DOCUMENT RESUME

ED 382 652 TM 023 086

AUTHOR Mislevy, Robert J.; And Others
TITLE Resolving Mixtures of Strategies in Spatial Visualization Tasks.
INSTITUTION Educational Testing Service, Princeton, N.J.; Polytechnic South West, Plymouth (England).
SPONS AGENCY Office of Naval Research, Arlington, VA. Cognitive and Neural Sciences Div.
REPORT NO ETS-RR-90-9-ONR
PUB DATE Jul 90
CONTRACT N00014-88-K-0304
NOTE 65p.
PUB TYPE Reports - Evaluative/Feasibility (142)

EDRS PRICE MF01/PC03 Plus Postage.
DESCRIPTORS *Learning Strategies; Models; *Problem Solving; *Spatial Ability; *Test Theory; *Visualization
IDENTIFIERS Mental Rotation; *Rule Application; Strategy Choice

ABSTRACT

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RR-90-9-ONR

RESOLVING MIXTURES OF STRATEGIES IN SPATIAL VISUALIZATION TASKS

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This research was sponsored in part by the
Program Research Planning Committee
Educational Testing Service; the
Cognitive Science Program
Cognitive and Neural Sciences Division
Office of Naval Research, under
Contract No. N00014-88-K-0304
Contract Authority Identification No.
R&T 4421552; and the
Polytechnic South West under a
Research Agreement with the
Army Personnel Research Establishment
Ministry of Defence, England

Robert J. Mislevy, Principal Investigator

Educational Testing Service
Princeton, New Jersey

July 1990

Unclassified
SECURITY CLASSIFICATION OF THIS PAGE

REPORT DOCUMENTATION PAGE

Form Approved
OMB No. 0704-0188

1a REPORT SECURITY CLASSIFICATION Unclassified	1b RESTRICTIVE MARKINGS
2a. SECURITY CLASSIFICATION AUTHORITY	3. DISTRIBUTION/AVAILABILITY OF REPORT Approved for public release; distribution unlimited.
2b. DECLASSIFICATION/DOWNGRADING SCHEDULE	
4. PERFORMING ORGANIZATION REPORT NUMBER(S) RR-90-9-ONR	5. MONITORING ORGANIZATION REPORT NUMBER(S)
6a. NAME OF PERFORMING ORGANIZATION Educational Testing Service 6b. OFFICE SYMBOL (If applicable)	7a. NAME OF MONITORING ORGANIZATION Cognitive Science Program, Office of Naval Research (Code 1142CS), 800 North Quincy Street
6c. ADDRESS (City, State, and ZIP Code) Princeton, NJ 08541	7b. ADDRESS (City, State, and ZIP Code) Arlington, VA 22217-5000
8a. NAME OF FUNDING/SPONSORING ORGANIZATION 8b. OFFICE SYMBOL (If applicable)	9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER N00014-88-K-0304
8c. ADDRESS (City, State, and ZIP Code)	10. SOURCE OF FUNDING NUMBERS PROGRAM ELEMENT NO.: 61153N PROJECT NO.: RR04204 TASK NO.: RR04204-01 WORK UNIT ACCESSION NO.: R&T4421552

11. TITLE (Include Security Classification)
Resolving Mixtures of Strategies in Spatial Visualization Tasks (Unclassified)

12. PERSONAL AUTHOR(S)
Robert J. Mislevy, Marilyn S. Wingersky, Sidney H. Irvine, Peter L. Dann

13a. TYPE OF REPORT	13b TIME COVERED	14. DATE OF REPORT (Year, Month, Day)	15 PAGE COUNT
Technical	FROM ______ TO ______	July 1990	50

16. SUPPLEMENTARY NOTATION

17. COSATI CODES			18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)
FIELD	GROUP	SUB-GROUP	EM algorithm, item-solving strategies, mental rotation, mixture models, spatial visualization
05	10		

19. ABSTRACT (Continue on reverse if necessary and identify by block number)

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20 DISTRIBUTION/AVAILABILITY OF ABSTRACT ☒ UNCLASSIFIED/UNLIMITED ☐ SAME AS RPT ☐ DTIC USERS	21. ABSTRACT SECURITY CLASSIFICATION Unclassified	
22a. NAME OF RESPONSIBLE INDIVIDUAL Dr. Charles E. Davis	22b TELEPHONE (Include Area Code) 202-696-4046	22c OFFICE SYMBOL ONR 1142CS

DD Form 1473, JUN 86 Previous editions are obsolete. SECURITY CLASSIFICATION OF THIS PAGE

S/N 0102-LF-014-6603 Unclassified

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The work of the first two authors was supported by the Program Research Planning Committee of Educational Testing Service and by Contract No. N00014-88-K-0304, R&T 4421552, from the Cognitive Science Program, Cognitive and Neural Sciences Division, Office of Naval Research. Polytechnic South West has a Research Agreement with the Army Personnel Research Establishment, Ministry of Defence, under which the data collection and the work of the latter two authors were carried out. We are grateful to Murray and Irit Aitkin for their counsel on mixture distributions, to Peter Pashlt and Kentaro Yamamoto for their comments on an earlier draft, and to Officer Commanding, Army Personnel Selection Centre, Sutton Coldfield, for permission to collect and coordinate data from recruits in October, 1988.

Resolving a Mixture of Strategies in
Spatial Visualization Tasks

Abstract

The models of standard test theory, having evolved under a trait-oriented psychology, do not reflect the knowledge structures and the problem-solving strategies now seen as central to understanding performance and learning. In some applications, however, key qualitative distinctions among persons as to structures and strategies can be expressed through mixtures of test theory models, drawing upon substantive theory to delineate the components of the mixture. This approach is illustrated with response latencies to spatial visualization tasks that can be solved by mental rotation or by a nonspatial rule-based strategy. It is assumed that a subject employs the same strategy on all tasks, but the possibility of extending the approach to strategy-switching is discussed.

Key words: EM algorithm, item-solving strategies, mental rotation, mixture models, spatial visualization.

Introduction

Recent research in cognitive and educational psychology reveals the central role of strategies, mental models, and knowledge structures in learning and problem-solving. Clancy (1986) describes the shift in perspective to "describing mental processes, rather than quantifying performance with respect to stimulus variables." To this task, standard test theory, with its focus on overall level of proficiency, is ill suited.

Consider as an example a test in which different persons employ different solution strategies. By characterizing examinees simply in terms of their propensities to make correct responses, the models of standard test theory (item response theory [IRT] as well as classical test theory) cloud analyses in several ways: information about subjects' mental processes is obscured, relationships between tasks' features and difficulties are confounded with strategy choice, and comparisons of subjects in terms of overall proficiencies are equivocal.

Mislevy and Verhelst's (in press) mixture model approach to test theory can be employed to handle certain distinctions of this type, as when different subjects employ different solution strategies. The following assumptions are made:

1. Potential strategies can be delineated a priori.
2. An examinee uses the same strategy on all tasks.

3. An examinee's responses are observed directly but strategy choice is not.

4. The responses of examinees following a given strategy conform to a response model of a known form, possibly characterized by unknown parameters.

5. For each strategy, psychological or substantive theory delineates relationships between the observable features and the difficulties of tasks.

The present paper illustrates this approach by modelling response latencies for spatial rotation tasks that can be solved by either a rotational or a rule-based strategy. Simplified versions of processing models from the visualization literature are the basis of inference about strategy usage. An empirical Bayesian approach provides maximum likelihood estimates of the "structural" parameters of the problem and, for each subject, posterior probabilities of membership in each strategy class and conditional estimates of proficiency under each.

The Data

The data were gathered with a computer administered test of what are typically called "mental rotation tasks." This area was selected by virtue of its long history of research in both psychometrics (e.g., Michael, Guilford, Fruchter, & Zimmerman, 1957; Thurstone, 1938) and cognitive psychology (e.g., Just Carpenter, 1985; Lohman, Pellegrino, Alderton, & Regian, 1987). Of particular interest is the finding that tasks of some tests designed to measure spatial visualization abilities can be solved with nonspatial, analytic strategies (French, 1965; Kyllonen, Lohman, & Snow, 1984; Pellegrino, Mumaw, & Shute, 1985).

The tasks addressed in this paper concern a right-angled triangle whose vertical height was 150 units on a computer presentation screen, and whose horizontal side adjacent to the right angle was 80, 100, 120, or 140 units (see Figure 1). The model triangle was presented on the top half of the screen. Immediately below it was a second triangle whose sides were identical in size, but which had been rotated from the vertical by 40, 80, 120, or 160 degrees. This target was either an exact match to the original or a mirror-image, and the subject was instructed to indicate whether the two triangles were the same or different. Response latency and correctness were recorded. The stimulus set was constructed from the Cartesian product of four factors: side lengths (4), rotations (4), identical or mirror-image (2), and hypotenuse of the model triangle left or right of the vertical (2), for 64 distinct stimuli in total. Because systematic differences are found routinely between patterns on rotations tasks whose correct answer is "same" compared to those for which the correct answer is "different" (e.g., Cooper & Podgorny, 1976), we address only the 32 stimuli in which the target triangle is identical to the model.

[Figure 1 about here]

Subjects were male recruits to the British Army, between 18 and 24 years of age, undergoing selection and allocation in the

Army Personnel Selection Centre. They were tested throughout by a single presenter during the week of October 17, 1988. They were assigned in groups of approximately 40 subjects by daily cohort of recruits to one of three experimental groups. All groups were presented two replications of the entire task set, but they received different instructions. Two groups started under the "standard testing condition" described below, and the third group was additionally shown a nonspatial rule-based strategy for solving the tasks. One of the two standard groups was given the rule-based strategy instruction before their second replication, but because only data from the first replication is addressed here, the two standard groups will not be distinguished.

The standard testing condition was derived from the customary way of demonstrating this class of rotational tasks to naive subjects. An overhead projector was used to display two cardboard right triangles vertically, as in Figure 1, creating solid black images on a white background. The bottom triangle was rotated to several positions to show that it could be "shuffled around on the page" to match the top one. It was physically lifted to show that it could be fitted exactly over the top one. By similar means, the mirror image of the top triangle was shown to be different from the top one, no matter how it was moved about in the plane. The same demonstration was given for both the hypotenuse-left and hypotenuse-right versions of the original stimulus. This introduction was devised to encourage the subjects to use a mental rotation strategy for problem solution, a strategy described by

Shepard and Meltzer (1971) as "isomorphic." Under this strategy, degree of rotation is a primary determiner of task difficulty, in terms of both response latency (Shepard & Meltzer, 1971) and accuracy (Tapley & Bryden, 1977).

