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AUTHOR	Watterson, Tricia
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ABSTRACT

This report describes an experiment to provide an inservice program involving problem solving and inquiry, in line with the new Scottish National Guidelines in mathematics, that would have a lasting effect on teachers. The program featured two stages: (1) In the first year teachers were involved in a pair of workshops, were visited at their individual schools by the tutor, and were assigned various projects; and (2) In the second year contact with the teachers was extended via electronic mail. Results showed that teachers can continue to benefit for long periods of time from a short period of out-of-school in-service if some kind of interest network is set up and maintained both by themselves and by the tutor. Chapters include: "How Do You Make the Effects of Inservice Last," "Developing Professional skills," "The Visits," "Communications," "New Research Stage," "The Children's Questionnaire," "The Children's Own Assessment," "The Challenges," and "Closing the Classroom Door." An appendix contains letters from the tutor to the teachers; an explanation of the game, Strategy 31; sample challenges; children's questionnaire format; questions for early and upper primary; children's solution for Leapfrog; and the Ten Face Game. (MKR)

Going for a Lasting Inservice Effect

Research Report by

Tricia Watterson Senior Lecturer University of Strathclyde

Going for a Lasting Inservice Effect

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Preamble

Two primary four teachers in a school in Scotland are good friends and have a very straight forward way of assessing maths progress in their classes. They make a point, about once a month, of asking each other which page their children have reached in Stage 3 of the SPMG mathematics scheme. (Of course not every child is using the same book. There are some very slow learning children who are on individual programmes made up by the teachers.)

This would be considered a reasonable tactic in Japan, Korea and Hong Kong where children of a wide range of ability are taught at the same level in each class using the same state designated mathematics text book. These are countries that do well in international mathematics tests and countries that also have high rates of students continuing to study maths in upper secondary and staying on to higher education. The teacher's task in mathematics teaching there is clear: to make sure that every pupil successfully completes the set book at each stage.

How is this done? It is the duty of every pupil in the class to support fellow pupils to achieve this end. It is the parents' duty to contribute to the achievement. Thus many children spend up to three post school hours in tutorial school completing tasks that were unfinished in day class! (We might argue that if that is the way to come top of the international mathematics league then we want no part of it.)

We have different expectations of our teachers and pupils and we organise our teaching differently. Yet the primary four teachers in our example are using an identical assessment yardstick.

Why do we want to change the assessment system of our two Scottish teachers?
What is wrong with using 'the place in the book' as a measure of success or failure?
What would be better questions to raise?
What have these questions to do with the new National Guidelines in Mathematics?

These are some of the ideas and questions that are raised when a new wave of ideas comes to the fore in mathematics education. The new National Guidelines have been distributed to every school in Scotland, parents have been informed of the fact and local authorities feel the duty to offer inservice courses to help teachers implement them. The guidelines did not appear out of the ether but are declared to be based on the best of present practice. They are designed to match in layout the guidelines for the other main areas of study and to form a common basis for communicating classroom practice across Scotland. For the first time in Scottish education the curriculum is presented in the one subject area for all teachers of that subject of pupils aged between 5 and 14. The purpose is to ensure an unbroken line of teaching across the primary secondary divide.

This paper describes a novel approach to the implementation of the spirit of the mathematics guidelines which is to promote in pupils a **problem solving and enquiry approach** to the understanding of the subject.

Patricia R Watterson,
Inservice Department, Faculty of Education
September 1994

Chapter 1

How do you make the effects of Inservice last?

The purpose of this report is to study the problem of how to ensure that class teachers change their teaching method from mainly instructional to a convincingly investigative one in mathematics in line with the recommendation in the Scottish National Guidelines where a **'problem solving and enquiry'** approach is advocated to underpin all teaching.

It was a problem devised for me by the coordinating tutor of a small local authority in Scotland. She presented me with the challenge to do something better than the standard two day course with a follow-up recall day which had proved in the past to effect only a very short term change in methods and approach. **Could some form of inservice be devised that would bring about a lasting change of attitude and practice?**

In research one starts with a literature search : in inservice planning one starts with a summary of the clients and their circumstances. In this case there were 36 primaries ranging from one teacher and four pupil schools to a four stream town school. Some teachers were running their classes as bilingual units and had already done a great deal of work on developing appropriate mathematical activities, purchasing and translating material after a period of considerable debate. Others were running bilingual units within their classes whilst others were using either Gaelic or English as the sole teaching medium for the whole class. A small fraction of the teachers were recently qualified but the majority were very experienced and a good proportion were in the forty to sixty age range.

The request began by asking for a focus on the teaching of younger children but the composition of the client group meant that there would be a need to provide a method equally adaptable to the teaching of older children.

Primary teachers teach a lot of different subjects. This affects the design of inservice to extend their professional skills. They are not subject specialists but child development specialists. They are mindful at all times of the need to strike a balance amongst the many things they teach and, to simplify their organisation, they are constantly looking for connections across the teaching that they do.

The skeleton of a plan devised to solve the problem **did** include a two day starter workshop course and a follow up day after an interval. **However, inserted into this programme were two innovations.** The first was a working visit to every single teacher involved in the programme in her classroom. The second was a series of letters and notes with activities sent to each teacher during the first year and based on the experience of needs pinpointed during the visits.

Amongst many of the teachers there already was an informal network of exchange of ideas because a number of them were related to each other or had been to college together. The network consisted of much more than the exchange listed in the opening preamble. Some of them even shared long term lesson preparation, particularly those in bilingual units where there was a spirit of enquiry and new adventure in their teaching.

Gaelic language poses a particular problem for bilingual children where number patterning is concerned because it counts in scores. (For example, children can clearly discern that 31 is named as a score and ten, with one, when listening to every clearly discern that 31 is named as a score and ten, with one, when listening to every day Gaelic.) The Gaelic language teachers were highly motivated to succeed, feeling as they did empowered in the use of their own language for the first time. The special difficulties ordinary language poses in mathematics learning was obvious to them and they had been attending to it as part of their share in developing a parallel curriculum. They had been supported with specially translated or commissioned teaching materials in Gaelic and by working visits from local staff tutors.

This is a key point.

In such a small community the benefits accruing to one group tend to spill over through the informal network to the others not directly involved in the initiative. In the teaching profession that benefit is the debate that begins to centre on teaching methods, resources, the particular skills of teachers and the lottery of the kind of class a teacher is given. If there is a strong cultural and emotional desire for one kind of change in teaching to succeed then there is an equally strong desire by those not involved in the initiative to be considered at least as successful. **A spirit of enquiry is fostered.**

My own involvement with the authority had begun some years before when I was approached to allow my computer assisted materials to be translated into Gaelic. Unlike the translation of a text, translation of software requires giving technical information that is usually kept secret. As all the teachers using the software are fluent in English the expensive teachers' books did not need to be translated. However, different teaching support guides could easily be tagged to the English versions to develop particular language outcomes in Gaelic teaching. Again, copyright of the original material became an issue and complementary teaching material opened up an educational debate about types of material which could be translated to support bilingual work in other languages, perhaps in Europe or in mainland Britain, based on the same computer assisted materials.

The computer assisted topic that was translated for the youngest age group had a mathematical underpinning and had been developed over many years of practical research into the ways that young children develop mathematical ideas. The familiarity of the Gaelic teachers with the software translation of the **Fantasy Islands** topic was the reason that I was approached to interpret for all of the teachers how to implement the problem solving and enquiry aspect of the new national guidelines in mathematics.

How would I convince the other teachers that they would have more success in mathematics if they widened their instructional approach?
Would the general ethos of enquiry make it easier?

The initial workshop took place either in a teacher centre or a local hotel, that is local to where I was lodged. The clients came from a wide geographical area, some so far away that they had to be flown in to the course. The resources for the course were brought from nearby schools and involved a considerable amount of organising by the local staff tutors. It was my task on the courses to share with the teachers the philosophy underpinning the new guidelines, to convince them of the soundness of it and to inspire in them a need for change or an encouragement to continue if they were already flexible in their approach to teaching mathematics.

I used two mini topics as exemplars of a quite different way of approaching maths teaching which makes the child's thinking in and talking through an activity more important and revealing than sets of answers in a commercial work book. In the course of the topic it was an easy matter to find opportunities to explain why the emphasis in maths teaching has changed, how it has changed through the forties to eighties to match technological change in society and the clearer understanding of how children develop. It is not only the older teacher who needs such clarification ; younger teachers are heavily pressured by parents with an out of date notion of what kind of mathematics needs to be taught. **Only convinced teachers teach with conviction.**

To catch a spirit of enquiry that leads teachers **to find a problem to solve** can be difficult when teachers are looking at early mathematics. They think they know it all already, it is so easy. So it was fun for all of us when I succeeded in demonstrating how much elegant mathematics was hidden in materials, activities and small numbers that they commonly used in more mundane fashion and how this could so easily be uncovered in the context of a topic. It is the breath of life for any teacher to get a "Eureka!" from the pupils, the more so when the pupils are teachers.

Spark a teacher and all her classes in the future will share the spark. **Step 1**

It is simple for an outside lecturer to 'enthral the teachers' with some unexpected insight into a subject area in which they do not specialise: that is her job. The harder thing to do is to enthral the teachers with their own professional skill. Good teachers seem unaware of the skills experience has brought them. The topics offered opportunities to highlight children's misconceptions and the difficulties of sorting these out. Games were introduced, innovative versions created, discussed and perfected in short order as teachers brought individual examples of children's difficulties to bear on the designs. This made it easier for the teachers to convince themselves that the process was of more interest and evaluative use to them than the final written answer.

Let the teachers discover a line of enquiry using their own pupils' difficulties. **Step 2**

Once a good mathematics workshop is underway participants develop enough steam to keep going for a week and two short days, a generous allocation from the authorities, seems miserly. To keep up that head of steam the teachers were given the homework task of creating a further game that solved a learning problem for their own pupils. This would be a game an appropriate group of their children could demonstrate on the planned visit.

Give teachers a simple task which converts in the classroom to a research problem. **Step 3**

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Chapter 2
Developing Professional Skills

We cannot ask teachers to stop teaching formally if they have never experienced any other form of teaching. **How can they imagine it?**

We cannot ask teachers to develop a problem solving approach if they have no experience of looking for problems to solve. **How can they imagine it?**

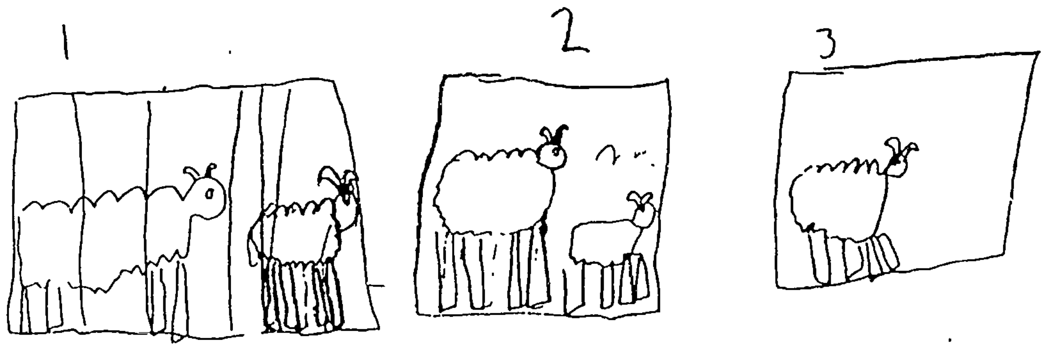
We cannot ask teachers to teach in the spirit of enquiry if they rarely raise questions about their own learning. **How can they imagine it?**

That is why the groundwork of the course had to release statements from the teachers such as :-
Slower children always confuse the subtraction sign with the equal sign.
That's always a sticky page in the SPMG.
Some children never give up counting on their fingers.
Some of them never get it no matter how often you tell them.
The children like doing pages of sums.

Without these contributions there could be no way of leading the teachers to restate them in the form of **"I wonder why........?"**
The course itself had to be an exemplar easy to mimic in the classroom. Teachers, like all human beings, cannot stop themselves teaching what they are taught : the difference is that teachers are more effective.

Expose teachers to effective teaching and they will improve through imitation **Step 4**

The teachers took away outlines of the topics, together with extracts from teaching materials, references, games sheets and analyses of children's learning difficulties used on the course. They were promised further materials by post and left with me a list of the range of stages and numbers of children that they were teaching during the session.



Chapter 3
The Visits

The visiting schedule seemed at first to be unsatisfactory because the first set of teachers were visited quite soon after the course. However, it turned out to be another type of opportunity. The enthusiastic teacher pulled out all the stops and had plenty of activity to demonstrate, discuss and share. The reluctant teacher had a good excuse and her normal modus operandi was clear to see and evaluate as a true baseline. The moderate teacher would be in a good position for me to help with a tentative start. The early feedback of the immediate effect of the course would influence the materials and tasks that could be sent to all the teachers during the session. It is a very different thing to see a teacher in her classroom compared with working with her on a course. The early set of visits kept expectations realistic and were a good reminder of the multiplicity of things that have to be 'minded' in a primary classroom.

The organisation of the visits was planned so that I could spend up to a half day working in the classroom of each teacher. Travel to and around the far flung schools was an adventure in itself. I tried to remember to photograph each class on the visits so that I would have a proper reference in retrospect. I tried, too, to write down the names and ages and some comments for each group of children I worked with as I went along. In the excitement of contact and teacher expectation from each visit it was a plan not always adhered to leading me sometimes to append the wrong names to a remembered situation. I have only a little Gaelic and so found the match between the spoken and written placenames difficult at times.

