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ABSTRACT

This report describes a program for improving higher-order thinking skills in mathematics of (n=17) third-, (n=27) fifth-, and (n=27) sixth-grade students in a middle class community. Three interventions were chosen: (1) cooperative learning to develop student self-confidence and to improve student achievement, (2) the instruction of students in mathematical problem-solving strategies, and (3) curriculum revision with the addition of a supplementary program on mathematical problem solving. All strategic solutions were related to improving student cognition and advancing student achievement on higher-order thinking skills. All of the components that contributed to the original problem were reduced as projected: student acquisition of mathematical problem-solving strategies became evident, student confidence levels in mathematics increased, and student achievement on non-routine problems requiring higher order thinking skills improved. Appendices include: problem-solving pre- and post-test results, sample problems, student survey, teacher questionnaire, teacher observation checklist, sample test questions, attitude survey, student evaluation form, student reflection sheet, and sample activities. (Contains 55 references.) (Author/MKR)

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ED 383 526

IMPROVING STUDENT HIGHER-ORDER THINKING SKILLS IN  
MATHEMATICS

by

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Submitted in partial fulfillment for the  
requirements for the degree of Master's  
of Arts in Education

Saint Xavier University-IRI  
Field-Based Master's Program

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Site: Wheeling  
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## Abstract

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SITE: Wheeling  
DATE: April 1994

TITLE: Improving Student Higher-Order Thinking Skills in Mathematics

**ABSTRACT:** This report describes a program for improving higher-order thinking skills in mathematics of third, fifth, and sixth grade students, in a middle class community located in a suburb of Chicago, Illinois. The problem was noted by the teaching staff, the principal, and the district administrators, who found students unable to solve non-routine problems in which higher-order thinking skills were necessary to find the solutions. Teacher anecdotal records, standardized test scores, district problem solving criterion reference tests (CRT), and the Illinois Goal Assessment Program (IGAP) test in mathematics confirmed the problem and described its extent.

Analysis of the probable cause data revealed that students were primarily exposed to a curriculum that was historically based in computational skills. Non-routine problems were rarely addressed by the textbooks presently in use. In conjunction with the inadequate textbooks, teaching methods were limited to direct instruction that emphasized the product rather than the processing of the problems.

Three interventions were chosen: cooperative learning to develop student self-confidence and to improve student achievement; the instruction of students in mathematical problem solving strategies; and curriculum revision with the addition of a supplementary program on mathematical problem solving. All strategic solutions were related to improving student cognition and advancing student achievement on the higher-order thinking skills.

All of the components that contributed to the original problem were reduced as projected: student acquisition of mathematical problem-solving strategies became evident, student confidence levels in mathematics increased, and student achievement on non-routine problems with higher order thinking skills improved.

## Chapter 1

### STATEMENT OF PROBLEM AND DESCRIPTION OF CONTEXT

#### Problem Statement

The students at a suburban elementary school exhibit a deficiency in higher-order thinking skills in mathematical problem solving as evidenced by standardized tests, the Illinois Goal Assessment Program (IGAP), district criterion reference testing (CRT), published tests, and teacher observation of student performance on daily activities.

#### Description of Immediate Problem Setting

The elementary school consists of 469 kindergarten through sixth grade students. The racial/ethnic background of the school population includes 87.4 percent Caucasian, 1.5 percent African-American, 4.5 percent Hispanic, 6.6 percent Asian/Pacific Islander, and 0.0 percent Native American. The attendance rate of the students is 96.1 percent without any chronic truants in the school.

The mobility rate is 12.1 percent. The number of low-income families who have children attending this school is 4.1 percent (The State School Report Card, 1993).

Of the 469 students, 17 are identified with specific learning disabilities (LD), 19 have behavior disorders (BD), 33 have speech or language problems, and 34 are identified as gifted. The LD students receive regular classroom instruction with resource help as needed. The BD students have been identified in their home school within the district and they are bussed to this site for remediation. Here they are divided into three self-contained classrooms. They are mainstreamed into the regular classrooms when progress in their behavior has been exhibited. The children who need speech or language services are seen by the speech therapist. The amount of time the child is seen by this teacher depends on the severity of the problem. The students identified as gifted receive extra instruction for one to two hours a week in math, reading, or both.

The full-time staff of the school includes: a principal, 19 grade level teachers, a language arts/math specialist, a resource teacher of LD students, two teachers of self-contained BD students, a physical education teacher, a music teacher, a media resource

specialist, three special education assistants, a library assistant, and a nurse's assistant. Part-time staff includes: a school psychologist, two social workers, a computer teacher, an art teacher, a speech therapist, and a nurse. Auxiliary personnel include a secretary and three custodians. The school personnel are 97.5 percent Caucasian and 2.5 percent Asian/Pacific Islander. Ten percent of the staff are male and 90 percent are female.

There are 80 third grade students at this school. They are divided heterogeneously into four homerooms. The average number of students in each class is 20. There are two identified LD students. There are no students in the BD program at this grade level. There are nine children who receive speech and language services, and three students identified as gifted. The children remain with their homeroom teacher all day. The academic subjects are not departmentalized.

There are 54 fifth grade students at this school. They are divided heterogeneously into two homerooms. The average number of students in each classroom is 27. There are six identified LD students and one BD student in the regular classroom who receive resource services. There are two self-contained BD students who

are not mainstreamed into the fifth grade classrooms. There is one child who receives speech and language services, and three students identified as gifted. During the first quarter, the students are divided into two groups for math instruction, otherwise they remain with their homeroom teacher for all other subjects. The students will receive math instruction in two heterogeneous groups during the remainder of the year.

There are 54 sixth grade students at this elementary school. These children are grouped into two self-contained homerooms. One of the 54 students has learning disabilities. Three BD students are mainstreamed into the regular classrooms for daily instruction in specified subject areas. Ten students are identified as being gifted. One student in the sixth grade receives speech therapy.

The sixth grade homeroom in this research study on mathematical problem solving has 27 students. This class has seven students identified as gifted, twelve average students, and eight low-average students. Within this group, two students have been diagnosed as having attention-deficit-disorders. Two BD students are mainstreamed into the classroom at designated times by the BD teacher.

Mathematics is taught within the self-contained homeroom. The class is divided into two separate groups for instruction. A small group of gifted math students was intentionally placed into this homeroom in order to receive instruction in pre-algebra on a daily basis. These students also leave the homeroom for specified blocks of time during each grading period to work in a gifted math class. The sixth grade level math group consists of the remaining 21 students. Direct instruction is used when new concepts are introduced. Small group work or individualized help is given to the low students to strengthen and reinforce their math skills. The accelerated pre-algebra students and the sixth grade level students are given whole group instruction and are combined for problem solving activities once a week.

#### Description of Surrounding Community

The school is within a large district located approximately 35 miles northwest of a metropolitan area. There are 6,355 students enrolled in the nine elementary and two junior high schools. The junior high schools feed into two high schools. The population within these schools is 74.5 percent Caucasian, 6.8 percent

Asian/Pacific Islander, 2.9 percent African-American, 15.6 percent Hispanic, and 0.2 percent Native American.

The community encompasses 8.28 square miles. The population of the community is 37,909. It has seen a 40 percent growth over the ten year period from 1980 to 1990. It is estimated to grow to 43,200 by the year 2005. The per capita income is \$23,718 and the mean income per household is \$64,797. The number of people living below the poverty level is 1.5 percent. The community consists of 94.2 percent Caucasian, one percent African-American, 0.1 percent Native American, 4.4 percent Asian/Pacific Islander, two percent Hispanic, and 0.4 percent other.

The majority of the land use is residential with 33.9 percent being single family homes, six percent for single family attached homes, and 3.6 percent for multi-family dwellings. Open space in the community accounts for 14.5 percent with 3.4 percent of the community comprised of unoccupied buildings and residences. The remaining land is used for industry, office buildings, and commercial businesses. The mean value of a house is \$175,529 and the median rent is \$754.

The majority of the population (51.4 percent) within the



community hold a college degree, and 23.3 percent have some college background. Of the remaining community, 19.4 percent are high school graduates and 5.9 percent have less education (1990 Census of Population and Housing).

### Regional and National Context of the Problem

Historically, American elementary school children have performed poorly on mathematical problem-solving activities and tests in comparison to their achievement on arithmetic computation problems and tests. This sub-standard performance has prompted the allocation of funds to improve the teaching of mathematics. Many commissions have been formed to study this issue and provide solutions to the teaching strategies that must be employed to strengthen the problem-solving skills of America's children. However, much of the work of these committees has been devoted to secondary school students. Those that work at the elementary level know that problem solving needs to be addressed before the students enter high school (Stevenson, Lee, and Stigler, 1986).

Stevenson, Lee, and Stigler (1986) researched the mathematical achievement of elementary students, kindergarten through grade

five, in Japan, China, and the United States. The mathematics tests were based on the content of the textbooks used in the three countries. Some items on the test required only computational skills and others needed to have problem-solving strategies utilized. The results of the research showed the children in American kindergarten as being slightly behind the Japanese children in their ability to understand mathematics. By the time these children were in fifth grade, they were surpassed by both Japanese and Chinese students.

Mayer, Tajika, and Stanley (1991) conducted research on the findings of Stevenson, Lee, and Stigler (1986). The findings of this study concur with previous studies that involved international comparisons. The Japanese children in this research achieved higher levels of performance on mathematical achievement tests in comparison to their American peers.

To keep American students in competition with other industrialized countries, problem-solving skills need to be addressed. According to Stevenson, Lee, and Stigler (1986, p. 698):

Although a small proportion of American children perform superbly, the large majority appear to be falling behind their

peers in other countries. The poor performance of children in mathematics thus reflects a general failure to perceive that American elementary school en are performing ineffectively and that there is a need for improvement and change if the United States is to remain competitive with other countries in areas such as technology and science which requires a solid foundation in mathematical skills.

## Chapter 2

### PROBLEM DEFINITION AND EVIDENCE

#### Problem Background

As indicated in Chapter 1, the need for improvement of mathematical thinking skills for American school children has become a growing concern in the United States. Declining standardized test scores and poor student attitudes have caused researchers and educators to closely address this issue. The National Council of Teachers of Mathematics (NCTM, 1989, 1991) and the National Research Council (1989, 1990) have contributed to influential reports which represent the thinking of many people interested and involved in the change of mathematics education. These reports criticize the current emphasis on the acquisition of facts and technical skills found in today's mathematics instruction. The research contends that this type of mathematical expertise is deficient in today's changing technological world. Proficiency with factual knowledge and algorithms is now found to be of less

importance than the ability to pose and solve non-routine problems while learning both independently and in collaboration with others (Siegel and Borasi, 1992).

Locally, student problem-solving deficiencies have been measured by the Stanford Achievement Tests, the Illinois Goals Assessment Program, district criterion reference testing, published tests, and teacher observation of student performance. Steps have been undertaken, at the district and building levels, to improve mathematics instruction and improve the curriculum.

Several math programs have been reviewed by district committee members to assess their conformity with the district objectives, state goals, and the standards proposed by the NCTM. Everyday Mathematics was adopted for the primary grades. This program is not yet completed for grades five and six, and other programs did not fulfill the needs of the district. Therefore, it was decided that the intermediate students would continue to use the present textbook.

The mathematics scores for the IGAP tests in third and sixth grade showed weaknesses in problem solving. Students in school buildings which piloted the Everyday Mathematics program scored significantly higher than those in buildings which were not in the

pilot study. The sixth grade scores were comparable across the district. The target site was not one of the school buildings in the pilot program. The principal has therefore set a building goal to have the third and sixth grades improve their math scores. The projected outcome is to have ten percent of the students move from the "meets district goals" category to the "exceeds district goals" classification. The entire staff has been called upon to address this issue and formulate a plan to improve problem-solving skills for all students. To achieve this goal, the primary teachers are using Everyday Mathematics; a floating teacher has been assigned to teach one-third of the fifth grade class for math to alleviate large class sizes, thus increasing the teacher-student interaction time for problem-solving; and the sixth grade target group engages in flexible grouping and whole-group activities focused on non-routine problem solving. The IGAP test given in March of 1994 will indicate any improvements made because of these program implementations.

### Problem Evidence

Teachers and administrators observed problem solving to be a weakness on the Stanford Achievement Test. This standardized test

is given at the third through sixth grade levels during the first week of October. The math section is grouped into three sections: Concepts of Number, Mathematics of Computation, and Mathematics Application. The problem-solving skills are tested in the Mathematics Application section.

Table 1  
The Percentages of Third, Fifth, and Sixth Grade Students in scoring categories on The Mathematics section of the Stanford Achievement Test for 1993

Section of the Stanford Achievement Test	Below Average			Average			Above Average		
	gr. 3	gr. 5	gr. 6	gr.3	gr.5	gr.6	gr.3	gr.5	gr.6
Problem-solving Subtest	13	6	6	57	55	45	30	40	48
Mathematical Application	11	6	2	51	49	41	38	45	58
Mathematical Computation	8	4	2	57	29	57	36	40	62

On the fourth, fifth and sixth grade tests, the problem-solving questions comprise more than 50 percent of the mathematical application test section. On the third grade test, less than 50 percent of the mathematical application questions are problem solving. The problem-solving section of the test has 6 questions at third grade, 20 at fourth grade, and 22 at fifth and sixth grades. The Mathematics of Computation section has 36 questions at the third

grade level, and 44 questions at fourth, fifth, and sixth grades.

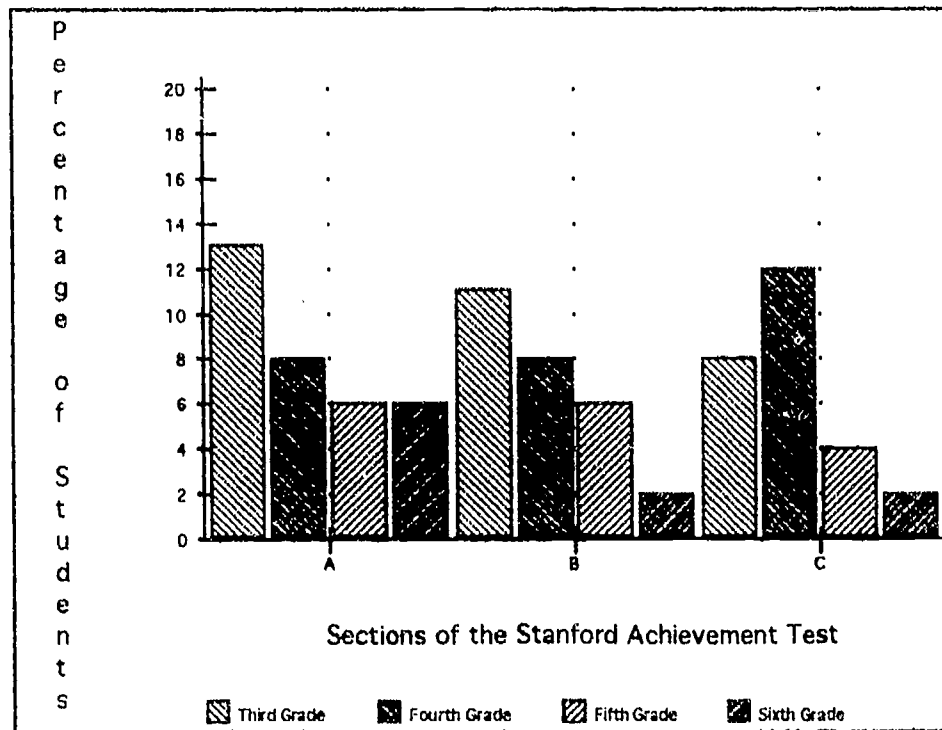


Figure 1  
 Percent of Students Performing Below Average  
 on the Stanford Achievement Test  
 A = Problem-solving Subtest of the Mathematical Applications Section  
 B = Mathematical Applications Section  
 C = Mathematics Computation Section

According to the criteria established for the Stanford Achievement Test, the above average range is 70 percent to 99 percent of correct responses, the average range is 30 percent to 70 percent, and the below average range is one percent to 30 percent. On last year's test, only the fourth grade scored lower in computation than in application. More students scored in the below-



average range in problem solving than did in the application section in third and sixth grade. The same number of students in fourth and fifth grade students scored in the below-average range in the subtest of problem solving as in the application section. The scores have shown improved student performance in each subsequent year.

The pattern of improved test scores on the Stanford Achievement Test from the third to the sixth grade may be partially attributed to the yearly repetition of the mathematical concepts being tested. The students, as they progress through school, also become more accustomed to the format and the skills being tested. However, the subtest on problem solving indicates that students at grades three and six are weaker in applying problem-solving strategies in comparison to their ability to solve basic computation and application problems. The fourth and fifth graders scored below expectations on the problem-solving subtest and applications section. The percentage of fourth grade students below average on computation is unproportionately high in relation to the other grade levels. Teacher observation purports the discrepancy between third and fourth grade scores to be a result of the contrasting levels of difficulty between the two grade levels.

A 10 question pretest on non-routine problem solving was administered to 38 third grade students, 27 fifth grade students, and 27 sixth grade students in the three target groups. The ten strategies upon which the students were tested comply with the curriculum standards presented by the National Council of Teachers of Mathematics (NCTM) in the Curriculum and Evaluation Standards for School Mathematics (1989). Table 2 shows an analysis of the pretest results for grades three, five and six in relationship to the 10 problem-solving strategies being focused upon. The Problem Solver at levels two, three, four, and six (Moretti, Stephens, Goodnow, and Hooegeboom, 1987) was used as the source for the questions on the three grade level pretests.

The third grade pretest consisted of second grade materials. The third grade materials seemed to be at too high a level for this target group. The researchers did not want the students to be overly frustrated while taking the pretest. During the testing, a few students asked for help whereas, the rest solved the problems independently. Only a few students expressed any discomfort or frustration about taking the test. There were not any comments about having to take the test.

There were five areas on the third grade test in which at least 73 percent correctly answered the question. On the remainder of the test, a maximum of 38 percent of the students answered correctly. The majority of the students who took the test responded correctly on four or five of the questions. The students performed better than anticipated on this pre-test. The researchers view this performance due as a result of the students being taught with a new mathematics series called Everyday Mathematics (from the University of Chicago Mathematics Program) which is based upon the NCTM standards.

Table 2  
Problem-solving Pretest Administered to the Target Groups  
September, 1993

Concepts Tested	Percent Correct		
	Third Grade	Fifth Grade	Sixth Grade
Logical Reasoning	76	48	7
Organized List	38	0	4
Use or Make a Table	77	7	26
Use or Make a Picture	30	22	0
Guess and Check	78	0	12
Use or Look For a Pattern	84	15	7
Act Out or Use Objects	73	4	19
Work Backwards	38	0	0
Make it Simpler	5	4	0
Brainstorm	5	30	0

N Third Grade = 38 Students  
N Fifth Grade = 27 Students  
N Sixth Grade = 27 Students

After previewing problem solving materials for the fifth grade

pre-test, it was decided that Problem Solver 4 would be used as the source of the test questions. It was determined through teacher observation that the fifth grade level would be too difficult for many of the children in the group. Upon observing the reactions of several students during the testing session, even the level four material appeared to raise the level of concern in many students. A few children expressed frustration and anger due to their inability to find solutions to the problems. Comments such as "I hate this!", "I can't do this!", and "I quit!" were heard.

An analysis of the results of the pretest showed that the students lack the skills needed to solve mathematical problems that require the use of higher-order thinking skills. Only four percent of the students were able to answer four of the ten problems correctly. Four percent of the students made three correct responses and 33 percent of the students made two correct responses. Thirty-seven percent of the students were able to make only one correct response and 22 percent were not able to solve any of the problems correctly.

Each of the ten strategies was analyzed to measure the percentage of correct responses. Three of the strategies (make an organized list, guess and check, and working backwards) were not

answered correctly by any of the students in the target group. The strategy of using logical reasoning had the highest percentage of correct responses with 48 percent of the students getting it right. The analysis of the fifth grade pretest indicates a deficiency in the students' ability to perform higher-order thinking skills in mathematics.

The sixth grade pretest was given in three sessions during the students' regular math period. The students were allowed 15 minutes to work on each problem. (This time allotment was chosen due to the complexity of the problems at this grade level.) One problem representing each strategy was selected for this test. Only one problem at a time was issued to the students to eliminate the possibilities of students working ahead or going back to previous problems that they might want to alter. Attached to each question was a student evaluation form for the problem that was either just completed, attempted, or abandoned. The purpose of the evaluation form was to let the researcher assess the students' familiarity with the type of strategy needed to solve the prescribed problem. Also in compliance with the NCTM curriculum standards, the researcher wanted to evaluate the target group's ability to communicate their

mathematical ideas in written form.

The students worked diligently on the pretest. However, their facial expressions and body movements exhibited the confusion they were experiencing while doing some of the test items. Sighs of relief and smiles were evident when the students felt they were being successful in their endeavors. The children were told that no assistance would be given to them during the pretest sessions. The high-ability students did not give credence to this information and were persistent in asking in-depth questions about the problems. Their requests for further explanations were denied in order to prevent the results of the pretest from becoming invalid.

The assessment of the pretest showed that a total of 20 correct responses were given by the sixth grade target group. Thirty-three percent of the students were able to arrive at the correct solutions for the problems. Nineteen percent of these students are participants in the gifted math and reading programs, four percent of the students are only in the gifted reading program, and the 11 percent of the remaining students are not members of any accelerated group. Seventy-five percent of the correct responses were given by boys.

The students were the most successful in employing the use of a table to solve a problem. Twenty-six percent of the students got this problem correct. No correct solutions were found for the problems that required the students to: make a picture, work backwards, break problems down into simpler ones, and brainstorm ideas. Further results are shown in Figure 2.

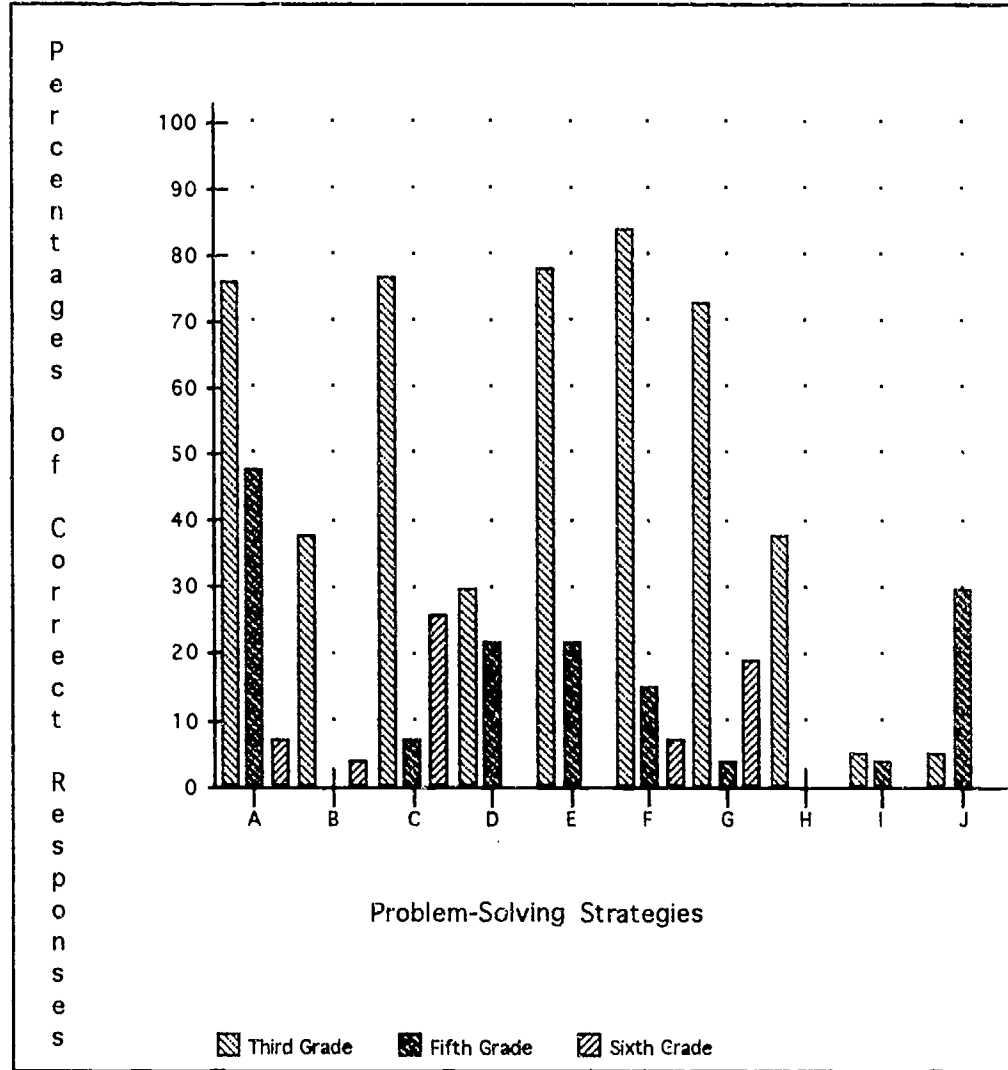


Figure 2  
 Results of the Problem-Solving Pretest  
 A = Logical Reasoning  
 B = Organize a list  
 C = Use or Make a Table  
 D = Use or Make a Picture  
 E = Guess and Check  
 F = Use or Look for a Pattern  
 G = Act Out or Use Objects  
 H = Work Backwards  
 I = Make it Simpler  
 J = Brainstorm

The student evaluation forms indicate the students in the sixth



grade target group generally found the non-routine problems to be interesting or tolerable. Seven percent of the students found all the problems uninteresting and seven percent of the low-ability students also found some of the problems to be dull. The students' viewpoints as to whether the problems were easy, average, or difficult in nature reveals their attitude that they are capable of solving non-routine problems correctly. The pretest results contradict their beliefs. The questions the students often felt to be easy or average in difficulty were the ones on which they had very few or no correct answers.

It can be noted that the third grade was administered second grade problems on the pretest. This may explain the third grade success relevant to the scores of the other target groups in fifth and sixth grade.

#### Probable Causes of Problem

Data to indicate probable cause factors was collected from three sources within the setting. The researchers created a teacher questionnaire to determine teaching methods, a student survey to understand attitudes toward math, and a pretest on problem solving

to assess problem-solving ability.

To determine the teaching methods of the staff, a questionnaire was created and distributed to the classroom teachers. Of the nineteen classroom teachers, sixteen returned the survey. The special education teachers were given the survey, but declined to respond. Due to the low academic ability and behavior of the special education students, the teachers utilize direct instruction methods for skill remediation. The following is a summary of the responses.

The first question dealt with teaching methods. This question was asked to ascertain whether the teachers are keeping up-to-date with the latest trends in education. Eighty-one percent of the teachers have changed the way they teach math in the past three years. The majority of these responses came from primary teachers and their responses may reflect the recent adoption of the University of Chicago Mathematics Program. Because of this new program the following changes have been made to the teaching and learning process: a focus on problem solving and higher-order thinking skills, a use of manipulatives, looking for patterns, more hands-on activities, games, discovery, cooperative learning, mental math, student-generated problems, integrated curriculum,

and application to real-life situations, fewer paper and pencil tasks, and direct instruction.

The teachers who are using the computational based curriculum use these teaching methods: direct instruction, cooperative groups, inquiry method, journals, looking for patterns, and analyzing, estimating, and checking. The teachers who use the inquiry method have found that the students tended to want more teacher direction and were less willing to try the problems. Also, the students with low computational skills had their progress hindered by their lack of ability.

The teachers did not find anything that proved to be unsuccessful for the students in their present teaching methods. Eighty-one percent of the teachers were satisfied with the students' responses to the teaching methods. Seventy-five percent of the teachers were satisfied with present student progress as compared to progress in the past.

Sixty-nine percent of the teachers incorporate higher-order thinking skills with their computational skills. Sixty-three percent of the teachers frequently or consistently use supplemental materials along with the curriculum. Sixty-nine percent of the

teachers feel students respond favorably to higher-order thinking assignments. Eighty-one percent of the teachers occasionally or frequently believe the students to be teacher dependent when working on higher-order thinking assignments.

The incorporation of problem solving in math class occurs once a week or more according to 75 percent of the teachers. The teachers were divided in their use of Arithmetic Developed Daily (A.D.D.) which is a daily supplemental program. It contains a variety of math problems at the student's grade level. On half of the sheet there are computational problems. The other half is problem solving. One-half of the teachers surveyed implemented A.D.D. at least once a week. The remaining half of the teachers used A.D.D. once a month or less. Those that do employ the use of A.D.D. more frequently, find it helps the students to think problems through, to process information, to pick an appropriate strategy, and to perform better at mental math.

At least 63 percent of the teachers use the following strategies to teach problem solving in the classroom: multi-step problems, estimating and rounding, working backwards, calculators, drawing pictures, manipulatives, guess and check, tables, graphs, charts,

cooperative learning, and direct instruction. Less than 63 percent use these strategies: computers, journal writing, and the inquiry method.

Table 3  
Percentage of Responses to the Student Math Attitude Survey

Questions Asked	Rarely		Sometimes		Often	
	Third	Fifth	Third	Fifth	Third	Fifth
Can do math on own	3	12	19	25	7	63
Difficulty due to readability	47	48	47	48	6	4
Difficulty due to application	53	36	42	56	5	8

The target group which consisted of 38 third grade students and 27 fifth grade students were administered a survey. The intent of the survey was to determine the students' perception of their mathematical ability, their feelings toward computational math and word problems, the amount of time spent on math homework, and the ways the students perceive using math outside of school. Tables 3 and 4 show the information given by the students on this survey.

An analysis of the third grade responses shows fifty-eight percent of the students who filled out this survey answered that they liked math. Sixty-one percent feel they are good at math. Only

twenty-one percent of the students felt they could not do math problems on their own.

Overall, the students felt the concepts dealt with in math were easy for them. The two areas in which most student responses were in the 'okay' category were multiplication and division, which are still new to students in third grade. Sixty-three percent of the students felt word problems were easy for them which was more than their responses for multiplication and division.

Seventy-six percent of the students received help with their homework. The current math curriculum from the University of Chicago Math Program, Everyday Mathematics, has assigned daily homework in which the parents are supposed to help to make the connection from school math to real-life math. Most students spend approximately fifteen minutes on their math homework each night.

The majority of students responded that they sometimes do math at home for fun. Based upon the student responses, the students' idea of doing math at home is just working out problems for no reason, or using flash cards. The student responses were answered more favorably than anticipated. The majority of the students gave answers such as: multiplying, adding, subtracting, or using flash

cards, but did not give the reason for doing these operations.

A little more than half the students see the connection between working with money in school to using it outside of school. About ten percent of the students see math outside of school as shopping, cooking, dividing things to share, measuring, or counting teeth. Three students stated they did not use math outside of school. The students used the University of Chicago Math Program called Everyday Mathematics, last year which is more real life orientated than the series used in the past with the target group.

Table 4  
Percentage of Students' Feelings Toward Math Concepts  
as Reported on the Student Survey

Math Concepts	Hard		Okay		Easy	
	Third	Fifth	Third	Fifth	Third	Fifth
Addition	3	0	16	20	81	80
Subtraction	5	0	28	40	68	60
Multiplication	13	0	55	48	32	52
Division	10	8	45	40	45	52
Measuring	8	8	24	36	68	56
Telling Time	3	0	13	28	79	72
Geometry	3	16	13	68	84	16
Money	0	0	21	28	79	72
Word Problems	0	4	33	68	67	28

N Third grade = 38 Students

N Fifth Grade = 27 Students

The analysis of the fifth grade student surveys shows that more than 50 percent of the students like math and 62 percent feel they

are able to do math without help. Sixty-two percent of the students feel the basic applications (addition, subtraction, multiplication, and division) are easy. Ninety-five percent feel that they are good or at least okay at math. Forty-seven percent of the students rarely find word problems difficult because of the readability of the problems. Thirty-six percent rarely find word problems difficult because they could not figure out what application to use. The data summary of the survey is illustrated in Figure 3.



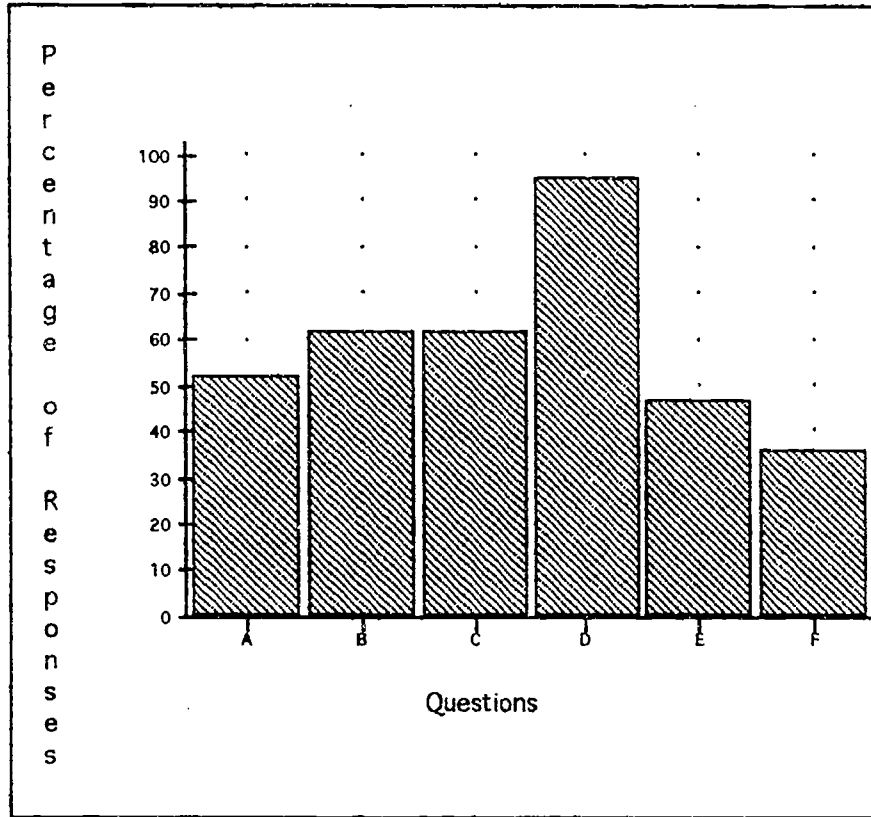


Figure 3  
 Fifth Grade Students' Attitude Toward Mathematics  
 A = Percent of students who like math.  
 B = Percent of students who feel they can do math on their own.  
 C = Percent of students who think the basic applications (addition, subtraction, multiplication, and division) are easy.  
 D = Percent of students who feel they are good or okay at math.  
 E = Percent of students who rarely think word problems are difficult because their readability.  
 F = Percent of students who rarely think word problems are difficult because they can't figure out the application to use.

A student mathematics survey was administered to the sixth grade target group during the second week of school in September 1993. As in the third and fifth grade student surveys, the sixth graders' present attitudes about mathematics were expressed.

However, the sixth grade survey, in addition to multiple-choice questions, also had questions that required the students to give written responses. A longer and more in-depth survey was given to the sixth graders since they have been receiving mathematics instruction longer than the other students. This increased experience allowed the sixth graders to reflect upon their strengths and weaknesses in the mathematics classroom, and how this will affect them as they become functioning adults in society.

Forty-four percent of the 27 students surveyed stated they enjoyed mathematics in school. Forty-one percent were neutral in regard to their like or dislike of mathematics. The remaining 15 percent did not like math. The students within the target group claimed to experience little or no stress when they were assigned word problems. The low stress factor may be attributed to the students' beliefs about their ability to be successful in mathematics class. See Table 5.

Table 5  
Correlation of Stress to Students' Mathematical  
Confidence Level at Sixth Grade in Percentages

Level of Stress	Extreme	Slight	None
Doing Word Problems	0%	37%	63%

Ability to Be Successful	Difficult in All Areas 0%	Difficult in One or Two Areas Only 37%	Easy 63%
Attitude if Incorrect Answer is Given	Embarrassed 22%	Depressed 22%	Not Concerned 56%

N = 27 Students

The majority of the students, 52 percent, conveyed they sometimes needed assistance with their mathematics homework. One percent of the students said they usually needed help with assignments, whereas 47 percent showed they consistently did their work without seeking guidance from anyone. Parents were chosen 80 percent of the time as the individuals who helped the children with their work at home. The other 20 percent of the students relied upon their older siblings to help them with their mathematics lesson.

The students were asked about the type of grouping practices they preferred for mathematics classes. The target group was equally divided with 48 percent of the students enjoying cooperative groups and another 48 percent of the students opting to learn and work independently. The other four percent replied that there were advantages to both ways of learning. The proponents of cooperative learning liked the idea that help could be found immediately within a

group while engaged in problem-solving activities. The sharing of ideas and working with friends was appealing to the sixth graders. Other students chose cooperative learning with different motives in mind, such as: one can get done faster; one does not have to work hard, and one doesn't have as much homework.

The children who preferred to work independently concurred that they liked to work under quiet conditions. Their concentration was not broken while working alone. These students were not in favor of cooperative groups because they have had bad experiences where: only part of the group works, and the other part withdraws from the activity; disputes develop within the groups; and socializing takes precedent over the assignment and the group does not complete the work at hand.

The gifted math students, in addition to having the former frustrations (as do the other students in class), have another list of disadvantages with cooperative learning. These students feel they are being academically restrained in these groups. They desire to work faster and prefer not to listen to the other students' input. Listening to others and then being made to explain the correct focus to those who were incorrect initially has become aggravating to this

fraction of students in the target groups. If these gifted students continue to isolate themselves from their classmates with these attitudes, they may find assimilation in the real-world a difficult process.

Class participation can often be used by a teacher to assess the students' levels of confidence. The sixth graders were asked to reflect upon their participation in math class. A variety of responses were given to this question ranging from little or no participation to participating in all areas of math in order to learn as much as possible. Many of the students only felt comfortable participating when they were positive they knew the correct answer. Others stated they would participate in class right away to get it over with so they would not have to answer questions later on. A few students (all girls) stated that participation in class consisted of being a good listener, learning the lesson, and doing their work. Those students who actively participate in class showed an enthusiasm for mathematics and a higher confidence level about themselves. See Table 6.

Table 6  
Sixth Grade Self-evaluation of Class  
Participation in Percentages

Amount/Type of Participation	None	Little	Bored	Enthusiastic
	15%	53%	2%	30%

N = 27 Students

In conjunction with the student participation question, students were asked how they felt when they gave an incorrect answer in class. The majority of the class felt no discomfort when they gave a wrong response during a class discussion. This seems to correlate to their apparent low levels of stress in class. See Table 5.

