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ABSTRACT

C. L. Olson (1976, 1979) suggests the Pillai-Bartlett trace (V) as an omnibus multivariate analysis of variance (MANOVA) test statistic for its superior robustness to heterogeneous variances. J. Stevens (1979, 1980) contends that the robustness of V, Wilk's lambda (W) and the Hotelling-Lawley trace (T) are similar, and that their power functions are highly sensitive to slight covariance inequalities. Yet under conditions of diffuse noncentrality structures, V is a clear choice. A Monte Carlo simulation of V, W, and T as omnibus tests under conditions of covariance heterogeneity and variance homogeneity investigates the robustness of each test. Conditions of concentrated covariance and noncentrality structure were imposed to compare power. Results indicate that the assumption of homogeneous variance-covariance matrices in the form of covariance inequalities does not affect the robustness of V, W, or T, while T is slightly more powerful under such conditions. Five tables are included. (Contains 14 references.) (Author/SLD)

Choosing a MANOVA Test Statistic When Covariances are Unequal

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ABSTRACT

Olson (1976, 1979) suggests the Pillai-Bartlett trace (V) as an omnibus MANOVA test statistic for its superior robustness to heterogeneous variances. Stevens (1979, 1980) contends that the robustness of V, Wilk's λ (W) and the Hotelling-Lawley trace (T) are similar and that their power functions are highly sensitive to slight covariance inequalities. Yet under conditions of diffuse noncentrality structures, V is a clear choice. A Monte Carlo simulation of V, W, and T as omnibus tests under conditions of covariance heterogeneity and variance homogeneity investigates the robustness of each test. Conditions of concentrated covariance and noncentrality structure were imposed to compare power. Results indicate that the assumption of homogeneous variance-covariance matrices in form of covariance inequalities does not affect the robustness of V, W, or T, while T is slightly more powerful under such conditions.

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Choosing a MANOVA Test Statistic When Covariances are Unequal

In applied research with a single dependent variable, the *F*-ratio is the uniformly most powerful test that is invariant to linear transformations (Scheffe', 1959). It is therefore the most flexible and most used test statistic. Due to trends in both computer technology and the philosophy of science over the past three decades, behavioral researchers have adopted a belief in a multivariate reality (e.g., Fish, 1988). Thus, research which utilizes multivariate statistics has become more prominent; however, there is no unique multivariate analog to the *F* test. Only in two special cases do the four most popular MANOVA test criteria lead to identical results. That is, when the number of variables $p = 1$ and/or when the numerator degrees of freedom (df_h) equals one, the criteria are equivalent to a univariate *F*-ratio. Thus, in one of the most common educational research situations (i.e., multiple group comparisons), when the null hypothesis is tested against a completely general alternative, no multivariate test has both the required invariance and the property of uniformly greatest power. Therefore, considerable debate has occurred among statisticians over which tests to recommend.

Olson (1976, 1979) has argued that the Pillai-Bartlett trace (*V*) is superior to other test criteria as an omnibus test in MANOVA because of its greater robustness to violations of the assumption of homogeneous variance-covariance matrices. Olson noted that when groups differ on only a single dependent variable, a *concentrated noncentrality structure*, Roy's maximum root (*R*) is generally most powerful. However, because *R* is based on a maximum eigenvalue, severe problems with Type I error exist, and therefore, *R* is rarely recommended under conditions in which assumptions have been violated. Thus, the Hotelling-Lawley trace (*T*) or Wilk's λ (*W*) is usually preferred under such conditions. Olson contends, however, that in educational and psychological research, a concentrated noncentrality structure is rare, in that it is more likely for groups to differ in a more diffuse manner (i.e., in more than one group and/or on more than one dependent variable). Thus, although di.ference in power are slight because *V* is preferred under conditions of *diffuse noncentrality structures* (Olson, 1976).

