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ABSTRACT

This report describes an intervention designed to assist students in creating and accepting divergent solutions to problems in mathematics. The targeted population consisted of fifth-grade students from a multicultural setting with varied economic backgrounds, and the sample included 171 fifth graders in two school districts. Underdeveloped critical thinking was documented by teacher observation, tests, and student surveys. Analysis of probable cause data revealed that students lacked adequate strategies for problem solving, lacked confidence in their math ability, and were unable to communicate their thinking. Reviews of curricular content and previous instructional methods revealed an overemphasis on basic skills and computation, with minimal attention to higher-order thinking and problem solving. One major intervention was selected, a change in the instructional approach with three essential characteristics: (1) Students were taught the steps to use when critically thinking about problem solving; (2) Manipulatives were used to enhance instruction; and (3) Metacognitive abilities were developed through writing about math skills. Results showed that students demonstrated improved attitudes about their mathematical ability and increased ability to write about their thinking. Appendices contain an autobiography sample, a problem solving pre-test, teacher survey, three-story intellect verbs, student activity samples, and a problem-solving post-test. (Contains 17 references.) (Author/MKR)

IMPROVING CRITICAL THINKING AND PROBLEM SOLVING IN FIFTH GRADE MATHEMATICS

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AN ACTION RESEARCH PROJECT SUBMITTED TO THE GRADUATE FACULTY OF THE SCHOOL OF EDUCATION IN PARTIAL FULFILLMENT OF HE REQUIREMENTS FOR THE DEGREE OF MASTERS OF TEACHING AND LEADERSHIP

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Table of Contents

ABSTRACT

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DATE: September 1993

TITLE: Improving Critical Thinking and Problem Solving in Fifth Grade Mathematics

ABSTRACT: This program describes an intervention designed to assist students in creating and accepting divergent solutions to problems in mathematics. The targeted population consisted of fifth grade students from a multi-cultural setting with varied economic backgrounds in a city in northern Illinois, and in a growing upper middle class suburban community west of Chicago. The underdeveloped critical thinking, related to problem solving, was documented by teacher observation and tests, and student surveys which revealed the need for instruction in critical thinking strategies and the development of metacognitive abilities.

Analysis of the probable cause data revealed that students lacked adequate strategies for problem solving, lacked confidence in their math ability, and were unable to communicate their thinking. Reviews of curricula content and previous instructional methods revealed an over-emphasis on basic skills and computation with minimal attention to higher-order thinking and problem solving.

Solution strategies suggested by knowledgeable others combined with an analysis of the problem setting resulted in the selection of one major intervention; a change in the instructional approach with three essential characteristics. Students will be taught the steps to use when critically thinking about problem solving, manipulatives will be used to enhance instruction, and metacognitive abilities will be developed through writing about math skills.

Chapter 1

PROBLEM STATEMENT AND COMMUNITY BACKGROUND

General Statement of Problem

Fifth grade students' abilities to use critical thinking to understand that there is more than one solution to a mathematical problem are inadequately developed as evidenced by teacher evaluation, pretest, and by student survey.

Immediate Problem Context

Illinois Park School is a public school in a unit district in Elgin, Illinois, with 425 students in kindergarten through sixth grade. The staff consists of one principal, 15 classroom teachers, a secretary, two teacher clerical assistants, one Chapter I resource teacher, one Spanish liaison, and one learning disabilities resource teacher. The speech and language therapist, nurse, and social worker serve the school on an itinerant basis. Art, music, and physical education are taught by specialists once a week.

There are two sections of each grade, with an additional Spanish bilingual classroom at fourth and fifth grades. Twenty percent of the student population is Limited-English-Proficient (LEP). By definition, these are students who have been tested and found to be eligible for bilingual education.

The student population is ethnically and racially mixed, with 62.8 percent of the population white, 20.9 percent Hispanic, 11.7 percent Black, and 4.6 percent Asian/Pacific Islander. Twenty-three students are serviced by the learning disability resource teacher. Thirty-two students receive Chapter I reading support, and 14 students receive Chapter I mathematics from a half-time teacher. Students are on the waiting list for these services.

Family socio-economic status covers a wide range, from upper-middle class to families on public aid. Twenty-six point eight percent of the students receive free and reduced lunch. Eleven percent of the students are bused from outside the neighborhood. Many of the students live in apartment complexes and low income housing areas. The mobility rate of the student population is 35.4 percent. The attendance record for the school is 95.9 percent. A statistical computer analysis indicates more than a year's growth in mathematics in both years of a two year testing project for students who stayed in the school (School Report Card, 1992).

The fifth grade students in this study are representative of the school population. The 72 students are assigned to three homerooms. Two classrooms are heterogeneous English speaking students, while the third classroom is Spanish bilingual transition students with a bilingual teacher. For mathematics instruction the 72 students are placed in three classes according to mathematics achievement based on a math pretest inventory given in September, and on their fourth grade teacher's recommendation. This study was done with 28 of the students receiving highest scores and recommendations in mathematics. Six of the mathematics students are Spanish bilingual transitional students mainstreamed for daily mathematics instruction.

The team of three fifth grade teachers work closely together evaluating student progress and developing an integrated curriculum for the classes to facilitate transitioning of the bilingual students. Materials, manipulatives, books, and equipment are shared. Release time during the day is scheduled to provide common planning periods for the three teachers. Additional planning sessions take place after school hours on a regular basis.

Illinois Park School was built in 1907, with additions built in 1954 and reconditioning in 1988. It is a four-story brick structure with restrooms, office, and library facilities at ground level. Fifth grade classrooms are on the top floor. The school is located at the intersection of two busy streets across from a shopping center. A fence and narrow blacktop separate the building from the four-lane street. Noise from the street requires that windows remain closed during instruction. There is no air-conditioning nor cooling system in the building. In spite of difficult physical conditions, a positive attitude exists within the school.

The Surrounding Community

Illinois Park School is located in Elgin, Illinois, in a city of 77,000 in the Fox River Valley 38 miles west of downtown Chicago. The community has undergone changes in the past decade. The population of the city is 70 percent white, 7.1 percent Black, 18.9 percent Hispanic, three percent Asian/Pacific Islander, and 0.3 percent other. An investigation into the population characteristics shows a decline of 10.6 percent in the white population, an increase of 0.6 percent Black, 8.7 percent increase in Hispanic, and a 2.2 percent increase in the Asian/Pacific Islander population since the previous census. Population projections are for 99,755 people by 2010. The unemployment rate was 5.9 percent in 1990. The per capita income in 1987 was $12,060 which was lower than the five surrounding communities (School Report Card, 1992).

In addition to the 44 public schools, Elgin has one parochial high school, eight parish elementary schools and three private schools. Elgin Community College is a fully accredited two-year public community college serving 20,000 students per year. Judson College, a private Christian four year college, and National Louis University also serve the community.

School district U-46 is the second largest in Illinois with 1992 enrollment of 29,559 students. It services students from ten communities covering an area of 90 square miles.

There are three high schools, six two-year middle schools, 35 elementary schools, one special education school, and one alternative high school. The district employs 1700 teachers, 130 administrators, and 1440 full-time and part-time support staff. The student population is representative of the cultural composition of the city. More than 60 different native languages were noted in a recent U-46 bilingual census (School Report Card, 1992). District U-46 scores well ahead of national norms in most categories on nationally standardized achievement tests. Approximately 60 percent of U-46 graduates pursue higher education (School Report Card, 1992).

Educational policies for the district are determined by an elected school board that meets biweekly to gather public opinion, to discuss financial matters, curriculum, educational policies, staffing and facilities. The district anticipates a deficit in excess of twenty-four million dollars in the fiscal year 1994. The district is administered by an appointed superintendent of schools, whose central office staff includes: three area superintendents, an assistant superintendent of finance, an assistant superintendent of human resources, a director of curriculum, a director of instructional programs, and a director of special education.

The district does not have a favorable history of passing school referendums. Labor relations have been an area of concern for many years. U-46 is the largest Illinois

Teacher Association affiliate in the state. The district has had seven teachers strikes in 21 years. It could be said that recent increased community concern has resulted in an effort on the part of the district to make changes to improve community-school relations.

The second target school, Wild Rose School, is a public school located in St. Charles, IL, with 620 students in kindergarten through fifth grade. Most students are bused while a few live in the neighboring areas. Wild Rose staff consists of one principal, twenty-six full time classroom teachers, two secretaries, three custodians, and one teachers' aide. The speech pathologist, nurse, social worker, and psychologist serve the school on an itinerant basis. Art, music, and physical education are taught by specialists at least once a week. There are four sections of each grade with one primary and one intermediate self-contained Learning Disabilities classroom.

Wild Rose student population is 98.9 percent White, 0.7 percent Hispanic, 0.0 percent African American, and 0.5 Asian. Family socioeconomic status is middle to upper-middle class. The attendance record for 1990-1991 was 96.0 percent, no chronic truants (School Report Card, 1992).

The fifth grade students in this study are representative of the school population. The 99 students are assigned to four heterogeneous homerooms, where they remain for mathematics instruction. Students who test in the top one percent on the Stanford Achievement Test are assigned to a gifted counselor from whom they receive their mathematics instruction. This study was done with 24 of the students in the regular classroom.

The four fifth grade teachers work very closely together as a team to develop, integrate, and evaluate the curriculum offered to the students. The students have homeroom in the morning where they receive language arts and mathematics instruction. The afternoon sessions are rotated between the four teachers where a hands-on science, social studies, writing, and health program is taught. The teachers meet regularly once a week to share ideas, materials and concerns, as well as to set up the academic program and goals for the upcoming units of instruction.

Wild Rose was built in 1967 with additions in 1978. It is a bi-level brick structure located in the Wild Rose subdivision on Red Haw Lane. On the first level there is an office, teachers' lounge, two restrooms, teachers' workroom, boiler room, kindergarten and the Learning Resource Center. The building has two main wings on the ground floor. The south wing contains the primary grades and the north wing is for intermediate grades. In each of these wings there is a community space with restrooms for the children.

