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AUTHOR	Barringer, Mary S.
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ABSTRACT

Researchers are becoming increasingly aware of the advantages of using multiple regression as opposed to analysis of variance (ANOVA) or analysis of covariance (ANCOVA). Multiple regression is more versatile and does not force the researcher to throw away variance by categorizing intervally scaled data. Polynomial regression analysis offers the investigator a method of analyzing curvilinear relationships with relative ease through commonly available statistical packages such as the Statistical Package for the Social Sciences (SPSS). This paper explores polynomial regression as an alternative to ANOVA for exploring nonlinear relationships. Challenges in interpreting the individual coefficients produced by polynomial regression are examined. Nine tables and four figures illustrate the discussion. Appendixes A and B illustrate SPSS output. (Contains 8 references.) (Author/SLD)

Curvilinear Relationships in Special Education Research: How Multiple Regression Analysis Can Be Used to Investigate Nonlinear Effects

Mary S. Barringer

Texas A&M University

Paper presented at the annual meeting of the American Educational Research Association, San Francisco, April 18, 1995.

Abstract

Researchers are becoming increasingly aware of the advantages of the using multiple regression as opposed to ANOVA or ANCOVA. Multiple regression is more versatile and does not force the researcher to throw away variance by categorizing intervally scaled data. The present paper explores polynomial regression as an alternative to ANOVA for exploring nonlinear relationships. Challenges in interpreting the individual coefficients produced by polynomial regression are examined.

In a seminal article, Cohen (1968, p. 426) noted that ANOVA and ANCOVA are special cases of multiple regression analysis, and argued that in this realization "lie possibilities for more relevant and therefore more powerful exploitation of the research data." Since that time researchers have increasingly recognized that conventional multiple regression analysis of data as they were initially collected (no conversion of intervally scaled independent variables into dichotomies or trichotomies) does not discard information or distort reality, and that the general linear model

> ...can be used equally well in experimental or non-experimental research. It can handle continuous and categorical variables. It can handle two, three, four, or more independent variables... Finally, as we will abundantly show, multiple regression analysis can do anything the analysis of variance does--sums of squares, mean squares, *F* ratios--and more. (Kerlinger & Pedhazur, 1973, p. 3)

One challenge is using regression to investigate curvilinear relationships. More and more of an intervention does not always monotonically yield more and more of an effect.

In conducting special education research, it is our desire to determine not only whether a given treatment is effective, but also which specific levels of the treatment are most effective and at what point our treatment becomes ineffective or even detrimental. We would also hope to be able to use our results to make predictions about the effect of our intervention for individuals who we believe are similar to those included in our test sample, but for whom we do not have scores on the dependent variable.

While ANOVA can be used to determine (a) whether the means of the treatment groups increase or decrease in a linear fashion with an increase in the level of the independent variable; (b) whether the trend is linear or nonlinear; and, if the trend is nonlinear, (c) what degree equation is required to fit the data (Hinkle, Wiersma, & Jurs, 1994, p. 372), the vital piece of information which such an analysis cannot provide is where the transitions in effectiveness occur (because the data for the predictor variable must be categorized to enable us to perform the ANOVA analysis and because ANOVA tests only the difference between group *means*). The only exception to this rule is when we used quantative and equally-spaced levels within an ANOVA way (e.g., 10, 20, and 30 minutes of intervention. Multiple regression can be employed to explore where these transitions occur. As noted by Pedhazur

> If the data depart significantly from linearity, one can do a multiple regression analysis in which the continuous variable is treated as a categorical variable. All that such an analysis can tell, however, is whether there is some trend in the data. When it is desired to study the nature of the trend, it is necessary to resort to nonlinear models... By an appropriate transformation, a model that is nonlinear in the variables may be reduced to a linear model...(Pedhazur, 193, p. 404).

An example of such a transformation is polynomial regression, in which the independent variable values are raised to powers. Table 1 describes commonly used polynomial equations involving a single predictor variable.

Insert Table 1 about here.

In polynomial regression, the transformed (powered) variables are entered into a hierarchical multiple regression analysis. A hierarchical multiple regression analysis is one which is carried out in steps. The first-degree vector (X) is entered, followed sequentially by the higher-degree polynomials. As shown in Table 1, the first-degree polynomial tests for a linear relationship between our predictor variable (X) and the dependent variable (Y). The second order polynomial tests for a quadratic relationship, and a third order polynomial tests for a cubic relationship between the independent and dependent variables.

At each stage of the polynomial regression analysis, the increase in variance accounted for by the new (higher order) polynomial, above that explained by the lower order term(s), can be tested for statistical significance. Given a continuous predictor variable with four distinct values, the increase in variance accounted for at each stage is calculated as follows:

Linear: $R^2_{y.x}$

Quadratic: $R^2_{y.x,x_2} - R^2_{y.x}$ (or R^2 quadratic-R^2 linear)

Cubic: $R^2_{y.x,x_2,x_3} - R^2_{y.x,x_2}$ (or R^2 cubic-R^2 quadratic)

and the F ratio for testing the statistical significance of these increments in variance accounted for is calculated as follows:

Linear:

$$F=((R^2_{y.x})/(k1))/((1-R^2_{y.x})/(N-k1-1))$$

Quadratic:

$$F=((R^2_{y.x,x_2}-R^2_{y.x})/(k1-k2)\}/((1-R^2_{y.x,x_2})/(N-k1-1))$$

Cubic:

$$F=(R^2_{y.x,x_2,x_3}-R^2_{y.x,x_2})/(k1-k2)/((1-R^2_{y.x,x_2,x_3})/(N-k1-1))$$

where N = number of subjects;

k1 = degrees of freedom of the larger R^2; and

k2 = degrees of freedom of the smaller R^2.

Although they may appear a bit complex, these analyses can be performed easily using readily available statistical packages such as SPSS. Thus there is no need to categorize the intervally scaled independent variable data, and in the process discard variance, in order to perform an ANOVA trend analysis.

To facilitate discussion of the process involved in polynomial regression, a small heuristic data set will be analyzed. The analysis is performed using SPSS commands presented in Appendix A. Tables 2 through 4 present the data to be analyzed.

Insert Tables 2-4 about here.

Table 2 shows the values for the predictor variable (X) and a set of values for a dependent variable (Y1). Table 3 includes the same values for X but a different set of dependent variable (Y2) values. Table 4 shows yet another set of dependent variable values (Y3) which might be obtained with the same values of the dependent variable (X). These are three different hypothetical data sets which have no relationship to one another. The values of the variables have been selected in order to demonstrate different possible outcomes of polynomial regression analysis. Figures 1 through 3 show the scatter plots of the three data sets.

Insert Figures 1 through 3 about here.

As has been made abundantly clear (Anscombe, 1973), a quick examination of the scatter plot for a given set of results can provide important clues about the nature of the relationship between two variables. Examination of the scatter plot of the values of X and Y1, for example, reveals that the relationship between X and Y1 is positive and linear. Figures 2 and 3 show that the relationship between X and Y2 is apparently quadratic (there is probably one bend in the regression line) and the relationship between X and Y3 appears cubic (there are probably two bends in the regression line).

Now we turn to describing the procedures used to determine the prediction equation for each of these three sets of data. A complete analysis of all possible levels of the polynomial equation is performed for each data set (even X with Y1, which is obviously linear) for purposes of explanation and comparison.

Performing the polynomial regression analysis

The first step in polynomial regression is to determine the highest degree polynomial which could be applied with the given data set. The highest degree polynomial which can be used for a set of data is g-1 where g = the number of distinct values of the independent variable. Whenever the highest-degree polynomial for a given set of data is used, $R^2=\mu^2$. (This relationship will be discussed later--however, remember that our interest, ultimately, is not in the highest order polynomial, but rather the *lowest* order polynomial which adequately describes our data. To find the lowest order polynomial which adequately describes the data, all possible levels are tested in the preliminary polynomial regression analysis. In most data sets we can test many different polynomials, but usually we do not go beyond 4 or 5, even when we could.) Since there are 5 distinct levels of the independent variable (X) in each of the three data sets, the highest degree polynomial possible for any of the data sets is fourth-degree, or quartic.

