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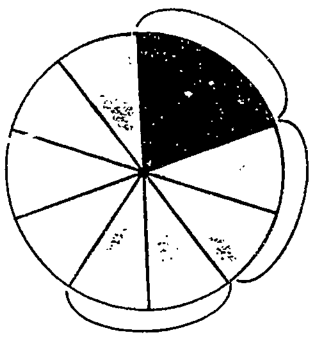
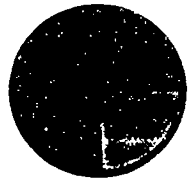
ABSTRACT

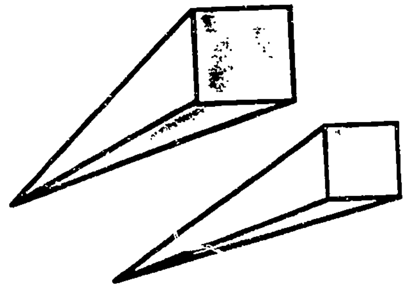
This document contains the revisions to the 5-8 Mathematics curriculum produced for Manitoba (Canada) teachers. It contains an introduction and seven content sections: Problem Solving, Algebra, Data Management, Geometry, Measurement, Number Concepts, and Number Operations. The introduction includes discussion of: rationale, goals, big ideas (mathematical reasoning; patterns and problem solving; mental mathematics; communication; connections; and review, drill, and practice), the seven strand areas, instructional practices, use of technology, changes from previous curriculum guides, assessment and evaluation, resources, and information for parents. Each of the seven strand sections contains a discussion of the topic, goals, and examples. Appendices include: "Kinds of Problem Solving," "Principles to be Used in Calculation," "Subtraction Algorithms," and "Division Algorithms." (MKR)

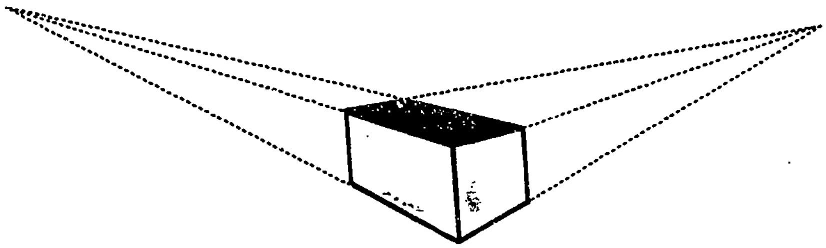
1994

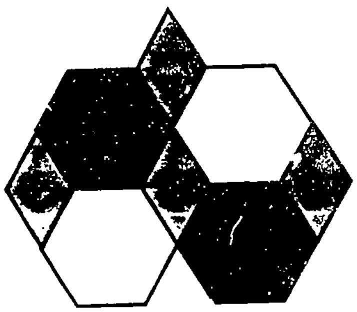
5-8 Mathematics

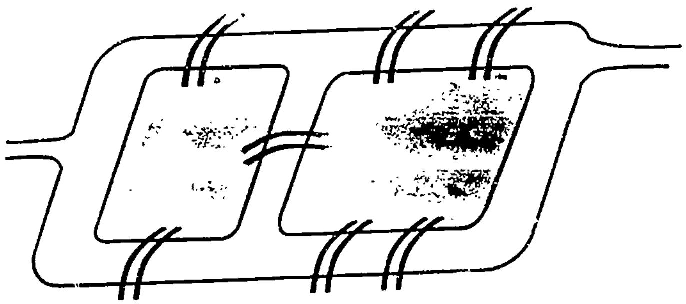












Interim Guide

Manitoba Education and Training

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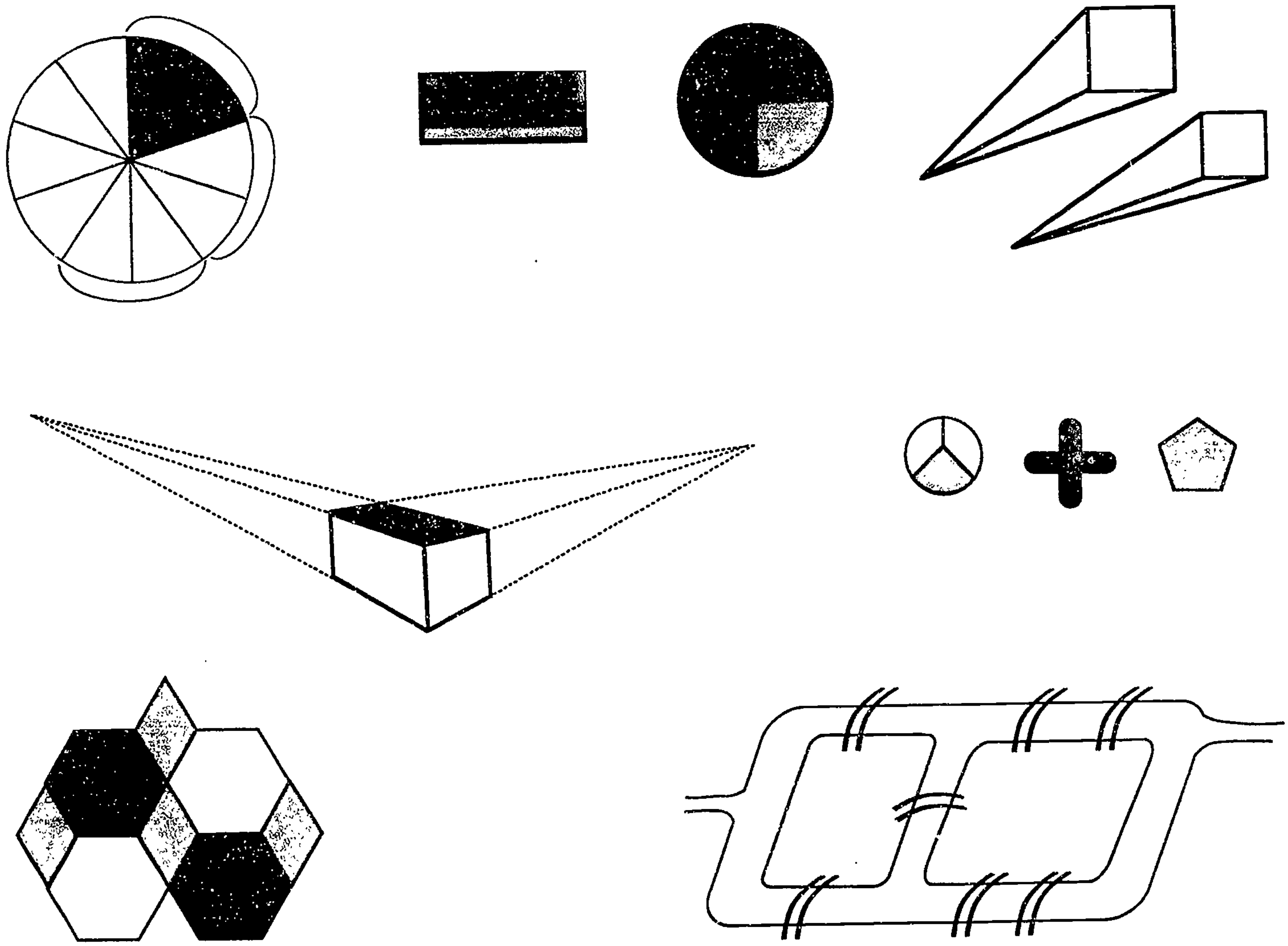
5-8 Mathematics



Interim Guide

Manitoba
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Contents

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Introduction to 5-8 Mathematics

Revisions to the Manitoba 5 – 8 Mathematics curriculum began in 1991. Scope and sequence charts were distributed to Regional Reaction Panels in 1992. The results of those meetings guided the preparation of pilot documents for classroom testing in 1993-94. Reactions of the pilot teachers led to current revisions and clarifications and the release of this interim guide in August 1994.

The reactions of classroom teachers to these proposals have been positive. However, in view of the fundamental transformation of the school mathematics curriculum, begun in this guide, ***it is expected that further reactions will necessitate ongoing revisions every two years.***

Rationale

The way the world uses mathematical knowledge has changed dramatically in the past 50 years. Technical innovations have reduced significantly the need for "shopkeeper arithmetic." At the same time, these innovations have broadened the ways most school graduates are expected to use mathematics.

In addition to completing some calculations by hand, students in the middle years now need to solve routine and novel problems, manage and interpret data, and use technical aids intelligently.

The revised curriculum responds to a variety of needs in reasonable and manageable procedures. Above all, it is intended to make mathematics a way of thinking which can be understood, not just "done" – to provide students with quantitative and geometric ways of thinking about their world.

Goals

Manitoba has a history of evolving the mathematics curriculum in a thoughtful and deliberate manner. The present revision represents another step in the continuing process of adapting the curriculum to meet students' needs

Manitoba is not alone in preparing students to meet these needs. Across North America, similar revisions are under way largely influenced by the ***Curriculum and Evaluation Standards for School Mathematics*** published in 1989 by the National Council for Teachers of Mathematics. This document identifies five major goals for students. Manitoba has adopted these goals and they are highlighted below.

Students should:

- learn to value mathematics
- become confident in their mathematical abilities
- become mathematical problem solvers
- learn to communicate mathematically
- learn to reason mathematically

Big Ideas

A number of "Big Ideas" have been identified in order for students to reach the goals of the revised 5 – 8 Mathematics curriculum. It is intended that these interrelated "Big Ideas" permeate all strands of the curriculum.

These "Big Ideas" include:

Mathematical Reasoning

More emphasis should be placed on thinking and reasoning in mathematics and less on memorization and the use of rote algorithms. In the middle years, mathematical reasoning should involve thinking, conjecturing, and validating, which help students to recognize that mathematics makes sense.

Patterns and Problem Solving

Students should be exposed continuously to a variety of routine and novel problems in all strands by:

- exploring problems with multiple solutions and multiple methods of solution
- creating their own problems
- exploring multiple variations on the same problem

Mental Mathematics

Students should, at every opportunity, be asked to answer questions orally and to estimate answers. Estimation is only one aspect of mental math. It is, however, a process that is important for everyday activities and, therefore, should be developed through frequent practice. As well, students need opportunities to practise routine calculations without paper, pencil, or calculator.

Communication

Stress oral and written communication in mathematics. Students should communicate daily (both orally and through diagrams and writing) about their mathematics learning in order to reflect upon, to validate, and to clarify their thinking.

Connections

Students should view mathematics as an integrated whole, rather than as a collection of separate units. To achieve this view, it is important to make connections both within and outside of mathematics. Within mathematics, connections should be made among modes of representation – concrete, pictorial, and symbolic. In addition, these links should connect the various strands, algorithms, and facts. Outside of mathematics, connections must be made to the real world and to other curricular areas.

Review, Drill, and Practice

A number of the objectives in 5-8 Mathematics require that students have immediate responses to some basic facts, procedures, and algorithms. It is important, therefore, that students have opportunities to practise these skills. It is recommended that much of the review, drill, and practice be disguised rather than presented in traditional worksheet fashion with many similar questions together. Also, it is important that review, drill, and practice be done on a regular and cumulative basis so that students are continually developing the facts and procedures they need to apply in problem-solving situations.

The Strands

The revised 5-8 Mathematics curriculum guide contains seven strands:

1. Problem Solving
2. Algebra
3. Data Management
4. Geometry
5. Measurement
6. Number Concepts
7. Number Operations

All seven strands are significant. Considerable time should be spent on the concepts and skills identified in each strand. To make this possible, within current time allotments, several considerations are important:

1. By decreasing emphasis on rote calculation and the size of numbers used in paper-and-pencil calculations, more time is available for concept development, problem solving, and mathematical reasoning.

2. By increasing emphasis on communication, both oral and written, integration with language arts is encouraged and expected.

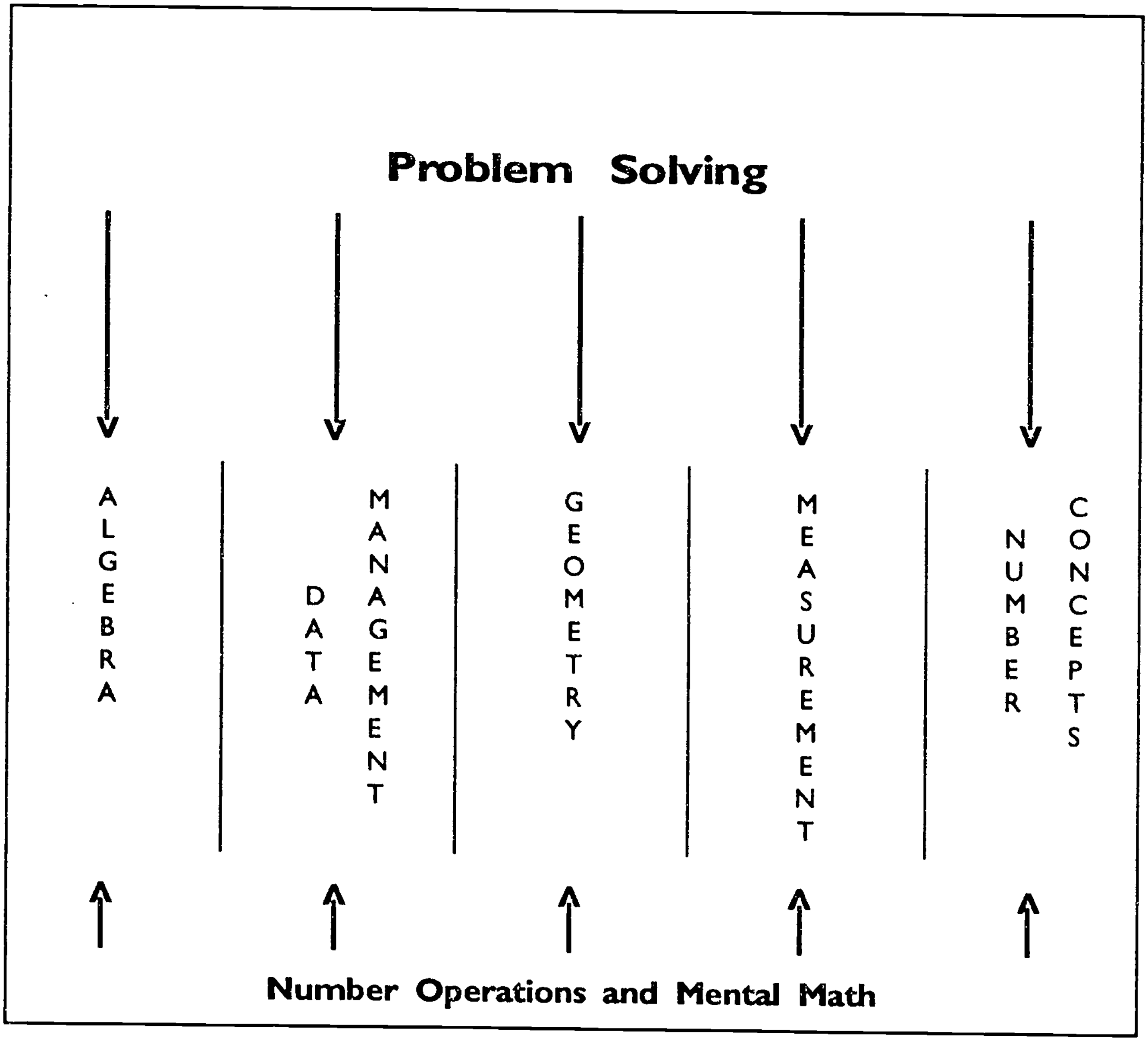
3. ***Problem solving*** is vital to increasing a student's mathematical power and ***should be integrated throughout the program.*** Approximately half the available time within all strands is to be dedicated to problem-solving activities.

4. All strands, outside of Problem Solving, require approximately the same amount of time. This will necessitate a great deal of disguised arithmetic with the teaching and learning of Number Concepts and Number Operations in Algebra, Data Management, Geometry, and Measurement.

5. There is to be a balance between mental arithmetic, paper-and-pencil practice of skills and procedures, and the appropriate use of technology including calculators and computers.

The following are two models which may clarify how the time alloted for mathematics is to be spent. Both models represent the curriculum in a slightly different way. Neither is meant to be a rigid prescription for any grade level nor class, but, rather, each should be looked at as an overview indicating emphasis.

Model A



Problem Solving and Number Operations are to be integrated throughout each strand with approximately 50% of available time for Problem Solving and 10% of available time for Number Operations. Remaining strands are of equal value so each should receive about 8% of the available time.

Model B

		Problem Solving 50%	Number Operations and Mental Math 20%	Application and Practice of Skills, Concepts, and Procedures 30%
20% Each	Algebra			
	Data Management			
	Geometry			
	Measurement			
	Number Concepts			

The categories across the top of the grid identify classroom activities with an approximate time breakdown. Categories down the left side identify the strands or content to be developed and they are expected to receive equal emphasis with about 20% of the time available for each of them.

The Guide

This curriculum guide is designed to identify the mathematical processes, concepts, and skills necessary to establish an appropriate foundation for mathematics in the senior years. Expectations are presented for 5-8 Mathematics using a two-page format. This allows teachers to see at a glance previous, present, and future requirements of students.

The following symbols and icons are used throughout the document:

■ expectations for average students

- how the expectations are to be demonstrated and/or examples to clarify them

[C] connections within the mathematics curriculum or to other disciplines

[?] problem-solving situations

⇒ indicates that students have previously worked with that concept, skill, or process if placed before the Grade 5 column. If placed after the Grade 8 column then extension will occur during the senior years.

Instructional Practices

In keeping with the goals of this middle years curriculum, a range of instructional practices designed to encourage development of the "Big Ideas" is recommended. By using a variety of strategies teachers will:

- provide students with opportunities to improve their understanding of mathematical concepts
- increase their proficiency with basic skills and procedures
- strengthen students' problem-solving abilities

These instructional practices have been influenced by the ***Curriculum and Evaluation Standards for School Mathematics*** (1989). Increased time and attention should be given to:

- exploring independently
- working with concrete, manipulative materials
- working in groups and learning interactively
- talking and writing about mathematics
- justifying thinking and conclusions
- problem solving as a way of teaching concepts and procedures
- inferring principles, procedures, and other mathematical understandings from analysis and exploration

Decreased time and attention, therefore, should be given to:

- rote memorization of rules and algorithms
- teaching by telling
- one answer and one method
- the use of repetitive worksheets

Use of Technology

Calculators and computers can be used effectively in the middle years to augment students' thinking about mathematics. These tools are not meant to replace thinking or to discourage students from learning important and useful facts and skills. They are to be used as aids in the learning and exploring of mathematics.

Calculators

Calculators should be used as arithmetic tools. It no longer makes sense to do large quantities of arithmetic or to deal with large and complex numbers by hand. This time is better spent on the development of mathematical concepts or the solving of important problems. Students can use calculators to test mathematical ideas that would take far too long to do by hand. It is most important that students have a good understanding (as appropriate for their grade level) of the four arithmetic operations as well as a high level of proficiency with the "basic facts." This enables them to estimate prior to using the calculator in their study of mathematics.

It is unnecessary for teachers to insist that all students in a class have the same kind of calculator. There are considerable benefits in having a variety of calculators in any class. Beyond the simplest operations, different calculators may treat the same string of keystrokes differently. Inferring how they treat input can be a superb problem-solving exercise.

Computers

A wide range of computer software programs are available for use at the 5-8 level. The use of computers can greatly enhance the learning of mathematics in any classroom. In much the same way as calculators, computers are not meant to replace students' thinking, but, rather, to act as tools for the management of data and the solving of non-routine and previously inaccessible problems. A list of software programs useful for the middle years is included below.

Note that the list is not meant to be complete and has been generated from suggestions by pilot teachers and/or committee members. Individual pieces of software or courseware have not been formally evaluated by Manitoba Education and Training.

Tool Software

Microsoft Works
Clarisworks
LOGO
or other similar tool software packages

Courseware from Sunburst

- Building Perspective
- The King's Rule
- Royal Rules
- Data Insights (7/8)
- The Right Turn
- Predictions from Samples
- Keep Your Balance!
- Statistics Workshop (7/8)
- Interpreting Graphs
- The Factory
- Creature Cube
- A Chance Look
- What to Do With a Broken Calculator
- Puzzle Tanks
- Data and Decisions
- Safari Search
- Teasers by Tobbs with Whole Numbers
- Teasers by Tobbs with Integers
- More Teasers from Tobbs

Couseware from MECC

- MECC Math Link (print package correlating math objectives with MECC products)
- Amazing Arithmetrics
- Deuling Digits
- Take a Chance
- Probability Lab (7/8)
- Problem Solving with Nim
- Conquering Mathematics Series
- TesselMania!
- DinoPark Tycoon
- Estimation Activities
- Estimation Strategies
- The Geometric Golfer (7/8)

Mathematics Activities Courseware (MAC), *(Houghton Mifflin Canada)*

Separate binders for each of Grades 5-8. Available through Nelson Canada.

Changes from Previous Curriculum Guides

Changes from previous curriculum guides have been made in response to the growing body of mathematical knowledge and to increased information about how students learn mathematical concepts. The major change is the requirement of increased emphasis on mathematical reasoning with a corresponding decrease in the use of rote algorithms. In addition, students are required to demonstrate their knowledge of mathematical concepts through discussion and the use of concrete materials, pictures, diagrams, and symbols. Changes have been made in both the content and the methodology of the curriculum.

Some specific changes recommended are to:

- increase emphasis on thinking or reasoning with a corresponding decrease in rote learning
- increase emphasis on patterns and problem solving
- include Data Management in Grades 5-8. Data Management consists of formulating the data, communicating, analyzing, and interpreting the data, and performing experiments or activities in probability
- introduce algebraic concepts by using number patterns beginning in Grade 5
- limit the size of numbers on which students perform pencil-and-paper calculations. For example, pencil-and-paper multiplication of whole numbers is limited to 3 digits x 2 digits. After that, use a calculator
- increase the emphasis on concept development and mental mathematics
- stress multiple techniques including manipulatives, diagrams, calculators, alternate formats, and a variety of algorithms
- introduce concepts and operations simultaneously with decimals and fractions
- delay formal operations with fractions until Grades 7 and 8. Addition and subtraction of fractions in symbolic form are introduced as new work in Grade 7 with multiplication and division in Grade 8
- introduce in Grade 7 addition and subtraction of integers. Multiplication and division of integers are covered in Grade 8

Assessment and Evaluation

Assessment allows for the continuous monitoring of student difficulties and progress, and can assist in making ongoing instructional decisions. Assessment occurs at the intersection of important mathematical concepts that deal with how they are taught, what is learned, and how they are learned. It is recommended that a **wide variety of assessment strategies** be used so that appropriate decisions regarding student evaluation and reporting to parents can be made. The following approaches are suggested:

Performance Assessments

Traditional tests of mathematical content may not reveal enough information about students' abilities in mathematics. Well-constructed performance tasks allow for the examination of the processes used by students, not simply the answer or end product.

Observations and Interviews

Teachers can gather a wealth of information about students' abilities in mathematics through observations and interviews. Some teachers find it helpful to jot down notes while interviewing or observing students. These notes can then be collected and are helpful when communicating to parents.

Portfolios, Journals, and Projects

Assessing the work that students do on projects and the records they make of their work in their journals are two additional ways to vary assessment methods. Samples of a wide range of a students' work, such as solutions to problems, tests, journal entries, writing about mathematics, and group tasks can be kept in a portfolio. When considered along with other assessment tools, the portfolio can provide a more complete picture of student progress and understanding.

Assessment Resources

Artzt, Alice and Newman, Clare. *How to Use Cooperative Learning in the Mathematics Class*. Reston, VA: National Council of Teachers of Mathematics, 1990.

Mathematical Science Education Board. *Measuring Up: Prototypes for Mathematics Assessment*. Washington, DC: National Academy Press, 1993.

Stenmark, Jean (ed.). *Mathematics Assessment: Myths, Models, Good Questions, and Practical Suggestions.* Reston, VA: National Council of Teachers of Mathematics, 1991.

Tunis, Harry B. (ed.). *Assessment* (Focus issue of the *Arithmetic Teacher*). Reston, VA: National Council of Teachers of Mathematics, Feb. 1992.

Tunis, Harry B. (ed.). *Alternative Assessment* (Focus issue of *Mathematics Teacher*). Reston, VA: National Council of Teachers of Mathematics, Nov. 1992.

Webb, Norman (ed.). *Assessment in the Mathematics Classroom*. Reston, VA: National Council of Teachers of Mathematics, 1993.

Non-Print Resource to Support the Implementation of 5-8 Mathematics

Burns, Marilyn. *Mathematics: Assessing Understanding (Grades 2-6)*. Set of 3 videotapes. Don Mills, ON: Addison-Wesley Publishers, 1993.

Resources

The following manipulative materials and print/non-print resources are necessary for resource-based learning. However, they may need to be purchased or collected over a one- or two-year period, depending on available funds.

Manipulative Materials Needed for Each Classroom

The following materials are recommended for use in each classroom. However, some of the materials can be shared or drawn from a central supply. Schools need to decide which materials are needed in each classroom on a permanent basis.

Essential Materials

Base 10 blocks and overhead set
Geoboards (10 x 10 pin or larger) and coloured elastic bands
Pattern blocks and overhead set
Coloured tiles, rods, or counters
Fraction pieces and overhead set
Interlocking cubes
Dice (varied and multi-sided)
Blank cards and/or playing cards
Mira
Measuring tools (tape measures, metre sticks, rulers, centicubes, measuring spoons, graduated cylinders, equal-arm balance, spring scales)
Geometric solids
Tangrams (can be made) and overhead set
Algebra tiles (can be made) and overhead set (Grade 8)
Calculators (school set or supplied by students). Overhead calculator is useful.
Overhead manipulatives (Recommended in Grades 7 and 8 where hands-on materials have been used extensively in K-6.)

Additional Recommended Materials

Pentominoes and overhead set
Geometry set (supplied by students)
Attribute blocks (4 attributes) with overhead set
Hinged mirrors
Money sets
D.I.M.E. solids
Dominoes
Decimal chips for Base 10 blocks
Googolplex
Place value mats (can be made)
Pentacubes
3-D divergent thinking puzzles
Probability spinners (can be made)

Print Resources to Support the Implementation of 5-8 Mathematics

Burns, Marilyn. *About Teaching Mathematics (K-8)*. White Plains. NY: Cuisenaire Materials, 1993.

Burns, Marilyn and McLaughlin, Cathy. *A Collection of Math Lessons (3-6) or (6-8)*. New York, NY: Math Solution Publications, 1990.

Farrell, Margaret. *A Bridge to the Classroom for Grades 5-12: Implementing the NCTM Standards*. Agincourt, ON: Gage Educational Publishing, 1994.

Fridriksson, Thor. *Common Terms: A Dictionary of Mathematics*. Morongo Valley, CA: Sagebrush Publishing, 1990.

National Council of Teachers of Mathematics. *Curriculum and Evaluation Standards for School Mathematics*. Reston, VA: NCTM, 1989.

National Council of Teachers of Mathematics. *Professional Standards for School Mathematics*. Reston, VA: NCTM. 1991.

Nichols, Eugene and Schwartz, Sharon. *Mathematics Dictionary and Handbook*. Honesdale, PA: Nichols Schwartz Publishing, 1993.

Thornton, Carol and Bley, Nancy (eds.). *Windows of Opportunity: Mathematics for Students with Special Needs*. Reston, VA: National Council of Teachers of Mathematics, 1994.

Tunis, Harry B. (ed.). *Mathematics Teaching in the Middle School* (Journal published four times per year). Reston, VA: National Council of Teachers of Mathematics, Apr. 1994.

Wagner, Sigrid (ed.). *Research Ideas for the Classroom: Middle Grade Mathematics*. Reston, VA: National Council of Teachers of Mathematics, 1993.

Non-Print Resources to Support the Implementation of 5-8 Mathematics

Burns, Marilyn. *Mathematics for the Middle School.* (Staff development videotapes for Grades 6 to Senior 1.) Aurora, ON: Spectrum, 1990.

Lovitt, Charles, et al. *Mathematics Curriculum and Teaching Program (K-10).* (Two volumes, three videotapes, and three diskettes [Macintosh, IBM, or Apple II] based on 114 classroom activities developed in Australia.) Reston, VA: National Council of Teachers of Mathematics, 1988.

Information for Parents

Teachers need to meet with parents to identify differences and/or points of emphasis in the revised 5-8 Mathematics curriculum. Most new print resources provide some information for parents. The following is intended to summarize major thrusts in the proposed revisions; it may be useful in a letter to parents.

The 5-8 Mathematics curriculum has been revised to help students meet the challenges of a society that is becoming increasingly dependent on widespread applications of mathematics and technology. To meet these demands, students need to become creative thinkers, problem solvers, data managers, and collaborative workers.

If students are to become creative thinkers, problem solvers, etc., then changes are needed in the way that mathematics is presently taught and learned. Worksheets, using only one text resource, and repetitive exercises are being de-emphasized. Instead, the focus is being placed on problem solving, communicating, reasoning, and developing proficiency with important skills and procedures. A balance is required among paper-and- pencil practice of skills and procedures, mental mathematics, and the appropriate use of technology.

Students will be given many opportunities to explore and investigate concepts and relationships in mathematics. At times, they will use concrete materials, calculators and/or computers for these tasks. The use of materials and technology in classrooms is designed to help students develop greater understanding and confidence. To develop skills and procedures, teachers will provide students with appropriate games, puzzles, and problems which are recommended ways for students to review and practise mathematics.

The proposed revisions represent new ways for students to learn mathematics. It follows, therefore, that teachers will use new methods for assessing students' ability and understanding in mathematics. Teachers may employ a wide range of assessment approaches, such as anecdotal records, projects, journals, performance tasks, tests, written work, and portfolios. These are all methods of assessment that are in line with the instruction that students are to receive in mathematics.

The most notable changes in the revised 5-8 Mathematics curriculum include an increase in high level thinking and reasoning, a decrease in the size of numbers for paper-and-pencil calculations, and the inclusion of a Data Management strand. Another important change of focus is toward connecting concepts and skills within mathematics and connecting mathematics to other subject areas. The cumulative nature of mathematics will be enhanced with students continuing to use skills and concepts previously learned as they progress through all the strands.

Problem Solving

The area of mathematics known as problem solving is a complex one, not only because problems themselves are complex but also because there is no clear-cut approach to teaching problem solving. **The intention is that students learn to be problem solvers by doing problems.** Thus, 50% of the time available for mathematics is to be devoted to problem solving. Problem solving must encompass more than traditional "word problems" where basically one method and one answer is emphasized. For example, working through the process outlined in the Data Management strand requires a great deal of thinking, organizing, and analyzing and is, in its own right, problem solving. Both the templates and strategies outlined as Type I and Type II problem solving later in this strand are intended to be used as tools and **not** as directives to be memorized for use in rote exercises.

Characteristics of Effective Problems

Students should be exposed to a wide variety of problems which:

- encourage many different approaches to solving a particular problem
- combine multiple arithmetic operations
- require students to complete the problem, that is, where the students must either collect more information or identify assumptions before solving
- cause students to formulate their own problems because of the open-ended nature of the problem
- require application of multiple concepts, processes, or skills being studied
- encourage students to develop procedures for solving complex problems over time, for example, Data Management processes, Algebra patterning, or Measurement projects
- encourage students to work in groups or teams
- emphasize multiple solutions

Students should be required to verbalize their solutions and the processes used to solve problems.

Types of Problem Solving

Solving problems can be separated into two distinct categories.

Type I: This type of problem solving is characterized by the solver's ability to recognize a form or template and then to apply the template in solving the problem, e.g., if Mary bought 3 candies each day for 5 days, how many candies did she buy? To most people this situation is recognized as being one of applying the template a X b = ? and getting the solution 3 candies/day for 5 days is 3 x 5 = 15 candies.

This guide identifies 13 basic templates to be used in Type I problem solving. Students need to be able to manage problems where the structure matches all 13 templates identified in Appendix A on page 22. However, emphasis is to be placed on having students solve problems and not on naming the particular template(s) to be used. **Students must learn to solve problems involving the combined use of multiple templates.** This is best accomplished by having students solve problems and learn by doing.

Type II: This type of problem solving encourages creative and logical thinking as well as divergent thought processes.

In Type II problem solving, the solution is not directly obvious. The solver has no obvious approach that can be applied. Rather, he or she must try a variety of strategies until a solution is achieved. Strategies included in 5-8 Mathematics are listed below and described in Appendix A on page 22:

- Partition the Problem
- Examine Cases
- Make a Related but Simpler Problem
- Work Backwards
- Draw a Picture
- Reflection
- Fermi Problems

At first, the choice of strategies will be rather arbitrary. As students' skills improve with practice, the choice of strategies will be increasingly varied. When a strategy becomes obvious to a student, the problem has migrated from a Type II to a Type I for that individual. This curriculum emphasizes both Type I and Type II problem solving. Teachers need to call attention to alternate strategies and/or templates which could be used in solving particular problems.

Students should realize that problems are a natural part of living and that many problems do not have clear-cut answers. The "right" answers often depend on who is asking the question or why the question is being asked.

For example, reflect on the following question: "How many people were in attendance at the hockey game tonight?"

- a rough estimate of 20 000 might be correct if an individual talks to an acquaintance
- a reporter who is writing about the game might want a slightly better estimate of 23 000
- the accountant for the team may need the precise answer of 23 207 for the owners to make decisions affecting the team

The teacher can often present an investigative type of problem as a means of introducing a particular concept or developing a particular skill. "What is the relation between the sizes of the various parts of the body?" could be a good problem to begin an investigation of measuring skills, ratios, fractions, and/or data management concepts. A problem such as "How can we estimate the number of books in our library or school?" could lead the class to investigate problem-solving strategies or to extend their counting skills and number concepts.

For a more detailed treatment of problems and problem solving, refer to Appendix A on the next page.

Kinds of Problem Solving

"Problem solving" has many different meanings.

Take, for example:

1. *John has six pencils. Mary has seven pencils. How many pencils do they have altogether?*
2. *John has more pencils than Mary. Mary has more pencils than Frank. Frank has more than one pencil. Together they have less than 10 pencils. How many pencils does each person have?*

Some would read the first question and say, "That's not a problem. It's just arithmetic." Just about everyone would agree that the second question is a problem; it is not immediately clear what arithmetic may have to be done.

This curriculum guide includes both of the above kinds of problems. This requires a distinction between the two kinds of problem solving – routine problem solving and the solving of non-routine or novel problems. They are called Type I problem solving and Type II problem solving.

The mental processes involved in the two kinds of problem solving seem to be different. Suppose, for example, a teacher tells a student: "There are 200 children in a school. The school intended to create classes of 25. How many classes would be created?" Beyond the first introduction of such problems, a student would think, "Oh, it's one of those," and proceed to the appropriate arithmetic. If he or she took this approach, it would be called Type I problem solving. Using cues, the student recognized a relationship between quantities that may be called a pattern or a template. He or she slots the given numbers into the pattern or template and continues. Since the word "pattern" is commonly used in other contexts within this document, the patterns that students recognize are called "templates."

Suppose, on the other hand, the same student was asked to make a list of all the whole numbers less than six that divide six evenly (1, 2, and 3) and to add them up. After pointing out that their sum is six, the teacher might ask, "Are there any other numbers like that?" Most students would think, "Gee, I've never seen anything like that before." If the student did not stop and say, "You haven't shown us how to do those," the student would engage one or more strategies; he or she would be into Type II problem solving. Some students might first use the simple strategy of testing other numbers more or less at random, but many would soon see the advantages of the more powerful strategy of testing a sequence of numbers, probably beginning with some small number like 2 or 3. In time, many would discover that the next such number is 28 and that the next one after 28 is 496.

Note: It is problem solving, not problems, that can be Type I or Type II. Teachers may give a student a problem with the expectation that he or she will say, "Oh, it's one of those" and solve it quickly. The student may not recognize any template in the problem and may use some strategy to solve it. Similarly, teachers may give a student a problem that they imagine to be novel and the student may have solved several like it before and may solve the new one immediately.

Which Kind of Problem Solving Should We Teach?

There is a simple answer to that question – both. Everyone, from students in First Grade to graduate courses in Engineering and Mathematics, needs to be able to solve problems both ways.

Consider two examples that are remote from mathematics. Imagine a dentist peering into a patient's mouth. Would you prefer the dentist to say, "Oh, it's one of those," or "Now that's an interesting problem"? There are many circumstances in which our society prefers others to recognize templates in problems and to solve them in routine ways.

Suppose, however, that a patient with ongoing sharp abdominal pains visits a doctor. The doctor, who conducts a sequence of diagnostic tests, later says, "I don't recognize any pattern in your symptoms. Sorry. There's nothing I can do. Good luck." There are many circumstances in which an individual expects those around him or her to use strategies to make sense of apparently novel problems.

Quite analogous circumstances occur with applications of mathematics. If a person was to bring, say, three identical shirts and a pair of children's shoes to a cashier with the knowledge that a federal tax on the entire purchase would be included in the total cost and that a provincial tax would be included only for the shirts, the cashier would not be expected to say, "Now that's an interesting problem. Let's see if we can work it out." You might even expect the whole transaction to be programmed into the cash register. If the store manager wandered by as the shopper reached the cashier and noticed a flaw in one of the shirts, he or she might say, "If this customer still wants it, take 40 percent off that shirt." The shopper and the manager would hope that even if the cashier had never met a similar problem before, that he or she could use a simple strategy to work it out. In this case, the cashier might partition the problem – a common and effective strategy.

There are, of course, many circumstances similar to the above transaction in which neither the customer nor the cashier has to think at all as everything is programmed into the cash register. In the mathematics curriculum, however, teachers must be concerned with situations in which students have to decide which calculator buttons to push and, later, how to program the computer. Therefore, it is essential that both kinds of problem solving are taught.