In such circumstances it was important to have a fairly standard approach to working in each class, at least at the outset. I carried with me materials suited to the different age groups of the whole primary stage and I brought a range of papers and coloured pens. In each class I let the teacher tell me what she wanted me to know and tried to appraise how best I could fit in to the way she was running her classroom and yet give the children a chance to open out to me ; to demonstrate for us both how they worked things out in a mathematical situation.

My intention was twofold. First, I wanted to encourage the teacher and make clear my interest in sharing the unique situation each teacher faces with a particular class. Second, I wanted to help her assess the difficulties I perceived in the children's learning as an outsider who was in a position to offer a different challenge. It is well known that children 'tune-in' to teacher's requirements and thus disguise their difficulties. It is easier for the teacher to assess her children when she observes them responding to someone else in a teaching situation with which she is familiar.

This is much the same as observing children at the computer, if the software is good and the group of children are in harmony with the task. It is a more difficult task if a teacher is also concentrating on teaching a whole group herself as a good part of her energy and attention are given to keeping the lesson flowing and everyone participating.

My method was mainly to devise situations where children would not feel judged but be quite happy to share with me how they worked things out. This proved not to be too difficult because for the most part I was accepted as an interesting visitor both by the children and the teacher. The children were accustomed to visiting teachers, particularly in the smaller schools, and were happy to work with a new teacher.

Most of the activities I shared with the children involved them in a game where they had a number pattern to spot, needed to range up and down a series of numbers, find quick ways of combining numbers, use some simple logic, arrange shapes or compare measurements. Sometimes I used models they had constructed in class, or an electronic gadget of my own, or simple counters and grid paper, or lolly sticks, or a miniature set of dominoes, or folded paper or their own drawings.

With the help of each teacher I very quickly spotted the main difficulties the teachers had to tackle and it was rewarding to discuss with them how they might be or were going about remedying the problems. Along the way I collected up many ideas to exchange with the other teachers I visited. Often I uncovered surprises for the teachers. Some children reveal their true worth in such situations. In no class was there nothing worth discussing.

There were examples of text book pages which undermined a good approach by falsely declaring examples of problems that were in fact thinly disguised attempts to force children to apply particular algorithms. In such cases we could demonstrate to the teacher with the children **'why a certain page was always difficult'** ; the child could spot a more efficient way to solve the problem, perhaps by adding on where a decomposition was demanded. Where teachers were encouraging children to work things out and talk about their methods it became clear that the text book not the child was out of step.

There were examples of children devising elaborate, restrictive patterns of counting which had been long hidden from their teachers but in a spirit of shared conspiracy were happily demonstrated to me.
There were interesting examples of young bilingual children with well founded notions of cardinality in Gaelic but still unsure in the same situation in English, exhibiting a distinct lag in the less familiar language.
There were examples of teachers trying out practical activities suggested in a new maths text and wanting an explanation from me of the real purpose they served in helping children to think mathematically.
There were surprises when a child 'good at maths' was beaten hands down by a 'poor soul' who spotted the logic in a game ; or when a child able to do arithmetic beyond most of the class, and able to name any multiple in the 'tables', was discovered counting on his fingers to solve a small number problem.
There was the delight of seeing classes where the children thoroughly enjoyed mathematics and were eager to share discoveries and the elaboration of mathematical games they had worked out for themselves.

It was exhausting being picked over, in the nicest possible way, in the staffrooms, and spirited into classes of other teachers to 'look over' the work of 'blocked' or exceptional children. It was good to feel wanted. Like the children, some teachers absolutely loved maths and others did it because it was necessary. For the second category it was important to build on the teaching strength and reserve some energy to try to share some of the joy.

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Chapter 4
Communications

Before the first set of visits I sent a letter to all the teachers referring to the analysis of teaching in the 5-14 National Guidelines and including for them a sample version of such an analysis using the topics we had begun to develop during the workshop together with a blank version for their own forward planning. I summarised the purposes of the games they had created during the workshop and made clear the purpose of my planned visit.

After the first set of visits, when about one third of the group had been visited, I sent another letter with a summary of how the first visits had gone and presented everyone with a second task to complete. **This was a small action research project** to be carried out with children who counted-on-in-ones either on their fingers or by some other means out of habit. A detailed procedure was presented to encourage the teachers to follow through some of the 'talking' approach I had used in their classes and demonstrated in the workshop.

I included a humorous article from the TES on all the misunderstandings that arise from finger counting. I asked the teachers to read the booklet I had given them on **"Talking Mathematics"** to find examples of good dialogue to draw out understanding and I made references to the use of software which might help to diminish the 'counting on in ones' practice.

Before the recall day I sent a letter inviting the teachers to bring the results of their research, sample games and examples of children's work to share with each other. From all the visiting experience I pooled some common ideas and gave the teachers a further experience of the joy of finding and solving a problem. The ideas they were able to share on that day were impressive.

I had noticed that some of the more remote schools had used the modem to keep in touch with each other via a bulletin board run for all the schools in the authority. I had suggested that two of the smallest schools, which had not been in contact with each other might use the modem as a part solution to their small number problem. The main use of the local electronic network was accessing the attractive teletext like bulletin board to 'tune-in' to local education events, extend Gaelic vocabulary and scan the ongoing bird watch database to which many schools contribute. (The area is fortunate to be under the path of major bird migratory routes.)

The enthusiastic response of so many of the teachers on the recall day prompted me to ask if they were willing to maintain contact with me, perhaps via modem, and continue with the mathematical connection by trying out challenges over the coming session.

Chapter 5

New Research Stage

The purpose of the continued connection was to foster a long term change from the mainly instructional to more of an investigative approach to teaching mathematics. Some of the teachers were considering studying for a higher degree and others were keen to remain supported as they opened up their teaching methods. Many had enjoyed their experience of a small scale action research project and wanted to try an other example.

At the beginning of the 1994-95 session my computer was linked by ethernet to JANET and worldwide Internet as part of the process of Jordanhill becoming the Faculty of Education of Strathclyde University. There was hope that I might be able to link with the schools network via Internet but in the event I was only able to do so by telephone link via modem ; a more expensive route. The local authority System Operator, Kenny Mathieson, gave me a mailbox in the local network and arranged the schools in two mailing blocks to reduce my downloading of challenges to two telephone calls that covered all the schools for each distribution.

My next problem was to devise suitable challenges for teachers to use with their whole class. I decided to use the challenges as a means of highlighting the notion that children develop mathematical understanding by revisiting a similar idea over the years and working it out at more and more sophisticated levels.

This reduces the quantity of mathematics a primary teacher needs to know. It makes her teaching more manageable if she can **grasp the underlying principles of the mathematics** she is presenting. Each example would have to illustrate this idea.

Also, every example would have to differ in order to cover all the **aspects of mathematics** in the 5-14 National Guidelines over the set of challenges. Mindful of the fact that so many of the teachers were teaching multi-composite classes I tried to devise the same challenges on three levels so that presentation in class would be simplified by the similar context at each level.

The activity was voluntary. Teachers might read the challenges and not do them or do them and not report them. They might not even read them. Would I have any of the teachers still with me by the end of the next session? This was my challenge.

The System Operator kindly downloaded my challenges and sent the first set out by local authority mail to make sure that the teachers knew they were in the system. For him it was to be an interesting experiment, too. How well was his bulletin board being used? Would too many of the teachers find it difficult to download my messages? Had any of them forgotten how to do it? Would BT be a stumbling block?

A further limitation was placed on me by the need to make the challenge all text and precise enough to fit on a single computer screen. I decided that following each challenge I would send a commentary of the kind of questions the challenge might raise with each age group.

The slowness of the modem and the vagaries of telecommunication turned the exciting adventure into something quite tedious as many an afternoon the system 'went down' on me or scrambled my messages after an interminable dark screen pause. We do not yet have the speedy technology.

One of my first replies was by post or 'snail mail' with a note appended by the teacher saying that she had tried the challenges that came by post and would not be able to download from the computer because she was too busy to get the hang of it. The teacher included samples of the children's responses and her own comments of surprise at the uncovering of a difficulty she had never noticed before. That was typical of the responses through the year. The local staff tutors would telephone me from time to time about some other matter and comment on their own observations of teachers trying out the ideas in the challenges. It appeared that maths teachers in the locality took an interest too, sometimes commenting that a problem was too difficult for primary children. They would be spotting the maths at their own level embedded in a practical problem.

Some teachers waited till the end of the year and then sent me a summary of how they had fared. Others kept a collection of samples and appended comments at the time of each challenge, noting in particular any surprises they experienced and then sent these on at the year's end to speak for themselves. **Nobody replied to me via the electronic mail.** At conferences during the year teachers in other Gaelic speaking areas gave me their electronic mail addresses and asked to be put on the 'challenge list'.

Parents form a very strong pressure group and, in a small community, teachers have to exercise great tact in dealing with them. Sometimes it seems easiest to bow down to the pressure and remain ultra conservative in one's teaching. To support teachers in this respect I had sent them a set of home and school notes I had devised for inservice on the mainland. These notes helped parents to understand where best they could help and why maths was being taught differently from their remembered experience. In those notes there were suggestions for maths activities that were related to everyday tasks in the home which could be done practically. In sending challenges I wanted children to have problems that they would **talk** about at home. My hope was that the discussion started in school about each challenge would spill over in to the home.

An example I used on the course for the teachers was a homework assignment they might devise which asked the children to find the weight of a cat. How can parents help with such an assignment and yet let the child find a way of making the measurement by him or herself? After all there are many ways to weigh fat cats even if the kitchen scales *are* out of bounds.

What if you do not own a cat? Why would anyone need to know the weight of a cat?
What is a 'normal' weight? How accurate does it have to be?
All these are great questions.

I highlighted this funny scenario as being more suitable than the bad habit of sending children home with some more of the sums they have just been tackling in class. We all can remember sad scenarios of how interested, helpful parents come to grief over unrealistic expectations about sharing poorly remembered algorithms for 'sums'.

(Teachers need the comforting reminder of how to deal with parents' unrealistic demands for their children.
Insist that the family loft is searched for a 'sum' jotter with a date on it. Parents never return to show 'what I was doing at his age' evidence once they uncover the real, rather than the remembered, item.)

Chapter 6
The Children's Questionnaire

As the session drew to a close I was aware that I would have to have some more concrete proof of whether teachers were succeeding in opening out their mathematics teaching. Anecdotal references and staff tutor comments would not be enough.

So I devised a questionnaire for the children to fill in. As the main purpose of the inservice and classroom research was to keep open the debate on what mathematics we teach; how we can teach it in the spirit of problem solving and enquiry, and how to free the teaching from page to page dependence on a text book, it would be important to give the teachers questions to ask the children that helped **not only in evaluating but in making clear where the next teaching steps were to be placed.**

A pictorial format was devised so that young children could fill in part of it themselves. An example is appended. The teachers had agreed after their research that a dependence on finger counting indicated a too rapid rush into narrow algorithms at the early stages: matching the book again. So a question was laid out to check which counting method children preferred. If the child felt obliged to say that s/he counted 'in the head' then a further question gave him/her a chance to show if counting mentally was preferred or not.

Separate from the child's sheet there were a set of questions for each stage of primary school which could be read to the child by the teacher or a friend. The covering letter asked that the questions be put as a letter from me to each child. The child's answers could be scribed by the teacher or, where indicated, recorded by the child.

The questions were planned so that a development at different stages would be clear to the teacher and differences spotted between children who 'remembered' answers and those who were able to work them out. I knew that the results sent to me would convey far less information to me than would be presented to the teacher when she posed the questions. I would have the bare answers but the teachers would observe the body language and know the time taken to find an answer AND whether or not the child was idly boasting as s/he filled in the questionnaire.

However, there was still much for me to find in the answers both for my own interest and to pass back to the teachers.

I had one instance of a keen teacher who used the questionnaire on her whole school, not just her own class. Most teachers who sent back the questionnaires also sent comments about the year's interchange and expressed an interest in continuing.

What were my findings?

I sent puzzles to you through the computer.

1. If you remember doing a puzzle, draw a  in its box.
2. If you did some of it at home, draw a 
3. If you were happy about it, draw a ✓
4. If you found it too hard, draw a ✗

Questionnaire presented June 1994

There were two parts to the questionnaire. The first page was pictorial and designed to give me feed back on how many children tackled and could remember the various challenges I had set; whether or not they enjoyed them and took them home; what their favourite counting method relied upon. The second page was numbered using words with the expectation that answers to questions presented orally would be recorded there or scribed by the teacher.

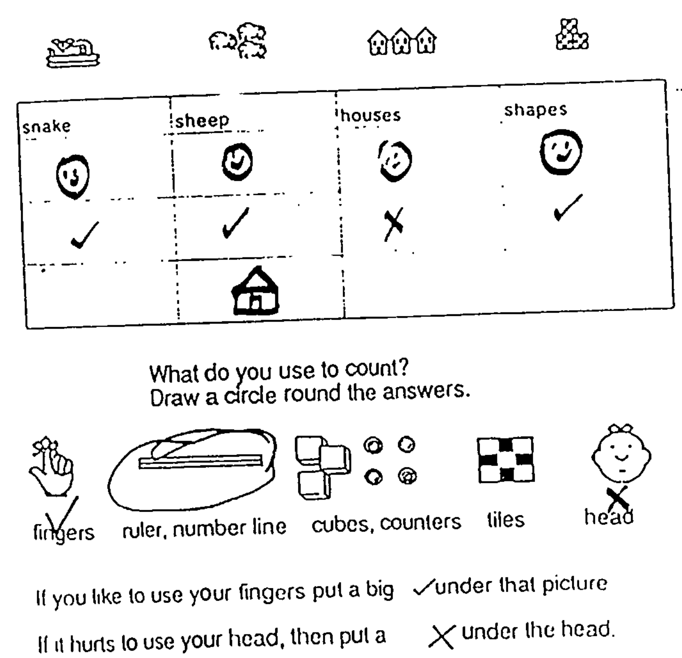
I wish to present the analysis of the second page first as it is a check that teachers have affected the children's thinking when tackling mathematics. The main purpose of the questions was to trace the development of children's ability to count in increasingly sophisticated ways as they progressed through primary. Could a real understanding of place value be discerned? It was expected that the oldest children would still be reflecting the results of a more formal teaching and that younger children would be showing some insights made possible through the opening out of mathematics teaching.