Once the highest-degree polynomial that can be tested has been determined, the next step is to compute the vectors for each polynomial equation. The values for X and Y may be entered directly into the data file, and the powered vectors (X^2, X^3, and X^4) can be generated using COMPUTE commands (see Appendix A, lines 9-11).

Next, these vectors are entered into the polynomial regression equation hierarchically, beginning with the first-order polynomial (X) and entering at the next three steps the second-, third- and fourth-degree polynomials (X^2, X^3, and X^4)

sequentially (see, for example, Appendix A, lines 13-15).

Interpreting the results of the polynomial regression

Data Set #1

Insert Table 5 about here.

Table 5 is a summary the results of the preliminary polynomial regression analysis for the relationship between X and Y1. (For complete SPSS output, see Appendix B.) As discussed previously, examination of the scatter plot of X and Y1 reveals that the relationship between these two variables is strongly linear. Thus one would expect the results to show a statistically significant R^2 for the linear relationship. Furthermore, assuming a linear relationship between the two variables, one would also expect that each of the remaining steps of the equation (entering X^2, X^3, and X^4) would *not* produce statistically significant increases in variance accounted for above what is explained by the first-degree polynomial. An examination of Table 5 reveals that the results of the regression analysis conform to these expectations.

In examining R^2, it is noted that $R^2 y.x, x_2, x_3, x_4$ = .75000, which means that 75% of the variance in the dependent variable (Y1) is accounted for by the fourth-degree polynomial of the independent variable (X). The F test for R^2 is:

$$F = (.75000/4)/((1-.75000)/(15-4-1)) = 7.5$$

with 4 and 10 *df*, $p < .01$. The same result will be found by using the Sum of Squares and Mean Square Residual.

$$F = (30/4)/(10/(15-4-1)) = 7.5$$

This is the same F ratio that would be obtained if the data had been analyzed using a one-way ANOVA. As noted by Pedhazur,

> Testing the highest-degree polynomial possible in a set of data is tantamount to a test of whether the means of the arrays are equal, which is equivalent to testing whether the means of the treatments differ from each other when a one-way analysis of variance is applied. (Pedhazur, 1982, p. 409)

The test of statistical significance indicates that there is, in fact, a statistically significant trend in the data. What we still do not know, however, is what degree polynomial best fits these data. To find the answer to this question, one must further examine the results of the polynomial regression analysis.

The increment in variance accounted for by each degree of the polynomial equation is reported in Table 5 in the fifth column as R^2 Change and in the seventh column as Sum of Squares Change. Neither of these numbers is produced as output from SPSS, but both are easy to calculate. To calculate R^2 Change, simply find the R^2 for the step of interest and subtract the R^2 for the previous step. (These values are produced in the SPSS Output.) For example, for the second step using Data Set #1, the R^2 Change is found by finding the R^2 for Step 2, which is the R^2 for the linear and quadratic components, and subtracting the R^2 for Step 1, which is the R^2 for the linear component alone. In this example, the R^2 Change for the second step is .75000 - .75000 = 0. In fact, the second-, third- and fourth-degree polynomials each result in an increment of zero because the relationship between X and Y1 is perfectly linear.

Sum of squares ci: ..ge is calculated in the same manner. Simply locate the Sum of Squares for the step of interest and subtract the Sum of Squares for the previous step. In this case, for Step 2 (the test of the quadratic relationship), 30 - 30 = 0. As was the case with R^2 Change, calculation of Sums of Squares Change shows that the second, third and fourth steps of the regression equation result in zero increase in variance accounted for beyond that explained by the linear component because the relationship between X and Y1 is perfectly linear. For this reason, the test of statistical significance of the increment in variance accounted for by higher-order equations will be discussed using the second data set.

Data Set #2

The results of the polynomial regression for the second data set are summarized in Table 6. As was the case for the first data set, the highest-degree polynomial which can be used for this data set is fourth-degree, or quartic. Table 6 shows that for the final step, $R^2 = .54545$. For the final step, $F = 3.00$ with 4 and 10 *df*, (not statistically significant). We now turn to the test of R^2 Change.

Insert Table 6 about here

R^2 Change and Sum of Squares Change have been calculated following the same procedures described for the first data set, and are presented in Table 6. The next step is to test the statistical significance of R^2 Change or Mean Square Change for each step. Each step of the polynomial regression has 1 *df*, therefore Sum of Squares Change = Mean Square Change. Thus, the F test for each step of the polynomial regression is

F = Mean Squares Change/Mean Squares Residual

with 1 *df* for the numerator. The *df* for the denominator of each step is the *df* Residual for the last step of the equation. For this data set, *df* Residual = 10. Thus, for the first step of Data Set #2,

$$F = 0.300/1 = .3$$

with 1 and 10 *df*. For the second step of the regression,

$$F = 10.5/1 = 10.5$$

with 1 and 10 *df*, $p < .01$. (Remember that **each step has 1 *df*. *Do not* use the *df* reported for each step by the SPSS Output because these df's are cumulative, and we are interested in incremental and not cumulative effects.**) These and the remaining F ratios for the data set are found in the tenth column of Table 6. The reader can see that for Data Set #2, only the second step (quadratic) of the regression equation resulted in a statistically significant R^2 Change.

Data Set #3

The procedures described above for the first two data sets have been followed in creating the contents of Table 7. The polynomial regression analysis of Data Set #3 results in a statistically significant R^2 Change for the linear ($p < .01$) and cubic ($p < .01$) levels only.

Insert Table 7 about here

Performing the hierarchical regression

After the highest order polynomial which produces a statistically significant increment in R^2 has been identified, the regression is run again, this time using only the vectors up to and including the highest-order vector which produced a statistically significant R^2 Change (including all lower order vectors regardless of whether or not they produced a statistically significant R^2 Change).

For Data Set #1, a simple linear regression of Y1 on X can be performed because only the linear vector produced a statistically significant change in variance accounted for. For Data Set #2, vectors (X) and (X2) will be entered into the final regression analysis because R^2 change for X2 was statistically significant and R^2 Change for X3 and X4 were not. Finally, for Data Set #3, where the highest order polynomial which resulted in a statistically significant increment in R^2 was found at the third step (the cubic relationship), vectors (X), (X2), and (X3) will be entered.

Impact on Sums of Squares, *df*, and Mean Squares

Since for Data Set #1, only the linear term (X) will be included in the final regression equation, the Sums of Squares and *df* associated with the quartic (X2), cubic (X3), and quadratic (X4) terms of the original polynomial analysis will be pooled with Sums of Squares and *df* Residual in the final regression analysis. Following the same principle, for Data Set #2, the Sums of Squares and *df* associated with X3 and X4 will be pooled with Sums of Squares and *df* Residual, and for Data Set #3, the Sums of Squares and *df* associated with X4 will be pooled with Sums of Squares and *df* Residual.

The result of this pooling is that, in general, Sums of Squares Residual will increase slightly, but *df* Residual will also increase (when compared with the Sums of Squares and *df* Residual for the final stage of the original polynomial regression) resulting in a smaller Mean Square Residual (error). This decrease in Mean Square Residual is important because Mean Square Residual is used in the test of statistical significance of R^2. Since Mean Square Residual is the denominator of the equation, decreasing the size of Mean Square Residual increases the likelihood of finding a statistically significant F ratio.

The regression equation

Data Set #2 will be used as an example for discussion of the regression equation. Table 8 shows the results of the final regression analysis for Data Set #2.

Insert Table 8 about here.