The Place of Problem Solving in the Curriculum

The teaching of problem solving is interwoven with mathematical ideas, rules, algorithms, and facts. Unless teachers aim to teach only repetitive skills for standardized tests, problem solving cannot proceed far without mathematical ideas, rules, etc., particularly with the rapid expansion in the use of electronic calculating tools.

As a result, it is generally agreed that the main focus of contemporary mathematics curricula ought to be on problem solving. This curriculum guide is consistent with that aim. While continuous attention must be given to mathematical ideas, rules, algorithms, and facts, the focus of what teachers do should be directed to teaching students to solve problems.

Using paper and concrete objects, teachers can structure learning experiences which familiarize students with situations relevant to the present and future with the intent of providing them with a sufficient set of templates to later apply mathematics in their daily lives while, at the same time, developing a sufficient set of strategies to manage simple but apparently novel problems.

In the following three sections, teachers are provided with some st: :cture concerning the teaching of both kinds of problem solving. The first section, below, is devoted to Type I problem solving, the next to Type II problem solving, and the brief third section to how they are blended.

The third section is brief but important. Suppose a student were to be familiar with the 13 simple templates discussed in the first section. Given problems like, "They need 53 cans of pop at the picnic and already have 32. How many more will they need?" and "How many 6-packs will they have to buy to have 48 cans of pop?" He or she would think, "Oh, it's one of those," and tackle the arithmetic. But suppose the teacher was to give the student the problem, "Susan brought three cans of pop to the cashier, gave the store employee two dollars, and got 20 cents in change. What did each can of pop cost?" It is not a difficult problem, but to find a solution is likely to entail the use of a simple strategy, partitioning the problem. Once it is partitioned, the student can use one template to decide that the pop costs $1.80. Using another strategy, he or she can decide that each can of pop costs 60 cents.

Beyond the beginning stages, ***templates and strategies are blended in this way continually.*** In time, students come to recognize overall templates in the problems and to shift to using strategies for yet more complex problems. That is how their problem-solving skills develop.

I. Teaching Type I Problem Solving

Ultimately, for example, in engineering and architectural practice, it is expected that those who apply mathematics must be familiar with a large number of templates. Whether designing a bridge across a chasm, with a given width and quality of supporting banks, or developing a continuous process for treating and partially recycling waste, or creating a computer package for massaging input in specified ways, a professionally-trained person is trained to most often say, "Oh, it's one of those." He or she would then write down suitable formulas, equations, or flowcharts, and get on with finding a solution. Teachers set the foundation for this kind of thinking in the middle years.

Fortunately, at this level, teachers do not have to cope with a large number of templates. About 13 will suffice.

Before turning to the task of teaching students about templates, there are two points to consider:

1. If teachers are to talk to one another and to students about templates, it is sometimes (but not always) useful to have some ways of representing them. This can include things that can be written down, pointed to, and talked about – "See? This is the (structure, pattern, template) that we are talking about."

   Keep in mind, however, that a template is a sense of relationship in a person's mind, not something that is written down. Prior to university level, many quite successful problem solvers rarely write down any representations of the templates they sense in problems.

There are several ways of representing templates that can be useful for teaching purposes. The writing down of representations of templates, however, should never become the main objective. ***The main objective is sensing templates in situations in order to solve problems.***

2. This guide identifies the 13 templates that underpin virtually all Type I problem solving in the middle years. This runs the risk that the exercise will deteriorate into 13 "types" of problems, taught independently, each with its own "magic words" cueing a template as though there were strong partitions between them.

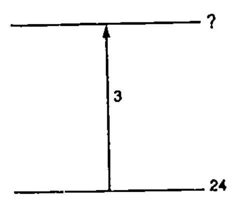
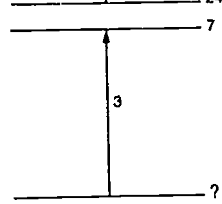
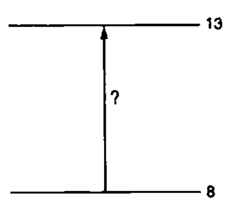
   What follows helps prevent that from happening, but nevertheless the teacher must watch for the following:

   - there is often more than one template that will fit the problem
   - students commonly transform the templates of problems almost as they detect them and in ways they themselves are unable to explain
   - some more able students invent their own more encompassing templates

Knowing what the templates are can be useful. It ensures that students are familiar with all of them, as well as assists them to diagnose difficulties. Avoid over-emphasizing the templates as this would be contrary to the spirit of problem solving.

Additive-Subtractive Templates

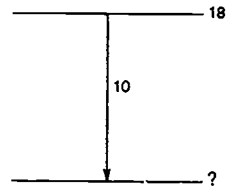
At the outset, there are six additive-subtractive templates which students soon blend and transform on their own. There are two ways to represent them using equations and using arrow diagrams. The six additive-subtractive templates, shown below, use both.

Sample Problem	Representation		Comment
1. There were 24 students in the class, but three new students arrived. How many were then in the class?	24 + 3 = [ ]		Students "talk" this template so these tend to be "easy" problems.
2. After the librarian gave her three more references on horses, Gail had seven of them. How many did she have to begin with?	[ ] + 3 = 7		Some students will transform this problem as they read it to 3 + [ ] = 7 or to 7 - 3 = [ ].
3. In the first term, the school volleyball team won eight games. By the end of the year they had won 13. How many games did they win in the second term?	8 + [ ] = 13		As in sample 2, some students will transform this problem to fit the template 13 - 8 = [ ]

Sample Problem (Cont.) | **Representation** | **Comment**

4. The cooking class made 18 pizzas and sold 10 at a noon-hour sale. How many pizzas did they have left?

18 - 10 [ ]

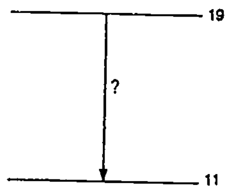


When teaching the regroup-take-away subtraction algorithm, "talk" this template.

Problems of this kind are then generally "easy."

5. The chess club began with 19 members, but by Christmas there were only 11. How many members left?

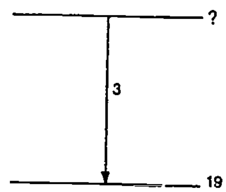
19 - [ ] = 11



Problems of this kind are generally a little more difficult, because teachers do not talk about the subtraction algorithm this way. Students must learn to bring subtraction to this template.

6. Three students in Ms Black's class moved away. Then there were only 19 students left in her class. How many students did she have to begin with?

[ ] - 3 = 19



This is often the first template to go as students begin to transform problems as they read them.

From near the beginning, if they are asked to write down a representation, they will write 19 + 3 = [ ] or 3 + 19 = [ ].

Recognizing Templates

It was noted above that, early on, some students will begin to transform problems as they read them, often so as to make them more amenable to the arithmetic that will follow. There is nothing wrong with this practice – it should be encouraged. The teacher should present problems in all six ways. Students must recognize all six relationships between quantities before they can transform problems.

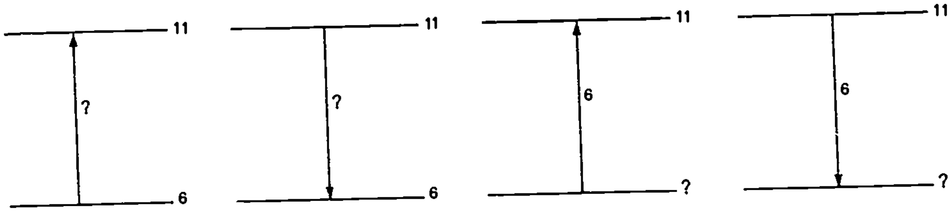
Imposing Templates

All of the above problems implied some action – an action matching the template that is being considered. Before long, however, students meet problems in which no action is implied. Consider the following problem:

> *Susan scored six points in the game. Megan scored 11. How many more points did Megan score than Susan?*

Now the students must impose a template on the situation, and now there is no telling which one they will impose.

If the students were asked to represent the problem somehow, they might draw any of the arrow diagrams which follow or any of the corresponding equations.



It sometimes pays to have students compare and discuss how various individuals think about a problem in order to represent it in specific ways.

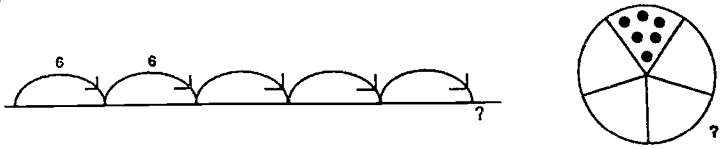
Multiplicative/Divisive Templates.

Again, begin with six templates. Students must learn to sense all six of them, even though most end up transforming problems as they read them so as to use only three or four. For each problem, consider three possible representations because no one kind of representation serves all six templates well.

Sample Problem | **Representations**

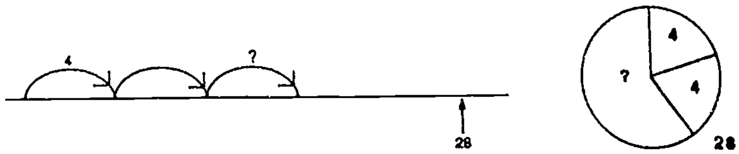
1. How many cans of pop are there in five six-packs?

$5 \times 6 = [\ ]$



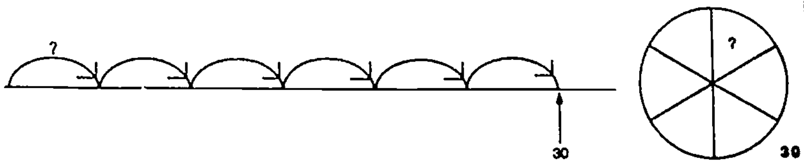
2. How many packages of four crackers will we need to have 28 crackers?

$[\ ] \times 4 = 28$



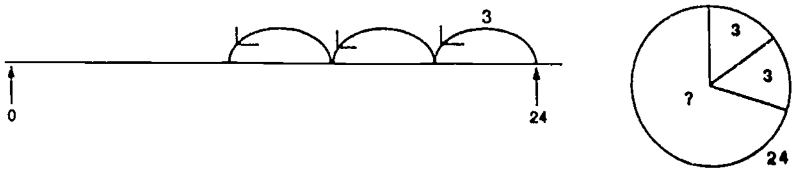
3. All relay teams have the same number of students. The six teams use all 30 students in the class. How many students are there on each team?

$6 \times [\ ] = 30$



4. How many packages of three oranges can we make from a bag of two dozen oranges?

$24 \div 3 = [\ ]$

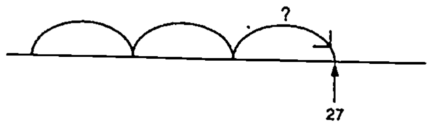


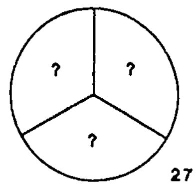
Teaching Note: Students will soon transform this problem and see it as $[\ ] \times 3 = 24$. The templates for problems 2 and 4 are often called the quotitive templates and the problems are called quotitive problems.

Sample Problem | **Representations**

5. Frank, Henry, and Julie decided to collect discarded pop cans and to divide those they found equally. They found 27 cans. How many did each get?

$27 \div [\ ] = 3$



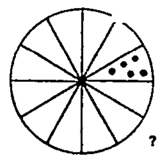


Teaching Note: This template will often be blended with the template from problem 3, and might be represented as $3 \times [\ ] = 27$. Together, these two templates are often called the partitive templates.

Students generally find partitive problems to be more difficult than quotitive problems. The reason has to do with the way division is taught. Teachers generally "talk quotitive" as division is taught. Write $7\overline{)98}$ and say, "We want to find how many sevens we can get out of 98." That is quotitive talk. This instructional dilemma can be solved by balancing quotitive talk with partitive talk.

6. Preparing for a party, Joan wanted to put five jelly beans on each plate. When the bag was empty, she had used 12 plates. How many jelly beans were in the bag at the beginning?

$[\ ] \div 5 = 12$



Teaching Note: This is commonly the first kind of multiplicative/divisive problem to get transformed as it is read. Whether or not they are asked to represent the template, many students will immediately think or write $12 \times 5 = [\ ]$ as they did for the first problem. Even though this and similar problems will often be transformed, they must still be recognized before they can be transformed and should certainly be presented with wording that, taken as it arrives, implies the template being considered. These two templates are commonly called the multiplicative templates.

There are, then, three clusters of multiplicative/divisive templates: the quotitive, the partitive, and the multiplicative. Whether or not they are represented at all on the way to being solved, students should encounter problems worded so as to suggest all six templates.

Ratio

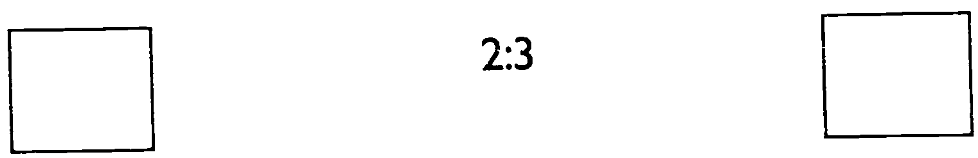
The ratio template is easily understood well before the middle years, but it has often remained in a confused state well into the senior years. This guide deals with three reasons for the confusion:

1. ratios have often been confused with numbers, particularly fractions
2. in recent decades, teachers have often not introduced a good notation for ratio
3. teachers have generally dealt with ratio implicitly rather than working with it explicitly

What Is Ratio?

A ratio is a rule, not a number. In more advanced mathematics such rules are called functions, but there is no need to introduce that word at middle years.

A ratio can be well represented concretely, as shown below



Bin controlled by the left-hand part of the rule.

Bin controlled by the right-hand part of the rule.

By building up and reducing the quantities in the bins under the control of a variety of rules, almost all students can become sensitive to the template and can find and use it in a range of problems from the beginning of the middle years.

Teaching Note: The word "proportion" is a "street" word, not a mathematical term. As a "street" word, it has at least two common meanings. It is sometimes used as a vague synonym for "ratio" and later to assert that the fractions formed from two different sets of quantities produced by a ratio are equivalent. With the first meaning, the word is not needed and using it with the second meaning without having first built a clear sense of what a ratio is has been a major contributor to the confusion referred to above.

Notation

Since the 1960s, partly because of the abstract and premature introduction of proportions, it has been common to name ratios using fractions. In retrospect, that was an unfortunate decision. It confused ratios with numbers, and the consequences of that confusion are still felt wherever ratio is subsequently applied.

This curriculum guide reverts to the more traditional : notation for ratio, for example 2 : 3.

Ratios and fractions are, of course, ultimately related, but those relationships can only be understood after students are well acquainted with ratios. For that purpose, ratios need a unique notation.

Implicit and Explicit Ratios

For at least two centuries, students were taught to use "the rule of three." The following problem provides an example of this rule:.

> Joanne can skate around the rink five times in four minutes. If she can maintain that speed, how long will it take her to skate around the rink 15 times?

Students were taught to use the template

$$\text{DEE} \ldots\ldots\ldots\ldots \quad \frac{\text{DEE}}{\text{DAH}} \times \text{DUM}$$

Notice that the ratio itself was left ***implicit***. Students had to rely on the pattern of words and quantities in the rule of three to guide them to the correct arithmetic. Further, that particular template is quite rigid and it is often not easily detected in applications.

Following the suggestions above, students should begin with the concrete representation. For example, the above problem can be represented as follows:

Times around the rink		Minutes
[ ]	5:4	[ ]

Enter the number of times around the rink and the number of minutes in the appropriate bins, and complete solving the problem.

Note that the ratio is now explicit and that in this form the template is much more easily found in a broad range of applications.

Grade Levels

It is the arithmetic that they entail rather than the templates themselves that determines the grade levels at which they are best used.

Students will have encountered all of the templates in simple forms by the time they enter the middle years. For example, all students will have met the templates a + b = [ ] and c - d = [ ] by the end of Second Grade. As they become able to add and subtract larger whole numbers, problems involving such numbers can be introduced.

It is commonly found in Fifth Grade that some students are less comfortable with the other additive/ subtractive templates. They should become comfortable with them at that level and throughout the middle years should extend their use as their arithmetic skills develop.

The same can be said of the multiplicative/divisive templates. For example, it is expected that most fifth graders be able to solve a problem built around the template [ ] x 3 = 12. Solving such problems involving larger whole numbers must wait for a division algorithm, and then division involving fractions and decimals. The domains in which the template may be applied, therefore, continue to expand throughout the middle years.

The ratio template provides a good example of that kind of development. Provided they work with explicit ratios and a suitable notation, at if not before the Fifth Grade level, almost all students can manage ratio problems that call up the concrete representation

[8]	2:3	[ ]

even if they must think, "I need four 2s on the left, so I will need four 3s on the right. That's 3, 6, 9, 12."

In time, virtually every other template and algorithm that students learn to work with should be brought to this template. Before long they should see the template [ ] x 2 = 8 on the left and use the "4" found to get 4 x 3 = [ ] on the right. As the division algorithm becomes available, larger numbers can be used. After fractions have been renamed, students can learn that ratios can be renamed in the same way – the ratio 15:10 can be renamed as 3:2.

Finally, although it layers abstraction on abstraction and is not suited to all students, they can be encouraged to discover that by examining the pairs of quantities produced by a ratio, equivalent fractions can be generated.

Consider, for example, the lists of quantities produced by the ratio 3:5

[ ]	3:5	[ ]
3		5
6		10
9		15
12		20
15		25
18		30
⋮		⋮

Whether teachers work left-right or up-down, the fractions generated by any pairs of similarly-placed numbers form equivalent fractions. For example, 6/10 = 15/25 and 9/12 = 15/20. The statement that two such fractions are equivalent is called a proportion.

From that point on, the ratios themselves can become ***implicit***. Students can work directly with the quantities.

But understanding all this takes considerable mental gymnastics, and no great harm is done if it is omitted at middle years. The fact that at one time the teaching of ratio often began at this point makes it clear why it was often considered to be a difficult topic, suited only for the senior years.

In summary, then, the templates themselves do not have grade placements. Using appropriate arithmetic, middle years students should become familiar with all 13 basic templates as early as possible. The range of problems to which they can be applied should evolve with students' skills.

II. Teaching Strategies

When it came to using strategies to find relationships in or to impose order on apparently novel situations, it was common for centuries to suppose that doing so required innate "mathematical talent" and that there was little that could be done to improve one's ability.

Beginning with Henri Poincaré early in this century and progressing through Georg Polya, Oscar Schaff, and others, it can be concluded that students' Type II problem-solving skills can be enhanced by identifying some simple strategies explicitly and encouraging students to use them.

Over the past few decades, lists of strategies have been published. They vary but also overlap extensively. The 5-8 Mathematics Revision Committee felt it would be useful to compile as many strategies as possible, to identify their common and unique suggestions, and to provide in this one place a list that is likely to contain whatever a teacher is apt to encounter.

What follows is based on that compilation.

In the chart that follows, a list of strategies is presented on the left and on the right are some examples of problems for which the strategies may be effective. This does not imply any one-to-one matching between strategies and problems. Students should find that various strategies may be used in solving a problem.

The labels provided are common, but practice in naming the strategies is not uniform. Following each description are other labels which are sometimes used.

Description

Sample Problems

A. *Partition* the Problem

If a problem is taken apart, it may be found that the parts can be managed and then put back together. This is a very useful strategy for solving multiple step arithmetic problems.

Susan's "three cans of pop" problem in Section III of this Appendix.

What is the total surface area of an open box?

Jacques won six marbles on Monday, lost eight on Tuesday, and won four on Wednesday. He then had 14. How many did he begin with on Monday morning?

B. Examine *Cases*

To test a hypothesis or to generate one, it is often productive to test a few cases.

In time, this strategy becomes a common non-formal way to "prove" things at the school level. It is sometimes called *probable induction.* The argument goes, "it is true for every case we tried, so it is probably true." For example, in beginning algebra it is noted that $2 + 3 = 3 + 2$, $17 + 5 = 5 + 17$, etc. It can be said or implied, "It seems to be true no matter what numbers we try. So we will assume it is true and say that addition is commutative."

The "sum of the divisors of six" problem in Section I.

There is something interesting about the sum of the measures of the angles of all triangles. Discover what it means.

There is something interesting about the ratio of the circumference to the diameter of a circle. Discover what is it.

After having inferred the 180^0 rule for triangles and having drawn all the diagonals at a vertex of a convex polygon, "See if can find a rule for the sum of the measures of the angles in any convex polygon."

(Before having done any algebra), "when frames are the same shape, the same number must go in every frame. What numbers will make $(\square \times \square) + 12 = 7 \times \square$ true?"

Alternative Labels and Extensions

This strategy is also known as "Guess and Check."

As is suggested above, the strategy can be refined by examining the cases (or guesses) in a systematic way. When students are encouraged to do that, the strategy is sometimes called "Make a Table," "Look for a Pattern," or "Make an Organized Table."

Description

Sample Problems

C. Make a Related but *Simpler Problem*

There are three common ways of using this strategy.

First, try crossing out any words or numbers that seem to be unnecessary and re-read the problem.

Second, replace all the big or complex numbers by simple ones and re-read the problem.

Third, try tying down a part of the problem that is "floating around." See if that problem can be solved, and then return to the main problem.

Joe's father owns an electronics franchise. One Monday he put 23 radios and 14 calculators on sale. On Thursday he had only three calculators left. How many calculators did he sell by Thursday?

One strand of insulated wire has a diameter of .086 mm. How many strands of that wire could be laid side-by-side in a gap that is 1.41 cm wide?

Can you invent a rule for finding the area of any triangle? (Make one angle a right angle, and compare the triangle with a rectangle. The problem is not then solved, but progress has been made.)

D. *Work Backwards*

Begin at the end of the problem, ask what is needed and if necessary repeat the strategy.

This is an old recommended strategy in secondary geometry. It is also applicable at earlier levels.

Provide the cost of various ingredients in a cake recipe or provide sufficient information to work them out, and ask for the cost of a cake.

A simplified version of the ancient "Sailor – monkey – banana" problem. For example, "A bunch of more than 20 bananas floats ashore on an island where there are three marooned sailors and a monkey. Since it is late, they agree to divide them up in the morning and go to bed. During the night one sailor, not trusting the others, divides the bananas into three equal piles and finds that there is one banana left over. The sailor gives it to the monkey and hides his share. In the morning they divide the remaining bananas into three equal piles. Again, they have one left over, and give it to the monkey. How many bananas could have been in the original bunch?"

Alternative Labels and Extensions

At the senior years, this strategy is sometimes called "Inverse Deduction"; the thinking goes, "What would I need to know to find this/prove this/test this speculation?"

While the strategy is commonplace at the senior years, it should not be assumed that it begins there. Pre-schoolers think this way. The main challenge in teaching is to distinguish it from the erroneous logic of "A implies B. B is true. Then A must be true."

Description

E. *Draw a Picture*

This strategy is not the same as representing a template. The idea here is to somehow draw a picture that focuses the solver's attention on the key elements of a problem all at once.

Sample Problems

Often useful on multi-step problems like the Jacques' problem in Strategy A.

Francine is three years older than George. The difference between George's age and Jim's age is 8 years, and George is the youngest. What is the difference between Francine's and Jim's ages?

Ulf, Grant, Samantha, and Joy live in a town with north-south and east-west streets. Samantha lives on the same street as Grant, but 7 blocks to the west. Ulf lives 3 blocks west and 3 blocks south of Grant. Joy lives 2 blocks east and 4 blocks north of Samantha. How many blocks does Ulf have to walk to get to Joy's house?

Alternative Labels and Extensions

At early levels, this strategy is often called, "Act it Out" and "Build a Model." Both these strategies are more concrete than drawing a picture but with the same intent – to focus students on the key relationships in the problem.

F. *Reflection*

In mathematics, this strategy goes back to Poincaré and it has attracted a good deal of attention in education during the past few decades. A problem should not merely be solved and then abandoned. Probably the greatest benefit to be gained from solving any problem is obtained by reflecting on how it was solved.

That reflection can be teacher-directed, but it can also be an excellent focus for group work. A group of students who first solve a problem and then reflect on how they solved it will become more sensitive to the strategies that proved to be effective and will be more likely to think of using them in the future.

Alternative Labels and Extensions

This strategy is also known as "Brainstorming," a label that students may find more attractive than "Reflection."

G. *Fermi Problems*

The idea is named for Enrico Fermi (1901-1954), an Italian-born American physicist. He first saw the full potential for teaching purposes of the common practice of estimating answers.

Students might, for example, be shown a photograph of a large crowd on one side of a stadium and be asked how many people are in the stands. They might think, "Well, there are 10 sections, and there are about 40 rows in a section, and, taking empty seats into account, there seems to be about 10 x 40 x 20 = 8000 people. We can't see the other side of the stadium, so let's assume there are another 8000 people. The end zones are smaller, but there are two of them. Let's say there will be about 6000 people there. Altogether, there should be about 22 000 people in the stadium."

This answer may, of course, be considerably in error. Fermi argued that, on average, errors tend to cancel one another and such estimates are often closer than one might first suppose. More important, the process of making such estimates brings students into close contact with both strategies and with the basic templates, stripped of all the complexities of calculation. As electronic calculators become more common, Fermi problems will increasingly be necessary.

Fermi problems are often enhanced if students have access to some data and if there is some way to check the accuracy of the estimate later.

The following list presents a few examples of how to generate Fermi problems:

- about how many students are there in your school?
- about how far is it from Winnipeg to Toronto? Vancouver? (An atlas can be a useful resource.)
- about how many coins are there in a jar?
- knowing that there are about one million people in Manitoba, about how many school-age children are there in Manitoba? So, about how many teachers are there in Manitoba? (The answer is about 12 000 teachers.)
- about how many automobiles are there in Manitoba?
- about how many cans of pop are sold in Manitoba each day?

Grade Levels

As with the templates, it is impossible to assign particular strategies to particular grade levels.

Most students use rudimentary forms of all of the identified strategies. In school, they use them in increasingly sophisticated ways as they reflect on how they solved problems.

What matters, then, is that the teacher be sensitive to the fact that students' individual repertoires of strategies become refined and extended over the years. This occurs either under teacher direction or in group work. Students reflect both on the problems they have solved and on the strategies that they have found to be effective.

III. Using both Templates and Strategies

Some strategies are effective only if students are first familiar with the basic templates. For example, there is little to be gained by partitioning the following problem unless the solver can recognize the basic templates once the problem has been partitioned.

> *Susan brought three cans of pop to the cashier. She gave the store employee two dollars, and received 20 cents change. What is the cost of one can of pop?*

On the other hand, middle years students are expected, as in the above example, to do more than solve problems that call for detecting a basic template.

Templates and strategies, therefore, evolve together. The teacher should consistently call attention both to what strategy proved to be effective and to what basic templates were used.

Summary

Type I problem solving involves learning to detect at least 13 basic templates in problems. The teacher should ensure that students learn to manage problems whose structure matches all 13 templates. But that teaching should not be done in a rigid way by identifying Type 1, Type 2, Type 3, ... , Type 13 problems. Students will transform problems as they read them or meet them in the real world and they will add to their repertoires of templates in different ways.

Type II problem solving requires becoming familiar with a collection of strategies for finding or imposing structure on apparently novel problems. Again, as they gain confidence, students will discover more advanced strategies for themselves, often without verbalizing exactly what it is that they have done.

Finally, these two kinds of problem solving are intertwined, and neither can progress very far without the other. For a strategy to be effective, students must often recognize basic templates as a strategy exposes them and the applicability of templates will always be seriously restricted until students can use simple strategies to extend them.

Teachers should call attention to the basic templates as they are found and should encourage students to reflect on the strategies they have used to solve problems, but neither should become codified to the point where students come to regard problem solving to be analogous to following the steps in some repair manual.

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Stevenson, Fred. *Exploratory Problems in Mathematics*. Reston, VA: National Association of Teachers of Mathematics, 1992.

Trowell, Judith (ed). *Projects to Enrich School Mathematics: Level I (Grades 4-6)*. Reston, VA: National Association of Teachers of Mathematics, 1992.

Algebra

GOALS	5	6

AL1. *Identifies and Describes Number Patterns* ⟹

Note: Students are expected to work through each goal using multiple examples. Use of the language of mathematics (equilateral triangle, trapezoid, volume, rectangle, perimeter, etc.) is expected.

Teacher Reference

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The Ideas of Algebra, K-12 (1988 Yearbook). A. Coxford, Editor. NCTM, 1988.
Algebra Thinking: First Experiences (5-8). Linda Charles. Available from Addison-Wesley.

Grade 5

■ explores and extends number patterns:

- using concrete materials to construct and expand 2-D and 3-D patterns

[?] Use stir sticks to copy these Δ shapes. Build the next 3 shapes. How many stir sticks are needed to build 4Δs? 5Δs? 6Δs? Predict how many stir sticks are needed to build 10Δs.

- developing a chart to record and reveal number patterns. Encourage mental calculations

# of Δ	1	2	3	
# of sticks	3			

Copy this chart. Complete the chart to record the number of Δs and the matching number of stir sticks in your constructions. Can the chart help you make predictions? Can you extend the pattern in your head?

- using natural language to state and write how a pattern grows

Grade 6

■ —>

- —>

[?] Use pattern blocks to copy these banquet tables and the guests sitting at each table. Build the next 3 terms (tables) in the pattern. How many guests can sit at 4 tables? 5? 6? Predict how many guests can be seated at 11 tables? 25?

- —>

# of tables	1	2	3	
# of guests	5			

Copy this chart. Complete the chart to record the number of tables and the matching number of guests for each of your constructions. How does the chart help you to make predictions?

- —>

Encourage and build upon student-generated statements. Students need to understand that there are often several equally valid ways of stating how a pattern grows. This helps to build a foundation for the later work in translating from algebraic expressions to English statements and vice versa.

Grade 5

[?] "The number of triangles increases by 1 from 1. The number of stir sticks increases by 2, starting from 3."
"As the number of Δs grows by 1, the number of stir sticks grows by 2."
"The difference between the number of triangles is 1. The difference between the number of stir sticks is 2."

Grade 6

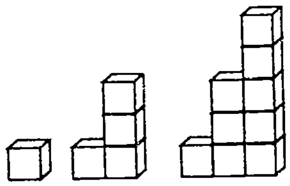
[?] "In the top row of the chart you count-on by ones, but in the bottom row you begin with 5 and count-on by 3s."
"While the tables increase by 1, the number of guests increases by 3."
"The difference between the number of tables is 1, the difference between the matching number of guests is 3."

7	8

7

■ explores and extends number patterns:

- using concrete naterials and diagrams to construct and expand 2-D and 3-D patterns



[?] Use cubes to copy and extend these hotels, according to the pattern. Predict how many cubes are needed for the 10th hotel. 25th. Explain why.

- developing a chart to record and reveal number patterns

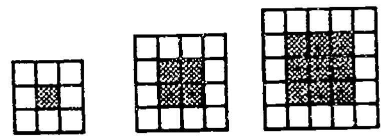
Create a chart which reveals the number patterns in your hotel constructions. Explain how to use your chart to check your predictions.

- using natural language to state and write how a pattern grows

8

■ —>

- —>



[?] Use two colours of tiles to copy and extend this pattern of patio blocks surrounding a pool. Predict how many blocks are needed to surround the 10th pool in the series. 15th.

- —>

Develop a chart and study the number patterns. Explain how your chart can be used to check your predictions.

- —>

Encourage and build upon student-generated statements. Students need to understand that there are often several equally valid ways of stating how a pattern grows. This helps to build a foundation for the later work in translating from algebraic expressions to English statements and vice versa.

Hotel model #	# of cubes
1st	1
2nd	4
3rd	9
4th	16
5th	25
⋮	⋮

As the hotel model # increases by one, the number of blocks needed is the model # times itself.

length of pool	1	2	3	4	5
# of patio blocks	8	12	16	20	24

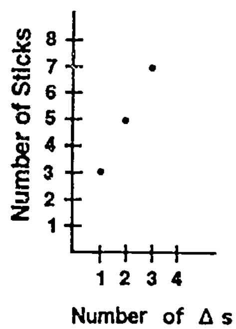
"While the size of the pools begins at 1, and increases by 1, the # of patio blocks begins with 8 and increases by 4."

GOALS	5	6

5

- making predictions regarding the nth pattern
- generating large class graphs to make relationships (patterns) visual and to verify predictions

Patterns with Triangles



[C] Data Management

6

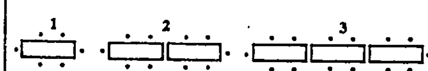
- —>
- generating small group and individual graphs to make relationships (pattern) visual and to verify predictions and to extrapolate beyond the graph

[C] Data Management

- building concrete constructions and developing a number table from an oral or written pattern description

[?] For one limousine, there are six motorcycle policemen. As the number of limousines grows by one, the number of police grows by four. Develop a number table to represent this pattern description. Then construct models to show the first three terms.

# of Limos	1	2	3	4
# of Police	6	10	14	



AL2. ***Describes and Mathematically Records Generality in Patterns***

6

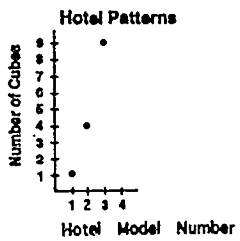
■ expresses relationships in number patterns by:

> Accept and build upon all student attempts to describe how number pairs are related. Encourage students to become more precise and concise in their use of natural and mathematical language.

7	8

7

- making predictions regarding the nth pattern
  How many cubes in the 10th hotel model?
- graphing number patterns and extrapolating beyond the data

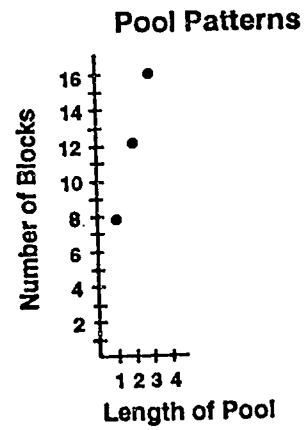


- building concrete constructions or diagrams, and developing a number table from an oral or written pattern description
  [?] One cube has 5 exposed faces. While the number of cubes increases by 1, the number of exposed faces increases by 3. Arrange cubes so you get this pattern. Develop a table to match this pattern description. Construct the first 4 terms.
- analyzing and describing patterns in a table (without concrete representations)
  [?] Look at this table. What is happening to the numbers. Describe how the pattern grows.

Total # of copies	10	20	30	40		
Total Cost of copies	$6	$7	$8	$9		

8

- —>



- —>
- —>

The number of triangles in the trapezoid shape is twice the number of the base minus one.

Base	2	3	4	5
# of Triangles	3	5	7	

- —>

Length of telephone calls	1 min	2 min	3 min		
Total Cost	$1.65	$2.10	$2.56		

■ expresses relationships in number patterns by:

■ —>

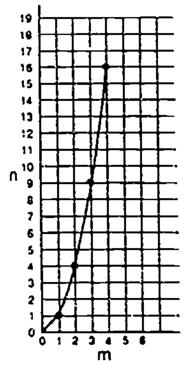
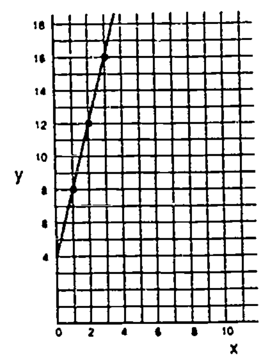
⟹

Accept and build upon all student attempts to describe how number pairs are related. Encourage students to become more precise and concise in their use of natural and mathematical language.

GOALS	5	6
		• using natural language in oral and written generalizations [?] Describe a rule which makes it possible for you to predict the number of guests that can sit at 11 banquet tables. 25 tables. Any number of tables. (The number of guests is 3 times the number of tables plus 2.) The number of stir sticks is double the number of Δs and add 1. • writing mathematical frames (open number sentences in which the variables are written as □ or Δ to shorten generalizations) 3 x □ + 2 = Δ where Δ is the # of guests and □ is the number of tables Note: identical frames must represent identical numbers: □ + □ = 6 implies □ = 3. Different frames may represent identical numbers: Δ + □ = 6 Δ = 1, □ = 5 Δ = 2, □ = 4 Δ = 3, □ = 3 etc. Write a description for this open number sentence (equation): 3 x □ + 1 = Δ when Δ means # of marbles and □ means size of container. Fill containers with marbles by this rule.