The set of questions as they were presented to the teachers can be found in the appendix but it is my intention here to group the questions to highlight the results.

The first question for all classes from primary 3 to 7 was:
What is the highest number you can count to?

And for the same classes the second question was;
What is the number after that?

It was obvious from the amount and kind of rubbing out on a number of the scripts that the children were influenced in their answer to the first question by either spotting the second question or relating back to it when they heard or read the second. So I shall consider them both together.



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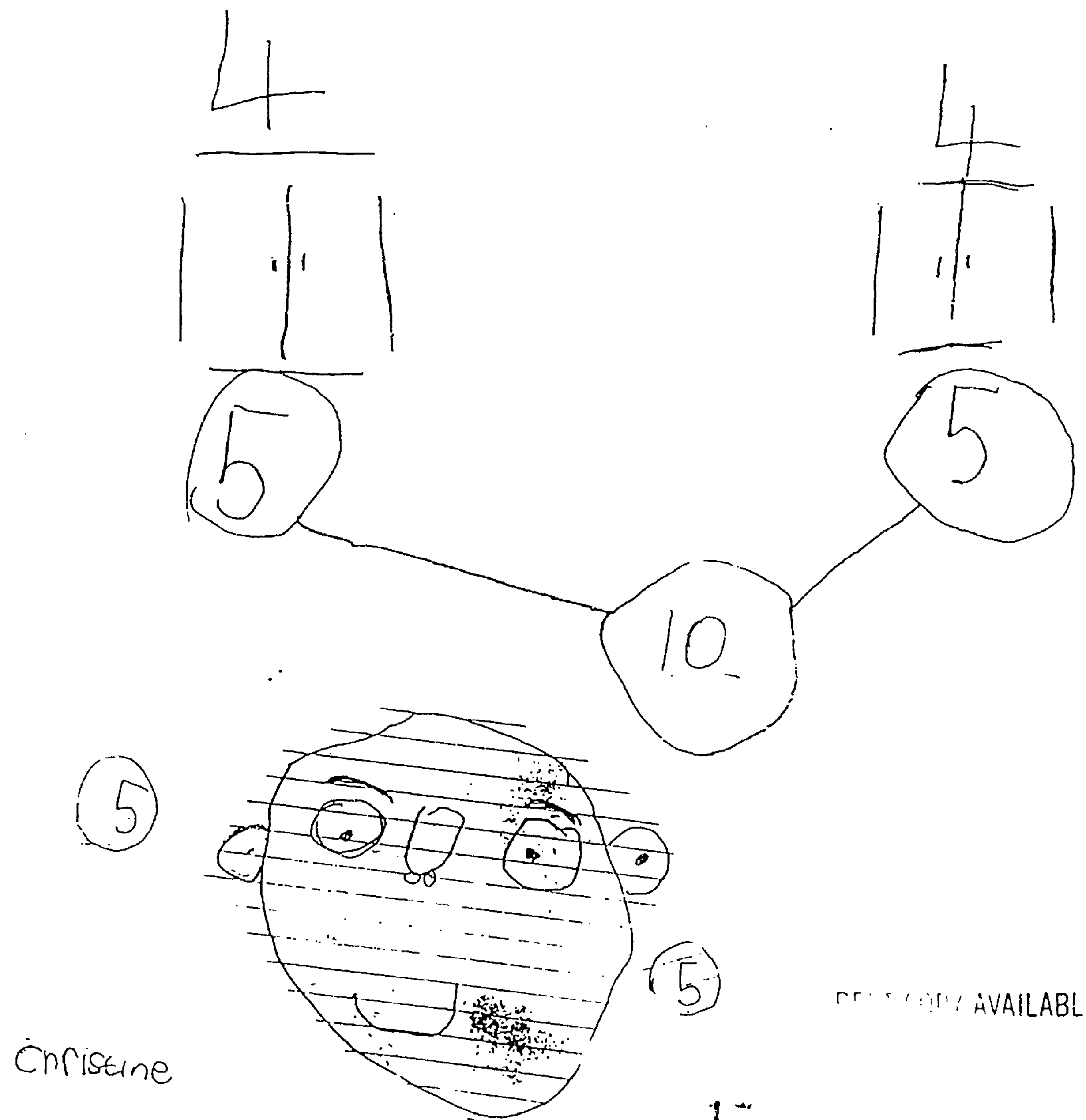
Primary 1 children were asked to count out loud until they got muddled.
Their answers ranged from **29** to **109**

Primary 2 had only to state the highest number they could count to, though some insisted on demonstrating, according to the teachers' comments.
Their answers ranged from **50** to **110** and in their case all answers were scripted by the teachers.

Exceptions

There was a seven year old child who was given the primary one questions and the highest number he could count to was **21**.
One primary 2 child wrote down her own highest count as **10090**, probably meaning **190**



Primary 3

The choice of first number lay between **100** and **1 000 000** with **no particular favourite** within the range and a strong possibility that children's own recordings were a misrepresentation of the number they chose. For example, **1004** and **1005** were probably meant to be **104** and **105** respectively in answer to questions one and two.

One delightful recorded answer to question one read,
"I've been counting past 100 in Inverness."
That child decided in the end that **101** was her highest countable number and **102** was the number that came after it.

Odd examples were **999** scored out and replaced by **1005** as the highest number that could be counted followed by **1006** as the next number to that: or **100** as the highest number with **102** as the following number.

Teacher's comment under one child's recording of **1000 : 1001**
"He counted to 199 and then got muddled."
In that case the teacher had double checked that the child could do what he said. However, I presume that older children interpret such a question as referring to the highest number they know.

Primary 4

The lowest number chosen in answer to question one was **51** but that was scored out and substituted by **100.** The highest number chosen was one million. **The most common number chosen was 1000.**

Primary 5

The lowest number in answer to question number one was **100** and the highest, recorded partly in words, **10 million**.

Primary 6

The **lowest** number in answer to question number one, recorded in words, was **two million** and the **highest** was **nine hundred and ninety nine thousand nine hundred and ninety nine million**. There was no particular favourite and a great variety of choices within that range.
(i.e. 2 million to 999 999 million)

Primary 7

The lowest number in answer to question number one, recorded in words, was nine thousand nine hundred and nine and the highest **nine hundred and ninety nine thousand nine hundred and ninety nine million**.
(i.e. 9909 to 999 999 million)

A general comment on the answer to question 2 by all primaries 3 to 7 is that **most** of the children chose a number in the first question ending in a **zero** and successfully answered the second question by **adding on a 1**.
e.g **1000 : 10001**
or **a million** : a million **and one**

However some children preferred to make their highest count end in a **nine** and then attempted **to add on one**, a more difficult task.
Top marks went to two pupils, one in primary 6 and one in primary 7, who gave
999 999 million : one billion
as their answers to the first two questions.

The most common error was to add a digit to the left hand side of the first answer.
e.g. **1)** 500 000 **: 2)** 600 000
or 1) nine thousand nine hundred and ninety nine million
2) ten thousand nine hundred and ninety nine million

Children who decided to choose numbers with **lots of nines** tended to make the error of forgetting one of the places in the sequence.
e.g. 1) nine million nine thousand **and** ninety nine : 2) ten million

The more cautious did better on this one and the more daring created difficulties for themselves. What I wonder is how many spotted, by answering the question, that there was no way of putting a ceiling on your highest number once you realised the rule of adding one. Perhaps some did but were signalling in the answers that speaking or recording a very high number was a demanding linguistic and sequential feat.

Only one child answered,
"I don't know the next number because I can't count any higher."

Neither primary one nor primary two children were asked the question,
"What is the number after that?"

Recent research has shown that children who love counting, count often and early, do well in mathematics. (Fuson) So the first question was designed to discover what primary one children could do by way of oral counting and with the older children to get a notion of what they considered to be their counting limit. For the older ones the second question was a search to see if they had thought about how the number system works.

Primary 1 children were asked to count out loud until they got muddled.
Their answers ranged from **29** to **109**

Primary 2 had only to state the highest number they could count to, though some insisted on demonstrating, according to the teachers' comments.
Their answers ranged from **50** to **110** and in their case all answers were scripted by the teachers.

Exceptions

There was a seven year old child who was given the primary one questions and the highest number he could count to was **21**.
One primary 2 child wrote down her own highest count as **10090**, probably meaning **190**

Primary 4 Jacob

The Way You Count

one. 500

two. 501

three. 150

four. 1099

five. 321

six. 49/39/29/19/9

20

Question 3 was designed to check children's ability to sequence, so too was question 2 for primary 1 and two. These will be dealt with together.

Primary 1

2) What number is between **two** and **four**? All the children managed this.
3) What number is between **seven** and **nine**? There was only one error. **10**

Primary 2

2) What number is between **fifteen** and **seventeen**?
3) What number is between **thirty six** and **thirty eight**? No errors recorded.

Primary 3

3) What is the lowest number between **90** and **100**?
Only one error recorded - **93**

The question for the early years made plain that most children are well versed in the word order of the smaller number sequences and can find an in-between number or next number from the contiguous number name sequence they carry mentally.

The isolated error in primary 1 points to a **'next'** rather than **in-between** number which is not a terrible error at that stage. The primary 3 error is in the lower half of the number set between 90 and 100. So in neither case is the answer 'wild'.

The questions for primaries 4 to 7 were increasingly testing of an understanding of place value. It might become clear with their results that earlier teaching had denied them a true understanding of how the number system works.

Primary 4

3) What number is **half way** between **100** and **200**?
The most common error was to give **50** as the answer (24% of the sample did so).
One child wrote **105.** Whether that is a recording or a place value error I do not know.

Primary 5

3) What number is **half way** between **1 000** and **2 000**?
60% of the sample wrote **1 050**
One child wrote **1099** - no doubt answering an imaginary question about the nearest number to **2 000.**

The middle primary pupils in our sample are clearly having difficulty with the recording of the **worth** of a number. During my school visits I spent some time trying to persuade teachers at primary two and three of the lasting benefit of children 'inventing' the number system as I demonstrated in the **Fantasy Islands** topic and in the workshop. Using the electronic 'guess my number' game during my visits I was able to pick out the children as far as primary five who did not have a good 'feel' for where numbers were placed over a sequence of tens when no near number clues were available. So it is understandable that children will need help to 'place' the numbers in the higher sequences. **So often the concept is left to chance,** the more so when the concentration in number work is mainly on an algorithmic approach. Children so deprived of understanding how the system works will not make good use of calculators and will find estimating in measure increasingly difficult.

Primary 6
3) What number is **half way** between **4 000** and **5 000**?

All but one child in the small sample chose **4 500.**
The odd one out chose **3 000.**

Primary 7
3) What number is **half way** between **1 million** and **2 millions**?

Only one child answered in words, **one and a half million**, making it the easiest answer in the whole set! Unfortunately he spoilt it by adding a second sentence in explanation.
Just over half the sample set managed to get the correct answer.
The error variations were, strangely enough,
not using the digit **5**
e.g. **1 100 000** or **1 000 100**

or misplacing the value of the five
e.g. **1 005 000**

and one child gave **1 million** as the answer.

The errors in this set were very similar to those of the primary 5 pupils.

The Way You Count

one. Nine thousand nine hundred and ninety nine million

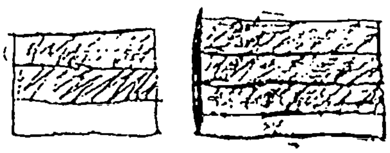
two. Ten thousand nine hundred and ninety nine million

three. One and a half million
One million five thousand

four. Two million ninety nine thousand nine hundred and ninety nine

five. A third of 270 = 90
A third of 2700 = 900
A third of 27000 = 9000

six Three quarters.



Question 4 for primaries four to seven was designed to check that they knew the number just before a change to the next place value position (power of ten). An understanding of this gives the child flexibility in calculation and frees the pupil from a crazy use of the decomposition method.

Primary 4
4) What number comes just before **one thousand**?

Around 75% of the sample offered correct answers.

The most common error was **199**
Other error variations were **991** (reversal compounding error) **:1099 :1000 : 900**

Primary 5
4) What number comes just before **one million?**
This proved too difficult and only 20% of the sample gave correct answers.

The most common error was **10 000**

Other error variations were **10 099 : 10 199 : 9 099 : 1 099 : 1 000 014** showing marked similarities to the error types in the primary 4 sample set.

Primary 6
4) What number comes just before **2 million**?
This proved too difficult for all but a tiny fraction of the sample set.
Half the set answered with **1 009 999** in words showing a common belief that a million is equivalent to ten thousand.

Other error variations were **1 000 099** (one million and ninety nine) :
and an improbable **900 000 900 999**
(nine hundred thousand million nine hundred thousand nine hundred and ninety nine) which if written in figures, and the initial one million had been remembered, might have been close, in that there were five mentions of the word **nine**
1 099 999, just one short of the correct answer **1 999 999** a mere **900 000** short!
And a clear case of linguistic mismatch.