The survey also was directed towards the students' awareness of problem solving. Students were asked to state: why word problems were difficult for them; what problem-solving strategies they were familiar with and how many they had used; and why they would or would not like to spend more time on problem-solving in school. Students were given a checklist of possible reasons which might contribute to their inability to solve problems. They were allowed to choose the causes that were applicable to them. Twenty-two percent of the students responded that they did not understand the question being asked. Their weak reading comprehension skills interfered with their ability to solve math problems. Zero percent,

however, felt that their vocabulary recognition and their knowledge of word meanings did not interfere with their problem-solving skills. Forty-one percent of the target group said they experienced confusion about the selection of the proper mathematical operation when solving a problem. The choice of operations is unclear to them because they are not comprehending what is actually being asked in the problem. Seven percent of the children did not know what problem-solving strategies were, and 30 percent responded that they did not know which problem-solving strategies to use. These two responses indicate that the students are having difficulty comprehending a given problem. Forty-one percent of the students answered that they had no difficulty with problem solving. (The junior high schools within the district are having the reading teachers come into the math classes to teach reading skills for mathematical problem solving. The integration of reading and math skills has only been attempted in a few classes within the target school. As a result, the students within the sixth grade target group have had little instruction on the utilization of reading skills in a mathematical context.)

The students were given a list of problem-solving strategies.

They were to circle the ones they had used and underline those they had never heard of before. Most of the children indicated they had employed the procedures necessary for multi-step problems and could use their estimation skills to solve problems. This is not a surprising finding to the researcher since these strategies are the ones most commonly found in the students' textbooks each year . See Table 7. When asked if they could use manipulatives to assist them in problem-solving, 74 percent of the students responded that they did not know what manipulatives were. (The sixth graders have used manipulatives in the primary grades, but they are unfamiliar with this mathematical term.)



Table 7  
Percentage of Familiarity and Use of Mathematical  
Strategies in Sixth Grade

Strategy	Used in Class	Familiar- Not Used	Unfamiliar to Student
Multi-step Problems	78	21	7
Estimation/Rounding	59	41	0
Drawing a Picture	37	56	7
Working Backwards	30	59	11
Graphs	37	63	0
Guess and Check	37	52	11
Logical Reasoning	22	37	41
Create a Model	7	37	56
Orally Describe the Problem	19	40	41
Tables	22	56	22

The class was closely divided in opinion concerning the issue of spending more time on problem-solving in mathematics class. Fifty-six percent of the students said no to this idea. Various reasons were given to support their viewpoint. Some children already felt they knew how to problem solve. Therefore, more problem-solving would be an inefficient use of time, and it would keep them from doing other problems (algorithms). Others contend that these types of problems were long and boring. The longevity of the learning experience tended to make their hands hurt, and they developed headaches. Some students did not like these problems because this was one of their weakest areas in math.

The remaining 44 percent of students, who indicated they wanted

more problem-solving activities, also stated that problem solving was hard for them. This group's goal, however, was to do more practice in this area so they might improve their skills in problem-solving. Some of the students also viewed these problems as being more fun to do and being more relevant to real-life situations.

The issue of gender dominance in mathematics was surveyed. The students were to give their opinions as to whether boys or girls do better in mathematics, or if there is mathematical equity between boys and girls. Four percent of the sixth graders responded that boys are superior to girls in mathematics. Their reasoning for this is that the pre-algebra students within the target group are all boys. (This grouping was based on standardized test scores and teacher recommendations.) Another cause for their beliefs is that the entire gifted math program for sixth grade only has three girls enrolled in it as opposed to six boys. The majority of the class, 96 percent of the students, believe that there is no cause for one gender to be more successful than the other. The students' responses were quite similar in nature. They all revolved around the main idea that there are boys and girls who: do very well in math, do average work in math, do poorly in math, volunteer answers in class, and are quiet in

class. To these children gender is not a factor in mathematical performance.

Students were asked if mathematics was a part of their lives outside of school. The frequency of mathematical involvement done away from school was recorded. Many of the responses given gave homework as an activity outside of school. Other students used mathematics while shopping and computing prices. One student remarked that he did math while he was swimming. The students were also asked if they ever did mathematics for fun at home. See Table 8. Students of all ability levels formed the group that did math for enjoyment. They played games, worked on the computers and calculators, played school, did extra math dittos, and worked out of a textbook.

Table 8  
Percentages of Sixth Grade Students  
Doing Mathematics Outside of School  
and for Enjoyment

Math Done Outside of School	Never 7%	Once a Month 7%	Weekly 52%	Daily 34%
Math for Enjoyment	Rarely 41%	Sometimes 52%	Often 7%	

N = 27 Students

Children are greatly influenced by their parents' expectations.

The sixth grade target group indicated in the survey that their parents desired them to be successful in mathematics. Seventy-seven percent of the students felt their parents expectations were very high, and that success in mathematics was extremely important. The remaining 39 percent stated that their mathematical success was of some importance to their parents.

The parents serve as role models for the children in mathematics. Children who see their parents doing mathematics at home and know it is also incorporated into their occupations will more readily realize that being proficient in mathematics is essential for survival in the adult world. Eighty-one percent of the children surveyed stated that their parents did types of math at home. Many of the responses were centered around paying bills and balancing checkbooks. Measuring and figuring out dimensions, paying employees, and paying taxes were also mentioned by the students.

The uses of mathematics at their parents' place of work left some students in a quandary. Twenty-six percent of the students did not know if their parents used mathematics at work. Fifteen percent stated that their parents did not use mathematics in their occupations. The 59 percent of the students whose parents did

employ mathematics at work listed their occupations as being: a teacher, an exterior designer, a pharmacist, a builder, an accountant, a bank cashier, and a computer technician.

The final portion of the survey had the students discuss the reasons why they should be good mathematicians and problem-solvers. Seventy-four percent of the responses focused upon the necessity of mathematics in the adult world. Securing a good job and keeping it may be dependent upon their problem-solving capabilities. Nineteen percent of the students centered their answers around report cards and paying attention in class to learn the steps of problem-solving. These students are still concerned with day-to-day experiences and are not yet cognizant of how these skills fit into the larger scheme of life. Seven percent were not sure why they needed to be good problem solvers.

The information drawn from the survey has increased the researcher's awareness of the probable causes which may be responsible for the students' inability to solve non-routine problems in mathematics. A closer look at the research and related studies will provide more insights into the reasons for student deficiencies in problem solving.

Reports issued by the National Council of Teachers of Mathematics (1989, 1991) and the National Research Council (1989, 1990) find fault with the methodology of teaching mathematics according to traditional means. The prevalent pattern of classroom teaching consists of the review of homework, teacher explanation and student practice (Siegel and Borasi, 1992). The NCTM (1989, p.1) cites Welch in regard to NSF case studies on teaching practices.

In all math classes that I visited, the sequence of activities was the same. First, answers were given to the previous day's assignment. The more difficult problems were worked on by the teacher or the students at the chalkboard. A brief explanation, sometimes none at all, was given of the new material, and the problems assigned for the next day. The remainder of the class was devoted to working on homework while the teacher moved around the room answering questions. The most noticeable thing about math classes was the repetition of this routine.

This sequence of instruction is no longer appropriate according to the research done on mathematical learning and problem solving. However, recent studies support Welch's findings that little change has transpired over the past decade (NCTM, 1989).

Most teachers, who are currently teaching, have not been trained to be constructivists. They have never been taught to allow students to construct their own meanings of mathematical concepts and

procedures. Traditional teachers are unaccustomed to being facilitators to the students (Prevost, 1993). They conceptualize mathematics as it was when they themselves were students. Mathematics was a discipline with a static set of rules, definitions, and algorithms. Students watched the teacher model problems on the board, took notes on the steps, and did homework on similar problems. There was never any evidence of invention, discovery, analysis, or group work. The students were passive, and the teachers supplied all of the information. This was the procedure by which current teachers learned mathematics, and this is the methodology employed by them presently in their classrooms unless they have already begun to follow the NCTM standards (1989, 1991). Teachers, therefore, feel uncomfortable with the new approaches to mathematics and find the alternative assessments to accompany them to be burdensome. They are also resistant to change since they believe their teaching methods to be successful as documented by student grades, standardized test scores, and subsequent achievement levels reached by former students in mathematics (Prevost, 1993). Hence, teachers are unwilling or are unable to teach problem-solving skills to their students.

Wasserman (1987) discusses the supportive data gathered by Rath during his studies on thinking skills. The findings indicated that the utilization of thinking exercises over time enables the individual to become a more competent and self-sufficient thinker and problem solver. However, theorists have not been able to change the educational practices that hinder the development of higher-order thinking skills and problem solving. There are a variety of reasons why little has been done to implement the teaching of higher-order thinking skills in mathematics. These reasons stem from educational, societal, and personal beliefs and practices. The following considerations are examples of why educational methods have not been significantly changed (Wasserman, 1987).

Teachers have held the belief that the development of thinking skills was a by-product of subject-matter teaching. Following the prescribed curriculum was, supposedly, the key to improving the students' higher-order thinking skills (Wasserman, 1987). This is an inaccurate assumption, as discussed by Beyer (1984), for in reality inappropriate instruction is a factor which contributes to students' weaknesses in thinking skills.

Many teachers assume that children will learn how to think



critically by merely answering a set of questions. Without direct instruction on how to respond to these questions, the students may work on problems without knowing the correct strategy to employ in order to arrive at the final solution (Beyer, 1984).

Historically, teachers have been concerned with the end result of the problem. Every student was expected to find the same answer using the same methods. Today mathematics educators are stressing the importance of the process of finding a solution rather than just the answer itself. Many students who have only been members of traditional classroom settings will have difficulty in changing their beliefs about the correct answer as being their ultimate goal (Meyer and Sallee, 1983). Garafalo (1992, p.47) quotes Lesh (1985) as stating:

. . . good solutions tend to focus on non-answer-giving stages of problem solving . . . students who spent large portions of their time on answer-giving activities (without thinking about the nature of givens and goals) usually produced inferior solutions. These weaker solutions generally took into account less information, recognized fewer and less complex relationships or the data, and failed to compensate for sources of uncertainty.

The students' learning activity can only be carried out properly if the teacher goes through the step-by-step explanation that is

necessary to process and understand the question (Beyer, 1984).

It is evident, therefore, that teachers need to promote thinking skills while interacting with their students. However, research studies conducted by Flanders, Parsons, and Simon Fraser University (as cited in Wasserman, 1987) indicate that several types of teacher responses actually inhibit or virtually cut-off student thinking. This is sometimes the result of the teacher inadvertently putting closure on the students' cognitive processing.

Closure occurs in a variety of ways, and often the teacher is not cognizant of its implications. Commonly, closure occurs when the teacher agrees or disagrees with the student. Closure is also evidenced when the teacher does not give the student a proper amount of time to think through a question. The impatient teacher will do the thinking for the student by showing or telling the pupil what to do (Wasserman, 1987). The problem will be read once and then the teacher will go into a discussion of the data given, the question being asked, and the strategies that could be used to solve the problem. The problems are often described in such detail that the only step left for the student is to calculate the answer to the problem. Teachers are overexplaining problems, thereby undermining

the students' capabilities to plan strategies and to find solutions on their own (Kersh and McDonald, 1991).

Closure is also present when the teacher abruptly stops the student's response with either the issuing of a positive or negative comment. The negative comments also undermine the student's idea and cause the child to lose confidence in himself or herself.

Some questioning techniques, even though they do not cause immediate closure, tend to limit the students' higher-order thinking skills. A prevalent type of questioning is for the retrieval of information previously learned. This line of questioning only utilizes the lower levels of cognition which require pure recall without having to process or analyze information. Teachers also limit their questions by leading students in the direction of the desired response. This type of questioning narrows down the students' thinking processes and the number of responses. These restrictive questioning techniques prohibit students from becoming independent learners (Wasserman, 1987).

Attempts have been made to change educational methodology in regard to thinking skill instruction. Colleges and universities advocate to preservice teachers that more effective teaching

practices need to be employed in the classroom. However, these programs rarely provide the means to help these young teachers learn from these practices. Inservice workshops and staff development programs that are offered to experienced teachers usually do not stress the long-term studies and the in-classroom support that are necessary to learn these processing skills and how to effectively follow through with them (Wasserman, 1987).

Personal teacher beliefs and everyday teacher practices are often in conflict. These contradictory positions are also deterrents in the teaching and utilization of higher-order thinking skills. Teachers need to ask themselves what their expectations are of their students. Idealistically, teachers may believe they are in favor of students developing critical thinking skills and questioning techniques. Realistically, though, teachers do not necessarily want more critical debate in the classroom with less reliance on the teacher as the authority. Teachers today may agree that thinking skills need to be improved upon, but in actual practice teachers still tend to reward students who sit quietly and do not ask questions that make them feel uncomfortable. Teachers still prefer students who give the desired answers and do as they are told. They want to

keep their classrooms running smoothly through conformity and the avoidance of controversy (Wasserman, 1987).

In addition to the ineffective teaching methods and the insufficient training of teachers, another obstacle is present when trying to improve the students problem-solving skills in mathematics. This obstacle is the current and prevailing curriculum. The National Council of Teachers of Mathematics believes, ". . . problem solving should be the central focus of the mathematics curriculum" (1989, p.23). However, the adoption of problem solving instruction is being suppressed due to the lack of materials provided to teachers for non-routine problems in mathematics (LeBlanc, Leitze, and Emenaker, 1992).

Textbooks, currently being used, mislead teachers about the relationship between factual content, questioning, and thinking skills. Textbooks have instructional omissions in their lessons. They do not provide instructional guidelines for the teacher, so that he or she may know the necessary teaching skills to engage their students in for the purposes of active and productive thinking (Beyer, 1984).

The third grade teachers and their target group of students work

daily with manipulatives in Everyday Mathematics and are no longer required to use a textbook. The teachers and students from the fifth and sixth grade target groups, however, do have textbooks, such as those described by Beyer.

The fifth and sixth grade textbooks are similarly structured. The sixth grade text reviews and further develops the concepts taught in fifth grade. However, specific concepts such as, probability, percentages, geometrical figures, ratios, and integers, are studied in far greater depth at the sixth grade level. The frequency and types of problem-solving activities presented by both the fifth and the sixth grade textbooks are also very comparable.

Problem-solving activities comprise 19 percent of the fifth grade textbook. Eighteen percent of the sixth grade text is devoted to problem solving. These percentages can be broken down into four general categories: assorted problems covering a variety of problem-solving strategies; application problems which provide scenarios in careers and consumerism; problem formulation opportunities to allow the students to create their own problems with a given set of criteria; and the Thinkers Corner where the student is given a separate extension problem at the end of a page.

See Table 9.

Table 9  
Percentages of Problem-solving Activities  
in the Fifth and Sixth Grade Textbooks

Grade	Assorted Problems on Strategies	Applications		Problem Formulation	Thinker's Corner
		Consumer	Career		
5	8%	2%	1%	2%	6%
6	8%	1%	1%	3%	5%

An analysis of the fifth and sixth grade textbooks indicated that nine strategies for problem solving are interspersed throughout the books. These strategies include: estimating, making choices for multi-step problems, finding patterns, guessing and checking, using logical reasoning, making a drawing, using tables, working backwards, and using graphs. The problems employing these strategies, however, are not taught to the students sequentially. The students are not afforded the opportunity to have direct instruction and sample lessons on these strategies before they are expected to successfully work on these problems. The teacher's manual gives very little information to the instructor about the strategy to be utilized. It basically restates the information on the student's textbook page and supplies the desired student responses.

The textbook has the teacher lead the students through the necessary steps allowing no time for the student to process the information. The solution is the desired and valued component of the problem according to the text instead of the planning and processing stages of the problem.

Follow-up activities on any designated strategy occur approximately every 100 pages. There is no continuity of instruction present within the text to effectively increase the students' higher-order thinking skills in mathematics.

Another weakness identified in the problem-solving sections within the text is the type of questions posed in the Thinker's Corner. In this section, the students are often asked to solve problems for which they are not yet mathematically ready to do. No directions or explanations are given to the students on these problems. Therefore, it is expected that they are to identify the proper problem-solving strategy and solve the problem even if they have not been instructed on the necessary strategy needed to find the solution or instructed on the mathematical concept being worked upon in the problem.

Putting a wide array of skills into a textbook and labeling them



problem-solving strategies does not necessarily mean that the students are improving their higher-order thinking skills. Hanna (1992, p.438) raises the following questions in regard to the amount of curriculum being taught in schools today.

How can the pace and range of work in the mathematics classroom be adapted to allow for increased understanding by all students?

Does the mathematics curriculum necessarily have to be so overloaded that the quantity tends to control the pedagogy?

Unless the teacher has a strong background in mathematical knowledge, the material presented in the text will be ineffective since little assistance is given to the teacher for the implementation of these random problem-solving activities.

This type of skills overload may be attributed to the students' inability to solve non-routine problems with higher-order thinking skills. Attempting to teach too many skills in too short a time is detrimental to the students' learning. Skills overload is caused by an attempt to correct deficiencies in skill achievement. To correct these weaknesses many curriculum developers emphasize the teaching of more skills to improve the weak ones, rather than putting an emphasis on the quality of the teaching necessary for

their improvement (Beyer, 1984).

It is more beneficial to have students practice a few problems employing the major mathematical problem-solving strategies than to have them practice a multitude of problems. If too many problems are being worked, the students who are struggling with what is going on in the reasoning process will become overwhelmed and confused. They will have difficulty transferring information from previously learned problems to the new set of problems at hand (Meyer and Sallee, 1983). A brief exposure to many problem-solving strategies, within a short period of time, will not enable the children to learn the new skills and attain significant levels of competency with them (Beyer, 1984).

Published materials currently available for problem solving are limited, just as textbooks are, in their usefulness to the classroom teacher. They are usually found in printed form and do contain non-routine problems. These problems, though, are usually only classified by grade level and the suggested strategy. They do not contain information regarding the other characteristics of the problem and the skills which are a prerequisite to solving the problem (LeBlanc, Leitze, and Emenaker, 1992).

In addition to the lack of proper materials to satisfy the curriculum needs, the constraints of time also pose a problem for mathematics teachers. They do not always have the time to locate problems which are appropriate for their students' mathematical abilities. As a result, they may assign a problem to the class and later discover that the solution requires a knowledge of numbers and/or operations beyond the students' mathematical level of development (LeBlanc, Leitze, and Emenaker, 1992).

The emphasis placed on standardized tests also affects the teaching of thinking skills. These tests are the determinants as to what is taught in the actual classroom. Teaching to the test interferes with the quality of education that is given to students. Students are merely taught isolated skills rather than sequential cognitive operations (Beyer, 1984).

Criteria reference tests (CRTs) are given in all subject areas in grades one through eight in the target groups' school district. The purpose of the tests is to show the individual student's knowledge of the curriculum objectives within any designated content area. In the early years of testing, a small sampling of word problems was given in each mathematics CRT. An analysis of these tests indicated that

a few word problems on each test were not sufficient for evaluating the students' problem-solving skills. A problem-solving CRT was created, therefore, to more accurately assess the students' abilities in this area of mathematics. The researcher's observations indicate, however, that students at all grade levels are often given practice questions quite similar to those given on the test. Therefore, the students have been given strategies in advance of the testing; thereby eliminating the processing element of the test items which should be one of the essential components in the assessment of the students' thinking skills.

In addition to the students being prepped for these tests, the test items do not fully measure the students' higher-order thinking skills in mathematics. The majority of questions require that the students only employ their lower-cognitive thinking skills. See Appendix K. The adoption of Everyday Mathematics in the primary grades has created a need to revise the problem-solving tests previously given. (These tests will be completed during the 1993-1994 school year.) The intermediate grade-level tests, however, remain unchanged. Therefore, due to teacher preparation and lower-level questioning, when the test results come back they are not truly representative of

the student's ability to do problem solving with higher-order thinking skills. It must also be noted, that even though the students have had preparatory experiences and have been asked questions which do not necessarily require the use of higher-order thinking skills, weaknesses in problem solving are still evident. See Tables 10, 11, 12, and 13.

Table 10  
 Percentage of Third Grade Students Not Passing Criteria  
 on District Objectives on Problem-solving CRTs  
 May, 1992 and May, 1993

Objectives	Passing Criteria	Non-passing Students	
		1992	1993
Find the Question	66%	11%	11%
Identify Missing Information	75%	9%	11%
Determine the Operation	75%	22%	24%
Check the Answer	75%	33%	40%
Draw a Picture	66%	11%	16%
Make a Table/Graph	75%	9%	9%
Add/Subtract Using a Calculator	75%	2%	16%
Multiply Using a Calculator	75%	0%	7%

1992 N = 46 Students

1993 N = 55 Students

The data in Table 10 indicate that a group of third grade students, in both years, was unable to meet the target district's passing criteria on the problem-solving CRT. Determining the mathematical operation and checking the answer were the students'

weakest areas. Drawing a picture was also an area of weakness for the third graders. This may be attributed to the students' unfamiliarity with this concept. The students tested in 1992 and 1993 were instructed from the textbook and were, therefore, only briefly exposed to this problem-solving strategy.

Table 11  
 Percentage of Fourth Grade Students Not Passing Criteria  
 On District Objectives on Problem-solving CRTs  
 May, 1992 and May, 1993

Objectives	Passing Criteria	Non-passing Students	
		1992	1993
Finding the Question	75%	5%	8%
Identify Missing Information	75%	3%	9%
Determine Operation	75%	19%	11%
Check Answer	75%	1%	5%
Estimation/Rounding	75%	20%	30%
Writing the Equation	75%	15%	15%
Reading a Graph	66%	1%	2%
Adding and Subtracting	75%	8%	8%
Using a Calculator			
Multiplying Using a Calculator	66%	14%	15%
Dividing Using a Calculator	66%	20%	23%

1992 N = 59 Students

1993 N = 53 Students

Table 11 reflects the percentage of fourth grade students who did not pass the district criteria for problem solving. Three new objectives were added to the third grade test to form the fourth

grade CRT. These objectives are: estimation/rounding, writing the equation, and dividing using a calculator. One objective from the third grade test, draw a picture, was not included on the fourth grade test. The elimination of this objective does not afford the researcher or the classroom teacher the opportunity to measure the growth of the non-passing students in this area from the previous third grade. The data does indicate, however, that the third graders of 1992 improved their non-passing percentage by 11 percent on the concept of determining the operation on the 1993 fourth grade CRT. Estimation/rounding and dividing with the use of a calculator posed the most difficulties for the fourth graders on the problem-solving CRT.

Table 12  
 Percentage of Fifth Grade Students Not Passing Criteria  
 On District Objectives on Problem-solving CRTs  
 May, 1992 and May, 1993

Objectives	Passing Criteria	Non-passing Students	
		1992	1993
Identify Unnecessary Information	66%	0%	8%
Identify Missing Information	66%	3%	4%
Determine Operation	66%	2%	8%
Multi-step Problems	66%	5%	10%
Estimation/Rounding	66%	5%	10%
Find the Pattern	66%	2%	8%
Tables and Graphs	66%	0%	6%
Adding and Subtracting Using a Calculator	75%	2%	4%

Objectives	Passing Criteria	Non-passing Students	
Multiplying Using a Calculator	66%	8%	29%
Dividing Using a Calculator	66%	14%	14%
Multiplying Decimals with a Calculator	66%	2%	6%
Dividing Decimals with a Calculator	66%	8%	12%

1992 N = 59 Students  
1993 N = 49 Students

The information presented in Table 12 shows improvement from the fourth grade scores. However, it must be noted that the required passing criteria score was lowered from 75 percent to 66 percent on three objectives: identify the missing information, determine the operation, and estimating/rounding. Three objectives, tested at the fourth grade level, were not present on the fifth grade test. They included: finding the question, writing the equation, and check the answer. The non-passing scores on the concept of writing the equation in fourth grade were most evident to the researcher. However, due to the elimination of this objective from the fifth grade CRT, the researcher and the classroom teacher were unable to assess the progress of the fifth grade students in this area.

Five new objectives were added to the fourth grade material to form the fifth grade problem-solving test. These were: identify



unnecessary information, multi-step problem, find the pattern, multiplying decimals with a calculator, and dividing decimals with a calculator. All five of these objectives appeared to give the students in 1993 more problems than the 1992 group of students. However, the higher percentages of non-passing students may have resulted from a smaller group of students being tested in 1993.

Table 13  
 Percentage of Sixth Grade Students Not Passing Criteria  
 On District Objectives on Problem-solving CRTs  
 May, 1992 and May, 1993

Objectives	Passing Criteria	Non-passing Students	
		1992	1993
Identify Unnecessary Information	66%	5%	1%
Identify Missing Information	66%	5%	3%
Determine Operation	66%	17%	7%
Write the Equation	66%	13%	12%
Multi-step Problems	66%	11%	7%
Rounding/Estimation	66%	2%	0%
Find Unasked Question	66%	9%	17%
Working Backwards	66%	42%	49%
Check Reasonableness	66%	3%	9%
Adding and Subtracting Using a Calculator	75%	5%	3%
Multiplying Using a Calculator	75%	6%	4%
Dividing Using a Calculator	75%	23%	14%
Multiplying Decimals Using a Calculator	66%	11%	19%
Dividing Decimals Using a Calculator	66%	2%	16%

1992 N = 64 Students  
 1993 N = 69 Students

The data provided on Table 13 indicate that many sixth graders did not meet the district objectives on the sixth grade problem-solving CRT. One objective which posed difficulty for the students was writing the equation. This objective was tested at the fourth grade level but was eliminated from the fifth grade test. Its reappearance on the sixth grade test revealed that the students in the sixth grade scored similarly to the children in fourth grade in both 1992 and 1993. See Table 11.

Two objectives from the fifth grade CRT, find the pattern and using tables/graphs, were not present on the sixth grade test. Three new objectives which were added on to those from the fifth grade test were: find the unasked question, working backwards, and checking reasonableness. Working backwards had a very high percentage of non-passing students. In 1992, 42 percent of the students did not meet the passing criteria of 66 percent. In 1993, 49 percent of the students were below the passing criteria for the target district.

Dividing with a calculator was also identified as a weakness for the sixth graders. The percentage of students who scored poorly on this objective remained constant in the fourth through sixth grades.

These scores indicate to the researcher that the teachers in the target school need to implement a program to assist students with the application of calculator skills to problem solving and real life situations. A one or two day lesson on calculators is not a sufficient amount of time to instruct those students who continually do not meet the district goals in this area. Ongoing lessons throughout the course of the school year will provide a knowledge base for the students which they can transfer and apply to new learning situations.

Baron (1991), reporting on the findings of Frederikson and Collins, affirms that current practices of teaching to the test only result in students who are effective test takers, as opposed to being genuine or authentic readers, mathematicians, and problem solvers. These types of tasks are not exposing the students to the thinking skills which will allow the transference of knowledge to new learning situations.

The state of Illinois has developed the Illinois Goals Assessment Program (IGAP) to measure student achievement levels in mathematics, reading, writing, social studies, and science. The IGAP tests for mathematics are given in grades three, six, eight, and

eleven. These tests were designed to evaluate the students' abilities to use higher-order thinking skills in mathematics. The analysis of this test by the researcher revealed a lack of the basic computation problems which are usually prevalent on mathematics achievement tests. The problems on the IGAP test focus more on the application of concepts to problem solving. Students are often asked to make choices, comparisons, or select strategies in the solving of problems. Working with graphs, tables, charts, and percentages makes the students call upon their prior mathematical knowledge. They then interpret the new information and incorporate it with their previously learned skills in order to solve the problem at hand. Thus, the students' scores are a reflection of their ability to effectively utilize the basic skills in higher-level thinking in mathematical problem solving.

Information on individual student scores was made available for the first time during the 1992-1993 school year. (Previously, only grade level scores were given to each school.) The individual scores are now provided to make the students more accountable for their work.

The old way of reporting the scores provided little information

about the students' strengths and weaknesses in mathematics. An overall score was provided without mentioning the specific goals tested. The teachers were given a percentage of the students in each quartile and a percentage of those students who had fallen behind, met, or exceeded the district goals. See Tables 14, 15, and 16. These generalities did not afford the teachers the opportunity to reflect upon their teaching so as to make adjustments in the mathematical discourse in order to meet the needs of their students. This information is still being given to teachers, but more informative materials are also being distributed now.

Table 14  
 IGAP Mathematics Scores for Grades Three and Six:  
 Quartile and Mean Percentages  
 April, 1992

Grade	National Quartile	School IGAP Math %	% Students Above and Below the Mean
3	4	72%	
	3	12%	84%
	2	7%	
	1	9%	16%
6	4	47%	
	3	23%	70%
	2	26%	
	1	5%	31%

Grade 3 N = 49 Students	Legend: Otr	%-ile
Grade 6 N = 65 Students	4	76-99
	3	51-75
	2	26-50
	1	1-25

It appears that the third graders did better than the sixth graders on the 1992 IGAP mathematics test. The researcher can find no apparent reason for this disparity between grade levels since the state did not provide specific results for the individual teachers and students.

Table 15  
IGAP Mathematics Scores for Grades Three and Six  
Quartile and Mean Percentages  
March, 1993

Grade	National Quartile	School IGAP Math%	% Students Above and Below the Mean
3	4	43%	
	3	20%	63%
	2	19%	
	1	18%	37%
6	4	31%	
	3	27%	58%
	2	23%	
	1	20%	43%

Grade 3 N = 57 Students	Legend: <u>Qtr</u>	<u>%-ile</u>
Grade 6 N = 68 Students	4	76-99
	3	51-75
	2	26-50
	1	1-2

The IGAP mathematics scores for 1993 had a lower percentage of third and sixth grade students in the upper quartiles. The percentage of third grade students below the mean increased by 21 percent, and the percentage of sixth grade students below the mean increased by 12 percent in 1993. It must be noted that the students who took the

1992 and 1993 tests were different. Therefore, varying mathematical abilities, skills, and attitudes were present during the teaching sessions for each year.

Table 16  
 IGAP Assessment April, 1992 and March, 1993  
 Percentages of Third and Sixth Grade Students  
 Meeting State Goals

Grade	Year	% Do Not Meet Goals	% Meet Goals	% Exceed Goals
3	1992	5%	67%	28%
	1993	4%	62%	34%
6	1992	12%	54%	34%
	1993	0%	57%	43%

Grade 3 1992 N = 49 Students  
 1993 N = 57 Students  
 Grade 6 1992 N = 65 Students  
 1993 N = 68 Students

Table 16 reflects that the majority of the third and sixth grade students who were tested in 1992 and 1993 either met or exceeded the state goals for the IGAP mathematics tests. The percentage of the third and sixth graders in the exceeds goals classification increased from 1992 to 1993. The percentage of students who did not meet the state goals for their grade level decreased from 1992 to 1993.

The current means of reporting the students' achievement is most

comprehensive and useful to both teachers and parents in assessing the students' abilities in math. An individual record sheet is made for each student with their score for each goal tested. Upon reviewing the individual goal (concept) scores for the 1992-1993 school year, there is clear evidence that students need to be instructed further in mathematics to improve their higher-order thinking skills. See Table 17. The individual scoring will allow teachers to assess the progress of the students over subsequent years.

Table 17  
Percentages of Incorrect Responses Given on the  
March, 1993 IGAP Mathematics Test for  
Grades Three and Six

Goals Tested	% Incorrect	
	Grade Three	Grade Six
Number Concepts and Skills	37%	22%
Percent, Ratio, and Proportion	*	25%
Measurement	37%	21%
Algebraic Concepts and Skills	24%	20%
Geometric Concepts and Skills	31%	25%
Data Collection and Analysis	27%	18%
Estimation and Approximation	26%	17%

Grade 3 N = 57 Students

Grade 6 N = 68 Students

\* Not Assessed at Grade 3

The data presented in Table 17 indicate the third and the sixth grade students are deficient in problem solving according to the



results of the 1993 IGAP mathematics tests. The percentages of incorrect responses on the concepts tested reflect the students' inabilities to employ higher-order thinking skills while taking a mathematics test. The new way of reporting the IGAP scores, which gives individualized test results for each student and each concept tested, provides the classroom teacher with the opportunity to assess the effectiveness of the teaching strategies used in the classroom and how these can be best adapted to improve the students' mathematical problem-solving skills.

Mathematics, therefore, can no longer be thought of as one of the basic skills in education. Siegel and Borasi (1992), while reporting on the work of Carnegie, Holmes, and Kuhn, find that traditional definitions of the "basics" are inadequate and counter productive when the goal of teachers is to educate students to be critical thinkers. The most recent NAEP reports, as noted by Siegel and Borasi (1992), contend that the acquisition of mechanical skills and algorithms necessary to read, write, and do arithmetic computations produce only literal students as opposed to critical thinkers. Siegel and Borasi (1992, p.32) discuss the findings of the NAEP reports as follows:

These reports show that students work successfully with basic arithmetic operations in mathematics and can understand the details of what they have read and even connect these details to the overall meaning. But when asked to engage in mathematical reasoning or explain and elaborate their initial interpretations of a written passage, they have trouble. These results are not so much a failure of student learning as they are a failure of our current curriculum and teaching practices. In short, students seem to be learning exactly what we are teaching in school. If we want future citizens to be able to deal with the novel, ill-structured situations that characterize thinking and problem solving in context (Lave, 1988), a new view of the "basics" must be forged.

When studying the process in which students learn, both the learner and the conditions of instruction must be considered. Math anxiety is a problem for students to overcome to be proficient in mathematics. This has been defined as an emotional and cognitive dread of mathematics (McCoy, 1992 citing the work of Williams, 1988). This has shown to cause avoidance of mathematics, and a negative impact on achievement in mathematics (McCoy, 1992 citing the work of Cluyte, 1984; Fennema and Sherman, 1976; Hembree, 1990; Tobias, 1987). In a study done by McCoy, 1992, he found that few manipulatives and a tactile/kinesthetic learning style increased math anxiety. Most current series use very little manipulatives and are basically designed for the auditory learner.

To help children reach higher levels of achievement in

mathematics there must be an increased willingness by parents to be of direct assistance to their children (Stevenson, Lee, and Stigler, 1986). Research consistently shows that when parents take an active role in their children's learning, student achievement and attitudes improve (Maeroff, 1986; Henderson, 1987; Martz, 1992).

Since students' attitudes are greatly influenced by their parents; attitudes toward school in general, and math specifically, the teacher needs to take the initiative to get parents more involved (Henderson, 1990). There must be more cooperation and communication between the school and parents (Stevenson, et al., 1986).

Becher (1986) suggests several methods of involving parents in the educational process. Included are: parent meetings and workshops; parent-teacher conferencing; written and personal communication; and encouraging parent visits to the classroom. When parents receive assistance from teachers as to how to help their children's progress, they become a more active participant in their children's education (Sheridan and Kratochwill, 1992).

What parents do in the home to help their children achieve in school has a profound affect on their academic success. Parents

should create a positive home learning environment by maintaining an upbeat attitude toward school and education. Parents should let their children know that they have high expectations for their success (The All-USA Academic First Team, 1993).

Parents must assume some of the responsibility of teaching their children. Their child's ability or inability to function in academic areas is their concern. Parents must make the time and place to supplement and encourage classroom activities (Schimmels, 1982). Thompson was cited by Kelly (1992, p. 6D) with the following strategies for parents in the area of developing math proficiency.

Don't bad-mouth math at home.

For every parent who says that math is difficult, there is a child who believes it.

Remember that the process of doing the problem is more important than arriving at the correct answer.

Practice estimation with your children. It helps develop a better numerical and spatial sense.

Don't insist that math be done by a specific time or in a specific way.

De-emphasize flash cards. Instead, explore math in real life

Use a calculator with an addition feature, which allows children to see multiplication patterns.

When parents become more involved with their children's education it has been found to be related to significant academic progress and improved positive attitudes toward school (Sheridan

and Kratochwill, 1992).

There are no known physical or intellectual barriers in regard to the participation of women in mathematics. However, questions are raised concerning the involvement of women in this area. It is theorized that if no physical or intellectual obstacles are present, then there must be social and cultural factors which lead to the underrepresentation of women in this field of study. These barriers, for the most part, have not been intentionally formed. They are merely an integral part of a discriminating social order which still prevails in today's society (Hanna, 1992).

The process of attaching a gender to everything begins at birth, is reinforced by the schools, and is further carried out in the workplace. Behavior becomes gender-related, as do objects and thought processes according to societal beliefs. Lee (1992, p.37) refers to Cockburn's discussion of Lloyd's work which states: "Gender is part of our tools for thinking, for ordering and understanding the world." Cockburn found that regardless of what different societies conceived to be feminine that one belief remained a constant: the inequality found between men and women was to the benefit of the man. Lee (1992, p.37) cites Cockburn as

saying:

We have no choice but to suppose that the social process of gender construction, formulations of gender difference, are important mechanisms in sustaining male dominance.

Parents need to be cognizant of the crucial role they play in their child's mathematics education. Sex-role stereotyping at birth, and the socialization patterns of boys and girls which are developed in early childhood are reinforced as children progress through school. This is evidenced by the differential expectations and treatment of boys and girls by teachers, parents, peers, instructional materials, and the media (Hanna, 1992).

Historically, many writers and researchers have believed that women were incapable of doing well in mathematics. In the United States and Canada, as well as other countries, a lot of media attention has been given to girls' supposed inferiority in mathematics, science, and technology. Popular magazines have had articles which express the view that women have inferior spatial skills and cognitive abilities along with low aptitudes for mathematics. It has been publicly claimed that women are "emotionally minded" and are incapable of comprehending

mathematics or science. Messages such as these by the press are influential in discouraging girls from pursuing higher-level mathematics and mathematics-related careers (Hanna, 1992).

The claims by the media are supposedly based upon achievement studies. However, the reports of Deaux, Hyde, and Linn, as discussed by Damarin (NCTM, 1990), have found only slight differences in achievement between boys and girls which are not educationally significant or are nonexistent in some cases. However, these findings have not deterred the study of this issue nor have they reduced the publicity which this topic evokes. Studies have been conducted to show that lower achievement scores for girls often receive wide publicity, whereas low achievement scores for boys are not highly publicized. Research done on the International Education Association (IEA) mathematics results indicate that boys and girls from 20 countries at the Grade 8 level (age 13) are about equal in achievement (Hanna, 1992).

Alan Feingold also challenges the viewpoint of girls as being inherently inferior in mathematics. Feingold's studies on research results of cognitive gender differences over a period of 30 years in the United States show that differences had declined over the three

decades preceding his study. His research conveys the message that the problem of gender differences and mathematics achievement is a socially constructed one (Hanna, 1992).