The second experimental condition was devised to encourage the use of an "analytic" or "rule-based" strategy. Such strategies employ procedural learning rules rather than mental visualization to determine correct answers. Subjects were shown how to judge whether the target was identical or mirror-image of the model by attending to a specific feature of the triangle, namely the length of the side clockwise adjacent to the right angle. The triangles are different if one encounters the long side of one triangle but the short side of the other. It can be hypothesized that under this strategy, difficulty is nearly independent of the degree of rotation, but depends primarily on the saliency of the key feature; that is, the strategy should be more difficult to implement as the right triangle becomes more nearly isosceles.

The rule was derived by asking British Naval Engineering cadets at the Royal Naval Engineering College at Plymouth how they habitually solved such tasks. While some replied they knew just by looking at them (i.e., their strategies were not consciously available), others said they never rotated the objects themselves; they instead moved their gaze around the model from a fixed point on it, noting the presence or absence of some salient feature. They then found the same starting point on the target and checked for the presence or absence of the feature. If it was there, the target had to be the same as the model. If not, it had to be different.

Data from 244 subjects were initially made available for this study. The analyses reported here concern the 196 remaining after trimming those with the highest and the lowest 10-percent of within task-type variance, as pooled over the 16 rotation/side-length pairs, so as to leave the remaining 80-percent nearly homoscedastic with respect to this component of variation. Of these, 131 had not been instructed in the rule and 65 had. Note that the 2:1 ratio reflects equal trimming of noninstructed and instructed subjects.

Figures 2 and 3 plot median latencies to "same" tasks against coded angular displacement and side length, respectively. Throughout the paper we work with natural logarithms of response times, standardized over all tasks and subjects. Figure 2 shows that average response latency tends to increase linearly with angular displacement, which is coded as -1.5, -.5, .5, and 1.5 along the x-axis. The plot symbols are coded side lengths, from shortest (most acute) to longest (most nearly isosceles) as -2.25, -1.25, .75, and 2.75. Coding side length in these unequally spaced intervals produces the nearly linear relationship shown in Figure 3, where the x-axis gives coded side length and plot symbols give coded angular displacements. Table 1 gives three representative subjects' observed means, averaging over left- and right-hypotenuse tasks within rotation and side-length.

[Figures 2 and 3 about here]

[Table 1 about here]

The Model

A mixture model for the data described above is laid out in two phases. First is the response model, which concerns the distribution of response latencies conditional on choice of strategy and proficiency in using that strategy. Second is the population model, which concerns the proportions of subjects that employ the various strategies and the distributions of proficiency within strategy classes. The generic model described below is for a mixture of K strategy groups; a single strategy can be modelled by setting K to one. After presenting the model, we give the results of fitting three single-strategy models and a mixture of two strategies.

The Response Model

Suppose that K potential strategies are available to solve tasks in a test, and each subject uses one strategy exclusively on all tasks. Each subject will be characterized by two vector-valued unobservable variables. The first is the indicator variable $\underset{\sim}{\phi} = (\phi_1,\ldots,\phi_K)$, where ϕ_k takes the value 1 if Strategy k is the one the subject uses, 0 if not. The second is $\underset{\sim}{\theta} = (\theta_1,\ldots,\theta_k)$, where θ_k is proficiency under Strategy k. Only one of the elements of $\underset{\sim}{\theta}$ plays a role in producing the data observed from a given subject, namely the one for which $\phi_k=1$. Task j is characterized by the vector of difficulty parameters

$\underset{\sim}{\beta}_j = (\beta_{j1},\ldots,\beta_{jK})$, where β_{jk} determines the difficulty of Task j for subjects who employ Strategy k.

We work with a log normal distribution for the response latency t_j of a subject to Task j, given that Strategy k is employed. Defining x_j as $\ell n\ t_j$ and δ_k as a scale parameter of the log-normal density pertaining to Strategy k, the response model is given by

$$f(x_j|\underset{\sim}{\phi},\underset{\sim}{\theta},\underset{\sim}{\beta}_j,\underset{\sim}{\delta}) = \prod_k f_k(x_j|\theta_k,\beta_{jk},\delta_k)^{\phi_k} \quad , \qquad [1]$$

where

$$f_k(x_j|\theta_k,\beta_{jk},\delta_k) = \frac{1}{\sqrt{2\pi}\ \delta_k} \exp\left\{- \frac{1}{2\delta_k^2} [x_j-(\theta_k-\beta_{jk})]^2\right\} . \quad [2]$$

Thus [2] gives the density for log response time to Task j for a subject employing Strategy k, with proficiency θ_k under that strategy. The difference $(\theta_k-\beta_{jk})$ is the mean and δ_k is the standard deviation. Note that lower values of θ signify faster expected response times (i.e., more proficiency on the part of the subject), while higher values imply slower response times. Lower values of β signify slower expected response times (greater difficulty on the part of the task), while higher values imply faster response times. The product over k that appears in [1] serves merely to select the f_k that applies, since ϕ_k is 1 in this case and 0 in all others. In the sequel $f_k(x_j|\theta_k,\beta_{jk},\delta_k)$ will be abbreviated at times as $f_k(x_j|\theta_k)$, suppressing the dependence on β_{jk} and δ_k.

Equations [1] and [2] posit individual differences among subjects as to strategy selection and speed-within-strategy. Individual differences corresponding to slopes might also be entertained, but to do so here increases complexity without adding insight into the inferential approach. We note in passing, however, that recent studies suggest that at least some of the slope differences among individuals found in standard analyses may be related to the use of different strategies (Carter, Pazak, & Kail, 1983; Just & Carpenter, 1985).

Psychological theory about what makes tasks easy or hard under various strategies appears in the form of models for the β_{jk}'s. Following Scheiblechner (1972) and Fischer (1973), let $\underset{\sim}{q}_{jk}$ be a known vector of coefficients expressing the extent to which Task j exhibits each of M_k features that determine task difficulty under Strategy k; let $\underset{\sim}{\alpha}_k$ be a parameter vector, also of length M_k, that conveys the influence of those features. Assuming a linear relationship between task features and task difficulty,

$$\beta_{jk} = \sum_{m=1}^{M_k} q_{jkm} \alpha_{km} = \underset{\sim}{q}'_{jk} \underset{\sim}{\alpha}_k . \qquad [3]$$

For the rotational strategy, for example, angular displacement is a major determiner of difficulty. Tasks rotated to the four equally-spaced displacements will have q values for this feature coded as -1.5, -.5, .5, or 1.5. The corresponding element of α would indicate the incremental difficulty associated with an increase of one additional unit of angular displacement.

or a strategy under which side length is posited to contribute to difficulty, tasks will have q values of -2.25, -1.25, .75, or 2.75 that indicate the saliency of this key feature. The corresponding element of α would reflect the additional difficulty resulting from an additional increment toward being isosceles.

Task responses are assumed to be independent given $\underset{\sim}{\alpha}$, $\underset{\sim}{\theta}$ and $\underset{\sim}{\phi}$. Letting $\underset{\sim}{x}=(x_1,\ldots,x_n)$ be a vector of log latencies to n tasks,

$$P(\underset{\sim}{x}|\theta_k,\phi_k=1,\underset{\sim}{\alpha}_k,\delta_k) = \prod_j f_k(x_j|\theta_k) = f_k(\underset{\sim}{x}|\theta_k) ,$$

so that

$$P(\underset{\sim}{x}|\underset{\sim}{\theta},\underset{\sim}{\phi},\underset{\sim}{\alpha},\underset{\sim}{\delta}) = \prod_k f_k(\underset{\sim}{x}|\theta_k)^{\phi_k} . \qquad [4]$$

The Population Model

Suppose that subjects are a representative sample from a population in which the proportion employing Strategy k is π_k, with $0<\pi_k<1$. Denote by $\underset{\sim}{\pi}$ the vector $(\pi_1,\ldots,\pi_K)$. Denote by $g_k(\theta_k)$ the density function of θ_k for members of Class k. We assume normal distributions (although other distributions or even nonparametric approximations could be used), so that $g_k(\theta_k|\mu_k,\sigma_k)$ has the form

$$g_k(\theta_k|\mu_k,\sigma_k) = \frac{1}{\sqrt{2\pi}\,\sigma_k} \exp\left\{-\frac{1}{2\sigma_k^2}(\theta_k-\mu_k)^2\right\} .$$

For brevity, denote the population distribution parameters by $\underset{\sim}{\Gamma} = (\underset{\sim}{\gamma}_1,\ldots,\underset{\sim}{\gamma}_K) = (\mu_1,\sigma_1,\ldots,\mu_K,\sigma_K)$.