In the **Talking Mathematics** booklet that I distributed to the teachers there is a section describing interesting examples of activities to help children grasp an idea of such large numbers linked to collections of such tiny and beautifully patterned things as butterfly eggs. Software designed for the **CATO** topic **Animal Lands** has examples of the same idea in the same context which helps children to build on earlier place value ideas so that they garner a sense of these large numbers, now so commonly used in the media in relation to populations, scientific discoveries and environmental studies. It is clear that the children's results vindicate my view that this is an area that needs attention.

Primary 7

4) What number comes just before two million one hundred thousand?

Only a very small fraction of the sample was correct.

Error variations that were most common was a lack of completeness in understanding each place value below one hundred thousand.
e.g. **2 999 000 : 2 990 999 : 2 099 999 : 2 090 099**

Then there were the errors where 'nine' did not figure as part of the solution.
e.g. **2 million and 2 001 000**

and the oddball answering a different question
e.g. **1 999 999**

Primary 1

4) Tell your teacher the numb~~er~~ that comes before ten.
All the children knew the ans[wer to t]his one. So there is a great start.

Primary 2

4) Tell your teacher the number that comes after **one** hundred.
In one school 50% of pupils said **102**
Could this be the beginning of a confusion in language? Two must **come after** one. There were many who said **200** and some wrote their answers, giving me either **1001** or **1100**, the first being a mis-writing of **101** and the second being a conviction that eleven must figure next as 10 is already there.
These were interesting and unexpected results, mainly because I made an error in the question which should have read,
"Tell your teacher the number that comes just before 100."
to fit in with the pattern of the others. In their answers the children are giving us their hypotheses about how the number sequence continues at a stage where the teacher has not yet dealt with it.

Primary 3

I did not include a similar question for this stage and think now that I should have asked them what number comes just before **200.** Instead I was more interested in a common place value confusion that occurs at the primary three stage and the results of that are gathered in the next section.

Conclusions on Place Value Understanding

It is too small and incomplete a study to draw conclusions about the 'dawnings' of place value but it **is** clear that there is not a good link for older children between the language used to mark place value and the worth of the numbers. The older children are clearly not tallying off the number words against a sequence of places. It would have been interesting to have presented these questions with a milometer type instrument available. In the same way that children need practice in creating or inventing the number line to one hundred as demonstrated in the **Fantasy Islands** topic, they need similar language and practical activities before they can truly understand the worth of the higher numbers and the way we name them.

Primary 3

The equivalent place value questions for this group concentrated on their understanding of the place value of similar looking digit pairs and whether or not they understood their worth. There were two related questions.

5) Write down the numbers seventeen and seventy one.

6) What is the difference between them?

Only one child in the sample reversed the digit **7** to make a symmetric numeric palindrome of

71 1Γ but this was scored out and written again as **17 71**

The same child had an interesting written sentence that had been erased rather poorly and read something like this,

"...65 has 71 that 65 is hayer... " with **64** written on top as the answer to question six. I would like to imagine that the child was trying to say that 17 and **65** is 71and so that is how much higher 71 is than 17. Although she got the amount wrong the placing notion would have delighted me as an insight into how she saw the difference. I suspect she looked at a neighbour and thought better of her unusual answer.

Another child had attempted to answer question 6 with an equation which looked like this :- **17 - 17,** corrected to **71 - 17 = 11 the other 7** Then all of this was scored out and a vertical decomposition 'sum' was laid out beneath with the 7 in 71 scored out and replaced by a 6 with the resulting answer of 54.

That equation is the only indication in the whole sample that the **10** and the **1** had been spotted giving the **11**. Unfortunately, the child was recording her own answer and we have no comment from the teacher of what she was attempting to say.

A further pupil in the same class wrote out a similar algorithm but omitted to score out the figure 7 resulting in an answer of 64, a daft answer if the worth of the two numbers was known. **64** is too close to **71** to make **17** a reasonable difference between them.

In that particular subset of the sample not one child mentioned the order of the digits.

In the remainder of the sample set practically every child had comments scribed for them by their teachers concerning the order of the digits in the two numbers

71 and **17**

"They are back to front or swopped around" being most common whilst others pointed to the digits and stated which was first and last in each pair. One pupil mentioned that 71 was a much higher number.

In one subset of the sample no pupils attempted to work out a numerical difference apart from the comment of 71 being a higher number. In the other samples some children set out the decomposition algorithm with a variety of replacing and scoring out of the various digits.

In truth, I had not expected them to do that. **No child commented on the relative value of the two numbers** on a number line by pointing out that 17 was near twenty and 71 near seventy; or on the actual value of the digits in the pair, i.e. that there was a ten and there was a one or there was a seven and a seventy, which is a little surprising at the end of primary three.

Primary 1
There were two further questions for this stage. One was to check how many children were still reversing single digits when recording and the other a similar check on 'next number' for the primary two and three classes.

5) Write down number **six**. All were correct
6) What number comes **after twenty**?
Most children were correct but one child had three attempts, one written over the other so that it was hard to decipher the final choice out of **19, 20 and 21.**

Only a small proportion of the sample set wrote the answer to the last question the rest had it scripted for them. The answers indicated that the children had a good grasp of the counting order of number names beyond those commonly written down at that stage.

One child omitted to answer the question.

Primary 2
The last two questions for this stage were to check for written reversals and to check recall of number order in the number set outwith that commonly used at this stage.

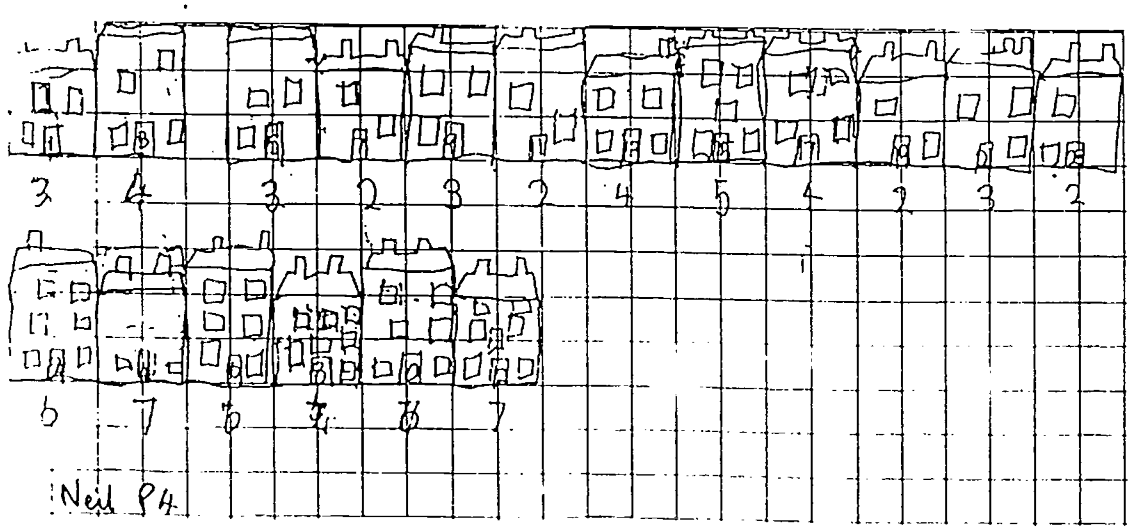
5) Write down the numbers sixteen and sixty one.
6) Tell your teacher the highest number betwen twenty and thirty.

All the children managed to write **sixteen** correctly in figures.
One child reversed the digit **6** in **sixty one** though the digit order was correct.
One child wrote **60** and then overwrote the zero with a one, showing clearly that it was to be recorded as a two digit number.

Most of the children gave 29 in answer to question six and only one child omitted to answer it.

There was a single error of **21** and one child wrote **301** (presumably meant to be 31), scored that out and replaced with **29.**

The only errors or omissions were from a sub sample set where the children had written out their own answers and where it appeared that they had worked on the questions on their own.



Sequencing

There was a similar question for each stage from primary three to primary seven on sequence patterning to see whether children would simplify numerical problems by attending to a number pattern. An understanding of place value is implicit in the questions which offered a check on the gradual formal application of the four operations as children progressed through the school. The examples will clarify this.

Primary 3

Tell your teacher the answer to **9+6, 19+6, 29+6, 39+6, 49+6**

Almost 100% correct. One child answered **49+6** as **56**, the only error in the whole sample. The pattern was clearly spotted and the operation of addition in this layout was a doddle for primary three children.

Primary 4

Tell your teacher the answer to **56 -7, 46 -7, 36 -7, 26 -7, 16 -7**

Only 62% of the sample were absolutely correct. The primary four returns were the largest in the sample and so this was a relatively large set. Most of the errors clearly displayed pattern but more often one consistent with subtracting ten than seven.

e.g. **49, 38, 28, 19, 9**
49, 36, 26, ---, 9

There was one figure reversal of the figure 3 in a correct pattern sequence.

There were two sequences giving **16 - 7 = 8** and one **16 - 7 = 10** indicating that the children were calculating each subtraction and not paying regard to the pattern. Of course there is no way of telling how many correct sequences were worked out laboriously but there is a higher possibility that the pattern was spotted.

Two very odd responses were :-

49, 11, 18 and **49, 36, 29, --- ,9**

Although subtraction is giving more problems than addition, the sequence chosen would be a really easy one if children had created their own number lines and used them a great deal in oral play. The simple pattern would be to see **seven** as **a six and a one**, then each subtraction in turn would be one less than the next lower ten number. A firm knowledge of the sequences **49, 50: 39, 40: 29, 30: 19, 20 and 9, 10** would render the answers without strain.

Primary 5

Tell your teacher the answer to

3 X 9, 30 X 9, 300 X 9, 3000 X 9

This question revealed an amazing variety of answers.

100% of the sample set correctly answered **3 X 9** and then the results plummeted.

22% knew the answer to **30 X 9**
and the most common error was **127** instead of **270.** Why?
Other variations in the errors were **207, 120** and **990!**
Has there been a confusion between adding and multiplying? **9 + 3 = 12**
Clearly place value is at the root of the children's difficulties.

One third of the sample did not answer **300 X 9** or **3000 X 9** at all and those correct so far continued with their pattern successfully.

Error variations were first, most commonly, **927**, then **1020** and an oddball **600**
Similar error variations followed with **9027**, **1209** and **6000**, the last pupil seemingly correctly answering the question **200 X 3** and **2000 X 3!**

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Primary 6
Tell your teacher the answer to
27 000 divided by **9, 2 700** divided by **9, 270** divided by **9**

This was more heartening as just over 70% of the sample set were correct for all three. However, one error was most intriguing and I would like the reader to become a sleuth with me to understand the child's difficulties.

27 000 divided by 9 = 17 100
2 700 divided by 9 = 180
270 divided by 9 = 18 r 2

Let's try talking it through with the child.

27 000 divided by 9		
2 divide by 9	I cannot do, go on to the next figure.	
2 + 7 = 9		
9 divide by 9 = 1	(write this down)	1
70 divide by 9 = 7	(write this down)	17
And there's 100 still to go	(write this down)	17 100

Let's try the next one, too.

2 700 divided by 9			
2 divide by 9	I cannot do, go on to the next figure.		
2 + 7 = 9			
9 divide by 9 = 1	(write this down)	1	
Child looks back at the sum in a right to left direction and sees **70 and 2**			
72 divide by 9 = 8	(write this down)	18	
Child looks at the sum and realises one zero has not been worked	(write this down)	180	**Eureka!**

To the child all of the above is perfectly logical. Here we see manifest an amalgum of well learnt facts. This is the result of building on sand. The quick fix and wee rigmarole worked at the time but one thing it has never done is provide a stepping stone to understanding. Dear Reader, I leave you to try to figure out how this child came by his last answer.

Primary 7
5) Tell your teacher the answer to
a third of **270** , a third of **2 700,** a third of **27 000**
All of the small sample set managed this correctly.

Notions of Measurement

The final question for primaries four to seven was designed to assess children's notion of measurement in its broadest sense.

Primary 4
What is the biggest number you can write with the figures one, two and three.

About 87% of the sample set answered **321**.

There were a small number of errors of other combinations of the number, **123** (most common), **312** and **231**.

The best answer of all was spread right across the page and then was erased. It was

33333333333322222222222222111111111

Once more a lateral thinker caved in to conformity.

Primary 5
6) Which is bigger, a third or a half? Show me why.

80% of the sample set were correct and offered a variety of illustrations and comments in response to the demonstration request.

Most of the children drew circles or half circles which **in no way** corroborated their answers. One child drew a circle and a triangle, the circle showed a diameter line, and added the comment that **a third has three pieces.**
Another child drew a bisected circle alongside a circle with three 'jagged teeth' inside and added the explanatory sentence that a third needs 3 piceics (sic) and a half only needs two. Only two children drew equivalent bars and correctly subdivided them to create a proper comparison. One child who chose the third as the bigger fraction wrote that 3 is more than 1.

I think we can safely conclude that the primary five sample had a good intuitive notion of which was bigger but no real understanding of how to prove it.

Primary 6
6) Which is bigger, a third or a quarter? Show me why.

90% of the sample set chose the third and were correct but most of them drew circles as their proof. In the main they inscribed the similar sized circles side by side in thirds and quarters reasonaby accurately but did not shade in the appropriate fraction assuming that the 'look' was sufficient to differentiate. Only two children drew a measuring bar. One was accurate and clear but the other showed two subsets in line below, the third appearing much smaller than the quarter, yet the third was given as the bigger fraction.

Chapter 7
The Children's Own Assessment

The children were asked what they used to count and this is an analysis of their responses taken class by class.

Two further questions were asked to try to find out the preference between counting on the fingers and calculating 'in the head'. So there are two percentages given for the head. The one under the √ indicates that the head cartoon was circled and the one under the **x** indicates that the child did not like to count 'in the head'. Some children circled the head and placed a x, just as I expected. They would not admit that they could not or did not count mentally but were happy to show me a preference.