In reply, Stevens (1979) concluded that the conditions Olson used to demonstrate the superiority of *V* had extreme differences in subgroup variances which are unlikely to occur in most research. In a review of several related studies, Stevens showed that under four conditions of subgroup invariance that are more likely to occur, the Type I error rates of *V*, *T*, and *W* are very similar. Furthermore, he reported that for concentrated noncentrality structures with heterogeneous variances, the slight robustness advantages of *V* are offset by the greater power of *T* and *W*. For diffuse structures, however, *V* remains the clear choice. Thus, under conditions of subgroup invariance, one may choose a MANOVA test statistic accordingly.

The MANOVA assumption of homogeneous variance-covariance matrices is not fully addressed by Stevens nor Olson, however. That is, the issue of heterogeneous *covariances* (and covariance/variance ratios) regardless of subgroup variances remains a concern. Stevens (1980) showed that the power of MANOVA test statistics generally increase as the intercorrelation among variables increases, but that the power of such tests are highly sensitive to small covariance inequalities with *equally* sized groups. Since little debate exists over the use of V under conditions of diffuse structures, the present paper focuses on the robustness and comparative power of *T*, *V*, and *W* as omnibus tests under a variety of conditions of covariance inequality with concentrated structures.

Methods

Conditions

Using K = 3 and 4 groups, p = 2, 4, and 6 variables, and n = 10 and 20 subjects per cell, the Type I error rate of *T*, *V*, and *W* were compared under two conditions of *concentrated covariance inequality* across groups, while group variances on all variables remained homogeneous at $s^2 = 1$. Furthermore, these combinations of heterogeneous covariance structures were examined under two conditions of *concentrated noncentrality structures* to compare the power of each statistic as an omnibus test.

Under *Type 1 concentrated* structure conditions, the population location on ***all*** variables is different in a ***single*** group (Olson, 1974). For this one group, constants of c = .3 and .6 were added to all variable vectors, which resulted in small to moderate effect sizes, respectively. For the conditions Type 1 concentrated covariance inequality (C1), all but one group had identical covariances of r = .10 on all covariance elements of the within-group variance-covariance matrix while the remaining group had different covariances (r = .30 or r = .50) on all variable pairs. In the case of p = 2 variables, there is no C1 concentrated covariance structure. This resulted

Under conditions of *Type 2 concentrated* structures, the population location differed on only ***one*** variable in ***one*** group (Olson, 1974). For this one group, constants of c = .3 and .6 were added to one variable vector, which resulted in small to moderate effect sizes, respectively. For the Type 2 concentrated covariance inequality (C2), this means that all but one group had identical covariances of (r = .10) on all variable pairs while the remaining group had a different covariance (r = .30 or r = .50) on one element of the variance-covariance matrix..

All conditions were crossed so that Type 1, Type 2, and no differences in location occurred under conditions of C1, C2, and equal covariances. Furthermore, under different conditions, location constants were added to a group with r = .10 and to a group with an aberrant covariance.

Procedures.

A normally distributed $n \times p$ data matrix **Z** with a mean of zero and variance of one for each variable (column vector) was randomly generated using the RANNOR function in SAS/IML (SAS Institute, 1990). Based on the fundamental postulate of principal components analysis (Forster & Dickman, 1962), a SAS/IML algorithm suggested by Beasley (1994) was used to impose a correlation/covariance matrix on to **Z** while each variable (column vector) of **Z**. From this transformation of **Z**, the various differences in location were imposed by the addition of the given parameters. This involves a linear transformation of the variable vectors; therefore, no changes in covariance/variance ratios (r) should occur. For each of the conditions elaborated, 1,000 replications of the data generation and transformation processes were completed. A SAS/IML MANOVA algorithm created by Sheehan (1994) calculated T, V, and W as an omnibus test from the $\mathbf{E}(\mathbf{H} + \mathbf{E})^{-1}$ matrix. Critical values derived from Seber (1984) and Timm (1975) were used to avoid precision problems associated with F approximations of these test statistics in simulation studies. The number of rejections at the α = .05 level of significance was used as an index of empirical robustness and power.