The Surrounding Community

Wild Rose is located in Community District 303 in St. Charles, Illinois, which covers 57 square miles and currently serves over 8,700 students who reside in the city of

St. Charles, Wasco, unincorporated St. Charles Township, a portion of Campton Township, the westernmost portion of unincorporated DuPage County, and portions of South Elgin, Plato township, and the village of Wayne. Approximately 50 percent of the present student enrollment comes from the city of St. Charles, with the other half living outside the city, but within the district.

Between 1950 and 1970 St. Charles and St. Charles Township experienced rapid growth; the population increasing by 100 percent. There was strong growth into the 1970's and through the 1980's. The final 1990 census count for the city of St. Charles was 22,501 and for St. Charles Township 33,112. Enrollment in District 303 presented a mirrored image of new housing trends and population increase. Enrollment mushroomed during the 1960's and the 1970's with the total number of K-12 students expanding from 3,191 in 1960 to 5,451 in 1970 and 6,902 in 1980. Due to the recession in 1980 and a drop in new single family housing construction, there was a decline in K-5 enrollment during the first part of the 1980's. Since that year total enrollment has increased steadily reaching 8,768 (including special education) in 1992-1993 (Kasarda, 1992).

On the District 303 State Report Card, St. Charles scores well above the norm in all academic categories on nationally standardized tests. Eight-five to eighty-seven percent of District 303 graduates go on to higher education (School Report Card, 1992).

Policies for the district are determined by an elected school board of six members that meet once a month to obtain public opinion, decide on financial matters, curriculum, educational policies, staffing and facilities. The district is administered by an appointed Superintendent of Schools, Assistant Superintendent for Curriculum and Instruction, and Assistant Superintendent for Support Personnel (School Report Card, 1992). Due to the growth and overcrowding in the St. Charles Schools, a referendum in the amount of $41,000,000 was passed in the Spring of 1993 to assist in building a new high school, one junior high, and several additions to existing elementary buildings.

State and National Context of Problem

The educational goals of both Illinois and the nation emphasize the need for critical thinking and problem solving skills. Illinois State Goal I states that "... each student will demonstrate the ability to solve problems and perform tasks requiring higher-order thinking skills, and be prepared to succeed in our diverse society and the global work force." The first of the stated National Goals says that "...by the year 2000, U.S. students will be first in the world in science and mathematics achievement (State Goals, 1992).

Students' underdeveloped abilities to use metacognitive or higher-order thinking skills in relation to mathematics has had much attention on the state and national level. The scope of this problem is growing and is an area of great concern. The Conference of Governors of the United States in 1990 noted that the educational system of the U.S. ranked last in mathematics and science when compared to other industrialized nations. The conference set a goal of becoming number one educationally by the year 2000. For this to happen, significant changes need to take place (Mitchell, 1991).

A Nation at Risk concludes that society's future depends on a citizenry that can "...think and reason creatively and deliberately..."(National Commission on Excellence in Education, 1983, p. 487). The study urged schools to develop curricula that emphasizes higher-level thinking skills. New technology and the need to learn quickly and effectively require that learners become instrumental in their own learning (Lippert, 1987).

In 1988 National Council of Teachers of Mathematics (NCTM) expanded its goals promoting problem solving as a curricular focus. It showed a shift from emphasis on rules and routine problem solving dominated by teacher talk and passive learning, to active student participation in which reasoning and communication are stressed (Lippert, 1987).

There appears to be agreement among educational researchers about the importance of critical thinking and the need for redirecting the emphasis of traditional mathematics instruction toward problem solving that applies to real-life situations. Teachers must begin teaching mathematics on a thinking level rather than a as a "drill and review routine." Researchers have concluded that many mathematics programs focus too much on the development of routine computation skills and too little on the mathematics concept.

> Students need to learn for more than basic skills. Children who have just started school may be in the labor force in the year 2030;.. To be practical, an education should prepare man for work that does not yet exist and whose nature cannot even be imagined (Fogarty and Bellanca, 1991, p. 9).

As many educators and authors agree, this can only be done by teaching students how to learn and how to apply and transfer their learning to real-life problems.

A number of studies reveal critical weaknesses in ways educators teach mathematics. Instruction is often textbook driven, with an emphasis on drill. Only 14 percent of the time is devoted to concept development (Good & Grouws, 1987). Educators often assume that academic content at each level of material has to be mastered before a student moves to the next level. It is a misconception that a child needs to memorize all the facts before problem-solving can be introduced (Chancellor, 1991). "Too often we give children answers to remember, rather than problems to solve" (Fogarty & Bellanca, 1991, p. 9).

Another concern about the traditional mathematics instruction is that teachers expect children to learn and solve problems, yet they seldom teach them about their learning, about how they think, and how they know the answers. Even when students are able to do problem solving they are not inclined to communicate their thinking unless this skill of metacognition is taught and practiced.

Educational literature states the concern about problem solving and critical thinking. Yet researchers are still attempting to define thinking and the nature of problem solving (Szetela & Nicol, 1992). The difficulty of assessing students' abilities to solve problems is complicated by their inability to communicate clearly what they have done or what they are thinking. Success in problem solving depends on metacognitive processes. These are difficult to measure, but assessment can be improved by creating problem situations that facilitate student thinking and communication of their thinking. If we can devise methods for improving communication of students' thinking, we can perform more effective assessment of thinking (Szetela & Nicol, 1992).

Traditionally the higher-order thinking skills have been reserved exclusively for students who were identified as gifted and talented. These misconceptions may come from misinterpretation of Bloom's Taxonomy (1956). For years educators have promoted the idea that students must master academic content at each level before they move on to the next one. As a result, the at-risk students might remain at the knowledge level until they memorize their facts. Yet for many of these students, memorization is their greatest weakness. Although among experts the definition of what we call thinking varies greatly, educators seem to agree that changes in the traditional instruction of mathematics is overdue. As educators continue to define thinking and problem solving, significant changes are occurring, with an increased emphasis on problem solving in text books and in the staff development of teachers. There is a direction toward the integration of curriculum and critical thinking skills. Lippert summarized this new theme:

> If you want students to desire knowledge,
> then give them a reason.
> If you want students to understand content,
> then define concepts.
> If you want a student to see usefulness,
> then let them try it.
> If you want understanding to lead to higher-level questioning,
> then let them create the questions
> (Lippert, 1991, p. 49).

Kleiman summed up the need for change in the philosophy of mathematics instruction by saying, "If we taught music as we teach mathematics, students would practice musical scales for years without ever getting to play a song" (Kleiman, 1991, p. 48).

Chapter 2

PROBLEM EVIDENCE AND PROBABLE CAUSE

Problem Background

As pointed out in Chapter 1, the educational goals of both Illinois and the nation emphasize the need for critical thinking, problem solving, and higher-order thinking skills to prepare students for the future. Research supports the need for changing traditional teaching methods to include instruction on how to think and solve problems and to encourage students to be able to communicate their knowledge.

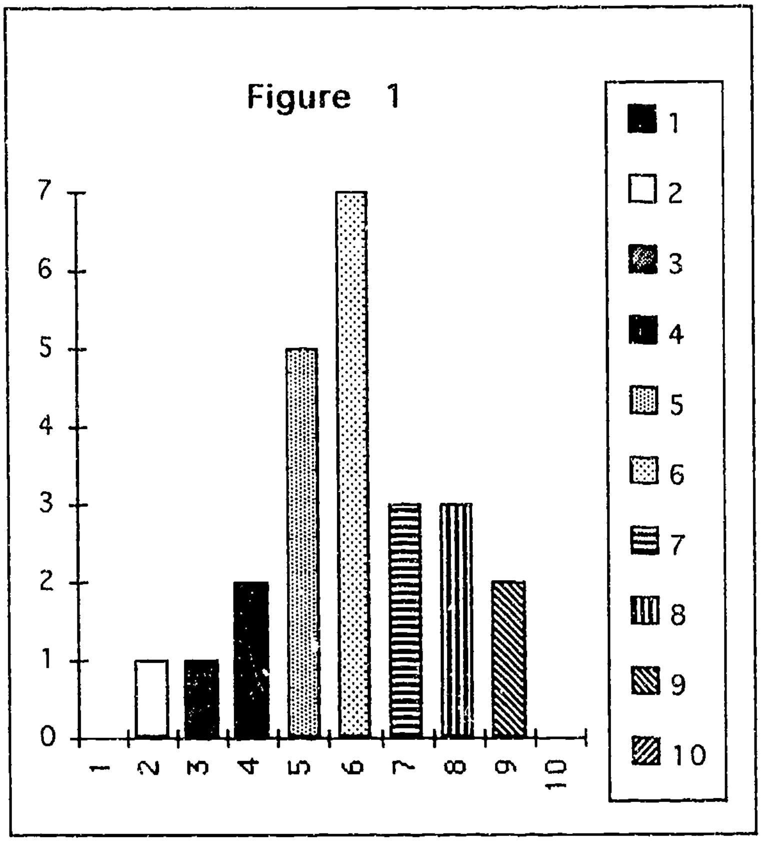
National and state goals have been set to include this emphasis. At the local level the implementation of this instruction depends on the individual teacher's ability to adapt traditional methods of instruction to the changing needs of students.

The adopted text books for elementary mathematics have traditionally placed the emphasis on skills of computation, with minimal attention to higher-order thinking and problem solving. District U-46, which includes Illinois Park School, has adopted a new mathematics series of texts, *Connections,* for the Fall of 1993 (Heath, 1993). This text changes the focus from skills and computation to the process of learning mathematics and problem solving. Wild Rose School in the St. Charles district currently uses *Addison Wesley Mathematics,* which is the more traditional approach to instruction (Addison Wesley, 1989). However, for the period of this study both researchers will be using the Heath materials.

Problem Evidence

Both subjective and objective means were used with the fifth grade students in this study to document the need for revised instruction in critical thinking and problem solving. The first week of school students were asked to write a mathematics autobiography. They wrote about their experiences in learning about mathematics as far back as they could remember. They were to describe their strengths and weaknesses, their likes and dislikes, their fears and apprehensions, and to explain how they think and reason in mathematics. The activity was selected to enable the teacher to assess student attitudes about mathematics, to examine their mathematics history, and to evaluate where to begin instruction. (See Appendix A.) The teachers observed the students' limited experience in writing about mathematics and problem solving.