What do You Use to Count?

	fingers	ruler or number line	cubes or counters	tiles	head √	head x
P1	100%	0%	100%	0%	100%	90%
P2	100%	0%	100%	0%	0%	100%
P3	75%	5%	50%	0%	75%	25%
P4	75%	25%	12%	0%	60%	40%
P5	**20%**	**0%**	**0%**	**0%**	**100%**	**0%**
P6	90%	0%	0%	0%	90%	10%
P7	60%	0%	0%	0%	100%	0%

The primary five self assessment is interesting in the light of the difficulties demonstrated at that stage in the number assessment. On my visits I was so surprised by the range of resources used by the children, including those at the primary five stage, to count on in ones that I made a study of that factor the basis for a small action research study by the class teachers. Children nodded, tapped, patted heads, counted along ceiling tiles, up blackboard rulers, across floor tiles, around fingers; you name it, they tried it, to support this most basic of counting strategies. It is interesting that large numbers of children in upper primary are happy to admit to finger counting and yet deny that mental counting is difficult for them, as shown in the above table.

It has been my experience in other schools that primary five children strongly resist the use of any counting materials at precisely the point when structured materials such as Dienes or Tillich bricks would consolidate an understanding of place value. The children's belief is that using such materials is 'babyish'.

I have tried to counter this resistance by simulating the use of such material in computer programs within contexts that disguise the 'wood' and lead children to use the material in imaginary contexts. I have found it necessary to encourage teachers to supervise such use, in particular the ten units for one ten exchange activities, to minimise children's tendency to 'count-on-in-ones' even here, thus undermining the worth of the material. In fact I think it is this time wasting tendency that disheartens teachers from persevering against the resistance. Once the teachers see the value of training the children to lay out prepared sub unit sets of ten in patterns of 4 plus one, a procedure profitably placed in the hands of a slower learner in the class, then they become enthusiasts.

So often, children are held back at this stage by the poor links made from their sub-factoring in primary three and four; this is particularly so of the younger children in each class. Whilst they are making the transition to using larger powers of ten, they need to see visual props that helped them see common factors in the smaller number sets. It is too easy to hurry them on in order to get written results. See page twenty four of this report for a not untypical example of this process.

If there is an ingrained use of finger counting holding back children's ability to use number sequencing and patterning, it might be better as a remedial action in the later primary years to deliberately teach the children more sophisticated forms of finger counting. There is a good base five, single handed version illustrated in Karen F. son's book, "Children's Counting and Concepts of Number"

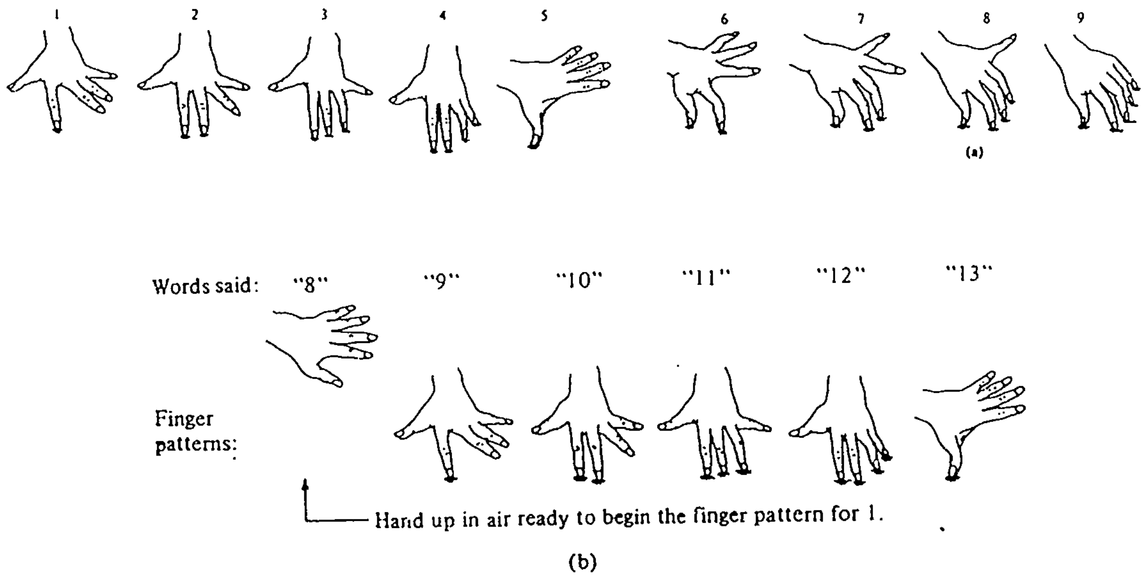

Figure 8-2. Counting on and counting up with finger patterns. (a) The finger patterns for 1 through 9 are made by touching certain fingers and/or the thumb to some surface such as a table. Thus, there is kinesthetic as well as visual feedback for the finger patterns. The finger patterns use a subbase of 5. The thumb is 5, and 6 is the thumb plus the 1 finger (6 = 5 + 1), 7 is the thumb plus the two fingers (7 = 5 + 2), etc. The motion from 4 to 5 is a very strong and definite motion—the fingers all go up and the thumb goes down, all in one sharp motion with the wrist twisting. The finger patterns in (a) are the patterns used in Chisanbop (Lieberthal, 1979). We use the finger patterns differently from the way in which they are used in Chisanbop. We use them (b) in the way that children spontaneously use fingers on both hands to keep track—the fingers just match the counting words that are counting on through the second addend (or counting up to the sum in subtraction).

Addition
8 + 5 = ?

The counting-on procedure:
1. Count on 5 more words from 8
2. Stop when finger pattern for 5 is made.
3. Answer is last *word* said (13).

Subtraction
13 − 8 = ?

The counting-up procedure:
1. Count up from 8 to 13.
2. Stop when say 13.
3. Answer is what the *hand* says—the finger pattern for 5.

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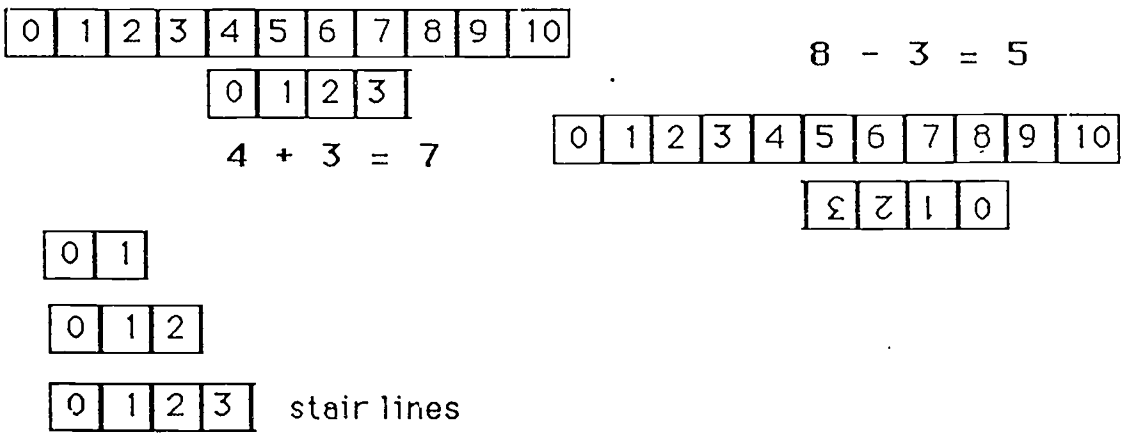
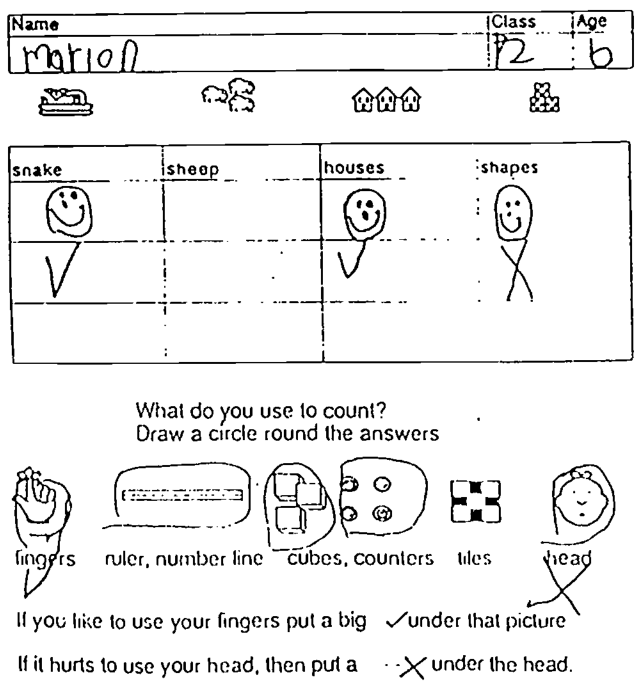
Finger Counting Strategies

During the workshops I demonstrated the method of using fingers for the nine times table which turns the hands into a miniature place value machine and leads to lots of number formation discussion. Also, I taught the teachers how to multiply any pair of numbers between 5 and 11 using the hands as base five machines, a common method in the Mediterranean countries which has an elegant mathematical explanation teachers can be helped to work out

The lack of indication of use by the children of ruler or number lines is curious because it is common to see small number lines pasted on the desks of the younger children and all children have access to rulers in class. Are the hand made number lines being used only for number recognition? Perhaps the teachers should make more use of them by making available sets of 'stair lines', with zero start places marked in, as a further embodiment to encourage connecting the smaller number bonds, thus making finger counting too unreliable by comparison.



Note that in primary 4 and primary 6, see page 26, a fair number of children admitted that they disliked counting mentally, yet it should be one of the comfortable achievements of childhood. Perhaps the greater lack of need for regular small number counting in our everyday life is detracting from this common and often joyful experience of childhood which is necessary to support confidence in a developing understanding of how our number system works.



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Chapter 8
The Challenges

An example of how the first challenge was downloaded and sent to teachers follows. On the computer screen it would not be so well defined, being simply a screen of text without colours or lines. All the children who tried this challenge marked their sheets as having remembered it and enjoyed it.

To T Watterson, Jordanhill Campus

From K A Matheson
Computer Resources Coordinator, Ed. Dept.

Date: 21 October 1993

Tricia Watterson, Strathclyde University (Jordanhill Campus), has arranged to correspond and maintain contact with staff attending her previous INSET courses, via the Bruetel viewdata system. Below is a printout of her first correspondence which you will also find in your school's mailbox on Bruetel.

To reply to her Bruetel Mailbox use User Group 292# and Short Name JORDAN.
Should you require any practical assistance communicating via Bruetel please contact myself or Aneas Maclean, Computer Programme Development Staff Tutor.

From Tricia Watterson
Strathclyde University Jordanhill Campus
Date 15/10/93
Time 11:24

Challenges that raise questions give opportunities to practise skills already acquired and so build confidence. I hope to send an interesting challenge most weeks. Please read them to your children.

They can be started in school and taken home to be talked about. If anything interesting happens in your class through a challenge pop the child's work or comment in the post to me - name, age, class and school: or drop a note in my mailbox on by Bruetel.

Many thanks.
Tricia Watterson.

CHALLENGE 1	Autumn - wet days, falling leaves and wiggling worms.
P1/2/3	Make a worm of plasticine as long as your desk. What is long? Why? If you join the tail to the tip what shape do you think the worm will make? Tell someone or show them a shape in the room it will be like BEFORE you try it. Do it and draw a picture of what happened.
P2/3	Will the closed shape be too big or too small for your desk top? What size is it?
P3	How did you find out? Draw a picture and write a story to show me.
P4	Ask 2 more questions of your own and try them out on your friends.
P5/6/7	Size is a tricky word. Use a tape measure as your 'worm'. Choose a length with lots of factors, eg 36. Make the longest rectangle you can enclose with it, tip touching tail. On grid paper mark out the shape and give all its measurements. How many are there and how do you label them? How did you decide what was the length? Will another 'fatter' rectangle made with the same length of tape be different in all its dimensions or not? Talk this through with a friend at home or in school. Show your thinking with a plan, a table of results and a story or explanation. (Dressmakers think 'long' means up and down the way.)

The accompanying questions for the first challenge which would be sent by electronic mail about a week later follow. The week's gap was deliberate to give the teachers time to do their own thing with some of their children first. It would prompt them with questions that they might not have considered or reaffirm their belief that they were working along the correct lines. My purpose was to help teachers who had had little experience of a more open approach to teaching mathematics. They need to know how to do this but they need to grow into it by developing from where they are.

To T Watterson, Jordanhill Campus

From Tricia Watterson
University of Strathclyde Faculty of Education
Jordanhill Campus
76 Southbrae Drive
Glasgow G13 1PP

Date 29/10/93
Time 15:17

Subject Challenge 1 - Comments for Teachers
(The following is a printout of a message sent to your school's Bruetel Mailbox)

With Primary 1 we just want to know the words they use for size and exactly what each word means to them in practical terms.
Is long an across or up and down word.
Do they make the 'worm' straight across the desk, diagonally or quite wiggly?
Can they imagine a shape before they form it?
Have they names for closed shapes?
Is their picture an accurate representation of what they did?
When asked to tell what it shows do they use size words appropriately?
Did any of them take the 'worm' and measure it against themselves?

Primary 2 - all of the above and how did they tackle the ambiguous question?
Too big for what?
Or did they use comparative words and point out that the'worm' was bound to fit on when curled round?
Did any of them think the size had changed?
Did you ask to see?
Did any of them show some idea of measuring by laying rods or blocks alongside, inside or all the way round the 'worm'?