7	8
• using natural language in oral and written generalizations [?] Describe, using your table of values, how you would calculate the number in the right column from the numbers in the left column. "The number of cubes is the square of the model # of the hotel."	• —> [?] Describe, using your table of values how you would calculate the numbers in the "bottom row" from the numbers in the "top row." "The number of patio blocks is the square of the sum of the length of the pool plus 2, minus the number of the length of the pool squared." "Four times the length of the pool times 4."
• writing mathematical frames (open number sentences in which the variables are written as $\square$ or Δ to shorten generalizations, and vice versa) $\Delta \times \Delta = \square$ or $\Delta^2 = \square$ where Δ is the model number of the hotel, and $\square$ is the total number of cubes. It is important that students express the legend.	• writing mathematical equations to express the generalization [?] $b = 4n + 4$ or $b = (n + 2)^2 - n^2$ where n is the length of the pool and b is # of blocks
Write a description for this frame: $2 \times \square + 4 = \Delta$ where $\square$ is the base and Δ is the perimeter of a rectangle. (The perimeter is always double the number of units in the base plus 4.)	
• using variables (a place-holder, frame, or letter that might be replaced by a quantity) to write the generalization as an expression or formula $m \times m = n$ or $m^2 = n$ where "m" is the hotel model number and "n" is the number of cubes needed to build the model	• —>
• using mathematical conventions for writing mathematical expressions $m \times m$ becomes m^2 $s = 5 \times t$ becomes $s = 5t$ $n = 2 \times (t + 1)$ becomes $n = 2(t + 1)$ $\frac{1}{2}$ of n becomes $\frac{1}{2}n$ or 0.5 n or $\frac{n}{2}$	• —>

GOALS	5	6
AL3. ***Understands and Uses Various Methods to Evaluate Expressions from Patterns***		■ explains how to evaluate an expression or generalization by: • substituting numbers in frames and comparing the results to the original pattern 3 x □ + 2 = Δ where □ is number of tables and Δ is number of guests 3 x □ + 2 = 11 (matches 3rd construction) 3 x □ + 2 = 20 (matches 6th construction) [?] A squirrel can live 7 years. A rabbit lives 3 years longer than the squirrel. A fox can live twice as long as the rabbit. A deer lives 10 years more than the fox and a bear can live as much as the fox and the deer together. How long does each animal live? [C] Measurement, Time

7	8
	• relating formulas developed for finding perimeter, area, or volume of various shapes and solids to algebraic thinking [C] Measurement and Geometry
■ explains how to evaluate an expression or generalization by: • substituting numbers in mathematical frames and comparing the results to the original pattern $\Delta^2 = \square$ $2^2 = 4$ (matches 2nd hotel construction) $4^2 = 16$ (matches 4th hotel model) So $10^2 = 100$ cubes and $25^2 = 625$ cubes • substituting numbers for variables in expressions, and comparing the results to constructions or number tables $m^2 = n$ If $m = 5$, then $5^2 = n$ and $25 = n$  • graphing the relationships, analyzing the results and drawing conclusions [?] $m \times m = n$ or $m^2 = n$ m = hotel model # n = # of cubes Students should begin to generalize by including values between the points. Mathematically, this allows us to join the points on the graph. Note that this was not possible for the graph representing the concrete example. Students need to realize that the graph from the mathematics is similar to the graph from the concrete example.	■ —> • —> • substituting numbers for variables in expressions If a = 4 and b = 3, find the value of: a + b ab 2a − b 2ab − b $\frac{a}{b} - \frac{3}{4}$ (Later, let a = 1 and b = −3) • graphing and analyzing the relationships $(x + 2)^2 - x^2 = y$ x = size of square pool y = # of paving stones around the pool 

⇒

GOALS	5	6
AL4. ***Demonstrates Ability to Solve Given Equations***		■ solves simple, one-variable equations with whole number coefficients and solutions: • building intuitions with guess-and-check and cover-up techniques ***Guess and check:*** [?] If you remove 15 from my number, I have 9 left. What's my number. ☐ - 15 = 9 Guess 25. Check by substitution. 25 – 15 = 10. No, etc. OR ☐ + Δ = 11 (Terms can be the same.) ***Cover-up technique:*** [?] 19 + ☐ = 25 Cover "☐" Then ☐ must be 6. Check by substitution. 19 + 6 = 25. Correct OR ☐ + ☐ = 16. (Terms must be the same.)

7	8
■ generates tables of number patterns by systematically substituting values in expressions Given the expression $2(s + 4)$: [?] Generate a table of number patterns, starting with $s = 1$ and stopping with $s = 7$ Find the 15th term Find the 127th term Find a way to illustrate the first 3 terms with concrete materials	■ —> Given the expression $n^2 + n$: [?] Generate the pattern for the first 6 terms, beginning with $n = 1$ Find the 21st term Find the 45th term Find a way to illustrate the first 3 terms with concrete materials ⇒
	■ simplifies algebraic expressions by: • recognizing and combining like-terms $b + b + b = 3b$ $4t - t = 3t$ • removing grouping symbols and collecting like-terms using the distributive principle. Review the distributive principle using partitioning and the array model. [C] Number Concepts $3(a + 2)$ is $(a + 2) + (a + 2) + (a + 2)$ which is $3a + 6$ $2(a - b)$ is $2a - 2b$ ⇒
■ solves simple, one-variable equations with whole number coefficients and solutions: The objective throughout is that students will master algebraic notation, principles, and procedures, not merely "find the answer." Note: Equations such as $10 - n = 6$ and $\frac{8}{x} = 2$ will not be explored at this level as they are difficult to justify mathematically. Numbers will be manipulated, not variables.	■ solves one- and two-step equations with whole number (and later, integral) coefficients and solutions using algebraic reasoning and recording ⇒

While students are developing formal equation-solving skills, guess and check and cover-up may still be needed for some students to build further intuitions.

7	8
Order of Development: One operation equations $n + 7 = 12$ $n - 5 = 7$ $5n = 35$ $\frac{n}{7} = 3$ Two operation equations $2x + 1 = 5$ $2n - 1 = 3$	Order of Development: Extend to include $\frac{x}{4} + 3 = 6$ $4(x + 6) = 40$ $3x + 5 - 2x - 1 = 7$

GOALS	5	6
		Extend to include solving equations with like terms using guess-and-check. This will lead to the discovery of collecting like terms. $(2 \times \square) + (3 \times \square) = 30$ Then $5 \times \square = 30$ Finally, students can explore: $(4 \times \Delta) + (7 \times \square) + (2 \times \Delta) = 72$ Then $(6 \times \Delta) + (7 \times \square) = 72$

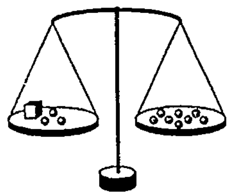
7	8

7

- using concrete materials

[?] The ***balance scale*** provides a concrete way for students to discover the principle of "balance" in equality.

ONE-OPERATION EQUATION



What can you do to isolate the variable but not distort the balance for $n + 3 = 8$?

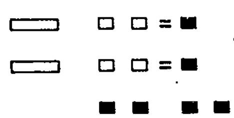
Remove 3 from each side.

$$n + 3 - 3 = 8 - 3$$
$$n + 0 = 5$$
$$n = 5$$

Check by substitution.

[?] It is difficult to represent operations other than addition on the balance scale. Therefore, a pictorial representation of the algebraic process can be developed through the use of ***algebra tiles***.

$n - 2 = 1$ where ▭ represents "n", ■ is +1, □ is (−1)



To isolate the variable, add 2 to each side of the equation.

▭ = 3

$$n - 2 + 2 = 1 + 2$$
$$n = 3$$

Extend to × and ÷ with two-step equations.

- using formal methods

$n + 7 = 12$	$n - 5 = 7$
$n + 7 - 7 = 12 - 7$	$n - 5 + 5 = 7 + 5$
$n + 0 = 5$	$n + 0 = 12$
$n = 5$	$n = 12$
check $n + 7 = 12$	check $n - 5 = 7$
$(5) + 7 = 12$	$(12) - 5 = 7$
$5n = 35$	$\frac{n}{4} = 3$
$\frac{5n}{5} = \frac{35}{5}$	$\frac{n}{4} \cdot \frac{4}{1} = 3 \cdot 4$
$\frac{n}{1} = 7$	$\frac{n}{1} = 12$
$n = 7$	$n = 12$
check $5n = 35$	check $n = 3$
$5(7) = 35$	$\frac{(12)}{4} = 3$

8

- —>

- using formal methods

Building upon experiences and intuitions developed earlier, students can make the transition to more formal Algebraic methods and shortcuts such as transposition. Transposition should not be introduced until students can recognize it as a shortcut procedure.

Using Operations	Using the Inverse	Shortcut
$p+5 = 7$	$p+5 = 7$	$p+5 = 7$
$p+5-5 = 7-5$	$p+5+(-5) = 7+(-5)$	(transposing)
$p+0 = 2$	$p+0 = 2$	$p = 7-5$
$p = 2$	$p = 2$	$p = 2$

Include equations of the form $2x + 5 = 9$.

GOALS	5	6
AL5. ***Applies Ability to Write and to Solve Equations for Mathematical Problems***		■ explains how to solve simple problems by pre-algebra strategies: • using concrete materials and guess and check strategy (encourage systematic listing) [?] There are 11 red and yellow cubes in a bag. There are 3 more yellow cubes than red cubes. What number of each colour are in the bag? Guess 3 Red, Then 3 + 3 = 6 Yellow Total = 9. No, 3 is too small. Guess 4 Red, etc.

7

$2x + 1 = 5$
$2x + 1 - 1 = 5 - 1$
$2x + 0 = 4$
$2x = 4$
$\frac{2x}{2} = \frac{4}{2}$
$\frac{x}{1} = 2$
$x = 2$
Check $2x + 1 = 5$
$2(2) + 1 = 5$
$4 + 1 = 5$

■ translates simple English terms to mathematical symbols and vice versa

Extend beyond language developed in patterns to commonly used English terms such as less, less than, the difference between, tripled, etc.

■ explains how to solve simple problems with informal algebraic methods:

- using concrete materials and guess and check strategy (encourage systematic listing)

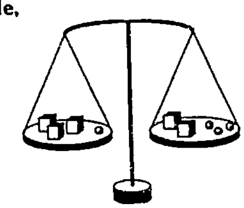
Problem #1

[?] Juan has some bingo chips in his hand. If you double the number of chips and add 7, you would then have 19. How many chips are in Juan's hand?

Guess 5 chips. Then
$2 \times 5 = 10$
$10 + 7 = 17$. No, 17 is too low. Guess must be higher. Try 6.

8

Extend to solving equations with like terms on both sides. (Use one of the previously identified two methods.) For example,



$3n + 1 = 2n + 3$
Take 1 chip from each side. Then

$3n + 1 - 1 = 2n + 3 - 1$
$3n + 0 = 2n + 2$
$3n = 2n + 2$

Take 2 cannisters from each side. Then,

$3n - 2n = 2n + 2 - 2n$
$n = 0 + 2$
$n = 2$

Check by substitution
$3(2) + 1 = 2(2) + 3$
$6 + 1 = 4 + 3$
$7 = 7$

[?] Allow students to discover there is not one first correct step but certainly a most efficient first step.

■ —>

Extend level of complexity.

[?] Side "a" of a triangle is 15 cm longer than the sum of the other two sides. Side "b" is 10 cm longer than 25% of side "c." Write algebraic expressions for each of the previous statements. Show how they could be combined.

⇒

GOALS	5	6

7

- visualizing the relationship

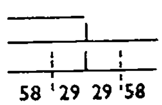
Problem #2

[?] If Bert's age is multiplied by 2 and divided by 3, you find the age of his third son, Walter. How old is Bert if Walter is 58?

Bert's age

2 times Bert's age

divided by 3



If ______ = 58 then

______ + ____ is

58 + 29 = 87

■ **explains how to solve simple problems using algebra**

For this purpose, a simple problem is defined as one in which the variable appears in only one term of the equation.

Problem #1 by formal methods

Let n = # of chips

$2n + 7 = 19$

$2n + 7 - 7 = 19 - 7$

$2n = 12$

$\frac{2n}{2} = \frac{12}{2}$

$n = 6$

Check by substitution

Juan has 6 chips

Problem #2 by formal methods

Let n = Bert's age

$\frac{2n}{3} = 58$

$\frac{2n}{3} \bullet \frac{3}{1} = 58 \bullet 3$

$\frac{2n}{1} = 174$

$2n = 174$

$\frac{2n}{2} = \frac{174}{2}$

$\frac{n}{1} = 87$

$n = 87$

8

■ —>

Extend to problems in which the variable may appear in more than one term of the equation.

Problem #4

[?] ·There are 12 vehicles in a parking lot. There is 1 more van than trucks. There are 5 more cars than trucks. How many of each vehicle are in the parking lot?

vans	trucks	cars
n	n-1	(n-1)+5

Let # vans = n

$n + (n - 1) + (n - 1) + 5 = 12$

$3n - 2 + 5 = 12$

$3n + 3 = 12$

$3n = 9$

$n = 3$

So 3 vans, 2 trucks, and 7 cars make up the 12 vehicles in the parking lot.

[?] Problem #5

Rex has 3 times as many sports cards as Karin. Jeff has 13 more cards than Karin. They have 128 cards in all. How many does each person have?

Let Karin's # of cards be x.

	Karin	Rex	Jeff
# of cards	x	3x	x+13

Karin + Jeff + Rex = 128

⟹

GOALS	5	6

7	8

[?] Problem #3
If the number of legs on a centipede is multiplied by 9, then reduced by fourteen, the answer is 400. How many legs does this centipede possess?

Let n be the # of legs.

$9n - 14 = 400$

$9n - 14 + 14 = 400 + 14$

$9n = 414$

$\frac{9n}{9} = \frac{414}{9}$

$\frac{n}{1} = 46$

$n = 46$

The centipede has 46 legs.

Make up a problem for the class to solve.

Then $x + (x + 13) + 3x = 128$

$5x + 13 = 128$

$5x + 13 - 13 = 128 - 13$

$5x = 115$

$\frac{5x}{5} = \frac{115}{5}$

$x = 23$

Karin has 23 cards
Jeff has 23 + 13 = 26 cards
Rex has 3(23) = 69 cards

- experiments and chooses other unknowns as the variable

[?] For example, in Problem #4 variables could be arranged as follows:

Vans	Trucks	Cars
t+1	t	t+5

Vans	Trucks	Cars
c–4	c–5	c

Let # trucks = t

$t+1+t+t+5 = 12$

$3t + 6 = 12$

$3t+6-6 = 12-6$

$3t = 6$

$t = 2$

Let # cars = c

$(c-4)+(c-5)+c = 12$

$3c - 9 = 12$

$3c-9+9 = 12+9$

$3c = 21$

$c = 7$

Data Management

DATA MANAGEMENT

Many discoveries can be made and many problems can be solved by collecting, analyzing, displaying, and interpreting data. Understanding the chance component of many daily activities helps students make better decisions and more accurately predict outcomes.

GOALS	5	6
DM1. *Formulates and Clarifies Questions* ⇒	■ selects wording which generates appropriate data [?] What's typical about a store-bought red potato?	■ —> How does weather affect our lifestyle? Extend to developing questions where comparisons are required or where a simple relationship exists. [?] Is there a relationship between the amount of gas used by a car and the distance travelled?
	Students should be involved in both short-term and long-term data analysis activities. Other subject areas and topics are good sources for Data Management (for example, sports events, see Olympic Education kits, science experiments, populations, economics, and Statistics Canada). Brainstorm for ideas that are pertinent to your school.	
	■ makes predictions before beginning the data analysis process	■ —>
DM2. *Develops and Implements a Data Collection Plan* __*Teacher Reference*__ *Dealing with Data and Chance* (Part of Grades 5-8 Addenda Series). Zawojewski, et al. NCTM, 1991. *Making Sense of Data* (Part of Grades K-6 Addenda Series). Lindquist, et al. NCTM, 1992.	■ identifies appropriate sources of information including: • first-hand (students collect data) • second-hand (use a census) • combination (newspaper and complete a survey)	■ —>
	■ specifies the population as: • total population • sample of population Each student takes any potato from a 10 kg bag picked randomly from a grocery store.	■ —>
	Students need experiences with population samples of differing size, composition, and bias in order to develop knowledge about the reliability of sampling as a technique for predicting what is typical or probable.	

7	8
■ selects wording which generates appropriate data Extend to questions which explore whether or not a relationship exists. What does it mean to be a "typical" Grade 7 student? [?] Is there a relationship between wrist circumference and height? [C] Measurement	■ —> [?] What are this community's most pressing sustainable development concerns? Do taller people have a higher vertical jump? [C] Measurement

Students should be involved in both short-term and long-term data analysis activities. Other subject areas and topics are good sources for Data Management (for example, sports events, see Olympic Education kits, science experiments, populations, economics, and Statistics Canada). Brainstorm for ideas that are pertinent to your school.

7	8
■ makes predictions before beginning the data analysis process	■ —>
■ identifies appropriate sources of information	■ —>
■ specifies the population as total population or a sample of the population	■ —>

Students need experiences with population samples of differing size, composition, and bias in order to develop knowledge about the reliability of sampling as a technique for predicting what is typical or probable.

GOALS	5	6
⇒	■ selects methods of obtaining data, including: ***First-hand:*** • by observation and counting How many eyes does each potato possess?	■ —> • by observation and counting What visual weather characteristics can be viewed and recorded each A.M. and P.M. over time? (cloudy, sunny, etc.)
	• by measurement tools How long? What is mid-length circumference? What is weight of each potato?	• by measurement tools What are the "average" high/low temps? sunrise/sunset times? wind velocity/windchill per month?
	• by survey What is your favourite way to eat potatoes? (mashed, fried, scalloped, baked)	• by survey What are the favourite activities per season for students in various grades? teachers? parents?
	• by experiment (controlling variables) [C] Science [C] Probability (see DM6)	• by experiment/simulation [C] Science [C] Simulation (see DM6)
	Students can apply their developing knowledge of the scientific method and the control of variables when collecting data through experimentation.	
	Second-hand: • using data banks E.g., census, tables, almanacs, stock market, newspaper, Statistics Canada, archives, records books, CD-ROM, Internet	• —>
⇒	■ establishes a method of keeping track of data Include tally, chart, and list	■ —>
⇒	■ implements the plan	■ —>

7	8
■ selects methods of obtaining data, including:	■ —>
• by observation and counting What colour of hair/type of shoe/genetic tongue curl do students possess?	• by observation What environmental and sustainable development issues do we see needing attention? [C] Sustainable Development
• by measurement tools What height/armspan/length of foot/ cupped-hand capacity do students possess? [C] Measurement	• by measurement tools How much household garbage is produced in our homes? Removed from our school? How much junk mail is received per month? Per year? [C] Measurement
• by survey What number of siblings/pets/types of collections/favourite snack food do students possess?	• by survey What issues concern students? Teachers? Parents?
• by experiment (controlling variables) [C] Science [C] Probability (see DM6)	• by experiment/simulation [C] Science [C] Simulation (see DM6)

Students can apply their developing knowledge of the scientific method and the control of variables when collecting data through experimentation.

• using data banks Opportunity to use technology including CD-ROM, Internet, etc.	• —>
■ evaluates the data collection plans of others	■ —> ⇒
■ establishes a method of keeping track of data	■ —> ⇒
■ implements the plan Students should conduct a trial run, and adjust if necessary.	■ —> ⇒

GOALS	5	6

DM3. *Organizes and Represents Data* ⟹

5

■ examines various criteria for classifications and groupings of data

Determine the groupings for the number of eyes on a potato with relevance to data collected

[?]

number of eyes
0 – 5
6 – 10
11 – 15
16 – 20

6

■ —>

[?] Create your own windchill groupings according to wind speed and temperature. Compare to Weather Bureau windchill criteria.

⟹

5

■ organizes the data for ease of quick representation, including:

- ordered list
- frequency distribution chart or table

6

■ —>

⟹

Teacher Reference

Developing Graph Comprehension: Elementary and Middle School Activities. Francis Curcio. NCTM, 1989.

Exploring Data. James M. Landwehr, et al. Creative Publications, 1986. [Available from Addison-Wesley]

Statistics Workshop (Grades 6+). Courseware available from Sunburst in Macintosh format only.

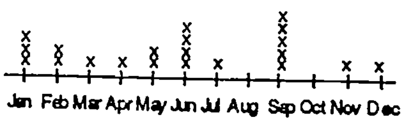
Data Insights (Grades 7-12). Courseware available from Sunburst in Apple II and MS-DOS formats.

5

■ constructs visual representations of the data (plots and graphs):

- using data representation appropriate to the data including
  - bar graphs
  - line plots

  Birthday months: Jan(3), Feb(2), Mar(1), Apr(1), May(2), Jun(4), Jul(1), Aug(0), Sep(5), Oct(0), Nov(1), Dec(1)



Students with Birthdays in Certain Months

6

■ —>

Extend to
- line graphs
- stem-and-leaf plot

  The following numbers represent the number of baseball cards in collections

  30, 18, 29, 7, 40, 16, 25, 32, 29, 9, 31, 25, 8, 26, 25, 30, 13, 27, 29, 25, 30, 40, 18, 18, 18

0	7 9 8
1	8 6 3 8 8 8
2	9 5 9 5 6 5 7 9 5
3	0 2 1 0 0
4	0 0

Baseball Cards

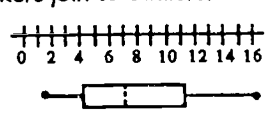
Extend to include double versions of plots and bar graphs, e.g., male/female data estimates vs actual measurements

Female		Male
4	1	5 2 7 7
2 2 3 7	2	0 8 1
2 6 9 1 4 0	3	4 5 5 2 9
3 9 9 6	4	1 0 0

Number of Chocolate Bars Sold by Students in Class 6

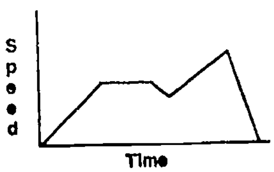
7	8
■ examines various criteria for classifications and groupings of data Determine groupings to show height changes throughout the school year (i.e., September – January/June).	■ —> [?] Determine the age groupings for the most listened to music. ⇒
■ organizes the data for ease of quick representation Extend to converting from one number system to another when necessary for comparison (use calculators).	■ —> [?] Use a simple computer spreadsheet. ⇒

Students may encounter situations where some data to be used has been collected/recorded using fractions, but plotting the data would be easier using percents.

7	8
■ constructs visual representations of the data (plots and graphs): • using data representation appropriate to the data including – bar graphs – line plots – line graphs – stem-and-leaf plots Extend to – circle graphs	■ —> ⇒ • —> Extend to – scatter plots on a coordinate system when two related variables are involved. Discuss the "best fit" line to predict future data. – box plots so 2 or more sets of data can be compared – box and whisker plot Draw a box and whisker plot for the data 1, 4, 4,5, 6, 7, 10, 10, 11, 13, 16. Find median for data (7). Find first quartile (median of left half of data which is 4). Find third quartile (11). Draw box as shown. Whiskers join to outliers.  Ensure students manipulate the data in several ways to better determine its characteristics.

Some computer programs allow a student to quickly view various representations of the data, and their use is encouraged and expected in Grades 7 and 8.

GOALS	5	6
	• provides title, legend and axis labels for plots and graphs	• —>
DM4. *Describes Data* ⇒	■ describes the general data distribution including: • range • smallest value • largest value • how many of each value • frequency • which value occurs most often? Least often? • which value is in the middle? • how many values have (name criteria), e.g., blue eyes? • is there a pattern in the data?	■ —> Extend to describing values that "don't fit" (outliers) • extremes • gaps in the data • clusters in the data ■ describes and calculates the measures of central tendency (average) including: • mode (occurs most often) • median (the middle value in an ordered list) • mean (arithmetic mean)
Teacher Reference *Interpreting Graphs* (Grades 5+). Courseware available from Sunburst in Apple II and MS-DOS formats.	■ relates descriptions of data to possible plots [?] Give an event and ask students to draw a plot which could represent that event. Axes should be labelled but no scale shown (e.g., general shape of data is wanted).	■ —> Extend to include situation where a plot is given and students are asked to tell or write a description for an event which fits the numerical plot.

7	8	
• providing title, legend and axis labels for plots and graphs	• —> ■ uses computer graphing to organize and represent data	
■ describes the general shape of data distribution in terms of: Extend to include: • quartiles ($\frac{1}{4}$ or $\frac{3}{4}$ values in data) • per cents	■ —> Extend to describing linear patterns in scatter plots; trends over time. E.g., tree growth vs. diameter, global warming	⟹
■ describes and calculates the measures of central tendency including: [?] Extend to determining which average is most appropriate to the data collection. For example: • mode (shoe purchaser) • median (housing costs) • mean (test scores) Determine the effect that outliers have on the average.	■ —> Extend to include the use of fractions, decimals, and per cents interchangeably in descriptions.	⟹
■ relates descriptions of data to possible plots Sketch a plot depicting the following situation: [?] Herbie accelerates from a stop, goes a constant speed for a time, and then slows for a red light. As he approaches the light it turns green so he accelerates again. While accelerating, Herbie notices a friend and quickly parks. Graph could look like: 	■ —> [?] Sketch a plot which shows the height of the liquid in a one litre bottle as you repeatedly add 100 mL until it is full.	⟹

<table>
<tr><th>GOALS</th><th>5</th><th>6</th></tr>
<tr>
<td>DM5. Analyzes and Interprets Data and Draws Conclusions ⟹</td>
<td>■ makes inferences based on data by:
• making predictions
[?] Is there a reason certain numbers don't appear? Or hardly ever occur?
Is there a reason certain numbers occur so often?
Are these predictions realistic or viable?
What does this graph mean?</td>
<td>■ —>
Extend to comparing sets of data by:
• finding relationships
[?] Does your graph seem to indicate any relationship between ______ and ______ ?
If so, what relationship do you notice? Did you expect to find a relationship?

• Hypothesizing
[?] If you had used a different source of data collection or sampling, would the results be different, similar?</td>
</tr>
<tr>
<td></td>
<td>■ determines the advantages and disadvantages of applicable graphs and plots</td>
<td>■ —></td>
</tr>
<tr>
<td></td>
<td>■ relates to real world situations through:
• application
Interpret, analyze, and draw conclusions based on graphs from second-hand sources</td>
<td>■ —>

• —>
[?] Each classroom in the school adopted a section of the playground to clean. The competition was fierce! Here are the results.
<table>
<tr><td>Room No.</td><td>5</td><td>6</td><td>7</td><td>8</td></tr>
<tr><td>No. of Students</td><td>25</td><td>25</td><td>30</td><td>22</td></tr>
<tr><td>No. of Pieces of Garbage</td><td>4000</td><td>3600</td><td>4600</td><td>3400</td></tr>
</table>
a) Graph the total garbage collected per class, arranging the bars from least to most. Label the graph.
b) Which class collected the most garbage? Justify more than one answer.</td>
</tr>
<tr>
<td></td>
<td>■ develops convincing written or oral arguments for decisions based on the data</td>
<td>■ —></td>
</tr>
</table>

7	8
■ makes inferences based on data by: Extend to making inferences where no apparent relationship exists. [?] If there is no relationship, explain why. Do you think there would be a relationship if .... Because there is no relationship, what other information can we retrieve from the data? Is there sufficient proof of a relationship?	■ —> Extend to using logical reasoning to rule out alternative explanations.
• Hypothesizing Would a larger sampling affect the results?	• —>
• Intrapolating and extrapolating to retrieve new information [?] Interpolate – reading between given values Extrapolate – reading beyond given values	• —>
■ determines the advantages and disadvantages of applicable graphs and plots	■ —>
■ relates to real world situations through:	■ —>
• application For what purposes do statisticians collect data?	• —> Extend to include use of reverse process, i.e., provide students with graphs; they write interpretation and vice versa – to recognize role of statistics in society
• examining misleading graphs to determine how or why the interpretation might be misused	• —> • analyzing second-hand sources to determine how data was collected How are stock market prices determined? How do governments determine percent of pollutants in air or water?
■ develops convincing written or oral summaries or arguments for decisions based on the data	■ —>

⇒

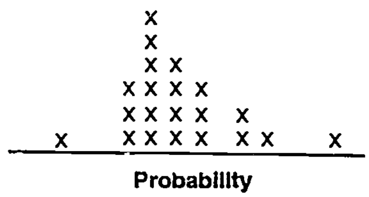
GOALS	5	6
DM6. *Develops an Appreciation of Probability (What are the chances of ...?)* ⇒	■ discusses events using the vocabulary of probability Terms to include: likely, unlikely, equally likely, certain, impossible, even chance, best bet, worst bet, probable, improbable	■ —>
⇒	■ determines experimental probability by:	■ —>
<u>*Teacher Reference*</u> *Data, Chance and Probability: Grades 4-6.* Available from Spectrum. *Data, Chance and Probability: Grades 6-8.* Available from Spectrum. *D.I.M.E. Probability Kit A/B (Grades 5-9).* Available from Spectrum. *Probability Jobcards: Intermediate (3-6) or Junior High (6-8).* Available from Addison-Wesley. *Predictions from Samples (Grades 5-9).* Courseware available from Sunburst in Apple II format only. *Organizing Data and Dealing with Uncertainty (Grades 5-8).* NCTM, 1979.	Use devices or events for which it is unlikely that students have any intuitions regarding the probabilities of the events recorded. For example, use devices such as tacks and paper cups. Avoid coins, spinners, and objects in bags and any other devices for which a theoretical probability can be calculated.	
	[?] Toss 20 tacks and record the number of points. The number of points divided by the number of tosses is called the ***probability*** of points. For example, with 13 points in 20 tosses, the probability equals $\frac{13}{20}$ or 0.65.	[?] Each student chooses a paragraph of similar length from a common text and counts the number of "e's" in the paragraph. For example, if there are 29 e's in 200 letters, the probability of an "e" is $\frac{29}{200}$ Using a calculator, the probability is 0.145.
	• making predictions Based on the above experiment, it can be predicted that if 60 tacks are tossed, the results will be about 13 x 3 = 39 points.	• —> Based on the above experiment, we might predict that a paragraph twice as long might contain 60 e's.
	• testing predictions Toss 60 tacks to test the prediction. Compare results with expectations. Repeat the procedure beginning with a larger number of tosses. Similar sequences of collecting data, inferring probabilities and testing them can be conducted using styrofoam cups which can fall T (top up), S (side up), or B (bottom up). More interesting data results from removing the top one or two centimetres of the cups.	

7	8
■ discusses events using the vocabulary of probability Extend to include: sample space, outcome, event, and percentages	■ —>
■ determines experimental probability in situations where theoretical probability can be calculated Devices could include spinners, dice, balls in a bag, coins, decks of cards, etc. Flip a coin numerous times. Calculate the $P(\text{Heads}) = \frac{\#\text{ of heads}}{\#\text{ of tosses}}$	■ —> Extend to include situations where outcomes are independent. Calculate P(2 ones) where 2 dice are rolled.

⇒

⇒

GOALS	5	6
		■ draws conclusions based on experiments Experimental probabilities converge as the quantity of data on which they are based is increased. Compare the probability in repeated trials using graphic organizers. Plot the values generated using a line plot. The converging point should emerge after repeated trials. See the sketch below.



7	8
■ draws conclusions based on experiments	■ —> ⇒
■ calculates theoretical probabilities using fractions between 0 and 1 A probability of 0 means that an event will not occur; a probability of 1 means that an event will occur. Coin < Heads / Tails $P(H) = \frac{1}{2}$ or 0.5 Determine the probability of getting 2 heads when 2 coins are tossed. Dime — Nickel H — H H — T T — H T — T P(Heads) or $P(H) = \frac{1}{4}$ or 0.25	■ —> ⇒ Calculates the probability of events from independent outcomes as follows: P(both events) = P(first event) x P(second event) Roll 2 dice. What is the probability of getting 2 ones? $P(2\text{ ones}) = \frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$
■ compares experimental results with theoretical results	■ —> ⇒

Students need time and experiences to understand that the more times an experiment is conducted (or the more data collected) the closer the results should be to the theoretical probability of the event.

7	8
■ uses simulation methods to determine outcomes using: • the Monte Carlo method Extend to establishing probability by collecting data from simulations – experiments which model real problems. Use the Monte Carlo method. [?] The Monte Carlo method of simulation uses random number devices such as number cubes, spinners, coins, computer random-number generators, etc., to represent real-world situations. An experiment is designed to model the problem situation, and data are generated and analyzed as though they are real data.	■ —> ⇒ • —> • Buffon Needle Experiment [?] Draw vertical lines on a large chart paper exactly 2 toothpick lengths (2 L) apart. Toss 1000 toothpicks randomly. Record any toothpick which touches a line as a "hit." Calculate the number of tosses divided by the number of hits. Compare results. As more trials are attempted, the outcome will converge on pi (π). Experiment with differing spaces between the lines such as L and 3 L as well as with different sticks. [C] Measurement

GOALS	5	6

7	8
■ relates probability to real life by:	■ —>

⇒

Students need to understand that there are some situations in which no theoretical probability can be calculated. In such cases, probabilities are estimated by sampling. Those emperical probabilities are used to make predictions or decisions for the whole population. The validity of the sample becomes a critical factor.

- examining the use of probability to make decisions

  Is it a good idea to buy 649 tickets?

  What are the chances of having at least one sunny day if the weather report indicates 50% chance of rain on Saturday and 50% chance of rain on Sunday? (This situation can be compared to flipping two coins where a quarter represents Saturday and a dime represents Sunday. A sunny day would be indicated by heads.

H	H	both days sunny
T	H	one day sunny
H	L	one day sunny
T	T	no day sunny

$$P\text{ (one sunny day)} = \frac{3}{4} = 0.75$$

- —>

  How could a quality control department test for defects?

- examining examples of subjective/intuitive probability

  Where community standards permit, see the card game "Montana Red Dog" in *"Dealing with Data and Chance,"* NCTM Addenda 5-8, p. 41.

  Explore the simplest form of what is called the "gambler's fallacy." The idea is that in a game in which there is 0.5 chances of winning, the gambler should wager $1, then $2, then $4, then $8 ....

  The gambler's argument is that sooner or later he will come out ahead by a dollar.

  In fact, the gambler's fallacy is, in the long run, a road to certain ruin. Have students discover why.

Geometry

Introduction

There is no set of commonplace conventions where informal geometry is concerned.

One traditional approach has been to prepare students for formal geometry by identifying the objects of that later study, such as lines, segments, triangles, circles, rectangles, parallelograms, and exploring some of their elementary qualities. Evaluation, therefore, was most often limited to the recall of definitions and properties.

A curriculum built in that way has two serious limitations. First, it excludes most interesting geometric objects. Students see ellipses, parabolas, catenaries, irregular polygons, and a variety of polyhedra wherever they look. However, because such objects have no place in a course whose prime objectives have to do with deduction and proof, they have often been ignored. A second related reason is that such a curriculum tended to be dull. The same triangles, rectangles, and circles often appeared, year after year, from kindergarten to the end of middle years.

In reaction to these weaknesses, teachers in particular, commonly went to the opposite extreme. Geometry became a "Friday-afternoon" experience – free-standing activities that were intrinsically interesting and which engaged students' curiosity. Sometimes, however, it was difficult to point to any specific cognitive outcomes.

A Solution

This curriculum guide blends the strengths of both of the above approaches and, of course, avoids their weaknesses.

Throughout the middle years, geometry instruction should be **activity-based.** As far as students are concerned, teaching and learning should comprise a sequence of free-standing activities. Each should present a challenge that invites the hands-on manipulation of materials, open-ended explorations, and the testing of speculations.

That, however, is insufficient. The activities must also be vehicles for the progressive development of identifiable geometric knowledge. They must not become animation without substance.

Four qualities of this guide are designed to assist teachers to reach the objectives:

1. The following set of **geometric themes** has provided a guide in identifying some sample activities. The particular activities suggested are known to be effective ones but they must not be taken to be the substance of the curriculum. **The themes are the substance of the curriculum.** It would make little difference if a teacher was to replace every suggested activity by others selected from elsewhere. It would matter very much if students were to leave the middle years with no notion of such mathematical concepts as congruence, similarity, or symmetry.

2. **Vocabulary should be seen as an aid to precise communication, not as an end in itself.** Every activity should be seen as an opportunity for the teacher to introduce and use correct terminology; teachers should encourage students to use terms correctly. That, after all, is how students learn most new words – in context. The various terms identified in this guide should be used in that way, rather than as a source list for spelling tests or quizzes on definitions.

3. Students learn about as much geometry between the elbows as between the ears. For example, teachers can define congruence in terms of translations, rotations, and reflections. It is pointless, however, to dwell on any such abstract definition in the middle years. In teaching so that students understand congruence, it would be better to agree that objects are congruent if there is any way to carry "this one" over to "that one" and see if they match. Better yet, students can talk of sliding, turning, and flipping objects so as to try for such matches.

   Activities selected should most often **encourage manipulating, measuring, cutting, and related hands-on qualities.**

4. The risk in recommending the above kind of activity-based learning is that it could either generate troublesome repetition or omit altogether some key themes.

   The extreme solution would be to mandate a full set of activities for each grade level. Doing so would transform this guide into a source book, consistent with neither the intent nor the constraints placed on the creation and publication of a curriculum guide.

   Our solution has been to identify the goals (G.1 to G.5) within which activities should be selected and, in each, identify some objects, some appropriate vocabulary, and some sample activities.

Geometric Themes

While the themes have guided both the selection of goals and suggested activities, they should not be taught explicitly.

There are two reasons for this recommendation. First, the themes overlap. It is rare that an activity involves only one of them. The teacher should seize whatever opportunities arise to provoke greater insight into a theme, but those opportunities generally cannot be identified with particular activities.

Second, it is not the intent that the themes become the objects of explicit independent instruction. They should, at most, be inferred from the outcomes of an activity. For example, students might construct and cut out some range of triangles, tear off the vertices, and discover that they can always be fitted together so as to create "a straight line." Remarkable. The teacher ought to notice the opportunity to call attention to the fact that they have found an invariant, something that does not change when all seems to be in flux. It is more important, however, that students be impressed with their discovery. They can also look for more invariants. Do not emphasize or think of finding the invariant as being the full intent of the activity. That activity can generate some useful vocabulary and some simple skills with rulers, drawing lines, and cutting.

In short, the teacher should look for opportunities to expose the themes but ought not to make them the obvious instructional focus of what is being done. Most teachers who embark on this kind of activity-based geometry instruction for the first time will find that their own familiarity with the themes grows over time.

The following eight themes are not exhaustive and are listed in no particular order.

1. Invariance

As suggested above, invariance embodies the idea that when all seems to be in flux, something behind the scene does not change. This is the most pervasive of all geometric themes. At the simplest level, the diagonal of a square is always longer than an edge, but not as long as two edges. Later, it may be found that the ratio of the diagonal to an edge is an invariant, as is the ratio of the diameter to the circumference of a circle.

2. Symmetry

Symmetry embodies the idea that some things seem to repeat themselves. There are three basic kinds of symmetry:

- Repetitive symmetry, as along a picket fence, in the rows of windows in an office tower, and in some music, etc.
- Bilateral symmetry, in which an object seems to be reflected across a line or plane, as on faces, inkblots, most clothing, birds, and airplanes, etc.
- Cyclic or rotational symmetry, in which it is found that some objects will fit with themselves if they are rotated. Take, for example, bottle caps, analogue clock dials, regular polygons, designs and around the edges of plates, etc.

3. Similarity

Similarity embodies the idea that one object is a magnified or reduced version of another. Take, for example, photographs, maps, scale drawings, overhead projectors, and large and small screen television receivers, etc. A study of similarity provides opportunities for measurement and elementary ratio on the way to trigonometry.