A written teacher created pre-test was administered at the beginning of the fifth grade to assess students' problem solving and problem writing abilities. (See Appendix B.) The test consisted of four story problems. Students were directed to show their work. The fifth exercise directed students to compose and solve a problem with given data.

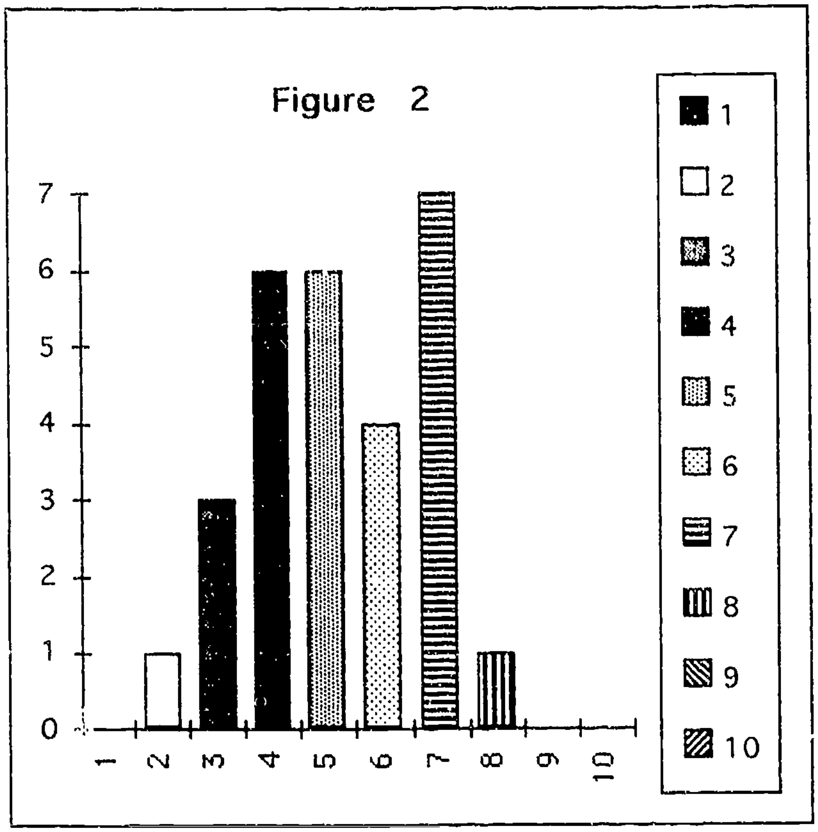


vertical axis = # of students
horizontal axis = # of correct responses

Figure 1

Mathematics Problem Solving Pre-Test Scores
Fifth Grade Wild Rose School

Figure 1 presents data on the problem solving pre-test in the heterogeneous fifth grade mathematics class in Wild Rose School. It should be noted that 15 of the 24 students scored above 50 percent and nine scored below 50 percent. No student scored zero points, and no student scored ten points. Each test questions was assigned two points; one for the answer, and one for showing their thinking.



vertical axis = # of students
horizontal axis = # of correct responses

Figure 2

Mathematics Problem Solving Pre-Test Scores
Fifth Grade Illinois Park School

Figure 2 presents data on the problem solving pre-test in a middle to high ability grouped fifth grade class in Illinois Park School. It should be noted that 12 of the 28 students scored above 50 percent, and 16 students scored below 50 percent. No student scored zero points, and no student scored nine or ten points. Each test question was assigned two points; one for the answer, and one for showing their thinking.

Probable Causes of Problem

Data to indicate probable cause factors was gathered from two sources within the setting. A questionnaire was administered to the third and fourth grade teachers at Illinois Park School and Wild Rose School to obtain information about the fifth grade students' previous type of mathematics instruction. (See Appendix C.)

In both schools the students' previous teachers used little or no writing during mathematics instruction. Manipulatives and cooperative groups were used to some degree by all of the teachers. Calculators were used occasionally for instruction by all the teachers, but not during testing. All of the teachers occasionally used the challenge problem solving assignments.

The fifth grade students in the study completed a metacognitive survey to determine how they approach problem solving and to assess their attitudes about mathematics.

Table 1

PROBLEM SOLVING METACOGNITIVE SURVEY
Given to 24 Wild Rose Students
28 Illinois Park Students ( )
September 1993

NO-No, I didn't do this.
MAYBE-I may have done this.
YES-Yes, I did do this.

BEFORE YOU BEGAN TO SOLVE THE PROBLEM-WHAT DID YOU DO?

		NO	MAYBE	YES
1.	I read the problem more than once.	38%(14%)	0%(21%)	79%(68%)
2	I tried to remember if I had worked a problem like this before.	42%(46%)	13%(21%)	50%(32%)
3.	I thought about what information I needed to solve the problem.	0%(14%)	0%(29%)	92%(57%)

Before beginning to work, the majority of students in both classes read the problem more than once, yet needed more information. Fiftv percent or less tried to remember if they had solved a problem like it before.

AS YOU WORKED THE PROBLEM-WHAT DID YOU DO?

		NO	MAYBE	YES
4.	I thought about all the steps as I worked the problem.	13%(36%)	29%(11%)	63%(54%)
5.	I kept looking back at the problem after I did a step.	25%(18%)	0%(14%)	71%(68%)
6.	I checked my work step by step as I worked the problem.	29%(29%)	21%(29%)	50%(43%)

The majority of students reviewed the steps of problem solving as they worked, but fifty percent or less checked their work step by step.

AFTER YOU FINISHED WORKING THE PROBLEM-WHAT DID YOU DO?

	NO	MAYBE	YES
7. I checked to see if my calculations were correct.	13%(36%)	46%(14%)	42%(50%)
8. I went back and checked my work again.	29%(39%)	25%(21%)	46%(39%)
9. I looked back at the problem to see if my answer made sense.	0%(21%)	17%(11%)	83%(68%)

After finishing the problem, fifty percent or less went back to check their work, however the majority of students went back to see if their work made sense.

DID YOU USE ANY OF THESE WAYS TO WORK?

	NO	MAYBE	YES
10. I drew a picture to help me understand the problem.	79%(93%)	13%(0%)	8%(0%)
11. I "guessed and checked".	46%(43%)	13%(0%)	46%(50%)
12. I felt confused and could not decide what to do.	75%(75%)	0%(0%)	21%(18%)

The majority of students in both settings did not draw a picture to help solve the problem. Nearly half of the students used the "guess and check" strategy. Seventy-five percent of all the students were not confused.

In addition probable cause data from the literature was collected and reviewed. Several authors stated similar concerns about the way mathematics is taught today. The

Conference of Governors of the United States (1990) noted that the educational system of the United States ranked last in mathematics and science when compared to other industrialized nations. The conference members set a goal of becoming number one educationally by the year 2000. For this goal to become a reality, vast changes must occur at all educational levels and particularly in the elementary school. Students need to learn far more than the basic skills. Goodlad (1987) showed that life in the classroom, for most students, consists of exercises that require lower-level thinking rather than the development of concepts and higher-order thinking.

A number of studies reveal critical weaknesses in the way teachers teach mathematics. Often elementary school mathematics is text book driven with emphasis on drill. In a study of 37 fourth-, fifth-, and sixth-grade classrooms in nine schools, Good and Grouws (1987) found that only 14% of a class period is devoted to the development of mathematics concepts. A difficult area for teachers is letting go of the skill and drill approach. The literature suggests that concepts could best be taught through problem solving and the development of metacognitive abilities. According to the literature, converting book knowledge into problem solving skills is a major stumbling block for many students. Lippert (1987) stated that instruction in problem solving remains a difficult task. The most critical elements in acquiring problem solving skills are procedural knowledge, problem solving skills are procedural knowledge, metacognition, and practice. He questioned, "How do students know when they understand something, and what do they know about what to do when they know they do not understand?" (Lippert, 1987, p. 480). Wasserman (1987) indicates that teachers who wish to emphasize higher-order thinking skills need not throw out old curriculum plans and build in new ones. They need rather to switch the curriculum to activities that require thinking.

A summary of probable causes for the problem gathered from Illinois Park and Wild Rose School, and from the literature include the following:

1. methods of previous instruction were traditional,
2. methods of previous instruction were text-driven with emphasis on memorization and rote learning,
3. instruction lacked emphasis on concept instruction, and
4. instruction did not require higher-order thinking.

Chapter 3

THE SOLUTION STRATEGY

Review of the Literature

Analysis of probable cause data suggested reasons for student inability to do problem solving and to use higher-order thinking skills. Students lacked adequate strategies for problem solving and metacognitive ability; and they were unable to use more than one way to solve a problem. In addition to these data the students previous mathematics instruction lacked emphasis on problem solving and higher-order thinking.

Research literature suggested the following probable causes: lack of direct instruction in more than one way to solve a problem, emphasis on text-driven curriculum, emphasis on the basic skills and computation, lack of emphasis on the higher-order thinking, difficulty for teachers to change instructional methods, lack of mathematical concept development, and student attitudes toward their ability to solve problems.

Analysis of the literature search for solution strategies suggested that the following questions be addressed in order to improve mathematical instruction that would meet current student needs.

The following questions relate to revised instructional methods: 1) How can we teach children that there is more than one solution to a problem? 2) How can teachers improve questioning techniques that promote higher-order thinking? 3) What strategies are necessary to the development of a problem solving repertoire? 4) Does the use of hands-on manipulatives increase concept development? 5) How can writing activities develop an understanding of mathematical concepts and students' metacognition?

How can we teach children that there is more than one solution to a problem?

Kleiman (1991) discussed the view of mathematics as an essential human experience. He states that mathematics provides a vehicle for thinking, a medium for creating and a language for communicating. Mathematics must be taught with real-life applications and must involve prediction, exploration, experimentation, and higher-order thinking.

A great deal has been written about thinking skills in relation to mathematics. Fredericks (1992) describes thinkers as those who are always searching for an abundance of answers. An important factor in promoting good thinking is to emphasize that there is not one "correct" response to a question, but rather a lot of possible responses. In order for students to realize there is more than one solution or approach to a problem, they must be encouraged to take risks, to explore differing opinions and solutions, and be able to defend their ideas in a safe, non-judgmental environment (Chancellor, 1991).

