in accomplishment. This is a uniquely positive technique for evaluation that is appropriate at all levels and for all subject areas. Portfolios can become a valuable assessment tool for a district, school, grade level or individual teacher's program.

I invite you to consider portfolios of mathematics work for your students.

—Shirley M. Frye, Past President,
National Council of Teachers of Mathematics

Student portfolios are well-known in art and writing, but until now have been rarely used to keep a record of student progress in mathematics. Teachers have always kept folders of student work, but portfolios may now have more focus and be more important for assessment.

What's in a math portfolio? The answer to this is that it's pretty much up to you and your children. Teachers and their students should be allowed to choose most of the items to include in a portfolio, since it gives a good indication of what is valued. Occasionally it may be desirable, for the sake of comparison, for some outside agency to ask for the inclusion of a certain type of item, but this should be the exception.

For the most part, teachers and students should be able to present and explain their portfolios to outside observers.

A math portfolio might include samples of student-produced: written descriptions of the results of practical or mathematical investigations; pictures and dictated reports from younger students; extended analyses of problem situations and investigations; descriptions and diagrams of problem-solving processes; statistical studies and graphic representations.

Other likely candidates for inclusion in a math portfolio are reports of investigations of major math ideas such as the relationship between functions, coordinate graphs, arithmetic, algebra and geometry; responses to open-ended questions or homework problems; group reports and photographs of student projects; copies of awards or prizes; video, audio and computer-generated examples of student work; and other material based on project ideas developed with colleagues.

The use of portfolios places an emphasis on documentation, of course. Putting dates on all papers becomes more important. First draft or revised writing should be acceptable, but with a note attached about which it is. The names of group members should probably be on papers done by a group; if not, there should be at least an indication that it was group work.

Obvious advantages. There are a number of obvious advantages that can be gained through the use of math portfolios. For example, such portfolios provide evidence of performance beyond the factual knowledge that has been gained; assessment records that reflect the emphases of a good mathematics program; and a permanent and long-term record of a student's progress, reflecting the lifelong nature of learning.

Less obvious advantages include opportunities for improved student self-image as a result of showing accomplishments rather than deficiencies; an active role for students in assessing and selecting their work; a clear and understandable picture, instead of mysterious test score number; and recognition of different learning styles, making assessment less culture-dependent and less biased.

A math portfolio may also contain important information about a student's attitude toward math In this sense at least, it can be considered a mathematical biography, one which is renewed each year.

Teachers should look at many portfolios before trying to establish a standard of assessment. Because portfolios should reflect the instructional goals of each situation, the detailed descriptions of assessment standards will vary.

Asking questions. An important part of assessment is asking questions. Asking the right question is an art to be cultivated by all educators. Low-level quizzes that ask for recall or simple computation are a dime a dozen, but a good, high-level, open-ended question that gives students a chance to think is a treasure!

Here are some helpful suggestions about assessment questioning:

- Prepare a list of possible questions ahead of time, but unless the assessment is very formal, be flexible. You may learn more by asking additional or different questions.

- Give students plenty of wait time so that they can give thoughtful answers.

- Do not use leading questions or feedback for formal assessment, although some assessment techniques include teaching during the examination.

- Make a written record of your observations. A checklist may or may not be appropriate.

The following is a starter list. You'll want to build a collection of your own good questions:

Problem Comprehension
Can students understand, define, formulate or explain the problem or task? Can they cope with poorly defined problems?

- What is this problem about? What can you tell me about it?
- How would you interpret that?
- Would you please explain that in your own words?
- What do you know about this part?
- Do you need to define or set limits for the problem?
- Is there something that can be eliminated or that is missing?
- What assumptions do you have to make?

Approaches and Strategies
Do students have an organized approach to the problem or task? How do they record? Do they use tools (Manipulatives, diagrams, graphs, calculators, etc.) appropriately?

- Where would you find the needed information?
- What have you tried? What steps did you take?
- What did not work?
- How did you organize the information? Do you have a record?
- Did you have a system? a strategy? a design?
- Have you tried (tables, trees, lists, diagrams...)?
- Would it help to draw a diagram or make a sketch?
- How would it look if you used these materials?
- How would you research that?

Relationships
Do students see relationships and recognize the central idea? Do they relate the problem to similar problems previously done?

- What is the relationship of this to that?
- What is the same? What is different?
- Is there a pattern?
- Let's see if we can break it down. What would the parts be?
- What if you moved this part?
- Can you write another problem related to this one?

Flexibility
Can students vary the approach if something is not working? Do they persist? Do they try something else?

- Have you tried making a guess?
- Would another recording method work as well?
- What else have you tried?
- Give me another related problem. Is there an easier problem?
- Is there another way to (draw, explain, say...) that?

Communication

Can students describeor depict the strategies they are using? Do the articulated their thought processes? Can they display or demonstrate problem situations?

- Could you reword that in simpler terms?
- Could you explain what you think you know right now?
- How would you explain this process to a younger child?
- Could you write an explanation for next year's students (or some other audience) of how to do this?
- Which words were most important? Why?

Curiosity and Hypotheses

Is there evidence of conjecturing, thinking ahead, checking back?

- Can you predict what will happen?
- What was your estimate or prediction?
- How do you feel about your answer?
- What do you think comes next?
- What else would you like to know?

Equality and Equity

Do all students participate to the same degree? Is the quality of participation opportunities the same?

- Did you work together? In what way?
- Have you discussed this with your group? with others?
- Where would you go for help?
- How would you help another student without telling the answer?
- Did everybody get a fair chance to talk?

Solutions

Do students reach a result? Do they consider other possibilities?

- Is that the only possible answer?
- How would you check the steps you have taken, or your answer?
- Other than retracing your steps, how can you determine if your answers are appropriate?
- Is there anything you have overlooked?
- Is the solution reasonable, considering the context?
- How did you know you were done?

Examining Results

Can students generalize, prove their answers? Do they connect the ideas to other similar problems or to the real world?

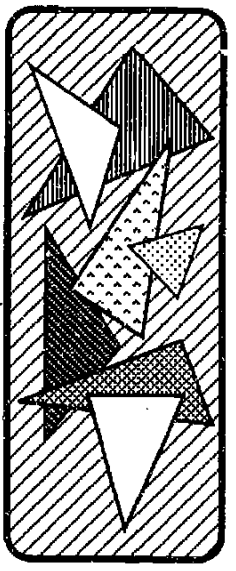
- What made you think that was what you should do?
- Is there a real-life situation where this could be used?
- Where else would this strategy be useful?
- What other problems does this seem to lead to?
- Is there a general rule?
- How were you sure your answer was right?
- How would your method work with other problems?
- What questions does this raise for you?

Mathematical Learning
Did students use or learn some mathematics from the activity? Are there indications of a comprehensive curriculum?

- What were the mathematical ideas in this problem?
- What was one (or more) thing(s) you learned?
- What are the variables in this problem? What stays constant?
- How many kinds of mathematics were used in this investigation?
- What is different about the mathematics in these two situations?
- Where would this problem fit on our mathematics chart?

Self-Assessment
Do students evaluate their own processing, actions and progress?

- What do you need to do next?
- What are your strength and weaknesses?
- What have you accomplished?
- Was your own group participation appropriate and helpful?
- What kinds of problems are still difficult for you?



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