In hierarchical regression, only the *b* weight associated with the highest order polynomial in the regression equation is tested for statistical significance. Thus, for this data set, only the *b* weight for the quadratic component is relevant. In other words, we are interested in a test of the squared semipartial correlation of the dependent variable (Y2) with the highest-order vector (X2) when all the other vectors have been partialed out from it, or, stated more simply, a test of the increment of variance accounted for by the quadratic term over and above the linear term. It is unnecessary to test the *b* for the linear term because of the hierarchical nature of this analysis. The F test for *b* for this data set is:

$$F = t^2$$

$$= (-0.1250/0.037268)^2 = 11.2498$$

with 1 and 12 *df*, $p < .01$.

Given the information in Table 8, it is now possible to determine the equation for the regression of Y2 on X. The equation is:

$$Y' = -.800 + 1.55(X) + (-.125)(X^2)$$

The regression equation can now be used to predict the dependent variable score (Y) based on knowledge of the individual's' score on (X). Table 9 shows Y' scores based on the regression equation. The regression line, along with the original data points for Data Set #2, is shown in Figure 4.

Insert Table 9 and Figure 4about here.

As is shown in Figure 4, the regression equation can now be used to predict Y' scores given an X value within the range of values of the predictor variable (X values) included in the data set on which the regression equation was developed. One should not, however, extrapolate beyond the values of the predictor variable included in the formulation of this regression equation. This regression equation may or may not apply equally well to a score of 20 on the predictor variable. To determine the regression equation to predict Y given an X score of 20, it is necessary to include higher values of the predictor variable (at least a value of 20) in another study and perform another regression analysis. Because it has already been shown that, within the given range of X values, the relationship between X and Y is curved, it stands to reason that at greater values of the independent variable, the regression line could have even more bends. The same caution applies to extrapolating below the lowest level of the predictor variable used in developing the regression equation.

Interpreting b weights

Interpreting polynomial regression results can be a bit challenging and requires a clear understanding of what each of the individual coefficients represents. The difficulty arises when one treats the powered vectors as if they were distinct predictor variables. For example, in the regression equation $Y' = a + b_1X + b_2X^2 + e$,

> By definition b_1 and b_2 measure the change in Y associated with each unit of change of X or X^2, respectively, controlling for the effects of the other... But this is impossible because X^2 *must* vary with X, for X^2 is merely a vector whose elements are squares of corresponding elements for X. (Stimson, Carmines & Zeller, 1978, p. 520)

Thus the usual interpretation of b weights does not make sense in polynomial regression. As Stimson et al. (1978) explain, polynomial equations "treat as two variables what is only one" (p. 520). In other words, Y is not a *multiple linear function* of two separate variables X and X2, it is a *curvilinear function* of X only.

A second difficulty in interpreting b weights in polynomial regression is that the magnitude of b1 and b2 cannot be compared because their variances are unequal. Generally, the variance of the quadratic (X2) term will be much greater than that of the linear (X) term. Furthermore, the variance of the cubic term (X3) will be greater than that of the quadratic term. Because of the inequality of the variances, a comparison of the coefficients will underestimate the impact of the more variable term. (Stimson et al., 1978)

A third challenge in interpreting b weights in polynomial regression is presented by the fact that powered vectors tend to be highly correlated. As pointed out by Smith and Sasaki (1979), as multicollinearity increases, the standard errors of the coefficients increase.

Several authors (Bradley & Srivastava, 1979; Marquardt & Snee, 1975; Smith & Sasaki, 1979) advocate the use of centering of the independent variable to reduce multicollinearity. Centering is the process of performing a linear transformation of X (such as X' = X - X) such that the independent variable has a zero sample mean. As Cohen (1978) points out, the R^2 (and the F value for R^2) for a polynomial remains invariant over linear transformation of X. Thus centering will reduce the correlation between the vectors to zero and reduce the variance of the powered vectors without changing the hierarchical regression analysis.

Conclusion

Polynomial regression analysis offers the investigator a method of analyzing curvilinear relationships with relative ease through the employment of commonly available statistical packages such as SPSS.

Researchers should be thoroughly familiar with the mechanics of any statistical procedure they employ. Polynomial regression is no exception to the rule. While interpretation of the results of polynomial regression presents certain challenges and requires that the researcher give careful consideration to the coefficients and what they represent, this is no more than would be expected of any researcher who hopes to accurately interpret results.

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Table 1. Polynomial Equations

Order		Polynomial Equation	Number of bends in regression curve
first-degree	(linear)	$Y'=a+b_1X$	zero
second-degree	(quadratic)	$Y'=a+b_1X+b_2X^2$	one
third-degree	(cubic)	$Y'=a+b_1X+b_2X^2+b_3X^3$	two
fourth-degree	(quartic)	$Y'=a+b_1X+b_2X^2+b_3X^3+b_4X^4$	three
fifth-degree	(quintic)	$Y'=a+b_1X+b_2X^2+b_3X^3+b_4X^4+b_5X^5$	four

Table 2. Data Set #1, Regression of Y1 on X

	Levels of Predictor Variable (X)				
	2	4	6	8	10
	1	2	3	4	5
Y1	2	3	4	5	6
	3	4	5	6	7

Table 3. Data Set #2, Regression of Y2 on X

	Levels of Predictor Variable (X)				
	2	4	6	8	10
	1	2	3	3	1
Y2	2	3	4	4	2
	3	4	5	5	3

Table 4. Data Set #3, Regression of Y3 on X

	Levels of Predictor Variable (X)				
	2	4	6	8	10
	1	2	3	1	5
Y3	2	3	4	2	6
	3	4	5	3	7

Table 5.

DATA SET 1--REGRESSION OF Y1 ON X

STEP	VARIABLE ENTERED	RELATIONSHIP TESTED	R SQUARE	R SQUARE CHANGE	SUM OF SQUARES	SUM OF SQUARES CHANGE	DF	MEAN SQUARE CHANGE	F
1	X	Linear	.750	0.750	30	30	1	30	30
2	X2	Quadratic	.750	0.000	30	0	1	0	0
3	X3	Cubic	.750	0.000	30	0	1	0	0
4	X4	Quartic	.750	0.000	30	0	1	0	0
	RESIDUAL				10		10	1	

Table 6.

DATA SET 2--REGRESSION OF Y2 ON X

STEP	VARIABLE ENTERED	RELATIONSHIP TESTED	R SQUARE	R SQUARE CHANGE	SUM OF SQUARES	SUM OF SQUARES CHANGE	DF	MEAN SQUARE CHANGE	F
1	X	Linear	.01364	.01364	0.300	0.300	1	0.300	0.300
2	X2	Quadratic	.49091	.47727	10.800	10.500	1	10.500	10.500
3	X3	Cubic	.54545	.05454	12.000	1.200	1	1.200	1.200
4	X4	Quartic	.54545	0.00000	12.000	0.000	1	0.000	0.000
	RESIDUAL				10.000		10	1.000	

Table 7.

DATA SET 3--REGRESSION OF Y3 ON X

STEP	VARIABLE ENTERED	RELATIONSHIP TESTED	R SQUARE	R SQUARE CHANGE	SUM OF SQUARES	SUM OF SQUARES CHANGE	DF	MEAN SQUARE CHANGE	F
1	X	Linear	.33716	.33716	14.700	14.700	1	14.700	14.700
2	X2	Quadratic	.38139	.04423	16.628	1.928	1	1.928	1.928
3	X3	Cubic	.62910	.24771	27.428	10.800	1	10.800	10.800
4	X4	Quartic	.77064	.14154	33.600	6.172	1	6.172	6.172
	RESIDUAL				10		10	1.000	

Table 8

VARIABLE	B	STD ERROR B	BETA	F
X	1.5500	.455826	3.6200	11.5628
X2	-0.1250	.037268	-3.354	11.2498
(Constant)	-0.8000	1.196291		

Table 9.