Fogarty and Bellanca (1991) emphasize the need for developing reasoning skills that focus on the why and how, instead of the who, what, when, and where. Ashley Montague as quoted by Fogarty advised, "..it is the method not the content that is the message...the drawing out, not the pumping in" (Fogarty & Bellanca, 1991, p. 11).

How can teachers improve questioning techniques that promote higher-order thinking?

To learn how to ask the right questions is a difficult skill for teachers and students to master. The types of questions that are asked by teachers in elementary classrooms determine to a large extent how children view mathematics. Proudfit (1992) agrees that if the only questions that are asked during mathematics instruction concern correct answers, children can only conclude that the most important things in mathematics are correct answers. To be successful problem solvers students must focus on more than just correct answers.

The National Council of Teachers of Mathematics in Professional Standards for Teaching Mathematics stressed the importance of teachers "posing questions and tasks that elicit, engage, and challenge each student's thinking." (NCTM, 1991, p. 35). One way to do this is to ask questions that focus on a variety of problem solving methods.

Marzano described five ways to get kids thinking, and presents teacher strategies for fostering student thinking.

1. Use the vocabulary of thinking.
2. Discuss controversial topics.
3. Have students analyze media accounts that express different viewpoints.
4. Ask students questions with multiple answers.
5. Have students read and discuss literature that reflects differing values and traditions (Marzano, 1988, p. 45).

Cook (1989) has developed an extensive list of "Cues for Effective Questioning." The study presented questions that build, bridge, and transfer learning. The questions may be appropriate for any problem solving task. Examples of Cook's suggestions follow:

1. Ask open-ended questions.
2. Ask students to think about thinking.
3. Ask "How can you decide what information is needed?"
4. Ask "What about this problem reminds you of yesterday's problem?"
5. Ask "What assumptions can be made..."
6. Ask "How would you plan to solve..."
7. Ask "How can you tell if your answer is reasonable?"

Cook's questioning cues are examples of "generic" leads that might apply to any problem solving situation. The list is a valuable tool for teachers wanting to improve their questioning techniques.

The National Council of Teachers of Mathematics lists the following sample questions:

1. What assumptions do we need to make in this problem?
2. How will you organize the information in this problem?
3. How will you show the action of the spider

moving up and down the spout?

4. What information will your picture need to to contain?
5. What do you think the data will look like when it is combined and organized?
(NCTM, 1991, p. 52)

Oliver Wendell Holmes, as cited in Fogarty and Bellanca (1991, p. 57), describes the three-story intellect that has become a model for questioning classrooms.

> There are one-story intellects,
> two-story intellects,
> and three-story intellects with skylights.
> All fact collectors who have
> no aim beyond their facts
> are one-story men.
> Two-story men compare, reason,
> generalize, using the labor of
> fact collectors as their own,
> Three-story men idealize,
> imagine, predict-
> their best illumination comes
> from above the skylight.
>
> -Oliver Wendell Holmes

The graphic of the Three-Story Intellect Verbs from Fogarty and Bellanca (1991, p. 58) is a useful model for reference in posing questions for developing higher-order thinking (Appendix D.) Questioning that promotes higher-order thinking is a necessary part of not only instruction, but in the evaluation and assessment of the learning. Mrs. Potter's questions recorded in Fogarty and Bellanca (1991) are a useful tool with which students can routinely "check" their thinking metacognitively.

<u>Mrs. Potter's Questions:</u>

1. What were you expected to do?
2. In this assignment, what did you do well?
3. If you had to do this task over, what would you do differently?
4. What help do you need from me?
(Fogarty & Bellanca, 1991, p. 227)

What strategies are necessary for the development of a problem solving repertoire for students?

In the report by Stonewater, Stonewater, and Perry (1988) the National Council of Teachers of Mathematics 1980 Agenda for Action lists problem solving as the priority goal for mathematics instruction. This is often a difficult area for the student and teachers.

> Some students seem to understand that problem solving is a process and that the techniques applied depend upon the problem at hand, while other students seem to collapse under the weight of too many alternatives and just want to know 'how to get the right answers' (Stonewater, 1988, p. 272).

One approach to teaching problem solving is found in the Heath Mathematics Program, *Connections*, (1993). A few simple, but specific strategies are designed to help students find help as they move through the problem solving process.

1. make notes
2. work backwards
3. make a plan
4. make an organized list
5. use simpler numbers
6. make a diagram
7. make a model
8. guess and check
9. write a word equation
10. make a table

These strategies allow students several options for problem solving and understanding. Throughout the Heath program, daily lesson plans for problem solving become an integral part of the mathematics experience. These strategies are used repeatedly. Morrow stated in the Heath

Connections (1993) that these strategies provide a meaningful way for students to connect steps, ideas and numbers in a manageable way that promotes understanding of mathematical concepts. These strategies empower students to use their higher-order thinking skills to become successful problem solvers in the real world.

Another effective strategy as noted by Fogarty and Bellanca (1991) is cooperative learning. In cooperative learning students work in small groups to help one another master academic material. As stated by Slavin (1990) the most successful approaches have two key elements: group goals and individual accountability. Achievement effects of cooperative learning have been found in all subject areas. Other important effects of cooperative learning have been found in improved self esteem, attitudes toward school, acceptance of academically handicapped students, and the ability to work cooperatively.

Does the use of hands-on manipulatives increase concept development?

The authors of Heath Mathematics Program, *Connections* (1993), have developed mathematics curricula that emphasize hands-on experiences that enable students to make connections between mathematics and the world beyond the classroom. The lessons are developed to provide exciting purposeful ways to get students actively involved by using manipulatives that will help students at all levels bridge the gap between understanding at the concrete level and understanding at the abstract level.

The National Council of Teachers of Mathematics (NCTM) describes changes that need to be made in mathematics instruction and ultimately in mathematics assessment. The standards stated that manipulative usage should be part of the formal evaluation process (NCTM, 1991). The states of Maine and Oregon are providing rulers, tangrams, counters, tiles, and calculators along with the standardized test booklets (NCTM, 1991). This should enable assessment to be consistent with the instruction. The NCTM Curriculum and Evaluation Standards stated, "Students must be permitted to use calculators; as the use of these 'fast pencils' becomes routine in classrooms, evaluation must also keep pace"(NCTM, 1991, p. 9).

Baratta-Lorton (1977) described the philosophy that computational skills are more easily learned when drawn from concrete experiences. Mathematics lessons designed as activity-centered learning experiences will provide mathematical and social skills that will enable students to handle problem solving situations

<u>How can writing activities develop understanding mathematical concepts and students' metacognition?</u>

As stated in Graves (1986) writing is a skill used throughout life in many different ways. Good writing skills require that students engage in practice every day at all grade levels and subject areas. Because writing is thinking and knowing, writing is an ideal way to develop higher-order thinking skills.

In the National Council of Teachers of Mathematics Curriculum and Evaluation Standards (1989), fifty-four standards covering grades K-12 presented new procedures and questioning techniques for teachers to use in the classroom. The standards on mathematics as communication reflect the view that "to know mathematics is to engage in a quest to understand and communicate" (Countryman, 1993, p. 51). With this emphasis on understanding and communicating it is not surprising that mathematics teachers are turning to writing.

Countryman (1993) believes that to learn mathematics, students must construct it for themselves. This can only be done by exploring, justifying, representing, discussing, using, describing, investigating, and predicting, simply by being active in the world. Situations need to be created for students to be involved, be creative and responsive, and to be involved in the physical world. Writing is an ideal vehicle for this process.

Szetla and Nicole (1992) indicated that the first three standards of the expanded goals of NCTM show a shift in the emphasis from rules and routine problem solving where the teacher talks and the students listen, to active student participation in which reasoning and communication are stressed. Students are prone to make calculations without explanation, and this alone fails to show the nature of the solvers work and thinking. Having students explain their work in writing may be critical for concept understanding and teachers may then assess comprehension based on this writing.

Countryman (1993) suggests several excellent techniques and activities for developing understanding in mathematics. She suggests using free writing where students write rapidly for a fixed period of time (five minutes) about anything that occurs to them. This writing encourages the students to allow their ideas, thoughts, and questions to flow freely, and to discover what they already know about any area of mathematics. Another writing activity is the learning log, a simple informal notebook, which students record examples and brief descriptions of lessons, or questions about assignments.

A mathematics autobiography can be a written history of the student's mathematics experiences in and out of the classroom. It can include a list of ideas and questions the student may bring from their personal experiences as far back as they can remember. This activity allows the student to acknowledge that mathematics does engender emotions. A mutual sharing of these autobiographies may help to establish a support system for the students when they discover that many of their emotions are shared by classmates.

Writing to explain concepts can clarify students' thinking and enable them to focus on what they truly understand. The use of writing as a metacognitive tool, as in the example of Mrs. Potter's Questions mentioned earlier, develops the students' ability to think about their thinking. Writing what they know will not only clarify the thinking and understanding, but can become a useful tool in the assessment of the learning.

Many of the strategies reported from the literature search were appropriate for the problem setting. The research indicated that many teachers use cooperative learning, hands-on activities, and develop thinking skills to some degree, but use very little writing in mathematics instruction. However, a consistent effort to organize and apply these techniques to improve mathematical instruction is necessary. Research suggests several solutions to revise mathematics teaching strategies that will improve students' problem solving abilities.

Project Outcomes

The first terminal objective of this problem was related to the students' ability to do problem solving and to understand that there is more than one solution to a problem. Probable cause data presented in Chapter 2 suggested the need for increasing the use of hands-on activities to develop mathematical concepts. Therefore:

> As a result of revised mathematics teaching strategies and increased hands-on activities during the period of September to December 1993, fifth-grade students at Illinois Park and Wild Rose schools will increase in their ability to do mathematics problem solving and to understand that there is more than one solution to a problem, as measured by teacher developed paper and pencil tests.

Probable cause data gathered from the site indicated that students had an inadequate understanding of how to solve problems and lacked strategies for problem solving, and therefore lacked confidence to experiment with different ideas leading to a solution. Therefore the second terminal objective stated that:

> As a result of revised math teaching strategies and increased hands-on activities during the period of September 1993 and December 1993, the fifth-grade students' attitudes and confidence in their problem solving abilities will improve as evidenced by a metacognitive survey and a student reflective journal.