Primary 3 - all the above. Did any of them show a distinct developmental change compared with primary 2 in how they tackled the measuring?
What was the distinction?
Did they mainly go for the width of the closed shape with a ruler or by chance open it out?

Primary 4 - all of the above AND were there any surprising questions?

Many thanks.
Tricia Watterson.

To reply to T Watterson on her Bruetel Mailbox use:
User Group 292# and Short Name JORDAN.
Should you require any assistance communicating via Bruetel please contact:
K A Matheson (0851 84 291) or Aneas Maclean (0851 703564 Ext 143)

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The original target group for the workshops was the group of teachers of younger children up to the primary four stage. However, many primary four teachers were teaching composite classes of four and five. So the remit was widened. Some of the smaller schools had teachers of the whole age range or of the stages primary four to primary seven. To make sensible suggestions in mathematics, therefore, I had to consider the whole primary range. It is obvious then why the sample numbers in primary six and seven were inevitably very small.

To	T Watterson, Jordanhill Campus
From	Tricia Watterson University of Strathclyde Faculty of Education Jordanhill Campus 76 Southbrae Drive Glasgow G13 1PP
Date	1/11/93
Time	12:33
Subject	Challenge 1 - Comments for Teachers relating to P5/6/7 activities (The following is a printout of a message sent to your school's Bruetel Mailbox)

Look at the notes for the younger children just in case you have a child with no concept of size.

Did any child make a rectangle with no area by simply folding the tape in half?

If so how did s/he explain that it was a rectangle? And, was the child challenged by other pupils?

Did the children compare with each other and try to find a single 'right answer'?

In Desperate Journey there is a computer investigation, page 21, along similar lines based on choosing of size when building a croft.

Try that as a follow up if you have sparked an interest. It gives support & a model for following through a problem.

Many thanks.

Tricia Watterson.

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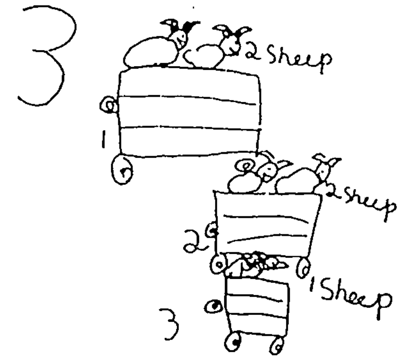


The kinds of comments that were returned were;-

If you stand up it's **long**.

The worm is **long** like Lionel school.

Long will be **thin** if you've only got a wee bit of plasticine.

It's a train if you put wee windows in.

It will be like a necklace....square.....flat thing....circle.

Primary four children thought that the worm would **not** be the same size if the head was joined to the tail.

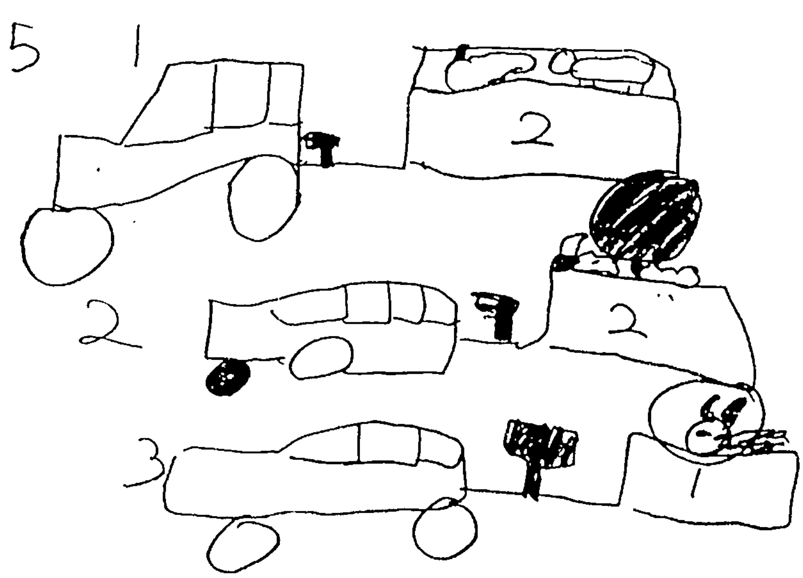
The most common answer about the size was simply that it is smaller now.

Some children could not believe that it would be possible to make a worm long enough to reach across the desk with a small bit of plasticine and when they succeeded they insisted that the plasticine was then a different size.

The second challenge was about sick sheep needing transport. One teacher was amazed that her primary three children did not know how to begin to solve the problem. Then the only child in the class whose father had a sheep fank set to and began to make suggestions.

Sometimes sheep catch cold too.
Primary 1, 2 and 3
We can get 2 sheep on our wee trailer. We have 5 sheep to bring down to the vet. How many trips do we have to make. Draw me a picture of your answer. Write in numbers, too.

The examples for the older children were similar but with more difficult number sets.



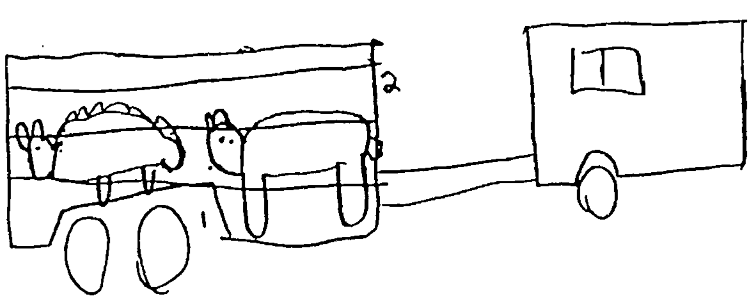
The supportive comments were:
What I want to know is how the children go about this one. Do they go successfully through the 3 stages -
1. Translate it into a maths calculation. 2. Perform it somehow. 3. Translate it back into a context.
The problem is arithmetically too difficult for primary 1 and 2 but they don't know this. How do they do it as a common sense problem? How do they record it if they bring numerals into their picture?
Do the older children make a reasonable guess? Do they get the correct arithmetic answer but the wrong practical one?

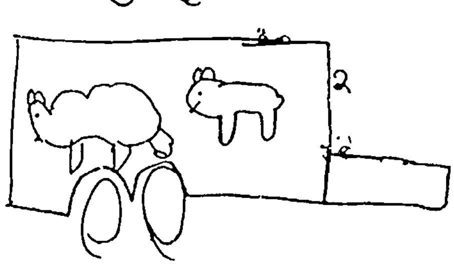
Its a trick son se 2 agus 2 agus an uairsin 1.

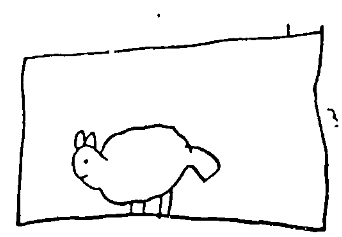
in English -

Its a trick because its 2 and 2 and then 1.

Three children in primary three spotted immediately that it was a 'divide sum' and shouted out ,
"It's 4 remainder 2!"
The error was ignored but they were asked what they would do about the 2 left over.
"Put them in the boot." "Squash them in"
Finally one of them suggested laying on the trailer again and that made three trips.







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Dark nights, lights in windows.
The **third** challenge was an open one and not liked so much by the teachers as there were many possible solutions. "We are baffled by this one!"
Yet it brought forth a rich variety of solutions from the children and, for me, was very revealing of teaching methods and children's thinking.

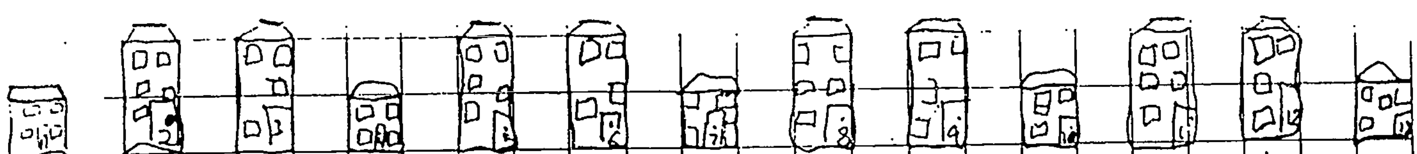
It differentiated clearly between those children who could work out patterns and those who were less attentive to details, or of a less persevering spirit.

Challenge 3

Dark nights, lights in windows. **Primary 1, 2 and 3.**
Draw a little house with a door and some windows. Next to it draw a bigger house with one more window. Next to that draw a house the same size with one less window. What kind of house comes next?

Can you go along the street keeping to your pattern? Write down how many windows are on each house. Write the number of each house on its door. Write or tell about your pattern.

Primary 4 and 5.
Following the same instructions draw two different streets of no more than 12 houses in each. The rent for each house is doubled for every window and trebled for each storey. If the window rent is two pounds, what is the rent for each house? Each street?



4 5 4 4 5 4 4 5 4 4 5 4 4

I have one house and with 4 window and thd

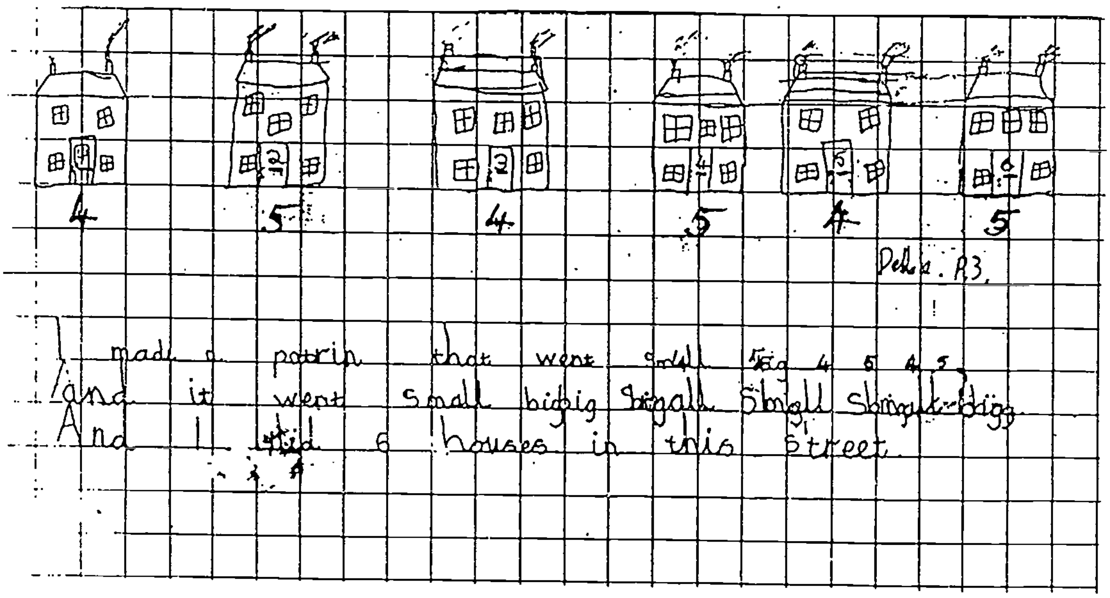
I have tow houses and one is divrant
because one has more window than the uther

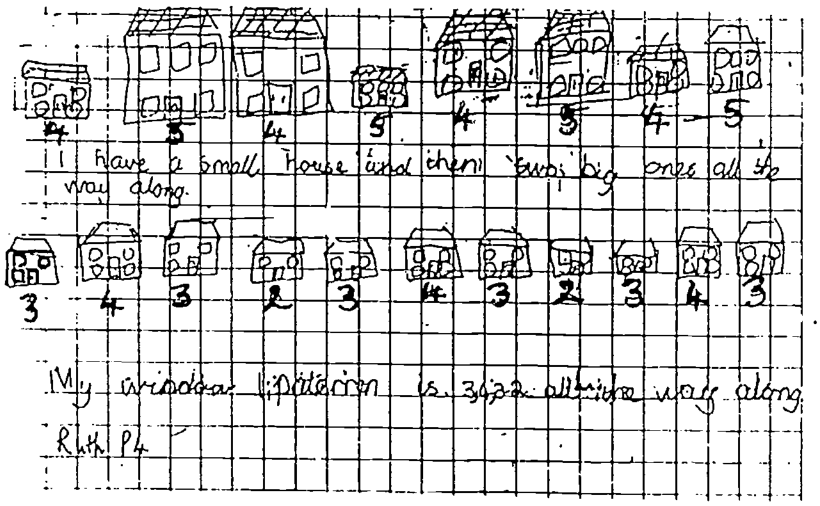
Billy P3

33

Amongst the younger group there was a good variety and a big effort made to write about the pattern so carefully drawn. Holding both a size and a number change in parallel provedtoo difficult, as expected, from some. Those that succeeded reiterated the starter pattern and did not innovate by extending the pattern. The children who put a lot of detail into their house drawing tended to lose their pattern as they went along.

Primary four children were asked to make two diffeent streets out of the basic pattern and so were forced to see another possibility. The question about rents was too difficult for them and was ignored. A number of them succeeded in producing two distinct streets either by varying the starter number of windows or by extending the pattern of size; the latter was the more testing. None of the primary four children succeeded in making both streets conform to a consistent pattern though many did with one. All of the children described the patterns they had created either by size or number pattern. No child attempted an explanation of how they decided upon a pattern. As expected, the streets were numbered consecutively and no child thought to make one street even numbered and the other odd.