X	X2	1.55(X)	-0.125(X2)	Constant	Y'
1	1	1.55	-0.125	-0.800	.625
2	4	3.10	-0.500	-0.800	1.800
3	9	4.65	-1.125	-0.800	2.725
4	16	6.20	-2.000	-0.800	3.400
5	25	7.75	-3.125	-0.800	3.825
6	36	9.30	-4.500	-0.800	4.000
7	49	10.85	-6.125	-0.800	3.925
8	64	12.40	-8.000	-0.800	3.600
9	81	13.95	-10.125	-0.800	3.025
10	100	15.50	-12.500	-0.800	2.200

Figure 1

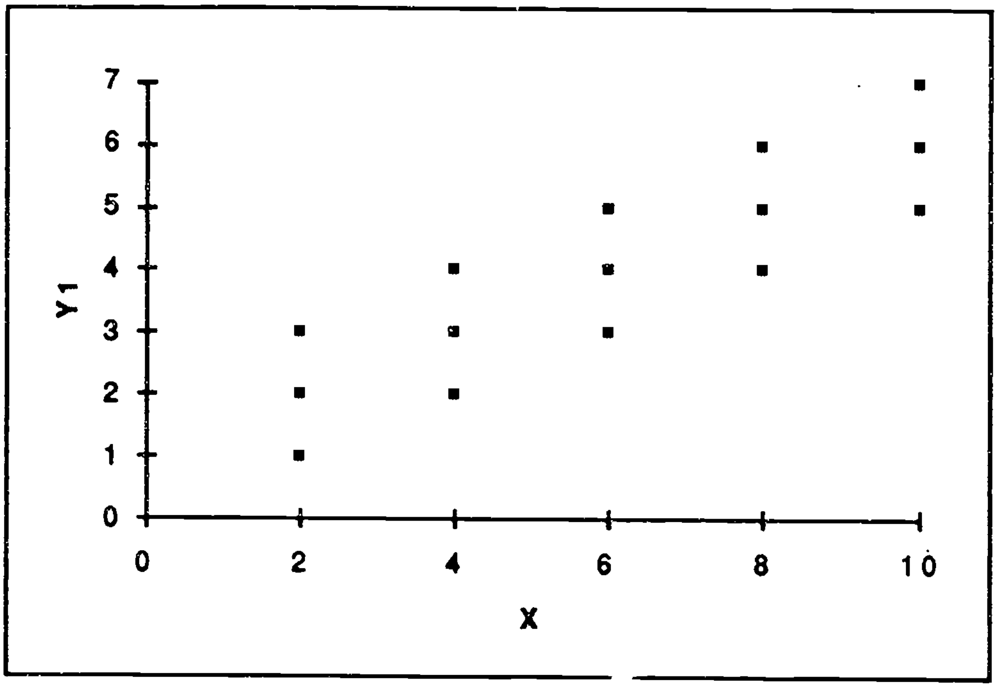


Figure 2

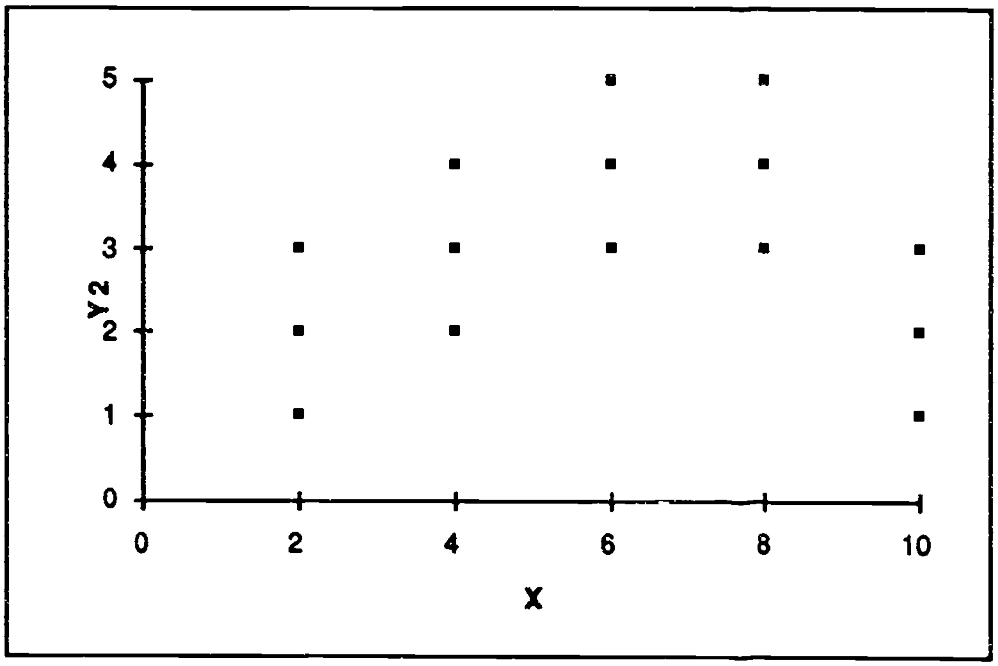


Figure 3

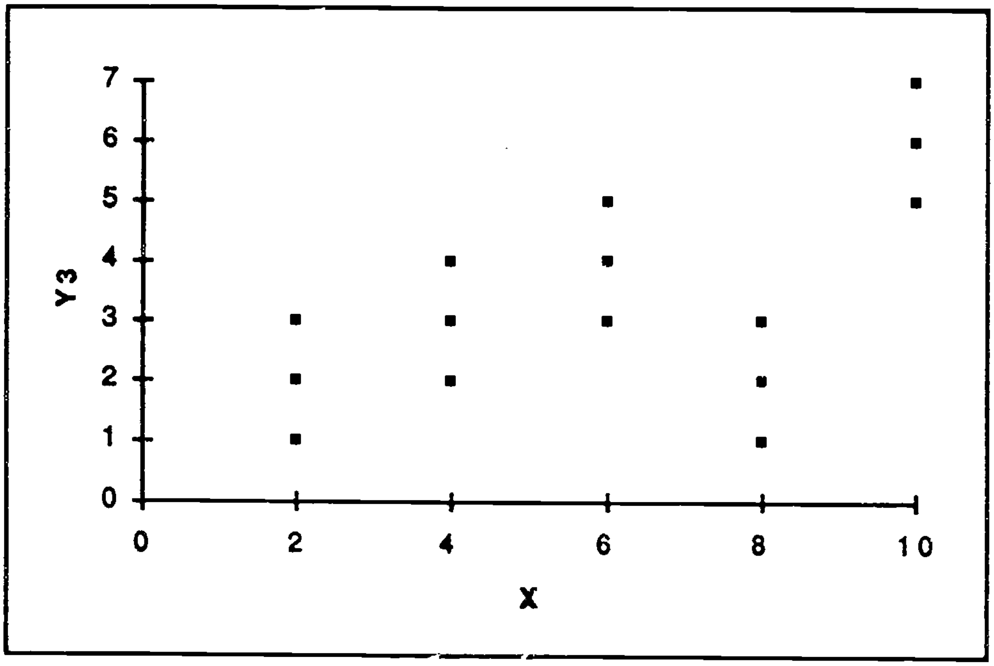
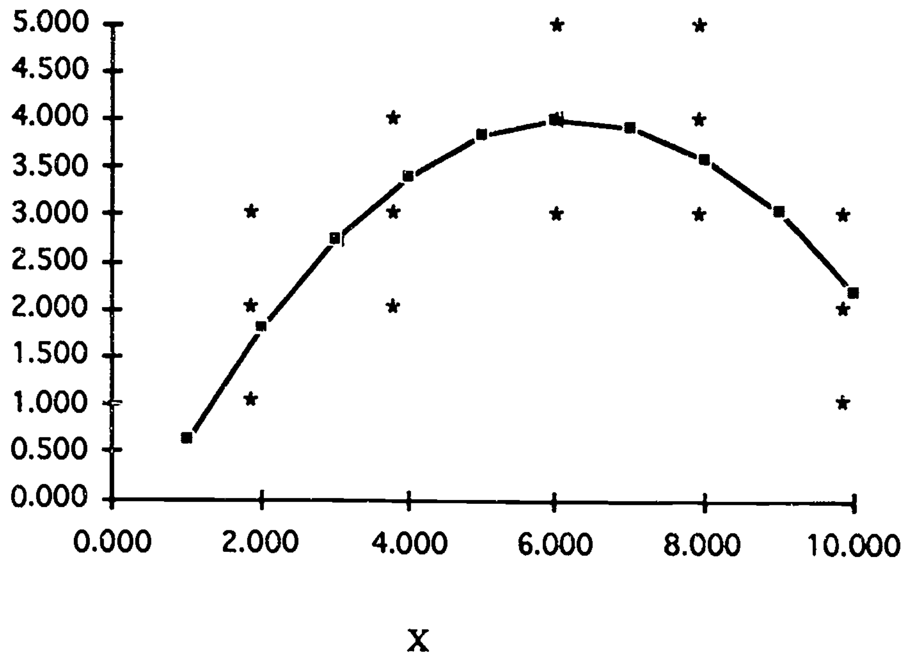