4. Maximum-Minimum

Both in nature and in human artifacts, people are often interested in both the geometry and arithmetic of maximums and minimums. Because soapy films "try" to shrink as much as possible, a sphere must have the minimum surface area for a given volume, otherwise soap bubbles would be some other shape. In marketing some products, an attempt may be made to construct boxes with the maximum apparent size for the minimum contents. Bridge builders seek designs providing the maximum strength for the minimum cost and weight of materials.

Partly by arithmetic and partly by construction, students could attempt to construct an open-topped box with the maximum volume from a single sheet of $8\frac{1}{2} \times 11$ paper.

5. Locus

The idea is that when a pencil (or a point) moves so as to obey some physical constraint or rule, it sometimes produces interesting shapes. For example, if a loop of string is kept tight around the shaft of a pin by a pencil, the moving pencil will trace out a circle. With the loop around two pins, another interesting shape (an ellipse) is created. A flexible chain (on a fence, around someone's neck, or on a bridge, etc.) hung from two points creates a shape called a catenary.

6. Congruence

Two figures or solids that have exactly the same shape and size are said to be congruent. The only way that two congruent figures or solids could differ would be in their position and orientation in space. Not a very exciting theme in two-dimensions, but all sorts of interesting explorations and measurements arise when students are asked to test whether or not two 3-dimensional objects are congruent.

7. Coordinates

The full use of coordinates must wait for senior years when they can be combined with algebraic skills but that is no reason why the concept cannot be introduced in middle years. Some graphing lends itself immediately to coordinates, and students can gain sharp insights from an exploration of what is sometimes called "taxi-cab" geometry. Vertical and horizontal lines are the "streets" and each intersection has an "address." Addresses can be plotted from simple open sentences with two variables.

8. Topology

Typology is a branch of geometry that concerns junctions and loops in a figure but has no restriction on size and shape.

 and  are topologically the same as the pair of glasses can be twisted into the second shape. (Each has 4 junctions, 3 branches, and 2 loops.)

GOALS	5	6

GI. *Develops an Appreciation of Geometry Through Describing Objects and Actions in the Physical World*

In all situations, tie into the physical world – natural and human-made. For example, how triangles are used to give structural strength, symmetries which exist in crystals, art, and construction, etc.

Over-emphasis on terminology and rigourous definitions are to be avoided. However, students are expected to provide both oral and written descriptors of geometric concepts and physical actions.

⇒

5

■ improves ability to identify and describe various real-world objects and their features through:

As much as possible, place emphasis on simple human-made structures, including the classroom and its furniture, school, home, community club, restaurant or other nearby buildings, mailboxes, street lights, bridges, etc.

Students use, or are introduced to appropriate geometric concepts and language within the context of exploring, constructing, measuring, comparing, classifying and discussing **human-made structures,** including:

- identifying examples of geometric "themes" such as symmetry, congruency, similarity, etc., present in structures
- identifying examples of 3-D figures and spaces such as specific prisms (cube, rectangular, hexagonal, etc.) pyramids (square, triangular, etc.) and cones, cylinders and spheres or hemispheres present in human-made structures
- identifying 2-D outlines, spaces or planes present in structures, including polygons and non-polygons

6

■ —>

Place emphasis on 3-D phenomena in nature (trees, flowers, crystals, erosion patterns, etc.) and on works of art (sculpture, crafts, painting, etc.) and graphic arts.

Students use, or are introduced to appropriate geometric concepts and language within the context of observing, comparing and discussing **objects in nature**, or observing, experiencing, comparing and discussing examples of artistic expression, including:

- identifying examples of geometric "themes." Focus on types of symmetry, presence or use of congruency and similarity, also coordinates
- identifying 3-D figures and spaces or impressions of them in nature and in artistic works

  locating 2-D figures and spaces or impressions of them in nature or in artistic works
- identifying various types of lines and the effect they have on creating mood and texture in artistic works

Teacher Reference

Geometry Around Us (2 posters). NCTM, 1993.

Geometry in Our World (200 slides). John Engelhardt, Editor. NCTM, 1987.

7	8

In all situations, tie into the physical world – natural and human-made. For example, how triangles are used to give structural strength, and symmetries which exist in crystals, art, and construction, etc.

Over-emphasis on terminology and rigourous definitions are to be avoided. However, students are expected to provide both oral and written descriptors of geometric concepts and physical actions.

Grade 7

■ improves ability to identify and describe various real-world objects and their features through:

Place emphasis on various manufactured goods, including the design, manufacturing process and marketing of various uniform items (coins, soup cans, cereal boxes, video cartridges, etc.), and custom-made items (individualized clothing, jewellery, pottery, automobile extras, and so cn).

Students apply or are introduced to appropriate geometric concepts and language within the context of viewing/ researching, analyzing and reporting procedures for **manufacturing various objects**, including:

- identifying examples of geometric "themes." Focus on similarity in manufacturing, size variations in clothing; maximum – minimum in packaging
- identifying and evaluting the reasons for 3-D shapes and spaces in manufactured items, the molds used to produce various items, etc. (consider function and aesthetics)
- locating and determining the reasons for 2-D shapes and spaces found in various manufactured goods
identifying lines of all types and discussing the need for them

Grade 8

■ —>

Place emphasis on architecture, particularly ancient and historical architecture, tracing changes and similarities in design, construction and decoration.

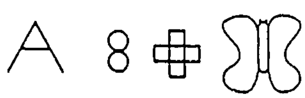
Students apply or extend appropriate geometric knowledge and vocabulary within the context of viewing/researching, analyzing and discussing interesting **types of architecture**, past and present, including:

- identifying examples of geometric "themes." Focus on maximum – minimum ideas in strength vs. weight/effort during construction; ancient methods involving locus
- identifiying, comparing and debating the merits of various 3-D figures and spaces in present and past architecture, construction methods, decoration features, etc.
- (consider function and aesthetics) identifying and comparing 2-D figures and spaces in present and past architectural
- design and decoration, etc. identifying lines and points and their use in present and past architectural design and decoration
- designing and constructing a 3-D structure which would be a credit to their community

⇒

<table>
<tr><th>GOALS</th><th>5</th><th>6</th></tr>
<tr><td>⇒</td><td>■ uses geometric vocabulary and conceptual understanding to give and receive directions for finding locations and for completing tasks.

Students use real or created opportunities for:
– providing oral or written directions and descriptions for others (May include direction, turns, distances, specific characteristics of structure, etc.)
– visualizing and acting upon oral or written directions and descriptions provided by others</td><td>■ —>
Extend to include LOGO. Limit to direction, location, and simple procedures.</td></tr>
<tr><td rowspan="3">G2. Develops Ability to Construct, Draw and Visualize Geometric Figures in 3-Dimensions and in 2-Dimensions

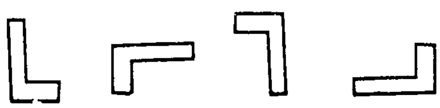
<u>Teacher Reference</u>
Geometry in the Middle Grades (Part of Grades 5-8 Addenda Series). D. Geddes, et al. NCTM, 1992.
Geometry and Spatial Sense (Part of Grades K-6 Addenda Series). John Del Grande, et al. NCTM, 1993. ⇒
Junior High Cooperative Problem Solving with Geoboards (Grades 6-9). Available from Addison-Wesley.</td><td colspan="2">Investigations/Explorations/Activities should be chosen for their potential to provide good opportunities for students to meet geometric shapes, solids, language, etc., and to develop greater insights into the conceptual themes. Most activities should engage both hands and minds as students work to make sense of form and space in our physical world.</td></tr>
<tr><td colspan="2">Students are expected to understand geometric concepts from a physical/ materials base reinforced with pictorial representations. Most concepts extend over several grades as complexity of expectation increases.</td></tr>
<tr><td>■ investigates and constructs 3-D structures and 2-D shapes involving:
• symmetry
Limit to <u>bilateral</u> symmetry with 2-D shapes and 3-D structures. (Require lines of symmetry.)
Examples
– How many lines/axes of symmetry? Draw in the lines of symmetry.
</td><td>■ —>

• symmetry
Extend to include <u>repetitive</u> (translational) symmetry. Include real world examples such as fences, apartment buildings, wallpaper, flooring, designs, tessellations, etc.
Examples:
– Investigate the symmetries of pentomino and tangram pieces.</td></tr>
</table>

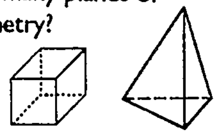
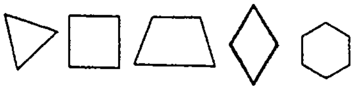
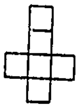
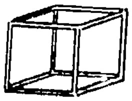
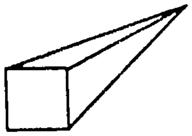
7	8
■ uses geometric vocabulary and conceptual understanding to give and receive directions for finding locations and for completing tasks Extend to flowcharts Examples could include charting: – actions from getting up in the morning and getting ready to go to school – going from home to school Design and sketch a piece of furniture that you would like to have in your home, and that can be mass-produced. Write a description so that the reader will be able to sketch your futniture design. Trade your description with a friend. Does the drawing match yours? Should you improve your description?	■ —> Extend to include flowcharts with decision box(es). Design the instructions for a robot to safely cross a street or road. Work with your group members to research, plan and construct a scale model of your favourite historical building, castle, monument, or .... Include an oral report which explains the function of the structure, the highlights of its construction, design and upkeep. Provide a written report on the significance today of this structure, and of how you feel towards it.

Investigations/Explorations/Activities should be chosen for their potential to provide good opportunities for students to meet geometric shapes, solids, language, etc., and to develop greater insights into the conceptual themes. Most activities should engage both hands and minds as students work to make sense of form and space in our physical world.

Students are expected to understand geometric concepts from a physical/ materials base reinforced with pictorial representations. Most concepts extend over several grades as complexity of expectation increases.

■ investigates and constructs 3-D structures and 2-D shapes involving: • symmetry Introduce ***rotational*** symmetry in nature and in design. Include order of rotation and the concept of point symmetry. (Two figures have point symmetry if one is a 180° rotation of the other about a point of symmetry.) Examples: – Which capital letters have rotational symmetry?	■ —> • symmetry Investigate combinations of symmetries and connect to Motion Geometry, G3. Examples: – Identify the symmetries in the following design. 

⇒

GOALS	5	6
	– Which letters of the alphabet have vertical symmetry? horizontal symmetry? – Find a trademark or company logo with one or more lines of symmetry. [c] Art	– How many planes of symmetry?  – If a solid has bilateral symmetry, will one or more of its nets have bilateral symmetry? – The word TOMATO when written in column form has a vertical axis of symmetry. Find another such word. T O M A T O
Teacher Reference *Introduction to Tessellations* (Grades 5+). Available from Spectrum. *"Try It" Activity Cards: Pentominoes* (Grades 4-9). Available from Spectrum. *Tessel Mania!* (MECC – Macintosh version only) *Introduction to Tessellations* (Grades 6-12). Dale Seymour. Available from Exclusive.	• tessellations (tiling) Tessellate using the following pattern block shapes:  Emphasize that tessellating involves covering area (with no gaps). Link to pattern and design. [c] Measurement (Area)	• tessellations (tiling) Extend to include all triangles and additional quadrilaterals (rectangle, parallelogram and kite) Example: – Create a tessellation using a scalene triangle. Identify the symmetries in the design. – Identify tessellations in nature. – Select a pentomino piece and tessellate. Colour each piece such that two pieces with a common edge must be in different colours. Use the minimum number of colours.
	• nets and skeletons Limit to cubes and other rectangular solids. Examples: – Draws other nets for any given cube or rectanglular solid. Net for a Cube  Skeleton (Frame) for a Cube 	• nets and skeletons Extend to include tetrahedra and pyramids Example: – Find all the nets of the solid shown.  Construct skeletons with straws or pipecleaners.

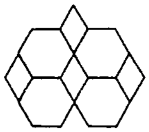
7

- Which common solids have rotational symmetry?
  Order of rotation – the number of times a figure would coincide with itself in being rotated through 360°.

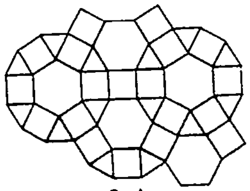
- Determine the order of rotational symmetry for any figure.

- Find all the symmetries for the following figures.

- tessellations (tiling)
  Extend to combinations of regular polygons.



2 shapes

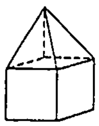


3 shapes

Colour each piece such that 2 pieces with a common edge are different colours. Use the minimum number of colours.

[C] Art

- nets and skeletons
  Extend to prisms and combinations.



"house" made from prism and pyramid.

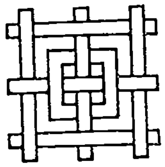
  - Given a net(s) identify the solid.
  - Indicate which net(s) would generate a given solid.

[C] Measurement, surface area

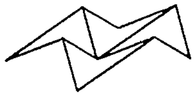
8

- Create a design, using a basic shape which demonstrates a combination of symmetries.

- Find all the symmetries for the following design.



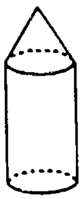
- tessellations (tiling)
  Extend to all quadrilaterals and shapes which lead to Escher-type designs.



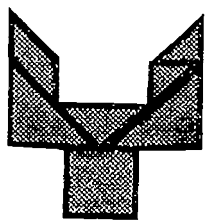
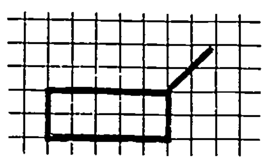
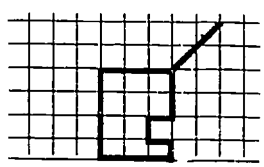
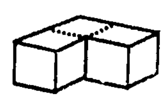
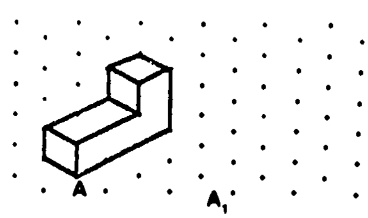
Research and reproduce a favourite tiling (tessellation) pattern found in ancient times. Analyze and report on the tessellation used.

[C] Art

- nets and skeletons
  Extend to cones, cylinders and combinations
  Examples:
  - Generate the net for the silo shown and vice versa.



Note: Students need to realize that nearly all solids have nets. What solid(s) do not have nets? (Spheres)

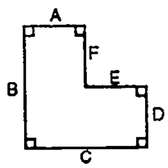
GOALS	5	6
	– Find all the nets of a cube. How many different nets are there for a specific cube? (11) – Which pentominoes fold to make an open box?	– Find a net for all pyramids whose bases are regular polygons and whose faces are unilateral triangles. – Given a net, identify the solid. Euler's formula Experimentally find the relationship among vertices, faces, and edges for 3-D solids with plane faces (V + F = E + 2).
Teacher Reference *D.I.M.E. Geometry Set* (Grades 4-9). Available from Spectrum. (This set contains 1 set of D.I.M.E. solids and 3 activity books.) *Tangram Geometry* (Grades 4-9). Available from Spectrum. *Points of Departure with Tangrams.* Available from Spectrum. *Super Tangram Activities, Books 1 and 2* (Grades 4-8). Available from Addison-Wesley. *Junior High Cooperative Problem Solving with Tangrams* (Grades 6-9). Available from Addison-Wesley. *Power Cubes – Adventures in Spatial Perspectives.* Rebecca Stanton and Elizabeth Miller. Stamina Associates: Los Altos, CA, 1987. *Spatial Reasoning with Soma Cube Activities.* Charlotte Mack and Constance Feldt. Janson Publications: Dedham, MA, 1993. Available from Gage Publishing.	• visual perception in 2-D and 3-D Covering shapes with tangram pieces and coloured rods or cubes.  Completing figures on a grid. Examples: – Complete the drawing of the skeleton of the rectangular solid.  – Complete the solid to show the perception of depth. 	• —> Use a complete set of tangram pieces to make a rectangle. Can you make any other rectangles? Sketching 3-D solids or skeletons with or without a grid. Examples: Sketch various geometric solids/skeletons. – Use a simple object such as that shown. Draw the object on dot paper.  – Given a drawing of the object below, reproduce the drawing with point A at A_1. 

7

Find all the hexomino pieces which are the net of a cube. [As an extension, find all the hexomino pieces (35).]

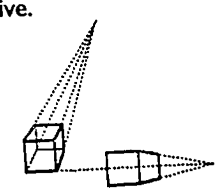
- visual perception in 2-D and 3-D

  What minimum dimensions are needed to draw the following figure?



(Six segments are needed to draw the figure. How many and which lengths would be needed to accurately reproduce the figure?)

Sketching solids using one point perspective.



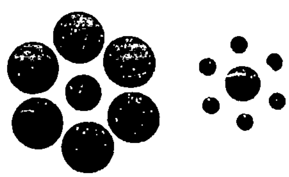
[C] Art

- optical illusions

  Example:

  – Which is shorter?

– Which centre circle is larger?



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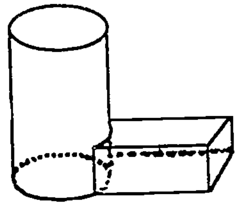
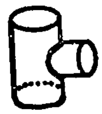
– Sketch the net for the following figure.



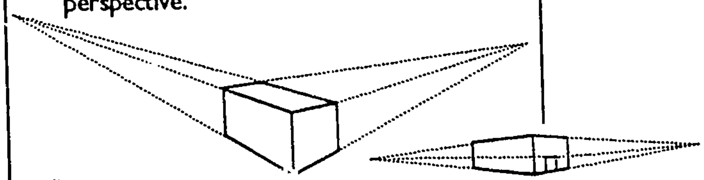
– What would the net for the pipe shown look like?



[C] Measurement (Surface Area)

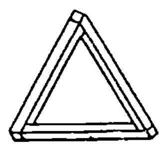
- —>

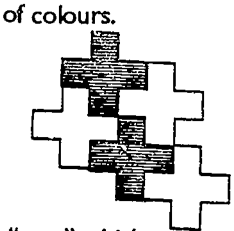
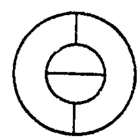
Sketching solids using two point perspective.



[C] Art

- Escher-type illusions

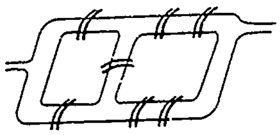


GOALS	5	6
Teacher Reference *Spatial Problem Solving with Cuisenaire Rods.* P. Davidson and R. Willcutt. Cuisenaire Company, White Plains, NY, 1983. Available from Spectrum.		• buildings, plans and elevations From a defined solid, create the plan and 2 elevations. • 4-Colour Problem Examples – Colour each area so that areas which have a common edge must be in different colours. Use the minimum number of colours.  – Design a "map" which requires only 3 colours? 4 colours? Can you design a map which requires more than 4 colours?  How many different colours are necessary for this logo?

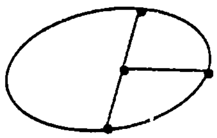
7

- buildings, plans and elevations
  From the plan and elevations, construct the solid.

- networks
  Konigsberg Bridge type problems
  – Konigsberg is located on the banks and two islands of the Pregel River. Parts of the city were connected by 7 bridges as shown below. Starting from home, is it possible to walk around the city in such a way as to cross each bridge only once?



  Euler's formula in 2-D
  – What is the relationship among the regions (R), lines (L) and points (P) in a network. (Note: The "outside" is a region.)
  $(R + P = L + 2)$



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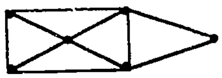
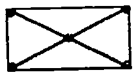
- —>
  Students construct their own solid(s) with appropriate plan and elevations and vice versa.

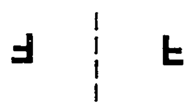
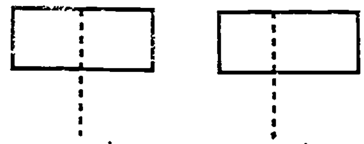
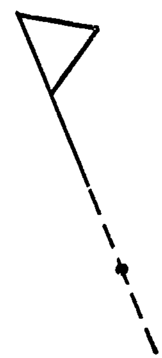
- networks
  Solving network problems using odd and even nodes/vertices.
  Examples:
  – Without taking your pencil off the paper and without drawing the same line twice, can you trace the following:

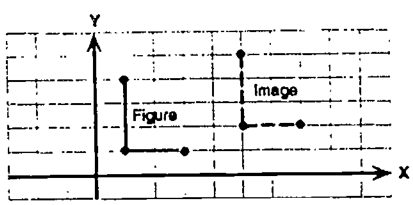
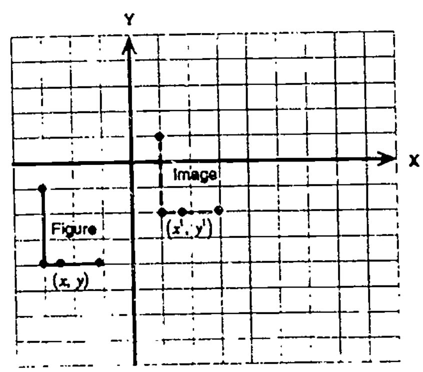




  – Create a chart for different figures.

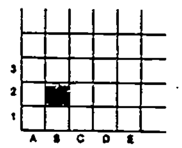
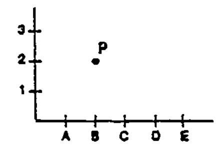
Number of Even Vertices	Number of Odd Vertices	Can Figure Be Traced?

GOALS	5	6
G3. ***Motion Geometry Visualizes, Applies and Explains the Effect of Motions and Transformations on Geometric Solids and Figures*** The language of motion geometry is important. Begin with simple language and increasingly use more sophisticated language: slide —> translation flip —> reflection turn —> rotation	■ performs and describes motions including: • slide – horizontal, vertical and combination • flip – line of reflection should be horizontal or vertical and limited to i) an "edge" of the object ii) external to the object • turn – $\frac{1}{4}$ (90°), $\frac{1}{2}$ (180°), $\frac{3}{4}$ (270°) and full (360°) turns clockwise or counter clockwise. Centre of rotation should be limited to simple internal or perimeter points for any object or figure. Measurement (Angles) C Students are expected to move an object or figure according to motion(s) and/or to describe the motion(s) required to move a figure to its image.  rotation and slide or rotation about A  flip about central line	■ —> Extend to: • multiple slides • flips where the line of reflection is internal to the figure. This line of reflection could be either symmetrical or asymmetrical.  symmetric line of reflection · asymmetric line of reflection • turns of $\frac{1}{4}$ (90°), $\frac{1}{2}$ (180°), $\frac{3}{4}$ (270°) and full (360°) where the centre of rotation is external to the figure (in simple positions such as to right, left, up or down from a corner)  external centre of rotation in simple position

7	8
■ performs and describes motions including: • rotations where the centre of rotation may be anywhere internal or external to the figure • exploring rotations of 30°, 45°, 60°, 120°, etc. • reflections using oblique lines (lines which are not horizontal or vertical)	■ —> Extend to combinations of motions and determining equivalent motions. Include solids in two- and three-dimensions.
■ describe motions using: • coordinate pairs (first quadrant) For translations, reflections and rotations, students are expected to determine coordinates for the image. Limit rotations to multiples of 90°.	■ —> • —> Extend to all 4 quadrants.
• verbal or written descriptions Limit to translations. Example: Given the figure and its image shown, describe the translation in terms of x- and y-coordinates.  Answer: The x-coordinate increases by 4, while the y-coordinate increases by 1.	• verbal or written descriptions as algebraic expressions. Limit to – translations  Answer $x' = x + 4$ $y' = y + 2$ – reflections where the line of reflection is vertical, horizontal or through the origin at 45° – rotations where the centre of rotation is the origin and the amount of turning is ±90° or 180°

⇒

⇒

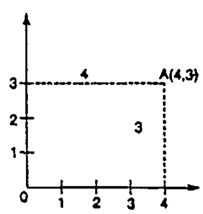
GOALS	5	6
G4. *Coordinate Geometry Discovers, Develops, and Uses Coordinate Geometry Concepts* ⇒	■ uses an alphanumeric scheme for naming a square on a grid Examples could include battleships, map reading, etc. In order to make such naming mathematically compatible label as follows.  Call the shaded square B,2 to later coincide with the coordinate pair (x,y). In other words, identify the horizontal value first and the vertical value second.	■ —> Extend to an ordered pair notation which identifies points rather than squares. The example from Grade 5 would now be labeled as follows:  (B,2) would be an ordered pair identifying the point P on the graph. ■ identifies and/or labels points on a coordinate grid Examples could include map reading, longitude, and latitude, etc. ■ traces a path from oral or written instructions or writes instructions to trace a given path From City Hall the fire station is 2 blocks East and 4 blocks South. Draw a map to show the City Hall and fire station. How many different paths could you take to get from City Hall to the fire hall on your map? Note: Students should be instructed to give the E-W distance first. This corresponds to later use of (x,y) where x is horizontal (E-W) distance and y is vertical (N-S) distance.

7

■ identifies real world uses for coordinate graphing and experiences those uses where possible

Uses could include map reading, fixing points on Forest Service maps, locating objects on radar screen, enlarging or reducing drawings, planning cheering section designs involving spectators at sporting events, drawing picture graphs on a coordinate system for needlework, commercial games such as Battleship, etc.

■ plots points and identifies coordinates of points in the first quadrant



The coordinates for point A are (4,3). The first coordinate gives the distance from the vertical axis while the second coordinate gives the distance from the horizontal axis.

8

■ —>

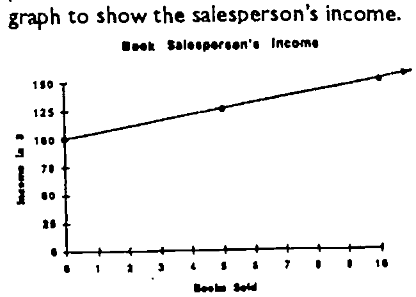
Extend to all four quadrants.

Students should be familiar with the following terms: coordinate plane, coordinate axes labeled as x and y, coordinates or ordered pairs, quadrants, origin

■ applies coordinate graphing techniques in the solution of related mathematical and real-world problems

Diagonally opposite corners of a rectangle are located at (-2,3) and (5,-2). Draw a graph and use it to help determine the coordinates for the missing corners of the rectangle. Which quadrant(s) contains the largest part of the rectangle? Justify your answer.

A book salesperson earns $100 a week plus $5 for each book sold. Draw a labeled graph to show the salesperson's income.



■ plots relationships of the form y = mx+b where "m" is the slope and "b" is the y-intercept (where the graph crosses the y-axis) from real-world examples ⟹

GOALS	5	6
G5. *Discovers, Develops and Begins to Apply Knowledge of Geometric Relationships* ⟹	This section must be taught using discovery and investigation. Relationships must be found by students not told to them. Use of materials and equipment is essential.	
	■ investigates polygons Investigation should introduce students to: • uses in real world (triangles for structural strength; shape of all stop signs, yield signs, etc.; geometric shapes seen in buildings) • names for polygons • properties of polygons including such factors as parallel lines and diagonals Students should understand that a square is a rectangle but a rectangle is not a square.	■ investigates triangles Investigations should be designed to lead to: • similarities and differences of different triangles • classification of triangles • properties of triangles, such as – sum of lengths of two sides is greater than length of the third side – longest side opposite largest angle – intersection of medians creates centre of gravity – intersection of altitudes may be inside, on, or outside the triangle. Determine when each is true. • interior angle sum of triangles and other polygons • exterior angle sum of triangles and other polygons

7	8
	In example 2, above, the equation would be I = 5B+100 where I is income of the salesperson and B is the number of books sold. Emphasis is to be on making sense of real-world examples not on the mathematical exercise of plotting curves from their equations. Limit examples to those which are linear (graph is a straight line).

7

■ investigates quadrilaterals

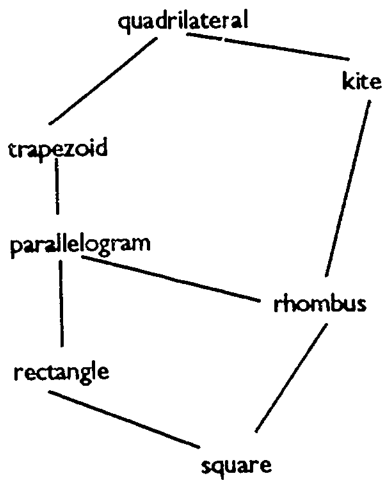
Investigations should lead to:

- similarities and differences among different quadrilaterals
- properties of different quadrilaterals

Students should not be told the various properties of quadrilaterals but need to identify them from group activities.

8

■ classifies quadrilaterals using similarities and differences among different figures

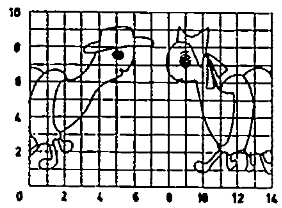
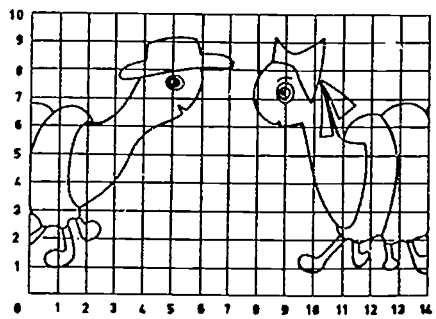


[C] Extend to diagramming a "family tree" which displays the relationships within your family.

■ constructs triangles through investigation and exploration

What information do you need to construct a specific, unique triangle? Is it sufficient to know the size of all three angles (AAA), etc.? Activities should lead to SSS, ASA, and SAS and must include the ambiguous case, SSA.

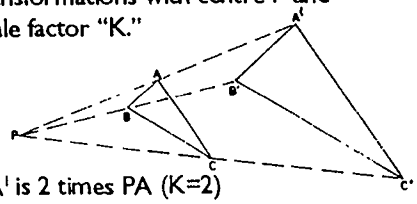
⟹

GOALS	5	6
	■ enlarges or reduces figures on grids   [C] Art	■ determines geometric solids or figures which are always similar or not always similar Examples which are always similar: equilateral triangles squares circles spheres cubes Examples which are not always similar: rectangles rectangular prisms (bricks) isosceles triangles cones Note: Similar figures are the same shape but different size. Congruent figures have the same shape and size.

7

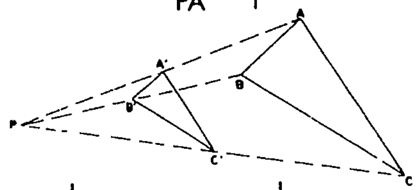
- **demonstrates understanding of similarity as an enlargement or reduction of geometric figures or solids using scale factors**

  The following diagrams illustrate size transformations with centre P and scale factor "K."



PA' is 2 times PA (K=2)

PA': PA = 2 : 1 or $\frac{PA'}{PA} = \frac{2}{1}$



PA' = $\frac{1}{2}$ times PA (K = $\frac{1}{2}$)

PA' : PA = 1 : 2 or $\frac{PA'}{PA} = \frac{1}{2}$

Note that for the illustrated examples:
- the image has the same shape but a different size if $k \neq 1$;
- the centre P, a point A, and its image A' all lie in a straight line.

If the scale factor is 1 figures are said to be <u>***congruent***</u>.

- **identifies real world uses for enlarging or reducing figures/solids and experiences those uses where possible.**

  Uses could include needlework patterns, some paint by number kits, dress patterns, house plans, blueprints, patterns for carpenters, photography, map scales, etc.

[C] Number Concepts, Measurement

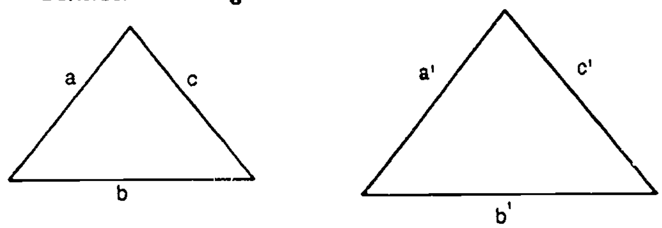
[C] Social Studies

8

- **discovers properties of similar figures by comparing measurements**

  Measurements could include linear and area. Volume could be used for geometric solids. Some properties to be discovered include:
  - similar figures have corresponding angles which are congruent
  - similar figures have corresponding sides which are proportional (i.e., in the ratio given by the scale factor)

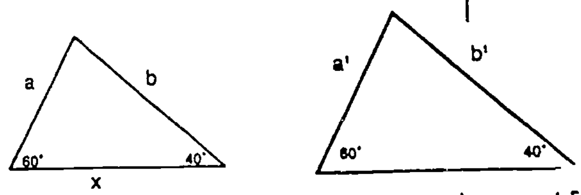
Consider the triangles below.



Through measurement and computation, compare the length of a to a', b to b', and c to c'.

How does $\frac{a}{a'}$ or $\frac{b}{b'}$ or $\frac{c}{c'}$ compare?

Extend to include the following situation.



px where p = 1.5, 2, 2.5, 3, ... and p is the scale factor

Students should measure the remaining sides to find possible relationships. <u>***Emphasis is on student discovery of the relationships.***</u>

⇒

Measurement

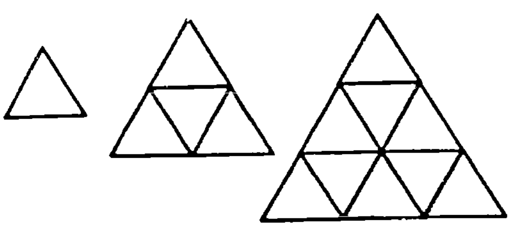
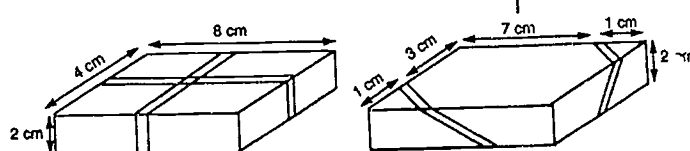
GOALS	5	6
M1. *Linear Measurement Extends Understanding by Applying the Processes of Estimating and Measuring Length*	■ recognizes and explains the meaning of length, width, height, depth, thickness, diagonal and perimeter in real-world contexts Students should recognize two meanings of length – length as a property of an object and length as the distance between objects or points.	■ —> Extend to include circle, diameter, radius and circumference.
<u>Teacher Reference</u> *Measurement in the Middle Grades* (Part of Grades 5-8 Addenda Series). D. Geddes, Editor. NCTM, 1994. *How to Teach Perimeter, Area, and Volume* (K-8). Beaumont, et al. NCTM, 1986. *Geoboard Activity Cards – Intermediate Set* (Grades 3-9). Available from Spectrum. *Junior High Job Cards: Geoboards* (Grades 6-9). Available from Addison-Wesley.	■ uses strategies to estimate various lengths including: • referring to a familiar referent Width of finger is 1 cm so width of hand is about 8 cm.	■ —> • familiar referent My hand is 9 cm wide. The fish caught was about 30 cm long because it measured about 4 hand widths.
	• unitizing – using a known unit A Cuisenaire rod is 10 cm long. A table is about 7 rods long. I think the table is about 70 cm long.	• unitizing Height of a table is approximately 1 metre. Height of a wall is about 3 tables high.
	• chunking – using a multiple unit object as a reference The length of the gym mat is approximately 3 m. The gym is about 10 mats in length. Therefore, the gym is about 3 x 10 = 30 metres.	• chunking Car = 6 m. Gym = 5 cars or about 30 metres.
⇒	■ demonstrates and explains the process for determining linear measurement by: • estimating, then using informal (non-standard) units or using a "home-made" informal tool for measuring	■ —> Include circular objects • —> Mark off and number a paper strip equal to five of your handspans. Use it to measure the height of the following: a) the blackboard ledge

7	8
■ recognizes and explains the meaning of length, width, height, depth, thickness, diagonal, perimeter, diameter, radius, and circumference in real-world contexts.	■ —> Extend to include hypotenuse.
■ uses strategies to estimate various lengths including:	■ —>
• familiar referent [?] Extend to situations where the referent is not clear. For example, students might be given a picture and asked to select a familiar referent in the picture in order to estimate the measurement of other objects in the picture.	• —> Continue to refine estimation and measurement skills using familiar referent, unitizing and chunking as developed in Grades 5-7. Extend to examining the estimate to increase accuracy.
• unitizing [?] Have students examine magazine photographs, paintings, etc. Estimate lengths by choosing a known object as a unit of measure (e.g., length of small car).	• —>
• chunking [?] Estimate the total length of the various kitchen countertops shown in your picture.	• —>
■ demonstrates and explains the process for determining linear measurement by:	■ —> Extend to measuring hypotenuse.
• estimating, then using informal (non-standard) units or using a "home-made" informal tool for measuring	• —> Extend to measuring indirectly. [?] Work with a partner. Find a way to determine the height of our school, the flag pole, and the goal posts without actually measuring each object. Explain your work and results to the class. [c] Geometry

GOALS	5	6
	Use a drinking straw to estimate, then measure the following: a) the width of a classroom door b) the long edge of a trapezoidal table c) the distance from the front to the back of the room [?] Record the difference between your estimate and measurement. Explain how each measurement helps you to be more accurate with your next estimate.	b) your chair seat [?] Work with 3 other students. Compare your handspan measurements. Find your group's "average" handspan length for each object. Are your "averages" the same as another group's average? Why? Prepare to justify your answers to the class. [C] Data Management
	• estimating, then measuring with standard units (mm, cm, dm, m) and standard tools (metre stick, metric rulers, tapes, trundel wheels) Work with your partner. Choose 5 items of various lengths. Select the unit you think best, then estimate, measure, and record your measurement to the nearest whole unit. Compare your estimates and your measurements, and record any differences. Discussion: What unit is best (remember context)?	• —> Extend to km [?] Work with 2 or 3 other students. Estimate and measure a track for a 1 km jog. Tell the class where your track is and how students will know when they have jogged a distance of 1 km.
	• renaming measurements if necessary by recognizing the relationships of various units to each other and to the place-value system. Include renaming m as cm, dm as cm, cm as mm, and vice versa. 2.4m is "two and four tenths metres" or two metres and four decimetres, or 240cm. Note: Have students develop, use, and refer to a place-value chart. [C] Number Concepts	• —> Include renaming m as dm or cm and vice versa. 2.41m is "two and forty-one hundredths metres" or two metres and forty-one centimetres or two hundred forty-one centimetres. • renaming linear measurements with decimals as fractions, and vice versa, when it's helpful to do so. 2.5 m is $2\frac{1}{2}$ m $3\frac{1}{4}$ m is 3.25 m

7	8
Work with a partner. Use your stride to estimate, then measure the length of objects such as your classroom, your school gym, your school grounds, etc. Estimate and measure the distance of various balls thrown (golf, basketball, ping-pong, baseball, etc.). Explain how the previous measurements help you better estimate the next length. Use fractions or percent in your explanation (e.g. "We think the length of the parking lot is about half the length of the soccer field.").	
• reviewing estimation and measurement using standard units (stress metric but include imperial) [?] Work with 3 or 4 other students. Estimate your armspans to the nearest half m and then measure. Calculate the "average" armspan. Then find and record three things which have about a) 5 times the length b) 3/4 the perimeter c) 1/3 the circumference of your group's "average" armspan.	• —>
• renaming measurements Include renaming m as dm, cm or mm and vice versa; also renaming km as m and vice versa. Use the calculator to assist with renaming.	• —> Include renaming any linear measurement if necessary [?] Explains how to use the place value system to rename any linear measurement in the metric system.