Research has been done to investigate the amount of attention males and females are given in a classroom. Lee (1992), discussing the work of Leder, reports on the differential treatment of boys and girls by teachers in an Australian study on gender issues in education. This research showed that the practice of calling on boys more often and asking them more higher-order thinking questions was prevalent in the mathematics classroom.

A study by Good, Sikes, and Brophy, as reported by Leder (1992), indicated subtle differences in the interactions between teachers and higher and lower achieving students in the elementary and high schools. They found that high achieving boys had more interactions than the other students with teachers in mathematics classes. Additional research suggests that high achieving students, or those who are perceived as being high, tend to have a greater number of contacts with the teacher than do the average pupils. They receive more constructive feedback for their answers, and they also initiate more contact with the teacher (Leder, 1992). Researchers believe



that this preferential treatment towards boys is a reflection and a reinforcement of society's gender beliefs and expectations. Lee (1992, p.28), citing Leder, states society still believes that "competing in mathematics is more important for boys than for girls."

Society must recognize that the concepts of schools, the teaching of curricula, the course of study, the development of some teaching methods, and the subject matter of mathematics was primarily created by men. Therefore, it is understandable that mathematics is a reflection of the life experiences and goals of men. The creation of schools and the development of subject matter began in a time period when many people believed women were incapable of any logical or rational thought. Although views have changed considerably today, history still plays an influential role in the approach to mathematics and its utilization in the classroom (Damarin, 1990).

Linn and Peterson examined the findings of sociologists and psychologists that boys were socialized to be more aggressive than the girls in their peer group. Mathematics has an abundance of vocabulary which reflects and supports this aggressiveness and

masculinity during its instruction. Damarin states:

Our vocabulary reflects goals of mastery and mathematical power. We teach students to attack problems and to apply strategies. Our instructional strategies include drill and the use of many forms of competition. In short, the ways we think about, talk about, and act out our roles as teachers of mathematics are heavily influenced by the masculine roots of the subject (1990, p.145).

Belenky and her colleagues (as documented by Damarin, 1990) addressed the outcomes of the female socialization process. They have identified the different stages present in female cognition. Research indicates that women learn mathematical abstractions best if they: 1) make silent observations; 2) listen to others; and 3) have personal mathematical experiences. After these three stages have been completed, women can relate to abstraction. The types of experiences that females need to understand abstractions often differ from those of men. These findings should suggest to educators that female students need to have opportunities presented to them which will promote their style of learning.

Research indicates that the females' learning styles have an effect on their test performance in mathematics. Lee (1992) discusses the studies of Kaur, Hanna, Kundiger, Larouche, and Rodgers in regard to comparable male and female performance levels

on mathematics tests. Kaur found, in her 1986 study of the O-level mathematics examination, that girls and boys had different preferences when it came to selecting problems in the choice section of the exam. More girls chose items related to algebra, graphs, and two-dimensional vectors, whereas more boys chose questions that required high-order critical thinking skills. Boys performed better on the mensuration and statistics problems than the girls. Male student performance was also higher on items dealing with probability and spatial relationships. Kaur concluded from this data that girls do better on rote learning and drill questions, whereas, boys out-perform girls on spatial visualization, fractions, and proportionality. Lee (1992, p.31) reports Kaur's citations of Wood and Jones. She quotes Wood as stating, "It appears that rote learning is more congenial to females." Kaur also quotes Jones on saying it is the,

. . . female tendency to keep methods they have been taught, to reproduce techniques, to show caution, to avoid being wrong and generally to use a method with which they feel confident and secure and which is approved by the teacher.

Kaur also raises the question of genetic factors which may influence spatial ability and quantification skills. Other variables

also need to be examined such as: motivation, intelligence, social class, teaching methods, integrated mathematics experiences in the curriculum, teacher attitude and differences between schools.

Lee (1992) also reviews the work of Hanna, Kundiger, and Larouche on the gender-linked differences in performance on specific questions and mathematical concepts while analyzing the SIMS test. The data found by the researchers concurred with Kaur's in that algebra was a stronger area of mathematics for girls and probability exercises were an area of strength for the boys.

Lee (1992, p.31) cites Maire Rodgers on the issue of sex-related preferences and academic performance in mathematics in regard to the courses she has taught.

It appeared that girls preferred what they considered to be more straight forward types of problems where they could follow recognized (sic) procedures and had most difficulty where the initial formulation of the problem was not so obvious to them. Boys preferred problems in which they encountered variety and which they found easy to visualize (sic) and disliked what they considered to be boring and repetitious.

Lee also discusses Rodgers's reflection upon Scott-Hodgetts's work which advocates that primary education causes girls to develop a step-by-step pattern approach to learning. "Rodgers wonders to

what extent girls' preferences are a part of a self-fulfilling prophecy" (Lee, 1992, p.31).

In addition to learning styles and problem preferences, female performance in mathematics is also dependent upon: the content presented to the students; the influence of female teachers; and the importance of female role models. Lee (1992) discusses the work of Barnes and Coupland which focused on the teaching of calculus in a more humanistic way. An analysis of the content of a traditional calculus course led the researchers to conclude that the abstractness and accent of problems concerning weapons, machines, and profits were unmotivating for women. This issue was addressed by avoiding discussions about limits and differentiation and replacing these with the use of computers and manipulatives which shift the focus from the theoretical to the experimental. The usual problems were eliminated and substituted with problems based on current issues such as population growth and endangered species.

Traditional teaching methods also have a negative effect on female students. Teacher-oriented instruction was found to favor the confident and assertive students. This authoritarian method fosters competitiveness within the classroom. Barnes and Coupland

see a need to complement lectures with discussions and small-group work to improve female success in mathematics (Lee, 1992).

Walkerdine's contention, as discussed by Lee (1992), is that both female students and female teachers are in damaging positions in classrooms where girls are given less attention than boys, and where girls are asked fewer higher-order thinking questions in mathematics. It is Walkerdine's belief that the classroom is the place where girls become suppressed and neutralized in mathematical achievement. Girls are thought to receive the same equality as boys in a progressive classroom, but they are denied this status due to their femininity. Girls are praised for their behavior and industriousness by their teacher and peers, but at the same time they are being classified as failures in relation to the theories behind progressive mathematics education.

Walkerdine (in Lee, 1992) points out that there is a conflict between behavioral expectations and academic expectations for girls. As young children, girls are expected to behave as boys do. They are encouraged to be active and open while exploring their environment. Additional expectations are also placed upon girls to be good, be kind, work hard, and be helpful. It is this set of

expectations that teachers either overtly or covertly promote to the female students in the classroom. The end result is that girls are always in a losing position whatever behavior they choose. If they behave like boys, they are considered to be unfeminine and are disliked by some peers; and if they demonstrate hard work and good behavior, they are considered good students, but incapable of having conceptual understanding and a high intelligence. Even if girls outperform boys on tests their achievement is not valued. For the girls who do well, ill-effects often accompany their accomplishments. Walkerdine, in her study of female academics, found the following characteristics to be persistent in their lives: the lack of self-confidence, the suppression of anger and hostility, silence, and apparent passivity (Lee, 1992).

Therefore, in conclusion, it may be ascertained that there are many cultural, social, and psychological variables that are related to the student's performance in mathematics. Damarin refers to the work of Fennema, Sherman, Reyes, and Stage which shows that:

Studies have revealed relationships between females' performance in mathematics and their score on measures of confidence, anxiety, fear of success, risk taking, attribution of success and failure to internal versus external factors, their perception of mathematics as a male dominion, parental

support for education, and related variables (1990, p.148).

It was shown in the study that independently these variables had little effect upon the females' learning. However, the cumulative effects of all of these variables working in conjunction with one another did have a significant impact on the females' learning in mathematics (Damarin, 1990).

Table 18  
Percentage of New Math Material Introduced  
at the Elementary Grades

Grade Level	Percent of New Material Presented
K	100
1	75
2	40
3	65
4	45
5	50
6	39
7	38
8	30
9	90

In evaluating mathematics series currently in use (see Table 18) new material was presented less often after first grade and continues to descend until ninth grade. The present series also focuses on operations taught in isolated segments at specific grade levels. It is also product oriented with only one way to achieve a right answer. Students work independently with the teacher being



the main dispenser of information. Students do paper pencil tasks on rote, drill exercises to master the basic skill. Each grade level has repetition of skills with no real-world application (Hiller, 1993).

A summary of probable causes for the problem solution gathered from the site and from the literature included the following elements:

1. a discrepancy exists between teacher perception and student performance on problem-solving activities,
2. the students had an inaccurate understanding of their problem-solving abilities,
3. the insufficient amount of time spent on the teaching of higher-order thinking skills in problem solving,
4. the curriculum is based on the acquisition of facts and algorithms rather than hands-on, real-life, and group problem solving,
5. a lack of parental involvement hinders students' academic achievement,
6. a limited exposure to new mathematical concepts causes boredom and prohibits students from reaching their potential,

7. a lack of varied teaching methods and the use of manipulatives causes math anxiety, therefore, hindering achievement,
8. the effects of gender bias upon females is a factor in lowered student achievement.

## Chapter 3

### THE SOLUTION STRATEGY

#### Review of the Literature

Analysis of the probable cause data at the target site, suggested reasons related to a discrepancy between teacher perception and student achievement, an inaccurate understanding by the students of their problem-solving ability, the duration and intensity of teaching problem solving was inadequate, and a curriculum which focuses on individual drill and practice rather than hands-on, real-life, and group problem solving. The review of the literature probable cause data suggested: the curriculum does not address higher-order thinking skills, has minimal exposure to new materials, does not employ the use of manipulatives and real-life problems; the lack of manipulatives caused anxiety towards math; minimal parent involvement causes a hindrance in student achievement; and gender bias is evidenced in the teaching of mathematics and in the curriculum.

The literature search for solution strategies was organized as suggested by these probable cause data. Analysis of these data suggested a series of questions related to curriculum design and teacher behavior.

The questions related to curriculum design included: 1) How often should problem solving be taught in the classroom? 2) What strategies should be used to help the students learn problem-solving skills? 3) What problem-solving strategies should be taught? 4) What supplements should be added to improve the present curriculum?

The questions related to the teacher included: 1) What teaching strategies should be employed to improve the students' problem-solving ability? 2) How can the teacher lower the students' anxiety level? 3) How can the teacher keep the student involved in the learning process?

The National Council of Teachers of Mathematics contends that teachers, as professionals, should develop their knowledge of mathematics to improve the evaluation of their teaching and to better assess the understanding of their students (NCTM, 1991). Prevost (1993), in this regard, discusses the work of Skemp which

addresses the vital role of teachers in a child's life. He quotes Skemp as saying, "A human child is at the most learning age of the most learning species that has yet evolved on this planet" (1993, p.75). It is Skemp's view that the extent to which a student's potential will be recognized is largely dependent upon their teachers. Teaching is any type of action that affects the learning process. Prevost (1993, p.75), citing Skemp, states, "A person who intervenes without an adequate mental image of what is going on inside is as likely to do harm as good." Therefore, it is essential that teachers of today know mathematical pedagogy and change their teaching to reflect this knowledge.

Teachers are in constant state of "becoming." Being a teacher implies a dynamic and continuous process of growth that spans a career (NCTM 1991, p.125).

Mathematics educators are now seriously supporting the idea that the acquisition of knowledge is a matter of individual construction rather than the result of the transmission of information from the teacher to the student. The NCTM has provided national direction and support for the constructivist approach to the teaching and learning of mathematics (Feldt, 1993). The Curriculum and Evaluation Standards (1989) views the students as active

learners and the teachers as their facilitators. The Professional Standards for Teaching Mathematics (1991, p.2) states that two roles are based upon the following assumptions:

Teachers are key figures in changing the ways in which mathematics is taught and learned in schools. Such changes require that teachers have long-term support and adequate resources.

This first step in changing teaching methods is to have the teacher reflect upon his/her current instructional practices. Prevost (1993, p.76) cites Freudenthal's advice that teachers must learn from "one's own and others' examples to analyze the instruction one is attempting to give, is giving, and has been giving." As teachers reflect upon what they do in the classroom they must also read current literature to supply insights into constructivism, effective teaching methods, changes in assessments, and the importance technology has in the classroom.

Another important resource for teachers are workshops and courses that will expand their knowledge and will offer an opportunity to hear what other professionals are doing in regard to the recommendation for change. Workshops should be chosen in which the presenter models active learning and teaching practices.

Suggestions from the workshops should be implemented in the classroom. Student reactions along with the teacher's feelings towards the lesson should be recorded and analyzed by the teacher (Prevost, 1993). Teachers also need to work with colleagues and supervisors to share ideas on lessons, and how to modify and make improvements in their teaching style (Feldt, 1993); thus, adjusting the lessons to fit the learning levels and needs of the students within the class (Prevost, 1993).

Therefore, reflecting upon traditional teaching practices, reading current literature, attending workshops, and staff development activities will assist teachers with pedagogical changes that will have a direct effect on the improvement of student learning. Although the process of change is discomforting for teachers, each small step taken will help to make significant changes in the classroom. Prevost (1993, p.78) cites Borko and her colleagues as stating the following:

It will never be possible, within the constraints of a single mathematics methods course or even an entire preservice teacher preparation program, to enable prospective teachers to learn all that they need to know and believe about mathematics and mathematical pedagogy in order to teach effectively.

In the constructivist tradition, we must reflect on teaching, and identify the discrepancies that exist between our methods and those we know we should be using. We are all aware that as novices or experts we can still do much to improve our teaching and that responsibility for improvement lies with us. We must do the learning, and we must reconstruct our own view of teaching.

In order for students to develop a proficiency in mathematics, a new curriculum and a risk-free classroom environment must be created and provided by the teacher. It is the role of the teacher to select materials that will be of high interest to the students while deepening their understanding of mathematics and its relevancy to the real-world. The math teacher of today should promote an investigative approach to mathematics in which students can plan mathematical strategies and analyze them to determine their effectiveness for finding problem solutions. The teacher should act as a facilitator to the students as they work through the problem individually, in cooperative groups, or as a whole class. The use of technology and other mathematical tools should be an integral part of the students' mathematical inquiries (NCTM, 1991). The classroom discourse also needs to be stimulating and well-managed, so that the teacher and student are clear about what is being learned. An ongoing analysis of the student learning, the



mathematical activities, and the classroom environment must also be done in order to assist the teacher with instructional decisions (NCTM, 1989).

Teachers need to view their role in the classroom from a different perspective. Rather than imparting a set of mathematical rules, procedures, and terms to the students, the teachers need to have students understand mathematical concepts and how these can be integrated and applied into the world around them. The NCTM (1989, p.2) cites the National Research Council as saying:

Effective teachers are those who can stimulate students to learn mathematics. Educational research offers compelling evidence that students learn mathematics well only when they construct their own mathematical understanding. To understand what they learn, they must enact for themselves verbs that permeate the mathematics curriculum: "examine," "represent," "transform," "solve," "apply," "prove," "communicate." This happens most readily when students work in groups, engage in discussion, make presentations, and in other ways take charge of their own learning.

This shift in mathematics education will take time. Teachers need to develop new instructional strategies, find good instructional materials, and select assessment materials that will more comprehensively evaluate the individual student's progress.

Curriculum changes are dependent upon teachers and their

willingness to change their role in the classroom. Initially, teachers involved in mathematics instruction need to formulate guidelines for a successful program. They should choose and develop mathematical tasks that will promote the students' understanding of concepts and procedures in order to improve their ability to solve problems and to reason and to communicate mathematically (NCTM, 1991). Specific goals need to be stated and mathematical activities need to be created to accomplish these goals (NCTM, 1989).

Good tasks incorporate mathematical thinking and mathematical concepts. They stimulate the students' curiosity and provide situations that allow the students to make speculations and pursue their hunches. Many of the tasks should be able to be approached in several ways, and some should have more than one reasonable solution. These tasks, therefore, facilitate classroom discourse. Students are encouraged to reason about strategies and outcomes, discuss alternatives, and pursue a particular plan for a solution. Teachers should base the development of these tasks on three components: the mathematical content, the students, and the ways in which students learn mathematics (NCTM, 1991).

In regard to mathematical content, the teacher must evaluate the

tasks as to their representation of the concepts and the procedures involved. The selected tasks should have underlying concepts that need to be addressed instead of requiring that just mechanical operations be performed. Teachers should consider the potential of a task to help the students progress in their cumulative understanding of a particular concept and how this new knowledge can be transferred and can be applied to the future.

Age-appropriate mathematical skills also need to be included in the tasks chosen by the teacher. The context of the math problems should promote skill development while the students are engaged in problem solving and reasoning. Essential skills are necessary at all age levels if effective problem solving is to occur (NCTM, 1991).

The students are also a major consideration when the teachers are deciding upon the appropriateness of a task. Several factors influence the teachers' decisions. One important aspect for teachers to assess is what their students already know and are able to do, what they need to improve upon, and how amenable they are to extend themselves intellectually. Well-chosen tasks give the teachers the opportunities to learn about the students' levels of understanding while they encourage them to strive for the

acquisition of knowledge at higher levels of thinking (NCTM, 1991). In order to promote problem solving, the tasks should be sustained and should allow for choices made by the students. The students should also be made responsible to design and carry out investigations which include opportunities for self-assessment and reflection (Baron, 1991). In the reflection segment of the lesson, the students should be taught to explicate the assumptions they have made about a problem, to analyze these assumptions for other interpretations, and then review and change these assumptions to generate alternative solutions (Kersh and McDonald, 1991).

Meaningful tasks should stress the importance of authenticity and optimal levels of challenge so as to provide maximum levels of the transfer of learning to real-world contexts (Baron, 1991). The NCTM (1991) also believes in making learning relevant, but contends that students also need tasks which are theoretically or fancifully based. Other factors of task selection are the students' interests, experiences, and dispositions. Teachers must also be knowledgeable and be sensitive to the cultural, sociological, psychological, and political diversities found within the classroom (NCTM, 1991).

Knowing the ways in which the students best learn mathematics

is another criteria for selecting tasks. The type of activity selected, the kind of thinking required, and the way the students are guided on their approach to the solution all contribute to the learning provided by the task. A teacher should also be aware of students' confusions and misconceptions on specific mathematics concepts. Tasks should be selected to afford students the opportunity to explore critical ideas that may clarify their confusion.

After the selection of appropriate tasks, student discourse can then be focused on mathematical reasoning and problem evidence. Student interaction amongst themselves as well as with the teacher is a vital part of classroom discourse. The teacher's role is to initiate this type of discourse and ensure that all students are participants in the learning process (NCTM, 1991).

Teachers who converse with their students help to build a thinking environment for the children. Through questioning, they probe for students' answers and make the students defend everything they say. The teacher, acting as a guide, must be knowledgeable of the concept being discussed since it is the teacher who must initiate proper questions before student discourse can begin

(Shalaway, 1990). Teachers need to plan explicit questions for the students who are novices at problem solving. These questions will serve as models for questioning and thinking as the students strive to become independent problem solvers (Theissen, 1988). Once the students have been taught the techniques and strategies to improve their understanding of non-routine problems, the teacher should encourage the children to use the strategies in a variety of activities to help them understand new problems. As their repertoire of strategies increases, students should feel more confident in attempting problem solving (Kersh and McDonald, 1991).

The teacher must also listen carefully to what the students are saying to check for understanding during problem-solving activities. The teacher's responses to the students' discourse should be put in question form to help the students construct their ideas and solutions (Shalaway, 1990). When teachers require students to perform tasks such as processing data and originating ideas, they are asking them to function at higher levels of thinking.

There are several responses that teachers may employ to promote their students' thinking. They include: student reflection, student analysis, classification of information, and data comparison.

Teachers can also promote thinking by asking questions that challenge the students' ideas. These types of questions push students beyond their present levels of learning. They will need to extend their thinking to levels which will produce tension and discomfort for them. Students must be made to feel secure in the classroom environment if such cognitive risks are to be taken by them.

Sadler and Whimbey (1985, p.202) discuss the importance of the classroom environment to student learning by stating that:

. . . a principle that we feel is crucial to teaching cognitive skills is the development of a social climate that is supportive of teaching and learning. Teaching cognitive skills is tough work. The area is largely uncharted; the work is risky; the opportunities for mistakes are many. Teachers and students need to feel that what they are doing is valued and that failure along the way is forgivable because it is part of the learning process.

Teachers must realize that this questioning in the classroom should be used thoughtfully so as not to inhibit the students' thinking. Responses that teachers may use to challenge students' thinking are: asking the student to make judgments and to specify the criteria for those judgments; asking the student to form a hypotheses; asking the student to apply principles to new situations; asking the student to make predictions; asking the student to formulate ways to test their predictions or hypothesis. Wasserman (1987, p.465) states: Teaching for thinking is more effectively carried out when thinking tasks are used in concert with teachers' reflective, analytical, and challenging responses. When activities and

interactions work together, teaching for thinking may indeed flourish in the classroom.

Students who are comfortable in the traditional setting where the teacher does most of the talking, while they remain participant observers, need to be guided and encouraged to become active learners within the discourse of the risk-free collaborative classroom (NCTM, 1991).

It is the teacher's values and sense of the students' ideas that determines the mathematical disposition of the class. If the teacher shows an interest in students' approaches and ideas and probes the students' thinking, whether it be valid or invalid, a positive tone will be present in the classroom. The teacher also needs to teach the students to value one another's ideas and to take risks in proposing their conjectures, strategies, and solutions. Students need to be assured that there is no place for ridicule in a mathematics class that is seriously engaged in the process of reasoning.

In order to ensure that the mathematical reasoning and justification are carried out properly, the teacher must stress two points. First, the students must learn how to justify their solutions without becoming defensive or hostile. Secondly, the students need



to learn how to question their peer's conjecture or solution with respect for that individual's knowledge and thinking (NCTM, 1991).

Emphasizing the explanation or elaboration of an idea rather than its correctness establishes norms of respect between the students and the teacher (NCTM, 1991). Stressing the importance of the processing of a problem allows students to generate new ideas, new questions, and new answers. While the students' discourse is in progress teachers need to understand their role in the mathematics classroom (Meyer and Sallee, 1983). The teacher's role, as a facilitator, is to filter out and direct the students' explorations that are best suited to the lesson. Doing this keeps the students focused on the task at hand. Teachers must also use their professional judgment to effectively guide the students. For in order to make the math program successful, the mathematical discourse may also dictate that the teacher go beyond the stage of asking clarifying questions or allowing students to struggle to make sense out of an idea. At times, the situation may necessitate that the teacher step-in and give direct information to the students (NCTM, 1991). The presence of an interested, yet unobtrusive teacher lends itself to a support working environment. This, in turn, allows students to

develop self-reliance and confidence in themselves in preparation for the real-world (Meyer and Sallee, 1983).

It is crucial to the students' success that they examine a few problems thoroughly rather than be involved with an abundance of problems. Experience has taught educators that several problems, quickly covered, teach the average student less than one or two problems which are completely investigated. Students need to see why a specific approach to a problem works better than another. They need the time to look at problems in different ways, so they can fully understand its underlying structure. Improving the quality of student thinking is the teacher's objective when increased time is allotted for mathematical problem solving. The time given to the student should be sufficient enough to allow for the connection between the problem under consideration and those already discussed which incorporated the same mathematical strategy. Working on one problem in depth versus several problems affords the student the opportunity to assimilate and integrate the new learning experience with his or her prior knowledge (Meyer and Sallee, 1983).

The teacher needs to encourage and to expect that the students will show perseverance while working through the designated task.

Students must be made to realize that problem solving takes time; that the use of higher-order thinking skills to analyze problems will greatly enhance their learning of mathematics (NCTM, 1991).

The new goals for school mathematics as established by the NCTM (1989) take a departure from the beliefs that mathematics is difficult, rigid, and an irrelevant part of the curriculum. They express the hope that future mathematicians will come to experience and value mathematics as an integral part of their cultural heritage and develop confidence in their ability to reason through and solve complex mathematical problems in collaboration with others. These goals for mathematics education necessitate that the students learn to mathematically communicate in oral discourse as well as in written form (Siegel and Borasi 1992). The authors of Curriculum and Evaluation Standards for School Mathematics (NCTM, 1989, p.6) write:

The development of a student's power to use mathematics involves learning the signs, symbols, and terms of mathematics. This is best accomplished in problem solving situations in which students have an opportunity to read, write, and discuss ideas in which the use of the language of mathematics becomes natural. As students communicate their ideas, they learn to clarify, refine, and consolidate their thinking.

Mathematics can be embellished if reading, as a form of communication, is explored by the teacher as a vital component to facilitate the learning opportunities that are created in mathematics instruction. Reading is a mode of learning which can be considered as a generative process in which the reader can use previous language experience as well as the context of the reading to make connections, form hypothesis, and raise questions in order to understand the context of the problem. Siegel and Borasi (1992) reported on Harste's twenty years of research on the reading process. Harste has substantiated his view of reading as an active process of constructing meaning. Siegel and Borasi (1992) also discuss the work of Carey (1985), Eco (1979), and Rosenblatt (1978). They concur with Harste that reading is a meaning-making process. The reader generates his or her own interpretation of the material read, rather than receive an interpretation from another source. Readers make predictions based on their prior experiences. These are confirmed or revised as the reader continues to interpret the reading passage. The interpretation is unique to the individual since it is a result of the reader's life experiences, social and cultural backgrounds, and beliefs and feelings. These factors, along with the

organization of the text and the set of language cues, are determinants in the reader's comprehension of the material. The social circumstances of the reader also affect the reader's interpretation. The social considerations involved are: the reader's interest in the material; the reason for the reading (e.g., enjoyment, student assignment); or who controls or assigns the reading. Comprehension can change when the context of the reading event changes. Therefore, the same reader may generate different meanings for a text when it is read for different purposes. Researchers thus believe that reading skills can be incorporated into the instruction of mathematics. If teachers are able to integrate reading and math skills, the students' learning opportunities will be enhanced and will be more meaningful for them (Siegel and Borasi, 1992).

Reading experiences should encourage the students to be active learners and to develop an understanding of mathematics. Positive reading experiences in mathematics are dependent upon three integral components: a variety of mathematical texts, transactional reading strategies, and a strong curricular framework.

Reading materials need to be made available to students which

address: technical content, the philosophy of mathematics, real-life applications of mathematics, and strategies used to solve specific types of problems. Exposing students to a wide-range of mathematical texts and programs would provide them with a variety of formats for mathematical thinking. This approach to material selection would provide students with insights into alternative solutions and the learning processes involved.

To maximize the effectiveness of the many texts and programs, sound reading strategies need to be employed by the mathematics teacher to improve instruction. Siegel and Borasi (1992, p.25) state:

Research in reading has made us aware that it is not only what students read, but also how they read that can make a difference in their learning. Students can use transactional reading strategies to learn from any kind of mathematical text. These strategies engage readers in active meaning-making in the sense that interpretations are constructed through reflective thought motivated by ambiguity.

Reading must transcend across the curricular framework. It should foster a context for meaningful learning which attaches value to risk-taking, processing, problem formation, and problem solving. This type of curriculum, as proposed by Harste, Rowe, Woodward, and Burke (in Siegel and Borasi, 1992), is based upon the idea that learning takes place through different channels (e.g., language,

mathematics, art, science, etc.) and meanings are created and learned about a topic in a purposeful context.

Reading, for the purpose of learning mathematics, may help students to develop a deeper understanding of it. This, however, cannot be accomplished through an isolated course of study. Having a knowledge and an understanding of mathematics requires ongoing discussions and demonstrations of concepts through daily learning experiences. "Teachers cannot just tell the students what mathematics is all about, students must experience it" (Siegel and Borasi, 1992, p.32).

In order for students to experience mathematics, it is the role of the teacher to expose their students to tasks that will elicit and extend their thinking in both oral and written discourse. The establishment of written and oral discourse should be based on the exploration of ideas using a broad and varied means of communication and approaches to mathematical reasoning. Teachers must encourage a variety of tools rather than placing an emphasis on conventional mathematical symbols. Students should be able to communicate ideas through the use of drawings, diagrams, invented symbols, and analogies. Teachers should assist the students in using

calculators, computers, and other technological devices as tools for problem solving (NCTM, 1991).

Teachers, therefore, need to monitor and organize their students' participation in class. They must assess their students' written skills as well as their abilities to verbalize mathematical ideas (NCTM, 1991). In order to stress the significance of the solution process, teachers will need to change their evaluation policies to reflect this viewpoint. Teachers will have to develop a set of evaluation criteria for the problems to be assessed. These should then be discussed with the students to emphasize the point that how they arrive at a solution is more important than the solution itself (Meyer and Sallee, 1983).

Ongoing analysis of the students' learning and the teaching that is transpiring in the classroom needs to be done. In order to make math instruction as effective and responsive as possible, the teacher needs to gather information about the students in a variety of ways. Assessing the students' abilities to communicate mathematically may be done while observing student discussions in small groups. Interviews with individual students will provide additional information about the students' conceptual and procedural



understanding. Student math journals are another means of evaluating progress. Whole-group discussions also give the teacher an opportunity to appraise the students' development. Working in small groups, however, is ideally suited for meaningful mathematical discourse (NCTM, 1991).

Duren and Cherrington (1992) noted that several recent journals have contained articles which promote cooperative learning as a way of improving problem solving and higher-level thinking skills (e.g. Johnson, Johnson, Holubec, and Roy, 1984). They cited Parker (1984) as finding that cooperative learning in small groups emphasized the development of problem-solving skills. Costa (cited in Bellanca and Fogarty, 1990) said that higher-order thinking skills are improved when students learn: to listen to the other members of their group, to value each other's contributions, to accept another person's point of view, to reach consensus, and to resolve conflict.

According to the Curriculum and Evaluation Standards for School Mathematics (1989), a portion of the middle school experience should be spent working in small groups. This enables the teacher to interact with the students. Cooperative grouping takes advantage of the students' social skills, and this provides opportunities for the

students to exchange ideas and develop positive interactions during the reasoning process. It also leads to the development of the students' ability to communicate and reason. Burns (1988) states that since many children feel more comfortable in small group settings, they are more willing to talk aloud about their ideas, to participate more actively, and are able to question and respond to the ideas of others.

Duren and Cherrington (1992) conducted a study in an urban middle school in Northern California. One hundred twenty six pre-algebra students in the seventh and eighth grades were randomly assigned to either a group that practiced solving problems in cooperative learning groups or a group that practiced solving problems independently. The results of the study showed significant differences between the two groups in the students' long-term retention of problem-solving strategies. Duren and Cherrington found that these differences might be due to the following observed behaviors: students in the cooperative groups were more willing to work on a problem for a longer period of time than those who worked independently; students were able to make qualitative verbalizations about the problem and were able to justify their

solutions; students as observed in their cooperative groups were seen to be more open to alternative strategies as observed and noted by the teacher, and they received more corrective feedback from members of their group; and students in the cooperative groups attempted to use a learned strategy seven percent more often than those in the independent practice classes when given a final test three months after instruction and practice.

According to Costa (cited in Bellanca and Fogarty, 1990), when employing cooperative groups, the teacher's role should be one of a coach or facilitator who gives structure to the classroom, presents the problems, mediates each group with positive feedback, and monitors each group's progress. The teacher should also model thinking skills and cooperation for the students.

Kroll, Masingila, and Mau (1992) state that if cooperative groupings are used on a regular basis, assessment of cooperative work should be made. Cooperative work can be assessed in several different ways. One way is by observing and questioning the students as they work in small groups. Another method is for the teacher to take notes of whole group discussions about the cooperative work. A third means of assessment is that teachers can

make comments on what each student records in their journals when writing about their cooperatively solved problems. Additionally, teachers can assign grades to the cooperative work.

Over 500 studies done by Johnson and Johnson and others have shown that cooperative learning is one of the most powerful teaching and learning tools (Bellanca and Fogarty, 1990). Johnson (1983) was cited by Bellanca and Fogarty, 1990, as saying that students improve their learning, self-esteem, liking of school, and their motivation due to the fact that cooperative experiences promote positive self-acceptance.

Bruce Joyce (cited in Bellanca and Fogarty, 1990 p. 242) wrote in his meta-study of research on various models of teaching:

Research on cooperative learning is overwhelmingly positive, and cooperative approaches are appropriate for all curriculum areas. The more complex the outcomes (higher-order processing of information, problem solving, social skills, and attitudes), the greater the effects.

Davison and Pearce (1988) find that writing in math helps students to acquire new information by writing their ideas on paper. They find five categories of writing and all five can be used in the mathematics class. The first one is direct use of language. This includes copying and recording information such as notetaking. The

second is linguistic translation. This puts symbols into the written language. This would involve taking a problem such as 234 times 23 and writing out in complete sentences the steps taken to solve the problem. The third is summarizing. This includes summarizing what was done, and paraphrasing to include reactions and reflections.

The fourth is applied use of language. This area has the students writing their own problems to show their understanding of a certain concept. When students write their own problems, they tend to create problems in which they had difficulty solving. The problems the students write can be grouped into four types: problems with a new concept, problems using a particular procedure, using problem solving knowledge not yet acquired, and problems students understand but make minor mistakes solving (Winograd, 1992).

The key to writing problems for students is to make them problematic. Students tend to make them problematic in these ways: adding extra non-numerical information, adding extra numerical information, adding pertinent information, using large perceived difficult numbers, making a sub-procedure a problem in itself, and avoiding standard problems (Winograd, 1992).

The last category of writing is creative use of the language. This

includes applying math to real-world problems. This doesn't necessarily mean using concepts learned in a mathematics class. It also could include doing research reports on a math subject or person (Davison and Pearce, 1988).

Journal Writing is considered an important element of any problem solving curriculum and can be used to cover the five types of writing suggested by Davison and Pearce (1988). It can be used to collect information about how the students' feel about math. It can be used to allow all students, especially quieter ones, to participate in math. When journals are used to reflect, students can explore alternatives, and gain confidence in math. Journal writing can give the teacher insight into the students and help open lines of communication with personal contact (Bagley and Gallenberger, 1992).

Another way to enhance problem solving in the classroom, is to use calculators and computers. If students are allowed to use calculators to solve problems, they can focus on the steps to reach the solution and not on computation. Students are less inhibited by larger numbers and are more apt to guess and check to find answers. They feel more comfortable guessing and checking because they can

hit the clear button to try again. Using the calculator can give the student a record of the thought process involved in solving the problem. Using calculators helps to make the problems more reality based because they do not have to fit a child's stage of development. Students confidence increases because they can solve problems with larger numbers. Students don't learn from problem solving if the problems are too easy or too difficult. Calculators are one way to ensure the students learning from problem solving (Duea, Immerzeel, Ockenga, and Tarr, 1980).

The National Council of Supervisors of Mathematics (1989) state that computers should be incorporated into all Kindergarten through grade twelve classes. Classrooms should have computers and projection devices or large screen monitors for demonstration purposes. Computer laboratories should also be available.

The software for the computer should not contain drill and practice, but should contain problem solving and concept development (Carl, 1989). Finding a data base for problem solving will save the classroom teacher time in fitting the problems to the students' ability level. The data base has non-routine problems with different combinations of skills or characteristics to fit the need of

the students. Problems found on a computer data base are categorized into three categories: process problems, project problems and puzzle problems. These problems are grouped by primary (Kindergarten through third grade), intermediate (fourth through sixth grade), and advanced (above sixth grade). There are three levels of difficulty. They are easy, medium, and hard. The data base also has the problems grouped by 13 strategies needed to solve the problem. These are: guess and test, work backwards, look for a pattern, use equations, use logic, draw a picture, make an organized list, make a table, act it out, make a model, simplify, compute, and use a calculator. The data base also identifies the operation used, the content area, the number of solutions, and if the problem is related to another problem (LeBlanc, Leitze, and Emenaker, 1992).

Revising the curriculum is a way to help solve the problem. Using a series such as Everyday Mathematics by the University of Chicago Math Program, would be most beneficial to the students. This series exceeds the NCTM standards in all areas. Because this exceeds the standards, more new tasks are introduced every year. Each task is taught five different times during the year and the skill is



introduced two years before mastery is expected. The series is more open-ended and integrates the math operations. It is process oriented and it teaches more than one right answer. Math is taught all day long with applications in the real-world. The students work cooperatively with mastery focused on doing, applying, and using basic skills in higher-order problem solving (Hiller, 1993).

Many researchers and educators are no longer willing to accept gender bias and permit its existence to go unnoticed in the classroom or workplace. Hanna (1992, p.434) quotes Judge Rosalie S. Abella, an advisor to the government in Ontario, Canada, as posing the problem as follows:

Systematic discrimination requires systemic remedies. Rather than approaching discrimination from the perspective of the perpetrator and the single victim, the systemic approach acknowledges that by and large the systems and practices we customarily and often unwillingly adopt may have an unjustifiably negative effect on certain groups of society. The effect of the system on the individual or group, rather than its attitudinal sources, governs whether or not a remedy is justified.

As a pluralistic, democratic society, we cannot continue to discourage women and minority students from the study of mathematics. . . . We challenge all to develop instructional activities and programs to address this issue directly (Damarin, 1990, p. 144 citing the NCTM).

In order to improve mathematics instruction of all students,

teachers must first raise their consciousness and evaluate their biases and structured beliefs about male and female students' abilities to effectively problem solve. Femininity and masculinity are socially developed constructs which are reinforced by the interactions of children with one another and with adults. Covert and overt assumptions and messages about male and female intelligence, needs, and inclinations seem to affect the students' attainment in mathematics. Gender differences in mathematics performance might therefore be a reflection of the differences in attitudes towards mathematics (Hanna, 1992).

Research has found that girls tend to avoid mathematics courses when they are no longer required. Female persistence in mathematics is determined by: the attitudes the girls have towards mathematics; their feelings as learners of mathematics; and the values that shape their attitude. Hanna (1992, p.436) states, "Girls who are aware that mathematics will be relevant to their lives and useful in their future careers are far more likely to remain in mathematics courses."

Therefore, educators need to closely analyze the factors that contribute to the discrepancies found in the involvement in higher-

level mathematics. Strategies need to be developed to encourage both genders to stay in mathematics courses, thus providing a wider-range of career and job options (Hanna, 1992).

Many issues need to be addressed if there is to be an equity between males and females in mathematics. Among these are: the significance of the parents and teachers in the socialization and enculturation processes of boys and girls; the outcomes of male-oriented discourse which is present in mathematics; and the content of the curriculum and its means of assessment. Hanna (1992, p.436-437) advises that the following questions be closely scrutinized in order to eradicate the inequality between the two genders in mathematics.