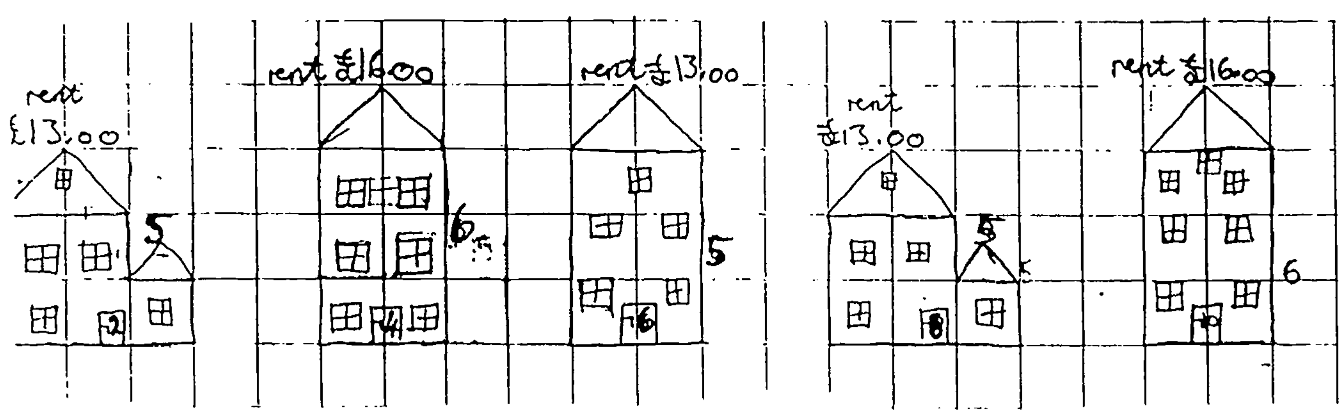
Primary five children did make a good attempt at working out the rents and about 50% of them numbered the street houses conventionally. Although some of them listed the rents in a way that clearly displayed patterning there was no evidence of use of it to make the total calculation.
e g. **£16, £18** and **£16** was a repeat pattern in one child's street of nine houses yet the **20+30** pattern within that was not spotted. Instead the answer offered was £78 when it should have been a much easier to calculate **£50 X 3** or **£150.**

I had no way of telling how the children would choose their starter window number but I had predicted that if they followed either the given pattern or some obvious extension of it, they would end up with some version of the patterned counting just described. Children in the habit of problem solving around number patterns would display their ability in such a task. However, the difficulty some children had in consistent pattern making served to undermine that end. Also, in attempting to be precise in my wording in order to fit the text of each challenge on a single computer screen, I made the possiblility of interpretation too wide. I had wanted the children to argue with each other and discuss with their teachers and parents, using their drawings, exactly what should be doubled or trebled, but I had not meant it to be as difficult as it turned out.

The children are told that 'a window rent is two pounds'. this should have read 'a single window rent is two pounds'. They are told that 'window rent for each house is doubled for every window and trebled for each storey."

I anticipated, as the children were either increasing or decreasing numbers of windows in each drawn house, they would either count the windows and multiply by two or argue that in a house of say three windows you start at 2, double to 4 and then double to 8. I was interested to note which way they interpreted this. If they chose powers of two, would they use the last value or total for each window; i.e. in this example would the rent be £8 or £14?

Next, I thought that they would draw 'small' houses as bungalows and 'big' houses with two storeys. Most children drew large and small two storey houses which meant that they had to consider what trebling meant for every single house. So they turned it into a more difficult task. Interestingly, some children worked it out to a quite different logic. In this example the observer must read **three** for **treble** and two for **double** and then add the threes and twos to reach the child's answers.



'ground floor £2 per window else £3'

I had expected the children to argue that if there was a bungalow then there was no trebling but for two storeys either the basic cost of the second storey windows was trebled or the total cost of all the windows was trebled. If the children were adventurous enough to draw three storey houses, a thing rarely seen in that local authority, then they might feel forced to argue that it would be £2 per ground floor window, £6 per next storey window and £18 per third storey window. The other logical sequence would be to double the ground floor windows, £2, £4, £8 etc., treble along the higher storeys, £2, £6, £18... The latter choice I did not expect middle primary children to make.

It is difficult to work out what combination the child used who gave £16 for a two windowed house and £18 for a three windowed house, each house appearing to be two storey type though the one with two windows had them placed on the ground floor. Was there a combination of powers and multiples? It is hard to tell.

The first street has one less one more one less one less. The second street is the same pattern but different numbers of windows.

street 1. number 1 £16 . . street 2 number 2 £18 street 1 Total £98
number 3 £18 number 4 £24 street 2 total £162
number 5 £16 number 6 £18
number 7 £16 number 8 £18
number 9 £18 number 10 £24
number 11 £16 number 12 £18
number 13 £16 number 14 £18
number 15 £18 number 16 £24
number 17 £16

Ann Po.

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The Power of Powers

The challenge for the primary six and seven classes was to play with the powers of two and discover the joys of so much pattern simplifying apparently difficult calculation. It may be remembered that as a result of exploring such patterns in the pre-calculator days tedious calculating problems involving long multiplication and division were simplified by the development of the principles of logarithms.

Challenge 3

Dark nights, lights in windows

Draw a little house with a door and some windows. Next to it draw a bigger house with one more window. Next to that draw a house the same size with one less window. What kind of window comes next?

Following the same instructions draw two different streets of no more than 12 houses in each. In your two streets the rents are fixed by a mad accountant. It starts at two pounds for the first window and doubles for each following window down the street until big house changes to small house. Then it halves until small house changes to big house when it doubles again. There is a shorthand way of writing these doubling numbers. Find out how to do it. The number of storeys makes no difference to the rents for each street. You can use a calculator to **check** your working.

Let me give the reader two examples to explore in a simple repeating patterned street that offers some of the excitement of finding quick ways to calculate. The first example shows a decision that **a move from small to big house means add a window** and **a move from big house to small house means remove a window.**
Keeping to the challenge, the move from a big house to a big house means remove a window. A further decision has been made that the move from a small house to a small house means add a window.
To summarise:

	small	big
small	**+1**	**+ 1**
big	**- 1**	**- 1**

Following the rule for assigning rent we produce a list of numbers:

2+4+ 8+16+32 +64+128+64 +32+16 +32+64+128 +256+512 +256

The child may decide to 'gather tens' or 'go for doubles'.
Let's **'go for tens'**

4+ 2+8 +16+64 +32+128 +64+32+64 +16 +32+128 +256+256 +512
4+10 +80 + 160 +160 **+16** +160 512+512
20+10 +80+320 +160+1024
+30 +400 +1024 +160
1454+160 = 1554 + 50 + 10 = **1614**

(Of course, it is clear that the houses improve immensely in quality as you progress down the street!)
It can be fun to **'go for doubles'**, too. Then it's best to start from the right.

2+ 4+8+16+32 +64 +128+64 +32+16 +32+64+128 +256+512 +256
2 +20+40 128+128 +16 +128+128 512+512
2+60 +256 +16 +256 +1024
2+60+512+1040 = 1102 + 512 = **1614**

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There are many permutations possible in the street arrangements starting from the basic plan in the challenge and continuing in a consistent pattern. Below would be a legitimate second street, beginning in identical fashion but resting on a different overall pattern. Here the rule followed for windows is simpler:

Move from small to big house - add a window.
Move from big to small house- keep same number
Move to same size house - subtract a window

	small	big
small	0	+1
big	+1	-1

This pattern, starting at two, will soon result in lots of no window houses!
Note the difference in the rent for the street.

2+4 +8+16+32 +64+128 +64 +32 +16 +32 +64 +128 +64
Either **'go for tens'**
10 +4 +80 +160 +80 +32 +160 +128
10+4 160 +160 +160 +160 = 320+320+14= 654

Or **'go for doubles'**
2+4 **+8**+16+32 +64+128 +64 +32 +16 +32 +64 +128 +64
rearrange
2+4 +8 +16+16 +32+32+32 +64+64+64+64 +128+128
2+4 +32+32+32+32 +128+128 256
2+4 +8 128 +256 +256 = 512+128+14 = 640+14 = 654
Which do you prefer? Would it be quicker on a calculator?

The following examples are further variations that the reader might check against the above for consistency of pattern and value to the mad accountant for rent. The rule making opened up by this activity is a valuable form of problem solving. It has the added merit of encouraging the children to continue pattern seeking and calculating in a self correcting manner. It offers extension work for the brighter child and a manageable challenge for the rest. In this type of activity the children set their own levels of difficulty.

	small	big
small	0	+1
big	+1	-1

	small	big
small	0	+1
big	0	-1

Now check out the children's examples and you will be more appreciative of their mental effort. It is not as easy as it looks. Unfortunately, the small number of pupils at the primary six and seven stage in the classes of the teachers involved in the project meant that I have nothing to show from them on this task.

Tessellation

The final challenge was a very simple shape making one using triangles and triangular spaces by tessellating to create number patterns. Uncovering parallel patterns with the square or rhombus was the development for older children. Many teachers wrote in to say that their younger or slower children excelled at this challenge whereas the children normally considered the best at mathematics found it very difficult.

Challenge 4

Take 1, then 2 then 3, then 4 triangles all the same size. Each time try to make a same shape triangle. You can colour in any spaces. Talk about the patterns you find.

(Primary 1, 2 and 3)

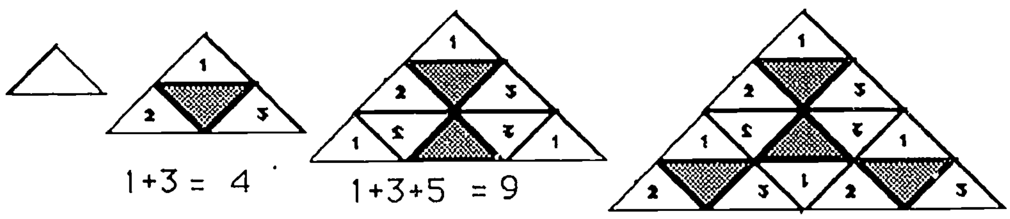
Do the same but then draw out a large triangle 4 times the size following your pattern. Talk and write about the patterns, sizes and numbers.

(Primary 4 and 5)

Do the same but repeat with squares. Talk and write about the patterns, sizes, numbers and angles.

(Primary 6 and 7)

I received many drawings both correct and incorrect but none of those that arrived had any numbers written on them and so I must conclude that the children did not spot the square number and odd number patterns in these different embodiments. It was obvious from the drawings that many children were very surprised to find that right angled triangles could be transformed in pairs to make similar triangles. The following shows the numerical connections I thought the children would be able to make if they worked right through the challenge. I do not doubt that there was a lot of discussion.

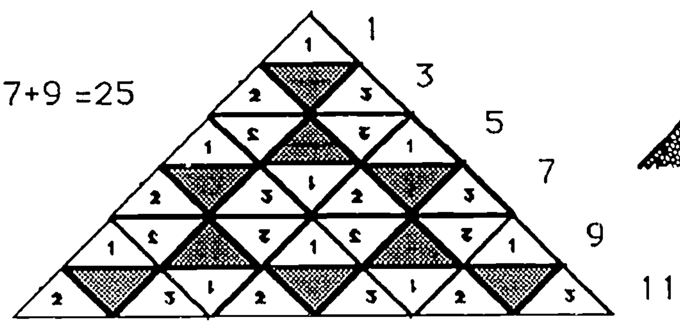


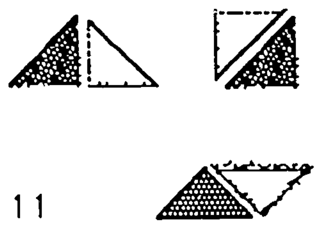
1+3 = 4

1+3+5 = 9

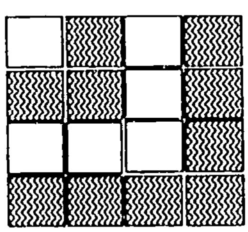
1+3+5+7 = 16

1+3+5+7+9 =25





1+3+5+7+9+11 = 36



1+3+5+7 = 4 x 4 = 16

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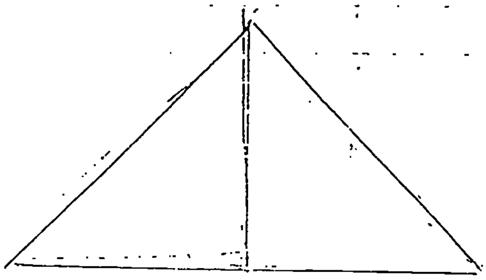
Problem solving

1



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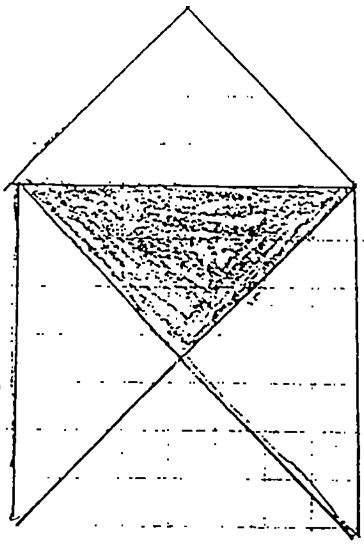

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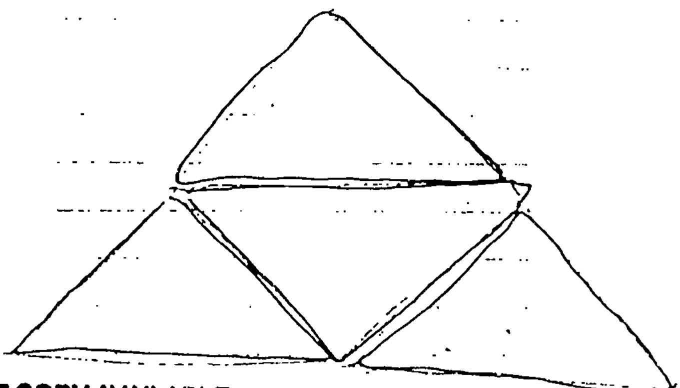