GOALS	5	6
⇒	■ uses linear measurements to solve problems by: • compiling data for comparing or ordering objects. [?] Compare and order the students in your room from least to greatest by measuring: a) height b) arm span Are the orders the same? Write about your findings. Are there any differences between boys and girls? [C] Data Management	■ —> • —> Include squares, rectangles, parallelograms and right-angled triangles [?] A ribbon is to be tied around each of several boxes in two different directions. Decide and record as a group, which box will take the least ribbon. Order the boxes from least to greatest length of ribbon used. Then check your order with a metric tape. Were you correct? Would the order remain the same if each box was wrapped in two other directions? Three directions? [C] Geometry
	• compiling data for investigations in other math strands [?] Work with your group to find, measure, and record every different length of line segment possible on the geoboard. Which standard unit did you use, and why? [C] Geometry	• —> [?] About how much crêpe-paper will you need to match the length from a corner to the middle of the ceiling in our classroom? How much do you think the length needed will change if the crêpe-paper strip is twisted into a streamer? What total length of crêpe-paper will be needed to hang a two-colour streamer from each corner to the centre of the classroom ceiling?
		• compiling data for investigations in other subject areas [C] Science: What happens to the distance the toy car rolls as you raise the ramp? [C] Social Studies: Choose appropriate tools to map out the size of the living quarters of an Iroquois longhouse. Write about your work and discoveries.

7	8
■ uses linear measurements to solve problems by: • compiling data for comparing or ordering objects. [?] Include all triangles, prisms, and regular polygons, e.g., measure the total length of all the edges of a prism. Build a doll house out of cardboard. [C] Geometry	■ —> • —> Include trapezoids Where possible, use LOGO to create and measure shapes.
• compiling data for investigations in other math strands [?] Start with a triangle or quadrilateral and add more of the same figure to create the next larger model of the original. Find the perimeter of each new polygon. Is there a pattern? Report to the class.  [C] Algebra, Geometry	• —> Practical use: cost of fencing, carpeting, and baseboard. [?] Which wrapping uses less ribbon? Find more than one way to solve this problem.  Justify your answer. [C] Pythagorean Theorem
• compiling data for investigations in other subject areas	• —>

⇒

GOALS	5	6
		■ discovers and generalizes relationships by: • writing an expression for finding perimeter of a shape S1 + S2 + S3 = Perimeter of a triangle (S1 = Side 1) L + W + L + W = Perimeter of a rectangle where L is long side and W is short side [c] Algebra • using materials and diagrams to show more than one rectangle with the same perimeter [c] Geometry
	■ develops awareness of other measurement systems Address imperial units as they come up in class. Help students learn about the history of linear measurement.	■ —>

7	8
■ discovers and generalizes relationships by: • finding formulas to describe the perimeter of polygons $P = S + S + S$ or $P = 3S$, for equilateral Δs [C] Geometry	■ —> • Given the value of 2 unknowns, solving for the third in $P = 2L + 2W$ [?] Find the length of a rectangular gym if the width is 16.4 m and the perimeter is 80 m. [C] Algebra • developing formula for circumference of a circle $C = \pi d$ or $C = \pi(2r)$ Diameter of a hula hoop measures 80 cm. The circumference of the hula hoop is unknown. Mary claims she can still tell you exactly what c/d equals. Jim says, "If measures are taken, then c/d won't come out a constant." Who is right and why? • developing formula for finding length of hypotenuse.
• using materials and diagrams to show more than one rectangle with the same perimeter [C] Geometry	• —> Extend to showing the relationships among radius, diameter, circumference, and area of a circle. Also, show relationship between length of hypotenuse and the sides of right-angle triangles. By the end of Grade 8, students should understand and generalize to include: – The more elongated a rectangular shape, the greater the perimeter. The more square a rectangular shape, the shorter the perimeter. – As the edge of a square grows by a factor of n, so the perimeter grows by a factor of n. – Circumference of the wheel is ___ ? What is the number of turns to go one km?
■ develops awareness of other measurement systems – address imperial units – address historic perspective, e.g., Egyptian measurement	■ —> Explore land measurement where applicable (township, section, range).

GOALS	5	6
M2. *Area* ⟹ ***Extends Understanding by Applying the Processes of Estimating and Measuring Area***	■ recognizes and explains the meaning of area (the number of identical units needed to cover an outlined shape or surface)	■ —>
	■ uses strategies to estimate areas of various ***rectangles***, then makes decisions according to estimates, using:	■ —> Extend to parallelograms and right-angle triangles.
	• familiar referent Use your hand, a milk carton; base 10 flat, etc., as a known area which can be used to assist with estimating.	• —> [?] Use the area of a regular (8 1/2 x 11) sheet of paper or newspaper page. Select 3 things in the room with an area of about 5 sheets of paper.
	• unitizing Set a milk carton in one corner and think in milk carton bottoms	• —> Lay down 2 metre sticks and use them to assist thinking in square metres.
	• chunking Estimate parts and then combine estimates to get a total area.	• —> Estimate 1/3 the area and triple your estimate.
⟹	■ demonstrates and explains the process for finding the area of rectangles by: • using estimation, then direct comparison (superimposing one area on another), and using correct "er"/"est" language [?] "This newspaper covers a greater area than my shadow."	■ —> Extend to finding the area of parallelograms, right-angled triangles and regular prisms. • —> [?] Compare the cut-out parallelogram and rectangle. How are the two areas related? Make and record sets of parallelograms and rectangles whose areas are identical on your geoboard.

7	8
■ recognizes and explains the meaning of area	■ —>
■ uses strategies to estimate areas of various figures, then makes decisions according to estimates, using: Extend to all triangles, the surface area of prisms, and the area of circles.	■ —> Extend to trapezoids, irregular shapes, and the surface area of cylinders.
• familiar referent Use your stride to estimate floor area in square metres.	• —>
• unitizing Estimate amount of carpet needed to cover room, classroom, etc.	• —>
• chunking Estimate the number of whole units and adjust for partial units (irregular shapes).	• —>

[?] Using materials, diagrams, and line graphs to compare perimeter with area.

Shape	Length	Width	Perimeter	Area
Rectangle A	10	5	30 units	50 sq. units
Rectangle B	25	2	54 units	50 sq. units
Rectangle C	50	1	102 units	50 sq. units

7	8
■ demonstrates and explains the process for finding area of various figures, by: Extend to find the area of all triangles and surfaces of prisms.	■ —> Extend to find the area of trapezoids, irregular shapes, circles, and the surfaces of cylinders.
• using estimation Compare a cut-out triangle and rectangle with the same base and same height. How are the areas related?	• —>

⇒

GOALS	5	6
	• estimating, then using informal (non-standard) units or a "homemade" tool to cover and count [?] About how many pattern block triangles would cover your notebook? Your desk? The teacher's desk? What strategy(s) could you use?	• —> [?] Work with 4 or 5 other students. You should each trace your hand to your wrist (fingers touching) on paper. Cut out, compare and order your handprints.
	• estimating, then measuring with standard units (square cm, square dm, square m) and covering. [?] Create a design which covers an area greater than 100 but less than 200 square cms.	• —> [?] Lay a transparent cm^2 grid over each handprint. Count the whole squares. Decide how to add on the part squares to get a total area. Find the difference between your estimates and your counts for each hand. Was your order correct? Prepare a presentation on your work for the class.
	• estimating, then measuring with standard units and tools to calculate area (numbers to be rounded to nearest whole unit) "This book is 17.8cm wide and 24.2cm long. We estimated using 20 rows of 20 square centimetres. That's 400 sq. cm. We multiplied 18 x 24 and found the area is 432 sq. cm." [C] Number Concepts	• —> (numbers may be written to two decimal places; use calculator to check estimation with rounded numbers; round calculator products to nearest whole numbers) Parallelogram: Height is 17.8 cm. Length is 24.2 cm. We estimated by rounding 17.8 cm to 20 cm and 24.2 cm to 20 cm. Our estimate for the area is a little more than 400 square cm. Area 17.8 x 24.2 = 430.76 (calculator) Round the area of the parallelogram to 431 square cm [C] Number Concepts, Number Operations
	• reading and writing area measurements Use square centimetres or square metres. Introduce formal abbreviation cm^2, dm^2, m^2 only after understanding is established.	• —> • renaming area measurements using recognized relationships ($cm^2 \leftrightarrow dm^2 \leftrightarrow m^2$) Work with a partner. Use Base 10 flats to cover: a) your textbook b) your writing folder

7	8
• estimating, then using informal (non-standard) units or a "home-made" tool to cover and count	• —>
• estimating, then measuring with standard units Extend to include hectare – ha², where applicable, and mm². Acknowledge historical units and include land area in acres where applicable.	• —> Extend to estimating area of circles, cylindrical surfaces, irregular surfaces; then use a grid to verify.
• estimating, then measuring with standard units and tools to calculate area (numbers to be rounded to nearest whole unit) Extend to rounding answers to the nearest tenth or hundredth.	• —>
• reading and writing area measurements Extend to include mm².	• —>
• renaming area measurements within the same system [?] Measure an object using two or more different but related units (cm^2 and dm^2) and compare. Look for patterns.	• —> [?] Find the surface area of a cereal box in cubic centimetres. Convert the answer to cubic decimetres. [C] Geometry

GOALS	5	6
		c) your desk Find, record, and report two different ways of expressing the area of each object.
	■ uses area measurements to solve problems by:	■ —>
	• compiling data to compare and/or order rectangles according to area	• —> Extend to include parallelograms and right-angled triangles
	• compiling data using other math strands (e.g., Geometry and Number Operations) [?] Find how many hexagons and part hexagons are needed to completely tesselate (cover) a page in your notebook. If every hexagon costs 45 cents, what is the cost of covering your page?	• —>
	■ discovers and generalizes relationships by:	■ —>
	• giving verbal explanation for finding area of rectangles "You just have to find out how many rows of units you need to cover the surface and how many units are in each row. Then, you can skip count or multiply to find the total units needed to cover the area."	• —> Extend to writing expressions for finding area of: rectangle parallelogram triangle
	• discovering the formula for the area of rectangles and squares	• using materials and diagrams to relate area to perimeter [?] Use 24 square tiles. Can you find more than one regular shape with an area of 24 square units? Record your work. Describe what you have found.

7	8
• renaming area measurements with decimals as fractions if it is helpful to do so, and vice versa "24.5 m^2 is 24 1/2 m^2" "65 3/4 cm^2 is 65.75cm^2"	• —>
■ uses area measurements to solve problems by:	■ —>
• compiling data to compare and/or order figures according to area Extend to include all triangles, trapezoids, circles and surfaces of prisms.	• —> Extend to include surfaces of cylinders.
• compiling data when using other math strands Extend to using data for solving problems from other subject areas The air pressure at sea level.... [c] Science, Soc' l Studies	• —>
■ discovers and generalizes relationships by:	■ —>
• giving verbal explanation for finding the area of rectangles Extend to expressions for calculating surface area, and to writing formula: Area of $\Delta = \frac{1}{2}$ b x h or $\frac{b \times h}{2}$ Surface Area = 2(lxw+lxh+wxh) for a rectangular solid	• —> Extend to writing formula for calculating area of a trapezoid, circle, and surface area of a cylinder.
• using materials, diagrams and line graphs to relate area to perimeter [?] Make a presentation to explain how area and perimeter are related.	• —> [?] By the end of Grade 8, students should understand and apply the following: – The more elongated the shape of a rectangle, the less the area enclosed. The more square the shape of the rectangle, the greater the area enclosed. – As the edge of a square grows by a factor of n, the area of the square grows by a factor of n^2.

Rectangle	Length	Width	Perimeter	Area
A	7	1	16 units	7 sq. units
B	6	2	16 units	12 sq. units
C	5	2.5	16 units	12.5 sq. units
D	4	4	16 units	16 sq. units

⇒

GOALS	5	6
M3. *Capacity and Volume* ⟹ ***Extends Understanding by Applying the Processes of Estimating and Measuring Capacity and Volume***	■ recognizes and explains the meaning of capacity and the volume of containers and cartons The amount you can pour into or out of a container is sometimes called its capacity, and sometimes its volume. The number of identical objects you can pack into a box is sometimes called its volume and sometimes its capacity. A capacity of 1 mL is equal to a volume of a 1 cm cube.	■ —> Extend to include volume of irregular solids (stones, full tins) [?] The space an irregular shape takes up can be found by measuring the amount of water it displaces if it sinks.
	■ uses strategies to estimate capacity and volume of various containers and regular shapes, and selects or makes decisions according to estimates, using:	■ —>
	• familiar referent Base 10 block is 1000 centicubes; 2L holds 2000mL; L milk carton holds 1000mL; cup holds about 250mL.	• —>
	• unitizing Pour and use a known capacity such as a 250mL cup into a container to assist with estimation	• —>
	• chunking [?] Use known capacity of oil truck to estimate volume of storage tank	• —> [?] How many cases (litres) of pop can be stored in your storage room? Classroom?
	■ demonstrates and explains the process for determining capacity and volume of rectangular containers and cartons by:	■ —> Extend to include irregular solids (stones full tins)
	• using estimation, then direct comparison [?] Will the water from a full jug fit into this can?	• —>

7	8
■ recognizes and explains the meaning of capacity and volume of containers and cartons	■ —> Extend to cylinders. (Cones and other 3-D shapes are to be considered enrichment)
■ uses strategies to estimate capacity and volume of various containers and regular shapes using:	■ —>
• familiar referent	• —> [?] To warm a room which measures 6.5 m by 5.2 m by 3.18 m we need 4.25 cubic metres of wood. How many cubic metres of wood will be needed to warm 2 rooms which measure 7.20 m by 6.75 m by 3.18 m each.
• unitizing	• —>
• chunking Estimate people in an arena	• —> Estimate cars in a parking lot
■ demonstrates and explains the process for determining capacity and volume of rectangular containers and cartons by:	■ —> Extend to include cylinders
• using estimation, then direct comparison Extend to include prismatic and irrigular solids. Water displacement could be used to measure irregular shapes.	• —>

⟹

GOALS	5	6
	[?] Will the cubes you packed into this cereal box fit if you rearrange them in this cookie box?	[?] Share a ball of plasticine equally among a group. (Use a scale to check.) Each person should form their clay into a shape different from everyone. Decide whose, if any, will displace the least/the most water. Record your order, then check, using a displacement pail and a clear jar. Put a different elastic band around the jar to show the level of water displaced by each person's shape. Does the order match your prediction? Report to the class.
	• estimating, then using informal (non-standard) units, or "home-made" tools [?] Use the smallest container in the collection as a measuring unit. Estimate, record and check the capacity of each container. Find and report the difference between estimates and measures. Use the drink carton to estimate and record the number of cartons needed to fill each box in this collection. Record your work, and tell how good you are at estimating volume. Cut one of the waterproof cartons to be your own 500ml measure. Then use your measure to find how close you can estimate capacity. Describe your work in an illustrated report.	• —> [?] Fill each container with water. Work together to decide on a group estimate for the number of times you can fill a coke glass with the water from each container. Choose one person to record your estimates and counts, and another to report how accurately you are able to estimate capacity.
	• estimating, then using standard units (mL, centimetre cubes, L and decimetre cubes) and tools (graduated containers) to find capacity and volume [?] Pour 100mL of water into each of the clear containers. Use this information to help you estimate the capacity of each of these containers. Keep a record of your work. Then check and record each capacity using a graduated container	• —> Extend to 1000 L and metre cubes [?] Work with 1 or 2 other students. Estimate the order of your stones from lightest to heaviest. Record your order by drawings in your notebooks. Then estimate how much water you think each stone will displace, and write your guess under each picture.
	[c] Data Management	[c] Science

7	8
• estimating, then using standard units and tools, to find capacity and volume Relate mL to cm^3 and 1000 L to m^3	• —> Extend m^3 and 1000 L to metric tonne.

GOALS	5	6
		Does the order of your volume estimates match your estimated weight order? Use the displacement pail and the graduated container to check you volume estimates.
	• reading and writing measures of capacity and volume	• —> Say and write mL, L, cm cubes, cubic cm and cm^3.
		• renaming capacity and volume measures using recognized relationships and the place value system "We renamed 3500 mL as 3.5 L because we know each 1000 mL is also called 1 litre, and 500 mL is one-half of a litre."
	■ uses capacity and volume measures to solve problems by:	■ —>
	• compiling data to compare and/or order various containers or cartons	• —> Extend to include irregular solids (stones, full tins).
	• compiling data to help solve problems in other areas	• —>

7	8
• reading and writing measures of capacity and volume Introduce abbreviated form for recording cubes (cm^3) and tie to introduction of exponents. "We wanted to calculate how many centimetre cubes it would take to pack our storage room full. We rounded 28.3 cubic metres to 28 and multiplied by 1 000 000 because we know that is how many centicubes it takes to fill a one-metre cube."	• —>
• renaming and rounding lengths when it is helpful in calculating volume Find the volume of each shipping carton in the storage room in terms of one metre cubes. "We measured the paper towel box Length = 124 cm – That's 1.24 m Width = 88 cm – That's 0.88 m Height = 55 cm – That's 0.55 m We changed the lengths to decimals. To estimate, we thought about 1 m by 1/2 m. That's about one half of a one-metre cube. We used the calculator and mutliplied using decimals. We got 0.60016. That's close to six tenths of a one-metre cube. Our estimate was good." [c] Number Concepts	• —>
■ uses capacity and volume measures to solve problems by:	■ —>
• compiling data to compare and/or order various containers or cartons	• —> Extend to include cylinders [c] Geometry
• compiling data to help solve problems in other areas [?] The dimensions of a classroom relate to each other in length, width and height as 8:5:3. What is the capacity of the room if its height is 3.6 metres. [c] Number Concepts, Ratio	• —> [?] How much oil was spilled in the **Exxon Valdez** disaster (or in the Persian Gulf during the Gulf War)? Work with your group to find ways to help visualize the amount of oil spilled. [c] Data Management

⇒

GOALS	5	6
		■ discovers and generalizes relationships including: • explaining volume in terms of the number of cubes packed in one layer multiplied by the number of layers • relating mL and centimetre cubes
M4. ***Angle Measure Develops and Extends Under-standing by Applying the Processes of Estimating and Measuring Angles*** ⇒ ⇒ ⇒	■ discovers and uses angles ■ identifies number of angles in closed shapes (internal angles) ■ names angles as being: • more than 90° • exactly 90° • less than 90° (use a right-angle finder – the inner corner of a paper folded in half; then folded in half again) ■ categorizes angles as: • right • acute • obtuse • straight	■ —> ■ —> Extend to open shapes. ■ —> ■ folds to create new angles ■ —> ■ uses strategies to estimate the size of angles • referring to a familiar referent (square corners, hour hands on an analog clock) • unitizing (square corner back to back) • chunking (90° and a little more)

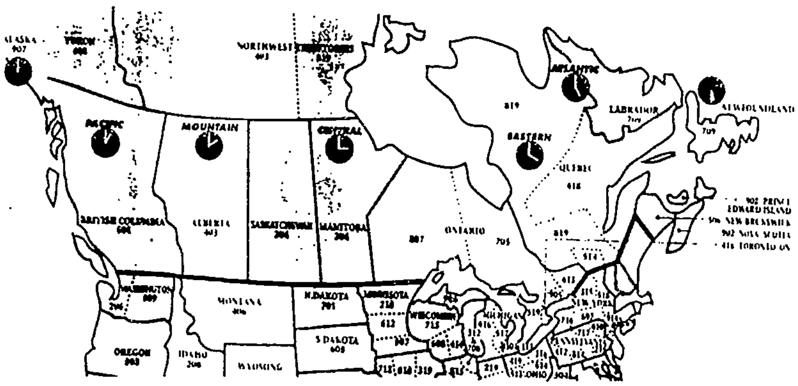
7	8
	Use illustrations or models to help your class presentation. (Extra – If you had to pay gasoline per litre prices for the spilled oil, what would it cost? What did the clean-up cost? What is the cost per litre?)
■ discovers and generalizes relationships by: • writing an expression for finding the volume of a prismatic solid • knowing volume of irregular objects is independent of its weight if the object sinks • relating mL and cm^3; L and dm^3 • relating volume and surface area of rectangular solids	■ —> • writing a formula for finding volume of a cylinder • beginning to account for volume of things which float • relating and using mL and cm^3 and g; relating L and dm^3 and kg • applying the relationship between volume and surface area [?] By the end of Grade 8, students should understand and apply the following: – The more cube-like a rectangular prism, the less the surface area, and vice versa. – As the edge of a cube grows by a factor of n, the volume of the cube grows by a factor of n^3.
■ uses angles	■ —>
Use LOGO programs and verbal LOGO commands to support the development of a dynamic perception of angle and angle measure.	
■ names angles	■ —>
■ folds to create new angles	■ —>
■ categorizes angles Extend to include reflex angles, corresponding angles, alternate interior angles (using grid paper).	■ —> Extend to include complementary, supplementary, and vertical angles.
■ uses strategies to estimate the size of angles	■ —>

GOALS	5	6
		■ discovers angle measure using various tools and logical reasoning [?] "Use the 90° corner of the square and a dot in the centre of your paper to help you find the size of every other angle in the pattern blocks." ■ demonstrates and explains the process of measuring angles with a protractor [C] Geometry
M5. ***Mass, Temperature, and Time. Applies the Processes of Measuring Mass, Temperature, and Time to Solve Problems***	Mass and time have been developed in K-4 mathematics, and are an integral part of the Science program. Temperature has been dealt with in science. It is anticipated that students will understand the important concepts and skills for these measurements. Therefore, students should be able to ***apply*** their knowledge of mass, temperature, and time in solving related problems.	

7	8	
■ uses various tools and logical reasoning to generate angles [?] "Use your hinged mirrors and the line and dot on your paper to create regular polygons. Explain how you can determine the size of each angle."	■ —>	⇒
■ demonstrates and explains the process of measuring angles with a protractor	■ —>	⇒
	■ recognizes similarity of angles in intersecting lines and perpendicular lines Students are expected to recognize similarity of angles in a combination of intersecting, perpendicular, and parallel lines	
	■ estimates and measures to determine complementary and supplementary angles	
	■ constructs angles to create known geometric shapes and designs	
■ discovers and generalizes relationships including: • relating total interior angle measurement to 3-, 4-, and 6-sided shapes • determining angle measure within regular polygons of 3, 4, and 6 sides	■ discovers and generalizes relationships for any regular polygon	⇒

Students are expected to apply knowledge of mass, temperature, and time in solving related problems.

GOALS	5	6



Source: Manitoba Telephone System, Winnipeg Directory. Winnipeg, MB, 1994.

' ɜ the accompanying map of Canada to assist in answering the following:

1. Write the names of the 10 provincial capital cities in a chart. Then give the time zone in which each city is located. If it is noon in the first city in the list, what time is it in each of the other cities?

2. It takes $3\frac{1}{2}$ hours to fly from Charlottetown to Wi^nnipeg. If Arin leaves Charlottetown at 14:45 Atlantic time what time will she arrive in Winnipeg? Give answer in terms of both Atlantic standard and central standard times.

3. At 07:30 Eric calls his friend Roger. Roger tells Eric that it is 10:30 where he lives. In which time zones could Eric and Roger be at that time?

7	8

Solve each of the following problems two different ways.

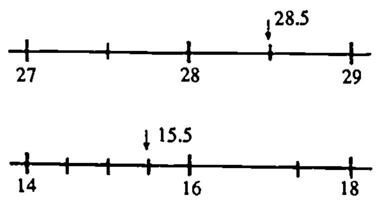
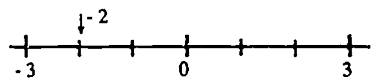
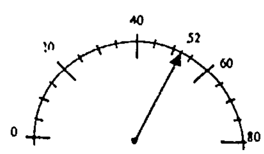
[?] 1. One litre of milk yields 35 grams of butter. How much butter would you get from 18.4 litres of milk? How many litres of milk would you need to get 1.4 kg of butter?

2. In a family of two parents and four children, the parents each consume 300 g of bread per day. Each child consumes 350 g of bread in a day. What quantity of wheat will this family need for a whole year if 130 kg of bread can be made from 100 kg of wheat?

3. A 10 cubic cm piece of iron weighs 7.5 kg. What is the mass of an iron beam that is 20 cm wide, 5 cm thick and 6 metres long?

4. A roof, consisting of two rectangles, each with sides of 8 m and 4 m, is covered with tiles. A tile weighs 200 g. The tiles are brought from a factory in a wagon which weighs 310 kg when it is empty and which weighs 790 kg when it is full of tiles. How many trips are needed to deliver all the tiles for the roof? What additional information is needed to solve this problem? Make up the required information and then solve the problem.

Number Concepts

GOALS	5	6
NC1. *Demonstrates and Extends Number Sense (Emphasis is on mental mathematics.)* ⟹	■ uses counting patterns by:	■ uses counting patterns by:
	• counting on orally and in writing from any number by 1s, 2s, ..., 9s, and multiples of 10s, 100s, 1000s up to 4 places; by tenths; and by unit fractions Count on by 5s –> 76, 81, 86, ... 200s –> 1100, 1300, 1500, ... 0.2s –> 5.6, 5.8, 6.0, 6.2, ... 1/4s –> 3 1/4, 3 2/4, 3 3/4, 4, 4 1/4, ...	• —> Extend to multiples of 10,000 up to 6 places, to hundredths and to non-unit fractions 500, 510, 520, ... 131 000, 141 000, 151 000, ... 6.14, 6.16, 6.18, ... 4 3/8, 4 5/8, 4 7/8, ...
<u>*Teacher Reference*</u> *Mental Math in the Middle Grades* (4-6). J. Hope, et al. Available from Addison-Wesley. *Mental Math in Junior High* (7-9). J. Hope, et al. Available from Addison-Wesley. *Developing Number Sense in the Middle Grades* (Part of Grades 5-8 Addenda Series). Barbara Reys, et al. NCTM, 1991. *Number Sense and Operations* (Part of Grades K-6 Addenda Series). Grace Burton, et al. NCTM, 1993. *Number Sense Now.* Directed by F. "Skip" Fennell. NCTM, 1993. [Set of 3 videotapes for teachers of K-6.] *Patterns and Functions* (Part of Grades 5-8 Addenda Series). NCTM, 1991. *Patterns* (Part of Grades K-6 Addenda Series). NCTM, 1992. *The Mental Math Collection* (Part of the Active Learning Series). Paul Lessard, et al. Exclusive Educational Products, 1994.	• counting back orally and in writing from any number by 1s, 2s, ..., 9s, and multiples of 10s, 100s, 1000s up to 4 places; by tenths; and by unit fractions Counts back by: 4s –> 29, 25, 21, ... 20s –> 196, 176, 156, ... 0.3s –> 2.9, 2.6, 2.3, ... 1/3s –> 5 1/3, 5, 4 2/3, ...	• —> Extend to multiples of 10 000, up to 6 places, to hundredths and to non-unit fractions 615, 605, 595, ... 4 750, 4 450, 4 150, ... 3.55, 3.35, 3.15, ... 12 7/10, 12 3/10, 11 9/10, ...
	• programming the calculator to count on or back Count on by 35: [C] [0] [+] [3] [5] [=] [=] [=] .... Count back by 15: [C] [1] [3] [5] [-] [1] [5] [=] [=]	• —> [C] [6] [1] [5] [-] [1] [0] [=] [=] ...
	• counting on and back with reference to time and money [?] <u>Time</u> – 0, 15, 30, 45, ... <u>Money</u> – 0, 25, 50, 75, ...	
	• reading and using counting lines and scales  NOTE: Students must deal with the difference between counting points and counting moves on a number line.	• —> Extend to include:   [C] Science

7

■ uses counting patterns by:

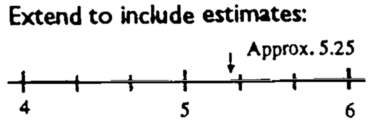
- counting on orally and in writing from any number by 1s, 2s, ..., 9s, and multiples of 10s, 100s, 1000s up to 4 places; by tenths; and by unit fractions
  Extend to multiples of 100 000s up to 7 or more places, to thousandths, and to integers
  1017, 1020, 1023, ...
  1 681 000, 1 781 000, 1 881 000, ...
  0.941, 0.946, 0.951, ...
  –13, –9, –5, ...

- counting back orally and in writing from any number by 1s, 2s, ..., 9s, and multiples of 10s, 100s, 1000s up to 4 places; by tenths; and by unit fractions
  Extend to multiples of 100 000, up to 7 or more places, to thousandths and to integers.
  1059, 1039, 1019, ...
  224 000, 216 000, 208 000, ...
  4.165, 4.115, 4.065, ...
  9 51/100, 8 31/100, 7 11/100, ...
  – 4, –7, –10, ...

- programming the calculator to count on or back
  [C] [1] [3] [+-] [+] [4] [=] [=] [=] ....

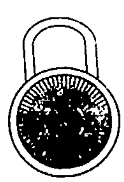
[?]

- reading and using counting lines and scales
  Extend to include estimates:



Locker combination:

[?]



8

■ uses counting patterns by:

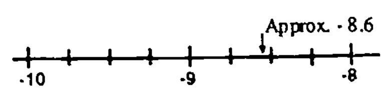
- —>
  Extend to include rationals
  2.5, 2, 1.5, ...
  17 751, 17 851, 17 951 ...
  1.5, 2, 2.5 ...
  –3.6, –3.3, –3.0 ...

- —>
  Extend to include rationals
  1 432 000, 1 332 000, 1 232 000 ....
  14.6, 14.1, 13.6 ...
  – 2.3, –2.7, –3.1 ...

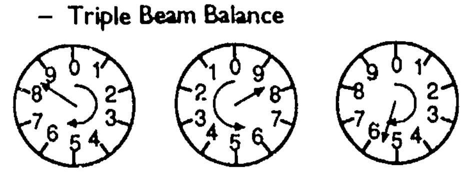
- —>
  [C] [2] [.] [3] [+-] [-] [0] [.] [4] [=] [=] [=]

[?]

- —>



[?] – Hydro Meters – Discuss need for rounding rather than exact reading
– Protractor
– Triple Beam Balance



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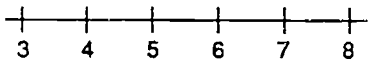
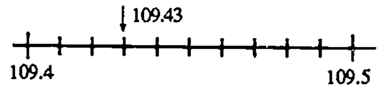
GOALS	5	6
	 There are 6 points from 3 to 8 inclusive. There are 5 moves between 3 and 8.	
⇒	■ uses place-value patterns in:	■ uses place value patterns in:
	• justifying, comparing and ordering numbers up to 4 places and tenths Why is 6 greater than 5.6?	• —> Extend to 6 places and hundredths
	• justifying rounding techniques in various estimation contexts	
	• explaining mental computation with <u>Multiples of 10, 100</u>: 10 + 40; 270 + 50; 345 + 400; 20 x 8; 7 x 300; 30 x 40 <u>Adding 9, 99</u>: 36 + 9 = 36 + 10 – 1; 54 + 99 = 54 + 100 – 1 <u>Subtraction</u>: 1000 – 276 = ▭ is 999 – 275 = ▭	• —> Extend to multiples of 1000, adding 999 970 + 80 450+999=450+1000-1 600 + 1700 2850 + 900 700 x 40 2000 x 7
	• explaining mental computation with division specialities 60 ÷ 2, 3, 6 80 ÷ 2, 4, 8 900 ÷ 3, 9 400 ÷ 10, 100	• —> Extend to larger numbers 600 ÷ 2,3,6 600 ÷ 20,30,60 800 ÷ 2,4,8 800 ÷ 20,40,80 1200 ÷ 2,3,4,6,12 1200 ÷ 20,30,.... 14000 ÷1000
	• explaining mental estimation of "midpoint," up to 4 places and tenths 14, _, 18 (16) 275, _, 175 (225) 1.3, _, 1.7 (1.5) 1400, _, 1900 (1650)	• —> Extend to 6 places and hundredths 60, __, 30 (45) 310, __, 390 (350) 2.65, __, 2.95 (2.8) 25 500, __, 30 500 (27 250)
	• explaining how to solve calculator challenges [?] Enter 725.3 and without clearing this number a) replace the 2 with 4 (add 20) b) replace the 3 with 9 (add 0.6) c) restore the original number in one step (subtract 20.6)	• explaining how to solve calculator challenges: [?] Enter 6925.74 and without clearing this number a) replace 9 with 3 (subtract 600) b) replace the 4 by 1 (subtract 0.03) c) restore the original number in one step (add 600.03)

7	8
■ uses place value patterns in:	■ —>
• justifying comparing and ordering numbers up to 4 places and tenths Extend up to and beyond 7 places and thousandths	• —> Extend to integers and rationals
• explaining mental computation Extend to mental computation with tenths and hundredths	• —> Extend to thousandths
• explaining mental computation with division specialties Extend to tenths and hundredths 60 ÷ 0.2 60 ÷ 0.05 120 ÷ 100	• —> Extend to thousandths 60 ÷ 0.002 180 ÷ 1000
• explaining mental estimation of "midpoint," up to 4 places and tenths Extend up to and beyond 7 places and thousandths 172, __, 186 (179) 1045, __, 1048 (1046.5) 5.678, __, 5.698 (5.688) 430 000, __, 415 000 (422 500)	
• explaining how to solve calculator challenges [?] Enter 96 325.841 and without clearing this number, a) change each digit which is even to half its present value (subtract 3010.42) b) replace the 9 by 5 (subtract 40 000) c) restore the original number in one step (add 43010.42)	

GOALS	5	6

5

■ develops understanding of multiples, factors, divisibility and primes

Use manipulatives and numerical explorations to demonstrate:
- odd and even numbers
- multiples and factors
- prime and composite numbers (to 25)
- divisibility by 2, 10, 5

6

■ —>

Extend primes and composites to 50. Extend divisibility to include 3, 6 and 9.

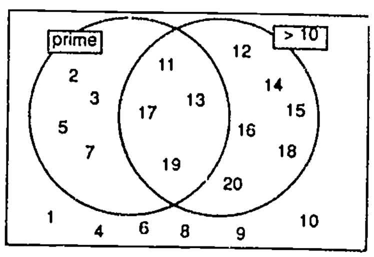
⟹

5

■ uses diagrams to sort numbers

For example, use a Venn diagram to sort the numbers from 1 to 20 by the rules shown. Describe the results.

[?]



6

■ —>

Categorize numbers in other ways.

For example, use a chart or Carroll diagram to sort the numbers from 1 to 20 using the rules shown on the diagram. Write a paragraph explaining your findings.

[?]

	composite	not composite
divisible by 5	10, 15, 20	5
not divisible by 5	4, 6, 8, 9, 12, 14, 16, 18	1, 2, 3, 4, 7, 11, 13, 17, 19

⟹

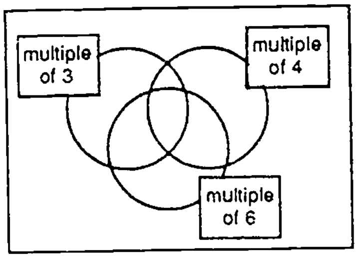
5

■ uses mathematical principles in calculation and mental arithmetic

6

■ —>

Refer to Appendix B on page 226 for a more detailed description of the principles to be used in calculation in the Middle Grades.

7	8
■ develops understanding of multiples, factors, divisibility and primes Extend primes and composites to 100.	■ —> Test larger numbers using a calculator. – discover that only smaller primes need be tried as factors – discover that when the divisor is larger than the quotient no larger factors need be tried Test 1237 to see if it is prime $1237 \div 19 = 65.105$ a) 19 is not a factor of 1237 b) $19 < 65$, so testing should continue assuming no factors of 1237 are found $1237 \div 37 = 33.43$ a) 37 is not a factor of 1237 b) $37 > 33.43$, so testing may cease c) 1237 is prime
■ writes composite numbers as products of primes $60 = 2 \times 2 \times 3 \times 5$	■ —> Uses exponents $1500 = 2 \times 2 \times 3 \times 5 \times 5 \times 5$ $= 2^2 \times 3 \times 5^3$
■ uses diagrams to sort numbers Extend to finding numbers with 3 attributes in common. [?] Sort the numbers from 1 to 50 using the rules shown. Explain why you can or can not use the results to predict all other numbers < 100 common to the rules shown.  	■ —> [?] Use a graphic organizer to illustrate the factors which 15, 24 and 42 have in common. Which common factor is greatest?
■ uses mathematical principles in calculation and mental arithmetic	■ —>

⇒

Refer to Appendix B on page 226 for a more detailed description of the principles to be used in calculation in the Middle Grades.