Estimation

Equation [4] is the <u>conditional</u> probability of a response pattern $\underset{\sim}{x}$, or the probability of observing $\underset{\sim}{x}$ from a subject having particular values of $\underset{\sim}{\theta}$ and $\underset{\sim}{\phi}$. Assuming the population model described above, the probability of observing $\underset{\sim}{x}$ from an examinee selected at random, or the <u>marginal</u> probability of $\underset{\sim}{x}$, is given as

$$p(\underset{\sim}{x}|\underset{\sim}{\alpha},\underset{\sim}{\delta},\underset{\sim}{\pi},\underset{\sim}{\Gamma}) = \sum_{k} \pi_k \int f_k(\underset{\sim}{x}|\theta_k,\underset{\sim}{\alpha}_k,\delta_k)\, g_k(\theta_k|\underset{\sim}{\alpha}_k)\, d\theta_k . \quad [5]$$

Let $\underset{\sim}{x} = (\underset{\sim}{x}_1,\ldots,\underset{\sim}{x}_N)$ be the response matrix of a random sample of N subjects to n tasks. A realization of $\underset{\sim}{x}$ induces the marginal likelihood function for $\underset{\sim}{\alpha}$, $\underset{\sim}{\delta}$, $\underset{\sim}{\pi}$, and $\underset{\sim}{\Gamma}$ as the product over subjects of factors like [5]:

$$L(\underset{\sim}{\alpha},\underset{\sim}{\delta},\underset{\sim}{\pi},\underset{\sim}{\Gamma}|\underset{\sim}{x}) = \prod_{i=1}^{N} p(\underset{\sim}{x}_i|\underset{\sim}{\alpha},\underset{\sim}{\delta},\underset{\sim}{\pi},\underset{\sim}{\Gamma}) . \quad [6]$$

We refer to $\underset{\sim}{\alpha}$, $\underset{\sim}{\delta}$, $\underset{\sim}{\pi}$, and $\underset{\sim}{\Gamma}$ as the <u>structural</u> parameters of the problem. Their number remains constant as N increases. The <u>incidental</u> parameters $\underset{\sim}{\theta}$ and $\underset{\sim}{\phi}$, whose numbers increase in direct proportion to N, have been eliminated by marginalizing over their respective distributions. Marginal maximum likelihood (MML) estimation finds the values of the structural parameters that maximize [6], say $\hat{\underset{\sim}{\alpha}}$, $\hat{\underset{\sim}{\delta}}$, $\hat{\underset{\sim}{\pi}}$, and $\hat{\underset{\sim}{\Gamma}}$. The Appendix gives a numerical solution for the present problem using Dempster, Laird, and Rubin's (1977) EM algorithm. The solution is for data in which strategy use is not known with certainty for any subject, as was the case in the present study. If strategy-use information is available for some subjects, however, as from follow-up interviews with a subsample, estimates of structural parameters can be improved--sometimes dramatically--by exploiting it (Titterington, Smith, & Makov, 1985, Section 4.2).

Once MML estimates of structural parameters have been obtained, one can obtain empirical Bayesian approximations of probabilities of class membership for any examinee, and estimates of θ_k conditional on membership in any class k. If the structural parameters were known with certainty, the posterior density for Subject i would be

$$p(\theta_k,\phi_k=1|\underset{\sim}{x}_i,\underset{\sim}{\alpha},\underset{\sim}{\delta},\underset{\sim}{\pi},\underset{\sim}{\Gamma})$$

$$= \frac{\pi_k\ f_k(\underset{\sim}{x}_i|\theta_k,\underset{\sim}{\alpha}_k,\delta_k)\ g_k(\theta_k|\underset{\sim}{\alpha}_k)}{\sum_h \pi_h \int f_h(\underset{\sim}{x}_i|\theta_h,\underset{\sim}{\alpha}_h,\delta_h)\ g_h(\theta_h|\underset{\sim}{\alpha}_h)\ d\theta_h} \quad . \qquad [7]$$

An empirical Bayes approximation substitutes MLEs for the structural parameters. The posterior probability that Subject i employed Strategy k, denoted $\tilde{P}_{ik}$, is approximated as

$$\tilde{P}_{ik} = P(\phi_k=1|\underset{\sim}{x}_i) \approx \int p(\theta_k,\phi_k=1|\underset{\sim}{x}_i,\hat{\underset{\sim}{\alpha}},\hat{\underset{\sim}{\delta}},\hat{\underset{\sim}{\pi}},\hat{\underset{\sim}{\Gamma}})\ d\theta_k \ . \qquad [8]$$

Conditional on membership in Class k, the posterior expectation and variance of θ_k are approximated as

$$\tilde{\theta}_{ik} = E(\theta_k | \phi_{ik}=1, \underset{\sim}{x}_i) \approx \tilde{P}_{ik}^{-1} \int \theta_k \, p(\theta_k, \phi_k=1 | \underset{\sim}{x}_i, \hat{\underset{\sim}{\alpha}}, \hat{\underset{\sim}{\delta}}, \hat{\underset{\sim}{\pi}}, \hat{\underset{\sim}{\Gamma}}) \, d\theta_k \quad [9]$$

and

$$\mathrm{Var}(\theta_k | \phi_{ik}=1, \underset{\sim}{x}_i) \approx$$

$$\tilde{P}_{ik}^{-1} \int \theta_k^2 \, p(\theta_k, \phi_k=1 | \underset{\sim}{x}_i, \hat{\underset{\sim}{\alpha}}, \hat{\underset{\sim}{\delta}}, \hat{\underset{\sim}{\pi}}, \hat{\underset{\sim}{\Gamma}}) \, d\theta_k - \tilde{\theta}_{ik}^2 . \quad [10]$$

Single-Strategy Solutions

This section uses single-strategy solutions to illustrate Q matrices, obtain baseline likelihood statistics, calculate task-difficulty estimates under different models, and give a feel for the data by looking more closely at the subjects introduced in Table 1. These solutions assume not only that a subject applies the same strategy on all tasks, but that all subjects are applying the same strategy. Three such solutions are considered:

1. Rule-based (difficulty depends on side length only),
2. Mental rotation without side-length effects (difficulty depends on degree of rotation only), and
3. Mental rotation with side-length effects (difficulty depends on both side length and degree of rotation).

Side-Length Only

Suppose that all subjects followed a rule-based strategy, under which difficulty is determined by side length alone. Under this scenario, the only systematic sources of variation in response times are subjects' overall proficiencies and tasks' differing side lengths. Proficiency is denoted by the univariate variable θ, assumed to follow a normal distribution whose mean μ and standard deviation σ are to be estimated. Task difficulty is given as $\beta_j = q_{j1}\,\alpha_1$, where q_{j1} is the coded side-length of task j (-2.25, -1.25, .75, or 2.75), and α_1 is the change in difficulty associated with an additional increment in side length toward isosceles. Finally, δ is the standard deviation of log response time within task type and subject.

The first panel of Table 2 gives the MLEs of the structural parameters. The value of -2 log likelihood, or -2 $\ell n\ \Sigma\ p(\underset{\sim}{x}_i)$, also appears, which will be used to compare the fit of alternative models. The estimate of α_1 is -.124, which translates into expected increases in response latency as side length increases. The first panel of Table 3 gives the resulting βs, with higher values corresponding to faster response times and lower numbers corresponding to slower ones. The modelled standardized log latency of a particular subject on a particular task type under this model would be obtained by subtracting the task's β value from that subject's θ value--thereby maintaining for all subjects the pattern of increasing difficulty with increasing side length without effects of degree of rotation.

[Insert Tables 2 and 3 about here]

This pattern is illustrated with the subject whose observed means data appear in the second panel of Table 1. The Bayes estimate $\tilde{\theta}$ is -.317, a measure of overall proficiency. Combining

$\tilde{\theta}$ with the βs gives the modelled latencies in the second panel of Table 4. The third panel gives residuals, which indicate faster averages than expected in some cells, slower in others, but no systematic trend in average residuals in either the side-length or rotation margins. (The residuals do not sum to zero because the expectations were based on a Bayesian proficiency estimate, as opposed to, say, least-squares within subject.) Table 5 gives residuals for all three sample subjects. Note the strong trend related to rotation for the last subject.

[Insert Tables 4 and 5 about here]

Rotation Only

Even though the side-length model fits the data of subjects like the second fairly well, the familiar relationship between difficulty and degree of rotation appears in the residuals of subjects like the fifth. An alternative simple model has difficulty depend on rotation only, neglecting the possible impact of differences in side-length. Under this model, $\beta = q_{2j}\ \alpha_2$, where q_{2j} is the coded rotation of Task j (-1.5, -.5, .5, or 1.5) and α_2 is the impact on difficulty of an additional increment in rotation. The results of fitting this model appear in the second panel of Table 2. The MLE of α_2 is -.108, signalling increasing difficulty as the degree of rotation increases. The resulting βs are in the second panel of Table 3, showing patterns of expected difficulty that depend on rotation alone.

The value of -2 log likelihood is 14907.78. To compare the fit of alternative, possibly nonnested, models to the same data set, Akaike (1985) calculates the index AIC ("an information criterion") for each model: -2 log likelihood plus twice the number of parameters estimated. The model with the smaller AIC is preferred. AIC for rotation-only is 14907.78 + 2×4 = 14915.78. AIC for side-length only, which also has four parameters, is 14432.40--smaller by 483. The side-length only model fits better than the rotation only model.

Subject θ estimates in this model combine with the βs to give expected response latencies that maintain the same relative relationships seen in the βs, but vary as to overall speed. Subtracting such expectations calculated with $\bar{\theta}$s from the observed data gives the residuals shown in Table 6 for the sample subjects. The third subject's strong linear trend in residuals associated with rotation has been dampened considerably, but a reverse trend in the rotation margin has been introduced for the first two subjects. Note also the tendency toward a consistent ordering in the side-length residuals.

[Table 6 about here]

Rotation and Side-Length

A final single-strategy model incorporates main effects for both rotation and side-length: $\beta_j = q_{j1}\alpha_1 + q_{j2}\alpha_2$, where the qs and αs have the same meanings as in the preceding models. This incorporates a more elaborate scenario for mental rotation, allowing for latency to increase not only with degree of rotation, but with the difficulty of comparison after rotation is complete (Cooper & Podgorny, 1976). Parameter estimates and -2 log likelihood appear in the third panel of Table 2. Because of the orthogonal experimental design, $\hat{\alpha}$s are identical to those of the previous single-feature models. AIC for this model is 14258.72, smaller than the side-length AIC by 173 and smaller than the rotation AIC by 661. The model incorporating both task features thus fits better than both models with a single feature only, even accounting for the additional parameters being estimated. The resulting βs appear in the third panel of Table 3.

The residuals for the sample subjects are in Table 7. Because of the orthogonal design, the rotation margins of the residual tables under this model match the corresponding margins under the rotation only model, and the side-length margins match those from the side-length only model.

[Table 7 about here]

A Two-Strategy Solution

This model posits a mixture of two types of subjects: those employing the rule-based strategy, whose response patterns can be largely captured with the side-length only model, and those employing mental rotation, whose responses depend on both side-length and degree of rotation. There are ten parameters to estimate in this model: the relative proportion of strategy use in the sample; a mean, standard deviation, and within task-type standard deviation for each strategy; an α for side-length for the rule-based strategy component; and two αs, one for side-length and one for rotation, for the rotation strategy component. The estimates appear in the final panel of Table 2. AIC improves over the rotation and side-length single-strategy model by 72.