Results

In any Monte Carlo study which compares the power and/or robustness of different procedures, one must consider the sampling error of the simulation process. Based on the nominal alpha of α = .05 and 1,000 replications, the standard error of each estimate is s_e = .007. To avoid the issue of Type I error rate within this study, the standard error is **not** used as a means to test whether one procedure is "significantly" better than the other. Rather, the standard error is used as general heuristic to compare methods.

Type I Error

Table 1 shows the empirical Type I error rates for K = 3 and 4 groups, p = 2, 4, and 6 variables, and a cell size of n =20. As can be seen, the actual Type I error rates were within two standard error units of the nominal Type I error rate (α = .05) under all conditions, even when heterogeneity of covariance was introduced. These results held regardless of the type of concentrated covariance structure introduced. Nearly identical results were found for a cell size of n = 10 but are not tabled.

Power

Tables 2 and 3 show the comparative power estimates for K =3 groups with location constants of c = .3 and .6 under all conditions of covariance contamination for cell sizes of n =10 and 20, respectively. Table 4 shows these power estimates for K = 4 groups and n = 10 subjects per cell. The results for K = 4 groups and cell size of n =20 were consistent with the other three situations and are not tabled.

Overall, the effect of assigning differences in location to the group with the aberrant covariance was not clear-cut. Under a few conditions, it appeared that when the group with the aberrant covariance also had differences in location more power was exhibited as compared to differences in location for the group with the base covariance of r =.10. Yet under other conditions, these tendencies were slightly reversed or made little difference. Therefore, in reporting these results this distinction is not made and the results were averaged. Therefore, these results for the C1 and C2 conditions are based on 2,000 replications which gives a standard error of s_c = .005. Whether the power of any of these omnibus tests, based on Type 1 or Type 2 concentrated covariance structures, is affected by which group has differences in central location needs more systematic investigation.

Type of noncentrality structure. The two different concentrated noncentrality structures were affected differently by the heterogeneity of variance-covariance matrices. The Type 2 noncentrality structure was not affected by the introduction of covariance heterogeneity in a consistent manner under the low effect size condition. Further, in most cases the rejection rates remained within two standard errors of the power levels without assumption violations. The pattern became more consistent when the effect size was increased. The empirical power values almost always increased when heterogeneity of covariance was introduced when the effect size was moderate. The increased power was at least two standard errors greater than the power without assumption violations with at least one of the types of violation. In contrast, the rejection rates of the Type 1 noncentrality structure decreased as heterogeneity of variance-covariance was introduced. The decrease was greater for a moderate effect size than for a small effect size.

Degree of contamination. The effect of the degree of contamination was not consistent with a low effect size. However, when a moderate effect size was introduced the effects of contamination were greater when the degree of contamination increased, with few exceptions. For the Type 2 noncentrality structure, this meant that as the covariance inequality increased from r = .3 to .5 in the contaminated group, the rejection rates increased. For the Type 1 noncentrality structure, as the covariance inequality increased from r = .3 to .5 the rejection rates generally decreased.

Concentration of contamination. The relative effects of the two concentrated levels of contamination depended on the type of noncentrality structure and the level of the effect size. With a low effect size there was not a consistent pattern. With a moderate effect size in the Type 2 noncentrality structure, the C1 contamination had slightly more of an effect than the C2 contamination. In other words, the increase in the empirical power was greater when contamination involved unequal covariances on all variable pairs. Thus, when a single group differs on a single variable, more power is demonstrated when covariance inequalities occur across all variables.

The reverse was true with the Type 1 noncentrality structure. The effects of contamination were greater with the C2 rather than the C1 levels of contamination. That is, the decrease in power that was greater when the covariance contamination involves all variables. Thus, outside of equal covariances, the most powerful situation is when, when a single group differs on all variables, but covariances differ on a single variable. This is probably because the C2 situation presents a lesser contamination of variance-covariance heterogeneity.

Test criteria. When the variance-covariance matrices were equal, the ordering of the test criteria was typically $T > W > V$ in terms of power. When this pattern did not hold, the difference in rejection rates among the test criteria was usually less than two standard error units. When heterogeneity of covariance was introduced the order of the rejection rates of the test criteria typically remained the same as without assumption violations; $T > W > V$.