Figure 4


5.000
4.500
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2.000
1.500
1.000
0.500
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0.000
2.000
4.000
6.000
8.000
10.000
X

Appendix A

```
TITLE 'OUTPUT'
DATA LIST FILE=ABC /1
  X 1-2 Y1 4 Y2 7 Y3 11
COMPUTE X2=X**2
COMPUTE X3=X**3
COMPUTE X4=X**4
LIST VARIABLES=ALL/CASES=500/FORMAT=NUMBERED
REGRESSION VARIABLES=Y1 X X2 X3 X4/DESCRIPTIVES=DEFAULT/
 CRITERIA=TOLERANCE (.000001)/DEPENDENT=Y1/ENTER X/ENTER X2
 /ENTER X3/ENTER X4
REGRESSION VARIABLES=Y2 X X2 X3 X4/DESCRIPTIVES=DEFAULT/
 CRITERIA=TOLERANCE (.000001)/DEPENDENT=Y2/ENTER X/ENTER X2
 /ENTER X3/ENTER X4
REGRESSION VARIABLES=Y3 X X2 X3 X4/DESCRIPTIVES=DEFAULT/
 CRITERIA=TOLERANCE (.000001)/DEPENDENT=Y3/ENTER X/ENTER X2
 /ENTER X3/ENTER X4
```

Appendix B

SPSS Output

```
For MVS/XA/JES3           TEXAS A&M UNIVERSITY: CSC         License Number 1267
This software is functional through August 31, 1994.


>Warning # 2032
>Your license has expired.  Please notify your local SPSS coordinator.

Try the new SPSS Release 4 features:

* LOGISTIC REGRESSION procedure              * CATEGORIES Option:
* EXAMINE procedure to explore data          *   conjoint analysis
* FLIP to transpose data files               *   correspondence analysis
* MATRIX Transformations Language            * New LISREL and PRELIS Options
* GRAPH interface to SPSS Graphics           *

See the new SPSS documentation for more information on these new features.

   1  0  TITLE 'OUTPUT'
   2  0  DATA LIST FILE=ABC /1
   3  0     X 1-2 Y1 4 Y2 7 Y3 11

This command will read 1 records from 'MSB0869.PAP.DATM'

Variable    Rec   Start    End        Format

X             1       1      2        F2.0
Y1            1       4      4        F1.0
Y2            1       7      7        F1.0
Y3            1      11     11        F1.0

   4  0  COMPUTE X2=X**2
   5  0  COMPUTE X3=X**3
   6  0  COMPUTE X4=X**4
   7  0  LIST VARIABLES=ALL/CASES=500/FORMAT=NUMBERED

There are 185,264 bytes of memory available.
The largest contiguous area has 178,208 bytes.


351 bytes of memory required for the LIST procedure.
120 bytes have already been acquired.
231 bytes remain to be acquired.
```

	X	Y1	Y2	Y3	X2	X3	X4
1	2	1	1	1	4.00	8.00	16.00
2	2	2	2	2	4.00	8.00	16.00
3	2	3	3	3	4.00	8.00	16.00
4	4	2	2	2	16.00	64.00	256.00
5	4	3	3	3	16.00	64.00	256.00
6	4	4	4	4	16.00	64.00	256.00
7	6	3	3	3	36.00	216.00	1296.00
8	6	4	4	4	36.00	216.00	1296.00
9	6	5	5	5	36.00	216.00	1296.00
10	8	4	3	1	64.00	512.00	4096.00
11	8	5	4	2	64.00	512.00	4096.00
12	8	6	5	3	64.00	512.00	4096.00
13	10	5	1	5	100.00	1000.00	10000.00
14	10	6	2	6	100.00	1000.00	10000.00
15	10	7	3	7	100.00	1000.00	10000.00

Number of cases read: 15 Number of cases listed: 15

```
Preceding task required .07 seconds CPU time;  .29 seconds elapsed.

   8  0  REGRESSION VARIABLES=Y1 X X2 X3 X4/DESCRIPTIVES=DEFAULT/
   9  0   CRITERIA=TOLERANCE (.000001)/DEPENDENT=Y1/ENTER X/ENTER X2
  10  0   /ENTER X3/ENTER X4

There are 185,360 bytes of memory available.
The largest contiguous area has 178,936 bytes.

     1612 bytes of memory required for REGRESSION procedure.
        0 more bytes may be needed for Residuals plots.
```

* * * * M U L T I P L E R F G R E S S I O N * * * *

Listwise Deletion of Missing Data

	Mean	Std Devi	Label
Y1	4.000	1.690	
X	6.000	2.928	
X2	44.000	35.809	
X3	360.000	377.480	
X4	3132.800	3857.750	

N of Cases = 15

Correlation:

	Y1	X	X2	X3	X4
Y1	1 000	.866	.850	.817	.782
X	.866	1.000	.981	.943	.903
X2	.850	.981	1.000	.989	.968
X3	.817	.943	.989	1.000	.994
X4	.782	.903	.968	.994	1.000

```
                          * * * *   M U L T I P L E   R E G R E S S I O N   * * * *

Equation Number 1    Dependent Variable..   Y1

 Descriptive Statistics are printed on Page    4

Block Number  1.  Method:  Enter      X


Variable(s) Entered on Step Number  1..    X


Multiple R           .86603        Analysis of Variance
R Square             .75000                          DF      Sum of Squares      Mean Square
Adjusted R Square    .73077        Regression         1            30.00000         30.00000
Standard Error       .87706        Residual          13            10.00000           .76923

                                   F =      39.00000       Signif F =  .0000

------------------ Variables in the Equation ------------------      ------------- Variables not in the Equation -------------

Variable              B          SE B        Beta          T  Sig T      Variable     Beta In  Partial  Min Toler          T  Sig T

X              .500000       .080064     .866025      6.245  .0000      X2          .000000  .000000    .037433       .000 1.0000
(Constant)    1.000000       .531085                  1.883  .0823      X3       -3.767E-16  .000000    .110529       .000 1.0000
                                                                        X4        6.039E-16  .000000    .183848       .000 1.0000

End Block Number   1   All requested variables entered.


                          * * * * * * * * * * * * * * * * * * * * * * * * * * * * *

Block Number  2.  Method:  Enter      X2


Variable(s) Entered on Step Number  2..    X2


Multiple R           .86603        Analysis of Variance
R Square             .75000                          DF      Sum of Squares      Mean Square
Adjusted R Square    .70833        Regression         2            30.00000         15 00000
Standard Error       .91287        Residual          12            10.00000           .83333

                                   F =      18.00000       Signif F =  0002
```

* * * * M U L T I P L E R E G R E S S I O N * * * *

Equation Number 1 Dependent Variable.. Y1

------------------ Variables in the Equation ------------------

Variable	B	SE B	Beta	T	Sig T
X	.500000	.430716	.866025	1.161	.2683
X2	.000000	.035215	.000000	.000	1.0000
(Constant)	1.000000	1.130388		.885	.3937

------------- Variables r t in the Equation -------------

Variable	Beta In	Partial	Min Toler	T	Sig T
X3	-3.004E-14	.000000	4.694E-04	.000	1.0000
X4	1.447E-14	.000000	.001562	.000	1.0000

End Block Number 2 All requested variables entered.