In order to accomplish the terminal objectives, the following process objectives defined the major strategic procedures for problem resolution. These techniques related to the terminal objectives as they revised teaching instruction to improve student abilities to solve problems in mathematics.

1) As a result of written mathematics autobiographies during the first week of school, the students will focus on their mathematics experience their likes and dislikes, their strengths and weaknesses, their fears and apprehensions, and will evaluate how they think and reason in mathematics.

2) As a result of writing daily for five to ten minutes in student mathematics logs, the students will reflect on their assignments, their difficulties and successes, and will increase their metacognitive ability.

3) As a result of on-going direct instruction of critical thinking skills as they apply to daily lessons, students will understand multiple strategies and be able to apply them to problem solving.

4) As a result of direct instruction on the use of manipulatives as they apply to daily lessons in problem solving, the students will improve comprehension of mathematics concepts, discover familiarity with a variety of tools to aid in problem solving.

Proposal Solution Components

The major elements of the approach used to increase mathematics problem solving and critical thinking fell into three categories; revised teaching techniques to develop a student repetoire of strategies; the use of hands-on manipulatives to increase concept development; and the development of metacognitive abilities through writing activities. These elements related to the terminal objectives in that they attempted to improve students' abilities to solve mathematics problems and increase critical thinking. Probable cause data indicated students' lack of strategies for problem solving, students' limited use of hands-on manipulatives, a lack of confidence in students' abilities to solve problems, and a lack of experience in writing to express thinking in mathematics.

Chapter 4

ACTION PLAN FOR IMPLEMENTING THE SOLUTION STRATEGY

Description of Problem Resolution Activities

The action plan is designed to address four major solution components: a mathematics autobiography, student journals, direct instruction of strategies, and the direct instruction in the use of manipulatives.

The mathematics autobiography will begin the first week of school in the fall of 1993 with the students writing about their experiences in learning mathematics, describing their strengths and weaknesses; their likes and dislikes, and their fears and apprehensions. Brainstorming ideas will precede the writing. This will assist the researchers in evaluation of the students' mathematics history.

The student mathematics journal activity will begin in September of 1993 and continue throughout the school year. This writing log will be used as a personal reflective journal and also for problem solving practice. The students will write for five to ten minutes daily resulting in improved metacognitive skills.

The direct instruction in thinking strategies will include six strategies used in mathematics problem solving: 1) work backwards, 2) guess and check, 3) make a table or chart, 4) make an organized list, 5) draw a picture or model, 6) do a simpler problem. Each strategy will be introduced at two week intervals during the twelve week treatment period, and will be followed by practice of the techniques. This will develop the students' repertoire of strategies for problem solving and increase higher-order thinking skills.

Direct instruction in the use of manipulatives will begin in the fall of 1993. It will involve six types: 1) tangrams, 2) unifix cubes, 3) links or colored counters, 4) base ten blocks, 5) pattern blocks, 6) calculators. One type of manipulative will be introduced every two weeks for the twelve week treatment period in conjunction with an appropriate lesson plan. This will develop familiarity with several different manipulatives to assist in problem solving.

The implementation plan is presented below in outline form and in chronological order, allowing for variation in the sequence in which the skills are taught.

1. Student mathematics autobiography.
   A. Who: Fifth-grade math students.
   B. What: Students will write mathematics autobiography.
   C. When: The week of September 6.
   D. Where: Illinois Park and Wild Rose fifth grade classrooms.
   E. How: Students will write about their experiences

in learning mathematics, describe their strengths and weaknesses, likes and dislikes, fears and apprehensions. Brainstorming will precede writing.

F. Why: To evaluate student mathematics history.

2. Student mathematics log.

A. Who: Teachers instruct students .

B. What: Writing log.

C. When: To begin the week of September 6 and to continue daily for five to ten minutes.

D. How: Instruction s on how to begin using a log for written reflection and practice.

E. Why: Writing their thinking will improve metacognitive skills.

3. Evaluation.

A. Who: Researchers.

B. What: Will review autobiographies and teacher surveys.

C. When: The week of September 13.

D. How: Read and evaluate student attitudes about mathematics and collect teacher survey data.

E. Why: To establish instructional needs and review students' mathematics history.

4. Problem solving pre-test.

A. Who: Target students.

B. What: Four written problems to solve individually and one problem to compose.

C. When: The week of September 13.

D. Where: Illinois Park and Wild Rose fifth grades.

E. How: Students take paper and pencil pre-test showing their thinking and solutions.

F. Why: To assess problem solving ability before treatment begins.

5. Evaluation.

A. Who: Researchers.

B. What: Review result of problem solving pre-test.

C. When: The week of September 16.

D. How: Make bar graph of test results.

E. Why: To evaluate test results and student abilities

6. Metacognitive survey.

A. Who: Target students.

B. What: Will complete survey of strategies they currently use to solve problems.

C. When: The week of September 20.

D. How: Will solve a written problem and show their solution then complete the paper and pencil survey.

E. Why: To gather data on students before treatment begins.

7. Evaluation.

A. Who: Researchers.

B. What: Review results of metacognitive survey.

C. When: The week of September 20.

D. How: Make table of survey results.

8. Direct instruction in critical thinking strategies.

   A. Who: Teachers with target students.

   B. What: Will do direct instruction in six strategies used in mathematics problem solving: 1) work backwards, 2) guess and check, 3) make a table or chart, 4) organized list, 5) draw a picture, 6) do a simpler problem.

   C. When: One strategy will be introduced every two weeks during the twelve-week treatment period, and will be followed by practice of the skill.

   D. Where: Illinois Park and Wild Rose fifth grades.

   E. How: Teachers will introduce the six strategies as part of the daily lessons, practice and review.

   F. Why: To develop a repertoire of strategies for problem solving.

9. Direct instruction in use of manipulatives.

   A. Who: Teachers with target students.

   B. What: Instruction in the use of six types of manipulatives: 1) tangrams, 2) unifix cubes, 3) links or colored counters, 4) base ten blocks, 5) pattern blocks, 6) calculators.

   C. When: One manipulative will be introduced every two weeks over a period of twelve weeks.

   D. How: Manipulatives will be introduced in conjunction with appropriate lessons.

   E. Why: To develop familiarity with several manipulatives to aid in problem solving.

10. Evaluation.

A. Who: Researchers.

B. What: Evaluate progress in student logs, lessons in critical thinking and manipulatives.

C. When: Every two weeks during the treatment period.

D. How: Teachers will read the written logs and check understanding with paper and pencil quizzes.

D. Why: To monitor progress and modify instruction as needed.

11. Problem solving post-test.

A. Who: Target students.

B. What: Will complete parallel post-test.

C. When: The week of December 6.

D. How: Take written test of four problems to solve and one problem to compose and solve showing written work.

D. Why: To assess progress in problem solving.

12. Metacognitive post survey.

A. Who: Target students.

B. What: Repeat the same survey.

C. When: The week of December 6.

D. How: Students solve a written problem and complete the paper and pencil survey.

E. Why: To compare students' abilities to use strategies and assess their attitudes about mathematics problem solving.

13. Student mathematics essay.

A. Who: Target students.

B. What: Will write essay reflecting on their mathematics experiences this school year.

C. When: The week of December 13.

D. How: Students will review their autobiographies and add to their mathematics history.

E. Why: To gather data on students attitudes on the mathematics instruction to this point.

Methods of Assessment

A variety of data collection methods will be used in order to assess the effects of the intervention. Changes in students' attitudes about mathematics problem solving will be determined through review of the mathematics essay and the administration of the metacognitive survey taken in September. Students' abilities to solve problems using a variety of strategies will be measured with the written post-test. Students' abilities to write about their thinking and about how to solve problems will be evaluated through the review of their written logs and their abilities to compose mathematical problems.

Chapter 5

EVALUATION OF RESULTS AND PROCESS

Implementation History

Terminal Objectives

The terminal objectives of the intervention addressed the fifth grade students' inadequate abilities to use critical thinking to understand that there is more than one solution to a mathematics problem. This was evidenced by teacher evaluation of a mathematics autobiography, a problem solving pretest, and a metacognitive survey about how students solve problems. Therefore, the terminal objectives stated:

> As a result of revised mathematics teaching strategies and increased hands-on activities during the period of September to December 1993, fifth-grade students at Illinois Park and Wild Rose schools will increase in their ability to do mathematics problem solving and to understand that there is more than one solution to a problem, as measured by teacher developed paper and pencil tests.

> As a result of revised mathematics teaching strategies and increased hands-on activities during the period of September 1993 to December 1993, the fifth-grade students' attitudes and confidence in their problem solving abilities will improve as evidenced by a metacognitive survey and a student reflective journal.

Pre Assessment Activities

The development of new teaching strategies to improve student problem solving began with the review of the assessment data. To begin the assessment in September 1993 students were asked to write an autobiography about their mathematics experiences. They were to tell about their strengths and weaknesses, their likes and dislikes, and their fears and apprehensions. Writing about mathematics was a new experience for most of the students.

In September the students were given a problem solving pre-test. The pre-test was composed of different types of problems which required the use of problem solving strategies. The students were then given a metacognitive survey to assess whether they used various strategies for problem solving, and to assess how they felt about problem solving. The students' response to the metacognitive survey on pages 50 and 51 showed that students varied in their abilities and confidence to solve problems.

Interventions

Students received direct instruction in the following six thinking strategies which they used in mathematics problem solving during the treatment period: a) work backwards, b) guess and check, c) make a table or chart, d) make an organized list, e) draw a picture or model, and f) do a simpler problem. These strategies were introduced and integrated into daily lessons. Students developed a repertoire of strategies for problem solving and used them on a regular basis. The "guess and check" strategy and "do a simpler problem" strategy were used several times during the week when story problems were a part of the lesson. Students were given direct instruction on how to "make a table or chart" using specific story problems. Graphs and tables were drawn in the students' mathematics log weekly, and students wrote reflections about their steps to problem solving and about their thinking.