Discussion

Past research on the effects of heterogeneity of variance-covariance on the omnibus MANOVA test criteria have focused on introducing heterogeneity of variance-covariance by creating heterogeneous variances (Olson, 1974; Sheehan, 1994). These studies have found that Type I error rates become greatly inflated in the presence of heterogeneous variances, and the test criteria are differentially affected. Further, these studies found that the power values are also differentially affected by introducing heterogeneous variances. This had led to the general recommendation of using the Pillai-Bartlett trace when heterogeneity of variance-covariance is suspected, because it tends to be more robust against inflated Type I error under these conditions (Olson, 1974). The important implications of this study is that these findings do not appear to hold under all types of violations of heterogeneity of variance-covariance. These results affirm that when heterogeneity of variance-covariance is introduced with unequal covariances across the groups, the Type I error rates of three of the MANOVA test criteria, the Pillai-Bartlett trace, the Hotelling-Lawley trace, and Wilk's' λ, are robust. Further, the relative power of the three test criteria remain consistent with the power levels without assumption violations.

These findings have implications for the choice of a MANOVA test statistic when heterogeneity of variance-covariance is suspected. If the heterogeneity is due to heterogeneous variances, the recommendations of Olson (1974) hold. However, if the heterogeneity is due to unequal covariances across the groups, the Hotelling-Lawley trace would be recommended. Since all of the test criteria were robust to Type I error under there conditions, the choice of a test statistic would be made based on power level, and the Hotelling-Lawley trace appears to have the greatest relative power among the test criteria investigated in this study.

Recommendations

The findings of this study indicate that it would be wise to reinvestigate the effects of heterogeneity of variance-covariance of Roy's Greatest Root. Since, this test statistic has greater power than the other MANOVA test criteria, it would be the preferred test statistic under conditions of unequal covariances if it too is robust to inflated Type I error. Also, a systematic investigation into the comparative power of MANOVA test criteria when differences in location occur in groups with covariance inequalities under concentrated and diffuse structures is warranted. Furthermore, this investigation into the properties of MANOVA test statistics has only addressed V, W, and T as omnibus tests. Ramsey (1980) commented that simultaneous test procedures (STP's) based on the overall multivariate test statistic as a means of multiple comparisons can be used to avoid the problems of "protected" univariate follow-up tests which disturb both Type I error rates and power. Thus, it has been argued that investigations into the choice of a MANOVA test statistic should be based on the power and robustness of MANOVA STP's rather than the omnibus tests (Bird & Hadzi-Pavlovic, 1983).

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Table 1.

Empirical Type I Error Rate for *T*, *V*, *W* under Conditions of Equal, C1, and C2 Covariance Structures for *K*= 3 & 4 groups, *p*= 2, 4, & 6 variables, and cell size of *n* = 20.

	Covariance Structure	Groups *K*=3			*K*=4		
Variables		*T*	*V*	*W*	*T*	*V*	*W*
p= 2	EQ	.043	.043	.038	.048	.048	.047
	C2(3)	.045	.044	.044	.048	.045	.046
	C2(5)	.050	.046	.044	.050	.054	.048
p = 4	EQ	.052	.046	.049	.044	.043	.042
	C1(3)	.048	.034	.043	.048	.047	.043
	C1(5)	.048	.038	.046	.054	.051	.052
	C2(3)	.051	.036	.048	.045	.046	.043
	C2(5)	.052	.034	.047	.050	.051	.049
p = 6	EQ	.049	.048	.044	.052	.048	.043
	C1(3)	.052	.047	.045	.053	.055	.051
	C1(5)	.056	.055	.051	.049	.051	.046
	C2(3)	.057	.058	.050	.052	.051	.049
	C2(5)	.047	.047	.043	.056	.055	.052

Note. EQ = Equal Covariance Structure; C1(3) = Type 1 Concentrated Covariance Structure with *r* = .3 as the aberrant covariance; C1(5) = Type 1 Concentrated Covariance Structure with *r* = .5 as the aberrant covariance; C2(3) = Type 2 Concentrated Covariance Structure with *r* = .3 as the aberrant covariance; C2(5) = Type 2 Concentrated Structure with *r* = .5 as the aberrant covariance.