GOALS	5	6
	Consolidation of: – adding and subtracting 0 – multiplying and dividing by 1 – order of addition with two addends – order of multiplication with two factors	Understanding that division by 0 is not allowed $\frac{n}{0}$ and $\frac{6}{0}$
	Refer to Appendix B on page 226 for a more detailed description of the principles. Teachers are reminded that the emphasis is to be on extending students' number sense NOT on naming mathematical principles. Students are expected to use the principles as part of mental and written computation and should not be expected to name them.	
	Develop an understanding of: – adding in any order (AWW for addition) $3 + 8 + 7 \Rightarrow 10 + 8 = 18$ – multiplying in any order (AWW for multiplication) $4 \times 9 \times 5 \Rightarrow 20 \times 9 = 180$ – The annexation principle 2 x 30, 20 x 30, 200 x 30 – distributing on the right with a factor up to 100's $6x7 \Rightarrow 6x(5+2) \Rightarrow 30+12 = 42$ $\begin{array}{r} 6 \\ x7 \\ \hline 30 \\ 12 \\ \hline 42 \end{array}$ $\begin{array}{r} 4 \\ x12 \\ \hline 8 \\ 40 \\ \hline 48 \end{array}$ $\begin{array}{r} 7 \\ x124 \\ \hline 28 \\ 140 \\ 700 \\ \hline 984 \end{array}$ – distributing on the left with a factor up to 100's $9x6 \Rightarrow (5+4)x6 \Rightarrow 30+24 = 54$ $\begin{array}{r} 9 \\ x6 \\ \hline 30 \\ 24 \\ \hline 54 \end{array}$ $\begin{array}{r} 23 \\ x7 \\ \hline 21 \\ 140 \\ \hline 161 \end{array}$ $\begin{array}{r} 246 \\ x4 \\ \hline 24 \\ 160 \\ 800 \\ \hline 984 \end{array}$ These two understandings merge with multiplication under number operations.	– annexation principle with any number of zeros – distributing on the left or right with a one-digit factor and any number of digits in the other factor

7	8

Students in Grades 7 and 8 are expected to proficiently use the principles identified for Grades 5 and 6. This use should be in terms of both mental and written computation.

■ Uses formal rules for order of operations including:

- brackets

  As in most computer languages, remove brackets from inside to out

  3 x ((6 x (2 + 3)) + 4)

  3 x ((6 x 5) + 4)

  3 x (30 + 4)

  3 x 34

  102

- traditional rules
  i) multiply and divide in the order given
  ii) then add and subtract in the order given

■ —>

Extend to exponents

When calculation is to be done, exponents have higher priority than x, ÷, +, or −.

For example, in $2 + 3^4$, 3^4 must be found first in $4 \times 3 \times 2^5$, 4 x 3 may be found first, but 2^5 "hangs together" until it is evaluated

GOALS	5	6

NC2. *Extends Understanding of Place Value Systems and Number Sense for Whole Numbers and Decimals* ⟹

Teacher Reference

101 Winning Ways with Base 10 (Grades 4-6). Marion Cross. Available from Exclusive.

Building Understanding with Base 10 Blocks: Middle School. Available from Spectrum.

Solid Sense in Mathematics for Grades 4-6/7-9. Available from Spectrum.

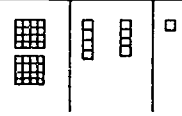
5

■ demonstrates and explains understanding of place-value systems for whole numbers by:

- representing small quantities in bases other than 10. Focus on bases 3, 4. Use concrete manipulatives such as bingo chips, blocks, or counters.

Base 4

Groups of 256	Groups of 64	Groups of 16	Groups of 4	Units



The number is read as two-two-one in base 4 or 221_4 which equals 41 blocks (base 10)

$221_4 = 41_{10}$

6

■ —>

Extend to base 2. In base 2 zeroes and ones are used to name a number.

> Emphasis is to be on using other bases to understand our base 10 system not to calculate in other bases.

5

■ explains estimations and calculations of large concrete and pictorial quantities (an appropriate unit must be adopted)

[?] Your group will be given a bucket full of bottle caps, a cup, and a balance scale. How could you make a sensible prediction for the number of caps in the bucket if you are not allowed to count? (One will give you your materials when you give me your plan.)

6

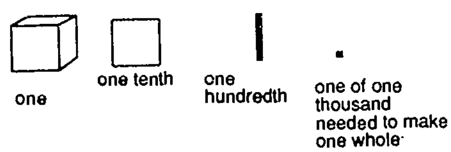
■ —>

Extend to a multistep process.

[?] Your group will be given a picture of a brick apartment block, and a 10 x 10 transparent grid. How will you go about estimating the number of bricks the contractor had to order? Write a description of your work, and include your estimate.

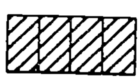
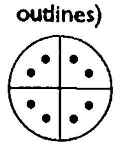
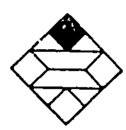
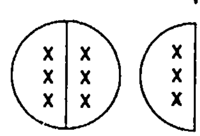
7	8
■ explains estimations and calculations of large quantities by: • conducting investigations and using place value to develop sense of large quantities [?] Let a centicube have a length of one cm. How many km will one million centicubes stretch? See how many zeros you can write in one minute. Use your calculator to find how many zeros you could write in one hour; one day, one week, if you kept up the pace. When would you reach one million zeros? Write a paragraph about your investigation.	■ —> [?] How many litres would there be in one billion ml of oil? If 200 litres fill a barrel, how many barrels can be filled from one billion ml of oil? Find how many dots a computer printer could print on one page. How many pages would you need to have one million dots? [?] Find the weight of one loonie. How many kg will one million loonies weigh? How many loonies will you need to have a metric tonne of loonies? How many loonies could you carry at one time? Tell why.

GOALS	5	6
	■ demonstrates and explains understanding of place-value for quantities less than one (tenths, hundredths, and thousandths) by: • using money dollars, dimes, cents (pennies)	■ —>
	• using measurements metre, decimetre, centimetre • using real objects which can be cut or separated Stress decimal point significance.	• using measurements metre, decimetre, centimetre, millimetre litre, millilitre • using revalued Base 10 blocks [?] If the flat represents one whole, what is the value of the other blocks in the set? Why?  ten — one — one of ten needed to make one whole — one of one hundred needed to make one whole
	• using calculators to enter numbers in expanded form	
⇒	■ demonstrates and explains relationships among place-values by: • explaining place value periods using models and diagrams • renaming numbers 3000 = 3000 ones 300 tens 30 hundreds 3 thousands	■ —> • —> Extend to charts • —> Extend to beyond 1000's and decimals [?] Find the smallest whole number with 5 digits which has the following properties: a) larger than 34 442 b) no repeating digits

7	8
■ demonstrates and explains understanding of place-value for quantities less than one (tenths, hundredths, thousandths and beyond) by:	■ —>
• using measurements including metric prefixes from milli to kilo	
• using revalued Base 10 blocks [?] If the big cube represents one whole, what is the value of the other blocks in the set? Why?  	• —>
[?] What value will we give our blocks if we want the centicube to represent one millionth? Use your models to prove. (Use cubic metre model.)	
• using calculators	• —>
■ demonstrates and explains relationships among place values by:	■ —>

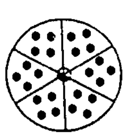
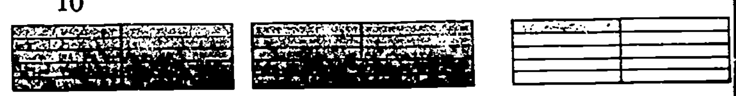
GOALS	5	6
	• using models to determine the significance of zero in naming numbers 014.2 -> The zero is not incorrect, but serves no purpose 108.2 -> As usual, the zero tells us that there are no tens 49.0 -> The zero does not change the value of the number, but in measurement it informs us that we have measured down to tenths	• —>
	• comparing and ordering numbers [?] Use the Base 10 blocks to build and order, from greatest to least: 2104, 2401, 2140, 2041 Choose someone from your group to explain your decisions to the class.	• —> [?] Use revalued Base 10 blocks to build and order, from least to greatest: 2.14, 2.01, 2.41, 2.04, 2.4, 2.1 Choose a reporter who will give the reasons for the way you have ordered these quantities.
	■ connects spoken and written symbols to manipulatives and diagrams (0.01 to 999 999 and 1/100 to 999 999) Note: Students may read 3.72 as three and 72 hundredths or as three point seven two. Both are useful. The first helps make the connection to fractions. The second can be more helpful making the connection to place value.	■ —> Extend from 0.001 and to 100 000 000, and 1/1000 to 100 000 000.
	■ identifies and names the place value of specific digits The first place to the right of the decimal point always represents tenths; etc.	■ —>
⇒	■ writes and uses numbers in expanded form. 435 062.7 = 400 000 + 30 000 + 5 000 + 60 + 2 + $\frac{7}{10}$ or 0.7)	■ —> Extend to further expansion 435 062.71 = (4 x 100 000) + (3 x 10 000) + (5 x 1000) + (0 x 100) + (6 x 10) + (2 x 1) + (7 x $\frac{1}{10}$) + (1 x $\frac{1}{100}$) [Last two terms could be written as (7 x 0.1) + (1 x 0.01)]

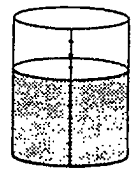
7	8
• using models to determine the significance of zero in naming numbers	
• comparing and ordering numbers [?] If a microprocessor is available, estimate the number of protozoa or red blood cells in a drop. [C] Science	
■ connects spoken and written symbols to manipulatives and diagrams (0.01 to 999 999 and 1/100 to 999 999) Extend from millionths to billion	■ —>
■ identifies and names the place value of specific digits	■ —>
■ writes and uses numbers in expanded form. Include brackets, exponents for whole numbers, and connect fraction and decimal notations for parts less than one.	■ —> Introduce negative integers to indicate decimal quantities. $62.718 = 6 \times 10^{1} + 2 \times 10^{0} + 7 \times 10^{-1} + 1 \times 10^{-2} + 8 \times 10^{-3}$

GOALS	5	6
	Students should also reverse the process. In other words, given a number express it in expanded form and vice versa.	
NC3. *Extends Understanding of Fractions as Used to Refer to* *– regions* *– parts of groups* *– measures* *– ratios* *– division* *and Relates Fractions to Decimals (Emphasis on halves, thirds, fourths, fifths, sixths, eighths, tenths, and hundredths)*	■ demonstrates and explains the meaning of proper fractions by: • identifying fractions in the environment • counting with fractions • using manipulatives and diagrams to represent: *fractions of a single object* (use Fraction Factory pieces; fraction circles; squares or bars)  "The black whole is covered by four equal purple pieces. Each is one fourth of the whole black." What other names for the black whole can be made from your fraction set? *parts of a group* (use various collections and fraction circle outlines)  Two marbles are one fourth of eight. Six marbles are three-fourths of eight.	■ —> Extend to improper fractions and mixed numbers • —> • —> *fractions of a region* (use Pattern Blocks)  "If the shaded area has a value of 1/3, what is the value of the whole design. Make a design with a value of eleven thirds." *parts of a group* 1 1/2 sets of 6 is 9 

7	8
435 062.798 = $(4 \times 10^5) + (3 \times 10^4) + (5 \times 10^3) +$ $(0 \times 10^2) + (6 \times 10^1) + (2 \times 1) +$ $(7 \times \frac{1}{10}) + (9 \times \frac{1}{100}) +$ $(8 \times \frac{1}{1000})$ or Last three terms could be written as $(7 \times 0.1) + (9 \times 0.01) + (8 \times 0.001)$ $(7 \times \frac{1}{10^1}) + (9 \times \frac{1}{10^2}) + (8 \times \frac{1}{10^3})$ write larger numbers using powers of ten 3 million = 3×10^6 or 30×10^5	

Students should also reverse the process. In other words, given a number express it in expanded form and vice versa.

7	8
	■ writes numbers using scientific notation 395 000 000 = 3.95×10^8 0.00000042 = 4.2×10^{-7} It is conventional to show one digit in front of the decimal point.
■ demonstrates and explains the meaning of proper fractions by: • identifying fractions in the environment • using manipulatives and diagrams to represent: ***fractions of a region*** $\frac{4}{10}$  ***fractions of a group*** What is 2/6 of 25?  "Eight and two sixths bingo chips are two sixths of our set of 25." Take a small handful of chips. Use all your fraction circles to find and record fractional quantities of your set.	■ —> Extend to rational numbers • identifying in the environment or in created contexts • —> ***fractions of a region*** $2\frac{1}{10}$  ***fractions of a group*** Use examples from probability experiments, statistical analysis, graphs. Half of the class scored between 60 and 70 per cent on the quiz.

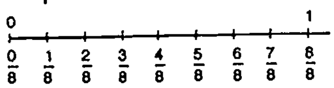
GOALS	5	6
Teacher Reference *Cooperative Problem Solving with Fraction Pieces* (Grades 4-6). Ann Roper. Available from Addison-Wesley. *The Fraction Factory Binder* (Grades 4-7). Linda Holden. Available from Addison-Wesley. *Fraction Pieces* (Part of Points of Departure Series). Available from Spectrum.		***fractions as measures***  "Six tenths of the cylinder will exactly fill the soup can with water. Find how much water every other can holds."

7	8

7

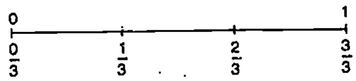
fractions as measures

Mark number lines and use them to compare fractions

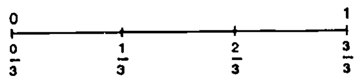


Partition and relabel number lines

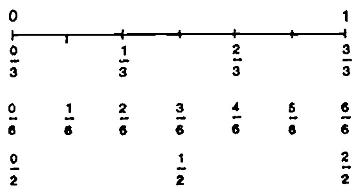
Mark in thirds



Partition in each part

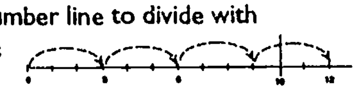


Relabel the line



fractions as division

Use a number line to divide with fractions



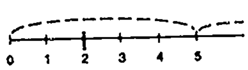
$10 \div 3$

= 3 and one-third of a step

$= 3\frac{1}{3}$

Then note that $\frac{10}{3} = 3\frac{1}{3}$

$2 \div 5$



$= \frac{2}{5}$ of a step

So, $\frac{2}{5}$ may be thought of as a fraction or as $2 \div 5$.

- using a calculator to explore decimal equivalents of fractions

  [c] [1] [÷] [8] [=]

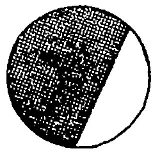
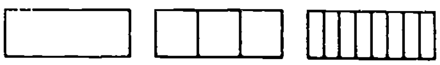
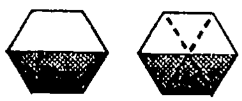
  The display shows [0.125]

8

fractions as measures

fractions as division

Become familiar with decimal equivalents of quarters, fifths, eighths, tenths, and twenty-fifths.

GOALS	5	6
	• reading and writing symbols for proper fractions represented by manipulatives and diagrams. The terms numerator and denominator are used. "five out of six – each part is $\frac{1}{6}$"	• —> Extend to improper fractions and mixed numbers.
	• estimating fractional quantities of regions  "about one half"	• —>  "about three fourths"
	■ explains the meaning of equivalent fractions by:	■ —>
	• using manipulatives and paper folding to generate equivalent proper and whole number fractions  "one whole is equivalent to three thirds, eight eighths, ..." "one half is equivalent to three sixths" 	• —> Extend to improper fractions and mixed numbers.  If Brown is one, then Orange is equivalent to five of the fourths (Red), ten of the eights (White)... Connect to equivalency in metric measurements 1 cm = 10 mm
	• connecting spoken and written symbols to models and pictures of equivalent fractions $\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8} = \frac{5}{10} = \frac{6}{12}$ $\frac{4}{12} = \frac{2}{6} = \frac{1}{3}$	• —> $\frac{2}{3} = \frac{4}{6} = \frac{8}{12}$ $\frac{9}{12} = \frac{6}{8} = \frac{3}{4}$

7	8

7

Notice some non-terminating decimals, such as

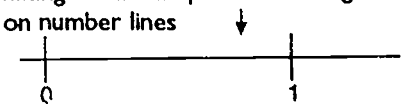
$\frac{1}{3}, \frac{2}{3}, \frac{1}{6}, \frac{5}{6}, \frac{1}{7}, \ldots$

Round to thousandths

<u>fractions as a ratio</u>

[C] Our class is $\frac{3}{4}$ as large as theirs. (For each 3 students we have, they have 4.) See the discussion of ratio and proportion later in this strand (NC4), and also the problem solving section of this guide.

- estimating fractional quantities of regions and on number lines



"about four fifths"

■ explains the meaning of equivalent fractions by:

- using manipulatives and paper folding to generate equivalent proper and improper fractions

At top, eleven-fifths is equivalent to two and one-fifth. Below, eleven-fifths is equivalent to one and six-fifths, twenty-two tenths, etc.

- renaming mixed numerals flexibly

$\frac{11}{5} = 1\frac{6}{5} = 2\frac{1}{5}$

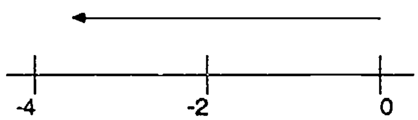
$3\frac{1}{4} = 2\frac{5}{4} = 1\frac{9}{4} = 1\frac{13}{4}$

8

Become familiar with rounded decimal equivalents for thirds, sixths, and ninths.

- —>

Extend to include negative fractions on number lines



"about negative three and a half"

■ —>

- justifying the formal rule for finding equivalent fractions

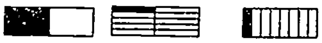
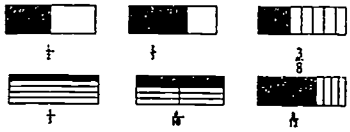
$\frac{3}{4} = \frac{3}{4} \times \_\_ = \frac{15}{20}$

$\frac{15}{20} = \frac{3}{4} \times \_\_ = \frac{3}{4}$

- using the formal rule in mixed numbers

$4\frac{6}{8} = 4\frac{3}{4}$ $\qquad$ $3\frac{5}{10} = 3\frac{1}{2}$

- —>

GOALS	5	6
	■ explains comparing and ordering fractions by: • using manipulatives to compare and arrange <u>unit</u> fractions in order from greatest to least, or vice versa. $\frac{1}{2}$ $\frac{1}{10}$ $\frac{1}{8}$  Put these fraction pieces in order from least to greatest. Explain your work.	■ —> • —> Extend to non-unit and improper fractions, and mixed numbers  Order these fractions from greatest to least. Explain your work.

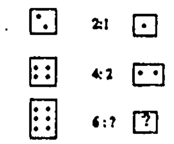
Manipulatives are to be used in testing at all grade levels.

GOALS	5	6
	• using estimation to compare fractions $\frac{1}{3}$ $\frac{1}{5}$ $\frac{1}{3}$ is closer in size to $\frac{1}{2}$ than to zero. $\frac{1}{5}$ is closer in size to zero than to $\frac{1}{2}$. Therefore, $\frac{1}{3}$ is a larger part or fraction than $\frac{1}{5}$. $\frac{1}{3} > \frac{1}{5}$ or $\frac{1}{5} < \frac{1}{3}$	• —> $\frac{1}{3}$ $\frac{3}{5}$ $\frac{1}{3}$ is less than $\frac{1}{2}$ $\frac{3}{5}$ is a little more than $\frac{1}{2}$ so $\frac{3}{5}$ is greater than $\frac{1}{3}$ $\frac{1}{5} < \frac{3}{5}$ or $\frac{3}{5} > \frac{1}{3}$
	• connecting words and symbols to models and diagrams of ordered <u>unit</u> fractions	• —>

7	8
■ explains comparing and ordering fractions by: • using manipulatives to represent, compare and order fractions like $\frac{1}{2}, \frac{5}{8}, \frac{2}{5}, \frac{4}{3}$	■ —> • —> Extend to rationals

The continuing use of manipulatives is recommended and encouraged.

7	8
• using estimation to compare fractions $\frac{1}{2}, \frac{5}{8}, \frac{2}{5}, \frac{4}{3}$ $\frac{2}{5}$ is less than $\frac{1}{2}$, $\frac{5}{8}$ is a little more than $\frac{1}{2}$ and $\frac{4}{3}$ is even more than one whole. The order is $\frac{2}{5}, \frac{1}{2}, \frac{5}{8}$ and $\frac{4}{3}$ from smallest to largest.	
• using the calculator and converting fractions to decimals	• —> Extend to paper/pencil calculations to verify and understand calculator results

GOALS	5	6
	■ relates fractions to decimals (and vice versa) by: • connecting words, symbols, manipulatives and diagrams when tenths and hundredths are used $\frac{35}{100} = 0.35 \quad 0.9 = \frac{90}{100}$	■ —> Extend to numbers easily related to 100 • —> $\frac{3}{4} = \frac{75}{100} = 0.75$
NC4. *Develops and Extends Understanding of Ratio, Rate, Proportion and Percent, and the Relationships Between Them*	The treatment of ratio here parallels closely the discussion of ratio in the problem-solving section of the guide. ■ demonstrates and explains the meaning of ratio by:	 ■ —>
	Note: Stress the idea that ratio is a rule (not a number) that controls constructing or thinking about two quantities at the same time. Care must be taken to distinguish between a ratio, which is a rule, and the numbers of the quantities produced by it. 2:1 2 in the left bin (hand) goes with 1 in the right bin (hand).	
	• using diagrams to represent spoken and written ratios You catch two fish for every three fish I catch me ☐ 3:2 ☐ you Find several possible total catches. Three cans of pop cost $2. cans ☐ 3:2 ☐ dollars Then enter either a number of cans or a number of dollars and solve the problem • using alternate language 2:5 "2 is to 5" "2 for every 5" "2 goes with 5"	• —> Extend to larger ratios, but restrict problems to those calling for whole numbers [6] 4:2 [?] The ratio of paperbacks to hardcover books read is 4:2. Now there are 6 paperbacks. How many hardcover books should there be? Number of Paperbacks / Number of Hardcovers  • —> Write the ratio for the number of squares to all figures.  • exploring other methods of generating quantities. These are explorations that will deepen understanding of ratio but are not regarded as objectives in the development of the ratio continuum.

7	8
■ relates fractions to decimals to per cent and vice versa	■ —>
• connecting words, symbols, manipulatives and diagrams	• —>
• using estimation and the calculator to express fractions as decimals and read as a per cent	• —>

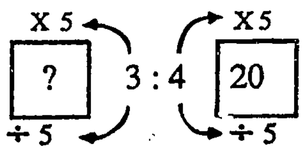
The treatment of ratio here parallels closely the discussion of ratio in the problem-solving section of the guide.

■ demonstrates and explains the meaning of ratio by:	■ —>

Stress the idea that ratio is a rule (not a number) that controls constructing or thinking about two quantities at the same time. Care must be taken to distinguish between a ratio, which is a rule, and the numbers of the quantities produced by it.

7

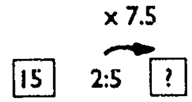
- generating quantities using a ratio (whole number quotients):
  pictorially/patterns
  Given a ratio and one of the quantities, use division and multiplication to determine the other quantity.



- using alternate language
  2:5
  "2 is to 5," "2 for every 5,"
  "2 goes with 5"
- exploring other methods of generating quantities.

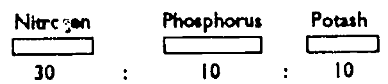
8

- generating quantities using a ratio (positive rational quotients):



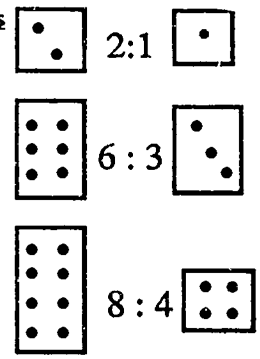
15 ÷ 2 = 7.5

- —>
- generating quantities using a three term ratio
  1 is to 2 is to 3 is written 1:2:3



GOALS	5	6
		Adding bins (see below); relating ratios to fractions

Adding bins



- relating ratios to fractions

white flowers	2:3	red flowers
2		3
4		6
6		9
8		12
⋮		⋮

Show that $\frac{2}{5}$ of the flowers are white and $\frac{3}{5}$ of the flowers are red.

Emphasize Ratio is a rule. In more advanced mathematics such rules are called functions.

7	8
■ demonstrates and explains understanding of proportion by:	■ —>
• using a ratio to generate proportions To this point, the ratio rule which generates quantities has been explicit (the rule is written down). Using the quantities produced by the ratio, we can write equivalent fractions. (The rule now becomes implicit.) For the ratio 5:3, the quantities generated are: 5 3 10 6 15 9 20 12 ⋮ ⋮ For similarily placed quantities, any of the following 4 pairs of equivalent fractions can be written. $\frac{10}{15} = \frac{6}{9}$ or $\frac{15}{10} = \frac{9}{6}$ or $\frac{10}{6} = \frac{15}{9}$ or $\frac{6}{10} = \frac{9}{15}$ Any such statement is called a proportion.	• —> Extend to multiple term ratios 4:3:1 = 8:6:2

⟹

Note that while this is the precise meaning of the word "proportion," the word is used less precisely in common speech. It is sometimes used as a synonym for "ratio" and sometimes with no exact meaning.

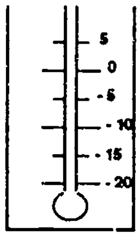
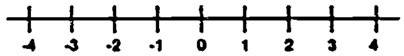
7	8
	• using calculators to solve more extensive problems with whole and positive rational quotients. scientific problems statistical analysis

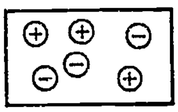
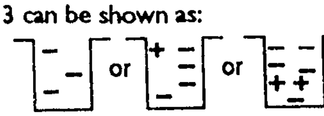
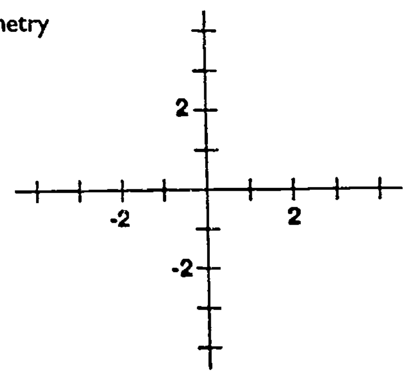
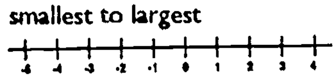
GOALS	5	6

7	8
• using the word "rate" to refer to ratios Ratios are commonly referred to as rates, particularly when the right hand number in the ratio is 1. For example, a rate of 20 km per hour means km ☐ 20:1 ☐ hours	
• renaming ratios or rates Use manipulatives to rename ratios or rates For example, ☐ 4:6 ☐ produces the same quantities as ☐ 2:3 ☐ Ratios can therefore be renamed in the same way as fractions.	• renaming and using ratios that are given using fractions For example $2\frac{1}{2} : 6$ 5:12 Susan can skate 4 times around the rink in $2\frac{1}{2}$ minutes [followed by a number of times around or a time] around ☐ $4:2\frac{1}{2}$ ☐ time ☐ 8:5 ☐
• using proportions to solve problems Without explicitly identifying ratios, use proportion to solve problems relating to mixing paint adding anti-freeze to water recipes scale drawings map scales comparing body parts	• —> Extend to currency exchange problems Begin with a table of values below a ratio frame but ***without identifying any ratio***. Cost in Canadian dollars ☐ : ☐ Cost in U.S. dollars $1 $0.70 Then enter the number of Canadian or U.S. dollars and complete the proportion. For example ☐ : ☐ $1 $0.70 ? $14 $\frac{1}{0.7} = \frac{?}{14}$ For more difficult problems, use a calculator.

GOALS	5	6
		■ demonstrates and explains the meaning of percent by: • using manipulatives to do conversions from % to ratio to fraction and vice versa Stress that percent can be thought of in two ways: as a ratio – 25% means 25:100 as a number – 25% means $\frac{25}{100}$, $\frac{1}{4}$ or 0.25

7	8
■ demonstrates and explains the meaning of percent by: • using manipulatives to do conversions from % to ratio to fraction and vice versa Stress that percent can be thought of in two ways: as a ratio – 25% means 25:100 as a number – 25% means $\frac{25}{100}$, $\frac{1}{4}$ or 0.25	■ —> ⇒
• recognizing and calculating percent in real world examples such as discounts, sales tax, media usage, etc. The school population is 450 students. The principal announces that 6% of the students were home sick. a) How many students are home sick? b) How many students are still at school? <u>Solution A</u> 6% as a ratio $\frac{6}{100} = \frac{x}{450}$ a) x = 27 b) 450-27 = 423 <u>Solution B</u> 6% as a fraction 0.06 x 450 = x or $\frac{6}{100}$ x 450 = x	• —> Extend to include examples involving compound interest versus simple interest commission, investments/savings, etc.
■ finds the percent of a number without and with a calculator 20% of $30 Emphasize that 20% is the same as $\frac{20}{100}$, $\frac{1}{5}$ or 0.2 Use real-life contexts	■ —> Extend to include increasing or decreasing a quantity by a given percent Increase $200 by 5% Decrease 150 by 20%
■ expresses one number as a percent of another $15 is what percent of $60 Use real-life contexts. Use estimation and verify thinking with a calculator.	■ —> ■ uses proportions to solve problems such as $15 is 25% of what number ⇒
■ renames percents as ratios, decimals, and vice versa $\frac{18}{20} = \frac{90}{100} = 0.9 = 90\%$	■ —>

GOALS	5	6
NC5. *Develops and Extends Understanding of Integers* [C] Number Operations	■ explains meaning of integers (opposite numbers) by: • using real-world examples such as: – thermometers (above zero or below zero) – scoring in games (golf – under par or over par) – east-west • using models or diagrams thermometer scale 	■ explains meaning of integers (extending counting numbers to less than zero) by: • using more real-world examples such as: – distance below/above sea level – riding elevators above/below main floor – programming the calculator to count forward/backward from zero on – timelines • using models or diagrams horizontal and vertical number line 
	■ connects spoken and written symbols of integers to representations (stress "negative" rather than "minus") by: • using thermometer scale	■ —> • using horizontal number line

7	8
■ explains meaning of integers (directed numbers) by:	■ —>
• using more real-world examples such as: – tides – profits and losses – BC and AD – time zones	• —>
• using models or diagrams – number lines – calculator – use bingo chip model and concept of net change (one color for positive, another colour for negative. An equal number of positive and negative chips results in zero change.) 	• —> The charges-on-a-jar model (two colours of chips and real or pictured jars to hold a "collection of charges". The ***charge on the jar*** is the integer which represents the collection of charges within it. Every integer can be represented in an infinite number of ways.) For example, 3 can be shown as: 
• using analogies (Postman brings or removes cheques [positive quantities] or bills [negative quantities]) ("Good guys" and "bad guys" and the effect of their arrival or departure on a town.)	• drawing coordinate pictures using four quadrant "Battleships" [C] Geometry 
■ connects spoken and written symbols of integers to representations, and vice versa by: • using number lines • using diagrams of models • using calculator	■ —>
■ continues and creates integer patterns	■ —>
■ explains relationship between or among integers by: • using the number line compare –5 and –3 (–5<–3) order –5, 1, –3, –1, 3 (–5, –3, –1, 1, 3) smallest to largest 	

Number Operations

Overview

Over the past few decades, there has been a major shift in the focus of middle years mathematics:

- since hand calculators and computers are now common, there is less need than was once the case for high-speed arithmetic using rote calculation, particularly where large numbers are involved

- there is an increased emphasis on problem solving, of both the routine kind and the kind which calls for using problem-solving strategies

- the time made available by the decreased emphasis on rote calculation makes it possible to give greater attention to those understandings which contribute to mastery of more complex arithmetic, algebra, data management, and geometry

This guide reflects the shift in focus. This does not mean that paper-and-pencil arithmetic will ever be obsolete. It does mean that teachers must continually ask what purposes paper-and-pencil arithmetic serves and how it should be taught so it serves those purposes. Broadly, there are three purposes achieved through paper-and-pencil arithmetic:

1. before leaving middle years, students should be sufficiently fluent in the use of algorithms to manage their ordinary day-to-day needs without a calculator. For example, they should be able to find 123 + 72 + 31, 12.65 – 7.81, 13 x 3.75, and 157 + 13 by applying paper-and-pencil arithmetic

2. students should see most of their algorithms as being linked to the templates of routine problem solving. For example $5 - 2\frac{1}{4}$ should not be just "something you do" but should be linked to solving any routine problem in which $2\frac{1}{4}$ objects are removed from a collection of 5 of them

3. most students should understand many of their algorithms in ways that prepare them for later algebra. For example, they should have functional understandings that addition and multiplication are commutative (4 + 73 = 73 + 4 and 113 x 9 = 9 x 113), associative (6 + 7 + 13 and in 9 x 14 x 10 it does not matter which pair of numbers we add or multiply first) and that they are related by the distributive property (in 14 x 9, teachers could begin by having students find 6 x 9 and 8 x 9 or 4 x 9 and 10 x 9)

Facts and Algorithms

At present, and for the foreseeable future, teachers will decide how to interpret this major shift in focus in the face of community standards and the needs of later curricula. Their decisions will relate to three facets of number operations:

1. most students will continue to require automatic responses for at least all of the basic addition, subtraction, and multiplication facts. Understandings are important, but the complete mastery of these facts as automatic responses will continue to be important

2. it is sufficient that some algorithms be worked step-by-step – thought out if necessary as the algorithm is completed. For example, it is now sufficient for a student to be able to find the total in a question, similar to the one illustrated at right, by thinking, "8 + 5 is 13; 13 + 9 is (3 + 9 is 12, so it's 22); 22 + 6 is (2 + 6 = 8 so it's 28) and so on

38
25
19
46

   Not many decades ago, and for good reason, students were expected to be able to find the answers to questions similar to the one above in more efficient ways. But those days are gone. Few students will now add such columns of digits often enough for it to be worth the considerable extra time it takes to equip them with a high-speed rote algorithm for multi-column addition.

3. some algorithms ought to be streamlined. Students should be able to perform them without deliberation. For example, even the simplest multiplication and division algorithms crumble if students are unable to find 264 + 1036 and 3000 – 275 without "thinking things out"

Prescribing now or in the future, where community standards and the needs of later curricula will allow the line to be drawn between step-by-step and streamlined performances of algorithms is not possible.

The determining factors are not all mathematical. Hopefully, any testing programs which are implemented will reflect contemporary objectives, but teachers may occasionally have to bend mathematical criteria. It may take some years for local non-professionals and shapers-of-opinion to adjust to the priority of contemporary social needs over traditional expectations.

How Curriculum is Changing

In the face of all of the shifts described above, it is reasonable to ask how this curriculum guide differs from earlier ones. Overall, the changed objectives of the middle yeaars mathematics curriculum affect number operations in six ways:

1. the size of the numbers on which students should be expected to perform paper-and-pencil algorithms has been decreased. For example, there is now little need to carry the algorithm for multiplication beyond 4 digits x 1 digit or 3 digits x 2 digits.

2. arithmetic is increasingly seen as being related to problem solving. As a problem is solved or an algorithm is performed, the relationship between them is emphasized.

3. more than in the past, arithmetic is to be seen as a vehicle for promoting mathematical understandings.

4. multiple methods using manipulatives, diagrams, calculators, varied formats, and a variety of algorithms are recommended. All of these methods can promote understandings and problem-solving skills.

5. particularly because of the gradual shift to metric units of measure, the shift to introducing and using decimals at earlier grade levels is continued.

6. fraction concepts are introduced early but formal operations with fractions are introduced in the seventh and eighth grades. Fraction algorithms now have little social use, but they provide a comparatively concrete arena in which to learn many of the same procedures that will be important in algebra. Formal work with fractions, therefore, is brought as close as possible to beginning algebra.

7. the addition and subtraction of integers is introduced in the seventh grade and multiplication and division of integers in the eighth grade, again to facilitate beginning algebra.

GOALS	*5*	*6*

NO1. ***Addition – Identifies, Understands, and Solves Problem Situations Involving the Addition Process*** ⟹
 	
Whole Numbers
 	
<u>Teacher Reference</u>
Estimate! Calculate! Evaluate! (Calculator Activities for the Middle Grades). Marjorie Bloom and Grace Galton. Cuisenaire: New York, 1990.
Fifth Grade Book: Addenda Series. Grace Burton, et al. NCTM: Reston, 1991.
Number Concepts and Operations in the Middle Grades. James Hiebert and Merlyn Behr, Editors. NCTM: Reston, 1988.
Sixth Grade Book: Addenda Series. Grace Burton, et al. NCTM: Reston, 1992.
Active Learning Series. Individual binders available from Exclusive on:
– Complete Book of Cube-a-Link (5-8)
– 200 Things To Do with Logic Blocks (K-8)
– Active Overheads (4-6)
– 101 Winning Ways with Base Ten (4-6)
– Pattern Blocks (6-8)
– Geoboard Collection (4-6 or 7-9)
– Exploring Geo-Blocks (4-11) ⟹
– Problem Solving: What To Do When You Don't Know What To Do (5-8)
– Kids 'N Calculators (1-8)
Connections: Linking Manipulatives to Mathematics. Linda Charles and M. Brummett. Addison-Wesley: Toronto, 1989. (Separate books for each of grades 1-8.)
	Maintain a balance between concepts and operations. This strand is best developed in the context of problem solving rather than as an isolated unit.
	