* *

Block Number 3. Method: Enter X3

Variable(s) Entered on Step Number 3.. X3

Multiple R	.86603
R Square	.75000
Adjusted R Square	.68182
Standard Error	.95346

Analysis of Variance

	DF	Sum of Squares	Mean Square
Regression	3	30.00000	10.00000
Residual	11	10.00000	.90909

F = 11.00000 Signif F = .0012

------------------ Variables in the Equation ------------------

Variable	B	SE B	Beta	T	Sig T
X	.500000	1.769891	.866025	.283	.7828
X2	2.42124E-15	.328461	5.129E-14	.000	1.0000
X3	-1.34514E-16	018133	-3.004E-14	.000	1.0000
(Constant)	1.000000	2.708013		.369	.7189

------------- Variables not in the Equation -------------

Variable	Beta In	Partial	Min Toler	T	Sig T
X4	6.884E-12	.000000	5.478E-06	.000	1.0000

End Block Number 3 All requested variables entered.

```
                         * * * *   M U L T I P L E   R E G R E S S I O N   * * * *

Equation Number 1    Dependent Variable..   Y1

Block Number  4.  Method:  Enter      X4

*  *  *  *  *  *  *  ***  WARNING  ***  *  *  *  *  *  *  *

The following variables in the equation have low tolerances:
          Variable     Tolerance
            X2            8.78585E-06
            X3            5.47806E-06
            X4                 .00003

*  *  *  *  *  *  ***  END OF WARNING  ***  *  *  *  *  *  *


Variable(s) Entered on Step Number  4..    X4


Multiple R            .86603        Analysis of Variance
R Square              .75000                          DF     Sum of Squares     Mean Square
Adjusted R Square     .65000        Regression         4          30.00000         7.50000
Standard Error       1.00000        Residual          10          10.00000         1.00000

                                    F =       7.50000       Signif F =  .0046


------------------ Variables in the Equation ------------------

Variable              B          SE B         Beta          T  Sig T

X               .500000      8.402146      .866025       .060  .9537
X2          6.00491E-13      2.517976    1.272E-11       .000 1.0000
X3         -7.25234E-14       .302502   -1.620E-11       .000 1.0000
X4          3.01620E-15       .012579    6.884E-12       .000 1.0000
(Constant)     1.000000      9.146948                    .109  .9151

End Block Number   4   All requested variables entered.
```

```
Preceding task required .14 seconds CPU time;  1.58 seconds elapsed.

  11  0  REGRESSION VARIABLES=Y2 X X2 X3 X4/DESCRIPTIVES=DEFAULT/
  12  0   CRITERIA=TOLERANCE (.000001)/DEPENDENT=Y2/ENTER X/ENTER X2
  13  0   /ENTER X3/ENTER X4

There are 185,360 bytes of memory available.
The largest contiguous area has 178,936 bytes.

     1612 bytes of memory required for REGRESSION procedure.
        0 more bytes may be needed for Residuals plots.
```

**** MULTIPLE REGRESSION ****

Listwise Deletion of Missing Data

	Mean	Std Devi	Label
Y2	3.000	1.254	
X	6.000	2.928	
X2	44 000	35.809	
X3	360.000	377.480	
X4	3132.800	3857.750	

N of Cases = 15

Correlation:

	Y2	X	X2	X3	X4
Y2	1.000	.117	-.019	-.127	-.205
X	.117	1.000	.981	.943	.903
X2	-.019	.981	1.000	.989	.968
X3	-.127	.943	.989	1.000	.994
X4	-.205	.903	.968	.994	1.000

```
                         * * * *   M U L T I P L E   R E G R E S S I O N   * * * *

Equation Number 1    Dependent Variable..   Y2

 Descriptive Statistics are printed on Page    9

Block Number  1.  Method:  Enter      X


Variable(s) Entered on Step Number  1..    X


Multiple R            .11677        Analysis of Variance
R Square              .01364                          DF      Sum of Squares      Mean Square
Adjusted R Square   -.06224         Regression          1              .30000           .30000
Standard Error       1.29199        Residual           13            21.70000          1.66923

                                    F =        .17972       Signif F =  .6785


------------------ Variables in the Equation ------------------      ------------- Variables not in the Equation -------------

Variable              B        SE B       Beta        T  Sig T        Variable     Beta In   Partial  Min Toler        T   Sig T

X               .050000     .117942    .116775     .424  .6785        X2        -3.570714 -.695608    .037433   -3.354   .0057
(Constant)     2.700000     .782337               3.451  .0043        X3        -2.143590 -.717566    .110529   -3.569   .0039
                                                                      X4        -1.686292 -.728885    .183848   -3.688   .0031


End Block Number   1   All requested variables entered.


                         * * * * * * * * * * * * * * * * * * * * * * * * * * * * *

Block Number  2.  Method:  Enter      X2


Variable(s) Entered on Step Number  2..    X2


Multiple R            .70065        Analysis of Variance
R Square              .49091                          DF      Sum of Squares      Mean Square
Adjusted R Square     .40606        Regression          2            10.80000          5.40000
Standard Error        .96609        Residual           12            11.20000           .93333

                                    F =      5.78571        Signif F =  .0174
```

```
                      * * * *   M U L T I P L E   R E G R E S S I O N   * * * *

Equation Number 1    Dependent Variable..   Y2

------------------ Variables in the Equation ------------------     ------------- Variables not in the Equation -------------

Variable              B          SE B         Beta          T  Sig T     Variable      Beta In   Partial  Min Toler       T   Sig T

X              1.550000       .455826     3.620020      3.400  .0053     X3          -6.273441 -.327327  4.694E-04   -1.149  .2750
X2             -.125000       .037268    -3.570714     -3.354  .0057     X4          -2.660813 -.326679    .001562   -1.146  .2760
(Constant)     -.800000      1.196291                   -.669  .5163

End Block Number   2   All requested variables entered.


                      * * * * * * * * * * * * * * * * * * * * * * * * * * * * *

Block Number  3.  Method:  Enter      X3

Variable(s) Entered on Step Number  3..    X3

Multiple R           .73855          Analysis of Variance
R Square             .54545                               DF      Sum of Squares      Mean Square
Adjusted R Square    .42149          Regression            3            12.00000          4.00000
Standard Error       .95346          Residual             11            10.00000           .90909

                                     F =       4.40000        Signif F =  .0289

------------------ Variables in the Equation ------------------     ------------- Variables not in the Equation -------------

Variable              B          SE B         Beta          T  Sig T     Variable      Beta In   Partial  Min Toler       T   Sig T

X              -.416667      1.769891     -.973124      -.235  .8182     X4          1.338E-11  .000000  5.478E-06    .000 1.0000
X2              .250000       .328461     7.141428       .761  .4626
X3             -.020833       .018133    -6 273441     -1 149  .2750
(Constant)     2.000000      2.708013                    .739  .4756

End Block Number   3   All requested variables entered.
```

```
               * * * *   M U L T I P L E   R E G R E S S I O N   * * * *

Equation Number 1    Dependent Variable..   Y2

Block Number  4.  Method:  Enter      X4

*  *  *  *  *  *  *  *  ***  WARNING  ***  *  *  *  *  *  *  *

The following variables in the equation have low tolerances:
          Variable    Tolerance
            X2          8.78585E-06
            X3          5.47806E-06
            X4               .00003

*  *  *  *  *  *  * ***  END OF WARNING  ***  *  *  *  *  *  *


Variable(s) Entered on Step Number  4..     X4


Multiple R           .73855         Analysis of Variance
R Square             .54545                          DF      Sum of Squares      Mean Square
Adjusted R Square    .36364         Regression        4            12.00000          3.00000
Standard Error      1.00000         Residual         10            10.00000          1.00000

                                    F =       3.00000      Signif F =  .0723


----------------- Variables in the Equation -----------------

Variable               B         SE B        Beta          T   Sig T

X               -.416667     8.402146    -.973124      -.050   .9614
X2               .250000     2.517976    7.141428       .099   .9229
X3              -.020833      .302502   -6.273441      -.069   .9465
X4           4.34756E-15      .012579   1.338E-11       .000  1.0000
(Constant)      2.000000     9.146948                   .219   .8313

End Block Number   4   All requested variables entered.
```

```
Preceding task required .14 seconds CPU time;  1.37 seconds elapsed.

  14  0  REGRESSION VARIABLES=Y3 X X2 X3 X4/DESCRIPTIVES=DEFAULT/
  15  0   CRITERIA=TOLERANCE (.000001)/DEPENDENT=Y3/ENTER X/ENTER X2
  16  0   /ENTER X3/ENTER X4

There are 185,360 bytes of memory available.
The largest contiguous area has 178,936 bytes.

     1612 bytes of memory required for REGRESSION procedure.
        0 more bytes may be needed for Residuals plots.
```