Two distinct sets of βs result in this model, one for each strategy. These appear in the last two panels of Table 3. The side-only component, like the side-length only single strategy panel, shows only a side-length effect. Interestingly, it is smaller than the corresponding effect in the side-length single-strategy model. The "side-length and rotation" component shows both effects. Compared to the corresponding single strategy solutions, both the side-length and the rotational effects are stronger. The interpretation would be that decreasing the distinctness of the salient feature of the stimulus hampers subjects employing rotation more than those employing the rule.

The point of mixture modelling is that we do not know with certainty which subjects are employing which strategy. One of the structural parameters is the proportion using each; MLEs are 57-percent for the rule-based strategy and 43-percent for the rotational strategy. These values can be expressed as the averages of subjects' posterior probabilities of being in one strategy group or the other (Equation 8). The histograms in Figure 4 show subjects' posterior probabilities for the rule-based strategy. Most have probabilities below .2 or above .8, indicating fairly good separation of the components of the mixture. To put this in the context of the statistical literature on mixtures (e.g., Titterington et al., 1985), Figure 5 gives the histogram of a closely matching "mixture of homoscedastic Gaussian components" problem. The mixing probability is .55 and the distance between the means of the components is 2.2 standard deviations. The information about the mixing proportion is about half what it would be if component membership were observable (Hill, 1963). Figure 6 breaks the information from Figure 4 down by whether subjects were shown the rule. The proportion of instructed subjects whose responses are strongly allied with the rule-based strategy is substantially higher--nearly half, as compared to just a fourth of those not instructed.

[Insert Figures 4-6 about here]

Table 8 gives posterior probabilities and residuals for the sample subjects. As suggested by patterns of residuals from the single strategy models, the posterior probabilities of Subjects 1 and 2 are concentrated on Strategy 1, the rule-based strategy ($\tilde{P}$s of .98 and .70), and that of Subject 3 is concentrated on the rotational strategy ($\tilde{P}$ nearly 1.00). The main factor that determines strategy assignment is the presence of a linear trend related to rotation. Of secondary importance is the strength of the trend related to side length, with stronger trend being associated with the rotational strategy. The high probability of the rule-based strategy for Subject 1 is a consequence of both a lack of trend for rotation and a weak trend for side length in the observed data (see Table 1).

For a decision-making problem that depends on identifying a subject's strategy use, the strategy with the higher posterior probability is the choice. For a problem that depends on predicting future performance, the optimal prediction is a mixture. For Subject 2, for example, predictions would be calculated under both strategies, using the task parameters and his posterior θ distribution within each; these distinct predictions would be averaged with weights of .70 for the rule-based strategy and .30 for the rotational one. The predictions for the residuals in Table 8 were calculated in this manner, using conditional within-strategy posterior means for θs.

[Insert Table 8 about here]

Discussion

This presentation was meant to demonstrate how a mixture-model approach can be incorporated into test theory to deal with different problem solving approaches. We readily concede that the psychological model for the visualization tasks is overly simplistic. There are some ways it could be made more realistic while preserving the assumption that a subject maintains the same strategy throughout observation. These include modelling subprocesses, relaxing distributional assumptions, or incorporating individual difference terms for residual variances or sensitivity to rotational angles. But more substantial steps toward reality lie beyond this framework.

It must first be admitted that the two strategies discussed here do not exhaust the variety of approaches that subjects bring to bear upon such tasks. They may be viewed as archetypes, one or the other of which may be sufficiently close to a given subject's data to provide a serviceable guide to selection or instruction.

In applied work, analyses of subjects' fit and patterns of residuals will be important to flag patterns that are not modelled, either for special consideration if they are few in number, or for incorporation into the model if they are recurrent.

Perhaps more importantly, we must consider the possibility of strategy switching, or the violation of the assumption that a subject follows the same strategy on all tasks. This assumption may be reasonable if strategy is defined through knowledge structures, so that response patterns associated with advanced structures are not accessible to learners in earlier stages. An example of this type is Siegler's analysis (1981) balance beam tasks, where competence is increased by adding rules to a repertoire in a largely predictable order. It may also be reasonable if strategies associated with alternative mental models of a domain are unlikely to coexist in a given subject.

The same assumption is less reasonable in a domain where alternative strategies are available to the same subject, so that the conditions and the frequency of strategy switching become additional sources of individual difference. Kyllonen, Lohman, and Snow (1984) note increasing use of non-spatial strategies as mental rotation tasks become more difficult. A more appropriate model in this case would characterize mixtures within subjects; not just that a subject followed Strategy A or Strategy B, but that she used Strategy A on 30-percent of the tasks and Strategy B on 70-percent--and that the characteristics of tasks that were relevant to her strategy choice were such-and-such. Given the vicissitudes of even the straight mixture model, it is clear that modelling mixtures within subjects will require richer information than the familiar correctness or response times, perhaps in the form of response protocols, intermediate products, or physical measures such as eye movement (as in Just & Carpenter, 1976).

It would appear that mixture models of the type illustrated here have most promise in contexts where the qualitative distinctions among persons are relatively few in number, stable during the period of observation, and distinguishable in terms of their implications for observable behavior. We have demonstrated by example that the calculations in this case are tractable and the mixtures can be resolved satisfactorily. Future work will focus on a more general computational scheme and a broader variety of more educationally relevant applications.

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Appendix: Estimating Structural Parameters via the EM Algorithm

Equation [5] is an "incomplete data" density function of the form addressed by Dempster, Laird, and Rubin (1977) in "Maximum likelihood from incomplete data via the EM algorithm." Estimating the structural parameters would be straightforward if values of the latent variables $\underset{\sim}{\theta}$ and $\underset{\sim}{\phi}$ could be observed from each subject along with his or her response vector $\underset{\sim}{x}$; this would be a "complete data" problem. The EM algorithm maximizes the incomplete-data likelihood [6] iteratively. The E-step, or expectation step, of each cycle, calculates the expectations of the summary statistics that the complete-data problem would require, conditional on the observed data and provisional estimates of the structural parameters. The M-step, or maximization step, solves what looks like a complete-data maximum likelihood problem using the conditional expectations of summary statistics. The resulting maxima for the structural parameters are improved estimates of the incomplete-data solution, and serve as input to the next E-step.

We employ the variation of the EM algorithm suggested by Mislevy and Verhelst (in press) to estimate the parameters of mixtures of psychometric models. The integration that appears in [5] is approximated by summation over a fixed grid of points. The E-step calculates, for each examinee, conditional probabilities of belonging to each component of the mixture (i.e., strategy class) and, conditional on component membership, the probabilities that θ takes various grid-point values. The grid points play the role of weighted pseudo-data points in the M-step. Numerical integration could be avoided in the special case considered above, but the approach described below also applies with alternative response models and distributional forms.

Solving the "Complete Data" Problem

This section gives the ML solution for $\underset{\sim}{\alpha}$, $\underset{\sim}{\delta}$, $\underset{\sim}{\pi}$, and $\underset{\sim}{\Gamma}$ that would obtain if values of $\underset{\sim}{\theta}$ and $\underset{\sim}{\phi}$ were observed for each subject along with $\underset{\sim}{x}$. Among the N_k sampled subjects from Strategy Class k, some number $L_k \leq N_k$ distinct values of θ would be observed, say $\Theta_{k\ell}$ for $\ell=1,\ldots,L_k$. Define the following statistics:

- $I_{ik\ell}$, an indicator variable that takes the value 1 if Subject i is in Strategy Class k and has proficiency $\Theta_{k\ell}$.

- N_k, the number of examinees observed to be in Class k:

$$N_k = \sum_i \phi_{ik} = \sum_i \sum_\ell I_{ik\ell} . \qquad [A1]$$

- $N_{k\ell}$, the number of examinees in Class k with $\theta=\Theta_{k\ell}$:

$$N_{k\ell} = \sum_i I_{ik\ell} . \qquad [A2]$$

- $\bar{x}_{jk}$, the mean of θ-centered log latencies for Task j from subjects in Class k:

$$\bar{x}_{jk} = N_k^{-1} \sum_i \sum_\ell (x_{ij}-\Theta_{k\ell}) \, I_{ik\ell} . \qquad [A3]$$

- s_{jk}^2, the average square of θ-centered log latencies for Task j from subjects in Class k:

$$s_{jk}^{2} = N_{k}^{-1} \sum_{i} \sum_{\ell} (x_{ij} - \theta_{k\ell})^{2} \, I_{ik\ell} \; . \qquad [A4]$$

The complete data likelihood for $\underset{\sim}{\alpha}$, $\underset{\sim}{\delta}$, $\underset{\sim}{\pi}$, and $\underset{\sim}{\Gamma}$ that would be induced by the observation of $\underset{\sim}{x}$, $\underset{\sim}{\theta}$, and $\underset{\sim}{\phi}$ can be written as

$$L^{*}(\underset{\sim}{\alpha},\underset{\sim}{\delta},\underset{\sim}{\pi},\underset{\sim}{\Gamma}|\underset{\sim}{x},\underset{\sim}{\theta},\underset{\sim}{\phi}) =$$

$$\prod_{k} P(N_{k}|\underset{\sim}{\pi}) \prod_{\ell} P(N_{k\ell}|N_{k},\underset{\sim}{\alpha}) \prod_{i} f_{k}(\underset{\sim}{x}_{i}|\theta_{k\ell},\underset{\sim}{\alpha}_{k},\delta_{k})^{I_{ik\ell}} \quad ,$$

whence the complete data log likelihood

$$\ell^{*}(\underset{\sim}{\alpha},\underset{\sim}{\delta},\underset{\sim}{\pi},\underset{\sim}{\Gamma}|\underset{\sim}{x},\underset{\sim}{\theta},\underset{\sim}{\phi}) = \sum_{k} N_{k} \log \pi_{k} \sum_{\ell} N_{k\ell} \log g_{k}(\theta_{k\ell}|\mu_{k},\sigma_{k})$$

$$+ \sum_{i} \sum_{k} \sum_{\ell} I_{ik\ell} \sum_{j} \log f_{k}(x_{ij}|\theta_{k\ell},\underset{\sim}{\alpha}_{k},\delta_{k}) \; . \qquad [A5]$$