	■ explains and verifies the process of addition using 4, 5 and 6 digit whole numbers.
	Students should become proficient with basic addition facts, adding with 10s, 100s, and 1000s, rounding, using associativity (for example, 39 + 26 = 40 + 25) and partitioning using place value (for example, 213 + 124 = 200 + 100 + 13 + 24). Students should be able to explain what they are doing, often by linking addition to problems.
	<u>Recommended techniques:</u>
	• creating story and application problems
	• estimating answers
	• using mental arithmetic
	• using manipulatives and diagrams
	• varying formats (vertical and horizontal)
	• using "free-style" non-traditional approaches such as
	  2375
	+ 3927
	  5000
	  1200
	    90
	    12
	  6302
	• estimating and then using a calculator
	
	■ generalizes for whole number addition
	– only digits in like places can be combined
	– more than nine in any place must be renamed
	– adding zero does not change the number
		■ —>
		Extend whole number addition. Emphasize place value rather than computation. Examples should be from real-life and could be selected from Science or Social Studies.
		
		Examples should involve multiple steps and multiple operations.
		[?] Paul has gone shopping for his mother. In his basket there are: 125 g of sausage, 1250 g of beef, 3 containers of yoghurt at 150 g each, 2 tins of vegetables at 450 g each. How much does the empty basket weigh if Paul is carrying a total of 3320 g?
		
		[C] Number Concepts, Measurement
		
		■ —>

7	8

Maintain a balance between concepts and operations. Many calculations should arise from problems that reflect real-life situations.

■ explains and verifies the process of addition through problem solving	■ —>

The bulk of ongoing addition should come from meaningful problem solving and investigations. Examples could include:
- Compiling measurements
- Compiling data from experiments involving probability
- Managing data

Some less practical explorations can also be of interest. For example, if numbers are attached to the letters of the alphabet (for example, A = 1, B = 2, C = 3, ...)
- find the "value" of names
- find words with given values

Examples:

1. Use the digits 1, 2, 3,4, 5, 6, 7, 8 and 9 exactly once to complete a magic square in which every column, row and one diagonal equals the same sum. Then use the same digits exactly once to complete a square where the sum of each row, column, and diagonal gives a different sum.

2. Replace the stars with digits such that the additions are correct.

```
 27**
 **75
 *823
 ----
 7654
```

3. Replace each letter with a digit so that the addition is correct.

```
   N A           U S A
 +   A      +  U S S R
   ---      ----------
   A N       P E A C E
```

GOALS	5	6
Decimals ⟹	■ explains and verifies the process of addition using tenths and hundredths.	■ —> Extend to thousandths in any context.

5

■ explains and verifies the process of addition using tenths and hundredths.

As with whole numbers, students should be able to explain what they are doing, often by linking the addition of decimals to simple problems.

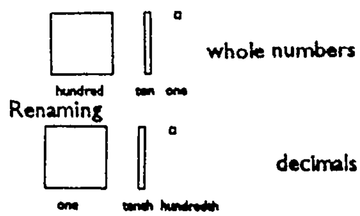
The ability to round, often to the nearest whole number, is central.

<u>Recommended techniques:</u>

- creating story or application problems using money, athletic events, and measurement contexts
- using manipulatives and diagrams

Similarities of whole number place value and decimal place value should be stressed for all operations.

For example:
Base 10 materials



Adding 47 + 56 is the same as adding 4.7 + 5.6. The decimal point simply defines the units column.

- using money and linear measurement examples involving metres, decimetres and centimetres
- using vertical and horizontal format
- using non-standard formats

	ones	tenths
	1	4
+	1	7
	2	11
	3	1

$$\begin{array}{r} 1.4 \\ +\ 1.7 \\ \hline 2.0 \\ \underline{1.1} \\ 3.1 \end{array}$$

- using calculators, often rounding and estimating before using a calculator

6

■ —>

Extend to thousandths in any context.

Example:

Louis and Susan spent all day Saturday shopping in the local store. Susan bought a pair of blue jeans for $32.60 and a shirt for $17.95. Louis had $23.50 to spend and only bought a shirt for $14.25. Finish the story and ask some mathematically interesting questions.

7	8
■ explains and verifies the process of addition using tenths, hundredths, and thousandths Extend indefinitely	■ —>

The following example is intended simply to show one way that operations with decimals can be presented.

Example:

The Fabulous Treehouse Restaurant is a favourite place for students.

Ken's Bill

1 snack	2.75
1 soup	2.00
1 salad	1.50
1 coke	1.25
	$7.30
PST	0.52
GST	0.52
	$8.34

Mark's Bill

2 slice	4.00
1 soup	2.00
1 cheese	0.25
	$6.25
PST	0.44
GST	0.44
	$7.03

Jennifer's Bill

1 salad	1.50
1 snack	2.75
1 juice	0.70
	$4.95
PST	0.35
GST	0.35
	$5.65

1. Check these bills and correct any mistakes.
2. Mark paid the bill for all three. He handed the cashier a twenty dollar bill and a five dollar bill. Is $25.00 enough to cover the meals for all three and, if so, what is his change?

<table>
<tr><th>GOALS</th><th>5</th><th>6</th></tr>
<tr><td>

Fractions
(Emphasis to be placed on halves, thirds, fourths, fifths, sixths, eighths, tenths and hundredths). Emphasis should be placed in groups of fractions, i.e.,

(a) $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}$

(b) $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{6}, \frac{1}{12}$

(c) $\frac{1}{2}, \frac{1}{5}, \frac{1}{10}, \frac{1}{100}$

</td><td colspan="2">

The formal arithmetic of fractions is not required in Grades 5 and 6, but fraction <u>concepts</u> should be built. Using teacher-made or commercial manipulatives, the following four objectives should be attained:

a) informal comparison

For example, $\frac{1}{3} > \frac{1}{4}$ because it covers more of a disc or a bar.

b) Informal equivalence

For example, $\frac{3}{6} = \frac{1}{2}$ because they exactly cover one another.

c) The names of 1

That is, $\frac{2}{2}, \frac{3}{3}, \frac{4}{4}, \ldots$ are different ways of talking about a whole object.

d) Informal calculation

For example, we can find $\frac{1}{6} + \frac{2}{6}$ by combining fractional pieces. Informal solutions to questions like $\frac{5}{6} - \frac{1}{6}$, $2 \times \frac{3}{8}$, and $\frac{6}{8} \div \frac{2}{8}$ can be found at the same time.

See the Fractions part of Number Concepts for the variety of meanings fractions may be given.

</td></tr>
</table>

7	8
■ explains and verifies the process of addition for proper fractions.	■ —> Extend to improper fractions and mixed numbers

7

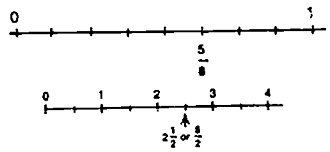
■ explains and verifies the process of addition for proper fractions.

Shift from informal to formal methods of adding fractions. Two models are recommended:

a) The ***cut model***, in which fractions are represented as parts of a whole.

$\frac{3}{4}$

b) The ***measure model***, in which fractions are represented by points along a line.



Students should continually explain what they are doing, often by relating it to a problem.

Recommended techniques:

- creating story and application problems
- estimating

For example

$\frac{1}{2} + \frac{3}{5} = ?$

$\frac{3}{5}$ is greater than $\frac{1}{2}$ so $\frac{1}{2} + \frac{3}{5}$

must be a little larger than 1.

8

■ —>

Extend to improper fractions and mixed numbers

GOALS	5	6

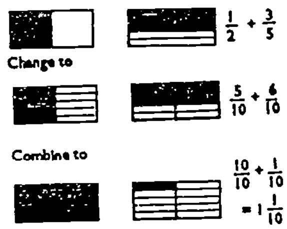
7	8

7

- using the informal concepts from Grade 6 to begin the shift to a formal algorithm

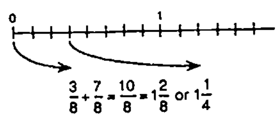
  For example,

  $\frac{1}{2} + \frac{3}{5}$



- using the measure model, which is slightly more abstract.

  For example, $\frac{3}{8} + \frac{7}{8}$



- using equivalence and the names of 1 to organize work in a systematic way

  $\frac{1}{2}$ becomes $\frac{5}{10}$

  $\frac{3}{5}$ becomes $\frac{6}{10}$

  $\frac{11}{10} = \frac{10}{10} + \frac{1}{10} = 1\frac{1}{10}$

- using both horizontal and vertical formats

  Fractions can have meanings beyond the "cut" and "measure" meanings. Of the others, the most important is the "indicated division" meaning.

  For example, $\frac{3}{5}$ implies $3 \div 5$

  It is possible to attempt to develop this meaning out of other understandings, but most teachers who have tried have found it to be too lengthy. It is generally best to teach it by probable induction – compare fractions where decimal equivalents are known with calculator outputs.

8

- after the rule for multiplying fractions is available,

  $\frac{1}{2} \times 1 = \frac{5}{10}$

  $\frac{3}{5} \times 1 = \frac{6}{10}$

  $\frac{11}{10} = 1\frac{1}{10}$

GOALS	5	6

7	8
Hand: $\frac{4}{5} = \frac{8}{10} = 0.8$ — Calculator: $4 \div 5 = 0.8$ Hand: $\frac{3}{4} = \frac{75}{100} = 0.75$ — Calculator: $3 \div 4 = 0.75$ • using calculators Calculators can manage questions like $\frac{3}{4} + \frac{5}{8}$ in several different ways. Depending on the calculators available, students should learn to use as many of them as possible. Note that this guide will use [MC] or [MRC] to indicate "clear memory" [C] to "clear registers" on screen ***Option 1*** – Use memory – most calculators allow us to add $\frac{3}{4}$ and $\frac{5}{8}$ as follows: [MC] [C] [3] [÷] [4] [=] [0.75] [M+] [5] [÷] [8] [=] [0.625] [M+] [MR] This display shows [1.375] ***Option 2*** –Using brackets. Some calculators allow: [C] [(] [3] [÷] [4] [)] [+] [(] [5] [÷] [8] [)] [=] The display shows [1.375] ***Option 3*** – Using order of operations. A few calculators observe the traditional rule of completing multiplication and division before adding and subtracting. On them, it is possible to enter [C] [3] [÷] [4] [+] [5] [÷] [8] [=] Note, however, that some calculators observe different conventions. If one is available it is a challenging and productive problem to figure out how it operates.	

GOALS	5	6

7

It helps to build understandings to teach students to convert mixed decimal quotients to whole number quotients with whole number remainders.

For example, beginning with 131 ÷ 8

[C] [1] [3] [1] [÷] [8] [=]

The display shows **16.375**

This means 16 eighths and 0.375 of 8. The 0.375 of 8 can be converted to a whole number.

[C] [•] [3] [7] [5] [×] [8] [=]

The display shows **3**

This means that

$131 \div 8 = 16\frac{3}{8}$ or 16R3

It is also a productive exercise to convert given decimal numbers to fractions. For example, 0.625 can be multiplied by a variety of trial factors until a whole number product is found. In this case, $8 \times 0.625 = 5$. Then $0.625 = \frac{5}{8}$

The lowest possible factor need not be found. For example,

$32 \times 0.625 = 20$. Then $0.625 = \frac{20}{32}$,

which can then be renamed.

8

- using calculators with mixed fractions
  Students learn to express mixed fractions as mixed decimals

  $4\frac{2}{3}$

  [MRC] [4] [M+]
  [2] [÷] [3] [M+] [MR]

  The display shows **4.6666666**

  Alternatively, students could add the whole numbers (in mixed numbers) separately from the fractions.

  $4\frac{2}{3} + 2\frac{5}{6}$

  [MRC] [C] [4] [+] [2] [M+]
  [2] [÷] [3] [M+]
  [5] [÷] [6] [M+] [MR]

  The display shows **7.4999999**

  The sum is $7\frac{1}{2}$

GOALS	5	6
Integers	■ discusses incidental real-life siutations involving negative numbers: For example: The temperature this morning was -20°C. Now it is 3°C. By how much did the temperature change? [C] Measurement	■ —> Death Valley in California is 85 m below sea level. How far from sea level is a helicopter hovering 68 m above the floor of the valley? [C] Social Studies

The prime objective is that students conceptualize that like integers on opposite sides of zero are opposites.
- positive 4 and negative 4 are opposites
- negative 6 and positive 6 are opposites
- 0 is its own opposite
- the sum of opposites is zero

7	8

Models for Integers

Integers can be modelled in several ways, some more concrete and some more abstract.

The key idea throughout is that:

AN INTEGER AND ITS OPPOSITE COMBINE TO MAKE 0.

For example, 4 + (-4) = 0 and -3 + 3 = 0

The "–" sign that is part of the name of a negative integer can be raised, or written in front of the digit, as in –7.

Concrete models include:

charges model

4 can be shown as

++ ++	or	–++ +++	or	+++ – +++ – +++ –

Note that -/+ pairs cancel.

chip model – Using bi-coloured chips, say black and red,

4 can be shown as

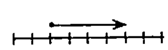
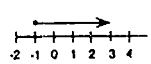
B B B B	or	B B B B B R

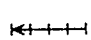
–4 can be shown as

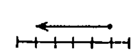
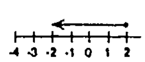
R R R R	or	R R R R R B

vector model

4 can be shown as anywhere on a number line

For example,  or 

–4 can be shown as  anywhere on a number line

For example  or 

GOALS	5	6

7

■ explains and verifies the process for computing sums involving integers.

Recommended techniques:

- using and creating story problems and applications

  Write a story using +4, −6, and +4 in that order.
- estimating answers
- using the charge, chip and vector models
- using both horizontal and vertical format
- using a calculator. Most calculators will not accept [−] [4] as an entry. The user must enter [4] [+/-]

  -4 + 3 is found as [C] [4] [+/-] [+] [3] [=]

  The display shows [− 1]
- introducing the pattern found below as it will be useful later

3 + 3	= 6
3 + 2	= 5
3 + 1	= 4
3 + 0	= 3
3 + (−1)	= 2
3 + (−2)	= 1
⋮	⋮
3 + (−6)	= −3

8

■ —>

Extend recommended techniques to include:

- using associativity and the zero property for addition to more formally justify the addition of integers

a) 6 + -4
   = (2 + 4) + -4
   = 2 + (4 + -4) = 2 + 0 = 2

b) 3 + -4
   = 3 + (-3 + -1)
   = (3 + -3) + -1 = 0 + -1 = -1

■ generalizes, in words, what happens when:

- a positive is added to a positive
- a positive is added to a negative
- a negative is added to a negative
- two negatives are added

GOALS	5	6
NO2. ***Subtraction – Identifies, Understands and Solves Problems Involving Subtraction***	Students must become proficient with all of the subtraction facts. Note, however, that there are two common subtraction algorithms (see Appendix A to this section of the guide), our traditional "borrow-take-away" algorithm and the "open addition" algorithm. The facts should be learned in a way that matches the algorithm to be used. For the former, facts are learned as 12 take away/minus 7 equals [5] . For the latter, facts are learned as 7 plus [5] equals 12 or [5] plus 7 equals 12. When subtraction is brought to problem solving and later mathematics, it should be understood in four ways. Students should become familiar with all four: – removing part of a set or group He has 53 hockey cards and gives 12 to his younger sister. – finding what must be added Her objective is to have 16 guests at the party. So far she has invited 11. – comparing two sets or groups Our class has 32 students. Their class has 23. – subtraction is related to addition 56 – [ ] = 27 ↔ 27 + [ ] = 56 or ↔ [ ] + 27 = 56	
Whole Numbers ⟹	■ explains and verifies the process for several meanings of subtraction involving up to 6 digit whole numbers. Recommended techniques: • creating story and application problems • encouraging mental arithmetic and estimation • suggesting alternative methods for mental subtraction For example 5926 – 2373 a) 5926 – 2000 = 3926 3926 – 300 = 3626 3626 – 70 = 3556 3556 – 3 = 3553 b) begin with 2373 Add 7 2380 20 2400 600 3000 2000 5000 926 5926 3553 Encourage "short cuts." [C] Number Concepts	■ —> Extend to 7, 8, and 9 digit numbers as applications require. Emphasize estimation rather than calculation. Use a calculator for most purposes. Maintain in the context of problem solving using multiple operations. [C] Problem Solving

7	8

Ongoing practice and explanations should come out of meaningful problem-solving activities and investigations:

- finding the differences between estimates and actual measurements of length, perimeter, area, mass, or volume
- finding "bank balances" when newspaper or catalogue shopping
- number investigations, such as investigating the numbers which give a difference of 6174, etc.
- finding differences in populations, land areas, distance in light years, etc.

7	8
■ explains and verifies the process for several meanings of subtraction involving up to 9 digit whole numbers. Extend to billions when application requires	■ —>

GOALS	5	6
	c) Begin with 2373, but accumulate differently 2373 + 3000 = 5373 5373 + 500 = 5873 5873 + 30 = 5903 5903 + 23 = 5926 3553 • using manipulatives and diagrams • using horizontal and vertical format • using a calculator, commonly first estimating to verify input	
⟹	■ generalizes for whole number subtraction – subtracting a zero does not change anything – subtraction and addition are inverse operations – addition can be used to solve subtraction problems	■ —>

Most computations should be connected to "real life" problems. Increasingly, calculators should be used. Continual reinforcement of basic facts via "mental math" helps to build both efficiency and self-confidence.

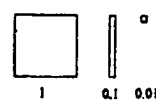
GOALS	5	6
	[C] Problem Solving	

7	8

Computations must arise from "real life" problems. Increasingly, calculators should be used. Students require continual reinforcement of basic facts by using "mental math."

You are offered the choice of the following ways to be paid for your new job at a fast food restaurant. Use a calculator to help you determine which way you would choose to be paid.

1. The first option is $5 a day. How much would you make in 25 years (calculate 365 days a year)? ____________
2. The second option is $3 an hour, 8 hours a day. How much would you make in 1 year (365 days)? ____________
3. The third option is $1000 for the first six months and a $200 raise after six months, and a $200 raise every following six months? How much would you make in 5 years? ____________
4. The fourth option is $1000 the first year, $2000 the second year, $3000 the third year, $5000 the fourth year, $8000 the fifth year and $13 000 the sixth year. How much would your salary be the tenth year? ________ the fifteenth year? ________
5. The fifth option is that you will receive one cent the first week, two cents the second week, four cents the third week, eight cents the fourth week, sixteen cents the fifth week. Each week your salary doubles. How much will you receive the tenth week? ________ the fifteenth week? ________ the twentieth week? ________ the twenty-fifth week ________ the twenty-eighth week? ________
6. Which salary would you prefer? ____________
   Why?
   ____
   ____
   ____

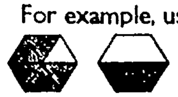
<table>
<tr><th>GOALS</th><th>5</th><th>6</th></tr>
<tr><td>Decimals</td><td>■ explains and verifies the process for several meanings of subtraction involving tenths and hundredths.
<u>Recommended techniques:</u>
• creating story and application problems (particularly involving money and measurement metres, decimetres, and centimetres)
• using estimation and mental arithmetic
• using manipulatives and diagrams
Base 10 blocks can be renamed as follows


• using horizontal and vertical format
• continuing the use of calculators</td><td>■ —>

Extend to use thousands in any context including milimetres

[C] Problem Solving, Measurement</td></tr>
<tr><td>Fractions (Emphasis to be placed on halves, thirds, fourths, fifths, sixths, eighths, tenths and hundredths). Emphasis should be placed in groups of fractions, i.e.,
(a) $\frac{1}{2}, \frac{1}{4}, \frac{1}{8}$
(b) $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{6}, \frac{1}{12}$
(c) $\frac{1}{2}, \frac{1}{5}, \frac{1}{10}, \frac{1}{100}$</td><td colspan="2">Note as with addition that the emphasis in fifth and sixth grades is on building fraction concepts, not on formal algorithms.

Using concrete materials and diagrams, attach subtraction to
a) informal comparisons
b) informal equivalence
c) the names of 1
d) informal calculation

For example, use pattern blocks to explore the difference between $\frac{5}{6}$ and $\frac{1}{2}$,
 and, having decided that $\frac{1}{2} > \frac{1}{3}$, explore configurations that may show that the difference between them is.</td></tr>
</table>

7

- explains and verifies the process for several meanings of subtraction involving tenths, hundredths, and thousandths.

  Extend to include ten thousandths in any context.

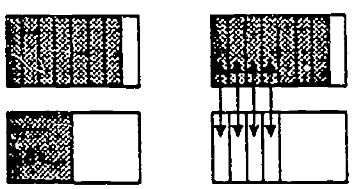
- explains and verifies the process for several meanings of subtraction involving proper fractions.

  <u>Recommended techniques:</u>

  - using a concrete model or a diagram

  <u>regions model</u>

  $\frac{7}{8} - \frac{1}{2}$ How much greater is $\frac{7}{8}$ than $\frac{1}{2}$?

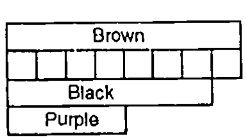


  $\frac{7}{8}$ is $\frac{3}{8}$ greater than $\frac{1}{2}$

  <u>coloured rods</u>

  a unit rod is white
  a purple rod is 4
  a black rod is 7
  and a brown rod is 8

  Then $\frac{7}{8} - \frac{1}{2}$ can be illustrated as



  Black - Purple
  $\rightarrow \frac{7}{8} - \frac{1}{2} = \frac{3}{8}$

8

- —>

  Maintenance should be in the context of problem solving preferably with multiple operations.

- —>

  Extend to improper fractions and mixed numerals. Place greater emphasis on number lines.

GOALS	5	6

7	8
On a ***number line***.	
• managing the algorithm in a more formal way $\frac{7}{8} \rightarrow \frac{7}{8}$ $\frac{1}{2} \rightarrow -\frac{4}{8}$ $\frac{3}{8}$	After the rule for multiplying fractions has been taught, it can be used here. $\frac{7}{8} \times \frac{2}{2} = \frac{14}{16}$ $-\frac{1}{2} \times \frac{8}{8} = -\frac{8}{16}$ $\frac{6}{16} = \frac{3}{8}$
• using a calculator [C] [7] [÷] [8] [=] [M+] [1] [÷] [2] [=] [M-] [MR] [0.375] Students can explore renaming 0.375 as a fraction as is suggested under addition of fractions.	
• creating story and application problems • approximating and estimating For example How much greater is $\frac{7}{8}$ than $\frac{1}{2}$? Since $\frac{7}{8}$ is "almost" 1, the difference will be a little less than $\frac{1}{2}$.	

GOALS	5	6
Integers		For Grade 7 teachers: Focus on a consequence of an ambiguity in our notation. Use "–" as a symbol for an operation (6 – 2 –> six take away 2) and also as part of the name for a number (–3 is the opposite of 3). The best strategy is to tell the truth; our notation is ambiguous, and the reader must sometimes decide which way to read it. For example, in 7 – –3, the first "–" must be an operation and the second "–" must be part of the label for a number. 6 – 4 could actually be read either way. It could be read as 6 + (understood) – 4 or as 6 take away 4. – ☐ is ambiguous. If it came from "4 – ☐" it would mean "take away." But it could also mean "take the opposite of whatever someone puts in the frame." Note: The above explanation is best understood if it follows the subtraction of integers in Grade 7.

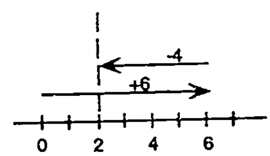
7	8
■ explains and verifies the process for computing differences involving integers. Refer to information under Grade 6 on previous page.	■ —>

The subtraction of integers has always been problematic. No one way of teaching this operation with integers is completely satisfactory. The methods that build on earlier meanings and intuitions generally break down in more complex cases, and the more formal methods generally omit some intuitions and meanings that remain important.

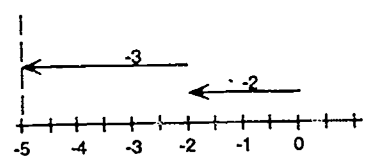
It is, therefore, best to use both methods, culminating in the formal rule shown in c) below.

a) Using earlier ***intuitions and meanings*** including:

i) vectors
best with questions like +6 – 4



or -2 – 3



ii) patterns

7 – 3 = 4
7 – 2 = 5
7 – 1 = 6
7 – 0 = 7
7 – -1 = 8
7 – -2 = 9
7 – -3 = 10

b) Using ***inverse operations***

Since 6 – 2 = 4 can be rewritten as 4 + 2 = 6
6 – -3 = [ ] can be rewritten as [ ] + -3 = 6
The missing number can be found by trial, ***adding*** integers.

c) Using a ***formal rule***

TO SUBTRACT AN INTEGER, ADD ITS OPPOSITE

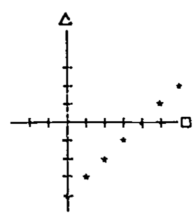
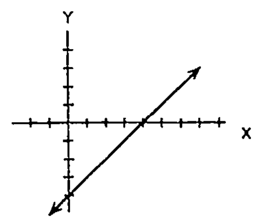
First test the rule

6 - 2 = [ ] –> 6 + -2 = [ ]

Then use the rule freely

5 – -4 –> 5 + 4
-2 – 6 –> -2 + -6
-3 – -6 –> -3 + 6

GOALS	5	6

7	8
Recommended techniques: • using real-life examples, stressing a) and b) on the previous page • estimating answers • using inverse operations and trial as in b) on the previous page	• using real-life situations that focus on both less formal methods of a) and b) above and the formal rule A golfer may say that she is at 5 under par, but then discover that she entered "two under par" incorrectly on his score card. Her true score is -5 – -2 –> -5 + 2 = -3 This solution "makes sense." Her recorded score was "two too low." It should have been -3. • practicing using the formal rule
• graphing open sentences using integers  Conclude that the points for □ – Δ = 4 are lined up.	• —> Introduce some fractions as part of solutions.  Begin to call □ – Δ = 4 (or x - y = 4) the ***equation for the line.***
• thinking a question through as a dilemma -7 – 3 = ? A move of 3 must end at -4 or -10. Which makes sense? Test speculations using the inverse operation.	• using the associative principle to formally subtract integers 7 + -3 –> (4 + 3) + -3 = 4 + (3 + -3) = 4 + 0 = 4 and then, among many examples -7 + (-3) –> -7 + (3 + -3) – 3 = -7 + (-3 + 3) – 3 = (-7 + -3) + (3 – 3) = -10 + 0 = -10 Test using the formal rule.

GOALS	5	6

NO3. *Multiplication – Identifies, Understands, and Solves Problems Involving the Multiplication Process*

Students require some basic skills and understandings including:

- Mastery of the multiplication facts to 10 x 10
- Rounding (for example, 93 ≈ 100 for estimation purposes)
- Using multiples of 10, 100, and 1000 both doing and undoing; 100 x 23 = 2300 and 160 = 10 x 16
- Reassociating, particularly with tens, as in

  3 x 70 = 3 x 7 x 10 = 21 x 10 = 210 and

  40 x 70 = 4 x 7 x 100 = 28 x 100 = 2800

  This use of factors of ten is sometimes called the annexation principle.

  Using the distributive property, at first in examples like

  12 x 7 –> 10 x 7 + 2 x 7 and

  4 1/2 x 8 –> 4 x 8 + 1/2 x 8

  and later in an algorithm

```
  23
  x7
  21 ← 7 x 3
 140 ← 7 x 20
 161
```

Nevertheless, the main emphasis remains in linking multiplication to problem solving. So far as is possible, the above skills and understandings should be developed in the context of suitable explorations and problems.

Whole Numbers ⟹

Grade 5:

■ explains and verifies the process for several meanings of multiplication involving whole numbers up to 3 digits by 1 digit and 2 digits by 2 digits.

The techniques recommended below are effective and entail only one difficulty. Using factors of ten and the annexation principle, a question like 231 x 7 can be managed as

```
  231
  x 7
    7 ←7 x 1
  210 ←7 x 30
 1400 ←7 x 200
```

But a question like 34 x 26 poses a problem. It can be thought of in steps like this

```
  26
 x34
  24 ← 4 x 6
  80 ← 4 x 20
 180 ← 30 x 6
 600 ← 30 x 20
 884
```

Grade 6:

■ —>

Extend to 3 digits by 2 digits. Use a calculator for large number applications.

Example

Replace the stars with digits. How many solutions are possible?

```
   2✡
 x ✡6
  1✡✡
  ✡5
  ✡✡0
```

7	8

Ongoing practice and explantions should come out of problem-solving activities and investigations:
- finding the weight of 100 000 loonies if the weight of 1 loonie can be determined
- finding one's age in terms of months; days; hours; minutes
- estimating part of a length; then predicting total length
- doubling/tripling a recipe
- determining cost of various quantities of supplies for class party
- determining two consecutive page numbers if their product is known

7	8
■ explains and verifies the process for several meanings of multiplication involving whole numbers up to 4 digits by 1 digit, and 3 digits by 2 digits.	■ —>

Maintenance should be in the context of problem-solving situations involving multiple operations.

GOALS	5	6

Many students have difficulty keeping track of what is happening. It is probably best to first have students learn to accumulate their working for one-digit multipliers into one row

```
 26
 x 4
 ---
104
```

and then use the annexation principle to manage two-digit multipliers

```
 26
x34
---
104 <-  4 x 26
780 <-  30 x 26
```

Recommended techniques:

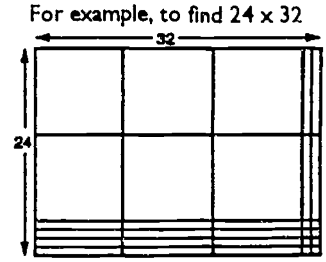
- creating story and application problems
- estimating and use mental arithmetic
- using the distributive principle for mental arithmetic

  For example

  $$7 \times 32 = (7 \times 30) + (7 \times 2) = 210 + 14 = 224$$

- using manipulatives and diagrams

  For example, to find 24 x 32



32 x 24 = 768

- using non-traditional procedures

  For example

```
          13   and   13      13
          x6         x6      x6
6x6 -->   36         18      26
6x7 -->   42         60      26
          78         78      26
                             78
```

Try the "double distributive" procedure

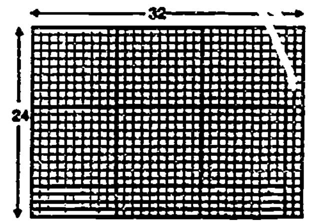
```
  32            32
x 24    or    x 24
 600           128
  40           320
 120           320
   8           768
 768
```

7	8

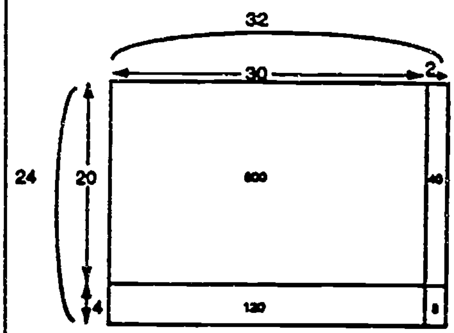
GOALS	5	6

5

- extending manipulatives and diagrams
  area model (using grid paper)



area model as a general diagram
24 x 32

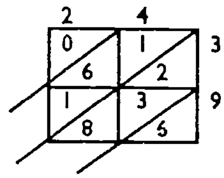


$$\begin{array}{r} 600 \\ 120 \\ 40 \\ \underline{8} \\ 768 \end{array}$$

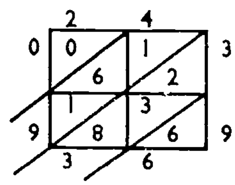
6

Some students may enjoy exploring the lattice or grid method for multiplication.

To find 24 x 39, prepare a grid, writing the products as shown



Then begin at the lower right adding down the diagonals, carrying as necessary



24 x 39 = 936

Try it with 3-digit numerals.

Ancient Egyptian Multiplication
or peasant multiplication. It was used by peasants until 1900.
The rule is to double and halve in two columns as shown below. When the number to be halved is odd, subtract 1 from it and place a tick beside the other number. End by adding the ticked numbers

$$\begin{array}{r} 25 \times 37 \\ 12 \times 74 \\ 6 \times 148 \\ 3 \times 296 \\ \underline{1 \times 592} \\ 925 \end{array}$$

Test that the method gives correct answers, and then try to explain why it works. (The distributive principle is involved!)

Find alternative methods.
For example

$$99 \times 43 = 100 \times 43 - 43 = \begin{array}{r} 4300 \\ \underline{-\ 43} \\ 4257 \end{array}$$

7	8

Computation should ordinarily arise from real-life problems. Increasingly, calculators should be used. Nevertheless, the basic facts and a paper-and-pencil algorithm should be periodically reviewed and used. Mental arithmetic is helpful in maintaining the facts.

Rainfall Example
In Morden, it was reported that 18 mm of rain fell overnight. How many litres of water would have fallen on a square metre area during this rainfall? A local store has a flat rectangular roof 20.3 metres by 24.8 metres and all the water from the rainfall is caught in barrels. If each barrel holds approximately 200 litres of water, how many barrels would be needed to catch all the water from the roof of the local store?

GOALS	5	6
	Using Calculators Students are to use sensible, rounded numbers and to estimate products before using the calculator. Build up this "mental math" skill.	
	■ rounds numbers, estimates, and then uses a calculator to perform multiplications.	■ generalizes for whole number multiplication Any number multiplied – by 10 has a zero in the one's place – by 100 has zeros in the one's and ten's places – by 1000 has zeros in the last three places Multiplying by 0 gives 0 Multiplying by 1 does not change a number
Decimals	■ explains and verifies the process for several meanings of multiplication involving decimals with tenths and hundredths multiplied by a whole number. Emphasize explaining what is being done and continually link the arithmetic to money and linear measurement. <u>Recommended techniques:</u> • using estimation and mental arithmetic For example: 5 x \$1.89 is approximately 5 x \$2	■ —> Extend to decimal numbers with tenths and hundredths multiplied by a decimal number with tenths or vice versa in any context. For questions like 0.4 x 0.14, some students can follow the demonstration $\frac{4}{10} \times \frac{14}{100} = \frac{4 \times 14}{1000} = \frac{56}{1000} = .056$ But it is difficult to carry that kind of demonstration very far. Most teachers (and students) prefer to revert to the traditional, "Ignore the decimal points and multiply. Then count up the number of decimal places in the factors and put that many in the product." If this is done, it is essential to devote sufficient time to establish that the rule is reasonable. For example, 0.6 x 7 Rule: 6 x 7 = 42 -> 4.2 Verify on Calculator: $\frac{6}{10} \times 7 = \frac{42}{10} = 4\frac{2}{10} = 4.2$ 1.3 x 0.6 13 x 6 = 78 -> 0.78 By estimating: 1.3 x .6 ≈ 1 x 1 ≈ 1

7	8

Using Calculators
Students are to use sensible, rounded numbers and to estimate products before using the calculator. Build up this "mental math" skill.

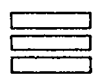
7	8
■ rounds numbers, estimates, and then uses a calculator to perform multiplications.	■ —>
■ explains and verifies the process for several meanings of multiplication involving decimals with tenths and hundredths multiplied by a whole number Extend to decimal numbers multiplied by decimal numbers where no whole numbers are involved Review the basic concepts from Grade 6.	■ —> Emphasis on problem solving. Students should use a calculator. Rounding and estimation skills should be in place. Your grandfather wants to know the cost of an imperial gallon of gasoline if he is currently paying 49.5¢ per litre. You check a reference book in the library and find that one gallon (imp.) is equivalent to 4.6 L. Based on this information, what did you tell your grandfather? (Give him all the details.)

GOALS	5	6

5

- creating story and application problems
- estimating answers
  using the distributive principle for
- mental arithmetic
  For example
  3 x $1.76
  = 3 x $1 + 3 x 70¢ + 3 x 6¢
  = $3 $+ $2.10 + 18¢
  = $5.28

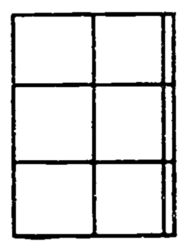
using manipulatives and diagrams
Let Base 10 flat be 1
Then long is 0.1
3 x 0.1 =



= 0.3

3 x 2.1

3 x 2.1 =



= 6.3

6

- multiplying on a calculator
  Rounding and estimation skills should be developed in conjunction with using the calculator for multiplying.

- —>

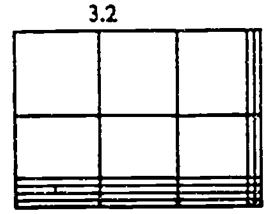
Have students revalue the Base 10 blocks. For example, let the "flat" equal one. Then the "long" equals one tenth, and so on. Students should investigate the equal sets model and the area model as ways to show multiplication involving decimals. 3.5 x 4.1 (Let one flat or 1 square on grid paper = 1.)

Concrete/Pictoral Base 10 Materials
Let Flat = 1
Long = 0.1
Unit = 0.01

3.2 x 2.4

3.2

2.4



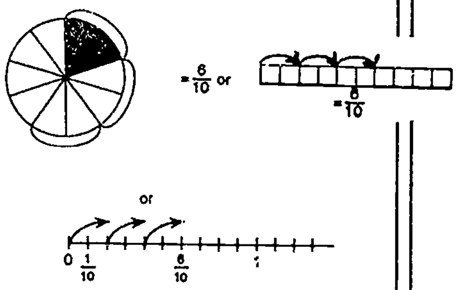
6 flats	⇒	7 flats
16 longs		6 longs
8 units		8 units

3.2 x 2.4 = 7.68

7	8

In Grades 7 and 8, students are expected to round numbers and estimate answers and then to use a calculator for multiplying decimals. However, for questions involving up to 4 digits x 1 digit or 3 digits x 2 digits, they should be able to find answers using pencil-and-paper.