Table 2.

Comparative Power for *T*, *V*, *W* under Conditions of Equal, C1, and C2 Covariance and Type 1 and Type 2 Noncentrality Structures for *K*= 3 groups, *p* = 2, 4, & 6 variables, *c* = .3 and .6, and cell size of *n* = 10.

Variables	Covariance Structure	Noncentrality Structure Type 1			Type 2		
c = .3		*T*	*V*	*W*	*T*	*V*	*W*
p = 2	EQ	.100	.088	.098	.076	.067	.076
	C2(3)	.090	.082	.089	.071	.064	.061
	C2(5)	.093	.082	.092	.084	.078	.085
p = 4	EQ	.115	.112	.117	.058	.054	.057
	C1(3)	.074	.070	.075	.099	.090	.086
	C1(5)	.106	.095	.103	.064	.059	.062
	C2(3)	.117	.107	.118	.060	.066	.065
	C2(5)	.104	.096	.100	.071	.066	.074
p = 6	EQ	.117	.121	.121	.064	.067	.067
	C1(3)	.095	.096	.094	.065	.064	.066
	C1(5)	.084	.086	.086	.068	.065	.073
	C2(3)	.117	.117	.119	.059	.067	.064
	C2(5)	.102	.100	.099	.068	.069	.070
c = .6		*T*	*V*	*W*	*T*	*V*	*W*
p = 2	EQ	.297	.274	.298	.184	.163	.181
	C2(3)	.286	.265	.286	.191	.177	.190
	C2(5)	.269	.247	.272	.186	.173	.188
p = 4	EQ	.357	.324	.348	.130	.121	.130
	C1(3)	.303	.275	.303	.127	.118	.130
	C1(5)	.289	.267	.289	.138	.126	.138
	C2(3)	.335	.297	.327	.138	.127	.139
	C2(5)	.321	.282	.309	.150	.145	.148
p = 6	EQ	.357	.317	.348	.103	.102	.105
	C1(3)	.283	.265	.285	.114	.109	.117
	C2(5)	.254	.239	.246	.127	.121	.125
	C2(3)	.342	.322	.346	.106	.106	.107
	C2(5)	.342	.322	.346	.126	.127	.127

Note. EQ = Equal Covariance Structure; C1(3) = Type 1 Concentrated Covariance Structure with *r* = .3 as the aberrant covariance; C1(5) = Type 1 Concentrated Covariance Structure with *r* = .5 as the aberrant covariance; C2(3) = Type 2 Concentrated Covariance Structure with *r* = .3 as the aberrant covariance; C2(5) = Type 2 Concentrated Structure with *r* = .5 as the aberrant covariance.

Table 3.

Comparative Power for *T*, *V*, *W* under Conditions of Equal, C1, and C2 Covariance and Type 1 and Type 2 Noncentrality Structures for *K*= 3 groups, *p* = 2, 4, & 6 variables, *c* = .3 and .6, and cell size of *n* = 20.

Variables	Covariance Structure	Noncentrality Structure					
		Type 1			Type 2		
c = .3		*T*	*V*	*W*	*T*	*V*	*W*
p = 2	EQ	.177	.173	.167	.111	.103	.106
	C2(3)	.162	.158	.152	.112	.113	.108
	C2(5)	.161	.159	.154	.118	.118	.112
p = 4	EQ	.197	.157	.212	.093	.071	.088
	C1(3)	.203	.154	.193	.077	.055	.070
	C1(5)	.175	.134	.164	.091	.069	.085
	C2(3)	.208	.158	.195	.107	.080	.097
	C2(5)	.172	.129	.159	.098	.073	.091
p = 6	EQ	.217	.216	.205	.085	.087	.075
	C1(3)	.190	.189	.175	.090	.088	.082
	C2(5)	.163	.162	.152	.096	.095	.087
	C2(3)	.200	.198	.190	.080	.081	.076
	C2(5)	.199	.198	.182	.088	.092	.082
c = .6		*T*	*V*	*W*	*T*	*V*	*W*
p = 2	EQ	.607	.603	.596	.349	.340	.332
	C2(3)	.622	.611	.608	.382	.370	.366
	C2(5)	.566	.553	.547	.381	.373	.367
p = 4	EQ	.744	.671	.726	.266	.213	.250
	C1(3)	.658	.578	.636	.294	.225	.279
	C1(5)	.587	.509	.567	.280	.218	.265
	C2(3)	.716	.629	.690	.288	.227	.274
	C2(5)	.656	.574	.633	.308	.238	.288
p = 6	EQ	.788	.765	.761	.216	.217	.201
	C1(3)	.682	.660	.657	.233	.235	.217
	C1(5)	.542	.559	.554	.258	.257	.240
	C2(3)	.754	.723	.722	.245	.235	.225
	C2(5)	.733	.713	.710	.244	.246	.230