Are there cultural patterns, such as social customs, family customs, customs in our educational system, and customs specific to mathematics, that discourage girls and women from pursuing mathematics?

Is there an implicit message in society that competence in mathematics is more important for the attainment of boys' career ambitions than it is for girls?

How can we increase the confidence of females in their ability to do mathematics?

Do specific teaching approaches and learning modes lead to more positive attitudes to mathematics?

What are the consequences in the theory and discourse of mathematics of the fact that it was constructed in predominantly patriarchal societies?

What features of mathematics as a discipline (e.g., the contribution it can make to developing creativity and enjoyment, and its value in developing reasoning powers) can be emphasized to make it more relevant to both genders?

Mathematics needs to be taught to all students. Therefore, in order to achieve gender equality in mathematics education, educators must closely look at the development and content of the curriculum. The teaching strategies employed to present the curriculum must also be evaluated. Hanna (1992, p.438) contends that the following questions need to be analyzed in regard to the curriculum.

What would a gender neutral curriculum look like?

Should different mathematics curricula be provided for different groups of students?

Does the mathematics curriculum fail to deal with topics of particular concern to girls and women?

How can different components of curriculum - instructional methods, assessment programs, and resources produced by teachers and by publishers - be designed so that the development of mathematics skills and knowledge becomes a prime aim for all children?

Teachers are extremely influential to the students' learning of mathematics. It is these men and women who create and maintain

the classroom environment which influences how students learn; what their expectations are for themselves in regard to mathematics; and what their perceptions and misapprehensions are in mathematics. Teachers need to be made aware at all levels of gender bias, and how they can eliminate it from current teaching practices. It is known that interventions by educators along with parents can modify the negative effects produced by stereotyping girls; and can provide for an equitable education for all students (Hanna, 1992).

A positive classroom setting has been found to contribute to female academic achievement in mathematics. Lee (1992) ascertains that girls are successful in a supportive, noncompetitive, and loving mathematical environment. She discusses the work of five researchers who concur with her viewpoint.

Lee first discusses the research of Zelda Issacson. She addresses the negative perceptions women have about school mathematics. For many women mathematics was synonymous with competition, fear, panic, and a lack of encouragement while they were in school. Parents and peers were not always supportive of this group studied, and the school organization itself also

discouraged women. Issacson then taught a mathematics course to these women which included confidence-building activities and the creation of a risk-free environment. This course, as observed by Issacson, helped women to develop positive attitudes about their abilities in mathematics.

Lee (1992) also reports that Maire Rodgers came to conclusions similar to those of Issacson. She too believes in a light and a supportive atmosphere for the teaching of mathematics. Lee (1992, p.29) cited Rodgers's contentions about learning styles whereby "the move toward collaborative group work and language-intensive processes might benefit girls." Rodgers's evidence is drawn from her case study in 1987 which tried to identify the factors that had encouraged female students to enroll in higher-level mathematics in three Northern Ireland schools. She found the girls who chose to continue their mathematics education had received support and encouragement from particular teachers. She also found that girls felt more comfortable doing problems with recognized patterns rather than those which were more complex. To ensure that girls were involved with higher-order thinking skills problems, Rodgers initiated a family math program. In the home environment, the girls

felt safe to take risks on difficult mathematics situations. The relaxed atmosphere turned mathematics into an enjoyable activity which helped to facilitate learning (Lee,1992).

Marr and Helme were researchers involved in creating a course to generate a positive and supportive classroom environment which would build the students' confidence levels in mathematics. This course was directed towards adult women who had three or less years of high school mathematics. Criteria used in their work were: confidence building, interactive and cooperative learning, practical hands-on activities, a relevant context, and the acknowledgment and the acceptance of individual differences. Also included with the criteria was the need to raise the students' social and economic consciousness in order for them to realize the impact these structures have on everyone (Lee, 1992).

Becker's research study, which also purports a positive learning atmosphere for females, was also discussed by Lee. The study performed by Becker involved 21 students in mathematics and computer science at the graduate level. Her research indicates that women have lower levels of confidence and need support for their endeavors in mathematics. It was found that both men and women

have similar reasons for liking their course of study. However, Becker's findings show that the men in the study were motivated to attend graduate school, and that women needed strong encouragement from at least one person to enroll in graduate level education (Lee, 1992).

Increased motivation by boys and men is not surprising to researchers. Recognizing that mathematics has evolved from men enables educators to see the depth of the gender issues which face present society. This increased awareness will assist teachers in identifying ways to improve gender equity in instruction (Damarin, 1990). One means to eradicate male dominance is to replace male-oriented words with a new set of vocabulary terms such as: the internalization of facts and concepts, the interaction with problems, the sharing of problems and working cooperatively, the resolution between two ideas, and the building of networks of facts, concepts, problems, and procedures.

Using cooperative learning for realistic problems and modifying the mathematics terminology would lessen the number of psychological barriers that girls and women encounter. Teachers will find, however, that de-emphasizing competition in mathematics



will be an ongoing process. New words may arise which infer gender bias, or new insights may be developed which will distance some students from mathematics. Even though it will take time to resolve new questions as they arise, teachers should continue to discuss these issues with their colleagues, so they continue to work collaboratively to make effective changes in instruction to enhance everyone's perception of mathematics (Damarin, 1990).

Not only does the male-oriented vocabulary need to change, but the curriculum, which focuses on male dominions, also needs to be addressed. The government of the Netherlands recognized a need for change in mathematics instruction. It mandated that a team of professionals revise the national mathematics curriculum. Lee (1992, p.29) quotes Verhage, a member of this team, as stating that the revision was,

. . . making sure that it will provide girls, too, with a favorable perspective on the job market. Curriculum development and gender describes the context of realistic mathematics education.

Subjects were introduced in the Netherlands to bring girls into the reality of mathematics. Topics such as embroidery, cooking, and knitting are being studied by boys and girls. Other diverse areas of

study for all students include tanagrams, tessellations, wallpaper, and crockery (Lee, 1992).

In conjunction with curriculum development is the means of assessment to be used with the students. The kind of information which is to be assessed, how it is gathered, and how it is reported are all factors in mathematics education. Major questions exist in regard to assessment as it relates to gender issues. A critical question, which envelops several others, is whether mathematics is taught equally well to different groups of learners. Hanna (1992, p.439) asks an array of questions which evolve from this concern.

What kinds of mathematical tasks are being assessed (short technical exercises, long tasks, extended problems, etc.)?

Are the methods of assessment used more favorable to certain groups of students?

How can we ensure that classroom materials and exam questions properly reflect gender equity? Should they include a wider range of human activities and interests than traditional materials and examinations?

Is the range of experiences provided in the mathematics classroom (or elsewhere in the school) biased in favor of one group of students to the possible detriment of others?

Interactions between teachers and students can be used for assessment purposes. Teachers, who are cognizant of gender bias, can remedy this problem in their classrooms by monitoring their

interactions with students. If teachers keep a tally for a few weeks of how frequently they interact with students of each gender, they can begin to work towards an equal distribution of interactions with students of both genders. Once a quantitative equity has been formed, the teacher needs to assess the quality of the interactions. Colleague support and technology can assist the teacher in this area. Peer coaching, audiotaping, and videotaping can alert teachers to any inadvertent messages of gender bias which they may be conveying to their students. (Damarin, 1990).

Lee (1992, p.36) quoting Hanna, Kunduger, and Larouche on their gender research states:

The study highlights the fact that the issue of gender differences in mathematics is very complex and should be explored from many different perspectives.

Therefore, it is essential for teachers to be familiar with the female student's background and the preconceived beliefs about mathematics that they bring into the classroom with them.

To fail to recognize a student's anxiety, uncertainty, or concern about whether women are mathematically inferior is to deny an important part of the mathematical reality of the student (Damarin, 1990, p.149).

Teachers need to discuss these issues with the female students.

Trust must be developed during these interactions to promote the major objective of mathematics instruction that "all students learn that they can learn mathematics" (Damarin, 1990, p.150). Therefore, it is necessary that mathematics educators encourage girls and young women to be active participants in mathematics since it is an integral part of the real-life experiences which they will encounter as adults.

According to the NCTM Curriculum and Evaluations Standards for School Mathematics (1989), if children are to see math as being practical and useful, they must understand that it can be applied to a wide range of real-world problems. Children need to understand that math is an integral part of real-world situations. The students' cultural backgrounds should be integrated into the math experience because students bring different everyday experiences to the mathematical class. A student from an urban environment might interpret a problem differently than a student from a suburban or rural environment.

There are many reasons why it is desirable to present mathematical problems in real-world scenarios. One reason is that students are more likely to find these types of problems more

interesting than others. Another reason is that real-world problems are more likely to facilitate transfer by showing the students that their knowledge and skills are useful in solving real problems (Baron, 1991 citing the work of Brown, J. S., Collins, and Duguid, 1989; Rogoff and Lave, 1984). Students may improve their abilities to transfer their knowledge and understanding to other situations if they have practice in solving real world problems (Baron, 1991).

Real-world situations should not consist of easy to solve problems. The problems should contain too much or too little information and have multiple solutions. They should require a substantial investment of time and be presented as extended projects that can be worked on for hours, days, or longer. This will better prepare the children for the types of problems they will encounter in their daily lives (NCTM Curriculum and Evaluations Standards for School Mathematics, 1989).

### Project Outcomes

The first terminal objective of this problem intervention was related to data collected from the problem-solving pretest. These scores indicated that the problem-solving ability of third, fifth, and

sixth grade students was below grade level. Probable cause data, presented in the latter part of Chapter 2, and solution strategies presented in the first part of this chapter suggested the need for improving the quality and quantity of problem-solving activities in the classroom with journals in order to increase the ability of solving non-routine problems.

Therefore:

As a result of curriculum revision, during the period, September 1993 to January 1994, the students will increase their ability to implement various problem-solving strategies in order to successfully solve non-routine mathematical problems which require the use of higher-order thinking skills, as measured by teacher observation and student log entries showing the steps taken to reach a correct solution.

Probable causes gathered from literature suggested a need to implement different teaching strategies in order to reduce anxiety and to incorporate the NCTM standards for the improvement of students' higher-order thinking skills.

Therefore, the second terminal objective stated that:

As a result of implementing different teaching strategies, during the period, September 1993 to January 1994, the students will increase their mathematical problem solving abilities by using higher-order thinking skills, as measured by teacher observation and a checklist of student approaches to problem solving.

In order to accomplish the terminal objectives, the following process objectives defined the major strategic procedures proposed for problem resolution.

- 1) In order to accomplish the terminal objectives, the teacher will prepare individual lessons on problem-solving strategies and implement them in student practice sessions using cooperative learning.
- 2) In order to accomplish the terminal objectives, the students will keep math journals and record what was learned and the processes that were involved in solving the problem.
- 3) In order to accomplish the terminal objectives, the students will be able to create their own word problems that require higher-order thinking skills.
- 4) In order to accomplish the terminal objectives, the teacher will assess the students with a problem-solving behavior checklist based upon teacher observation.
- 5) In order to accomplish the terminal objectives, the students will be put into cooperative groups once a week for problem-solving tasks that have alternate ways of finding the solution to the problem.
- 6) In order to accomplish the terminal objectives, the teacher will develop and implement a supplementary program on higher-order thinking skills in math for the target group.
- 7) In order to accomplish the terminal objectives, the students will develop an awareness of the relevance of mathematics in their personal lives by being

participants in real world scenarios that incorporate several non-routine problem-solving strategies.

- 8) In order to accomplish the terminal objectives, the teacher incorporate different teaching strategies into the mathematical problem-solving lessons.

### Project Solution Components

The major elements of the approach used to increase the target groups' higher-order thinking ability are: to teach the strategies necessary to become a problem solver, to supplement the curriculum with journals, to promote student-generated and real-life problems, and to incorporate different teaching strategies such as cooperative learning and the use of manipulatives. These elements are related to the terminal objectives in that they attempted to produce a change in the higher-order thinking skills, to reduce anxiety, and to eliminate the discrepancy between the student's perceived ability and the actual student performance. Testing data indicated a low degree of problem-solving ability and probable cause data indicated an inaccurate student perception of their problem-solving abilities, a discrepancy between teacher perception of effective teaching methods and student performance, a poor quality and a low quantity



of problem-solving activities in the present curriculum which are auditorily taught.

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## Chapter 4

### ACTION PLAN FOR IMPLEMENTING

#### Description of Problem Resolution Activities

The action plan is designed to address three major solution components: the development of a supplementary math program for problem solving, the teaching of problem solving strategies, and the implementation of cooperative groups to provide a supportive environment to facilitate the improvement of the students' problem-solving skills in mathematics.

The teachers will begin the development of a supplementary mathematics program in the summer of 1993. This program will follow the mathematics standards of the National Council of Teachers of Mathematics (NCTM, 1989). District objectives for problem solving will be reviewed and enhanced by the addition of non-routine problems and related activities which incorporate higher-order thinking skills in mathematics.

The preliminary phase of the implementation plan will begin with a student attitude survey in September 1993. A pretest on non-

routine problem solving will be administered shortly thereafter. The purpose of these evaluation tools is to: 1) determine probable cause for the students' deficiency in problem-solving skills; and 2) observe and record the students' reasoning capabilities when given non-routine problems which require the students to use a variety of problem-solving techniques. The teachers will then proceed with the implementation plan to instruct the students on the various strategies which can be employed to become effective problem solvers. Students will be encouraged to be active participants in their own learning as they experiment with the new strategies in a positive classroom environment. During the lessons, students will be required to give justifications for their solutions and use self-reflection to evaluate themselves.

The NCTM recommends that educators use cooperative learning to improve students problem solving. Working in cooperative groups, with the sharing of ideas on problem solving, benefits all members. Students will become comfortable with the idea that there are different ways of approaching a problem while arriving at the same solution. The acceptance of others' ideas and the use of self-reflection will strengthen the children's problem-solving abilities

to deal successfully with real-life situations.

The implementation plan is presented below in and outline form and in chronological order, allowing for the overlapping of strategies over time.

1. Supplement for the math curriculum.
  - A. Who: The third, fifth, and sixth grade teachers will provide the supplemental math materials.
  - B. What: The teachers will design a supplementary math problem-solving program consisting of: manipulatives (i.e., calculators, play money, counters, rulers, meter sticks, tape measures, base 10 blocks), real-life scenarios, graphic organizers (i.e., graphs, charts, grids), and journal writing. In addition, Third grade teachers will familiarize themselves with the University of Chicago Math Program called Everyday Mathematics, to enable them to implement this program in September.
  - C. When: This will take place during the summer of 1993.
  - D. Where: This work will take place at the teachers' homes and at the suburban elementary school.
  - E. How: The teacher will collect a variety of resources from

printed literature. These resources will be used to create lessons that will follow the national standards set forth from the National Council of Teachers of Mathematics and the Illinois state goals for mathematics. The third grade teachers will also read the manuals and ancillary materials for the Everyday Mathematics series.

- F. Why: The product will be used by the teacher to promote higher-order thinking skills in mathematical problem solving.
2. Designate prescribed days and times for the implementation of non-routine problem-solving skills.
- A. Who: The third, fifth, and sixth grade teachers will design a schedule.
  - B. What: The schedule will include 90 minutes a week over a fifteen week period.
  - C. When: The schedule will be created the first week of school in September of 1993.
  - D. Where: The program will be implemented at the suburban elementary school.
  - E. How: The teacher will integrate the supplemental program to fit into the current math program.

- F. Why: The schedule will provide continuity and increased frequency of instruction of problem-solving skills.
3. Determine students' ability levels.
- A. What: Attitude surveys on mathematics will be given to the target group. The teacher will administer a pretest to the students, and observations will be recorded on checklists and logs (refer to appendix ).
- B. When: Student attitude surveys will be given during the first week of school in September of 1993. The pretest will be administered during the first two weeks of school in September of 1993. The teacher observation checklists and logs will be done from September 1993 through January of 1994.
- C. How: The teacher will create an attitude survey, pretest, and observation checklist for the target group.
- D. Why: Probable causes and the students' beginning academic levels before implementing the supplementary programs will be analyzed and referred to in order to chart the growth of the students' problem-solving capabilities.
4. Teach the students problem-solving strategies to assist them

with non-routine problems.

- A. What: The following strategies will be taught: logical reasoning, making an organized list, using or making a table, using or making a picture, guessing and checking, using or looking for a pattern, acting out or using objects, working backwards, making it simpler, and brainstorming.
  - B. When: The target group will be instructed on problem-solving strategies from September of 1993 to November of 1993.
  - C. How: The students will receive direct instruction on each of the problem-solving strategies. Individual practice will be provided with whole group reflection following each lesson.
  - D. Why: Students will be exposed to a variety of strategies in order to assist and improve their problem-solving abilities both in the classroom and the real world.
5. Students will do learning activities incorporating their problem-solving strategies acquired.
- A. Who: Students will utilize the strategies for non-routine problem solving.
  - B. What: The students will use the program developed by the teachers.

- C. When: The students will be engaged in the learning process during the period of September 1993 through January 1994.
- D. How: Students will do independent and cooperative group work to become effective problem solvers.
- E. Why: Students will become independent learners and will transfer their mathematical knowledge to real-life situations.

#### Methods of Assessment

Data will be collected in a variety of ways to assess the students' growth as a result of the interventions. Progress in the target groups' problem solving abilities will be measured through the use of: published materials, teacher-made tests; observation checklists; teacher observation logs; student journals; and student-generated problems on problem solving. The posttest will be administered in January 1994, and the results will be used to compare the responses documented from the September 1993 test.

The ability of the students to plan a strategy, to formulate and find a solution, to justify their reasoning, and to reflect upon their work will be documented through formal classroom observations and journal entries based upon informal classroom observations and



conversations with the students. The changes found in the students' metacognitive abilities will be assessed in order to measure the effects of the interventions upon the target group.

## Chapter 5A

### THIRD GRADE GROUP A EVALUATION OF RESULTS AND PROCESSES

#### Implementation History

The terminal objectives of the intervention addressed the inability of third grade students to use higher-order thinking to solve non-routine problems. Test scores and observations indicated a weakness in the students' ability to use problem solving strategies. Therefore, the terminal objectives stated:

As a result of curriculum revision, during the period, September 1993 to January 1994, the students will increase their ability to implement various problem solving strategies in order to solve non-routine mathematical problems which require the use of higher-order thinking skills, as measured by teacher observation and student log entries showing the steps taken to reach a correct solution.

As a result of implementing different teaching strategies, during the period, September 1993 to January 1994, the students will increase their mathematical problem solving abilities by using higher-order thinking skills, as measured by teacher observation and a checklist of student approaches to problem solving.

The development of a curricular component to address the lack of

problem solving and higher-order thinking skills began with a review of the current curriculum to assess the problem solving content. After seeing the lack of problem solving problems using higher-order thinking, various printed problem solving curriculum were reviewed. There was one program that addressed the ten different problem solving strategies and included the use of different manipulatives. This was The Problem Solver published by Creative Publications (Goodnow and Hoogeboom, 1987). The district had adopted a new curriculum in the primary grades. This program Everyday Mathematics, from the University of Chicago Mathematics Program, (Everyday Learning Corporation, 1993) was also reviewed for its problem solving content. It was found that the approach of the series included manipulatives and problem solving on a daily basis. Along with adopting Everyday Mathematics and adding The Problem Solver, a journal writing book was also identified to help develop the target group's problem solving ability. This was Math Journal Writing and Problem Solving (Carson-Dellosa Publishing Company, Inc., 1992). The curriculum review was done by the classroom teacher during the summer of 1993.

After the schedule of special subjects (art, computer, music, gym,

library, and Discovery Center) was made, the classroom teacher of the target students adapted her schedule to include a time for problem solving four days a week for approximately 20 minutes.

The first week of the school year, the target students and the teaching staff at the target school filled out attitude surveys (See Appendix B and C) to assess possible causes of the problem. The target group was also given a pretest (See Appendix D) to assess their current ability level and to chart any growth made through this action plan. The questions for the pretest were taken from the second grade level of The Problem Solver. These questions were used because the direct instruction lessons were to be taught with the same book. The second grade level was chosen because the third grade level seemed too difficult for the group.

The strategies were taught in this order, through direct instruction, to the target group: logical reasoning, make an organized list, use or make a table, use or make a picture, guess and check, use or look for a pattern, act out or use objects, work backwards, make it simpler, and brainstorm. The instruction for the first nine strategies followed a specific order. The first lesson of the week reviewed the question (second grade level) given on the

pretest. This was done as whole group instruction. The second lesson was another second grade level question completed by students in small groups or on their own. This lesson was checked by the teacher. If the student missed the answer, they were told to try again. The students who solved the problem correctly were used to help the students who were struggling to answer the question. The third lesson of the week was direct instruction using a third grade level question. The last lesson of the week was a third grade level question in which the students worked in groups to solve the problem. Once again, if a student missed the answer, they tried again until they solved the problem correctly. Once they solved the problem correctly, they were to help others who couldn't solve the problem. This teaching technique was followed throughout the first nine weeks with one strategy taught a week.

The strategy of brainstorming was taught differently than the other nine. The students were given the sheet and it was read aloud. Then in groups the students would 'brainstorm' possible solutions to the question. After about 15 minutes, the students were given the answer along with an explanation.

Within each of the ten strategies, the students were taught a

Four-Step Strategy to follow to assure a correct answer. They were: find out, choose a strategy, solve it, and look back. In the first step, find out, the students were to look for information given in the problem. This included the question they needed to answer. For the step, choose a strategy, the students looked at what was being asked, and determined what they could do to help them solve the problem. Then they worked through the problem to find the correct answer. Lastly, they were to go back to the problem and reread it to see if the answer they found fits with the information given in the problem, and if it answered the question.

After the ten strategies were taught (instruction sheets), the students were given ten additional problems (practice sheets). One problem for each of the strategies given in a random order. A student would read the problem to the class. As a group, discussion was held to determine the possible strategies to use to solve the problem. The students had folders to keep all of their papers together. They could look back to other problems to help them. They also worked together to solve the problems. The students tried again if unsuccessful on their first try to solve the problem.

Along with the additional problem solving time and supplemental

curriculum, the University of Chicago Math Program called Everyday Mathematics was used during regular math instruction. The regular math instruction is daily from an hour up to an hour and a half. This program uses problem solving on a daily basis and puts it in the real world. It doesn't separate problem solving from other math skills being taught. The students have a "Home Link" to link the learning at school to direct uses at home. During math instruction, the children used a variety of manipulatives to solve the various problems including: calculators, rulers, slates, and counters. During the instruction time a checklist was completed to assess the students' problem solving abilities.

Incorporated in the Everyday Mathematics series are Mathboxes. These Mathboxes contain various math skills taught throughout the series. The math skills such as adding, subtracting, multiplying, dividing, measuring, and identifying fractions appear on the Mathboxes. At the bottom is a place to add problems in the areas in which the students need additional help. Starting in October up through January, this part of the Mathbox was used for the students to create their own story problem. The Mathboxes were given to the students four times a week, almost every week.

Also in Everyday Mathematics, the students participate in Explorations. This is a time where the students explore math. Some examples of the types of activities in Explorations include: cutting apart one figure and putting it back together in the shape of a square, taking a box of Base-10 blocks and coming up with a strategy to count the exact number in the box, and connecting dots to draw a figure and then counting the number of squares and triangles within the figure. The students create a group report to show what strategy they used or what conclusions they found from the information they worked with. These activities are done in small groups with little teacher intervention. The teacher acts as a facilitator to answer questions and clarify directions.

Every math lesson, except for Explorations, starts with a math message. The students had journals (spiral notebooks) in which they put the answers to the math message. The math message is somehow related to the math lesson. The math messages range from completing math equations, to thinking about given information, to writing their own math stories. Along with the math message, once a month the students were given a journal topic to write about. Journal writing was intended to happen more often, but the students



took at least a half hour to complete the journal writing assignment. This journal writing along with the additional problem-solving instruction took too much time away from the curriculum dictated by the district office. Therefore, it was too difficult to have the students write long journal entries more than once a month.

### Presentation and Analysis of Project Results

In order to assess the effects of the planned intervention, students' problem-solving ability was tested as stated in the objective. During September through January the interventions were implemented. The students were involved in direct instruction and cooperatively solved practice questions. The results of the pre and post test are presented in appendix A and summarized in table 19.

The students seemed to perform the same on the test as they did on the instruction and practice questions. During the instruction and practice portions of the action plan, the more difficult the problems became, the faster the students seemed to work. After the first three strategies were taught, most students were getting the problems wrong. At this point the students were given second and third attempts to solve the problem correctly. Students solving the

problem correctly assisted those who were having difficulty solving the problem. After the third try, the problem was discussed with the target group and the solution was shown in the step by step process.

The last problem for each strategy was at third grade level and the students worked together to solve the problems. Within five minutes after passing out the problem, the same three boys were finished. They barely had enough time to read the problem besides taking the necessary steps to solve it. When the student 'solved' the problem that quickly, the response was incorrect. In some cases the students' response didn't fit with the question being asked. If the question called for the students to identify two items (such as 'How many copper and how many silver coins are there?'), they would put one answer. If they put two answers, which was rare, they didn't identify which number was which.

Problem-solving instruction and the time to work on the problems was given in school, therefore, if a student was absent they didn't make up the missed question. Problem-solving questions were never to be taken home.

At the conclusion of the instruction and practice sessions, the

pretest was re-administered as a posttest. As shown in Table 19, the posttest scores are higher than the pretest scores. Questions for the pre and post test were chosen from the second grade level book. After the students scored higher than anticipated on the pretest, the second and third grade levels were used to instruct the students on the strategies. Therefore, a second posttest was given from the third grade level. The questions for the strategies of 'making it simpler' and 'brainstorming' were taken from the fourth grade book. These questions were used because of the limited number of questions in the third grade book. All of the questions at the third grade level were used during the instruction and practice part of the action plan.

As with the pretest, the students worked individually on the posttest. All ten strategies were given in the same sitting. The students weren't given any assistance in answering the questions. After the first few students were denied help, no other students asked. The first student finished with the posttest, completed it in 15 minutes while the last student was finished in 35 minutes. Two students had moved since taking the pretest.

Table 19  
The Percentage of Third Grade Students Answering  
Correctly on the Pre and Post test

Problem Solving Strategy	Pre	Post
Logical Reasoning	80	89
Organized List	50	78
Use or Make a Table	35	100
Use or Make a Picture	35	44
Guess and Check	75	89
Use or Look for a Pattern	85	94
Act Out or Use Objects	70	83
Work Backwards	25	39
Make it Simpler	0	28
Brainstorm	5	61

Pretest n = 20  
Posttest n = 18

Even though the posttest questions were the same as the pretest, on more than half of the strategies the percentage of students answering correctly increased less than 15 percent. Two of the areas in which the percentage of correct responses increased 10 percent or less were on 'use or make a picture' and 'work backwards'. During direct instruction 'use or make a picture' was the fourth strategy taught. On the second grade problem of this strategy, only 20 percent of the students answered correctly, and on the third grade level problem, only one student answered correctly.

Although the percentage of increase in half of the strategies was low, the number of problems each student answered correctly increased. On the pretest, 40 percent of the students correctly

solved six or more out of the ten problems. On the posttest, 79 percent of the students successfully solved at least six or more of the questions. On the posttest 39 percent of the students correctly solved eight out of ten questions.

The students performed much better at the beginning of the test than at the end of the test. The strategies, in the order they were given, became more difficult. The lower percentage of correct responses could be due to the students not putting in as much effort at the end of the pre and post tests than at the beginning. There are other possible causes. Seeing that others finished early could have caused a student to rush to finish as well. Taking the test in one sitting could have run the student down so there wasn't as much effort at the end. Some students may have hurried through this test in the same manner in which they hurried through the instruction and practice sheets. It was obvious that some students didn't take the time to check their answers or follow the Four-Step Plan. The last step was to check their answers.

After the ten weeks of direct instruction on the strategies, the students were given 10 additional practice problems. They worked on these in groups. The students scores on these sheets ranged

between five and eight correct responses out of the nine problems. This is higher than on the third grade posttest (See Table 20). The students were given a test at the third grade level since half of the direct instruction lessons and the practice sheets were at this level. The students were allowed and encouraged to work in groups on these practice sheets, but not on the test.

Table 20  
The Percentage of Correct Responses  
on the Third Grade Level Practice Questions  
and Posttest

Problem Solving Strategy	Practice Questions	Post Test
Logical Reasoning	82	53
Organized List	71	59
Use or Make a Table	100	18
Use or Make a Picture	17	35
Guess and Check	71	24
Use or Look for a Pattern	94	53
Act Out or Use Objects	100	6
Work Backwards	100	12
Make it Simpler	12	24

Posttest n = 17

Practice n = 15, 17, or 18

As shown in Table 20, on all but two strategies the students scored better when working in a group to solve the problems. On these same seven strategies, 13 or more students answered correctly. Due to absences, not all of the students worked on the practice questions. If absent, the students were not given the

question. During the practice on 'working backwards' and 'using or making a table', at least 12 students were given a second or a third chance to solve the problem. If the students' response didn't fit with the question asked on the practice, they were given an opportunity to correct it. This was not the case during the test. Several responses on the posttest didn't fit the question.

Brainstorming was left off of Table 20 due to the fact that the results of the instruction and practice sheets were not kept. This strategy was taught differently than the rest. The students were given the question and then they had approximately 15 minutes to 'brainstorm' the answer in groups. Then the students gave their answers orally. Each answer was reviewed by rereading the question to see if it fit with all of the given information.

### Reflections and Conclusions

The action plan seemed to increase the students' ability to use higher-order thinking skills to solve non-routine problems. This was accomplished through supplemental curriculum and adoption of a new math series.

The district-wide adoption of the new math series (Everyday

Math) which contains a hands-on approach to math, and incorporates problem solving on a daily basis, is key to increasing students' problem-solving ability. With the amount of current curriculum dictated by the district office throughout the day, it was difficult to add supplemental curriculum. By adding a time for a supplemental curriculum, the time used for another subject had to be reduced.

If the supplemental curriculum had been used by all teaching staff at the target school, there would be a more significant increase in students' problem solving ability. The students would be familiar with the four steps involved in problem solving. They would have been able to check back in the problem and write their answers to fit the question being asked.



## Chapter 5B

### THIRD GRADE GROUP B EVALUATION OF RESULTS AND PROCESSES

#### Implementation History

The terminal objectives of the intervention addressed the ability of third grade students to solve non-routine mathematical problems requiring the use of higher-order thinking skills. Test scores from the Stanford Achievement Tests, district criterion reference testing, published tests, and teacher observation of student performance indicated a deficiency in students' ability to implement problem-solving strategies. Therefore, the terminal objective stated:

As a result of curriculum revision, during the period, September 1993 to January 1994, the students will increase their ability to implement various problem solving strategies in order to solve non-routine mathematical problems which require the use of higher-order thinking skills, as measured by teacher observation and student log entries showing the steps taken to reach a correct solution.

As a result of implementing different teaching strategies, during the period, September 1993 to January 1994, the students will increase their mathematical

problem solving abilities as measured by teacher observation and a checklist of student approaches to problem solving.

The development of a curricular component to address the students' deficiency in problem solving began in the summer of 1993. Using the national standards set forth by the National Council of Teachers of Mathematics and the Illinois state goals as a guideline, the current curriculum was reviewed to assess its problem-solving content. After it became apparent that the current curriculum lacked an intensity of problem-solving activities, a search for supplemental materials was instituted. Various published problem-solving curricula were reviewed. After analyzing and receiving input from colleagues, The Problem Solver, published by Creative Publications (Goodnow and Hooegeboom, 1987), was selected to be the core component for teaching ten different strategies for problem solving. This program was available for various grade levels. After reviewing the materials for grades two and three, it was found that level three would probably be too difficult to start with, so level two was selected to be used for the pretest and introductory lessons, and level three material would be used for follow-up lessons.

The newly adopted district curriculum in mathematics for the primary grades was also reviewed for its problem-solving content. This program, Everyday Mathematics, (The University of Chicago Mathematics Project, 1993), was deemed appropriate and relevant in addressing the development of problem-solving skills. The approach of Everyday Mathematics includes problem solving about real-life events, hands-on activities, sharing ideas through discussion, and cooperative learning through partner and small-group activities.

In addition to The Problem Solver and Everyday Mathematics, a journal writing book was also adopted for use in developing the students' problem-solving ability. This published curriculum, Math Journal Writing and Problem Solving (Carson-Dellosa Publishing Company, Inc., 1992), was chosen for its practical application to everyday situations. Notebooks for the journal writing and folders for compiling activities were purchased by the classroom teacher for use by the students.

During August 1993, and September 1993, time was spent developing procedures and a schedule for implementation of the problem-solving program. The regular school day schedule was adapted to include problem-solving instructional time. The schedule

allowed for approximately 20 minutes a session, four days a week, for ten weeks. Initially, the program was scheduled to be implemented from September to January, 1993. However, due to days off, previously scheduled school activities, and the necessity of meeting district goals and objectives mandated by the district curriculum, it was necessary to extend the problem-solving program through February.

During the first week of the school year, the target students and the teaching staff at the target school completed attitude surveys to help the classroom teacher gain insights into the possible causes of the students' limited capabilities in problem solving in mathematics. Students in the target group were also administered a pretest during the first week of school to assess their current ability in problem solving. This test would then be used at the conclusion of the action plan, along with a posttest, to compare and assess the effectiveness of the program. The questions used on the pretest were taken from the second grade level of The Problem Solver. This pretest was administered to 17 students. One student was absent. Recognizing that the students had limited problem-solving abilities, it was decided that the third grade level questions

were too difficult, and would prove to be too frustrating to the students.

The action plan was then implemented through direct instruction of the ten problem-solving strategies. The strategies were taught following the sequence set forth by the guidelines of The Problem Solver. The ten strategies taught were: Logical Reasoning, Make an Organized List, Use or Make a Table, Use or Make a Picture, Guess and Check, Use or Look for a Pattern, Act Out or Use Objects, Work Backwards, Make It Simpler, and Brainstorm.

The students were taught a systematic approach to problem solving through the Four-Step Method, outlined in The Problem Solver. In the first step, the students must determine what the problem means and what question needs to be answered to solve it. Second, the students must choose a strategy that will help in solving the problem. The third step is to work through the problem to solve it, recording their work and changing strategies if necessary. The final step was looking back. The students were to reread the problem and review the solution to be sure it was logical and reasonable.

Instruction for each strategy started with reviewing the question

given on the pretest for each specific strategy. This was a second grade level question and was approached with whole-group instruction. The second lesson for the particular strategy was also at the second grade level and was completed independently. Students with the correct solution were then asked to work with a student who was not having success at solving the problem. The problem was then discussed in a whole-class setting, and the correct answer was given. Direct instruction of a third grade level question was the third lesson of the week. This was, again, taught by whole-group instruction. The fourth lesson pertaining to the given strategy was done with a partner or in a group of three. After reaching the correct solution, these group members then offered their assistance to struggling groups. At the end of the period the problem would be discussed and the correct solution would be given. This plan was followed throughout the teaching of all ten strategies.

In addition to The Problem Solver, Everyday Mathematics was used during regular math instruction. A variety of manipulatives were used in the presentation of math lessons. They included, but were not limited to, calculators, slates, rulers, meter sticks, counters, tape measures, coins and bills, straws, and pipe cleaners.

Everyday Mathematics starts each lesson with a math message. This is an independent activity related to the lesson. The students used their journals to complete the math message. Upon completion of the math message there was a discussion of the responses. The journals were also used to record activities from Math Journal Writing and Problem Solving. These entries allowed the students to create problems, solve problems, and reflect on mathematics or mathematically related activities. Though valuable and enjoyable to the students, the journals were not used as often as had been anticipated or desired due to lack of time during problem-solving sessions.

#### Presentation and Analysis of Project Results

In order to assess the effects of the planned intervention, students' problem-solving ability was tested as stated in the objective. Eighteen students were administered the posttest. During September 1993, through February 1993, the interventions were implemented. The results of the pre and post tests are available in Appendix H and summarized in Table 20.

Table 20  
The Percentage of Third Grade Students Answering  
Correctly on the Pre and Post Test

Problem Solving Strategy	Pre	Post
Logical Reasoning	71	100
Organized List	24	89
Use or Make a Table	88	100
Use or Make a Picture	24	56
Guess and Check	82	83
Use or Look for a Pattern	82	94
Act Out or Use Objects	76	94
Work Backwards	53	78
Make It Simpler	12	28
Brainstorm	6	17

Pretest n = 17  
Posttest n = 18

The data indicate pre to post test improvement in all ten strategies. The greatest improvement was on the Make an Organized List strategy. The students made a 65 percent gain in this area. The strategy of Use or Make a Picture showed an increase of 32 percent. Logical Reasoning increased by 29 percent, and Work Backwards gained 25 percent. Act Out or Use Objects rose by 18 percent, and Make It Simpler by 16 percent. Lower gains were achieved in Use or Make a Table and Use or Look for a Pattern, which both increased by 12 percent, and an 11 percent increase was noted in the strategy of Brainstorm. The strategy of Guess and Check, which was the area that the students had the most difficulty with during instruction, gained one percent.



The number of correct responses made by each student on an individual basis also showed increases. Table 21 shows the number of correct responses and percentage of students that responded correctly to individual questions on the pre and post tests.

Table 21  
Number and Percentage of Third Grade Correct Responses  
on the Pre and Post Tests

Number of Correct Responses	Number of Students		Percent of Students	
	Pre	Post	Pre	Post
10	0	0	0	0
9	1	4	6	22
8	0	4	0	22
7	2	7	12	39
6	5	2	29	11
5	3	0	18	0
4	3	1	18	6
3	3	0	18	0
2	0	0	0	0
1	0	0	0	0
0	0	0	0	0

Pretest n = 17  
Posttest n = 18

The posttest results for each of the ten problem-solving strategies indicate improvement. There were no students that scored zero, one, or two correct responses on either the pretest or the posttest. A total of six students scored three or four correct responses on the pretest, whereas on the posttest only one student scored four correct responses.

The data show that on the pretest 11 students scored 50 percent

or better, and six students scored below 50 percent. On the posttest 17 students scored 50 percent or better, and only one student scored below 50 percent. Overall, 95 percent of the students increased their problem-solving abilities, and six percent remained the same.

### Reflections and Conclusions

The practicum reduced the discrepancy by improving the problem-solving abilities of the target group. It also increased the group's knowledge of using a systematic approach to problem solving that can be used to address any problem they may come across.