7	8
• multiplyng on a calculator Note that beyond some point many calculators switch to exponential notation. For example, a student who explores questions like .00006 x .00006 may get 3.6 - 09 or 3.6 E-9 as an answer. At this level, it is sufficient to state that the "– 09" means that the decimal point has been moved 9 places to the right. The answer is 0.0000000036 Use a calculator to verify the annexation principle (avoid factors with a terminal "5" at first) Students should demonstrate the ability to find the products of any two numbers involving decimals and, in many simple situations, show that the answer makes sense. Refer to recommended techniques in Grades 5 and 6.	• —>

GOALS	5	6
Fractions ***(Emphasis to be placed on halves, thirds, fourths, fifths, sixths, eighths, tenths, and hundredths.)***		Emphasize halves, quarters, thirds, fifths, sixths, eighths, twelfths, and hundredths. The formal multiplication of fractions is not required in Grade 6, but emphasis should be placed on the four basic concepts: a) informal comparison b) informal equivalence c) the names of 1 d) informal calculation now in the context of multiplication. For example, using cuts or the measure model $3 \times \frac{2}{10}$ can be explored as: 

7	8

7

■ explains and verifies the process for a whole number times a proper fraction and vice versa.

Include proper use of the word "of" in multiplying.

"Of" comes from natural language and "x" is a mathematical operation. There can, therefore, be no formal way to prove that one goes with the other. In fact, "of" often does not mean "times."

For example, There are 7 students in the club. (Three of them are girls.)

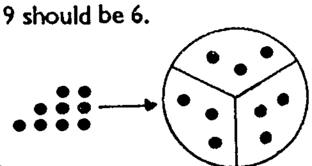
The best pedagogy is to develop both as suggested below, and then make the linkage between them seem reasonable.

Recommended techniques:

- using estimation

To find $\frac{2}{3}$ of 9, estimate $\frac{1}{3}$ of 9 is 3,

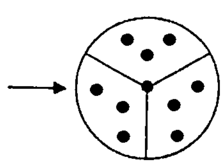
so $\frac{2}{3}$ of 9 should be 6.



Find $\frac{2}{3}$ of 10

Estimation – since $\frac{2}{3}$ of 9 is 6,

$\frac{2}{3}$ of 10 should be a little larger than 6



8

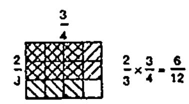
■ —>

Extend to multiplication of fractions by fractions and fractions by mixed numbers.

Attempts have been made for at least 50 years to find concrete models that lead to the algorithm.

$$\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$$

None of them work very well. Probably the most commonly recommended ends up with a diagram like this:



but even that model soon becomes difficult.

The best pedagogy is to introduce the rule and then make it seem reasonable.

– by estimation

$$\frac{3}{4} \times \frac{2}{3} \approx 1 \times \frac{1}{2} \approx \frac{1}{2}$$

– by rule

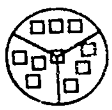
$$\frac{3}{4} \times \frac{2}{3} = \frac{6}{12} = \frac{1}{2}$$

GOALS	5	6

7	8

7

- using manipulatives and diagrams



Partition 10 into 3 equal parts. 1 is shown at the centre of the "region."

$\frac{1}{3}$ of 10 is $3\frac{1}{3}$

$\frac{2}{3}$ of 10 is $3\frac{1}{3} + 3\frac{1}{3} = 6\frac{2}{3}$

Blending "of" and "x"

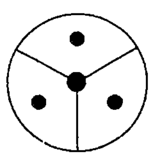
$5 \times \frac{2}{3}$ or $\frac{2}{3}$ of 5

Experience with this model soon leads to the generalization

$a \times \frac{b}{c} \rightarrow \frac{a \times b}{c}$

- using symbols in horizontal and vertical formats

Find $\frac{2}{3}$ of 5 and $5 \times \frac{2}{3}$



$5 \times \frac{2}{3} = \frac{10}{3}$

$= 3\frac{1}{3}$

Each sector has $1\frac{2}{3}$

So $\frac{2}{3}$ of 5 is $2\frac{4}{3} = 3\frac{1}{3}$

8

Once the rule can be used, note that some questions can be explored using the distributive principle

$75 \times 3\frac{4}{5} = (75 \times 3) + 75 \times \frac{4}{5}$

$= 225 + (75 \times \frac{4}{5})$

$= 225 + \frac{300}{5}$

$= 225 + 60 = 285$

Further explorations can be of this form

$6 \times 2\frac{3}{4} = (6 \times 2) + (6 \times \frac{3}{4})$

$= 12 + (6 \div 4) \times 3$, etc.

- using calculators

Questions like $\frac{3}{4} \times \frac{1}{2}$ can be managed on some calculators as:

[C] [3] ÷ [4] [x] [1] [÷] [2] =

Calculators with brackets can do this

[C] [(] [3] [÷] [4] [)] [x] [(] [1] [÷] [2] [)] [=]

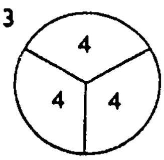
If a calculator has a memory, questions like $2\frac{1}{2} \times 3\frac{5}{6}$ can first be estimated and then managed this way.

[C] [1] [÷] [2] [=] [+] [2] [M+]

[5] [÷] [6] [=] [+] [3] [=] [x] [MR] [=]

GOALS	5	6
Integers		

7	8
• using a calculator $\frac{2}{3}$ of 10 or $\frac{2}{3} \times 10$ [C] [2] [÷] [3] [x] [1] [0] [=] The display shows [6.66666] = $6\frac{2}{3}$ 14 is what fraction of 42? [C] [1] [4] [÷] [4] [2] [=] The display shows [0.33333] = $\frac{1}{3}$ Where fraction equivalents of decimals are not known, they can be found by exploration as indicated earlier.	■ generalizes for all problems involving fractions in the multiplication process
	■ explains and verifies the process for computing products when integers are involved As with the subtraction of integers, this topic falls into two parts: a) Those things which can be built on intuitions b) Those situations in which it is best to introduce a formal rule. For part a) the recommended techniques include: • creating story problems For example On three plays, the football team lost 4 yards each time $3 \times -4 = -12$ Susan took $8 from her account each weekday $5 \times -8 = -40$ • thinking of multiplication as cumulative addition For example $6 \times -2 = -2 + -2 + -2 + -2 + -2 + -2 = -12$ For part b) consider the following techniques: Many attempts have been made to make $-2 \times -3 = 6$ seem reasonable. The following is probably the most commonly recommanded.

GOALS	5	6
NO4. ***Division – Identifies, Understands, and Solves Problems Involving the Division Process***	■ recognizes and uses division as: the removal of equal parts For example, 12 + 3 This view of division is known as ***measurement division*** or as ***quotitive division*** • equal partitioning For example, 12 + 3  12 + 3 = 4 This view of division is known as ***partitive division*** • open multiplication 12 + 3 = □ ↔ □ x 3 = 12	■ —> Ongoing practice should be related to problem solving and investigations. Examples could include: – determine the sizes of equal shares of a collection – particularly in measurement, finding how many equal sized parts can be removed from a larger (longer/more massive) object – dividing where the two basic meanings may be blurred For example: – finding the mean of data (distances, times, volumes, or spreads) – finding unit prices Students may enjoy puzzles and problems which are not related to practical application.

7	8
	• using a pattern First, learn that 2 x -3 = -6 and -2 x 3 = -6 Then, -2 x (3 + -3) = -2 x 0 = 0 Then, (-2 x 3) + (-2 x -3) = 0 Then, -6 + [ ] = 0 and, -2 x -3 must be 6 • introducing a formal rule When this argument is not persuasive, it is best to introduce the rule that "negative times negative is positive" and then make it seem reasonable as above or as below. 4 x 3 = 12 4 x -3 = -12 3 x 3 = 9 3 x -3 = -9 2 x 3 = 6 2 x -3 = -6 1 x 3 = 3 1 x -3 = -3 0 x 3 = 0 0 x -3 = 0 -1 x 3 = -3 -1 x -3 = 3 -2 x 3 = -6 -2 x -3 = 6 ⋮ ⋮ • using a calculator most calculators will manage -7 x -8 this way [C] [7] [+/-] [x] [8] [+/-] [=] [56] generalizes, in words, for all multiplication situations involving integers positive x positive = positive negative x positive = negative negative x negative = positive positive x negative = negative

Continue to practice division, linking it continually to problem solving and arithmetical contexts which expose:
– quotitive division
– partitive division
– transferring to multiplicative statements

Examples could include:
– determining the size of equal shares of a collection
– finding means of distances, speeds, scores, and times
– finding unit prices

GOALS	5	6
Whole Numbers	Intent: Grade 5 – concept development and understanding Grade 6 – an algorithm is developed. The traditional algorithm is optional. See Appendix D at the end of this guide for a discussion of formal techniques which could be used.	
	■ explains and verifies the process for several meanings of division involving 1 digit or simple 2 digit divisors (multiples of 10) and dividends up to 3 digits.	■ —> Extend to divisors with any 2 digits.
	<u>Recommended techniques</u>: • creating a story or application	• —>
	• using estimation and mental calculation strategies Students must become proficient with the basic facts involving division by 2 digit multiples of 10; with estimating based on rounding, compensating, and compatible number techniques.	• —> Students are to be proficient with the basic division facts, division by 10s, 100s or 1000s, and several estimating techniques.
	• using manipulatives and diagrams Equal emphasis is to be placed on using materials, then diagrams, to act out all meanings of division for single digit divisors, and to explain all actions using correct terms and meanings. Then, try simple 2 digit divisors (i.e., multiples of 10).	• —> Emphasis should be placed on <u>seeing</u> the relationship between the measurement (repeated subtraction) and partitive (equal shares) meanings of division. When students are sure the answer is the same if the numbers involved are the same, they can step out of context and use the easiest strategy for explaining and verifying thinking.
	• using calculators Students will meet decimal quotients if the problems involve remainders. They need to understand the meaning of the numbers in the problem, and the meaning of the calculator display. For example, 389 ÷ 12 [C] [3] [8] [9] [÷] [1] [2] [=] [32.416667] (This means a little more than 32) The student should recognize that 32 groups of 12 is almost 389; then key in these numbers to find how close.	• —> Extensive work on estimation and mental calculation should be done for division problems <u>before</u> the calculator is used to verify thinking. Also, keeping a "Trials Record" for "broken ÷ key" problems helps students become accurate estimators. For example: How might you find the answer to this problem if your calculator's division function key is broken? Keep a record of each attempt (trial). [?] 2072 ÷ 37

7	8
■ explains and verifies the process for several meanings of division involving 1 digit or 2 digit divisors and dividends up to 3 digits. Extend to larger divisors and/or dividends in meaningful applications. Emphasis to be placed on improving estimation and mental calculation strategies. Use a calculator.	■ —>

GOALS	5	6

5

C 3 2 × 1 2 =

384.

The remainder is 5, since 389 – 384 = 5. So 389 + 12 = 32.416667 or 32R5 or 32 $\frac{5}{12}$

Be sure to establish these concepts using small numbers. Students need to understand the relationships involved.

- explains and verifies the repeated subtraction algorithm for division. [Limit to 3 digits ÷ 2 digits]

> Note: Grade 4 division is limited to 2 digit + 1 digit.

186 + 7

	# of 7s removed
186	
70	10
116	
70	10
46	
42	6
4	26

186 + 7 is 26 Rem. 4 or 26 $\frac{4}{7}$

636 + 17

	# of 17s removed
636	
510	30
126	
102	6
24	
17	1
7	37

636 + 17 is 37 R7 or 37$\frac{7}{17}$

6

- learns to use and verifies a root algorithm for division. [Limit to 3 digits ÷ 2 digits]

186 + 7

```
    026
 7 /186
    0
    18
    14
     46
     42
      4
```

186 + 7 is 26 R4 or 26$\frac{4}{7}$

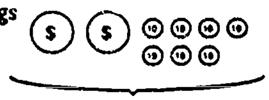
636 + 17

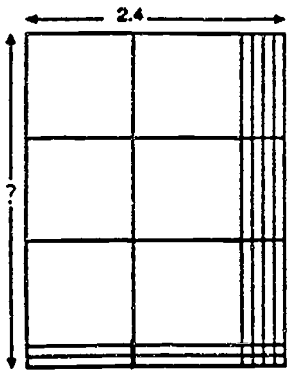
```
     037
 17 /636
     0
     63
     51
     126
     119
       7.
```

636 +17 is 37 R7 or 37$\frac{7}{17}$

> The placement of the 0 (zero) in the quotient is useful to maintain the integrity of the place value, and is useful in later work with decimals.

7	8

GOALS	5	6
Decimals	■ explains and verifies the process for several meanings of division with tenths and hundredths in the dividend. Whole number divisors should be 1 digit or 2 digit multiples of 10. Contexts should involve money or linear measurement. <u>Recommended techniques:</u> • creating story and application problems • using estimation and mental calculation For a problem like "$2.70 is to be shared among 6 people" Estimate – $2.70 is about $3.00, so each person will get a little less than 50¢ • using drawings  $2.70 ÷ 6 The key is to trade larger for equivalent smaller coins. • using vertical and horizontal formats for simple problems • using a calculator, estimating first Students should practise estimation and mental calculation strategies <u>before</u> using the calculator to verify thinking. Students should use the calculator to explore division of decimals by a whole number, and to note the placement of the decimal. Students must recognize that $\frac{2.70}{6}$ or 2.70/6 is entered as 2.70 ÷ 6 when using a calculator.	■ → Extend to divisors involving tenths • using diagrams or manipulatives For example, if 4.7 m of ribbon is to be cut into 0.5 m lengths, students could think that there are 2 lengths in each metre. The answer will then be 4 x 2 lengths plus 1 more length from the .7 metres remaining. The answer should be 9 with a bit left over. Beyond this point, students should move to using a calculator. It is most important that answers be estimated in advance. Extend exploration to include divisors with tenths.

7	8
■ explains and verifies the process for several meanings of division with tenths and hundredths. Extend to divisors involving hundredths Emphasize the use of problem contexts and the use of estimation. In most cases, the arithmetic should be done using a calculator. Where it is appropriate, use sketches and diagrams followed by the estimation of what is a reasonable answer. Students could use revalued Base 10 blocks and the equal sets or area models to verify their reasoning. Let the flat = 1. 7.68 ÷ 2.4  So 7.68 ÷ 2.4 = 3.2	

Students should explain what is happening in:
- division by hundredths
- division by mixed numerals like 3.25
- the placement of decimal points

GOALS	5	6
Fractions (Emphasis to be placed on halves, thirds, fourths, fifths, sixths, eighths, tenths, hundredths, and thousandths)	Work only such questions as can be answered using informal methods. For example, students should be able to think through questions like "How many quarters are there in $\frac{3}{4}$?" $\frac{3}{4} \div \frac{1}{4}$, $1 \div \frac{1}{5}$ and $2 \div \frac{1}{6}$	
Integers		

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■ explains and verifies the process for several meanings of division with fractions

Beyond those questions that can be answered using informal methods, the formal division of fractions is close to becoming obsolete. For example, in questions like

$4\frac{1}{2} \div \frac{2}{3}$

the preferred method is now to convert the numbers to decimals. Some teachers may have reason to introduce the common denominator algorithm

$\frac{8}{10} \div \frac{2}{10} = 4$, and in general $\frac{a}{d} \div \frac{b}{d} = \frac{a}{b}$

Then $\frac{4}{5} \div \frac{1}{2} = \frac{8}{10} \div \frac{5}{10} = \frac{8}{5} = 1\frac{3}{5}$

If the product of the original denominators is always used as the common denominator, this algorithm is identical in operation to the traditional "invert and multiply" rule.

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■ uses estimation and the calculator to solve problems involving integers

■ uses materials, diagrams or patterns to justify solutions

■ generalizes the outcomes by rewriting division statements as multiplication statements

In general

(+) ÷ (+) = (+)
(+) ÷ (–) = (–)
(–) ÷ (+) = (–)
(–) ÷ (–) = (+)

Examples
24 ÷ 6 = 4 since 4 x 6 = 24
24 ÷ –6 = –4 since –4 x –6 = 24
–24 ÷ 6 = –4 since –4 x 6 = –24
–24 ÷ –6 = 4 since 4 x –6 = –24

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Appendices

Appendix A, Kinds of Problem Solving (See page 22)
Appendix B, Principles to be Used in Calculation
Appendix C, Subtraction Algorithms
Appendix D, Division Algorithms

Principles to be Used in Calculation

When teachers say that students "understand" how they can transform arithmetic expressions they mean, for example, that students know that addition is commutative; that 2 + 7 or f + 3 or 3t + 6 can be rewritten as 7 + 2 or 3+ f or 6 + 3t. These are sometimes called "rules of the game" principles. By the time they enter the middle years, students have usually learned to use several such principles. For example, as early as in the First Grade it helps to know that 1 + 7 may be thought of as 7 + 1.

Mathematicians generally prefer to reduce the number of such rules. A century ago about 11 principles emerged, about the same list that is now used in algebra.

This list, however, is unable to be used in the middle years for the following reasons:

- The principles are too abstract. middle years students, for example, should understand that 9 + 5 can be rewritten as 10 + 4. "We can push 1 across the '+' sign" but the mathematical way of stating that principle [Addition is Associative – (a + b) + c = a + (b + c)] is of no help in the middle years. At this level, it is the idea that matters.

- The formal list is too condensed. Some principles that are useful in the middle years are not mentioned in the mathematicians' compressed lists because they can prove them as theorems which is not considered at this level. Two examples are provided.

  First, it is useful for middle years students to know that expressions like 19 – 13 and $6 - 3\frac{3}{4}$ can be transformed to 20 – 14 and $6\frac{1}{4} - 4$ (by adding the same number to both the minuend and the subtrahend). This is sometimes called the ***equal addends principle***. When used to subtract rather than add, it provides a method to manage questions like 200 – 124 –> 199 – 123.

  Second, in multiplication "zeros are like the raisins in a bowl of cereal. We can stir things around as we please, but in the end they will still be there at the bottom of the bowl." This may be called the ***annexation principle***.

  For example, 30 x 7 –> (3 x 7) with a zero –> 210 and
  80 x 200 –> (8 x 2) with 3 zeros –> 16 000.

  Mathematicians use the above ideas continually but neither the Equal Addends Principle nor the Annexation Principle are on their list.

- Some of the principles on the mathematicians' formal list are of little use in the middle years and need not be stressed. For example, it is important for some later purposes to know that every number other than nine has an inverse. The product of a number and its inverse is always one.

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For the previous reason, the list for middle years is a little different from what students will use in senior years. Formal algebraic statements of the principles are unimportant at this level. Functional understandings, which enable students to know that they can use them to transform expressions, are important.

List of Principles for Middle Years

It is not important that middle years students be able to name these principles or discuss them in the abstract. What matters is that they think of them as "things you can do when you see an expression."

1. **Order of Addition**

   (Addition is commutative.)

   2 + 3 = 3 + 2
   2t + 4 = 4 + 2t

2. **Order of Multiplication**

   (Multiplication is commutative.)

   7 x 2 = 2 x 7
   3f x 6 = 6 x 3 f

3. **Grouping of Addition**

   (Addition is associative.)

   9 + 6 -> 10 + 5
   18 + 9 -> 20 + 7

4. **Grouping of Factors**

   (Multiplication is associative.)

   6 x 14 -> 6 x 2 x 7 -> 12 x 7 = 84
   (A factor of 2 is shifted from the 14 to the 6.)

   This principle is particularly useful in multiplying by 5.

   16 x 5 -> 8 x 2 x 5 -> 8 x 10 = 80

5. Any Which Way (AWW) Principle

This principle does not appear in the mathematicians' list, but is important and is used from Second Grade onward.

In a string of numbers to be added or multiplied, addends or factors can be shuffled at random.

4 + 7 + 6	–> 4 + 6 + 7
	–> 10 + 7 = 17
2 x 9 x 5	–> 2 x 5 x 9
	–> 10 x 9 = 90

The additive AWW principle is used in column addition similar to the multiplicative AWW principle in questions like 5 x 13 x 2.

It is important that students learn that they are unable to use AWW on the "4," "3," and "6" in 4 + 3 + 6 x 2. More than one operation is involved. The "6" is "linked" with 2.

6. Annexation Principle

This principle does not appear in the mathematicians' list but it is important for the ***understanding*** of both multiplication and division. This principle is used in two ways:

200 x 6	(i)	record the "00"	00
	(ii)	write 12 in front of them	1200
or			
	(i)	find 2 x 6	12
	(ii)	write "00" after the 12	1200

Note that the principle can and should be used in questions like 60 x 50 = [30]00, but that 6 x 5 contributes a further "0."

7. Distributing on the Right

When 7 x 4 is thought of as 4 + 4 + 4 + 4 + 4 + 4 + 4, the 4s can be added in parts.

7 x 4 –>	4, 4, 4, 4, 4	5 x 4 = 20
	4, 4	2 x 4 = 8
		28

This principle can be used in the Fifth Grade to think of

	3	or		7
	x 8			x 13
5 threes	15		3 sevens	21
3 threes	9		10 sevens	70
	24			91

8. Distributing on the Left

When 5 x 7 is imagined as five sevens, the sevens can be separated as follows:

	☺	☺	☺	☺ |	☺	☺	☺		
	☺	☺	☺	☺ |	☺	☺	☺		5 x 4 =	20
5 X 7 ->	☺	☺	☺	☺ |	☺	☺	☺		
	☺	☺	☺	☺ |	☺	☺	☺		5 x 3 =	15
	☺	☺	☺	☺ |	☺	☺	☺			35

Distribution and the annexation principle is used to think of

	124
	x7
7 fours	28
7 twenties	140
7 hundreds	700
	868

Mathematicians usually combine the two distributive principles, but in the middle years, at least to begin with, it is best to teach them separately.

9. The Properties of 0

(a) Additive and subtractive. 7 + 0 = 7   8 - 0 = 8
0 + t = t   f - 0 = 0

After integers are introduced,

(a number) + (its opposite) = 0.

(b) Multiplicative. 5 x 0 = 0 (since 0 + 0 + 0 + 0 + 0 = 0)
0 x 4 = 0 (since multiplication is commutative)

(c) Divisive. Since $6 \times 0 = 0$

$$\frac{0}{6} = 0 \qquad 0 \div 6 = 0$$

$\frac{6}{0}$ is not allowed

$\frac{0}{0}$ is not allowed

10. The Properties of 1

(a) Multiplicative

$6 \times 1 = 6 \qquad 1 \times 7 = 7 \qquad t \times 1 = t$

(b) Divisive

$8 \div 1 = 8 \qquad t \div 1 = t$

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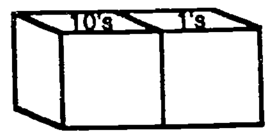
Subtraction Algorithms

Borrow-Take-Away Algorithm

Most individuals who attended school in North America began to subtract by "counting down" and then they learned some facts. Later, when they needed an algorithm, they learned the "borrow-take-away" algorithm. More recently, students learned the "regroup-take-away" algorithm, illustrated below.

This algorithm has a major strength and a major weakness. The strength is that it can be taught at about the same time as addition using the same physical device – a place value box. For questions, similar to the illustration, think of breaking up a package of ten, moving the 10 ones to the ones' bin, and then subtracting as necessary. This thinking can be matched, step-by-step, to the manipulation of sticks or counters. No other subtraction algorithm is so easily taught at the outset.

$$\begin{array}{r} 23 \\ -7 \\ \hline \end{array}$$



The major weakness of this algorithm shows up later. It does not work well on questions of the kind shown at right. (Go next door to "borrow" a package of 10, and there's nobody home.)

$$\begin{array}{r} 206 \\ -17 \\ \hline \end{array}$$

For close to a century, methods books, texts, and teachers have wrestled with solutions, most often beginning with multiple regroupings. The fact that virtually no adults subtract that way is proof that sooner or later someone tells the student, "Don't worry about it. Borrow ten anyway and pay it all back when you do find somebody home." Nevertheless, problems persist and many errors in both subtraction and division can be traced to this weakness of the borrow-take-away algorithm.

$$\begin{array}{r} 3000 \\ -276 \\ \hline \end{array}$$

Open Addition Algorithm

Three alternative algorithms, each designed to overcome the weakness of the borrow-take-away algorithm, have been tried from time to time. Two of them proved to have major weaknesses of their own and they are now obsolete. But one of them, ***the open addition algorithm, is gradually replacing the borrow-take-away algorithm.*** Middle years teachers should consider switching to it.

The open addition algorithm has one minor weakness. It cannot be taught until after students have some facility with addition. This algorithm cannot be built using a place value box.

Questions are first presented and worked as shown at right. The problem is to find the missing addend. At first, the student may run through his or her addition facts, looking for something that will fit. Later, the subtraction facts are learned in this form.

$$\begin{array}{r} 6 \\ +\square \\ \hline 13 \end{array} \qquad \begin{array}{r} 17 \\ +\square \\ \hline 100 \end{array}$$

Questions are then presented in the conventional form, as illustrated at the right, but are still thought of as addition questions with a missing addend.

$$\begin{array}{r} 13 \\ -6 \\ \hline \end{array} \qquad \begin{array}{r} 100 \\ -17 \\ \hline \end{array}$$

As soon as students can "carry" as they add, they can use exactly the same procedure to complete any subtraction question. In particular, 0s in the minuend do not raise any problems. The algorithm proceeds as usual.

Subtraction Algorithms and the Templates

The borrow-take-away algorithm is matched to the a − b = [ ] template. "Talk" that template when teaching the algorithm. It is not surprising that many early years students find problems such as

> *Tom had nine T-shirts but his mother threw out three of them. How many does he have left?*

to be "easy" and problems like

> *Tom had seven T-shirts but after Christmas he had 11. How many did he receive at Christmas?*

to be more difficult.

The first of the problems above matches the template that is at least implicit while teaching the borrow-take-away algorithm. The second does not. The ease or difficulty with which students detect the templates of such problems is not intrinsic to the templates; it results from the way subtraction is taught.

When teachers switch to the open addition algorithm, the above situation is reversed. From the beginning, students think, "What do we add to this to get that?" Now the way students "talk" about the algorithm matches the templates a + [ ] = b and [ ] + c = d. Problems similar to the second become "easy" while problems such as the first become more difficult for early years students.

By the time they enter the middle years, most students will have resolved these difficulties and will be able to detect any additive/subtractive template. Nevertheless, middle years teachers should be aware of the possible need to dwell for a time on using the open addition algorithm with the template a − b = [ ].

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Division Algorithms

Beginning at least four thousand years ago, scribes solved problems like 72 ÷ 4 by finding how many 4s can be subtracted from 72. They went on to invent a variety of streamlined subtractive algorithms so as to be able to find the quotient more efficiently than by subtracting the 4s one at a time.

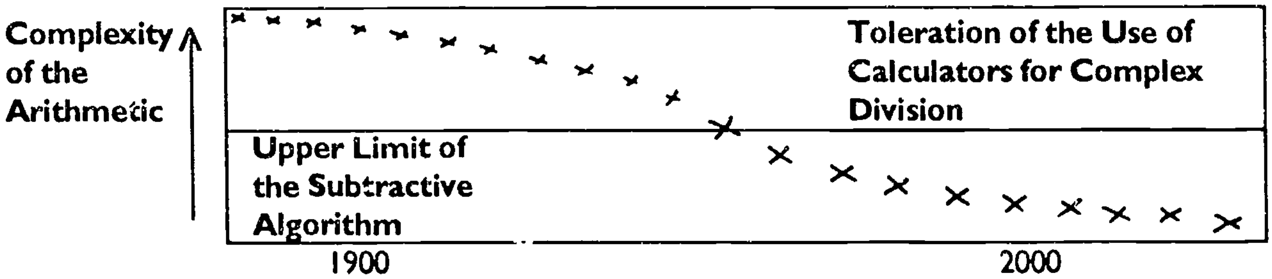
Those algorithms have the merit of linking the algorithm to problem solving; they match the problem that the learner is thinking about. They are easy to learn and work effectively (considerably better than the algorithm commonly taught) so long as teachers are not concerned to go much beyond whole number remainders.

At that point, they break down. Many attempts have been made to extend subtractive algorithms into that domain, but none of them work.

That is why, under pressure from the industrial revolution and the demand for large volumes of accurate human calculation, in the early 1800s people switched to the conventional ritual, division algorithm. It has always been taught as a ritual with no pretence that it might be understood. It is difficult to learn to manage both exact quotient digits and some special cases, but it has the virtue that once those things are learned there is, in principle, no upper limit to the questions.

Until quite recently, community standards have required that students completing middle years have a streamlined algorithm enabling them to manage questions like 246.79 ÷ 3.8. For good reason, then, schools ignored or quickly abandoned attempts to switch to the comparatively simple and more flexible subtractive algorithm for division. No subtractive algorithm can manage such questions well.

But community standards are changing rapidly. The situation is diagrammed as suggested below.



Community standards vary, but some students are still expected to be able to manage questions like 246.79 ÷ 3.8 by hand. So long as they are expected to do these questions, schools should provide them with a suitable algorithm – some form of the traditional division algorithm.

Given the rate at which community standards are coming to accept the speed, accuracy, and convenience of calculator algorithms, it is likely that the expanding toleration for their use will meet the upper limits of the subtractive algorithm while this curriculum guide remains in force. Many students may already be at that point.

It is recommended, therefore, that as soon as community standards allow, teachers should abandon the ritual division algorithm and teach the subtractive algorithm from the beginning. A procedure for doing so, together with some notes for those who have no experience with it, are provided below.

When that change is made, there may remain some students who will be required to perform complex division questions by hand. For that purpose only, this guide provides a slightly amended version of the traditional ritual division algorithm.

The Subtractive Division Algorithm

1. Two Necessary Components

   The subtractive division algorithm can begin without these components, but they are quite essential beyond an intermediate point.

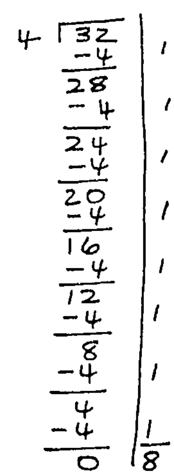
   - Students must be able to accumulate and detach zeros using what is called the annexation principle. They must be able to accumulate the zeros in questions like 4 x 20, 300 x 7, and 30 x 400 without thinking anything out. No formal language is required. It is perfectly acceptable for students to conclude that "zeros are like the raisins in a bowl of cereal. We can stir them around all we like, but they will still all be there at the bottom of the bowl."

     Questions similar to 60 x 50, in which an extra zero is contributed after the principle is used, should be delayed until after the approximation principle is well established.

   - Students must be able to decompose numerals so as to write and think of expressions such as 60 = 6 x 10, 1200 = 12 x 100 and 34000 = 34 x 1000, again, without thinking anything out. This procedure is sometimes called unmultiplying. A more general form of unmultiplying, as in 1200 = 30 x 40, is not required for the subtractive division algorithm.

     Teachers may have to supplement this material.

2. First Algorithm

   The algorithm begins as shown at right.

   If teachers choose to "talk" quotitive, which they usually do, how many 4s are in 32. Subtract them one at a time and tally them. When the original supply is exhausted, count up the tallies.

   The procedure may be matched, step-by-step, to removing sets of four from a stack of 32.

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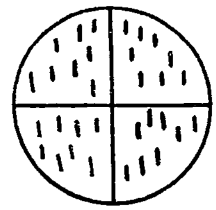
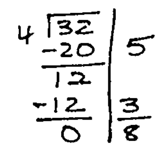
If teachers choose to "talk" partitive, the algorithm looks exactly the same but they "talk" differently. The problem is now to partition 32 objects into four equivalent parts. Now teachers use four objects to put one in each set, and again tally "1." When the original supply is exhausted, again add the tallies to find how many objects are in each set.



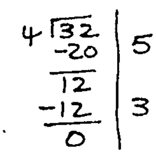
It is useful for problem solving but not for the algorithm itself to maintain both quotitive and partitive "talk" as the algorithm is developed. Students who are accustomed to thinking about the algorithm either way are more likely to detect both quotitive and partitive templates in problems.

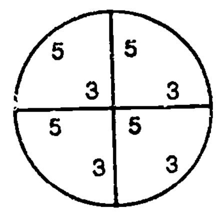
3. Developing the Algorithm

The next step is to remove several packages, all at once. Teachers say "Let's take out five packages of four. That means that we will be taking out 5 x 4, which equals 20, and that we can tally '5'."



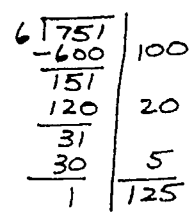
Talking partitive say, "Let's put 5 in each sector. That means that we are taking out 4 x 5, which equals 20, and we can tally '5'."



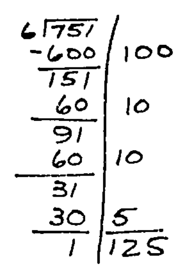


Note that remainders can be introduced at any point.

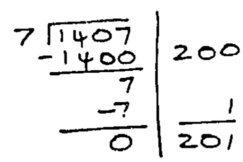
It is not long before students see the advantage of taking out 10s (and before long, 100s). At this point it is necessary that students be able to use the annexation principle and, before long, be able to unmultiply so as to be able to choose suitable partial quotients.



There is no rush to find the optimum number of, for example, tens for the quotient. In the ritual algorithm, the quotient digit must be exactly correct. In this algorithm, all that matters is that it not be too large. In the subtractive algorithm, if the number selected is too small, "It will come out next time."



Note: All of the problems once associated with zeroes in the quotient now evaporate. They turn up in the sum of the tallies with no special attention having been given to them.

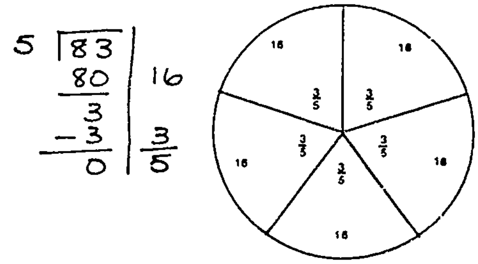


So long as remainders are of no great concern, then, this algorithm is considerably easier to learn, is more flexible, and is about as fast as the traditional ritual algorithm.

4. Taking it to the Limit

The subtractive algorithm can be extended to finding exact quotients but it is not always worth doing.

Suppose teachers talked partitive as the algorithm to the right was performed. They have, so far, sent 16 counters to each of five sectors. Since there are only three left in our original supply, they are unable to send even one more to each sector. But if students can manage the simplest multiplication of fractions, they can see that the supply can be exhausted by sending a further three-fifths to each sector.



Filling the Gap

The subtractive division algorithm has the virtue of being comparatively simple and flexible, and of linking the algorithm to problem solving. The ancient Egyptians, the Alexandrians, and the Romans used versions of it. In the past two centuries, the problem has been that there is no comfortable way to extend it to manage more complex questions such as 132.776 ÷ 13 and 242.551 ÷ 7.13 and that numbers of school graduates have been expected to be able to answer such questions by hand.

Now that hand calculators with speed, accuracy, and convenience are becoming common, the time is rapidly approaching when community standards will welcome a shift to the simplicity of the subtractive algorithm and its promotion of problem-solving skills. Those things that it does not manage well will be delegated to calculators.

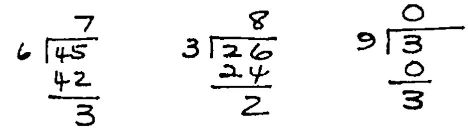
During the transition period, however, when many students (including those who go on to technical and university careers) find the subtractive algorithm/calculator combination quite sufficient for their needs. Others are expected to be able to manage more complex questions by hand.

It may be necessary, for that purpose, to equip some middle years students with a suitable ritual algorithm. It is, therefore, recommended that teachers use a modified version of the traditional ritual algorithm. The modifications allow the algorithm to be used in a completely ritual way. Most of the "special cases" and "traps" in the traditional algorithm are avoided.

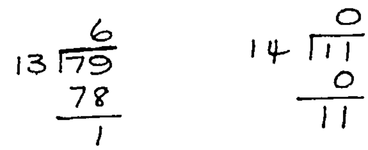
Keep in mind that a ritual algorithm is being addressed. There is no pretence that it is to be understood.

1. The Root Algorithm

The key to the modified algorithm is something that can be called the root algorithm. Beginning with 1-digit divisors, students learn to write out questions and answers as shown at the top of the following page. Note: questions are included that call for zeroes in the quotient.



Questions must be chosen that do not call for quotients greater than nine.

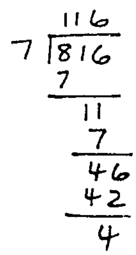


Immediately or soon thereafter, the root algorithm is extended to 2-digit divisors.

Again, some questions should call for zeroes as quotients, and no questions may call for quotients greater than nine.

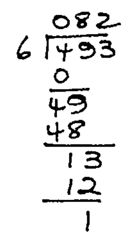
2. Looping the Root Algorithm

Students learn to loop the root algorithm according to the fixed and unvarying protocol shown at right.

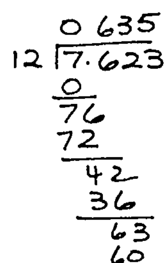


Always bring down 1 digit; then do it again.

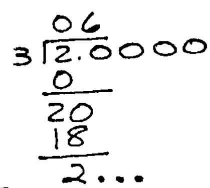
The procedure may generate leading zeros. Do not suggest the "short-cut" of omitting them. They can serve an important purpose, as is shown below. Where they serve no useful purpose, strike them off after the procedure is completed.



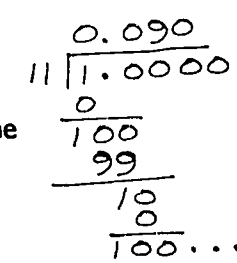
Next, introduce a decimal point in the dividend. Since this is a ritual algorithm, no harm is done by saying, "Go ahead. Do everything the way you did it before, but stick a decimal point in the quotient right above the decimal point in the dividend when you are done."



Now, if more "places" are desired, they are accommodated by simply writing more zeros at the right in the dividend.



All of a sudden, the advantage of maintaining leading zeros in the quotient is evident.



As in the traditional form of the ritual division algorithm, quotient digits must still be exact. The root algorithm, however, takes care of both leading zeros and any zeros that may turn up later in the quotient.