Note. EQ = Equal Covariance Structure; C1(3) = Type 1 Concentrated Covariance Structure with *r* = .3 as the aberrant covariance; C1(5) = Type 1 Concentrated Covariance Structure with *r* = .5 as the aberrant covariance; C2(3) = Type 2 Concentrated Covariance Structure with *r* = .3 as the aberrant covariance; C2(5) = Type 2 Concentrated Structure with *r* = .5 as the aberrant covariance.

Table 4.

Comparative Power for *T*, *V*, *W* under Conditions of Equal, C1, and C2 Covariance and Type 1 and Type 2 Noncentrality Structures for *K*= 4 groups, *p* = 2, 4, & 6 variables, *c* = .3 and .6, and cell size of *n* = 10.

Variables	Covariance Structure	Noncentrality Structure Type 1			Type 2		
$c = .3$		*T*	*V*	*W*	*T*	*V*	*W*
$p = 2$	EQ	.091	.091	.089	.080	.082	.082
	C2(3)	.096	.095	.098	.060	.059	.059
	C2(5)	.088	.089	.088	.079	.081	.079
$p = 4$	EQ	.105	.102	.098	.061	.055	.054
	C1(3)	.112	.109	.106	.057	.049	.048
	C1(5)	.084	.079	.077	.063	.057	.055
	C2(3)	.097	.091	.086	.063	.057	.058
	C2(5)	.098	.095	.091	.064	.061	.058
$p = 6$	EQ	.107	.101	.099	.072	.069	.064
	C1(3)	.099	.097	.098	.058	.057	.055
	C1(5)	.104	.096	.097	.072	.067	.068
	C2(3)	.102	.097	.094	.067	.063	.063
	C2(5)	.097	.091	.094	.060	.055	.053
$c = .6$		*T*	*V*	*W*	*T*	*V*	*W*
$p = 2$	EQ	.270	.271	.273	.159	.160	.160
	C2(3)	.273	.271	.271	.166	.170	.167
	C2(5)	.252	.256	.257	.150	.157	.151
$p = 4$	EQ	.343	.311	.319	.124	.112	.114
	C1(3)	.280	.263	.262	.124	.120	.115
	C1(5)	.268	.252	.249	.136	.135	.124
	C2(3)	.320	.291	.300	.125	.131	.117
	C2(5)	.310	.285	.290	.120	.114	.108
$p = 6$	EQ	.340	.312	.328	.108	.096	.098
	C1(3)	.297	.267	.285	.124	.114	.111
	C1(5)	.261	.235	.238	.103	.101	.096
	C2(3)	.332	.287	.307	.115	.110	.109
	C2(5)	.314	.267	.287	.111	.099	.102

Note. EQ = Equal Covariance Structure; C1(3) = Type 1 Concentrated Covariance Structure with *r* = .3 as the aberrant covariance; C1(5) = Type 1 Concentrated Covariance Structure with *r* = .5 as the aberrant covariance; C2(3) = Type 2 Concentrated Covariance Structure with *r* = .3 as the aberrant covariance; C2(5) = Type 2 Concentrated Structure with *r* = .5 as the aberrant covariance.