A critical component of this implementation plan was the involvement of the teachers who would implement the program. Two third grade teachers, one fifth grade teacher, and one sixth grade teacher worked collaboratively to establish the goals and objectives for the unit of study. These teachers met to produce a teacher and student survey to meet the needs of all grade levels participating in the problem-solving unit. They then researched various publications and materials to find ones suitable for the varying grade levels.

The cooperation of the various grade level teachers was an integral part of the success in implementing the problem-solving program. The use of these new teaching techniques was

implemented in an atmosphere of support. Ideas, approaches, successes, and failures were shared and solutions sought.

After the routine of the problem-solving lessons was established, the teaching and learning became more efficient and effective. It is unfortunate that time constraints and the full load of a preset district curriculum impeded further implementation of the problem-solving action plan. A full year curriculum in problem-solving would do justice to teaching and practicing problem-solving skills.

The cooperative learning aspect of the problem-solving curriculum was an invaluable component of the program. Students discovered that working together was enjoyable. They learned to listen to each other, and to be receptive to other's ideas. They learned to build on each other's mathematical discoveries. They discovered that reaching success as a team could be just as rewarding as being successful as an individual.

## Chapter 5C

### FIFTH GRADE EVALUATION OF RESULTS AND PROCESSES

#### Implementation History

The terminal objectives of the intervention addressed the ability of fifth grade students to solve mathematical problems which require the use of higher-order thinking skills. Scores from the Stanford Achievement Tests, district criterion reference testing, published tests, and teacher observations of student performance indicated student problem-solving deficiencies. Therefore the terminal objectives stated:

As a result of curriculum revision, during the period September 1993 to January 1994, the students will increase their ability to implement various problem solving strategies in order to successfully solve non-routine mathematical problems which require the use of higher-order thinking skills, as measured by teacher observation and student log entries showing the steps taken to reach a correct answer.

As a result of implementing different teaching strategies during the period September 1993 to January 1994, the students will increase their mathematical problem-solving abilities by using higher-order

thinking skills, as measured by teacher observation and a checklist of student approaches to problem solving.

The development of changes to the fifth grade mathematics curriculum at the target site began in the summer of 1993. Materials that followed the national standards set forth from the National Council of Teachers of Mathematics and the Illinois state goals were needed. The libraries and bookstores were searched in order to find materials which required the students to use higher-order thinking skills. Teachers from the target site and outside school districts were asked for possible suggestions of materials to use. After analyzing several programs, it was decided that The Problem Solver (Goodnow and Hoozeboom, 1987) would be used as a basis for teaching ten different strategies for problem solving. Since the program is published for every grade level, the next step was to determine which grade level of The Problem Solver would be appropriate for use with the fifth graders since the target group had not been exposed to any previous lessons in grades one through four. It was decided that level four instead of level five would be purchased since the problems found in level five seemed to be too difficult. Exercises from Problem Solver 3 would be used as

introductory lessons. This lower level would give the students a sense of accomplishment and success.

The next step was to accumulate enough material to teach math using real-life situations and cooperative groupings. The material also needed to follow the mathematics standards of the NCTM. Several published books and exercises developed by other teachers were purchased. Notebooks and folders to be used by the students for journal writing were bought.

Time was spent during August 1993 and the beginning of September 1993 designing procedures to implement the program. The program had two components; a series of lessons involving ten strategies for problem solving that required direct instruction, and a series of less teacher directed activities that involved real-life situations and cooperative learning groups.

The series of lessons that required direct instruction involved ten strategies for problem solving using higher-order thinking skills. These ten strategies were: Use Logical Reasoning, Make an Organized List, Use or Make a Table, Make a Picture or Diagram, Guess and Check, Use or Look For a Pattern, Act Out or Use Objects, Work Backwards, Make It Simpler, and Brainstorm.

It was proposed that 90 minutes per week over a period of 15 weeks would be dedicated to problem solving. The target group would be instructed from September to November of 1993. In the process of conducting the program, it was determined that the goal of 90 minutes per week was not feasible. Due to the amount of curriculum mandated by the school district and the development of a special thematic unit involving other teachers, the time had to be shortened to 60 minutes per week. With the reduction of time, the number of sessions proved to be inadequate. The program, initially scheduled for September to November 1993, had to be both delayed and extended. The administration of the student survey, the pretest, and all of the beginning of the year activities were causes that attributed to the delay. The instruction of the 10 strategies, the incorporation of real-life scenarios into the curriculum, and the use of cooperative learning groups was employed from October 1993 through February 1994.

The proposed program called for the use of a teacher observation checklist. The difficulty of the lessons, the need for direct instruction, and the need for explanation to individual students prohibited the use of a consistent checklist.

The use of student journals was proposed in order to give the students a chance to document their strategy by formulating and finding a solution and reflecting on the outcome of their work. The journals were also to be used for the writing of student-generated problems on problem solving. Due to the limited time per session and the increased time needed for direct instruction, the number of student-generated problems was reduced.

A pretest (Appendix I) was administered to twenty-seven students at the onset of instruction. The pretest consisted of ten problems taken from The Problem Solver 4. There was one question for each of the ten strategies to be taught. The program then began with an instruction of the ten strategies in a sequence that followed the guidelines of The Problem Solver. The children were given a copy of a problem and were guided by the teacher through the Four-Step Method. First, they were to find out what question was needed to be answered in order to solve the problem. Second, the children needed to choose a strategy that would help them solve it. Next, they were to work through the problem until they found an answer. Last, the students were to look back, checking the solution to see if the answer was logical and reasonable. The direct instruction was



followed by two or three similar problems that were to be done independently. A discussion of the answers followed and, when appropriate, a similar problem was created by each child. These problems were recorded in notebooks and shared by the class. The use of cooperative learning groups and real-life scenarios were interspersed throughout the implementation time. These lessons were incorporated into the program to accomplish the following objectives:

- 1) to enable the participants to become involved with situations that incorporate important life skills such as graphing, finding mean, median, and mode, and calculating.
- 2) to enable the participants to become active learners who share ideas and interact with all group members.
- 3) to enable the participants to investigate math content, develop strategies, interpret results, and formulate solutions.

Manipulatives such as meter sticks, tape measures, coins, graph paper, calculators, and objects such as candy were used during these lessons. The purpose was to give the children an opportunity to perform "hands on" activities.

A posttest was given at the completion of the instruction and practice of the ten strategies and after the implementation of real life scenarios and cooperative learning groups. Two students had moved since the administration of the pretest, therefore reducing the number of students taking the posttest to twenty-five. A comparison of the pretest and posttest scores was used as a measure to assess the effectiveness of the program.

#### Presentation and Analysis of Project Results

In order to assess the effects of the planned intervention, students were tested on their ability to solve mathematical problems that required the use of higher-order thinking skills. The proposed interventions were implemented from October 1993, through February 1994. The results of the pretest and posttest are compared in Table 22.

Table 22

Comparison of the Results of the Problem-Solving Pretest and Posttest

Strategy	Pre	Post
1. Logical Reasoning	48%	84%
2. Organized List	0%	16%
3. Use or Make a Table	7%	44%
4. Use or Make a Picture	22%	48%
5. Guess and Check	0%	16%
6. Use or Look For a Pattern	15%	44%
7. Act Out or Use Objects	4%	12%
8. Work Backwards	0%	4%
9. Make It Simpler	4%	52%
10. Brainstorm	30%	44%

N = 27 Pre

N = 25 Post

The data indicate pretest to posttest improvement in all areas. The greatest improvement in the number of correct responses was on the strategy Make It Simpler which increased by 48 percent. The concept of Use or Make a Table showed an increase of 37 percent and the concept of Logical Reasoning showed an increase of 36 percent in the number of correct responses. The lowest percent of increase was on the concepts of Work Backwards, which increased by only four percent, and the concept of Act Out or Use Objects which increased by only eight percent.

The number of correct responses made by each student also

increased. Table 23 shows the number of correct responses the students made on the pretest and posttest. It also shows the percentage of students that made correct responses on the ten questions.

Table 23  
The Number and Percentage of 5th Grade Students Choosing Correct Responses on the Pretest and Posttest

Number of Correct Responses	Number of Students		Percent of Students	
	Pre	Post	Pre	Post
10	0	0	0%	0%
9	0	0	0%	0%
8	0	0	0%	0%
7	0	0	0%	0%
6	0	3	0%	12%
5	0	5	0%	20%
4	1	4	4%	16%
3	1	6	4%	24%
2	9	3	33%	12%
1	10	2	37%	8%
0	6	2	22%	8%

N = 27 Pre

N = 25 Post

Table 23 reflects an increase in the students' ability to perform problem-solving strategies that require higher-order thinking skills. On the pretest, only two of the 27 students, or eight percent, answered more than one or two questions correctly. On the posttest, 18 of the 25 students, or 72 percent, answered more than two questions correctly.

Individual test scores on the pretest and posttest were analyzed and the results are shown in Appendix J. The data show that 84 percent of the students increased their ability to solve problems using higher-order thinking skills. Eight percent remained the same and eight percent showed a decrease.

### Reflections and Conclusions

The program increased the ability of the target group to solve mathematical problems that require the use of higher-order thinking skills. This was accomplished through curriculum revision and the implementation of different teaching strategies.

An important part of the curriculum revision process was the addition of materials that had specific guidelines and approaches to problem solving. The Problem Solver utilized a four-step method of approach and provided ten different problem-solving strategies. The goal of this particular program was to provide techniques that enable students to tackle future problems.

The use of different teaching strategies was also a critical factor which increased students' problem-solving abilities. Incorporating cooperative learning groups into the math lessons gave

the children a chance to become active learners, offering suggestions and sharing ideas. Students who lacked strength in one area of mathematics could learn from others who had a particular strength in that area. Cooperative learning let the students who were afraid of making incorrect answers have a chance to participate in the formulation of a solution without the fear of failure.

Using real-life scenarios and manipulatives gave the children a chance to perform math that was relevant to their personal lives. The opportunity to participate in "hands-on" activities was a way to address the different styles of learning of the children.

## Chapter 5D

### SIXTH GRADE EVALUATION OF RESULTS AND PROCESSES

#### Implementation History

The terminal objectives of the intervention addressed the inability of sixth grade students to solve non-routine problems in mathematics that required higher-order thinking skills. An evaluation of the mathematics materials and tests used in the target school indicated the sixth grade students had not been exposed to a curriculum that offered extensive lessons and activities in mathematics which necessitated the use of various strategies while employing higher-level thinking skills in problem solving. Therefore, the terminal objectives stated:

As a result of curriculum revision, during the period, September 1993 to January 1994, the students will increase their ability to implement various problem-solving strategies in order to successfully solve non-routine mathematical problems which require the use of higher-order thinking skills, as measured by teacher observation and student log entries showing the steps taken to reach a correct solution.

As a result of implementing different teaching

strategies, during the period, September 1993 to January 1994, the students will increase their mathematical problem solving abilities by using higher-order thinking skills, as measured by teacher observation and a checklist of student approaches to problem solving.

The sixth grade teacher began to develop a supplementary mathematics program during the summer of 1993. Materials on problem solving for levels six to eight were reviewed and purchased by the teacher. The items selected followed the standards of the National Council of Teachers of Mathematics (NCTM, 1989) and also enhanced the target district's objectives on problem solving for the sixth grade. Teachers from surrounding school districts were also consulted on the choice of materials. (These teachers were asked for advice because they were involved in curriculum revision, based upon the NCTM standards, within their own districts.)

An analysis of the published materials on problem solving allowed the sixth grade teacher to choose a program that best met the needs of the students in the target group. The Problem Solver 6 (Moretti, Stephens, Goodnow, and Hooegeboom, 1987) was selected to be used as the basis for instruction in problem solving. This book was chosen for the following reasons: it was closely aligned to the NCTM standards and the school district's objectives; the reading



level of the problems was suitable for the sixth grade; the problems and activities within the book employed a variety of strategies to arrive at a solution; the problems could have been done either independently or cooperatively in groups; and a wide range of problems was offered to provide the average student with opportunities for success while offering extension activities to challenge the gifted children. (See Appendix L.)

In addition to the collection of problem-solving materials, resources on real-life mathematics were also obtained. The real-life math activities were selected to give the students the opportunity to use their mathematical knowledge and problem-solving skills in situations that would be applicable to their own lives. A variety of manipulatives were necessary for the students to proceed through the lessons. The increased student involvement would help to make the learning experiences more relevant to the children.

Literature was also reviewed on the assessment techniques for mathematical problem solving. A teacher observation checklist and student reflection checklist were two tools selected from How to Evaluate Progress in Problem Solving (Charles, Lester, O'Daffer,

1987). These were employed for some of the problem-solving lessons. Self-reflection sheets were also created for the students by the teacher to coincide with specific lessons presented. Student journal writing was also decided upon as a means of metacognition for the students. Spiral notebooks and pocket folders were purchased for each student by the teacher for journal writing since this was to be one of the integral parts of the supplementary mathematics program on problem solving.

A student attitude survey was written by the teacher during the summer of 1993 and was administered to the sixth grade students in September 1993. The intent of the survey was to assist the teacher in evaluating the mathematical disposition of the sixth grade target group. (See Appendix M.) The findings of the sixth grade attitude survey are discussed in Chapter 2.

The sixth grade pretest was then devised by the teacher. Ten problems were taken from The Problem Solver 6 (1987) and were assembled into a pretest. (See Appendix N.) Each one of these problems focused upon the ten strategies to be taught in the intervention program. The ten strategies were: use logical reasoning, organize a list, use or make a table, use or make a

picture, guess and check, use or look for a pattern, act out or use objects, work backwards, make it simpler, and brainstorm. An evaluation form was written by the teacher to afford the students the opportunity to give their input on each problem of the pretest. (See Appendix O.) The pretest was administered to the sixth graders during the last week of September 1993. The results of the pretest and the student evaluation forms are presented in Chapter 2.

The intervention program for the improvement of mathematical problem solving began in October 1993. The program was comprised of three components: the direct instruction of the ten problem-solving strategies to be employed by the students and the related guided practice sessions; the use of cooperative learning with real-life application lessons; and journal writing for the purpose of student self-reflection and for the generation of new problems by the students.

The sixth grade problem-solving lessons were taught once a week for approximately ninety minutes from October to December of 1993. The scheduling for the class was built around the gifted math program, the remedial tutorial program, and the orchestra and band programs since students are removed from the classroom for

specialized group instruction. Friday was designated as "problem-solving day" because this was the only day that all of the students would be present in the classroom at the same time.

The first phase of the curriculum delivery was comprised of ten lessons on the problem-solving strategies given on the pretest. Each week the students were given one pretest item, and they were instructed on the various methods that could have been employed to solve each problem. The teacher guided the students step-by-step through this first problem. Secondly, during the lesson, the students were given a similar problem which required the use of the same strategy. The teacher read through the problem with the students, gave clues on approaches to the problem, but did not solve the problem for anyone in the classroom. After the class had completed the new problem, the teacher proceeded to show the class the strategy involved in finding the correct solution. Students who had devised alternate strategies were asked to share these with the class. They discussed their means of processing the problem and visually displayed the steps they went through on the chalkboard to help promote the understanding of the other students. The third segment of the lesson was to give the children another problem,

again one which was related to the strategy for the day. However, no teacher assistance of any type was given to the sixth graders at this point in the lesson. The teacher wanted to see if the students were able to transfer and apply the processing information they had received earlier in the lesson to a new problem. Upon the completion of this problem, the students discussed their answers and gave justifications for their reasoning processes. The teacher encouraged those students with different solution strategies to come to the board and discuss their findings with the class. Emphasizing the different ways of processing a problem and allowing students to share these methods with the class was done to lead the target group away from the traditional belief that there is only one correct way to solve a problem. The conclusion of each lesson, journal writing, provided the students with a time for self-reflection. The children were cued to respond to various questions in regard to: how they solved or attempted to solve the problem; how they felt about the lesson for the day; and how they perceived their problem-solving capabilities. The students also included writing activities concerning mathematics and wrote original problems of their own based upon the strategies taught for the day.

The second phase of the problem-solving curriculum delivery involved the use of cooperative grouping. Ten lessons over an eight week period were presented to the students in January and February of 1994. These lessons ranged from one hour sessions to all-day integrated activities. The cooperative groups varied in size from two to four students depending upon the activity. Groups were formed by student selection, teacher selection, or random selection based on various techniques. If the random selection process yielded extremely volatile group combinations, the teacher interceded and made group adjustments that produced more favorable conditions for the students' learning experiences. Roles were assigned to the students to make them interdependent and work for the common goals of the group. The teacher set appropriate time limits on each lesson/activity in order to keep the class on task. At the end of each lesson, each group would present its findings to the class. Visual aids were often required for these presentations. After the presentations, the students would reflect upon the success of their group for that day. This was either done on an individual or group reflection sheet (see Appendix P), in the students' journals, or during a whole group discussion.

The objective of the first cooperative activity was to take the students' conceptual knowledge about two geometric shapes, a square and a cube, and apply this knowledge to a hands-on activity. The students were randomly grouped into cooperative pairs. Each pair of students was given twelve toothpicks and was directed to make six squares with these manipulatives. The children proceeded to work and kept their figures two-dimensional. They created squares by overlapping toothpicks, but this did not follow the problem guidelines. Twenty minutes had elapsed before one pair of students decided to make their shape three-dimensional. Once the class overheard the words three-dimensional, they too were able to form the figure. All of the students were actively engaged in solving this problem. It was a positive learning experience for students of all ability levels in the sixth grade target group.

The following nine cooperative learning lessons were based on real-life situations. One of these was on balancing a checkbook. The objectives for this activity were to have the students: follow an organized list of data; make the appropriate calculations; and understand how information is recorded in a checkbook. The students employed calculators while working on this lesson. Many

students initially made errors because they were not as proficient with a calculator as they had thought. As a result of these beginning errors the lesson took longer than originally anticipated. The explanation, the worktime, and the student reflection activity took approximately an hour of time.

The sixth grade students also participated in an integrated math activity. The objectives of the project were to have the students: organize, graph, and present data; develop knowledge about and calculate percentages; understand the relationship between money and the percentage taxed; comprehend and use ratios; and create word problems using the specific data collected during this project. This day was designated as "Candy Bar Math Day." The manipulatives and supplies necessary for this lesson were: a full size candy bar with ingredients and nutrients listed, a calculator, chart paper, markers or colored pencils, and the activity sheet with the directions for the lesson. (See Appendix Q.)

The lesson proceeded smoothly until the students were expected to calculate percentages. It became necessary to give individualized instruction to many students on this section of the lesson. The students' base knowledge about percentages was not



developed enough for the questions being asked. The need for one-on-one instruction caused this three and one-half hour lesson to take more time than originally planned by the teacher. The lesson, presentation of the graphs, and the group metacognition took four and one-half hours. The inability of the students to calculate percentages and understand their use in everyday life led to the next series of cooperative lessons.

The objectives of the second cooperative activity, which incorporated percentages, were to have the students be able to: use a sales tax chart, understand the columns on an order form, multiply decimals, add decimals, and fill in a blank order form using the correct tax percentage for their town. The students were allowed to use calculators for this lesson. The outcome of this activity was successful and the next cooperative lesson was planned to expand and reinforce the sixth graders' knowledge about percents.

The subsequent cooperative lesson was on sales tax percentages of different states. The objectives of this lesson were designed to give the students the opportunity to: find percentages, change percentages to decimals, multiply decimals, round to the nearest cent, add decimals, and subtract decimals. The children

were permitted to use a calculator in conjunction with the activity sheet . This lesson achieved its purpose. The students became aware of the significance of sales tax in their lives and strengthened their ability to use the correct procedure to calculate percentages.

A lesson was conducted to evaluate the students on their ability to: make predictions, form ratios, calculate percentages, and interpret and graph collected data. All of these objectives were built around a package of plain M & Ms. The students formed cooperative groups of their own choosing. The activity sheet (see Appendix R) and a small package of M & Ms were given to each student. Directions were given by the teacher and the activity began. The students were allowed to use calculators to determine the percentages. They were also allowed to eat the M & Ms upon the completion of the graphing. Teacher observation indicates that the low-ability students had difficulty organizing their results into a graph and were dependent upon other group members to complete this portion of the lesson.

Another concept which was studied cooperatively by the students was the circumference of an object and its relationship to

volume. The objectives of this lesson were to have the students: improve their understanding of volume; measure accurately; become knowledgeable about the parts of a circle; and compute the volume of their balloon accurately using the given formula.

Before the activity began the students were randomly placed in groups. Roles were assigned and materials were gathered by the group's material manager. The materials and manipulatives required for this lesson were: a round balloon for each student, a long piece of string, a calculator, an activity and reflection sheet (see Appendix S), and four yard sticks taped down to a table and two desks to provide convenience while measuring.

The activity entailed blowing up the balloon five times and measuring its circumference after each time. The students were then asked to relate this activity to their lung capacity. The second part of the lesson required the students to apply the data from the first part of the lesson to find the volume of the balloon using a plan of their own creation in conjunction with the formulas provided on the activity sheet. This lesson was running smoothly until part two was introduced to the class. This segment required higher-order thinking skills. However, teacher observation noted that the target

group had very few students ready to undertake and comprehend this segment. Most of the students were still processing the information on finding the circumference of a circle. They were not ready to advance to this next level of learning. The gifted students were resistant to attempt to do part two, but were encouraged to do so by the teacher. They were able to solve the problem after much deliberation and appeared to have renewed self-confidence in their abilities. The teacher guided other students through the procedure, but observations indicated that the children did not fully comprehend the process. Therefore, the teacher of the target group needs to build the students' base of knowledge in these areas of mathematics before any further lessons are undertaken which require higher-order thinking skills for problem solving.

The concepts of mean, median, mode, and range are presented once in the sixth grade textbook. Therefore, the target group's teacher presented a cooperative lesson to the class to clarify the meaning of each term and improve the students' abilities to use these measures of data collection. The materials that were necessary to carry out this lesson were student worksheets (see Appendix T), long pieces of string, yard sticks, and calculators.

This activity was divided into two parts. First, the students filled out the "About Me" information sheet independently. When these were handed in the second part of the lesson began. The students were divided randomly and put into groups of four. This group size provided seven groups which coincided with the seven items on the activity sheet. The student sheets were cut apart and each group became responsible for one item on the data sheet. The students had to tabulate the mean, median, mode, and range for this one particular piece of information and report their findings to the class. Observations and notes reflect that the children were active learners and understood the different processes that were involved in the lesson. The teacher then wanted to advance the class to an activity which would reinforce their data collection skills and interpretations while making these skills seem applicable to their everyday lives.

The students in the sixth grade target group were very interested in sports. Therefore, a mean, median, mode, and range lesson was conducted using the children's shooting skills in basketball. This was a two day lesson. On the first day the class was divided in half and was sent to two basketball hoops in the gym.

Each group had a recorder, timer, counter, and ball retriever. Each student had one minute to shoot as many baskets as possible from the free throw line. The number of baskets made and the total number of shots taken were reported by the counter to the recorder. The data was collected and compiled by the teacher for the students' use on the following day.

The second day of the lesson was done in student selected cooperative groups. Each student received the data sheet with the students' scores, the activity sheet (see Appendix U), and a calculator. The objectives of this part of the lesson were to reinforce the students' abilities to calculate percentages; strengthen the students' skills in finding the mean, median, mode, and range; and provide practice in graphing a set of data. This activity was successful in producing the desired outcomes.

The last real-life intervention was focused upon the application of metric measurement skills. The manipulatives and materials for this lesson included: paper meter tapes, tape, gummi worms, calculators, and activity sheets (see Appendix V). The purpose for this activity was to: give students the opportunity to measure in metrics; provide practice with ratios and percentages;

strengthen calculator skills; improve the students' abilities to make metric conversions; collect and record data; and have students display their knowledge of metric measurement by generating their own problems. This lesson was comprised of two separate activities. They were done in student selected cooperative groups. The knowledge acquired from this lesson varied from group to group. The lack of student self-control interfered with the success of two groups as observed by the teacher.

#### Presentation and Analysis of Project Results

The effects of the intervention on problem solving were assessed by a posttest as stated in the objective. (See Appendix W.) This test was to have been administered in February 1994. However, due to the extensive sixth grade curriculum demands, the preparation time needed for the IGAP tests, and the inclement weather which closed schools, the problem-solving test was not given until March 1994.

The sixth grade posttest was given under the same conditions as the pretest. The test was taken over a three day period. New problems were selected from The Problem Solver 6 for the test. A

15 minute time limit was imposed on each problem. Only one problem at a time along with a reflection sheet was given to the students. The reason for this procedure was to keep students from rushing through the test just to finish it or to prevent them from going back to previous problems and changing their answers. Giving one test question every 15 minutes also provided for a more relaxed classroom environment where everyone of all abilities was working on the same problem. This ensured that the average and low students would not feel inadequate if they took longer to find the solution to the problem since it had been emphasized during the intervention period that everyone needed to work at their own pace in order to be successful. If the entire test had been passed out to everyone, the pressure of knowing how far ahead the gifted students were on the test may have altered the confidence levels and the successes of the other children.

The purpose of the reflection form (see Appendix X) was to assess each child's familiarity and confidence level in regard to the individual strategy being tested. The researcher also wanted to evaluate the student's ability to communicate their mathematical reasoning processes in written form since this was included in the



problem-solving intervention plan in order to comply with the NCTM standards. Additionally, each child was asked to rate how their present problem-solving ability, in each of the ten strategies, compared to their problem-solving methods before the intervention plan began. The researcher wanted to know if the children had developed a more realistic perception of themselves as problem solvers. Each child was also asked to comment if they enjoyed doing each problem and to justify their answers in written form.

The students worked diligently on the posttest just as they had done on the pretest. Teacher observations note, however, that the confusion that was displayed by the students during the pretest was replaced by an air of self-confidence. When the test items were distributed everyone began to formulate a strategy and started to work. Even the low math students, who had left their pretests blank or had put a large question mark on their sheet, knew that certain strategies had to be employed and started to process the problem. The researcher could see charts, tables, lists, pictures, etc. being readily used on the posttest. Whereas, these types of strategies were not even considered by the students for the pretest items. The posttest generated enthusiasm for the class. The students had a

look of determination upon their faces as they analyzed each problem. Smiles were evident as they concluded their solution process. The children could be heard saying to themselves such things as, "Yes, I got it!" or "Alright, that was easy!" Some students rushed to the teacher to get immediate feedback as to whether their answer was correct. This was not possible since the students had been told that no assistance of any type would be given during the posttest sessions. Two of the high-ability students thought these rules did not pertain to them and tried to ask in-depth questions about the test items. They were repeatedly told that the testing procedures applied to everyone, and their requests for further explanations were denied. One of the low achievers in the target group continually approached the teacher with complaints that he did not understand the problems and could not solve them. The teacher used a positive approach and stressed that he could be successful if he read each problem carefully and did it one step at a time. The student was reminded of the times he had been able to solve problems like these in class. The teacher tried to provide the encouragement that would allow him to proceed with his work. (This student attempted each problem and chose the correct process.

One problem was successfully worked to its correct solution. Being able to process all ten problems, even though nine of the answers were wrong, was a tremendous amount of growth for this student.)

The evaluation of the posttest showed that 70 correct responses were given by the sixth grade target group. This is a 19 percent increase from the pretest. Only nine students were successful at solving problems on the pretest. Twenty-two students were able to solve a least one of the ten problems on the posttest. Twenty-one students from this group increased the number of problems they got correct from the pretest to the posttest. One of the 22 students had the same amount correct, and five students remained at zero answers correct on the posttest. Therefore, the number of students who demonstrated growth in their problem-solving abilities went up by 48 percent. See Table 24.

Table 24  
Sixth Grade Student Pretest to Posttest  
Growth on Problem Solving

Percentage of Students Giving Correct Responses	Pretest 33%	Posttest 81%	
Percentage of Correct Responses Given by the Students	Decreased 0%	Stayed the Same 22%	Improved 78%

N = 27 Students

The gifted math students increased their accuracy by 32 percent on the posttest. The average students showed an improvement of 17 percent, and the low students had one correct response which yielded a two percent increase in the low ability group's score from the pretest to the posttest. See Table 25.

Table 25  
Correct Responses and Percentages by  
Ability Groups for the Sixth Grade  
Problem-solving Pretest and Posttest

Ability Level	# Students by Group	Pretest		Posttest	
		# Correct	% Correct	# Correct	% Correct
Gifted	6	15	25%	34	57%
Average	16	5	5%	35	22%
Low	5	0	0%	1	2%
Class Total	27	20	7%	70	26%

Gifted N = 60 Problems Possible  
Average N = 160 Problems Possible  
Low N = 50 Problems Possible  
Class N = 270 Problems Possible

The data indicate pretest to posttest improvement in varying degrees on all ten problem-solving strategies. The students showed the most gains in the areas of "logical reasoning", a 48 percent increase, and "organize a list", an improvement of 51 percent. The students also showed an improved ability to "work backwards", a rise from zero percent to 30 percent. The least amount of progress

was made in the areas of "make it simpler", a seven percent increase, and "brainstorm", a four percent improvement. Table 26 further summarizes the test results. The student scores for the pretest and posttest are presented in Appendix Y.

Table 26  
Comparison of the Percentages Correct  
for the Sixth Grade Pretest and Posttest  
on Problem-solving

Problem-solving Strategies	% Correct		% Improvement
	Pretest	Posttest	
Logical Reasoning	7%	55%	48%
Organize a List	4%	55%	51%
Use or Make a Table	26%	33%	7%
Use or Make a Picture	0%	11%	11%
Guess and Check	11%	19%	8%
Use or Look for a Pattern	7%	15%	8%
Act Out/Use Objects	19%	30%	11%
Work Backwards	0%	30%	30%
Make It Simpler	0%	7%	7%
Brainstorm	0%	4%	4%

N = 27 Students

The student reflection forms that accompanied each posttest problem indicated that the students had become more proficient at realistically assessing a problem's level of difficulty for themselves since metacognition had been incorporated into the non-routine problem-solving lessons. Before the intervention began, the students evaluated the pretest questions as being easy to do. The

pretest results indicated to the students that they were not accurate in their assumptions about each problem's difficulty.

The majority of the students stated that they enjoyed doing the problems on the posttest. Students responded that these problems were: challenging, fun, like a puzzle, cool, interesting, like drawing pictures, and easy. Four students, however, consistently said that they did not like the problems. Three of these children have a low mathematical ability and expressed that the problems on the posttest were: too hard, impossible to solve, boring, and too long to solve. The fourth student, who did not like the problems, continually stated that the problems were too easy. This student was from the average ability group and got four correct solutions out of the ten test questions. The only problem that was disliked by the majority of the students, 52 percent, was the brainstorm strategy question. The students responded that it was too much like a riddle and had nothing to do with mathematics.

Eighty-five percent of the students were able to explain their reasoning processes in written form. Only eleven percent of the children were able to explain their method of solving the problems on the pretest. This noticeable improvement may be attributed to

the math journal writing and reflections that accompanied the lessons on problem solving.

The students also were given the opportunity to rate their own ability in regard to how much improvement they felt they had made from the onset of the intervention. The children displayed more insight than the researcher thought possible. The students' self-evaluations of their problem-solving abilities for a designated strategy can be closely correlated to the students' processing procedures of the test items. Those who had selected the correct strategies and had come close to or had found the solution indicated that they had improved on this strategy on their reflection sheet. The children who had been able to solve similar items on the pretest stated that their ability had stayed the same. Students who had not solved this strategy on the pretest or on the posttest also marked that they had the same ability. Only three percent of the responses given revealed that the students believed they had done worse on the posttest.

The sixth grade target group did make progress at becoming better problem solvers during the intervention period. The students learned about the different strategies and how to employ them while

doing non-routine problems. The students' success rate cannot be based upon the number of correct solutions they arrived at on the posttest, however. It is the growth that they have made in reference to their ability to now analyze the components of a problem and comprehend what the problem is actually asking them to do. The students learned how to formulate a plan using techniques that they previously would not have ever imagined using in a mathematics class. Even though their computations may not be entirely accurate while finding the solution, the children no longer stare at non-routine problems in amazement. The fear and anxiety that many of the students expressed either verbally or through body language are no longer present. Observations show that the children still become frustrated but for different reasons now. Their frustrations arise from not being able to find the solution on the first try. It is the miscalculation of algorithms or the incorrect placement of an object that is causing the students some stress. Student reflection sheets and teacher-student conferencing notes indicate that the students know they are extremely close to the solution, but they are unable to determine the one error within the problem which is prohibiting them from finding the correct solution. (The students' uses of



correct strategies for the pretest and posttest are presented in Appendix Y.)

### Reflections and Conclusion

A review of the supplementary curricular content indicates that the children were exposed to a wide variety of activities which focused upon the ten problem-solving strategies being addressed in the intervention. However, the time constraints of the program did not give the researcher the opportunity to go into more depth in the students' weakest areas: make it simpler, use or make a picture, and brainstorm. Additional lessons would have strengthened the students' understanding of these strategies and would have allowed them to achieve more success on the problems which necessitated the use of these strategies. The increased use of cooperative grouping for the solving of non-routine problems would have also enhanced the students' learning experiences.

Cooperative grouping practices, however, were not employed as often as originally planned at the beginning of the intervention program. The social disposition of the target group restricted the use of this teaching strategy. There were often displays of hostile

behavior towards group members and feelings of animosity were present among some members of the class. Those students not engaged in negative and aggressive behaviors occasionally became silly and displayed off-task behaviors. Examples of these behaviors are as follows: drawing faces on their balloons instead of measuring the balloon circumference; asking to eat the M & Ms before they finished the ratio and percentage activities; eating their gummi worms before they measured them; or playing calculator games instead of finding percents with the 1. The researcher through observation and discussions with the students found that the immature and negative behaviors hindered the acquisition of knowledge for the entire target group. Lessons often took longer than anticipated because 48 percent of the target group took too long to become focused on the cooperative task.

Not only was a wide range of behaviors and attitudes present in class, but an extreme range of abilities was also present in the target group. Six of the students were the highest mathematical achievers in the entire sixth grade in the target school. In order to balance the target groups' classroom ability average, the lowest students in the sixth grade were also placed in the target group.

Having such a wide span of abilities made the selection of problem-solving activities a challenging task for the teacher. Activities were chosen that would promote success for all students, but yet could be extended to higher-levels of thinking to stimulate and enhance the advanced students problem-solving capabilities.

Teacher observations and notes indicate that the students' confidence levels improved as they worked through the strategy lessons. The perplexed facial gestures and sounds of frustration diminished after the class started working through and discussing the problems. The students, who were so conditioned to being evaluated on the one correct way to solve a problem, were initially struck with disbelief when they were told that these problems were risk-free. Their job was to learn how to process the information first and not to worry about grades. The teacher encouraged the students to experiment with ideas. It was emphasized that making errors was alright since these mistakes can help in the learning process. A positive classroom environment was provided for the sixth graders. Criticism was not accepted in the classroom. Strategies and solutions were analyzed for their correctness and signs of negativism were not permissible.

The students had been accustomed to solving problems in equation form, which only required a small space on their papers. The new problem-solving strategies now engaged them in activities that necessitated the use of a lot of space and enlisted the aid of tables, charts, pictures, diagrams, and lists along with numbers to formulate a solution. Once the students became comfortable with the reality of a risk-free classroom, they started to freely try out the strategies learned in the lessons. As their confidence levels rose, their smiles and comments expressing amazement at their ability to actually solve the problem, became most apparent to the teacher. Even though some students still had difficulty arriving at the final solution, their faces reflected a sense of satisfaction that they were at least able to select a strategy and begin the problem-solving process whereas, previously, they had left their papers blank. This improved confidence transferred over to the regular math class. Students who had been low achievers or those who did not do their homework on time because they were afraid to ask for assistance on their assignments actually started to get improved grades in math class. The problem-solving lessons emphasized keeping an open mind in math and trying your best. When these ideas

became instilled in the children, they themselves were often surprised at how well they were achieving in math class.

The researcher found the student reflection and evaluation sheets to be valuable tools for student metacognition and assessment. The children were very open and honest when evaluating a designated lesson and their individual/group participation in the lesson. Students also became accustomed to transferring their mathematical thought processes into words.

The children also became comfortable with the concept of journal writing in math class. It was a means to improve students' communication skills and allowed them to reflect upon what they had just done in math class. The math journals were also used for writing essays concerning math, and developing problems of their own using the strategy which had just been employed in class for the day. Most of the students enjoyed writing their own problems. They were extremely proud of themselves when the problem was completed. One of the gifted students wrote a problem-solving book which includes problems of varying degrees of difficulty. An answer key was also written to go along with the book. This student has always been outstanding in mathematics, but never had the need to

communicate his ideas in written form so that others might share his expertise. Therefore, his intense interest in higher-order problems in mathematics has evolved into a book which has caused him to focus upon the importance of having good writing skills in conjunction with an exceptional mathematical ability.

The problem-solving intervention proved to be extremely beneficial for the IGAP tests in mathematics. The sixth grade test is primarily based on the use of higher-order thinking skills. The students are required to use their prior knowledge about a concept, transfer its meaning and apply it to a new situation. While reviewing for the math test, with the state provided review materials, the students conveyed their thoughts to the teacher about the questions being asked. The sixth graders expressed the opinion that the lessons they had done for the real-life math activities and problem-solving strategy lessons were far more advanced than some of the questions on the state review questions. The intervention, therefore, helped to prepare the students to take the IGAP math test. The results of the test will not arrive at the target school until the fall of 1994. The researcher will analyze the results at that time to note if the target group's test performance was better than that of

the sixth grade class which did not participate in the intervention.

Timing the test questions on the pretest and posttest provided consistency and security for the sixth graders. The fifteen minute period allotted gave everyone the opportunity to finish each problem without feeling rushed or behind the other students. If all the students finished before the 15 minutes were over, the class went on to the next problem. The designated time period also reinforced the idea discussed and internalized by the students that everyone needs to work at his or her own pace to be successful.

The problem-solving intervention is viewed by the researcher as having positive results for the students. The children acquired new knowledge about problem solving and were able to apply this knowledge to other concepts in mathematics. The risk-free environment in which the lessons were presented and discussed gave the students a chance to explore the mathematical world and discover their hidden potential in this subject area. The real-life math experiences gave an importance to mathematics in the students' minds. The students who took advantage of the cooperative grouping lessons were also learning the social skills which are a necessity in order to function successfully as an adult

in the workplace.

The target group's exposure to the intervention afforded them the opportunity to realize how mathematics is a part of their lives every day whether they are in or out of school. Mathematical concepts are all around them and are inescapable. Journal writing activities allowed the children to personally reflect on these ideas. These entries and other metacognitive activities helped to build student self-confidence which enhanced their problem-solving abilities.