ML estimation for the complete data problem proceeds by solving the likelihood equations, which are obtained by setting to zero the first derivatives of [A5] with respect to each element of $(\underset{\sim}{\alpha},\underset{\sim}{\delta},\underset{\sim}{\pi},\underset{\sim}{\Gamma})$. That is, for a generic element u of $(\underset{\sim}{\alpha},\underset{\sim}{\delta},\underset{\sim}{\pi},\underset{\sim}{\Gamma})$,

$$0 = \frac{\partial \ell^{*}(\underset{\sim}{\alpha},\underset{\sim}{\delta},\underset{\sim}{\pi},\underset{\sim}{\Gamma}|\underset{\sim}{x},\underset{\sim}{\theta},\underset{\sim}{\phi})}{\partial u} \; . \qquad [A6]$$

For elements of $\underset{\sim}{\pi}$, one must impose the constraint that $\Sigma\pi_{k}=1$, say with a Lagrangian multiplier. One obtains a closed form solution for the proportion of subjects in each strategy class:

$$\hat{\pi}_k = N_k/N \ . \qquad [A7]$$

For elements of the population parameter vector $\underset{\sim}{\Gamma}$, the ML estimates in the normal case are, for k=1,...,K,

$$\hat{\mu}_k = N_k^{-1} \sum_{\ell} \theta_{k\ell} N_{k\ell} \qquad [A8]$$

and

$$\hat{\sigma}_k^2 = N_k^{-1} \sum_{\ell} (\theta_{k\ell} - \hat{\mu}_k)^2 N_{k\ell} \ . \qquad [A9]$$

For the parameters of the task response model,

$$\hat{\underset{\sim}{\alpha}}_k = - (\underset{\sim}{Q}_k \underset{\sim}{Q}_k')^{-1} \underset{\sim}{Q}_k \bar{\underset{\sim}{x}}_k \ , \qquad [A10]$$

where $\underset{\sim}{Q}_k = [\underset{\sim}{q}_{1k} \ldots \underset{\sim}{q}_{nk}]$ and $\bar{\underset{\sim}{x}}_k = (\bar{x}_{1k}, \ldots, \bar{x}_{nk})$, and

$$\hat{\delta}_k^2 = n^{-1} \sum_j \left[s_{jk}^2 + 2 \hat{\beta}_{jk} \bar{x}_{jk} + \hat{\beta}_{jk}^2 \right] \ . \qquad [A11]$$

Solving the "Incomplete Data" Problem

The likelihood equations in the incomplete-data problem, in which $\underset{\sim}{\theta}$ and $\underset{\sim}{\phi}$ are <u>not</u> observed, are the first derivatives of the log of the marginal likelihood function, [6], set to zero. Under mild regularity conditions, these take the form of a data-weighted average of the complete-data likelihood equations shown as [A6]. Letting $\ell(\underset{\sim}{\alpha},\underset{\sim}{\delta},\underset{\sim}{\pi},\underset{\sim}{\Gamma}|\underset{\sim}{x}) = \ell n\ L(\underset{\sim}{\alpha},\underset{\sim}{\delta},\underset{\sim}{\pi},\underset{\sim}{\Gamma}|\underset{\sim}{x})$, the incomplete-data likelihood equation for a generic element u of $(\underset{\sim}{\alpha},\underset{\sim}{\delta},\underset{\sim}{\pi},\underset{\sim}{\Gamma})$ is

$$0 = \frac{\partial\ \ell(\underset{\sim}{\alpha},\underset{\sim}{\delta},\underset{\sim}{\pi},\underset{\sim}{\Gamma}|\underset{\sim}{X})}{\partial u}$$

$$0 = \sum_k \int \frac{\partial \ell^*(\underset{\sim}{\alpha},\underset{\sim}{\delta},\underset{\sim}{\pi},\underset{\sim}{\Gamma}|\underset{\sim}{x},\underset{\sim}{\theta},\phi_k=1)}{\partial u}\ p(\underset{\sim}{\theta},\phi_k=1|\underset{\sim}{x})\ d\underset{\sim}{\theta}\ , \qquad [A12]$$

where

$$p(\underset{\sim}{\theta},\underset{\sim}{\phi}|\underset{\sim}{x}) = \prod_i \frac{p(\underset{\sim}{x}_i|\underset{\sim}{\theta}_i,\underset{\sim}{\phi}_i,\underset{\sim}{\alpha},\underset{\sim}{\delta},\underset{\sim}{\pi},\underset{\sim}{\Gamma})}{p(\underset{\sim}{x}_i|\underset{\sim}{\alpha},\underset{\sim}{\delta},\underset{\sim}{\pi},\underset{\sim}{\Gamma})}\ . \qquad [A13]$$

It is seen in [5] that evaluating the denominator of [A13] involves integration over the $N(\mu_k,\sigma_k)$ distributions. We approximated these integrals via Gaussian quadrature. Tabled values are obtained for L points $\Theta_1,\ldots,\Theta_L$, with associated weights $W_1,\ldots,W_L$. (We used L=80, ranging from -3 to +3). Calculation proceeds as though these were the only possible values for θ; $I_{ik\ell}$ indicates whether Subject i used Strategy k and had proficiency Θ_ℓ.

In the incomplete-data problem, neither the values of $\underset{\sim}{\theta}_i$ nor $\underset{\sim}{\phi}_i$ are known, so neither are the $I_{ik\ell}$ values used to calculate summary statistics. If the values of the structural parameters $\underset{\sim}{\Gamma}$, $\underset{\sim}{\pi}$, $\underset{\sim}{\delta}$, and $\underset{\sim}{\alpha}$ were known, however, it would be possible to calculate the expected values of the $I_{ik\ell}$s given $\underset{\sim}{x}_i$s as follows:

$$\tilde{I}_{ik\ell} = E(I_{ik\ell} | \underset{\sim}{x}_i, \underset{\sim}{\alpha}, \underset{\sim}{\delta}, \underset{\sim}{\pi}, \underset{\sim}{\Gamma})$$

$$= \frac{\pi_k \, W_\ell \, f_k(\underset{\sim}{x}_i | \Theta_\ell, \underset{\sim}{\alpha}_k, \delta_k) \; g_k(\Theta_\ell | \gamma_k)}{\sum_h \pi_h \sum_r W_r \, f_h(\underset{\sim}{x}_i | \Theta_r, \underset{\sim}{\alpha}_h, \delta_h) \; g_h(\Theta_\ell | \gamma_h)} . \qquad [A14]$$

In the E-step of the EM algorithm, one evaluates [A14] for each i, k, and ℓ using provisional estimates of $\underset{\sim}{\alpha}$, $\underset{\sim}{\delta}$, $\underset{\sim}{\pi}$, and $\underset{\sim}{\Gamma}$. One then obtains expectations of the summary statistics defined in [A1]-[A4], say $\tilde{N}_k$, $\tilde{N}_{k\ell}$, $\tilde{x}_{jk}$, and $\tilde{s}^2_{jk}$. The grid values Θ_ℓ in the incomplete-data solution thus correspond to the observed θ values in the complete data solution.

In the M-step, one solves facsimiles of the complete data likelihood equations, [A7]-[A11], with $\tilde{N}_k$, $\tilde{N}_{k\ell}$, $\tilde{x}_{jk}$, and $\tilde{s}^2_{jk}$ in place of their observed counterparts. Cycles of E- and M-steps continue until changes are suitably small. Convergence to a local maximum is assured, except from initial values that lie on boundaries of the parameter space (e.g., $\pi_1=0$). Repeated solutions from different starting values help identify the global maximum. Accelerating methods may be used if convergence is too slow.

Equation [A14] will be recognized as an application of Bayes theorem, giving the posterior probability that $\theta_{ik}=\Theta_\ell$ and $\phi_{ik}=1$ after observing $\underset{\sim}{x}_i$. The normalizing constant in the denominator is an approximation of $p(\underset{\sim}{x}_i)$ as given in [5]. During the E-step, one may therefore accumulate the sum $-2\ \Sigma \log p(\underset{\sim}{x}_i)$ to track the performance of improvement in fit over cycles, to examine the fit obtained with various values of structural parameters, or to compare the fit of alternative models.

Empirical Bayes Estimates of Examinee Parameters

The numerical approximations employed above to estimate structural parameters can be used for subsequent empirical Bayesian estimates for individual subjects. The expectations of the indicator variables $I_{ik\ell}$ are evaluated via [A14] with MML estimates of $\underset{\sim}{\alpha}$, $\underset{\sim}{\delta}$, $\underset{\sim}{\pi}$, and $\underset{\sim}{\Gamma}$. The empirical Bayes approximation of probability of membership in Strategy Class k is given as

$$\tilde{P}_{ik} = P(\phi_{ik}=1|\underset{\sim}{x}_i) \approx \sum_{\ell} \bar{I}_{ik\ell} \; . \qquad [A15]$$

Conditional on membership in Class k, the posterior expectation of θ_{ik} is approximated as

$$\bar{\theta}_{ik} = E(\theta_{ik}|\phi_{ik}=1,\underset{\sim}{x}_i) \approx \bar{P}_{ik}^{-1} \sum_{\ell} \Theta_{\ell} \; \bar{I}_{ik\ell} \qquad [A16]$$

and the corresponding posterior variance is

$$Var(\theta_{ik}|\phi_{ik}=1,\underset{\sim}{x}_i) \approx \bar{P}_{ik}^{-1} \sum_{\ell} \Theta_{\ell}^2 \bar{I}_{ik\ell} - \bar{\theta}_{ik}^2 \; . \qquad [A17]$$

Table 1

Examples of Observed Standardized Mean Log Response Times

Subject 1

Side Length	Rotation 40	80	120	160	Average
80	-0.069	0.059	-0.508	-0.676	-0.299
100	-0.724	-0.483	-0.242	-0.383	-0.458
120	-0.591	-0.828	0.406	-0.993	-0.502
140	-0.288	-0.287	-0.485	-0.483	-0.386
Average	-0.418	-0.385	-0.207	-0.634	-0.411
Within Cell Pooled Standard Deviation					0.779