The strategy of "draw a picture or model" was used for lessons with unifix cubes and pattern blocks every two weeks during the treatment period. Students also wrote organized lists in their logs to help solve problems several times weekly during the treatment period. (See Appendix E.)

Students developed a repertoire of strategies for problem solving. A chart of these strategies was posted in the room from which students selected the best method for problem solving. These six strategies were reinforced weekly over the twelve week period.

Six lessons with direct instruction in the use of manipulatives were introduced at two week intervals. A series of five lessons on the use of plastic tangram pieces developed the concept of tangram squares, geometric shapes, and the area of a tangram. Students explored the various geometric shapes that can be made with the seven tangram pieces. The class made a large organized chart to record this data, thus integrating the thinking strategies with the manipulatives. (See Appendix F.)

Base ten blocks were used in a two week unit on place value and decimals. Students used the blocks with graph paper to develop the concept of units, tens, and hundreds in the study of place value, and with ones, tenths, and hundredths in developing decimal concepts.

Colored counters enabled students to explore the concept of division with hands-on activities. This activity preceded the instruction of two-digit division computation. The colored counters were also used in three different lessons on probability and random sampling. In these lessons students made organized lists and charts, thus integrating the new strategies with the manipulatives. (See Appendix G.)

Unifix cubes were used with lessons teaching area and perimeter. In a four day lesson on perception and three dimensional figures students built a variety of cube configurations and drew their models using dot paper. (See Appendix H.)

Pattern blocks were used in lessons developing geometric concepts. They were used in the introduction lesson on fraction concepts, and in a three day lesson on perimeter. (See Appendix I.)

Calculators were used in specific lessons on how to use the M+ function and the constant function. Students also used calculators on a daily basis for figuring grade percentages, story problem practice, and warm up exercises. Calculators were allowed during some testing situations. (See Appendix J.)

Another aspect of the assessment was the student mathematics log. As students used these manipulatives in the lessons, they were asked to reflect in their journals. They described strategies, wrote examples of problems, described their thinking process, or expressed feelings and attitudes about their work.

Post Assessment Activities

After a twelve week treatment period, the following post assessment activities were conducted. Students wrote an essay explaining how they felt about mathematics after the treatment period.

A post-test was administered which contained parallel problems to the pre-test. Students were expected to use strategies that they had learned. (See Appendix K.)

After the post-test students completed a metacognitive survey which assessed their problem solving abilities. It also assessed their use of strategies and their comfort level with problem solving.

Students' mathematics logs were reviewed to note their progress in writing about mathematics. Students solved problems in the log, and reflected on their thinking.

Presentation and Analysis of Project Results

First Assessment

As part of the post assessment the students wrote another essay which was compared to the mathematics autobiography written in September. Students expressed more confidence in their mathematics abilities, and felt comfortable in the use of mathematics strategies. (See Appendix L.)

Second Assessment

Figures 3 and 4 compare the results of the problem solving pret-est and post-test. Figure 3 presents data on the outcomes for Wild Rose School. It should be noted that only two of the 24 students scored 50% or less on the post-test compared to ten students who scored 50% or less on the pret-est. Twenty-two of the 24 students scored 60% or more on the post-test as compared to 15 students on the pre-test.

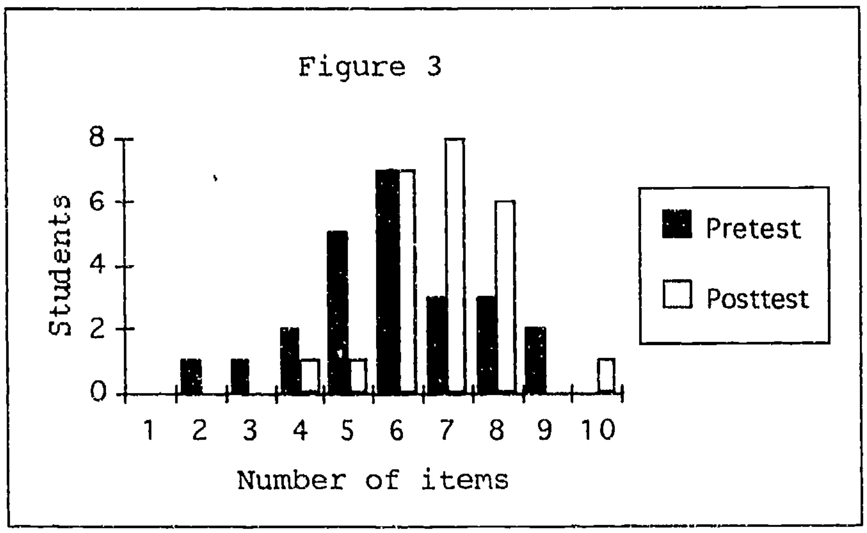


Figure 3

Mathematics Problem Solving Pre-Test and Post-Test Scores
Fifth Grade Wild Rose School

Figure 4 presents data on the problem solving pre-test and post-test given at Illinois Park School. It should be noted that only two of the 28 students scored 50% or less on the post-test as compared to 16 students who scored 50% or less on the pre-test. Twenty-three of the 28 students scored 60% or more on the post-test as compared to 12 students who scored 60% or more on the pre-test.

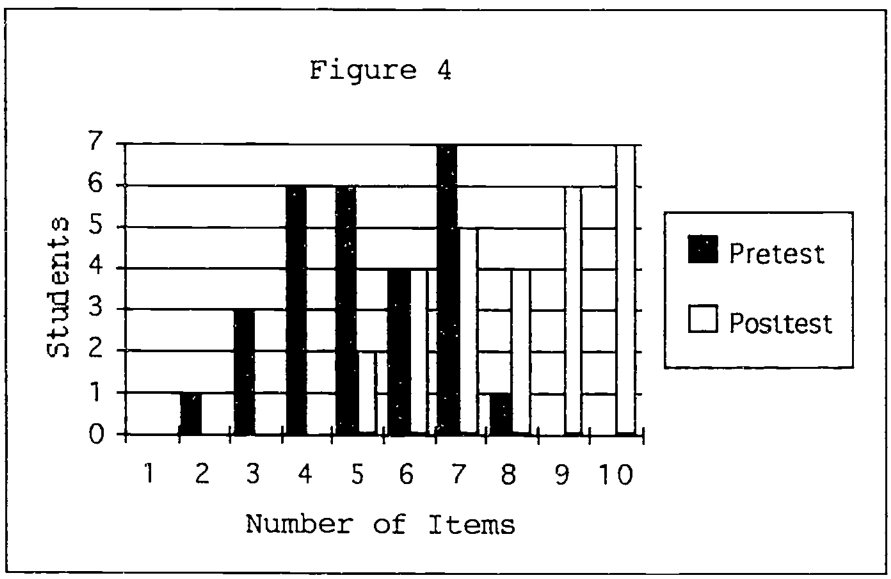


Figure 4

Mathematics Problem Solving Pre-Test and Post-Test Scores
Fifth Grade Illinois Park School

Third Assessment

The metacognitive survey referred to on pages 19 and 20 was repeated at the end of the treatment period to reassess problem solving skills and attitudes.

Table 2

PROBLEM SOLVING METACOGNITIVE SURVEY
Given to 24 Wild Rose Students
28 Illinois Park Students ()
September 1993
*January 1993

NO-No, I didn't do this.
MAYBE- I may have done this.
YES-Yes, I did do this.

BEFORE YOU BEGAN TO SOLVE THE PROBLEM-WHAT DID YOU DO?

		NO	MAYBE	YES
1.	I read the problem more than once.	38%(14%) *0 (*0)	0%(21%) *21(*15)	79%(68%) *79(*85)
2	I tried to remember if I had worked a problem like this before.	42%(46%) *20(*8)	13%(21%) *45(*21)	50%(32%) *35(*65)
3.	I thought about what information I needed to solve the problem.	0%(14%) *0(*0)	0%(29%) *10(*15)	92%(57%) *90(*85)

Post-test results showed that before beginning to work, the majority of students read the problem more than once, and only 15% or less needed more information to solve the problem as compared to 50% on the pre-test.

AS YOU WORKED THE PROBLEM - WHAT DID YOU DO?

		NO	MAYBE	YES
4.	I thought about all the steps as I worked the problem.	13%(36%) *12(*0)	29%(11%) *54(*31)	63%(54%) *34(*69)
5.	I kept looking back at the problem after I did a step.	25%(18%) *12(*0)	0%(14%) *37(*27)	71%(68%) *51(*73)
6.	I checked my work step by step as I worked the problem.	29%(29%) *16(*12)	21%(29%) *41(*38)	50%(43%) *43(*50)

More students showed that they thought about the problem before solving it. An increased number of students checked their work step by step, and thought about the information needed before working the problem as compared to the pretest.

AFTER YOU FINISHED WORKING THE PROBLEM-WHAT DID YOU DO?

		NO	MAYBE	YES
7.	I checked to see if my calculations were correct.	13%(36%) *0 (*0)	46%(14%) *50(*54)	42%(50%) *50(*46)
8.	I went back and checked my work again.	29%(39%) *0(*0)	25%(21%) *29(*19)	46%(39%) *70(*77)
9.	I looked back at the problem to see if my answer made sense.	0%(21%) *0(*4)	17%(11%) *29(*19)	83%(68%) *70(*77)

After finishing the problem the majority of students checked their work to see if it was correct and if their answers made sense.

DID YOU USE ANY OF THESE WAYS TO WORK?

		NO	MAYBE	YES
10.	I drew a picture to help me understand the problem.	79%(93%) *16(*12)	13%(0%) *37(*65)	8%(0%) *47(*23)
11.	I "guessed and checked".	46%(43%) *8(*23)	13%(0%) *21(*50)	46%(50%) *71(*27)
12.	I felt confused and could not decide what to do.	75%(75%) *54(*58)	0%(0%) *29((34)	21%(18%) *17(*8)

An increased number of students used one of the strategies to solve the problem. Fewer students felt confused about how to solve the problem than on the metacognitive pre-test.

Fourth Assessment

After the treatment period researchers assessed the students' mathematics log entries. It was noted that students had become more comfortable about writing in mathematics class. Their writing showed more specific vocabulary as they explained their problem solving process and metacognition. The writing became an expected part of the mathematics lessons. (See Appendix E.)