## Chapter 6A

### THIRD AND FIFTH GRADE DECISIONS ON THE FUTURE

#### The Solution Strategy

The data indicate that the supplemental problem-solving program and the implementation of different teaching strategies should be continued. However, modifications of the original design are suggested. Though commissions have been formed to study the lack of problem-solving skills in American children, these committees have addressed the problem at a secondary level. Elementary school teachers recognize that problem solving needs to be addressed before students enter high school (Stevenson, Stigler, and Lee, 1986). The instruction of problem solving needs to begin in kindergarten and continue throughout all grade levels. If instruction were to begin at a basic level in kindergarten, children would gradually learn the different strategies involved in problem solving. As they progress through the grade levels, they would be more comfortable solving problems requiring higher-order thinking skills.

Their frustration and anxiety levels would be lower when working with an approach that is more familiar to them.

We, as researchers, feel that the instruction of the ten problem-solving strategies in a four month period was an insufficient amount of time to truly do justice to the intervention. The students were often frustrated and confused. These ten strategies need to be taught and practiced throughout an entire school year in order for them to become inherent to the learner. A more relaxed pace would encourage students to become more enthusiastic problem solvers and encourage them to approach problems as interesting challenges.

The major focus of this intervention was to introduce the students to the Four-Step Method to solving problems in mathematics. Not only can this four step strategy be utilized in mathematical problem solving, it can also be applied to answering questions and solving problems throughout the school day and in social situations.

Students were encouraged to use methods that were best for them, as long as they could demonstrate a legitimate solution process. Some students approached the problems with their own strategies, and were successful in achieving a solution. This

flexibility in problem solving was encouraged, for it demonstrated active participation in problem solving and higher-level thinking by the learner.

### Additional Applications

Problem-solving skills should not only be taught during mathematics instruction. These skills should be integrated into the instruction of all subject areas. If problem-solving strategies were incorporated into all subject areas, the students would be more able, and more likely, to transfer these strategies to everyday situations. In today's world factual knowledge is deemed less important than the ability to pose and solve non-routine problems. Our children need to learn how to approach and solve these non-routine problems while working independently or in collaboration with others.

### Dissemination of Data and Recommendations

The results of this intervention should be shared with all other grade level educators. Efforts should be taken to work collaboratively with the district math coordinator to continue the adoption of the University of Chicago Math Program and inservice the teachers in grades four, five, and six. This program, currently being

used in Kindergarten through third grade, integrates problem solving while teaching basic math skills. Along with the adoption of UCMP in all grade levels, care should be taken to select materials that would utilize the same strategies and processes throughout the grade levels to ensure consistent instruction of the problem-solving strategies in all subject areas.

Resources within the community should be identified and tapped for their possible participation in an integrated unit of study. This would give students the opportunity to experience math in the real world.

The project has proven, to us, the importance of reviewing and revising the curriculum. Staff development of new teaching methods, strategies, and curriculum is needed to ensure teachers are enabling students to achieve their full potential. As educators, we have the responsibility to provide students with a comprehensive problem-solving background. With this knowledge, these students will be able to face the challenges that they may encounter throughout life.

## Chapter 6B

### SIXTH GRADE DECISIONS ON THE FUTURE

#### The Solution Strategy

The data indicate that the project on non-routine problem solving with higher-order thinking skills should be continued. However, modifications on the original plan should be considered. Students' inabilities to solve problems that entail critical thinking skills have been of great concern to educators both nationally and locally. The implementation of teaching problem solving in mathematics needs to be integrated into all curricular areas across all grade levels. Incorporating mathematical problem solving in all facets of education will impart to the students the knowledge that mathematics plays an important and useful part in their daily lives. In order to accomplish this goal teachers at each grade level need to put forth a concerted collaborative effort to engage their students in problem-solving activities that will promote the students' problem-solving skills and advance them to the next level of

learning. Staff members need to formulate a plan to provide consistent expectations for their students and set the tone for a risk-free classroom environment where problem solving is viewed as a positive, challenging, and rewarding activity. Only a percentage of students can be reached in individual isolated classrooms where the teaching of higher-order thinking skills transpires. The remainder of the school population is thus deprived from having the benefits of a problem-solving curriculum. Those students, who had been in classes where mathematical problem solving had been a part of their regular curriculum, will experience a void in their education if their next class in the subsequent year does not engage them in problem-solving activities employing the use of higher levels of thinking. The return to algorithmic procedures may diminish the students' enthusiasm for math and cause them to become apathetic towards a subject which had previously generated excitement for learning and had raised their confidence levels across the entire curriculum. Therefore, a consensus among staff members must be made to guarantee that objectives on problem solving follow a sequential pattern of development, and that these objectives be addressed to maximize the students' learning and academic growth.

A major focus of the intervention was to increase the students' awareness that there are many problem-solving strategies that can be employed to find the same solution to a problem. The students through direct instruction, guided practice, independent practice, and cooperative grouping became problem solvers and were able to accept each other's justifications for their problem-solving processes in written and oral form.

#### Additional Applications

In order to facilitate problem solving in mathematics or other content areas, teachers need to observe the students' reactions to the lessons presented and modify their instruction to adapt to the class's many abilities and learning styles. By being perceptive, a teacher can provide each child with the opportunity to understand the concept being taught. Teacher flexibility and patience is necessary to guide students to higher levels of learning.

Teaching methods for problem solving should utilize direct instruction, independent practice, open discussions with judicious use of criticism, and small group work to improve student success. Competition should be de-emphasized because it is difficult to learn

while competing, especially for those students who are apprehensive about mathematics.

Students of both genders should be required to participate in problem-solving activities. Higher-order thinking skill questions should be addressed to all students of all abilities. Girls, historically, have been hesitant to assert themselves in class discussions. However, they need to be called upon so that they too can be recognized as being effective problem solvers. The more success the girls experience in front of their peers, the more self-confidence they will display in class. Female success in mathematical problem solving can carry over to the other content areas as well.

Problem solving should also promote and reinforce the idea that everyone has different learning styles and can have alternate ways to arrive at a solution to a problem. Having students demonstrate their solution strategies in front of the class allows children to acknowledge one another's learning styles. Enthusiasm for the concept being learned is generated because students enjoy seeing the relationship between their work and that of the student giving the presentation. This activity enhances a positive classroom



environment and promotes self-esteem. Individual or group presentations could be utilized for a variety of purposes across the curriculum for any grade level.

The sixth grade target group was continually monitored during the intervention activities. Constantly monitoring the students gave them the security of knowing that I was available if they needed assistance, and that I would also reassure them if they were processing the problem correctly. Monitoring was done in a positive manner, never negatively. The monitoring and the acknowledgment of a risk-free environment built a sense of trust and security in the classroom. This interaction with students is applicable to any type of lesson and promotes learning for all students.

Student reflection sheets were used for metacognitive purposes during the intervention period. These were valuable learning tools for the students as well as for the teacher. The self-evaluation sheets let the children openly express their opinions about a specific lesson and their ability to work on it. It was evident from these sheets which problems were beneficial to accomplish the desired objective and which ones were un motivating to the students. Student comments will be helpful in doing

curricular revisions for the following year. The information stated on these sheets also provided insight into the individual student's classroom behaviors and personal anxieties. They allowed for immediate teacher intervention or modification in order to better guide the child through the problem-solving process, thereby raising the confidence level of the student.

#### Dissemination of Data and Recommendations

The results of the intervention should be shared with the elementary and middle school staff members along with the assistant superintendents involved in the target district's mathematics curriculum. Efforts should be made to strengthen the problem-solving curriculum within the district for all grade levels. A sequential program for teaching higher-order thinking skills needs to be established. Staff members should receive inservice training on the teaching of problem solving and the ways in which it should be integrated into the other subject areas. Emphasis should be placed on the linkage between reading comprehension skills, basic mathematical knowledge, and their application to problems that require higher-order thinking skills. This plan should be presented

to the Board of Education and the district superintendent. A committee to find resources and select appropriate materials should be created. Those on the committee (teachers, parents, and administrators) need to represent all the grade levels and be knowledgeable about the mathematics standards set forth by the National Council of Teachers of Mathematics. After a thorough examination of available resources and materials the committee would report to the School Board and make its recommendations.

The direct instruction of the ten problem-solving strategies and the subsequent guided practice activities were based upon The Problem Solver 6. In retrospect, it would have been more beneficial if the introductory lessons had come from the fifth grade level. The sixth grade target group had very little exposure in the previous grades to problem-solving strategies. Math had been focused on computation rather than on non-routine problems which necessitated the use of higher-order thinking skills. Therefore, by using fifth grade level material, improved student confidence could have developed earlier in the intervention program. The low and average students would not have felt so lost at the onset of the instruction. The fifth grade material could have also been expounded upon to

challenge the gifted students in the target group. Once initial successes had been achieved at the fifth grade level, the transition to the sixth grade material would have been a natural progression to the next level of knowledge based upon the concepts/strategies just learned by the students.

The schedule of doing problem-solving activities once a week for 90 minutes on Fridays was not conducive to learning the problem-solving strategies. The students were deprived of the daily continuity of instruction which reinforces the knowledge learned from the previous lesson. Too much time elapsed between the direct instruction lessons on the ten strategies. The class scheduling conflicts made it necessary to review for longer periods of time in order to afford the students the opportunity to retrieve the information they had learned the week before. Fridays were also inconvenient due to the number of days off for conferences, holidays, and teacher inservice meetings. Daily routine practice by all students on the ten problem-solving strategies would have strengthened the intervention program. The current system of removing students from the classroom needs to take into consideration the instruction and learning that are going on in the

classroom. Teachers need to have input as to a time preference when their students could leave the classroom for special activities. The band, orchestra, gifted, tutorial, and homeroom teachers need to formulate a scheduling plan to ensure that students are not being removed from homeroom instruction at a crucial time. Student abilities, needs, and content areas need to be taken into consideration.

Cooperative grouping lessons took longer to work through and process than originally anticipated. The wide range of academic abilities, the size of the class, and class behavior all contributed to the time extension of the lessons. Due to the scheduling factor, these lessons had to be done all on one day. Carrying the lesson from a Friday to a Monday would have interrupted the flow of the activity and weakened the objectives of the lesson. The low students would have had an extremely difficult time recalling how they got to a specific point in the lesson. The average and low students needed to complete the lesson and share in the metacognitive activity in order to fully grasp the concept being taught for the day. Additionally, if the lesson had been taken over to a Monday, many students would not have been present to finish it cooperatively due to the removal of

the band, orchestra, and tutorial students over the course of the day.

The use of teacher observation checklists during cooperative learning lessons became cumbersome and deterred me from the learning that was going on in the classroom. The continual necessity to check students off for displaying certain behaviors often delayed my giving assistance to a student in need. I tried using checklists for the entire class, but due to the class size of 27 this was an ongoing task. Only assessing two or three groups during one lesson was also done. However, due to the individual needs of the other students, this assessment practice was also found to be burdensome. Writing brief notes, as I monitored the classroom, was more meaningful to me when it came time to evaluate the students' progress. Further investigation needs to be done on evaluation tools. Teaching professionals who successfully employ checklists should be consulted. Literature on assessment tools for cooperative learning also needs to be further researched.

The use of manipulatives proved to be most beneficial in the problem-solving intervention. The primary grades and the fourth grade have manipulatives provided with the Chicago Math Program which the target district has adopted. However, the fifth and sixth

grades do not have manipulatives budgeted for their math programs. Until the Chicago Math Program has been developed for fifth and sixth grades, money should be allocated by the district, the individual schools, or the school Parent Teacher Organizations to supply manipulatives that will strengthen the children's conceptual knowledge of mathematics and enhance their learning experiences. Manipulatives make learning easier, more relevant, and motivating for the students. Presently teachers supply the manipulatives for math activities. However, some activities cannot be pursued because the cost of the manipulatives is too high for the teachers to personally purchase.

The issue of gender bias in mathematics needs to be addressed further in elementary, middle, and secondary schools. The number of female students with low confidence levels in mathematics and often low achievement levels continues to be of great concern for those educators who recognize that this is a problem often caused by blatant sexism or unintentional experiences in the classroom. Teachers are often unaware of the messages they are sending to their students in regard to a female's competency level in mathematics. Experienced teachers need to receive inservice

training on gender bias in order to come to the realization that some of them are indeed being biased in their classrooms without being cognizant of it. Those educators who openly proclaim the superiority of the male students in math, or any subject area, need to be closely evaluated by the school administration and be required to increase their knowledge of equitable practices in education and apply these practices in their classrooms. Teachers must set the same behavioral and academic expectations for boys and girls. Putting forth good effort, being kind and helpful, listening to others, accepting ideas and constructive criticism, and asking questions should be demonstrated by all students. Consistent and equitable expectations by the teacher for both boys and girls places value on everyone's intelligence and conceptual understanding in class.

The selection of curricula and materials was an important variable of the problem-solving intervention program. Problems that were interesting, challenging, and adaptable to the children's needs will become a permanent part of the supplementary problem-solving curriculum for the sixth grade. Uninteresting or poorly written problems will be removed from the curriculum materials. More problems will be sought that allow students to make steady



progress and to develop a cumulative understanding of a specific concept, as was done with percentages in the cooperative grouping lessons. This type of conceptual progression allows the student to see how each problem facilitates the transference of knowledge to more advanced applications of the concept.

Other elements that were important to the success of the intervention plan focused upon the number of problems to be worked on per class session and the time factor while working on them. As a result of the intervention program, I am fully in concurrence with the NCTM that a lesson with an abundance of problems serves no purpose. It only yields stress and frustration for the students and nothing is effectively learned. Devoting a class period to one or two problems evoked positive responses by the students. True learning transpired because time was spent discussing each aspect of the problem and how these different parts were all interrelated. Students enjoyed the class discussions and with the passage of time even the quieter students became active participants in the group discussions and student presentations of the different problem-solving approaches.

Problem solving can only be effective, however, if teachers are

willing to provide ample time to each student in order to formulate a solution. The teacher must be a source of encouragement as the students proceed to work through a problem-solving task. Students need to be reassured that the problems at the sixth grade level do take a long time, and that these problems also require a lot of workspace in which to solve them.

The non-routine problem-solving intervention assisted the sixth grade students in the target group in growing intellectually and emotionally. The lessons and activities met the varied needs of the students in the classroom. Those who already had mathematical expertise became enriched and were made aware that their reading and grammar skills were also connected to the aura of being a great mathematician. Those students with low confidence levels in mathematics were made to realize that an open mind and a willingness to learn could allow them to do well in a subject they had previously thought was beyond their ability to understand. Students no longer viewed their attempted, yet incorrect solutions, as failures. They took pride in their ability to discern the correct strategy and continued on with the problem until they did finally arrive at the correct solution. Students freely extended their help

to others who were experiencing difficulty at some point in a problem. Words of encouragement could be heard instead of the negative sarcasm which had been so freely expressed at the beginning of the intervention period. Students who had previously said they hated math or that math was their worst subject, were now saying how much fun math was and how much better they were becoming at this subject. Mathematics was now viewed as a relevant and essential subject which transcended all aspects of everyone's lives. The strengths and weaknesses of the students were openly discussed in class and everyone knew that their weaknesses could be remediated and continued growth would be achieved. All students of both genders and of all abilities were treated with equal respect during their problem-solving venture. Achievement levels of various degrees improved for each child in the target group. The improved confidence levels were not only evident in math class but in the other content areas as well. The intervention program created a positive risk-free environment for the student. The external pressures associated with competition, large workloads, time constraints, authoritarian teaching practices, peer pressure, and different learning styles were absent from the

students' learning environment; thus providing conducive conditions for children to aspire to advanced levels of learning through the development and use of higher-order thinking skills in problem solving.

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APPENDICES

## Appendix A

### Third Grade Group A Problem Solving Second Grade Level Pre and Posttest Results

Student Number	Logical Reasoning		Organized List		Use or Make a Table		Use or Make a Picture	
	Pre	Post	Pre	Post	Pre	Post	Pre	Post
1	Y	N	N	Y	Y	Y	N	N
2	Y	Y	Y	N	Y	Y	N	N
3	Y	Y	Y	Y	Y	Y	Y	Y
4	Y	Y	Y	Y	Y	Y	Y	Y
5	Y	Y	N	Y	N	Y	N	Y
6	Y	Y	Y	Y	Y	Y	Y	Y
7	Y	Y	N	N	N	Y	N	N
8	Y	N	Y	Y	Y	Y	N	N
9	Y	-	N	-	N	-	N	-
10	Y	Y	Y	Y	Y	Y	Y	N
11	Y	Y	Y	Y	Y	-	Y	Y
12	Y	Y	N	Y	Y	N	N	Y
13	Y	Y	Y	Y	Y	N	Y	Y
14	Y	Y	N	N	N	Y	N	N
15	Y	-	N	-	N	-	Y	-
16	N	Y	N	Y	Y	Y	N	Y
17	Y	Y	Y	Y	N	Y	Y	N
18	Y	Y	N	N	Y	Y	N	N
19	Y	Y	Y	Y	Y	Y	N	N
20	Y	Y	N	Y	Y	Y	Y	N

Student Number	Guess and Check		Use or Make a Pattern		Act Out or Use Objects		Work Backwards	
	Pre	Post	Pre	Post	Pre	Post	Pre	Post
1	Y	Y	Y	N	N	N	N	N
2	N	Y	Y	Y	Y	Y	N	N
3	Y	Y	Y	Y	Y	Y	Y	Y
4	N	Y	Y	Y	Y	Y	Y	Y
5	Y	Y	Y	Y	Y	Y	Y	Y
6	Y	Y	Y	Y	Y	Y	N	N
7	N	Y	Y	Y	N	Y	N	N
8	Y	Y	Y	Y	Y	Y	N	N
9	N	-	N	-	N	-	N	-
10	Y	Y	Y	Y	Y	Y	Y	N
11	Y	Y	Y	Y	N	Y	N	N
12	Y	N	Y	Y	N	Y	N	N
13	Y	Y	Y	Y	Y	Y	N	Y
14	Y	N	Y	Y	N	N	N	N
15	Y	-	Y	-	Y	-	N	-
16	N	Y	Y	Y	N	N	N	Y
17	Y	Y	Y	Y	Y	Y	N	Y
18	Y	Y	Y	Y	Y	Y	N	N
19	Y	Y	Y	Y	N	Y	Y	N
20	Y	Y	N	Y	Y	Y	N	Y

Student Number	Make It Simpler		Brainstorm	
	Pre	Post	Pre	Post
1	N	N	N	N
2	N	Y	N	N
3	N	Y	Y	Y
4	N	N	N	Y
5	N	Y	N	Y
6	N	N	N	Y
7	N	N	N	Y
8	N	N	N	N
9	N	-	N	-
10	N	Y	N	Y
11	N	N	N	Y
12	N	N	N	N
13	N	N	N	N
14	N	N	N	Y
15	N	-	N	-
16	N	N	N	Y
17	N	Y	N	N
18	N	N	N	N
19	N	N	N	Y
20	N	N	N	Y

N = Incorrect Response  
Y = Correct Response

Appendix B

Student Survey

1. Circle how you feel about math.

I dislike math.

Math is okay.

I like math.

2. I feel I am able to do math problems on my own:

often.

sometimes.

rarely.

3. Circle how you feel about:

Addition	hard for me.	okay.	easy for me.
Subtraction	hard for me.	okay.	easy for me.
Multiplication	hard for me.	okay.	easy for me.
Equal sharing (division)	hard for me.	okay.	easy for me.
Measuring things	hard for me.	okay.	easy for me.
Telling time	hard for me.	okay.	easy for me.
Geometry	hard for me.	okay.	easy for me.
Using money	hard for me.	okay.	easy for me.
Word problems	hard for me.	okay.	easy for me.

4. Circle the sentence that tells how you think you do in math.

I'm good at math.

I'm okay.

I need help with math.

5. I think word problems are hard for me because the words are difficult to read.

rarely

sometimes

often

6. I think word problems are hard for me because I can't figure out what application to use (addition, subtraction, multiplication).

rarely

sometimes

often

7. Does anyone help you with your homework?    Yes            No  
If so, who helps you?
8. How much time do you spend on math homework?  
15 minutes                      30 minutes                      45 minutes  
one hour                              more than an hour
9. Do you ever do math for fun at home?  
rarely                              sometimes                      often
10. List the ways you use math outside of school.



## Appendix C

### Teacher Questionnaire

1. Have you changed your methods of teaching mathematics in the past three years?

Yes                  No

If so, how have you changed your instruction?

2a. Intermediate Teachers:

What methods are you currently using to teach mathematics?

(Direct instruction, cooperative grouping, inquiry method, etc.)

2b. Did any of these methods prove to be unsuccessful for your students?

Please explain.

3. How satisfied are you with student response to your present teaching methods?

1	2	3	4	5
extremely dissatisfied	dissatisfied	neutral	satisfied	extremely satisfied

4. How satisfied are you with your students' progress in math with the present teaching methods in comparison with your former methods?

1	2	3	4	5
extremely dissatisfied	dissatisfied	neutral	satisfied	extremely satisfied

5. Do you incorporate higher order level thinking skills with computational skills?

1	2	3	4
rarely	occasionally	frequently	consistently

6. Do you use supplemental math materials with your present math program?

1	2	3	4
rarely	occasionally	frequently	consistently

7a. How do the majority of your students respond when given higher order thinking skill assignments?

1	2	3
Unfavorably	indifferently	favorably

7b. Are your students dependent upon you to work through problems?

1	2	3	4
rarely	occasionally	frequently	consistently

8. How often do you do problem solving (non-routine word problems) in math class?

1	2	3	4
less than once a month	once or twice a month	once or twice a week	daily

9a. How often do you use A.D.D.?

1	2	3	4
less than once a month	once or twice a month	once or twice a week	daily

9b. If you use A.D.D. at least once a week, how has it affected problem solving skills of your students?

10. Underline the following problem solving strategies you have taught and/or modeled for your students.

multi-step problems  
estimation/rounding  
working backwards  
use of a calculator  
draw a picture  
use of manipulatives  
use of computer  
logic of reasoning  
guess and check

tables  
graphs  
charts  
journal writing in math  
student generated problems  
cooperative learning  
direct instruction  
inquiry method



USE LOGICAL REASONING

Name \_\_\_\_\_

- 1 Kris likes caps! He wears caps to school. He wears caps to the park. He wears caps everywhere he goes. Today he found one more cap to wear.

It covers his ears.

It has two buttons on it.

One of the buttons shows a picture of an animal.

Which cap did Kris find today?

Draw a ring around the cap.

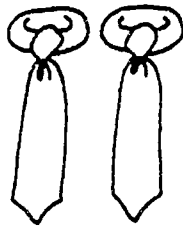


1. \_\_\_\_\_
2. \_\_\_\_\_
3. \_\_\_\_\_

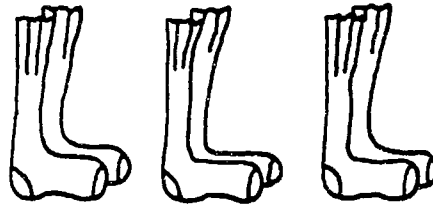
**MAKE AN ORGANIZED LIST**

Name \_\_\_\_\_

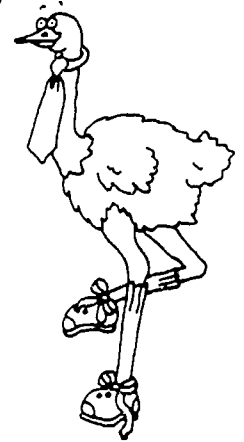
**4** Oscar the ostrich likes to show off his long neck and legs. When he goes to town, he puts on a necktie and a pair of socks. Oscar has a blue necktie and a red necktie. He has a pair of orange socks, a pair of green socks, and a pair of yellow socks. What are the 6 different sets of neckties and socks that Oscar can put on when he goes to town?



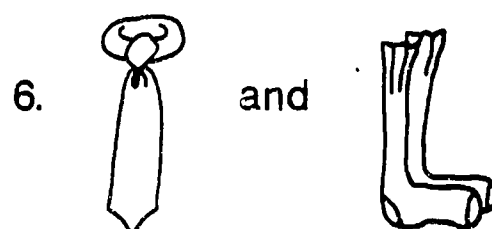
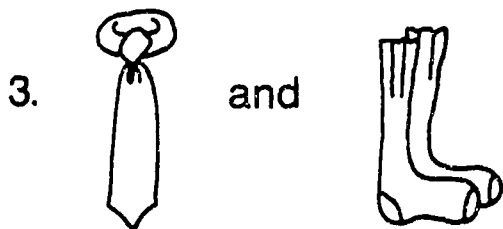
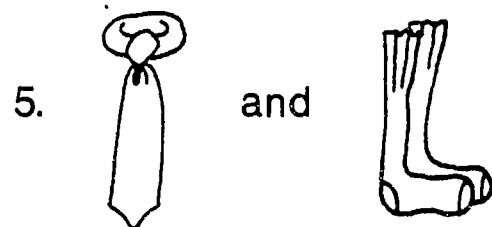
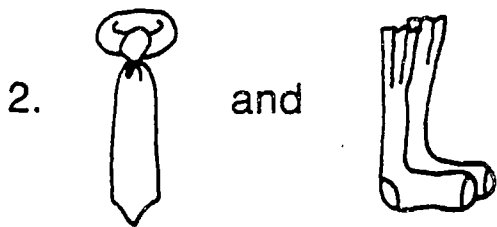
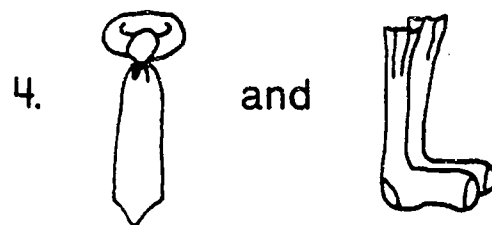
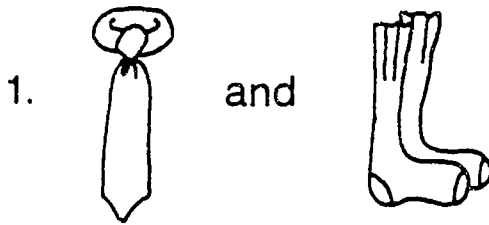
**neckties**



**socks**



Color the pictures to show the answer.



A	B
3	0
7	5

**7** The elves of Wimple Woods had a big summer picnic and day of fun. Elves came from all over Wimple Woods. Melva wanted to find out how many elves were at the picnic, so she counted them. (Melva is a good counter.) She gave these clues:

There were more than 15.

There were fewer than 24.

There was an even number of elves.

The number had an 8 in the ones' place.

How many elves came to the big picnic?

Use the table. Write the number. \_\_\_\_\_

15

16

17

18

19

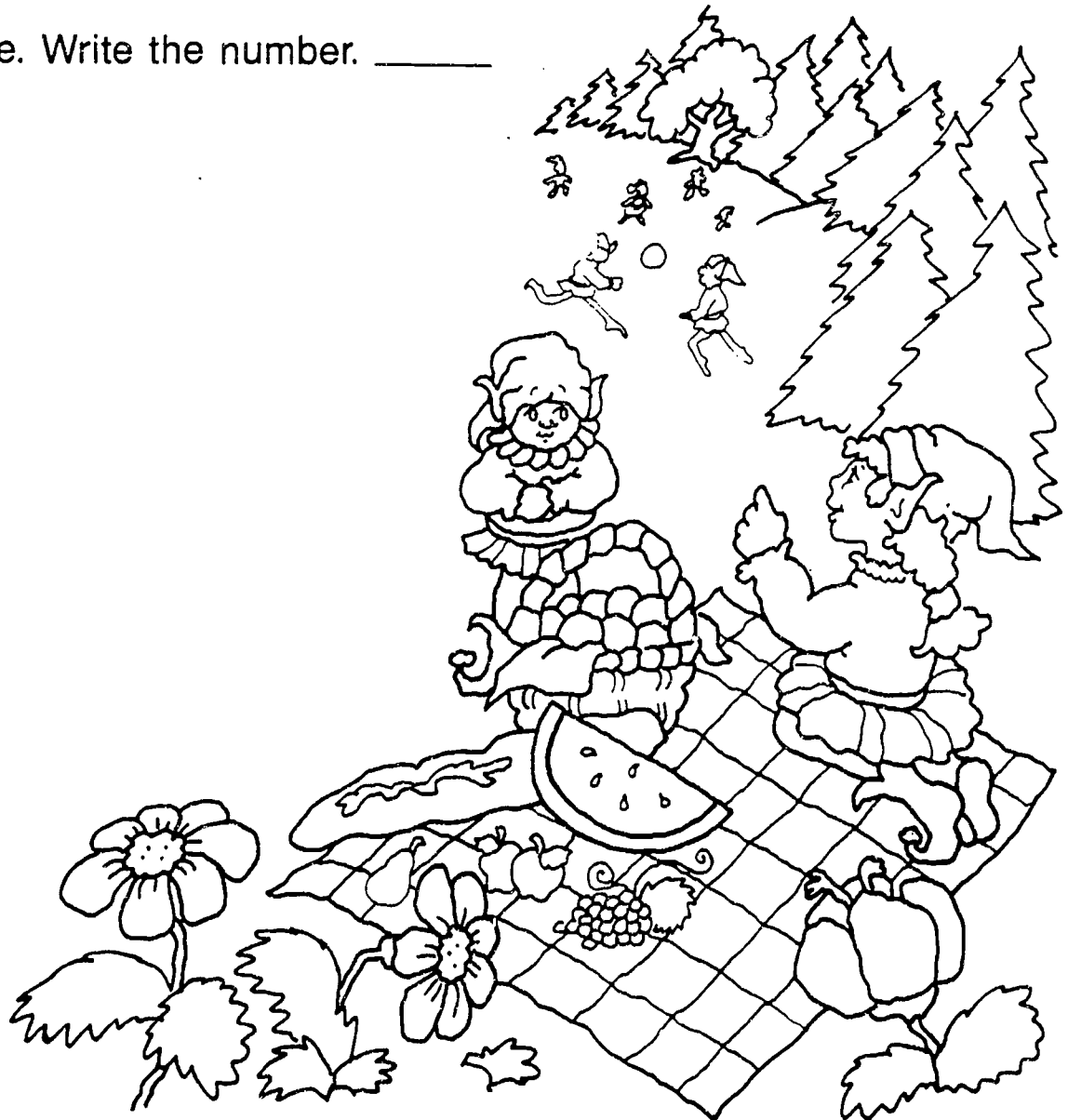
20

21

22

23

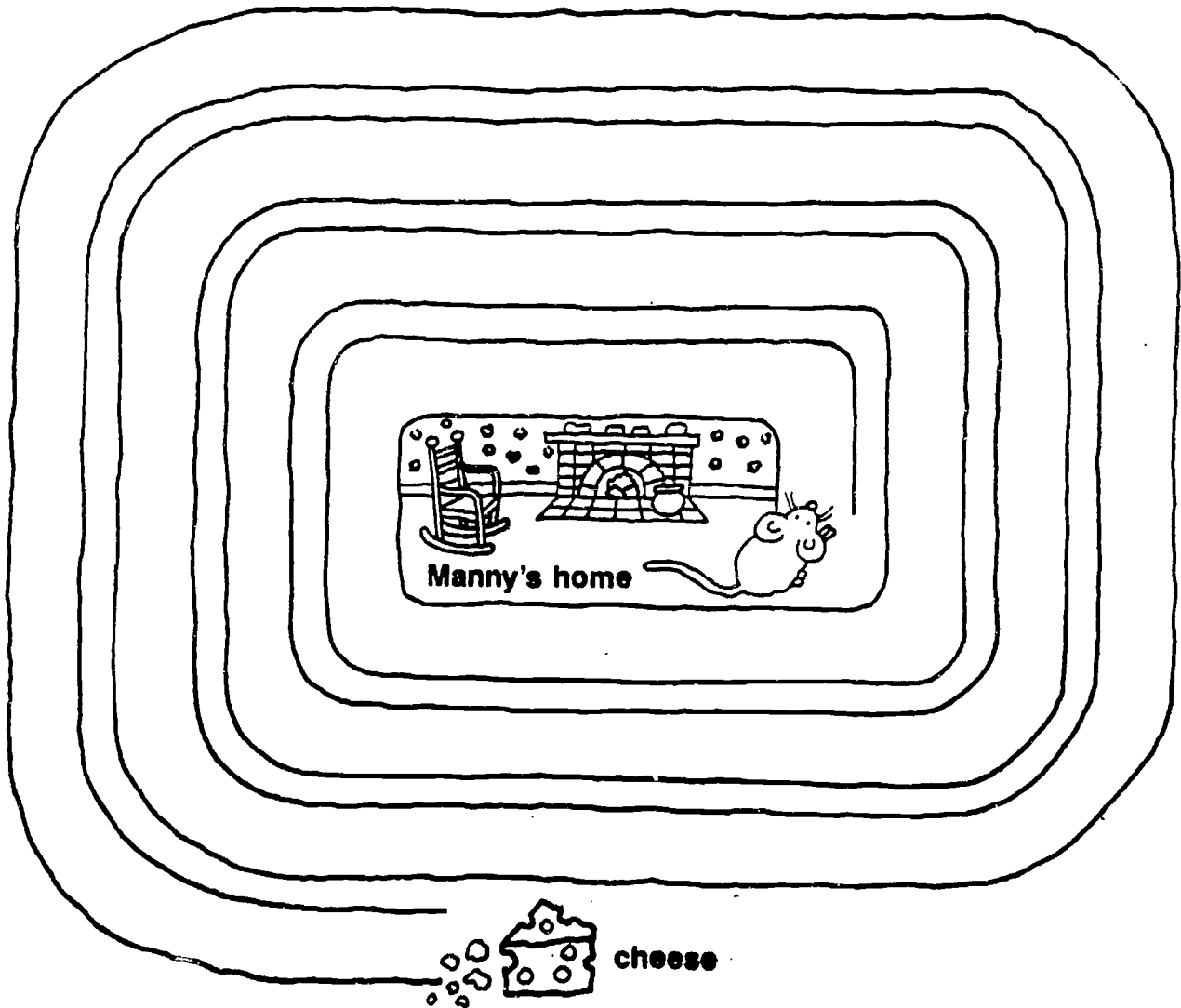
24





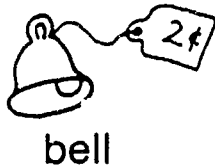
- 13** Manny Mouse is going out to get some cheese. He has to crawl through a long tunnel to get from his home to the cheese. Manny has to go around 15 corners on his way. He starts out from home and goes around 11 corners. Squeak! He dropped his crackers somewhere. Manny turns around and goes back around 5 corners. There are his crackers! He turns around again and goes around 7 corners. Where is Manny now?

Write an **M** to show where Manny is.





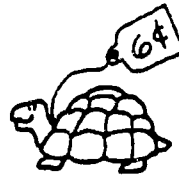
19 Soo Won saw lots of fun things in the Tiny Toys Shop. She saw a bell, a telephone, a turtle, a tiger, a frog, a lion, a horse, a raccoon, an owl, and a rabbit. Soo bought one toy for a friend, one toy for her sister, and one toy for herself. She paid 20 cents for the toys. Which three toys could Soo have bought?



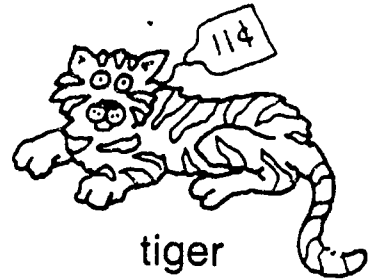
bell



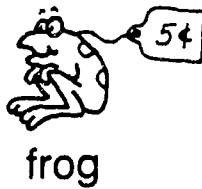
telephone



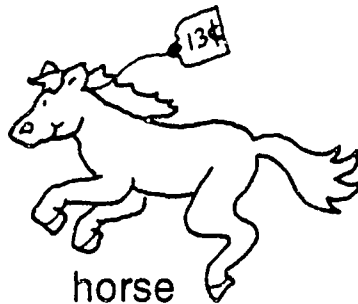
turtle



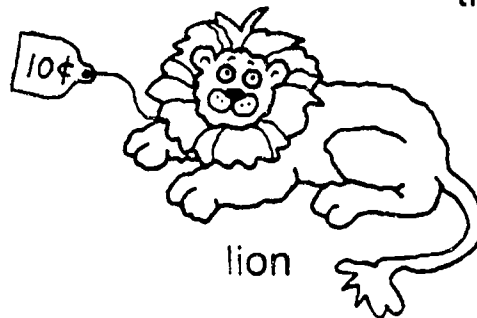
tiger



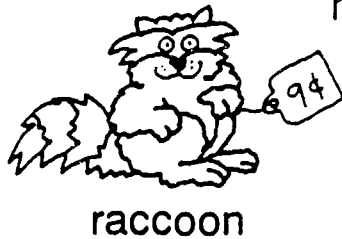
frog



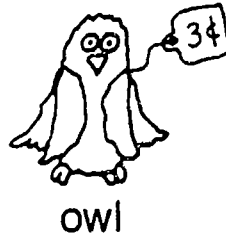
horse



lion



raccoon



owl



rabbit

Write the names of three things Soo could buy.