Subject 2

Side Length	Rotation 40	80	120	160	Average
80	-1.692	-0.277	-1.012	-0.863	-0.961
100	0.705	0.514	-0.450	-0.871	-0.025
120	-1.082	-0.912	0.224	0.126	-0.411
140	0.661	0.015	-0.316	-0.035	0.081
Average	-0.352	-0.165	-0.389	-0.411	-0.329
Within Cell Pooled Standard Deviation					0.713

Subject 3

Side Length	Rotation 40	80	120	160	Average
80	-0.413	-0.545	-0.192	-0.304	-0.363
100	-1.699	-1.849	-0.108	0.223	-0.858
120	-1.186	-0.019	-0.189	0.966	-0.107
140	-0.451	-0.815	-0.525	0.404	-0.347
Average	-0.937	-0.807	-0.254	0.322	-0.419
Within Cell Pooled Standard Deviation					0.687

Table 2

Parameter Estimates and -2 Log Likelihood

Side Length Only — -2 log likelihood = 14424.40

μ	σ	δ	$\alpha(1)$
0.000	0.645	0.726	-0.124

Rotation Only — -2 log likelihood = 14907.78

μ	σ	δ	$\alpha(2)$
0.000	0.644	0.756	-0.108

Side Length and Rotation — -2 log likelihood = 14248.72

μ	σ	δ	$\alpha(1)$	$\alpha(2)$
0.000	0.645	0.716	-0.124	-0.108

Two Strategy Model — -2 log likelihood = 14167.03

Strategy	π	μ	σ	δ	$\alpha(1)$	$\alpha(2)$
Rule	.566	0.099	0.745	0.668	-0.104	
Rotation	.434	-0.129	0.455	0.753	-0.151	-0.219

Table 3

Task Difficulty Parameters

Side Length Only Model

Side Length	Rotation 40	80	120	160
80	0.279	0.279	0.279	0.279
100	0.155	0.155	0.155	0.155
120	-0.093	-0.093	-0.093	-0.093
140	-0.341	-0.341	-0.341	-0.341

Rotation Only Model

Side Length	Rotation 40	80	120	160
80	0.162	0.054	-0.054	-0.162
100	0.162	0.054	-0.054	-0.162
120	0.162	0.054	-0.054	-0.162
140	0.162	0.054	-0.054	-0.162

Side Length and Rotation Model

Side Length	Rotation 40	80	120	160
80	0.441	0.333	0.225	0.117
100	0.317	0.209	0.101	-0.007
120	0.069	-0.039	-0.147	-0.255
140	-0.180	-0.287	-0.395	-0.503

(continued)

Table 3, continued

Task Difficulty Parameters

Two Strategy Model: Side Length Only Component

	Rotation			
Side Length	40	80	120	160
80	0.234	0.234	0.234	0.234
100	0.130	0.130	0.130	0.130
120	-0.078	-0.078	-0.078	-0.078
140	-0.286	-0.286	-0.286	-0.286

Two Strategy Model: Side Length and Rotation Component

	Rotation			
Side Length	40	80	120	160
80	0.667	0.448	0.229	0.010
100	0.517	0.298	0.079	-0.140
120	0.215	-0.003	-0.222	-0.441
140	-0.086	-0.305	-0.524	-0.742

Table 4

Modelling Subject 2 under Side Length Only

Observed Log Response Times

Side Length	Rotation 40	80	120	160	Average
80	-1.692	-0.277	-1.012	-0.863	-0.961
100	0.705	0.514	-0.450	-0.871	-0.025
120	-1.082	-0.912	0.224	0.126	-0.411
140	0.661	0.015	-0.316	-0.035	0.081
Average	-0.352	-0.165	-0.389	-0.411	-0.329

$\tilde{\theta} = -.317$

Modelled Log Response Times

Side Length	Rotation 40	80	120	160	Average
80	-0.599	-0.599	-0.599	-0.599	-0.599
100	-0.475	-0.475	-0.475	-0.475	-0.475
120	-0.227	-0.227	-0.227	-0.227	-0.227
140	0.021	0.021	0.021	0.021	0.021
Average	-0.320	-0.320	-0.320	-0.320	-0.320

Residuals

Side Length	Rotation 40	80	120	160	Average
80	-1.096	0.319	-0.416	-0.267	-0.365
100	1.177	0.986	0.021	-0.399	0.446
120	-0.859	-0.689	0.447	0.350	-0.188
140	0.636	-0.010	-0.341	-0.060	0.056
Average	-0.035	0.152	-0.072	-0.094	-0.013
Root Mean Square Error					0.619

Table 5

Residuals from Side Length Only Model

Subject 1 ($\tilde{\theta}$ = -.40)

Side Length	Rotation 40	80	120	160	Average
80	0.605	0.733	0.167	-0.002	0.376
100	-0.174	0.067	0.309	0.168	0.092
120	-0.289	-0.526	0.708	-0.691	-0.199
140	-0.234	-0.233	-0.431	-0.429	-0.332
Average	-0.023	0.011	0.188	-0.238	-0.016
Root Mean Square Error					0.426

Subject 2 ($\tilde{\theta}$ = -.32)

Side Length	Rotation 40	80	120	160	Average
80	-1.096	0.319	-0.416	-0.267	-0.365
100	1.177	0.986	0.021	-0.399	0.446
120	-0.859	-0.689	0.447	0.350	-0.188
140	0.636	-0.010	-0.341	-0.060	0.056
Average	-0.035	0.152	-0.072	-0.094	-0.013
Root Mean Square Error					0.619

Subject 3 ($\tilde{\theta}$ = -.40)

Side Length	Rotation 40	80	120	160	Average
80	0.269	0.138	0.490	0.379	0.319
100	-1.140	-1.291	0.450	0.781	-0.300
120	-0.877	0.290	0.121	1.276	0.203
140	-0.389	-0.753	-0.463	0.465	-0.285
Average	-0.534	-0.404	0.149	0.725	-0.016
Root Mean Square Error					0.703

Table 6

Residuals from Rotation Only Model

Subject 1 ($\tilde{\theta}$ = -.39)

	Rotation				
Side Length	40	80	120	160	Average
80	0.487	0.507	-0.168	-0.444	0.095
100	-0.168	-0.035	0.098	-0.151	-0.064
120	-0.035	-0.380	0.746	-0.761	-0.108
140	0.268	0.162	-0.145	-0.251	0.009
Average	0.138	0.063	0.133	-0.402	-0.017
Root Mean Square Error					0.374

Subject 2 ($\tilde{\theta}$ = -.32)

	Rotation				
Side Length	40	80	120	160	Average
80	-1.215	0.093	-0.750	-0.710	-0.646
100	1.183	0.884	-0.189	-0.717	0.290
120	-0.605	-0.543	0.485	0.280	-0.096
140	1.138	0.384	-0.055	0.119	0.397
Average	0.125	0.205	-0.127	-0.257	-0.014
Root Mean Square Error					0.694

Subject 3 ($\tilde{\theta}$ = -.40)

	Rotation				
Side Length	40	80	120	160	Average
80	0.150	-0.089	0.155	-0.064	0.038
100	-1.135	-1.394	0.240	0.462	-0.457
120	-0.623	0.436	0.159	1.206	0.294
140	0.113	-0.359	-0.177	0.644	0.055
Average	-0.374	-0.352	0.094	0.562	-0.017
Root Mean Square Error					0.623

Table 7

Residuals from Side Length and Rotation Model

Subject 1 ($\tilde{\theta}$ = -.40)

Side Length	Rotation 40	80	120	160	Average
80	0.768	0.788	0.113	-0.163	0.376
100	-0.012	0.122	0.255	0.006	0.093
120	-0.127	-0.471	0.654	-0.852	-0.199
140	-0.072	-0.178	-0.484	-0.591	-0.331
Average	0.139	0.065	0.135	-0.400	-0.015
Root Mean Square Error					0.457

Subject 2 ($\tilde{\theta}$ = -.32)

Side Length	Rotation 40	80	120	160	Average
80	-0.934	0.373	-0.469	-0.429	-0.365
100	1.339	1.040	-0.032	-0.561	0.447
120	-0.696	-0.634	0.393	0.188	-0.187
140	0.798	0.044	-0.395	-0.221	0.057
Average	0.127	0.206	-0.126	-0.256	-0.012
Root Mean Square Error					0.639

Subject 3 ($\tilde{\theta}$ = -.40)

Side Length	Rotation 40	80	120	160	Average
80	0.432	0.192	0.436	0.217	0.319
100	-0.978	-1.237	0.397	0.619	-0.300
120	-0.714	0.345	0.067	1.115	0.203
140	-0.227	-0.699	-0.517	0.304	-0.285
Average	-0.372	-0.350	0.096	0.564	-0.016
Root Mean Square Error					0.626

Table 8

Residuals From Two Strategy Model

Subject 1 ($\tilde{P}_1$ = .98, $\tilde{\theta}_1$ = -.40; $\tilde{P}_2$ = .02, $\tilde{\theta}_2$ = -.39)

	Rotation				
Side Length	40	80	120	160	Average
80	0.571	0.695	0.124	-0.049	0.336
100	-0.188	0.048	0.285	0.140	0.071
120	-0.265	-0.506	0.723	-0.680	-0.182
140	-0.171	-0.174	-0.377	-0.380	-0.276
Average	-0.013	0.016	0.189	-0.242	-0.013
Root Mean Square Error					0.405

Subject 2 ($\tilde{P}_1$ = .70, $\tilde{\theta}_1$ = -.32; $\tilde{P}_2$ = .30, $\tilde{\theta}_2$ = -.31)

	Rotation				
Side Length	40	80	120	160	Average
80	-1.011	0.338	-0.462	-0.380	-0.379
100	1.268	1.011	-0.019	-0.505	0.439
120	-0.755	-0.651	0.419	0.256	-0.183
140	0.752	0.040	-0.357	-0.141	0.074
Average	0.064	0.185	-0.105	-0.192	-0.012
Root Mean Square Error					0.629

Subject 3 ($\tilde{P}_1$ = .00, $\tilde{\theta}_1$ = -.41; $\tilde{P}_2$ = 1.00, $\tilde{\theta}_2$ = -.40)