Reflections and Conclusions

The mathematics autobiography was used in the beginning of the treatment period in September, and a math essay was written as a concluding activity. In both Wild Rose School and Illinois Park School the students expressed improved attitudes about their mathematical ability, and demonstrated increased ability to write about their thinking.

The mathematics log writing was received with some reluctance in the fall because the students were unaccustomed to writing about mathematics. As the treatment period progressed students developed mathematics literacy, and became more confident in their writing.

Based on the results of the problem solving pre-test and post-test at Wild Rose School, improvement was noted from September 1993 to December 1993. The students at Illinois Park School showed an even greater improvement between the pre-test and post-test. The researchers noted that this difference may have been the result of less instruction in problem solving prior to fifth grade. The students at Wild Rose School had a stronger background in problem solving in previous grades.

The direct instruction of problem solving strategies throughout the treatment period helped students to develop a repertoire of techniques to solve many types of problems. It was noted that students became more comfortable with the use of a variety of strategies as they used them in daily assignments.

The manipulatives used in the treatment were helpful in the development of concepts, and the students enjoyed using them. When the concepts became better understood, as in numeration and decimals, the counters and base ten blocks were used less frequently. Tangrams, unifix cubes, and pattern blocks were used as a necessary part of specific problems, and were a useful tool in the development of the concepts. Calculators became a part of the daily mathematics class.

Chapter 6

DECISIONS ON THE FUTURE

Solution Strategy

The data indicates that direct instruction of problem solving strategies, the use of manipulatives, and writing in mathematics should be a part of the mathematics program. This program should be further improved by spending more time establishing a routine for use with the writing log on a daily basis.

The increased use of manipulatives was enjoyable for the students, although it may present a time management concern for teachers. However, manipulatives should be an important part of the concept development. The use of tangragms, unifix cubes, and pattern blocks, in addition to the frequent writing, did increase students' critical thinking to understand that there is more than one solution to a problem.

Additional Applications

In reviewing the outcomes of this action research, we felt that each of the components should become a part of any mathematics instruction. As a result of this implementation, mathematics could be better integrated into the whole educational program.

The steps in the scientific method are nearly parallel to the steps in critical thinking in problem solving in mathematics. The development of predicting outcomes, the collection and graphing of data, and probability skills are used in mathematics and science. Mathematics cannot be separated from science.

The direct instruction of mathematics strategies for problem solving can by carried over into all areas of curriculum. For example, the strategy of "making an organized list" could be used in literature, social studies, and science. The strategy of "guess and check" is another method of predicting outcomes which is a necessary skill in all areas of learning. There are innumerable uses for making charts, tables and models. This repertoire of strategies provides the student with critical thinking skills for life.

We felt that the use of manipulatives was the key to concept development in problem solving. These hands-on activities allowed students to see the relationships between the abstract and the concrete.

Writing across the curriculum has become emphasized in recent years. However, this has not often included mathematics. We felt that we have shown the importance of writing in the development of thinking skills in mathematics.

Dissemination of Data and Recommendations

The results of this research will be shared with staff members in both schools at in service meetings. We will serve as a resource to all teachers and provide them opportunities for observation of our classes, and offer assistance in planning lessons that incorporate the activities developed in this action research project.

A copy of the action research project will be made available in the school resource center at Wild Rose School and at Illinois Park School. On May 12, 1994, this research project will be on exhibit at Larkin High School in Elgin, Illinois, at the Saint Xavier Field Based Master's Program exhibition. In addition, we felt that the outcomes of this research were significant enough to be made available to the district mathematics coordinators.

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APPENDICES

Appendix A

Autobiography Sample

Math Autobiography

Sep. 8, 14

"Why I'm Good In Math" Math

I was three when I started to count, But I didnt count right. This is how I counted, 1, 2, 3, 4, 6, 90, 3, 9, 16, and so on. Mom and Dad taught me how to count right.

I have a big family, so my brothers Rob and Dan tought me how to multiply in first grade.

I forget some math during the summer, but I lesen it agian very fast. My mom taught me to do a trick with nine times any thing up to nine. You do it with your hands. soppose the problem was nine times eight. You'd count up to your eighth finger then you'd count the finger before your eighth finger and after it. You'd get the answer seventytwo.

I like learning tricks about math. Mrs. Orth told us one called casting out the nines, a way of checking in division and multiplication.

I'm not afraid of any thing in math. I like trying new stuff and experimenting things. I like counting high, my goal is to make it to googol.

The End

Appendix B

Problem Solving Pre-Test

NAME ________________

DATE ________________

PROBLEM SOLVING PRETEST

Directions: Solve each problem. Show your thinking.

1. At the end of the sixth inning, the score at the baseball game was 8 for the White Sox and 5 for the Cubs. In the last inning the White Sox made 4 runs, and the Cubs made 6 runs. Which team won the game? By how many runs?

2. Your teacher has decided that you may have your choice of recess times:
   A. You may have 30 minutes a day for the next two weeks; or
   B. You may have 1 minute of recess on the first day, 2 minutes on the second day, 4 minutes on the third day, 8 minutes on the fourth day, and so on, for the next two weeks. Which recess would you like? Why?

3. A T-shirt shop has only four digits from which to print numbers on their shirts.

1 5 8 9

How many different 2-digit numbers can you choose from for your shirt?

4. There will be 142 fifth-grade students attending Jefferson School this year. There will be 5 fifth-grade classes. If each class should have about the same number of students, how many students should be assigned to each class?

5. Write your own story problem using the following data:

6 students 5 pizzas $10 per pizza

____

____

____

____

Solve and show your thinking.

Appendix C

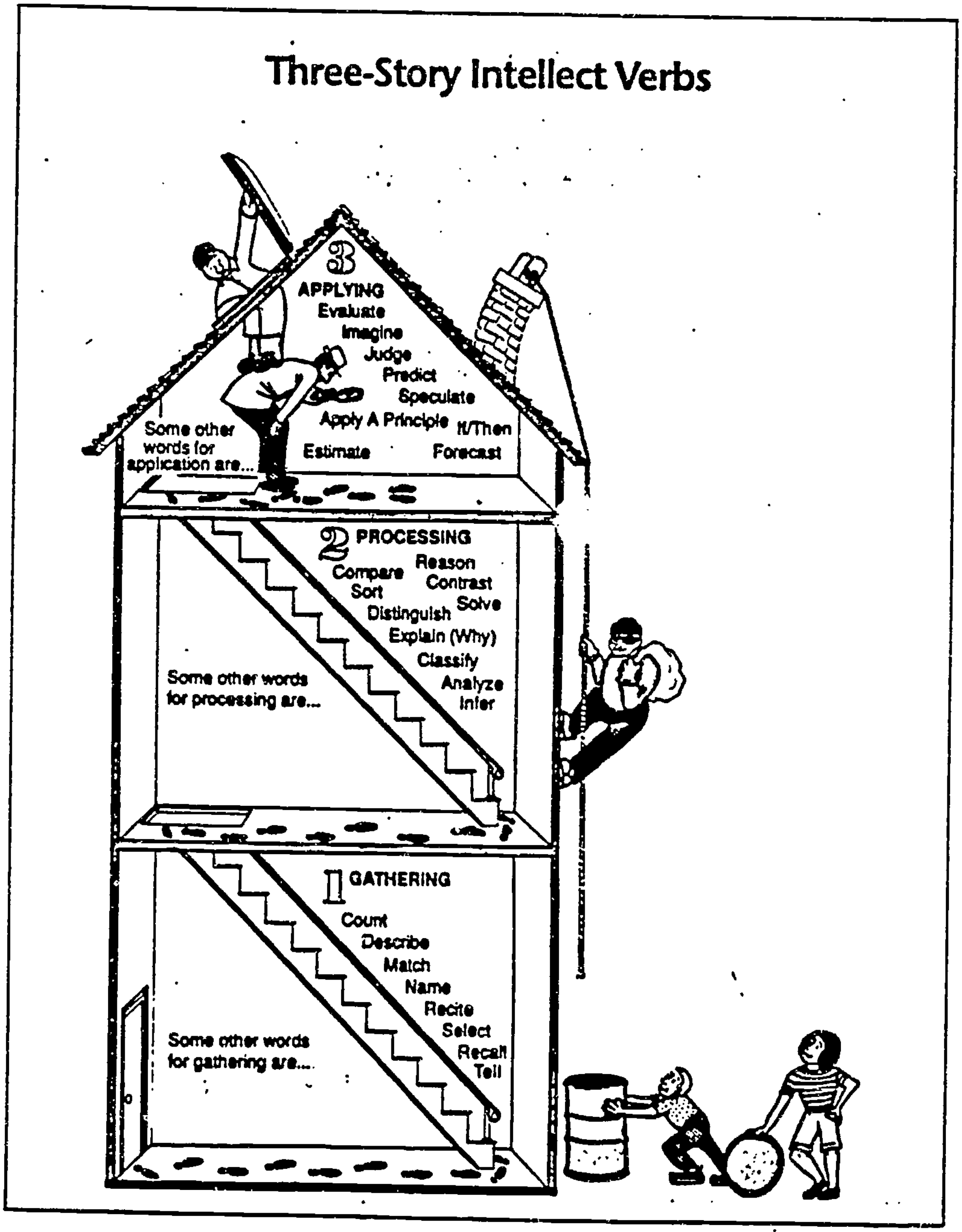
Teacher Survey

TEACHER SURVEY ON PROBLEM SOLVING

		Always	Sometimes	Never
1.	How often do you use cooperative grouping in problem solving?	______	______	______
2.	Do you spend time on problem solving in your math class?	______	______	______
3.	Do you have students write about problem solving during math?	______	______	______
4.	How often do you use manipulatives in problem solving?	______	______	______
5.	How often do you use calculators in problem solving?	______	______	______
6.	Do you allow calculators or manipulatives during testing?	______	______	______
7.	Do you use the story problems in the chapters of the text?	______	______	______
8.	Do you use the THINK assignments?	______	______	______

Appendix D

Three-Story Intellect Verbs



Appendix E

Student Mathematics Log

JAN 25-94

1. What we did: First of all we got seven tangrams with those we had to try and make a square. The second thing we did was make at first 4 peices with the sixteen parts to it out of a puzzle. The third thing we did was make a □ with peices 1-7 and did the same with △, ▭, ▱, and ⏢.