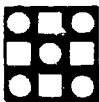
\_\_\_\_\_

Guess:

Guess:

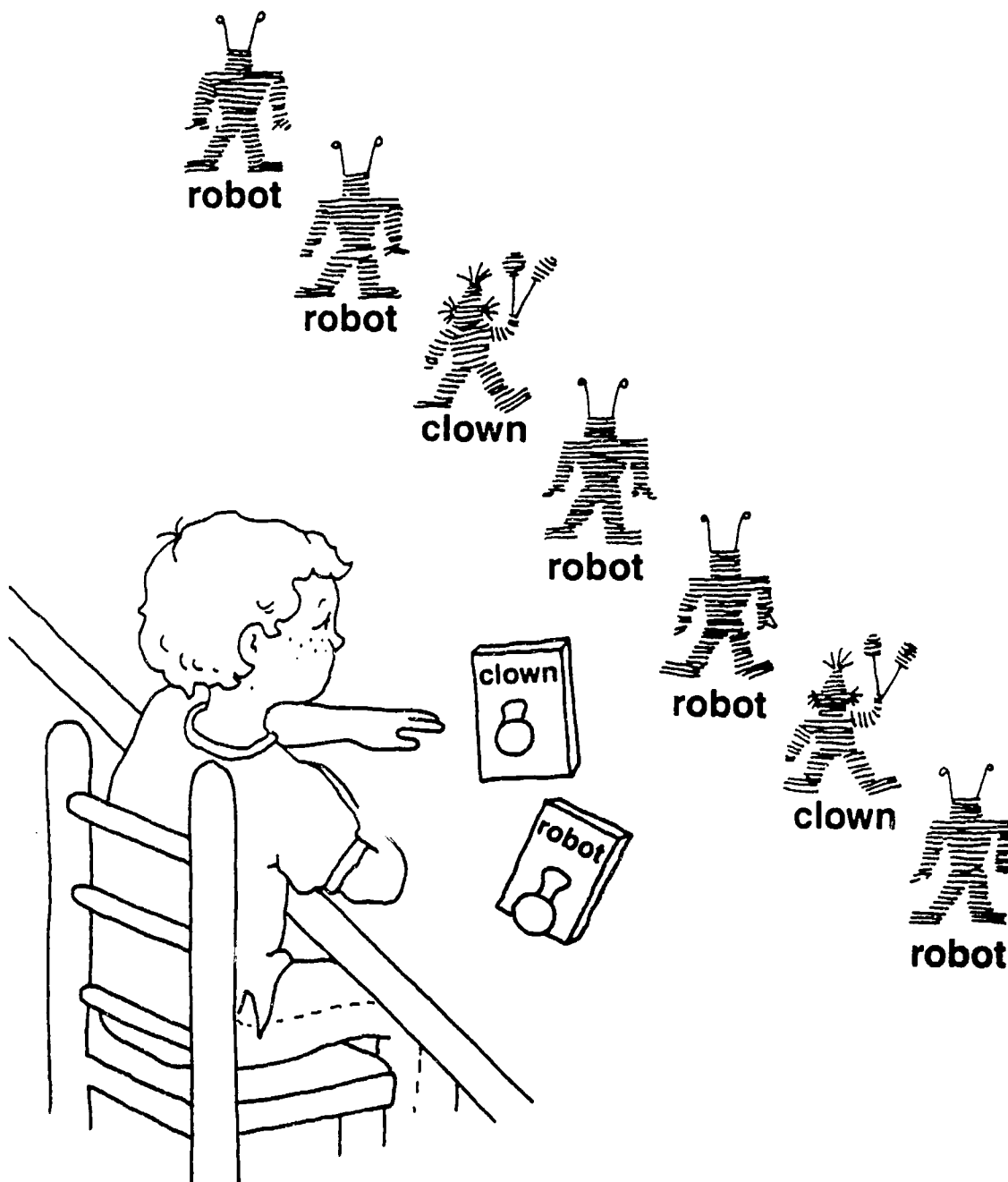
Guess:





- 22** Brian has two rubber stamps with pictures on them. Brian dips each stamp in ink. Then he stamps it on paper. One stamp makes a picture of a robot. The other one makes a picture of a clown. Brian is putting those pictures on paper in a pattern. What stamp will Brian use next?

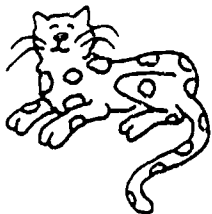
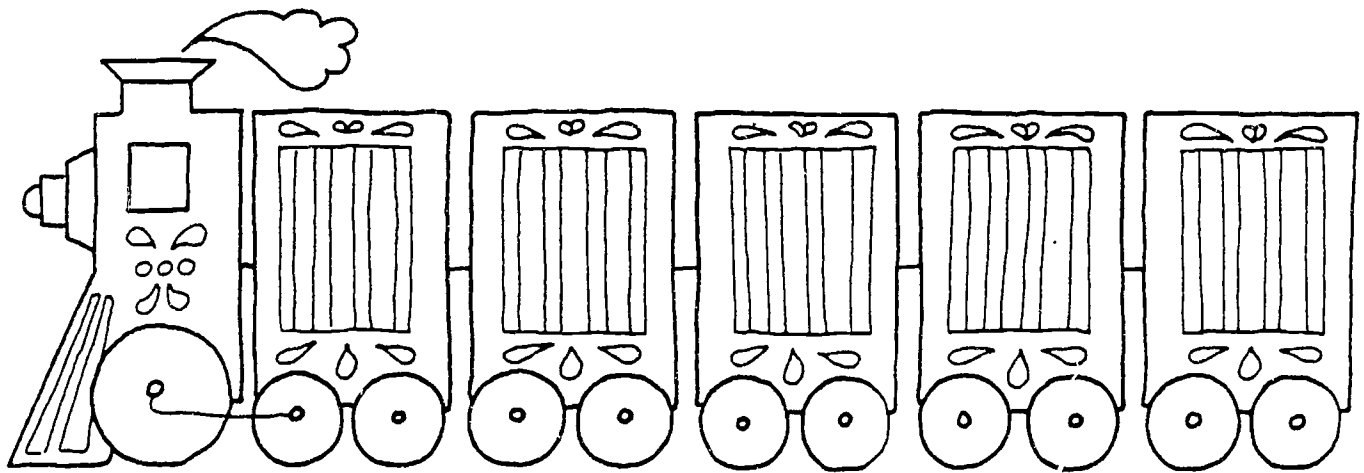
Draw a circle around the stamp.



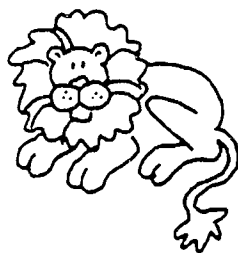


- 25** The Jumbo Circus is in town! Five circus wagons are going to carry animals to the circus tent. There will be one animal in each wagon. The leopard will be ahead of the lion. The monkey will be ahead of the leopard. The elephant will be behind the lion. The horse will be in the first wagon. Which wagon will each animal be in?

Draw a line from each animal to the wagon it will be in.



**leopard**



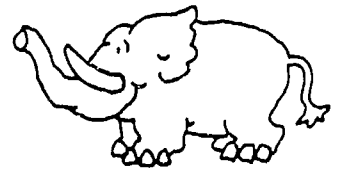
**lion**



**monkey**



**horse**

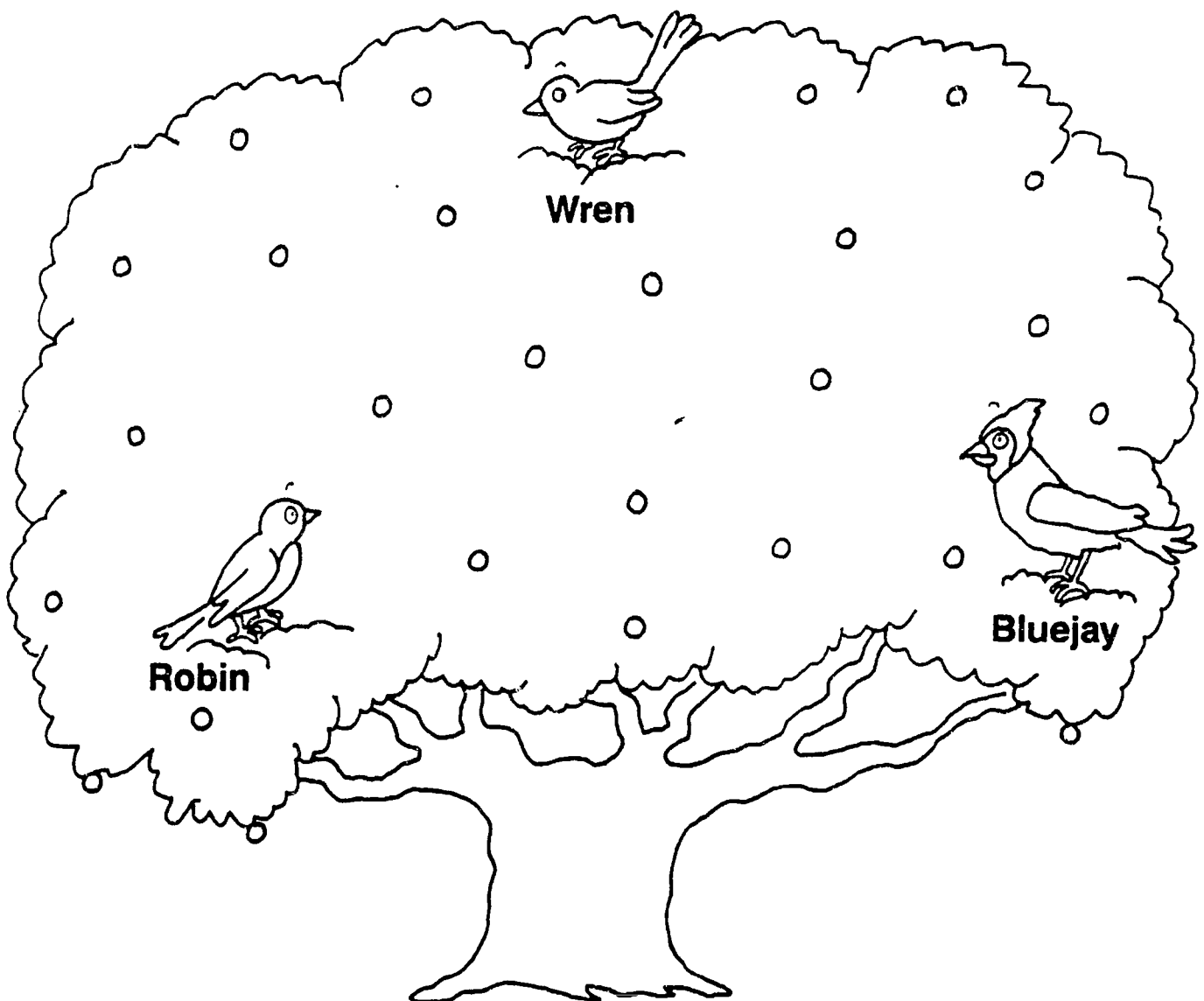


**elephant**



- 43** Do big, red cherries taste good? Yes! Robin sang about the wonderful cherries hanging from the tree. Bluejay heard Robin sing and he told Wren. Wren ate 10 more cherries than Robin ate. Bluejay ate 6 more cherries than Wren did. Robin was so busy singing that she ate only 2 cherries. How many cherries did Bluejay eat?

Write the number. \_\_\_\_\_



**49** "Let's all have our pictures taken together," said Linda. She was with her friends Megan, Nora, Olga, and Pearl. Linda's brother took pictures of them with his camera. He could only get two girls on each picture. So, each of the five girls had her picture taken with every other girl. How many pictures did Linda's brother take in all?

Finish the table. Write the number. \_\_\_\_\_

How many pictures did Linda's brother take for

2 friends? \_\_\_\_\_

3 friends? \_\_\_\_\_

4 friends? \_\_\_\_\_

5 friends? \_\_\_\_\_





**52** Reese is giving you a puzzle to solve. He says, "It weighs nothing. It can be seen. If it is put in a can, it makes the can weigh less. What is it?"

Write its name. \_\_\_\_\_

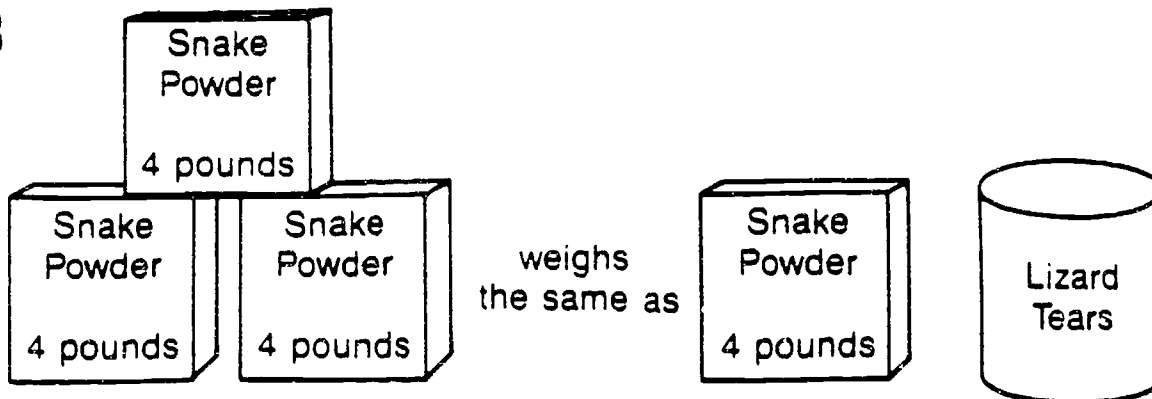




USE LOGICAL REASONING

Name \_\_\_\_\_

33



Then how many pounds does one can of Lizard Tears weigh?

- FIND OUT**
- What is the question you have to answer?
  - What weighs the same as three boxes of Snake Powder?
  - How much does one box of Snake Powder weigh?

- CHOOSE A STRATEGY**
- Circle to show what you choose.



- SOLVE IT**
- How much does one box of Snake Powder weigh? Then how much do three boxes of Snake Powder weigh together?
  - How many pounds do the box of Snake Powder and can of Lizard Tears weigh together? If that is so, then how can you find out how much the can of Lizard Tears weighs? How much does the can of Lizard Tears weigh?

- LOOK BACK**
- Look back to see if your answer fits with what the problem tells you and asks you to find. Read the problem again. Look back over your work. Does your answer fit?

**7** Anthony came home from school with a puzzle for his sister Teresa. He gave her three cards. One card had a 2 on it; one card had a 4 on it; and one card had an 8 on it. Anthony asked, "Teresa, how many 2-digit numbers can you make with these three cards?" Teresa surprised Anthony. She made six different 2-digit numbers. What numbers did Teresa make?



- FIND OUT**
- What is the question you have to answer?
  - What did Anthony give to Teresa? What number was on each card?
  - What did Anthony ask Teresa to do with the three numbered cards?
  - How many cards did Teresa use to make each 2-digit number?
  - How many different 2-digit numbers did Teresa make?

**CHOOSE A STRATEGY**

- Circle to show what you choose.



- SOLVE IT**
- What were the numerals Teresa could use to make the 2-digit numbers?
  - Could Teresa use the same numeral twice in one number? Why or why not?
  - Look at the list started below. What is the first 2-digit number? What numeral is in the tens' place? What numeral is in the ones' place?
  - What is the second 2-digit number? What numeral is in the tens' place? What numeral is in the ones' place? Could Teresa make any other 2-digit number with the numeral 2 in the tens' place?
  - Finish the list to find all six different 2-digit numbers Teresa made. What numbers did she make?

24	4	_____	_____
28	4	_____	_____

**LOOK BACK**

- Look back to see if your answer fits with what the problem tells you and asks you to find. Read the problem again. Look back over your work. Does your answer fit?



19

Carol's cat weighs 2 pounds. Doug's dog weighs 14 pounds. Both pets are gaining about a pound a month. If they keep on gaining weight like that, the dog will soon weigh three times as much as the cat. How many pounds will the cat weigh then?

**FIND OUT**

- What is the question you have to answer?
- How much does Carol's cat weigh now?
- How much does Doug's dog weigh now?
- About how much weight is each pet gaining every month?
- What will happen soon if the pets keep gaining weight like that?

**CHOOSE A STRATEGY**

- Circle to show what you choose.



**SOLVE IT**

- How many rows are there in the table started below?
- What are you going to keep track of in the first row?
- What are you going to keep track of in the second row?
- What is the first number in the cat's row?
- What is the first number in the dog's row?
- What is the next number in the cat's row?
- What is the next number in the dog's row?
- Is the dog's weight three times as much as the cat's?
- Keep adding numbers to the table until you find that the dog's weight is three times the cat's weight. How much does the cat weigh when that happens?

<b>Pounds the cat weighs</b>	2	3							
<b>Pounds the dog weighs</b>	14	15							

**LOOK BACK**

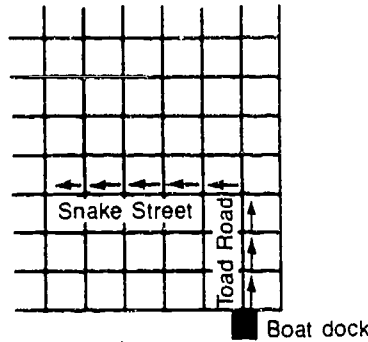
- Look back to see if your answer fits with what the problem tells you and asks you to find. Read the problem again. Look back over your work. Does your answer fit?





17

Juanita and Cole discovered a note in a bottle. It said, "Start at the Boat Dock on Toad Road. Go forward 3 blocks on Toad Road to Snake Street. Turn left and go forward 5 blocks to Snail Trail. Turn right and go forward 4 blocks to Mud Street. Stay there. Use your eyes. Look for a secret message under a big white rock." Can you show the path from the Boat Dock to the secret message?



- FIND OUT**
- What is the question you have to answer?
  - What happened to Juanita and Cole?
  - What did the note tell them to do?
  - What must they look for at Mud Street?
  - What would they find under the big white rock?

- CHOOSE A STRATEGY**
- Circle to show what you choose.



- SOLVE IT**
- Where does the path start? Can you find that place on the map? Use a pencil to trace the path on the map.
  - How many blocks must Juanita and Cole go forward on Toad Road? What street will they come to?
  - What direction must they go on Snake Street? How far must they go? What street will they come to? Write the street name on your map.
  - What direction must they go on Snail Trail? How far must they go? What street will they come to? Write the street name on your map.
  - What must they look for on the corner of Snail Trail and Mud Street?
- LOOK BACK**
- Look back to see if your answer fits with what the problem tells you and asks you to find. Read the problem again. Look back over your work. Does your answer fit?



**41** Oozy Pond is filled with crayfish and mud turtles who are there for an egg-laying contest. Betsy Beaver says she doesn't know how many crayfish and turtles there are, but she counted 76 legs in all. Each crayfish has 10 legs, and each turtle has 4 legs. How many crayfish and how many turtles could there be in Oozy Pond?

- FIND OUT**
- What is the question you have to answer?
  - Who came to Oozy Pond for the egg-laying contest?
  - How many legs does each crayfish have?
  - How many legs does each turtle have?
  - How many legs in all did Betsy Beaver count?

**CHOOSE A STRATEGY**

- Circle to show what you choose.



- SOLVE IT**
- What is the total number of legs?
  - How many legs does each crayfish have?
  - How many legs does each turtle have?
  - How many crayfish and how many turtles could there be in Oozy Pond? Make a guess. How many crayfish? Then how many crayfish legs would there be? How many turtles? Then how many turtle legs would there be? Then how many legs would there be in all?
  - Was your guess correct? Was it too high? Was it too low? Keep guessing and checking until you find an answer.

- LOOK BACK**
- Look back to see if your answer fits with what the problem tells you and asks you to find. Read the problem again. Look back over your work. Does your answer fit?



**25**

Mona is wearing her magic cape again. The first time she wore it, she found 5 pennies in a crack of the sidewalk. The next time she wore it, she discovered 9 pennies under an old barre!. The third time she wore it, she found 13 pennies in some sand. The fourth time she wore it, she discovered 17 pennies under the bleachers in a ball park. If Mona keeps finding pennies in this way, how many will she find when she wears her cape the eighth time?

**FIND OUT**

- What is the question you have to answer?
- What happens when Mona wears her magic cape?
- How many pennies did she find the first time she wore the cape? How many the second time? How many the third time? How many the fourth time?

**CHOOSE A STRATEGY**

- Circle to show what you choose.



**SOLVE IT**

- What are you going to keep track of in the table started on your paper?
- How many pennies did Mona find the first time she wore the cape?
- How many pennies did Mona find the second time? How many more is that than what she found the first time?
- How many pennies did Mona find the third time? How many more is that than what she found the second time?
- How many pennies did Mona find the fourth time? How many more is that than what she found the third time?
- Do you see a pattern in the numbers of pennies?
- Keep filling in the table. How many pennies will Mona find when she wears her cape the eighth time?

Time	1st	2nd	3rd	4th	5th	6th	7th	8th
<b>Pennies Mona finds</b>	5	9	13					

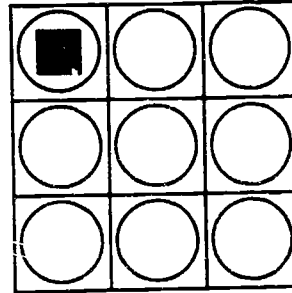
**LOOK BACK**

- Look back to see if your answer fits with what the problem tells you and asks you to find. Read the problem again. Look back over your work. Does your answer fit?



23

Here are 9 hamburgers on a grill. One hamburger has cheese on it.



Put cheese on 5 more hamburgers, but be sure to leave 1 hamburger without cheese in each row and in each column. Which hamburgers can you put cheese on?

- FIND OUT**
- What question do you have to answer?
  - How many hamburgers are on the grill?
  - How many hamburgers have cheese on them?
  - How many more hamburgers must you put cheese on?
  - How many hamburgers without cheese must there be in each row? How many hamburgers without cheese must there be in each column?
  - Can a hamburger without cheese be in both a row and a column?

**CHOOSE A STRATEGY**

- Circle to show what you choose.



- SOLVE IT**
- How many hamburgers must you put cheese on? So how many pieces of paper do you need?
  - How many hamburgers in row 1 must not have cheese? Then how many hamburgers in row 1 must have cheese? If 1 hamburger has cheese on it already, how many hamburgers must you put cheese on?
  - How many hamburgers in row 2 must you put cheese on?
  - How many hamburgers in row 3 must you put cheese on?
  - Now look at the columns. Is there 1 hamburger without cheese in each column? If not, move your papers around. Which hamburgers can you put cheese on?

- LOOK BACK**
- Look back to see if your answer fits with what the problem tells you and asks you to find. Read the problem again. Look back over your work. Does your answer fit?



**43**

Someone broke a bag of peanuts on Peep Street. Pigeons came from all over the city to feast on the peanuts. They ate lots of the nuts on Monday. They ate 2 fewer nuts on Tuesday than on Monday. They came back again on Wednesday, Thursday, and Friday. Each day they ate 2 fewer peanuts than the day before. On Friday they cleaned up the last 4 peanuts. How many peanuts in all did the pigeons eat?

**FIND OUT**

- What is the question you have to answer?
- What were the pigeons doing?
- How many peanuts did the birds eat on Monday?
- What do you know about how many peanuts the pigeons ate on Tuesday, Wednesday, Thursday, and Friday?
- How many peanuts did they eat on Friday?

**CHOOSE A STRATEGY**

- Circle to show what you choose.



**SOLVE IT**

- How many peanuts did the pigeons eat on Friday? Where will you write that number in the table started on your paper?
- How many fewer peanuts did the birds eat each day than the day before? Then how many peanuts did the birds eat on Thursday?
- How many peanuts did the birds eat on Wednesday?
- How many peanuts did the birds eat on Tuesday?
- How many peanuts did the birds eat on Monday?
- How many peanuts in all did the birds eat?

Day	Monday	Tuesday	Wednesday	Thursday	Friday
Peanuts eaten					

**LOOK BACK**

- Look back to see if your answer fits with what the problem tells you and asks you to find. Read the problem again. Look back over your work. Does your answer fit?



**47** Which is better, a new \$5 bill or an old one?

- FIND OUT**
- What is the question you have to answer?
  - What are the two bills in the problem?

- CHOOSE A STRATEGY**
- Circle to show what you choose.



- SOLVE IT**
- Do you think this is a hard question, or easy to answer?
  - If you think it is very easy, do you think there may be a trick to the question?
  - Try to think of all the different meanings the words could have. What are some of the things you can think of?
  - Is there more than one meaning for the words "an old one?"
  - Which is better, a new \$5 bill or an old one?

- LOOK BACK**
- Read the problem again. Look at the information given and the main question. Review your work. Is your answer reasonable?

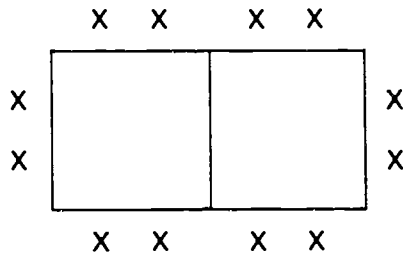
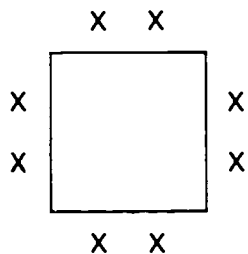
**45** Several soccer teams are having an end-of-the-season soccer party. The team captains are putting square tables together in a long row for the party. They can put two chairs on each side of a table. The tables are all the same size. If they put together ten tables in a row, how many people can sit down?

- FIND OUT**
- What is the question you have to answer?
  - What are the soccer team captains doing?
  - How many chairs can they put on each side of a table?
  - How many tables are they putting together?

- CHOOSE A STRATEGY**
- Circle to show what you choose.



- SOLVE IT**
- If you make an organized list, what do you want to keep track of?
  - To make the problem simpler, begin with one table. How many people can sit at one table?
  - Now put together two tables. How many people can sit at two tables?
  - Put together three tables. How many people can sit down at three tables?
  - Look at your organized list. Do you see a pattern?
  - If they put together ten tables in a row, how many people can sit down?



$$1 = 4$$

$$2 = 6$$

- LOOK BACK**
- Read the problem again. Look at the information given and the main question. Review your work. Is your answer reasonable?

# Appendix F

## Teacher Observation Checklist

Student	Restate Problem	Steps Involved	Solve more than one way	Identify needed Information	Identify not needed information	Correctly Solved
1						
2						
3						
4						
5						
6						
7						
8						
9						
10						
11						
12						
13						
14						
15						
16						
17						
18						
19						
20						
21						
22						
23						
24						
25						
26						
27						
28						
29						
30						



## Appendix G

### Third Grade Group A Scores on Third Grade Practice Sheets and Posttest

Student Number	Logical Reasoning		Organized List		Use or Make a Table		Use or Make a Picture	
	Pract.	Post	Pract.	Post	Pract.	Post	Pract.	Post
1	Y	N	Y	N	Y	N	N	N
2	Y	N	Y	N	Y	N	N	N
3	Y	Y	Y	Y	Y	Y	Y	N
4	Y	Y	Y	Y	Y	N	N	Y
5	Y	N	-	Y	Y	N	Y	Y
6	N	Y	Y	Y	Y	N	N	N
7	Y	N	N	N	Y	N	N	N
8	Y	N	Y	Y	Y	N	N	N
9	-	-	-	-	-	-	-	-
10	-	Y	Y	Y	Y	Y	N	N
11	Y	-	Y	-	Y	-	N	-
12	Y	Y	N	N	Y	N	N	N
13	N	Y	Y	Y	Y	N	N	Y
14	Y	N	N	N	Y	N	N	N
15	-	-	-	-	-	-	-	-
16	N	N	Y	Y	Y	N	N	N
17	Y	Y	Y	N	Y	Y	N	Y
18	Y	N	N	N	Y	N	N	Y
19	Y	Y	Y	N	Y	N	N	N
20	Y	Y	Y	Y	Y	N	Y	Y

Student Number	Guess and Check		Use or Make a Pattern		Act Out or Use Objects		Work Backwards	
	Pract.	Post	Pract.	Post	Pract.	Post	Pract.	Post
1	Y	N	Y	Y	Y	N	Y	N
2	Y	N	Y	Y	Y	N	Y	N
3	Y	N	Y	N	Y	N	Y	Y
4	Y	N	N	N	Y	N	Y	N
5	Y	Y	Y	Y	Y	N	Y	N
6	N	Y	Y	N	Y	N	-	N
7	N	N	Y	Y	Y	N	Y	N
8	N	N	Y	Y	Y	N	Y	N
9	-	-	-	-	-	-	-	-
10	Y	N	Y	Y	Y	N	-	N
11	Y	-	Y	-	Y	-	Y	-
12	Y	N	Y	Y	Y	N	Y	N
13	Y	Y	Y	Y	Y	N	Y	N
14	N	N	Y	N	Y	N	Y	N
15	-	-	-	-	-	-	-	-
16	Y	N	Y	N	Y	N	-	N
17	Y	N	Y	N	Y	Y	Y	N
18	Y	N	Y	N	Y	N	Y	N
19	N	N	Y	Y	Y	N	Y	Y
20	Y	Y	Y	N	Y	N	Y	N

Student Number	Make It Simpler	
	Pract.	Post
1	N	N
2	N	N
3	N	N
4	N	Y
5	Y	Y
6	Y	N
7	N	N
8	-	Y
9	-	-
10	N	N
11	N	-
12	N	N
13	N	Y
14	N	N
15	-	-
16	N	N
17	N	N
18	N	N
19	N	N
20	N	N

N = Incorrect Response

Y = Correct Response

## Appendix H

### Number of Third Grade Group B Correct Responses On Pretest and Posttest

Student Number	#Correct Responses Pre	# Correct Responses Post
1	3	6
2	4	7
3	6	8
4	5	7
5	4	8
6	5	7
7	-	7
8	6	7
9	5	7
10	3	4
11	6	8
12	7	9
13	3	7
14	6	9
15	4	6
16	9	9
17	7	9
18	6	8

274

284

Appendix I  
Fifth Grade Pretest and Posttest

**NAME** \_\_\_\_\_

**Date** \_\_\_\_\_

**MATH PROBLEM SOLVING PRE-TEST**

There are many different strategies to use when solving a mathematical word problem. Below are 10 word problems with 10 suggested strategies to use. Try to solve each of the problems. Show your work in the space provided for your answer.



**MAKE A PICTURE OR DIAGRAM**      Name \_\_\_\_\_

- h** The kitten climbed its first tree and got stuck on the top branch. First it went up the trunk of the tree and on up to the 6th branch. A big squirrel scared the kitten and it climbed down 3 branches. A bird flew at the kitten and scared it again. Now it climbed up 10 branches. The kitten climbed back down 2 branches and then went up 4 branches to the very top of the tree. How many branches were in the tree?

1  
2  
3

**MAKE AN ORGANIZED LIST**

Name \_\_\_\_\_

2.

Allen was at a neighborhood garage sale. He was standing at a table with all sorts of comic books divided into 3 piles. One pile was marked 10 cents, the second 5 cents, and the third 1 cent. Allen had 26 cents. How many different combinations of comic books could Allen buy for 26 cents?



**GUESS AND CHECK**

Name \_\_\_\_\_

3.

Monica and Marty are llamas at the petting zoo. They like to count their visitors, and the number of people who pet them. After visiting hours on Sunday Monica reported that 105 people in all had petted both of them. She bragged that  $2\frac{1}{2}$  times as many people petted her as petted Marty. How many people petted Monica and how many petted Marty?

**ACT OUT OR USE OBJECTS**

Name \_\_\_\_\_

**4.**

Donna was putting six new bears in the display case at the toy store. The case had three shelves, one on top of the other, with two spaces on each shelf. Each bear had a name: Abby, Bobby, Cathy, Dorothy, Eric, and Forrest. Donna put Dorothy next to Eric and above Forrest. She did not put Bobby next to Eric or Forrest. She did not put Abby next to Bobby. Where did Donna put each of the bears?

---

**USE OR MAKE A TABLE**

Name \_\_\_\_\_

**5.**

Melody and Mandy are circus elephants. They always lead the circus parade. Melody is 4 years old and Mandy is 13 years old. When will Mandy be twice as old as Melody?

---



**USE OR LOOK FOR A PATTERN** Name \_\_\_\_\_

6. Yolanda and Willie found that they were getting more and more snails in their garden. On the first day they counted 9 snails, then on the second day there were 17. On the third day they counted 24, 32 on the fourth day, and 39 on the fifth day. On what day did they count more than 90 snails?



**USE LOGICAL REASONING** Name \_\_\_\_\_

7. Amelia, Gigi, Evan, and Collin were dreaming about the circus. They wanted to be an animal trainer, a clown, a juggler, and a trapeze artist. Gigi is training her dog to be a seeing-eye dog for the blind. Evan is always telling jokes, and Amelia is afraid of heights. Which job in the circus do you think each friend would choose?



**WORK BACKWARDS**

Name \_\_\_\_\_

8. Slowly Katie peeked around the door. The huge monster wiped its mouth, smiled, went to sleep, and started to snore. Katie had been watching while the monster ate the cakes in the bakery. The first hour it ate  $\frac{1}{2}$  of all the cakes in the bakery; the second hour it ate  $\frac{1}{2}$  of all the cakes left; the third hour it ate  $\frac{1}{2}$  of what was left; and the fourth hour it ate  $\frac{1}{2}$  the cakes left again. Now there are 3 cakes left. How many cakes did the monster eat?

**MAKE IT SIMPLER**

Name \_\_\_\_\_

9. Several soccer teams are having an end-of-the-season soccer party. The team captains are putting square tables together in a long row for the party. They can put two chairs on each side of a table. The tables are all the same size. If they put together ten tables in a row, how many people can sit down?

**BRAINSTORM**

Name \_\_\_\_\_

10.

Which is better, a new \$5 bill or an old one?

## Appendix J

### Fifth Grade Problem Solving Test Results

#### Pretest and Posttest Data

Student #	#Correct Pre	#Correct Post
1	3	1
2	0	2
3	2	-
4	2	6
5	0	5
6	4	3
7	0	3
8	2	5
9	1	5
10	2	6
11	1	5
12	1	2
13	1	4
14	2	3
15	1	1
16	0	-
17	0	0
18	1	4
19	2	4
20	2	3
21	1	2
22	1	3
23	0	4
24	1	3
25	2	5
26	1	6
27	2	4

Appendix K

Criterion Reference Test (CRT) Sample Questions

Third Grade:

8. Kari had 12 stickers. Susie had 10 stickers. How many stickers did they have in all?

- A. Subtract
- B. Add
- C. Multiply
- D. Divide

13. There are 9 children in the reading group. Each child reads 3 pages. How many pages do the 9 children read?

- A. 6 pages
- B. 21 pages
- C. 27 pages
- D. 12 pages

19. Which item costs between 50¢ and 80¢.

- A. tablet
- B. folder
- C. eraser
- D. glue

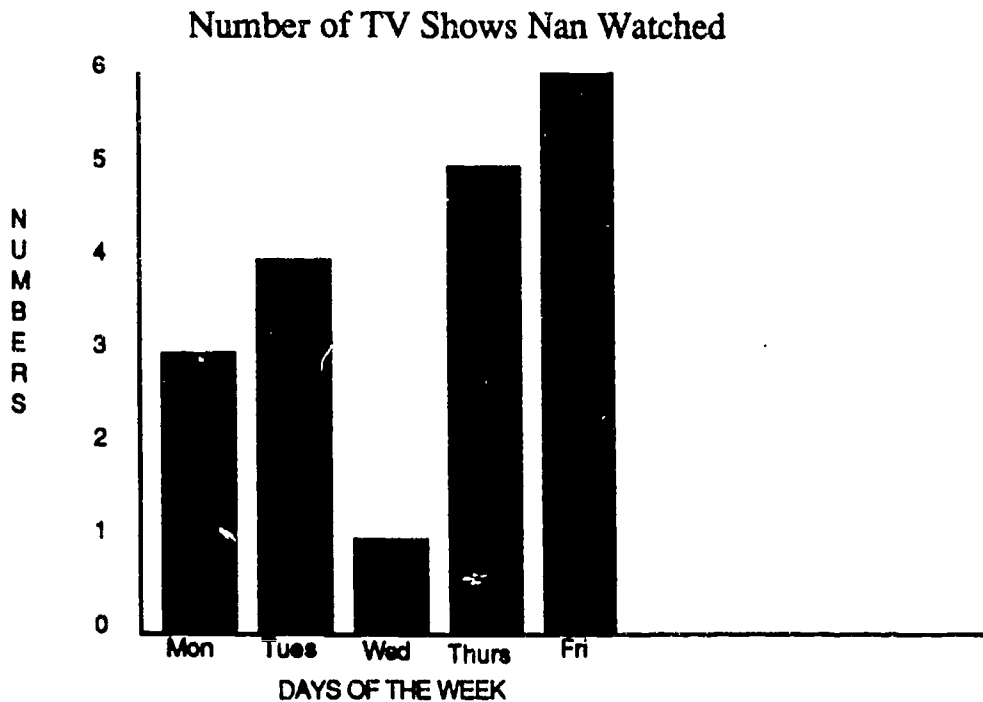
SCHOOL STORE PRICES	
Eraser	15¢
Folder	49¢
Tablet	79¢
Glue	35¢
Pencil	12¢

24. Subtract the numbers using a calculator.  $7,557 - 3,869 = ?$

- A. 4,868
- B. 4,098
- C. 11,426
- D. 3,688

Fourth Grade:

10. Bill had 428 football cards. Bob gave him 145 more cards. How many cards did Bill have then?
- A. divide
  - B. subtract
  - C. multiply
  - D. add
14. A book costs \$9.00. A tape costs \$6.00. How much does it cost for both?
- A. \$3.00
  - B. \$54.00
  - C. \$15.00
  - D. \$16.00



25. On which day did Nan watch the most programs on TV?
- A. Thursday
  - B. Friday
  - C. Tuesday
  - D. Monday

Fifth Grade:

**FOR QUESTIONS 1 - 3 MARK THE LETTER OF THE FACT THAT IS NOT NEEDED TO SOLVE THESE QUESTIONS.**

1. There were 15 Boston terriers, 12 collies and 22 Irish setters in the dog show. How many more Irish setters were there than Boston terriers.
- A. There were 15 Boston terriers.  
B. There were 12 collies.  
C. There were 22 Irish setters.
9. There are 12 seeds in each pocket. How many seeds are in 8 pockets?
- A. Multiply  
B. Divide  
C. Add  
D. Subtract
14. Sue jogs 5 miles a day. How many miles will she jog in June. (June has 30 days.)
- A. 150  
B. 180  
C. 121  
D. 90

22. Jeff bought  $\frac{3}{4}$  pounds of peanuts  
How much did he pay?

- A. \$1.35  
B. \$1.77  
C. \$ .89  
D. \$ .94

**COST OF SNACKS**

	1/2 lb.	3/4 lb	1 lb	1 1/2 lbs
Peanuts	\$0.89	\$1.35	\$1.77	\$2.21
M&M's	\$0.94	\$1.45	\$1.96	\$2.38
Jelly beans	\$0.18	\$0.27	\$0.36	\$0.45
Gummy bears	\$0.54	\$0.81	\$1.08	\$1.35
Licorice	\$0.06	\$0.09	\$0.12	\$0.15

26. Subtract the numbers using a calculator:      2,047,967 - 829,876.
- A. 1,218,091  
B. 215,065  
C. 418,724  
D. 4,186,240

Sixth Grade:

**FOR QUESTIONS 1 - 3 MARK THE LETTER OF THE FACT THAT IS NOT NEEDED TO SOLVE THESE QUESTIONS.**

1. A youth club charged \$2.75 to wash a car. They charged another 50¢ to dry it. Mr. Jones had a \$5 bill. How much change did he receive if he just had his car washed?
- A. Mr. Jones had a \$5 bill.
  - B. It cost 50¢ to dry the car.
  - C. The club charged \$2.75 to wash a car.