	Rotation				
Side Length	40	80	120	160	Average
80	0.650	0.300	0.433	0.103	0.371
100	-0.786	-1.156	0.367	0.479	-0.274
120	-0.575	0.373	-0.015	0.921	0.176
140	-0.140	-0.723	-0.652	0.058	-0.365
Average	-0.213	-0.301	0.033	0.390	-0.023
Root Mean Square Error					0.577

Captions for Figures

1. A Sample Task

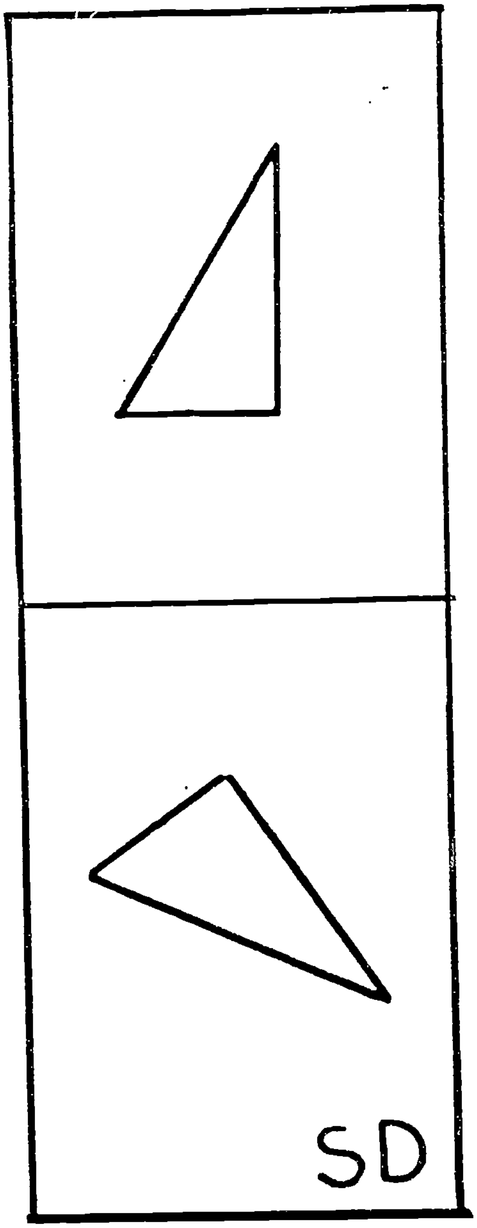
2. Median Standardized Log Response Time versus Rotation

3. Median Standardized Log Response Time versus Side Length

4. Posterior Probabilities of Use of Rule-Based Strategy

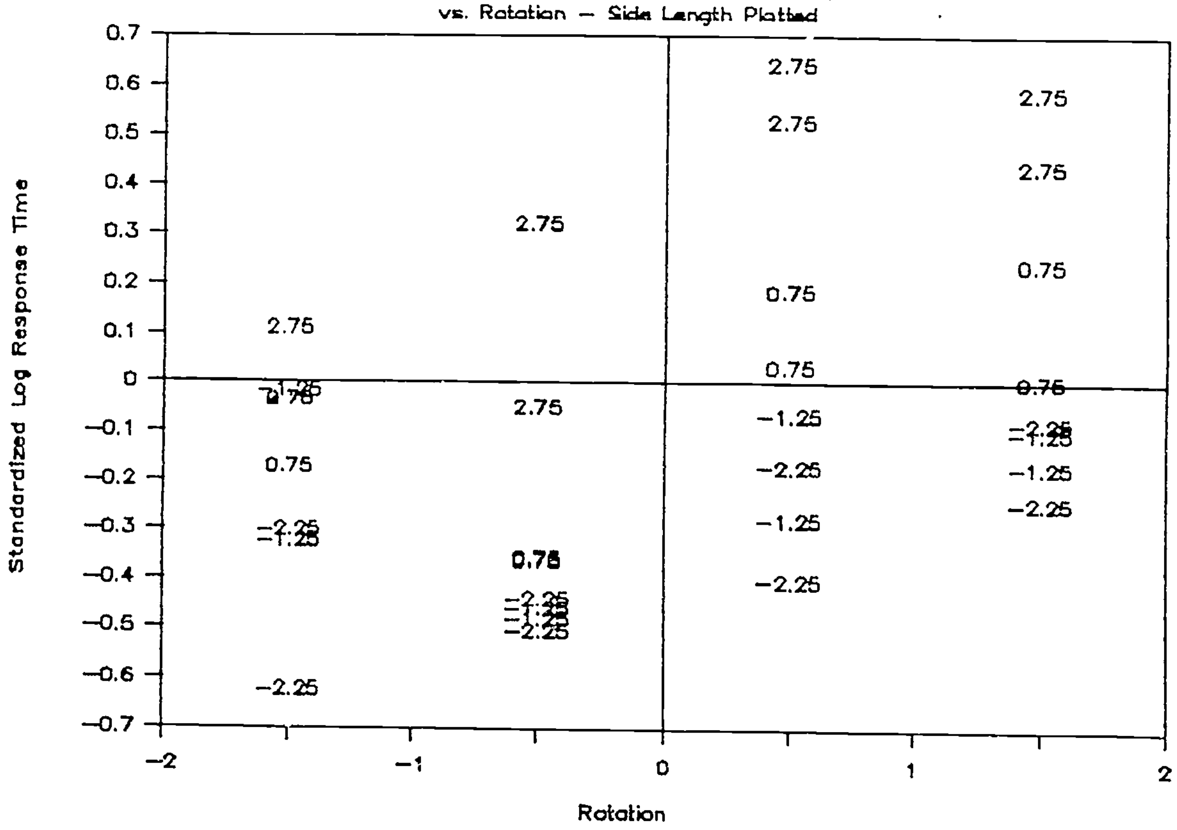
5. Posterior Probabilities of Component 1 Membership in a Mixture of Two Gaussian Components