* * * * M U L T I P L E   R E G R E S S I O N * * * *

Listwise Deletion of Missing Data

	Mean	Std Devi	Label
Y3	3.400	1.765	
X	6.000	2.928	
X2	44.000	35.809	
X3	360.000	377.480	
X4	3132.800	3857.750	

N of Cases = 15

Correlation:

	Y3	X	X2	X3	X4
Y3	1.000	.581	.610	.636	.658
X	.581	1.000	.981	.943	.903
X2	.610	.981	1.000	.989	.968
X3	.636	.943	.989	1.000	.994
X4	.658	.903	.968	.994	1.000

```
                         * * * *   M U L T I P L E   R E G R E S S I O N   * * * *

Equation Number 1    Dependent Variable..   Y3

 Descriptive Statistics are printed on Page   14

Block Number  1.  Method:  Enter      X


Variable(s) Entered on Step Number  1..     X


Multiple R           .58065        Analysis of Variance
R Square             .33716                              DF      Sum of Squares      Mean Square
Adjusted R Square    .28617        Regression             1            14.70000         14.70000
Standard Error      1.49100        Residual              13            28.90000          2.22308

                                   F =       6.61246       Signif F =  .0232

------------------ Variables in the Equation ------------------        ------------- Variables not in the Equation -------------

Variable              B          SE B        Beta         T  Sig T        Variable     Beta In   Partial   Min Toler        T   Sig T

X               .350000       .136109     .580651     2.571  .0232        X2          1.087042   .258326     .037433      .926   .3725
(Constant)     1.300000       .902845                 1.440  .1735        X3           .796266   .325156     .110529     1.191   .2566
                                                                          X4           .728106   .383459     .183848     1.438   .1759


End Block Number   1   All requested variables entered.


                         * * * * * * * * * * * * * * * * * * * * * * * * * * * * *

Block Number  2.  Method:  Enter      X2


Variable(s) Entered on Step Number  2..     X2


Multiple R           .61757        Analysis of Variance
R Square             .38139                              DF      Sum of Squares      Mean Square
Adjusted R Square    .27829        Regression             2            16.62857          8.31429
Standard Error      1.49921        Residual              12            26.97143          2.24762

                                   F =       3.69915       Signif F =  .0560
```

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* * * * MULTIPLE REGRESSION * * * *

Equation Number 1 Dependent Variable.. Y3

------------------ Variables in the Equation ------------------

Variable	B	SE B	Beta	T	Sig T
X	-.292857	.707363	-.485851	-.414	.6862
X2	.053571	.057333	1.087042	.926	.3725
(Constant)	2.800000	1.856435		1.508	.1574

------------ Variables not in the Equation ------------

Variable	Beta In	Partial	Min Toler	T	Sig T
X3	13.368884	.632790	4.694E-04	2.710	.0203
X4	5.940282	.661612	.001562	2.926	.0138

End Block Number 2 All requested variables entered.

* *

Block Number 3. Method: Enter X3

Variable(s) Entered on Step Number 3.. X3

Multiple R	.79316
R Square	.62910
Adjusted R Square	.52794
Standard Error	1.21249

Analysis of Variance

	DF	Sum of Squares	Mean Square
Regression	3	27.42857	9.14286
Residual	11	16.17143	1.47013

F = 6.21908 Signif F = .0100

------------------ Variables in the Equation ------------------

Variable	B	SE B	Beta	T	Sig T
X	5 607143	2.250717	9.302271	2.491	.0300
X2	-1.071429	.417694	-21.740844	-2.565	.0263
X3	062500	.023059	13.368884	2.710	.0203
(Constant)	-5.600000	3.443697		-1.626	.1322

------------- Variables not in the Equation ------------

Variable	Beta In	Partial	Min Toler	T	Sig T
X4	68.313246	.617758	5.478E-06	2.484	.0323

End Block Number 3 All requested variables entered.

```
                         * * * *   M U L T I P L E   R E G R E S S I O N   * * * *

Equation Number 1    Dependent Variable..   Y3

Block Number  4.  Method:  Enter      X4

*  *  *  *  *  *  *  ***  WARNING  ***  *  *  *  *  *  *  *

The following variables in the equation have low tolerances:
          Variable     Tolerance
           X2            8.78585E-06
           X3            5.47806E-06
           X4                .00003

*  *  *  *  *  *  ***  END OF WARNING  ***  *  *  *  *  *  *


Variable(s) Entered on Step Number  4..    X4


Multiple R           .87786          Analysis of Variance
R Square             .77064                             DF      Sum of Squares      Mean Square
Adjusted R Square    .67890          Regression          4            33.60000          8.40000
Standard Error      1.00000          Residual           10            10.00000          1.00000

                                     F =       8.40000       Signif F =  .0031


----------------- Variables in the Equation ------------------

Variable              B         SE B         Beta          T   Sig T

X             -14.750000    8.402146  -24.470306      -1.756   .1097
X2              5.125000    2.517976  103.993703       2.035   .0692
X3              -.687500     .302502 -147.05773       -2.273   .0464
X4               .031250     .012579   68.313246       2.484   .0323
(Constant)     16.000000    9.146948                   1.749   .1108

End Block Number   4   All requested variables entered.
```

```
Preceding task required .13 seconds CPU time;  1.21 seconds elapsed.

  17  0  REGRESSION VARIABLES=Y1 X/DESCRIPTIVES=DEFAULT/
  18  0   CRITERIA=TOLERANCE (.000001)/DEPENDENT=Y1/ENTERX

There are 185,392 bytes of memory available.
The largest contiguous area has 178,936 bytes.

      804 bytes of memory required for REGRESSION procedure.
        0 more bytes may be needed for Residuals plots.
```

**** M U L T I P L E   R E G R E S S I O N ****

Listwise Deletion of Missing Data

	Mean	Std Dev	Label
Y1	4.000	1.690	
X	6.000	2.928	

N of Cases = 15

Correlation:

	Y1	X
Y1	1.000	.866
X	.866	1.000

```
                              * * * *   M U L T I P L E   R E G R E S S I O N   * * * *

Equation Number 1    Dependent Variable..   Y1

  Descriptive Statistics are printed on Page   19

Block Number  1.  Method:  Enter


Variable(s) Entered on Step Number  1..    X


Multiple R           .86603        Analysis of Variance
R Square             .75000                          DF      Sum of Squares      Mean Square
Adjusted R Square    .73077        Regression         1            30.00000         30.00000
Standard Error       .87706        Residual          13            10.00000           .76923

                                   F =       39.00000       Signif F =  .0000


------------------ Variables in the Equation ------------------

Variable              B          SE B        Beta          T  Sig T

X               .500000       .080064     .866025      6.245  .0000
(Constant)     1.000000       .531085                  1.883  .0823

End Block Number   1   All requested variables entered.
```

```
Preceding task required .12 seconds CPU time;  1.05 seconds elapsed.

  19  0  REGRESSION VARIABLES=Y2 X X2/DESCRIPTIVES=DEFAULT/
  20  0   CRITERIA=TOLERANCE (.000001)/DEPENDENT=Y2/ENTER X/ENTER X2

There are 185,384 bytes of memory available.
The largest contiguous area has 178,936 bytes.

     1036 bytes of memory required for REGRESSION procedure.
        0 more bytes may be needed for Residuals plots.
```

**** MULTIPLE REGRESSION ****

Listwise Deletion of Missing Data

	Mean	Std Dev	Label
Y2	3.000	1.254	
X	6.000	2 928	
X2	44 000	35.809	

N of Cases = 15

Correlation:

	Y2	X	X2
Y2	1.000	.117	-.019
X	.117	1.000	.981
X2	-.019	.981	1.000

```
                    * * * *   M U L T I P L E   R E G R E S S I O N   * * * *

Equation Number 1    Dependent Variable..   Y2

 Descriptive Statistics are printed on Page   22

Block Number  1.  Method:  Enter      X


Variable(s) Entered on Step Number  1..     X

Multiple R           .11677        Analysis of Variance
R Square             .01364                          DF      Sum of Squares      Mean Square
Adjusted R Square   -.06224        Regression         1              .30000           .30000
Standard Error      1.29199        Residual          13            21.70000          1.66923

                                   F =        .17972       Signif F =  .6785

------------------ Variables in the Equation ------------------        ------------- Variables not in the Equation ------------

Variable              B          SE B       Beta          T  Sig T        Variable     Beta In  Partial  Min Toler        T  Sig T

X               .050000       .117942    .116775       .424  .6785        X2         -3.570714 -.695608    .037433   -3.354  .0057
(Constant)     2.700000       .782337                 3.451  .0043

End Block Number   1   All requested variables entered.


                    * * * * * * * * * * * * * * * * * * * * * * * * * * * * *

Block Number  2   Method:  Enter      X2


Variable(s) Entered on Step Number  2..     X2

Multiple R           .70065        Analysis of Variance
R Square             49091                           DF      Sum of Squares      Mean Square
Adjusted R Square    .40606        Regression         2            10.80000          5.40000
Standard Error       96609         Residual          12            11.20000           .93333

                                   F =       5.78571       Signif F =  .0174
```

AIIM

Association for Information and Image Management

1100 Wayne Avenue, Suite 1100
Silver Spring, Maryland 20910
301/587-8202

Centimeter

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 mm

1 2 3 4 5

Inches

1.0 1.1 1.25 1.4 1.6 1.8 2.0 2.2 2.5

MANUFACTURED TO AIIM STANDARDS
BY APPLIED IMAGE, INC.

**** MULTIPLE REGRESSION ****

Equation Number 1 Dependent Variable.. Y2

------------------ Variables in the Equation ------------------

Variable	B	SE B	Beta	T	Sig T
X	1.550000	.455826	3.620020	3.400	.0053
X2	-.125000	.037268	-3.570714	-3.354	.0057
(Constant)	-.800000	1.196291		-.669	.5163

End Block Number 2 All requested variables entered.

```
Preceding task required .13 seconds CPU time;  1.14 seconds elapsed.

  21  0  REGRESSION VARIABLES=Y3 X X2 X3/DESCRIPTIVES=DEFAULT/
  22  0   CRITERIA=TOLERANCE (.000001)/DEPENDENT=Y3/ENTER X/ENTER X2
  23  0   /ENTER X3

There are 185,376 bytes of memory available.
The largest contiguous area has 178,936 bytes.

     1316 bytes of memory required for REGRESSION procedure.
        0 more bytes may be needed for Residuals plots.
```

**** MULTIPLE REGRESSION ****

Listwise Deletion of Missing Data

	Mean	Std Dev	Label
Y3	3.400	1.765	
X	6.000	2.928	
X2	44.000	35.809	
X3	360.000	377.480	

N of Cases = 15

Correlation:

	Y3	X	X2	X3
Y3	1.000	.581	.610	.636
X	.581	1.000	.981	.943
X2	.610	.981	1.000	.989
X3	.636	.943	.989	1.000

```
                    * * * *   M U L T I P L E   R E G R E S S I O N   * * * *

Equation Number 1    Dependent Variable..   Y3

 Descriptive Statistics are printed on Page   26

Block Number  1.  Method:  Enter      X


Variable(s) Entered on Step Number  1..    X


Multiple R           .58065        Analysis of Variance
R Square             .33716                          DF      Sum of Squares      Mean Square
Adjusted R Square    .28617        Regression         1            14.70000         14.70000
Standard Error      1.49100        Residual          13            28.90000          2.22308

                                   F =       6.61246       Signif F =  .0232

------------------ Variables in the Equation ------------------       ------------- Variables not in the Equation -------------

Variable              B        SE B       Beta         T   Sig T       Variable     Beta In   Partial   Min Toler        T   Sig T

X               .350000     .136109    .580651     2.571   .0232       X2          1.087042   .258326     .037433     .926   .3725
(Constant)     1.300000     .902845                1.440   .1735       X3           .796266   .325156     .110529    1.191   .2566

End Block Number   1   All requested variables entered.


                    * * * * * * * * * * * * * * * * * * * * * * * * * * * * *

Block Number  2.  Method:  Enter      X2


Variable(s) Entered on Step Number  2..    X2


Multiple R           .61757        Analysis of Variance
R Square             .38139                          DF      Sum of Squares      Mean Square
Adjusted R Square    .27829        Regression         2            16.62857          8.31429
Standard Error      1.49921        Residual          12            26.97143          2.24762

                                   F =       3.69915       Signif F =  .0560
```

* * * * M U L T I P L E R E G R E S S I O N * * * *

Equation Number 1 Dependent Variable.. Y3

------------------ Variables in the Equation ------------------

Variable	B	SE B	Beta	T	Sig T
X	-.292857	.707363	-.485851	-.414	.6862
X2	.053571	.057833	1.087042	.926	.3725
(Constant)	2.800000	1.856435		1.508	.1574

------------- Variables not in the Equation -------------

Variable	Beta In	Partial	Min Toler	T	Sig T
X3	13.368884	.632790	4.694E-04	2.710	.0203

End Block Number 2 All requested variables entered.

* *

Block Number 3. Method: Enter X3

Variable(s) Entered on Step Number 3.. X3

Multiple R	.79316
R Square	.62910
Adjusted R Square	52794
Standard Error	1.21249

Analysis of Variance

	DF	Sum of Squares	Mean Square
Regression	3	27.42857	9.14286
Residual	11	16.17143	1.47013

F = 6.21908 Signif F = .0100

------------------ Variables in the Equation ------------------

Variable	B	SE B	Beta	T	Sig T
X	5.607143	2 250717	9.302271	2.491	.0300
X2	-1.071429	417694	-21.740844	-2.565	.0263
X3	.062500	.023059	13.368884	2.710	.0203
(Constant)	-5.600000	3.443697		-1.626	.1322

End Block Number 3 All requested variables entered.