**FOR QUESTIONS 4 - 6, WHAT FACT IS NEEDED TO SOLVE THESE PROBLEMS.**

4. In 1989, Mr. Lee kept a record of car expenses. He spent \$580.68 on gas, \$164.32 on maintenance and repairs, and \$365 on auto insurance. How much more did he spend in 1989 than in 1988?
- A. Mr. Lee plans to keep the car 2 more years.
  - B. Mr. Lee spent \$1,035 for car expenses in 1988.
  - C. Mr. Lee paid \$14,000 for the car.
  - D. The car was manufactured in 1986.

**FOR QUESTIONS 16 - 18, CHOOSE THE BEST ANSWER FOR ESTIMATING THE ANSWERS TO THE FOLLOWING PROBLEMS.**

16. A company sent 585 pencils to each of 210 schools. Estimate the approximate number of pencils they sent out.
- A.  $550 \times 300$
  - B.  $500 \times 200$
  - C.  $600 \times 250$
  - D.  $600 \times 200$

**FOR QUESTIONS 28 - 45, YOU MUST USE A CALCULATOR, THEN MARK THE CORRECT ANSWER ON THE ANSWER SHEET.**

28. A factory makes 24,725 posters for the roller rink, 85,476 posters for the football stadium, 92,583 posters for the baseball park and 496,872 posters for the circus. How many posters do they make altogether?
- A. 699,656 posters
  - B. 674,931 posters
  - C. 614,180 posters
  - D. 795,326 posters

Appendix L

Sample Sixth Grade Problems



**MAKE IT SIMPLER**

Name \_\_\_\_\_

**45**

Your sock drawer has 25 electric yellow socks, 30 blue striped socks, 17 orange socks, 13 magnetic magenta socks, 33 pale purple socks, 30 royal red socks, 11 gruesome green socks, 14 midnight black socks, and 23 bruin brown socks! If you reach into the drawer in the dark, how many socks do you need to pull out to be sure you have a matching pair?

**FIND OUT**

- What is the question you have to answer?
- How many yellow socks are in the drawer? blue? orange? magenta? purple? red? green? black? brown?
- How many different colors are there?

**CHOOSE A STRATEGY**

- Would it be easier to solve this problem with fewer different colors?
- Is there another strategy you can use along with the first one?

**SOLVE IT**

- Begin with 3 different colors and 2 socks of each color: yellow, red, and green.
- If you pull out 1 yellow, what are the possibilities for the next sock you pull out? What are your chances that it is yellow?
- If you pull out a green sock, you have 1 yellow and 1 green. What are the possibilities for the next sock you pull out? What are your chances that it is yellow or green?
- If you pulled out a red sock, then you have 1 yellow, 1 red, and 1 green. What are the possibilities for the next sock? What are your chances of pulling out either red, green, or yellow?
- Now consider the problem you started with. How many socks do you need to pull out to be sure you have a matching pair?

**LOOK BACK**

- Read the problem again. Look at the data, conditions, and the main question. Review your work. Is your answer reasonable?

**BRAINSTORM**

Name \_\_\_\_\_

**48**

Ruben ignored everyone at the table, and even his dinner when it arrived. He was leaning intently over the geometric shapes he was creating with toothpicks. Ruben was trying to make 6 squares with 12 toothpicks. Can you help him?

**FIND OUT**

- What is the question you have to answer?
- What is Ruben trying to make?
- How many toothpicks does Ruben have?

**CHOOSE A STRATEGY**

- When the strategies you know about don't apply to the problem, what can you do?

**SOLVE IT**

- What is your first reaction to Ruben's project?
- What is your visual picture of the 6 squares? How many toothpicks would you need for them?
- Do the squares have to be flat on the table?
- Can a toothpick be in more than one square?
- How can Ruben build 6 squares with 12 toothpicks?

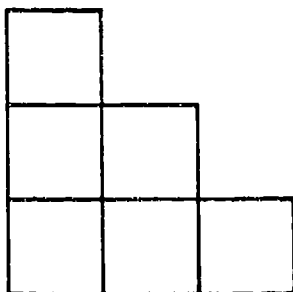
**LOOK BACK**

- Read the problem again. Look at the data, conditions, and the main question. Review your work. Is your answer reasonable?

Name \_\_\_\_\_

**73**

It is the last event in the sixth-grade math marathon. Kelly studies the final problem. She has been given an unlimited supply of identical wooden cubes, and directions to build a 30-step staircase with them. If a 3-step staircase looks like this:



how many cubes will Kelly need to complete the staircase?



Name \_\_\_\_\_

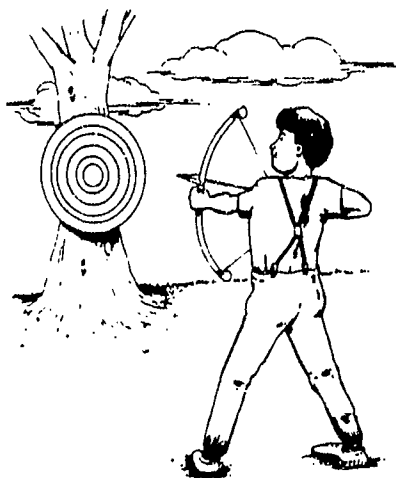
- 74** Two mothers and two daughters divided \$21 in dollar bills evenly among themselves. Each received an equal number of dollar bills. How could this be?

Name \_\_\_\_\_

- 75** It is the first school band concert of the year and Brett is responsible for setting up the chairs on the stage. His instructions are to set up the chairs in sections. One half the chairs will go in the brass section, one fourth in the wind section, one eighth in the percussion section, and 5 chairs are needed for the strings. How many chairs will Brett need to set up, and how many will he put in each section?

Name \_\_\_\_\_

- 76** In your dream you are competing in the semi-finals of the archery contest. You know you must make a total of 50 points to be in the finals. The bulls-eye is worth 50, and is therefore the smallest circle; the next circle just outside that is worth 30; the circle just outside that is worth 20, and the last and largest circle is worth 10. Anything outside the target is worth 0 points. You have 3 arrows in your quiver — 3 shots to make 50 points. Every eye is on you. How many ways can you score 50 points in three attempts? (Remember,  $0 + 0 + 50$  is different from  $0 + 50 + 0$ .)



Appendix M

Sixth Grade Attitude Survey

Name \_\_\_\_\_

Date \_\_\_\_\_

Student Mathematics Survey

- 1) What is your opinion about mathematics as taught in school ?
  - A. Dislike it
  - B. Neutral
  - C. Enjoy it
  
- 2) How do you feel when you are given word problems as an assignment ?
  - A. Extremely stressed
  - B. Slightly stressed
  - C. No stress
  
- 3) How would you evaluate your ability to be successful in math class ?
  - A. Difficult for me in all areas
  - B. Difficult in one or two areas  
( Name them: \_\_\_\_\_, \_\_\_\_\_ )
  - C. Easy to be successful
  
- 4) When do you do math entirely on your own ?
  - A. Rarely
  - B. Sometimes
  - C. Consistently
  
- 5) Do you like to do math assignments alone or in cooperative groups ? \_\_\_\_\_  
What are some of your reasons ?

- 6) Why can word problems be difficult for you ? ( Circle all those that you agree with.)
- A. I don't understand the question being asked in the problem.
  - B. I can't understand the problem because the vocabulary words used in the problem are too hard for me.
  - C. I am confused about which mathematical operation to use.
  - D. I don't know which problem-solving strategy to use.
  - E. I don't know what problem-solving strategies are in math.
  - F. Word problems are never difficult for me.
- 7) Do you know what is meant by the term non-routine problem ? \_\_\_\_\_ If so, please explain what it means to you.
- 8) Would you like to spend more time on problem solving in math class ? \_\_\_\_\_ Explain the reason for your answer.
- 9) Have you had an opportunity to do any math problem solving outside of school ? \_\_\_\_\_ If so, where were you and what did you do ?

- 10) How often do you use math outside of school ?
- A. Never
  - B. Once a month
  - C. Once a week
  - D. Daily
  - E. Other \_\_\_\_\_
- 11) Are you able to solve the same problem in different ways ?
- A. Rarely
  - B. Sometimes
  - C. Usually
- 12) Describe your participation in math class.
- 13) How do you feel when you answer an oral math question incorrectly ?
- A. Embarrassed, everyone might laugh at me
  - B. Depressed, I feel like a failure
  - C. Doesn't bother me
- 14) Have you ever had any bad math experiences in school ?  
\_\_\_\_\_ If so, please explain.

15) Who do you believe does better in math ?

- A. Boys
- B. Girls
- C. Both do equally as well

Explain your answer.

16) Approximately, how much time do you spend on math homework each school night ? \_\_\_\_\_

17) Do you participate in after school activities ? \_\_\_\_\_  
If so, what are they ? How much time do you spend on each activity per day ?

<u>ACTIVITY</u>	<u>DAY(S)</u>	<u>TIME PER DAY</u>
1.		
2.		
3.		
4.		

18) Does anyone help you with your math assignments at home ? \_\_\_\_\_ If so, who ? \_\_\_\_\_  
Why ?

19) Which of the following strategies have you used to solve word problems in school ? ( Circle all those that you have used.)

- |                          |                                  |
|--------------------------|----------------------------------|
| A. Multi-step procedures | G. Guess and Check               |
| B. Estimation/Rounding   | H. Logical reasoning             |
| C. Drawing a picture     | I. Create a model                |
| D. Manipulatives         | J. Describe the problem verbally |
| E. Working backwards     | K. Tables                        |
| F. Graphs                |                                  |

Underline the strategies you have never heard of before.

20) Do you ever see your parents doing anything involving math at home ? \_\_\_\_\_ If so, what ?

21) Does either one of your parents use math at their place of work ? \_\_\_\_\_ If so, what is it used for ?

22) Why do you think you need to be a good mathematician and problem solver ?

23) How do your parents feel about your mathematical success in school ?

- A. Not important
- B. Somewhat important
- C. Extremely important

24 ) Do you ever do math at home for fun ?

- A. Rarely
- B. Sometimes
- C. Often

If so, what do you do ?

25) What goals do you have for yourself in regard to math ? ( Be as specific as possible.)

Appendix N

Sixth Grade Pretest

Name

---

- 1) James ran down the stairs to the subway and got in line for a ticket. If the line didn't move fast, he and Tony would be late for the movie. James fumbled in his pocket and finally found a dollar. He gave the dollar to the ticket man, got his change of 36 cents, and sprinted for the subway car. What are the possible combinations of coins that James could have received for his change of 36 cents?
- 

Name

---

- 2) Bill, his sister Martha, and Ann are sitting down to learn a new game from Phil Hill. Each player has a partner and the partners are seated across the table from each other. Bill Dill is sitting to the right of Phil's sister. Phil Hill is sitting to the right of Bill's sister's partner. Where is each player sitting at the table?
- 

Name

---

- 3) Ann and Makiko like to swim laps at the community center pool. They are swimming together today, but they are on different swim schedules. Ann swims every 3 days and Makiko swims every 5 days. How many times will they both be at the pool on the same day during the next ten weeks?
- 

Name

---

- 4) Gino and Mark had found all the things on the list for the treasure hunt and had only a few minutes to get to the finish point. But they were lost! Gino said, "When we were at the bridge, we were 2 blocks west of the finish point. Can you remember where we went after that?" Mark recalled that they had gone south 3 blocks, then they went to their left 5 blocks, left again for 2 blocks, then north for 1 block. What is the quickest route from where they are to the finish point?
-



Name

---

- 5) How many minutes did Heidi, Saul, and Joy each travel to get to the skating rink on Saturday? Joy came by skateboard, Heidi came by bike, and Saul came on the bus. It took Heidi twice as long as Joy to get there. It took Saul 10 minutes more than it took both the girls together. All three skaters together took 64 minutes to get to the rink.
- 

Name

---

- 6) On July 5, in the area around Center Village, there was great excitement. Six different people reported to the police that they had seen Bigfoot, the large hairy creature sometimes seen but never captured. The next day, twice as many people called the police, sure they had seen the creature. Each day the police received twice as many calls as the day before. After they got a total of more than 300 calls, the police took the phone off the hook! On what day did the police receive their 300th call?
- 

Name

---

- 7) Andre's Blue Ribbon Cars & Trucks is having a big sale. Mike and Don are setting up the lot. The boss gives them a diagram of the lot and these directions: "Put the 4-door car in front of the van. Put the jeep between the truck and the van. Put the sportscar to the left of the 2-door and 4-door cars." How did Mike and Don set up the lot?
- 

Name

---

- 8) Kevin, Barbara, and their mother and father went backpacking in Yosemite National Park. On the first and second days, each hiker had a serving of food for breakfast, lunch, and dinner. A large, noisy, brown bear barged into camp the second night, got the food pack down from the tree where they had hung it, and ate one half of the food that was left. The next morning, after they all had breakfast, they found they had 4 food servings left. They decided they had better hike back to their car. How many servings of food did they begin the trip with?
-

Name

---

- 9) A group of 13 friends were planning a trip. On the night before they left they made a lot of phone calls. Each friend talked to every other friend at least once. What is the fewest phone calls that could have been made?
- 

Name

---

- 10) Juanita presented a problem to Karl. "If you can solve this," said Juanita, "I'll buy you the ice-cream cone of your choice! Here's the problem: Show how one half of five is four." Karl got his ice-cream cone. What was his answer?
- 

Answer Key

- |                                 |   |
|---------------------------------|---|
| 1. Make an Organized List:      | 24  |
| 2. Use Logical Reasoning:       | Martin is across from Bill. Phil is on Bill's right and is across from Ann.                                     |
| 3. Use or Make a Table:         | 4   |
| 4. Make a Picture or a Diagram: | west 3 blocks   |
| 5. Guess and Check:             | Joy 9, Heidi 18.<br>Saul 37   |
| 6. Use or Look for a Pattern:   | 6   |
| 7. Act Out or Use Objects:      | truck, jeep, van<br>sportscar, 2-door, 4-door<br><br><u>or</u><br>sportscar, 2-door, 4-door<br>truck, jeep, van |
| 8. Work Backwards:              | 40 servings of food   |
| 9. Make It Simpler:             | turkey 3, ham and<br>cheese 16, egg salad<br>23, pastrami 7,<br>submarine 15, tuna 3                            |
| 10. Brainstorm:                 | F <span style="border: 1px solid black; padding: 0 2px;">I V</span> E   |

Appendix O

Sixth Grade Student Pretest Evaluation Form

Do you think this problem is interesting?

interesting       neutral       not interesting

Do you think this problem is easy?

easy       average       difficult

Is this problem the same as the problems in your math textbook?

yes       no

In comparison to the problems in your math textbook, did you like this problem?

yes       no

Have you seen problems like this one before?

yes (Where? \_\_\_\_\_)

no

Can you find the answer to this problem in another way?

yes       no

If so, what method would you use?

Can you explain in words how you solved this problem?

yes       no

If so, please describe the reasoning you used when you solved the problem.

Appendix P  
Student Reflection Sheet

Names \_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

Date \_\_\_\_\_

MATHEMATICS GROUP REFLECTION

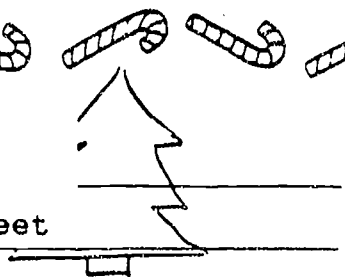
1. What things did your group do well?
2. What things could your group have done better?
3. What did you learn from the group presentations?
4. What was the purpose of doing this math activity?

5. What things did you learn from this math activity?

6. Did you enjoy this activity? Why or why not?

Appendix Q

Candy Bar Math Activity Sheet



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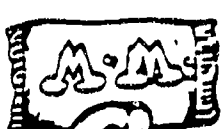
- ① Average the prices of all of the candy bars in your group.
- ② How much was your candy bar per ounce?
- ③ Make a graph of your candy bar ingredients for your entire group.
- ④ How much tax would be charged to you if you bought all of the candy bars in your group?
- ⑤ What is the ratio of plain candy bars to those with nuts?



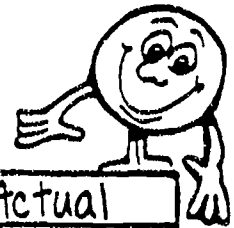
- ⑥ What is the ratio of cookie type candy bars to regular bars?
- ⑦ What percent of the candy bars in your group have caramel?
- ⑧ Have your group make up one division, multiplication, addition, and subtraction problem using the different candy bars in the group.

Name \_\_\_\_\_





# Count & Crunch



1. How many M&M's are in your package?...
2. What is the most common color?...
3. What is the least common color?...
4. What is the class favorite color?...
5. What is your favorite color?.....
6. How much does your package weigh?...
7. How much does 1 M&M weigh?.....
8. How much does your package cost?...
9. How much does 1 M&M cost?.....

Prediction	Actual
1.	
2.	
3.	
4.	
5.	
6.	
7.	
8.	
9.	

Fill in the Prediction part of the Data Table below. Next... open your package and count the number of candies. Fill in the Actual part of both tables.

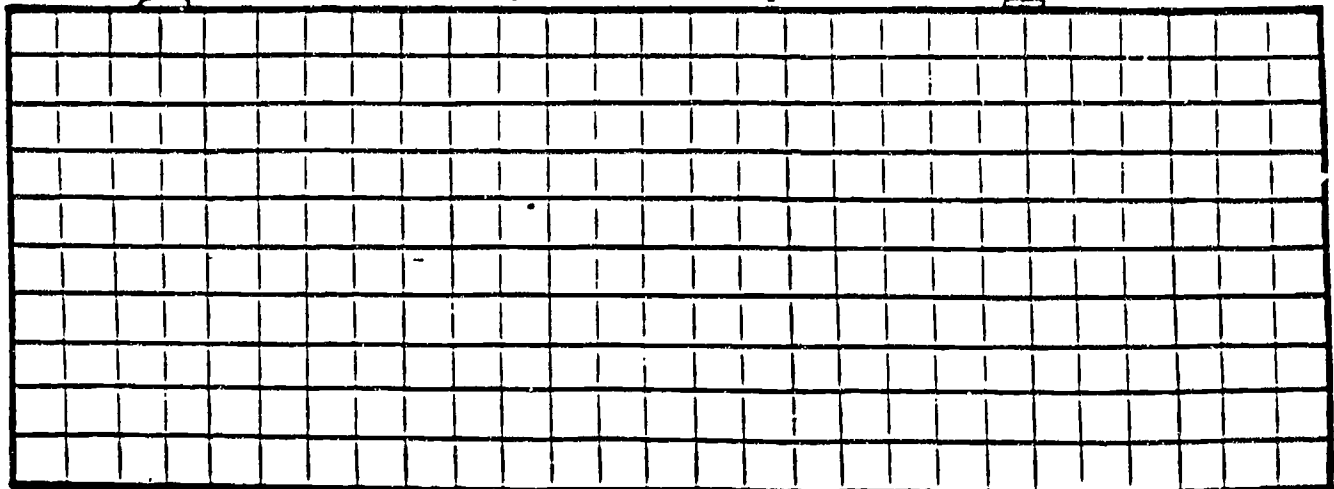


M&M Data Table

Total # of M&M's	GREEN		ORANGE		YELLOW		LT. BROWN		DK. BROWN		RED	
	#	%	#	%	#	%	#	%	#	%	#	%
Prediction												
Actual												



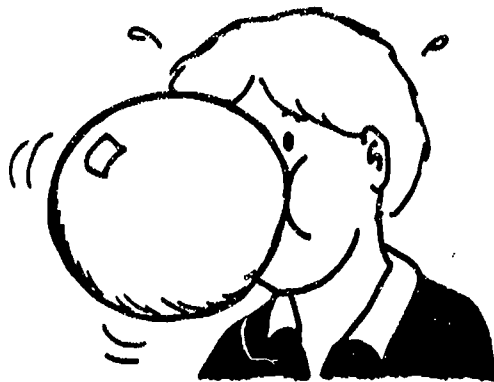
Design a graph to display your results.



## Are You Full of Hot Air?

How much air do you exhale with each breath? There are machines which measure the volume of air exhaled, but using a balloon can give a rough estimate.

Decide how to measure the circumference (distance around) of a round balloon. Discuss it with your group and write down your method.



To measure the amount you exhale, blow into your balloon with *one* breath. Hold the stem of the balloon closed while another group member measures it. Take turns within your group blowing up your own balloons and measuring them.

My balloon measured \_\_\_\_\_ around with one breath.

If you blow into the same balloon again, do you think the size of the balloon will be larger, smaller or the same? \_\_\_\_\_

Why? \_\_\_\_\_

Empty your balloon. Blow *one* breath into your balloon again. Was your prediction correct? \_\_\_\_\_

My balloon measured \_\_\_\_\_ around on the second try.

Repeat this procedure three more times. On the fifth time, tie off your balloon.

third measurement \_\_\_\_\_ fourth measurement \_\_\_\_\_ fifth measurement \_\_\_\_\_

Circle the trial that was the largest. Do you think this measurement gave the clearest idea of how much your lungs can hold? Why or why not? \_\_\_\_\_



## Are You Full of Hot Air?

To find out how much air you blew into the balloon, use the balloon you filled on the fifth trial. The air inside the balloon can be measured. A balloon is considered a sphere like any other ball. Because a sphere is three-dimensional, it is measured in cubic units.

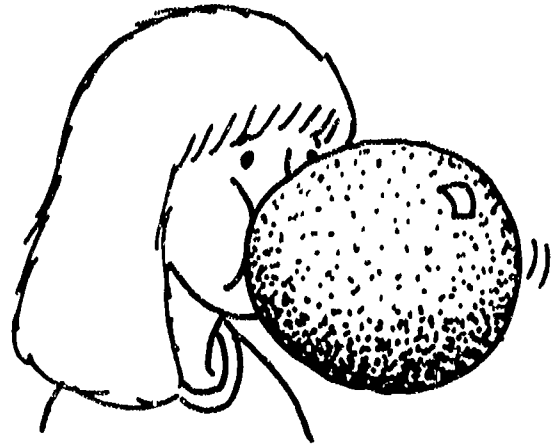
The formula used to find the volume of a sphere is:  $\frac{4}{3} \times \pi r^3$

Pi ( $\pi$ ) is the ratio of the circumference to the diameter of a circle. It is an irrational number, but for practical purposes it is rounded to 3.14.

The *radius of a circle* is represented by the letter  $r$ . The radius is the distance from the center to the circumference of a circle. It is half the length of the diameter.

*Three* as an exponent is the symbol for "cubed." It means that the number below it, called the *base*, is multiplied by itself three times. In this case, the base is the radius of the circle, so  $r^3$  means  $r \times r \times r$ .

You know the *circumference* of your largest balloon. You know the formula for the volume of a sphere. With your group, make a plan to figure out how to find the volume of your balloon with the information you have. (Hint: The formula for the circumference of a circle is  $C = \pi \times d$  when  $d$  means diameter.) Write your plan below.



Now carry out your plan.

The volume of my balloon is \_\_\_\_\_ cubic units.

Does this seem reasonable to you? \_\_\_\_\_

Why or why not?

**Extension:** Make a box that has close to the same volume as your balloon.

Name \_\_\_\_\_ Group Members \_\_\_\_\_

Date \_\_\_\_\_

### Balloon Math Reflection

1. What was the purpose of this lesson?
  
2. a) Did you understand this lesson?  
b) If you did not understand it, list the reasons why.
  
3. a) Did you enjoy this lesson?  
b) Why or why not?
  
4. What did you learn from the lesson?
  
5. What did your group do well today?

\*\*\*TURN OVER\*\*\*

304  
314

6. What could your group have done better today?

7. a) Do you like learning math this way?

b) Why or why not?

Mean, Mode, Median, and Range  
Activity and Reflection Sheets

We are going to find out a few things about the "average" student in this class. To do this, we need the following information about you. Work with your partner to measure your head circumference, armspan and resting pulse rate. The data will be shared with your classmates. If you are curious about other things, ask your teacher if they can be surveyed.

**Head circumference** \_\_\_\_\_

**Armspan** (fingertip to fingertip) \_\_\_\_\_

**Resting pulse rate** \_\_\_\_\_

**Shoe size** (check one)  Female  Male \_\_\_\_\_

**Vehicles in my household:**

number \_\_\_\_\_

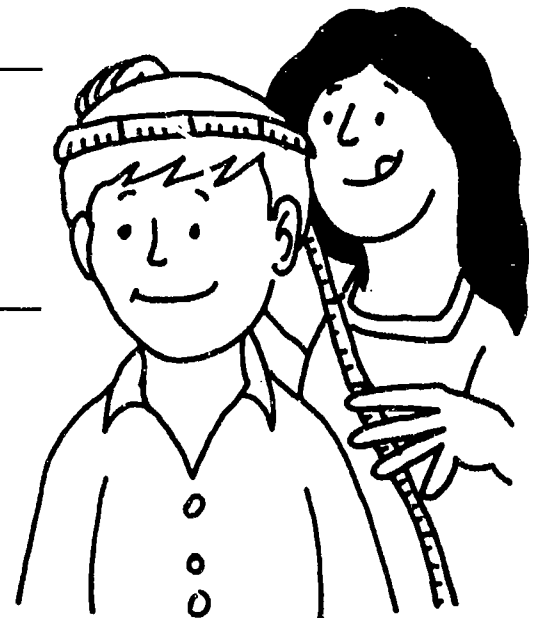
types \_\_\_\_\_

**Allowance/week** \_\_\_\_\_

**Pets in my household**

number \_\_\_\_\_

types \_\_\_\_\_



# Statistical Words

## (Mean, Median, Mode, Range)

---

When we hear about statistics, we usually hear about averages, but there are ways other than finding an average to look at data collected. Statisticians (people who work with statistics) have a number of different methods of finding what is typical from the data with which they are working. You will be able to find these measures using the data you have collected about your class.

**Mean:** The *mean* is the same as finding the average of a group. To get the mean, add up all of the items in your collection of data; then divide by the number of items you have.

**Median:** The *median* means the middle number. To find the median, organize your pieces of data from smallest to largest and find the one exactly in the middle. If you have an even number of items, you will have to find the number that comes exactly between the two middle numbers.

**Mode:** The *mode* means the number which occurs the most often. When you organize your data, the number that occurs the most times is the mode. There can be more than one mode.

**Range:** The *range* is the difference between the largest and the smallest items in your collection of data.

To see how to find these measures, look at the following problem.

Greg and 10 friends decided to have a frog jumping contest.

Their frogs jumped the following distances: 29", 12", 23", 31", 32", 23", 30", 17", 19" and 34".

**Mean:** Add the distances

$$(29" + 12" + 23" + 31" + 32" + 23" + 30" + 17" + 19" + 34" = 250).$$

Divide the sum by the number of frogs

$$(250 \div 10 = 25").$$

**Median:** Put the numbers in order from shortest to longest length jumped, then find the number in the middle.

12, 17, 19, 23, 23, 29, 30, 31, 32, 34

The two middle numbers are 23 and 29, so the median is 26".

**Mode:** The number that shows up most is 23". More frogs jumped 23" than any other length, but this is shorter than the average.

**Range:** The distances range from 12" to 34". There is a difference of 22" between the longest jump and the shortest jump.

Your Name \_\_\_\_\_

Other Group Members

\_\_\_\_\_

\_\_\_\_\_

\_\_\_\_\_

Date \_\_\_\_\_

" About Me "  
Reflection

1. What did your group do well?
  
  
  
  
  
  
  
  
  
  
2. What could your group have done better?
  
  
  
  
  
  
  
  
  
  
3. a) Did you like this activity?  
b) Why or why not?

\*\*\*Turn Over\*\*\*

4. What did you learn today?

5. What strategies did you use to solve this problem?

Appendix U

Basketball Mean, Median, Mode, and Range  
Activity and Reflection Sheets

Name \_\_\_\_\_

Date \_\_\_\_\_

**Basketball Math**

1. What was the ratio of your shots made to your shots attempted?
2. What was your percent of accuracy?
3.
  - a) What was the ratio of the girls' shots made to the shots attempted?
  - b) Ratio for the boys?
  - c) Ratio for the class?
4.
  - a) What was the girls' percent of accuracy?
  - b) The boys' percent?
  - c) The class's percent?
5.
  - a) What was the mean for the girls?
  - b) The boys?
  - c) The class?
6.
  - a) What was the median for the girls?
  - b) The boys?
  - c) The class?



7.
  - a) What was the mode for the girls?
  - b) The boys?
  - c) The class?
8.
  - a) What was the range for the girls?
  - b) The boys?
  - c) The class?
9. Graph the percentages of accuracy for the girls, the boys, and the class.
10. Graph the data on the mean, median, mode, and range for the girls, boys, and class.
11. How did your personal score compare to the data collected on the percent, mean, median, mode, and range for the class?

Your Name \_\_\_\_\_ Group Members \_\_\_\_\_

Date \_\_\_\_\_

### Basketball Math Reflection

1. What was the purpose of this lesson?
2. What did you learn today?
3. Did you enjoy this lesson? Why or why not?
4. Why is it important to learn this information?
5. Do you like graphing? Why or why not?
6. Why is it important to be able to create and read graphs?

**\*\*\*Turn Over\*\*\***

7. What did your group do well today?

8. How could your group have done better?

9. What manipulatives were used for this math activity?

10. What mathematical operations and strategies did you use in this lesson?

Appendix V

Gummi Worm Metric Math Activity  
and Reflection Sheets

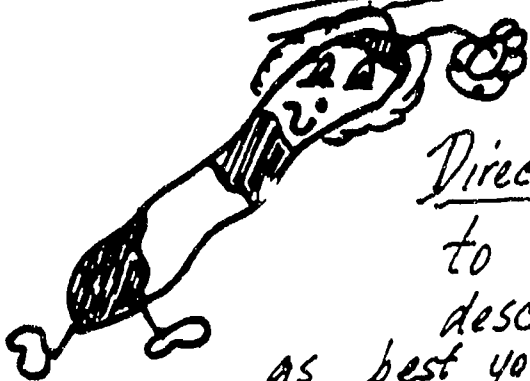
Name \_\_\_\_\_

Date \_\_\_\_\_

GUMMI WORMS

1.
  - a) How many centimeters long is your gummi worm?
  - b) How many millimeters long is it?
  - c) What part of a meter is it?
  
2.
  - a) What is the combined length of the gummi worms in your group when measured in centimeters?
  - b) How many millimeters is this length equal to?
  - c) What part of a meter is this measurement?
  
3.
  - a) What is the length in centimeters of your gummi worm when you stretch it to its full capacity?
  - b) How many millimeters is this?
  - c) What part of a meter is this equal to?
  - \* d) What part of a kilometer is this equal to?
  
4. Create a problem of your own using metric measurements which need to be converted from one unit of measurement to another. Try to incorporate your gummi worm into the problem.

# GUMMI WORMS



Directions: Use your gummi worm to answer these questions. Be sure to describe your specimen's characteristics as best you can. Measure to the nearest mm.

Colors	Shape	Smell	Texture	Thickness	Length

Measure the length of each section.

Length	$\frac{\text{part}}{\text{whole}}$ Ratio	Percent
Section 1		
Section 2		
Section 3		
Section 4		

In the space below make an accurate, full size drawing of your worm. Be sure to color it in. Write a paragraph about your gummi worm.

Name \_\_\_\_\_ Group Members \_\_\_\_\_

Problem \_\_\_\_\_ Date \_\_\_\_\_

### Metric Measurement Reflection

1. What was the purpose of this lesson?
2. What did you learn from this lesson?
3. What did your group do well during this lesson?
4. What could your group have done better in this lesson?
5. What manipulatives did you use in this lesson?
6. When might you use these manipulatives in everyday life?
7. Did you enjoy this activity? Why or why not?

Appendix W

Sixth Grade Posttest

Name

---

- 1) **28** Steven is leaning over the video game, concentrating on shooting down as many spaceships as he can within the time limit. Vulcan spaceships are worth 20 points each, and Android spaceships are worth 25 points. If he shoots down 21 spaceships for a total of 465 points, how many Vulcan spaceships and how many Android spaceships did Steven shoot down?
- 

Name

---

- 2) **36** It is 7:00 P.M. on Back-to-School Night. Mrs. Anderson is welcoming the parents in her first class: Mr. Black, Mr. Green, Mr. White, Mrs. Brown, and Mrs. Rojo. She sees five of her students: Peter, Mary, Jack, Sam, and Jill. Mrs. Anderson notices that Mr. Green's daughter did not inherit his freckles; Mrs. Brown has big dimples when she smiles; Mary and Sam both have freckles and dimples; Mr. Black's son looks just like him; and Sam's father and Peter's father were unable to attend. Can you match up each student with a parent?
- 

Name

---

- 3) **24** The Rialto Theater is celebrating its 11th anniversary. In honor of the occasion, they are giving away free passes! They have hidden a gold, silver, purple, or green star under every seat. Every person who sits in a seat with a gold star gets a free pass to the next show. For every 2 gold stars they hid 18 silver, 16 purple, and 12 green stars. If there are 384 seats in the Rialto Theater, how many people won free passes?
- 

Name

---

- 4) **59** Helga is trying to decide which way to go to visit her friend Marisa. There are 6 different streets that lead to her friend's apartment building. Then there are 3 stairways up from the street, where Helga can go into the building through 2 different doors. Once inside she can choose from 2 inside stairways or 3 elevators to get to the third floor where Marisa's apartment is. How many different ways can Helga go to visit her friend?
-

Name

---

- 5) **106** Ryan was working at the summer recreation program. His first job each morning was to lock up the bicycle cage after all the little kids arrived. He entertained himself by counting the number of wheels in the cage each day. Today, Ryan counted a total of 31 vehicles and a total of 84 wheels. If some of the vehicles were Big Wheels, some were bicycles, and some were wagons, how many of each type of vehicle were in the cage?
- 

Name

---

- 6) **58** Willy the Wizard has a special hobby, growing huge vegetables. His favorite is zucchini, which he likes to measure. He has one that started out at 6 inches, then grew by 9 inches to be 15 inches on the second day. Each day the zucchini grew by the same amount as the day before plus another 3 inches. On what day would the giant zucchini be longer than 11 feet?
- 

Name

---

- 7) **64** The Bickerton family is leaving for Yosemite Park in their van. Father Bickerton is driving, with Mother Bickerton up front next to him. On the two bench-style seats in the back of the van are the rest of the family: Grandmother, Brad, his two sisters Bernice and Betty, the baby, and Bow-Wow the dog. Grandmother is next to the window, because she gets carsick. She also likes to sit next to Bow-Wow. The baby is to the right of Brad and not far from Mother. About halfway there, Brad complains that Betty is kicking his seat! Where is each Bickerton sitting in the van?
- 

Name

---

- 8) **91** The Gadfly Gazette is published every day, rain or shine. Marion helps her sister Janet to get the paper delivered. Marion is on a schedule that includes folding, delivering, and collecting. Every 6th day she goes collecting, every 3rd day she delivers the paper, and every 4th day she folds the papers. If she helped Janet for 12 weeks, how many times did Marion do all three jobs in the same day?
-



Name

- 9) **84** Carla is excited because her big brother Joe has just come back from six weeks of fishing for salmon in Alaska. Joe earned money in bonuses, and has promised Carla \$10 if she can figure out how much he made. Each time Joe caught \$500 worth of salmon, he got a bonus. The first time he received a \$10 bonus. The second time, he got a \$30 bonus. The third time he received \$50, and the fourth time he received \$70. During the time Joe spent fishing, he caught \$500 worth of salmon on 21 days. If the bonuses continued at the same rate, how much bonus money did Joe make?

Name

- 10) **102** "Did you get the job, did you get the job?" asks Brian. His brother Jim just applied for a job parking cars at the largest hotel in the city. Jim hands Brian the written test he had to take in order to qualify for the job. "Yes," he says, "I got the job. But I had to answer this question first: How can you park twenty cars in these ten stalls without doubling up in any of the stalls?" Brian studied the question and finally came up with the solution. Can you?

Answer Key

1. Make an Organized List: 12 Vulcan, 9 Android
2. Use Logical Reasoning: Mr. Black - Jack, Mr. Green - Jill, Mrs. White - Mary, Mrs. Brown - Sam, Mrs. Rojo - Peter
3. Use or Make a Table: 16
4. Make a Picture or a Diagram: 180
5. Guess and Check: 5 wagons, 12 Big Wheels, 14 bicycles
6. Use or Look for a Pattern: 9th day
7. Act Out or Use Objects: Father                      Mother  
Brad                      Baby                      Bernice  
Betty                      Bow-Wow                      Grandmother
8. Work Backwards: 7 times
9. Make It Simpler: \$4,410
10. Brainstorm: 

T	W	E	N	T	Y	C	A	R	S
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Appendix X

Sixth Grade Posttest Reflection Sheet

Name \_\_\_\_\_

Date \_\_\_\_\_

Problem # \_\_\_\_\_

Reflection

1. Was this problem easy, average, or difficult for you?
2. Were you able to solve this problem? If so, how did you solve it? What strategy did you use?

3. Did you like this problem? Why or why not?

4. How would you rate your ability to solve this type of problem in comparison to your ability in the fall?

\_\_\_\_\_ extremely improved

\_\_\_\_\_ the same (no change)

\_\_\_\_\_ improved

\_\_\_\_\_ worse

\_\_\_\_\_ somewhat improved

Appendix Y

Sixth Grade Pretest and Posttest Data

Number of Problems Attempted and Number of Correct Solutions

Student#	Pretest		Posttest	
	#Problems Attempted	#Correct	#Problems Attempted	#Correct
1	4	0	10	1
2	4	0	10	1
3	7	1	9	2
4	7	0	10	2
5	7	1	10	3
6	8	4	10	7
7	5	0	10	0
8	8	3	8	3
9	5	0	9	3
10	3	0	8	0
11	5	0	10	2
12	7	0	10	0
13	4	1	8	4
14	2	0	3	0
15	6	0	10	1
16	7	2	10	4
17	10	3	10	7
18	5	0	10	1
19	4	0	10	4
20	4	0	10	2
21	2	0	10	0
22	7	1	10	6
23	8	4	10	7
24	9	0	10	3
25	6	0	10	2
26	0	0	10	4
27	2	0	10	1