6. Posterior Probabilities of Use of Rule-Based Strategy, Distinguishing Instructed and Noninstructed Subjects


SD

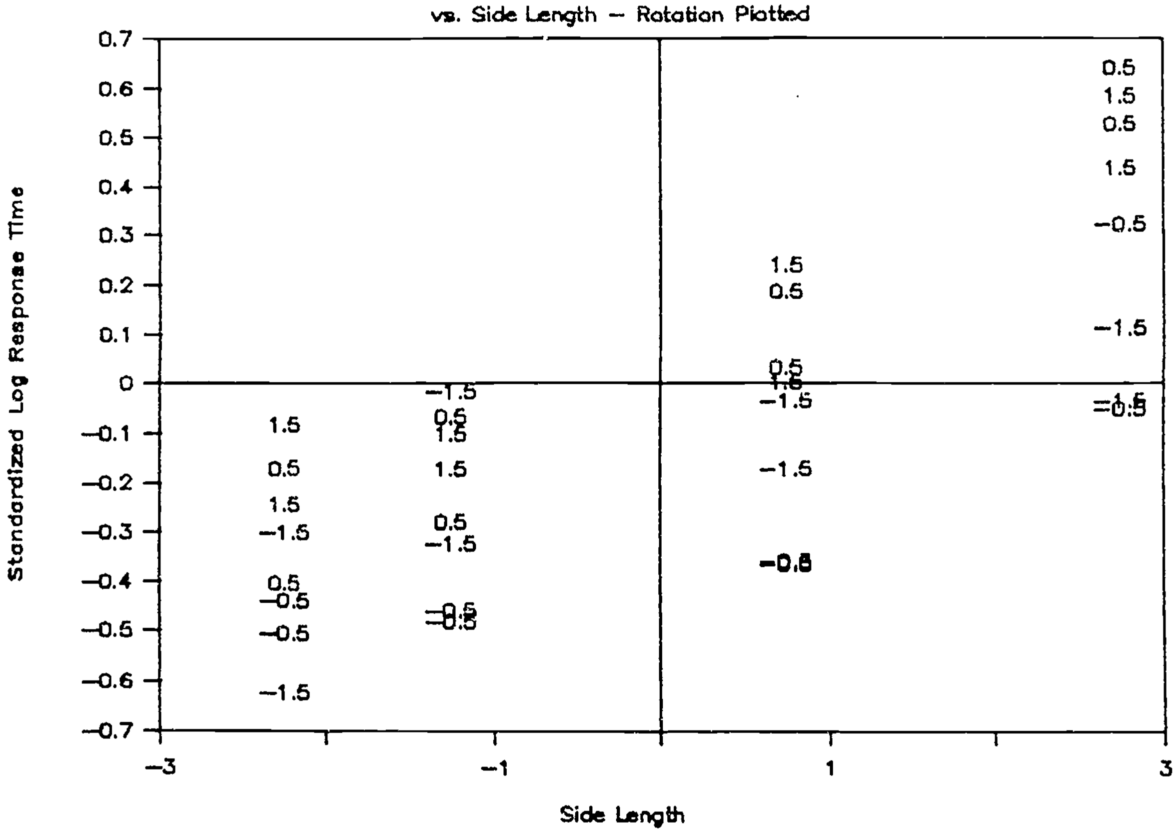
Median Standardized Log Response Time

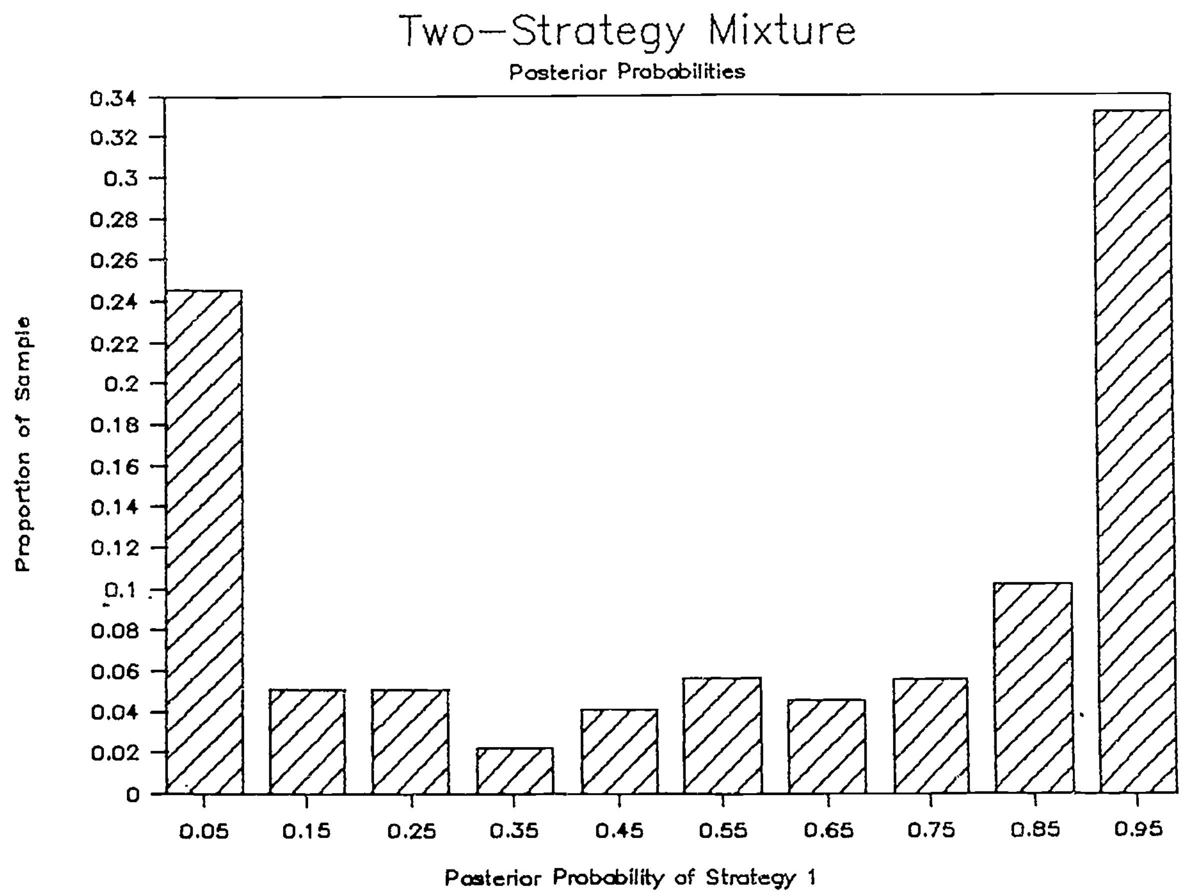
vs. Rotation — Side Length Plotted

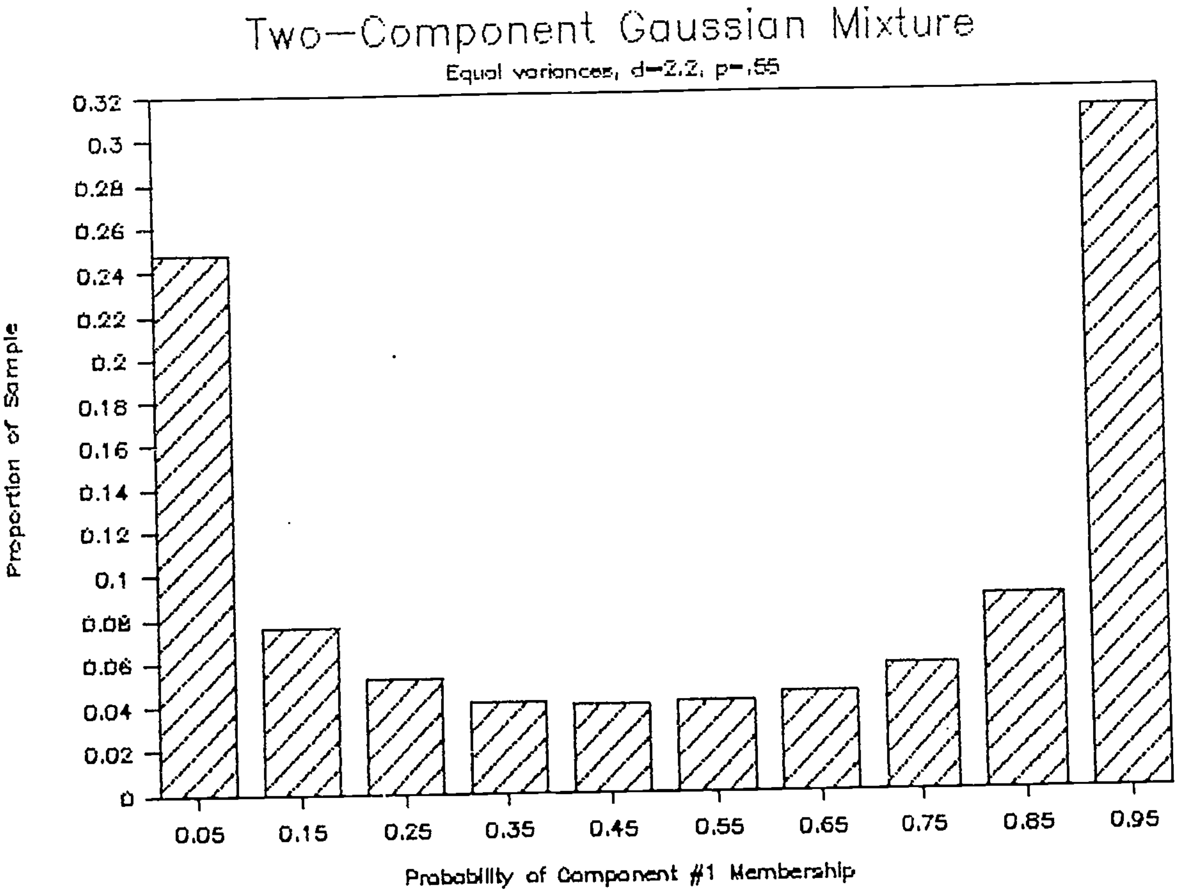


Median Standardized Log Response Time

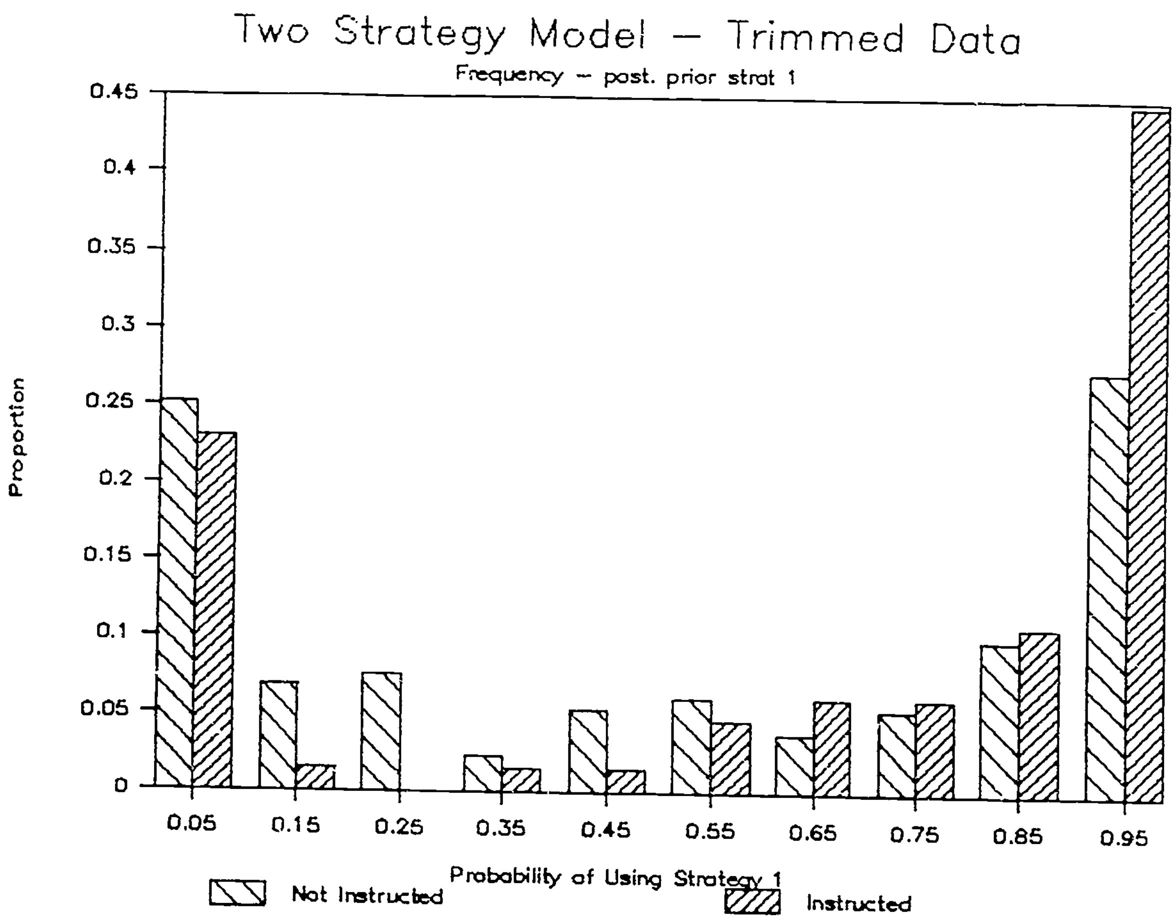
vs. Side Length — Rotation Plotted




Two—Strategy Mixture
Posterior Probabilities
0.34
0.32
0.3
0.28
0.26
0.24
0.22
0.2
0.18
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
Proportion of Sample
0.05
0.15
0.25
0.35
0.45
0.55
0.65
0.75
0.85
0.95
Posterior Probability of Strategy 1



Two—Component Gaussian Mixture
Equal variances, d=2.2, p=.55
Proportion of Sample
0.32
0.3
0.28
0.26
0.24
0.22
0.2
0.18
0.16
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
0.05
0.15
0.25
0.35
0.45
0.55
0.65
0.75
0.85
0.95
Probability of Component #1 Membership


Two Strategy Model — Trimmed Data
Frequency — post. prior strat 1
Proportion
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
0.05
0.15
0.25
0.35
0.45
0.55
0.65
0.75
0.85
0.95
Probability of Using Strategy 1
Not Instructed
Instructed

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