2. What I thought. I thought it was fun, confussing, agervating. Trying to put the pieces together to make the certain shapes was the most agervating.

P. 93 25-30

25. 5 ft 6 6%
26. 3 hr
27. 33 ft
28. 6 lb
29. 600 Lb
30. 2 five ounce coins and three 3 two ounce coins.

8 coins 4,1 Lb

$12\overline{)60}$ r1 $\quad 3\overline{)100}$ 33 r1 $\quad 60\overline{)180}$ 3 $\quad 2\overline{)16}$ 8 $\quad 16\overline{)288}$ 18 $\quad 8\overline{)4800}$ 600 $\quad 16 \times 9 = 144$

BEST COPY AVAILABLE

Appendix F

Tangram Activity Sample

puzzle 16
pieces 1-7

TANGRAMS

Jan. 25
p. 53

15
15
30

13
13
13
13
52

1. What we did the first day we got Tangrams we had to make a square out of them (7 peices). Then for homework we got a paper that had a whole bunch of little triangles on it we had to see how many shapes of 16 little triangles. The next day we got our Tangrams back & we had to try to see how many of the shapes we could fit with our Tangrams. Then today we tried to make □s, △s, ▭s, ▱s, & ⏢s of our tangrams.

What we learned: some of us learned how to do Tangrams.

Matt (-2) 66% Jan. 25

+25. 5 feet 66% 66%

~~26~~ 1 hour and 20 minutes

+27. 300 feet

+28. 18 pounds

+29. 600 pounds

Matt

Matt

Jan. 26

(-1)

1. 15
2. 42
3. 11
4. 16
5. 10
6. 20
7. 30
8. 52

When you have a problem like (14-7)+8= you should do the part in coreecencies first, but if it has no parenthesis do x or ÷ first

Appendix G

Make a Chart Sample

Randon Sample Feb 1

experament #1: yesterday we our names on a card and put them in a bag. Mrs. Long drew a card and Tia won a pencil.

Experament #2: We picked one out of three places to go for a day. We wrote it on the card and put the card in the bag.

Try 1

Camp	GA	Bulls
1	1	1
	1	
	1	

Camp	GA	Bull
1	1	1
1	1	

Camp	GA	Bull
1	1	1
1	1	

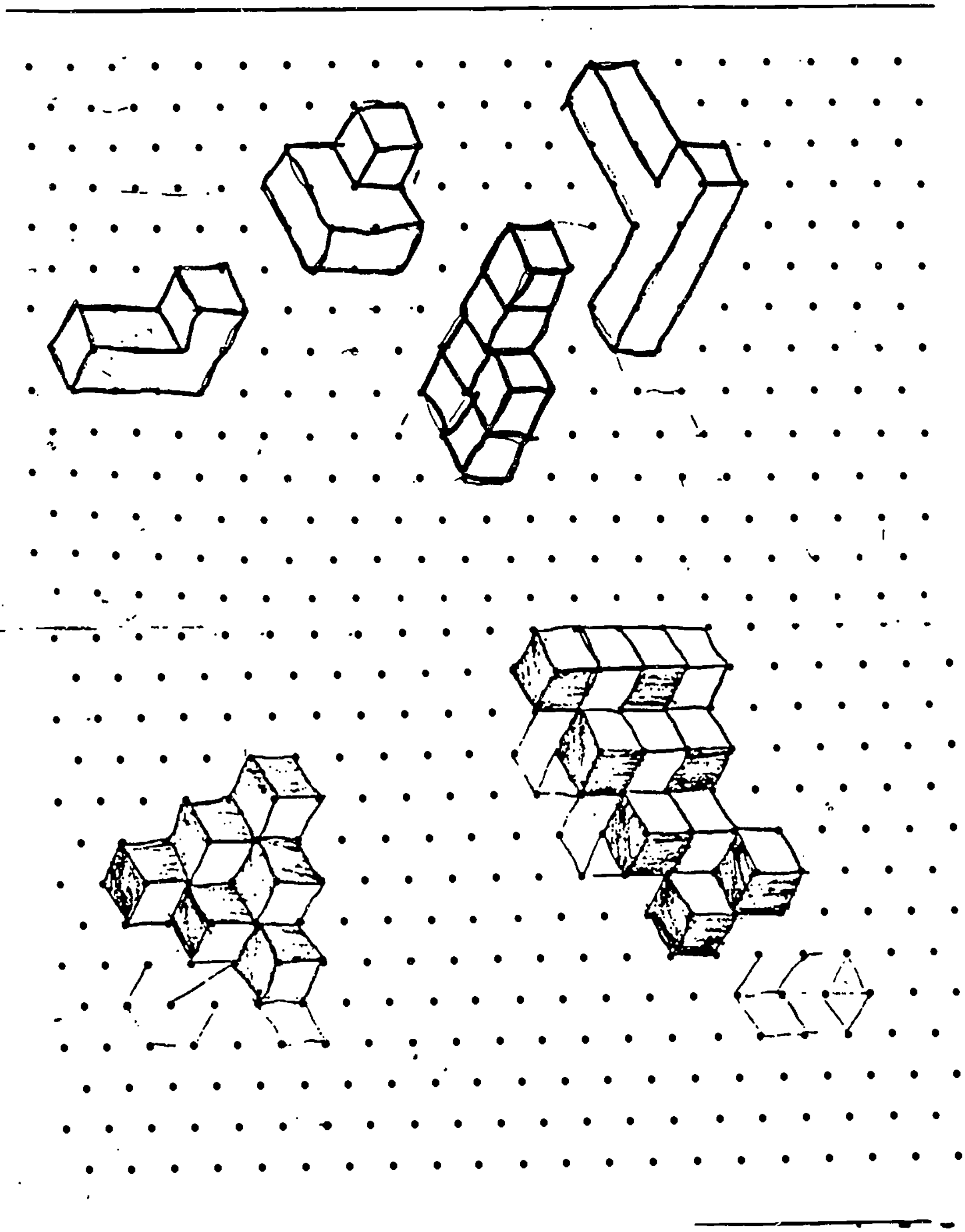
Camp	GA	Bull
11	111	111
11	11	11
111	11	11
1	111	1

I was close because I was right because it's to cold to go to Great Ameraca. A random sample is just a guess

Experament 3 There are 100 links in the bag red yellow blue

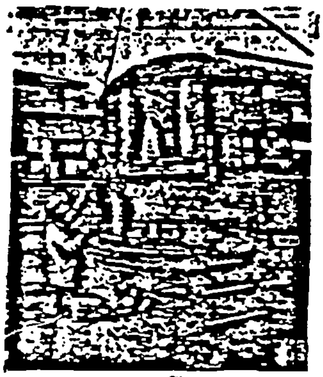
Appendix H

Unifix Cube Activity Sample



Appendix I

Pattern Block Activity Sample



Name ____

Date ____________

PERIMETER AND AREA TRAINS

							formula
TRAIN # CARS		2	3	4			
PERIMETER	3	4	5	6			P=N+2 P=3+n-1
TRAIN							
# CARS	1	2	3	4	5	6	
PERIMETER	4	6	8	10	12	14	P=2N+2
TRAIN							
# CARS	1						
PERIMETER	6						
TRAIN							
3 CARS							
PERIMETER							
TRAINS							
#CARS							
PERIMETER							

Appendix J

Calculator Activity Sample

5-K
Oct. 14th, 1993

Calculator Activity

Try the following activities with your calculator.

Key In:

[0] [+] [3] [=] [=] [=] [=] [=] [=] [=] 21

What are you getting? Predict what would be on the display if you pressed the equal key 3 more times? How would you use this procedure to find the first 6 multiples of 4? What is another way you could get the same result using multiplication?

Key In:

[7] [+] [5] [=] [=] [=] 22

How is the display changing? Predict what will be on display after you press the equal sign 4 more times. Check to see if you are correct.

Predict what will be on display after you key in the following sequence.

58

[8] [+] [10] [=] [=] [=] [=] 48

Check to see if you are correct.

[5] [+] [N?] [=] [=] [=] [=]

After keying in the sequence above the display of the calculator read 21. What number must I have keyed in for the N?

4

Key In: (Notice how the display changes)

[53] [-] [5] [=] [=] [=] [=] [=] [=] [=] 18

goes down by 5

What is happening? What do you think will be on display after you press equal three more times? Check to see if you are correct. What do you think will happen after you press the equal sign four more times. Were you correct? Why is the one's digit different than before?

Appendix K

Problem Solving Post-Test

NAME ____________________

DATE ____________________

PROBLEM SOLVING POSTTEST

Directions: Solve each problem. Show your thinking.

1. At the end of the third inning, the score at the baseball game was 2 for the Cardinals and 4 for the Cubs. The final score was 7 to 3. Who won the game? How do you know?

2. You have just won a contest and you may choose your prize:
   A. You may have $1,000,000; or

   B. You may have a penny a day and double your money each day for one month.

   Which prize would you choose? Why?

3. Each of 8 friends wants to take one ride on a "bicycle built for two" with everyone else. How many bicycle rides do they need to take?

4. There were 156 fifth graders at the carnival waiting for a roller coaster ride. If each roller coaster car holds 8 passengers, how many cars will they need for everyone to have one ride?

5. Write your own story problem using the following data:

9 students	sets of 12 baseball cards	$2 per set

____

____

____

____

Solve and show your thinking.

78

Appendix L

Post Essay

Good

As a math student I'm good 3/14/94
at long $x \div$ I'm also good at
graphs. I think I'm ok
at $(20+3\times9=5-4^2)$ doing prombles
like that. I'm good at shaps
and some tangrams.

Weekness

I'm not so good at some
kinds of tangrams & some
promblum sloving. I'm not
good at what patern will
come next, I'm not so good
at decimals. I'm not good at
mean & range & median

Likeness

I like making quilts. I like
doing geometry. I like writing
in our math log.

Fears

I afread I'm going to do
somthing wrong and get
a bad grade

dislike

I dislike it when I don't
know how to do somthing
and I don't know if I understan
it until I come home.