4

P4

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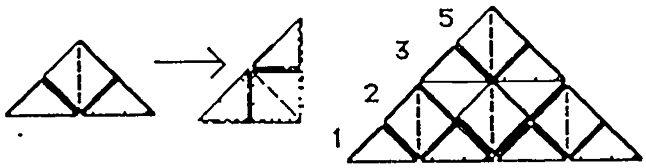
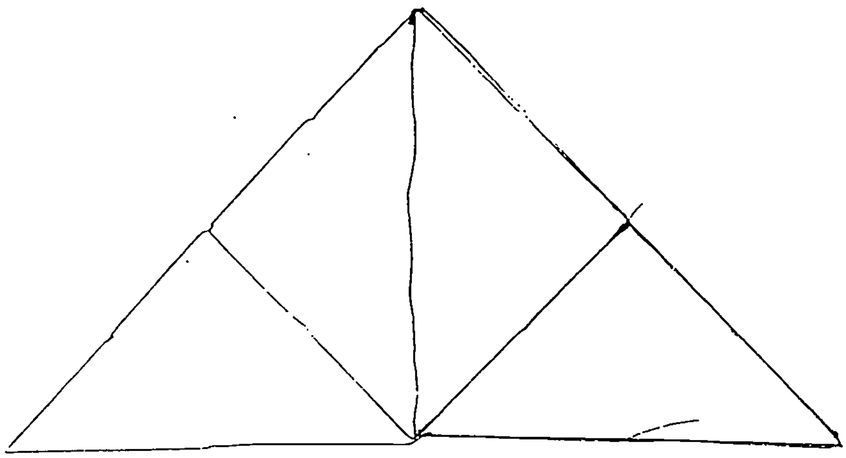
Children's Examples

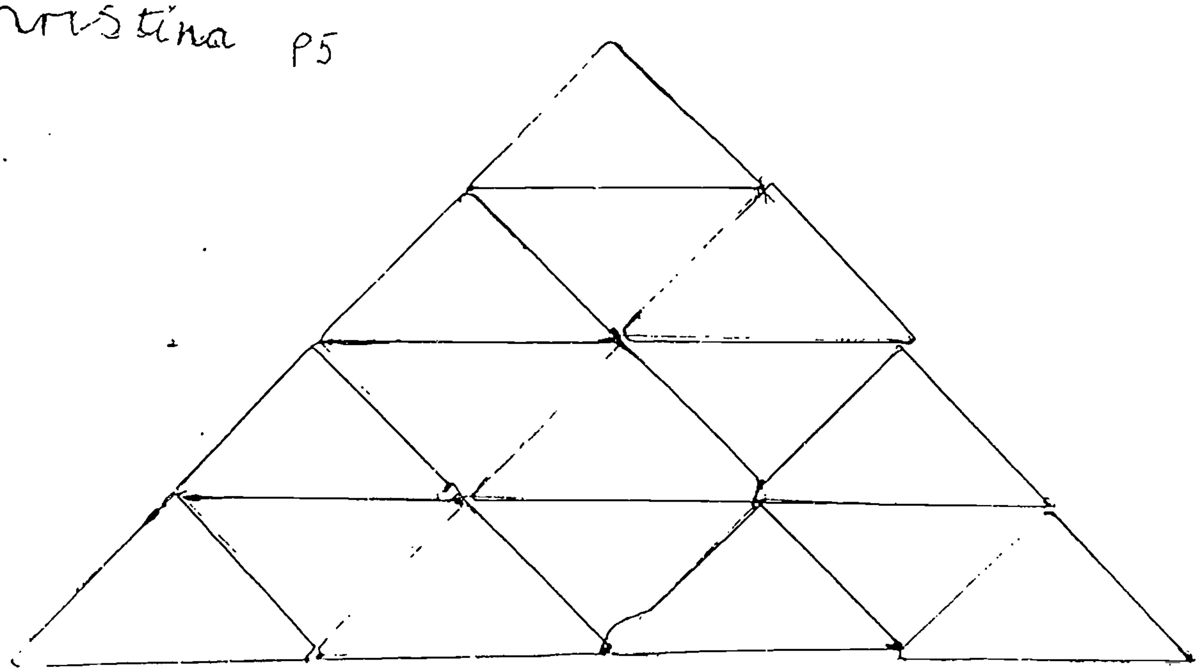
The example on the previous page demonstrates the difficulties the children have in in imagining a similar triangle with spaces. The first example on this page does not readily throw up the odd number pattern. It has to be turned through a quarter turn. It is also more difficult, in its orientation, to see how to 'make it grow'.



Annmairi P5



Christina P5



Conclusion

The moral of the challenges is that there is maths for everybody. As teachers we have to look for the right kind of problem in order to give every child the satisfaction of understanding just a little more every time.

Can I say that there has been a lasting effect?

I can say that nearly two years after the main workshop there are still a good number of teachers corresponding with me and each other about how their children are tackling mathematics even though the staff development emphasis has moved away from mathematics to the next area in the national curriculum guidelines. From the survey I can say that there is evidence of teachers talking more with their children about mathematics. From the worked examples of the challenges sent to me from the schools there is more drawing and writing being done by the children in maths lessons than before. From responses to my letter enquiring how the maths teaching had changed as a result of the two years of contact, the teachers tell me that they are using more games in maths classes and that they notice more, which in turn means more talk and real contexts for mathematical thinking.

Was the electronic mail, as a vehicle to reach a lot of teachers cost effectively, a success?

The problem was that many teachers in the remote areas suffer frequent breakdowns in telecommunications and are therefore not confident users at this time of the electronic mail. I have not given up my faith in this medium for my message but I, too, suffered a frequent loss of confidence when I had a run of 'dark pauses' on the screen. So I understand how classroom teachers just cannot afford the time to trouble shoot such problems.

I was disappointed that no one responded to me by electronic mail. Perhaps they thought they had to devise a formal letter. The returns by mail were full of small snippets of conversation pinned to children's work giving me a kind of instant replay of the action in the classroom. This was easy and interesting for the teachers to do. Typing something similar on electronic mail would bring teachers face to face with the problem I had; that there was no way to illustrate the mail snippets. However, I know my challenges were done out of interest by the teachers and not looked upon as chores. There were reports about the doing of challenges from the staff tutors who continued to support the teachers in their regular visits.

Lest this report gives the impression that my contribution was the sole input to staff development in mathematics in that local authority, let me set the record clear. Groups of teachers and staff tutors and a secondary principal teacher of mathematics had laboured over the previous year to collect and disseminate mathematical materials to the schools, producing in the process useful summary guidelines in chart form to support teachers in adapting their traditional programmes. My part was to complement and support their work by leading the teachers to appreciate in a practical way why the underpinning spirit of the mathematics guidelines was one of problem solving and enquiry, how this had come about and what adjustments they might have to make in their teaching approaches.

There is an unfortunate example in the national guidelines of a worked 'problem', known as **the crisp problem**, presented in order to lead teachers to appreciate how to change their methods from didactic to heuristic. The difficulty with the example is that any teacher who does enjoy using a problem solving approach with her pupils will see the example as an easy logical one that requires no more than a simple route finding and the comparison of two numbers in order to solve it.

Whoever chose the example must have overlooked the simple and obvious solution. As a result there is a tedious analysis of how children might be led to solve the crisp problem using quite unnecessary arithmetic.

The example highlights the very point that would undermine the confidence of the teacher who teaches in a traditional way and, at the same time, the working of the example would appal the mathematically oriented teacher by its contradictory pedantry. Underconfident teachers do not want children posing unexpected solutions to problems they set. They need to know that what they are doing in class is within their compass. **So the basic principles and joyful patterning of mathematics has to be shared with such teachers.** They need to be supported towards the view that children have a responsibility to communicate their thinking clearly and the teachers need to be shown how to develop such an approach based on their own bank of skills in language teaching. That was my remit.

The teacher who loves mathematics needs to be assured that her encouragement of unconventional recording of problems, and lack of willingness to 'shoehorn' children into early slickness in algorithmic setting out of number work, will not be condemned. She needs clear ways of presenting her evaluation of pupils' thinking and progress. So often, dedicated and high achieving teachers are unaware of how outstanding they are. They need praise too. That was my remit also.

Primary teachers have to teach so many things. It can be very demoralising to see each area of the curriculum in turn given scrutiny by 'experts' in that one field and then an expectation, made clear by the formal delivery of the 'navy blue' end product, that the teacher will unravel her integrated curriculum to adjust that particular aspect. When the navy blue book is written in analytic style the heart of the primary teacher sinks. Where is the inspiration to teach more dynamically? The message is hidden under a format so constrained to match each other curricular area that instead of flowing in harmony, all that is perceived is turbulence. The whole style of how a primary teacher operates appears to have been hijacked by a secondary approach.

The remit of bringing the inspirational.method was given to me. The attractive underpinning philosphy needed to be made clear by someone who understood the wholeness of the primary curriculum. Teachers' morale was in the front line. A simple way of linking the best ways that primary teachers operate already with the analysis within the 'navy-blue' guideline book had to be uncovered. Teachers want to teach well and be seen to do so. The guideline had to be turned from an administrator's analysis to a classroom tool.

Chapter 9
The Closing of the Classroom Door

Many people want to influence how and what a teacher teaches.

Psychologists may inform teachers about their findings; government analysts may try to standardise how teachers make assessments; school board members may provide resources they want teachers to utilise; politicians may promise the electorate that they will make teachers follow a new curriculum; but, in the event, the task of school teaching belongs to the teacher and not to anyone else.

After the class has gone home, each and every teacher has to find time before the next day's class commences, to complete all corrections, prepare materials and muster teaching plans for the following day. For the primary teacher the task is to teach many different things and yet make a coherent whole out of each day's learning experience for each of his/her pupils.

Out of the clamour of all the areas of teaching requiring attention I wanted to concentrate on a change in the teaching of mathematics. The very nature of the subject, with its specialised use of language, meant that it could not be isolated from general language teaching, an area where primary teachers feel rightly confident of their expertise.

There is only one way to influence a teacher so that s/he changes a well established teaching pattern. All I had to do was to make the subject of the teaching activity really interesting. I think for many of the teachers behind those closed doors I did succeed.

A suitable quote from Bruner, in his Theory of Instruction, p51...

"There are several quite straightforward ways of stimulating problem solving. One is to train teacher to want it, and that will come in time. But teachers can be encouraged to like it inte estingly enough, by providing them and their children with materials and lessons that **permit** legitimate problem solving and permit the teacher to recognize it.

....

Students being taught do not often have a sense of conjecture and dilemma. The task of the curriculum maker and teacher is to provide exercises and occasions for its nurturing. If one only thinks of materials and content, one can all too easily overlook the problem.

......

The most important aspect of evaluation is to provide intelligence on how to improve things.

Children tackling the logic game - Strategy 31

Contents

Letter 1

20 October 1992

Dear Colleague,

I enclose a blank 'Aspects of Mathematics' for your own use and a sample of my attempt to use that 5-14 map with the topic I started with you, **Mathie Street**. I followed the plan we tried out on the course of writing down the pupil tasks and then identifying what were the start points. Next I listed the main ways the starting points might be tackled and that assured me that I was following the underlying approach demanded by the document of problem solving and enquiry methods.

Thereafter it was a simple matter to flip through the document sections to see what levels were on offer from the tasks I had set. In class that would give me a quick check of how far ranging my topic was and it would also be an easy one page guide for me in assessing which children could tackle the different levels encompassed by the topic activities. The numbers after the A B C..levels refer to the sections of the story line shown on the left.

As we did not get very far into the Mathie Street topic I have enclosed a fuller outline and I am sure you will be able to use the topic for problem solving activities having experienced the style of approach in Safari Park. The Safari Park topic parallels Fantasy Islands very closely and so all the the maths teaching notes in there will serve for both topics. The computer programs will serve just as well for Safari Park. The same holds good for Mathie Street. Notes on sharing and the Cash & Carry program can be used in both topics.

The storyline for Safari Park was based on developing the counting numbers from activities with sets, tackling the teen number language/order problem and leading in to the operations of addition and subtraction. The storyline for Mathie Street overlaps some of the activities in the previous topic but takes them in an upward spiral so that slower learners are still firmly in the picture. The base for Mathie Street has to be a storyline using sets of sets in preparation for moving on to the operations of multiplication and division.

All the topics use a story to involve the children in creating and feeling in charge of their own mathematical development. The story approach makes it easier for the teacher to stand aside and listen, observe and then discuss. It also leads to more sustained 'time on task' by the children which in turn lessens the class management burden for the teacher. The hope is that you will develop your own version of this approach and gradually feel more satisfied with your maths teaching.

My task when I visit you is to help you with this personal development and to share with you ways of developing children's mathematical thinking. The task you start with is to run a small audit of your children's levels of understanding by mapping your current activities for mathematics, perhaps for a period of a fortnight, on to the blank **Aspect of Mathematics** sheet which I have sent you. I have filled in more headings than we did on the course to make the exercise less time consuming. To help swing you into more of a listening and observing mode for part of your teaching time, you have been asked to devise a game which highlights a specific piece of mathematics you want to check. You will recall that the small sample we tried on the course covered a range of possibilities. The paddock stepping game checked for an ability to hold a number, representing steps left, in memory and compare it with a die throw for match or 'less than'. The seed sowing game checked for ability to count out groups of four, move in one direction, follow a simple rule of collect only if less than three items were in the landing site and estimate how far a handful of beads might last in a 'seeding'. The choose a number for a set game checked for ability to make simple rules and agree that they were carried out. It also checked for a variety of ways of finding a total. Judge what stage your children, or a group of them, have reached and see if you can find a suitable game to check if you are correct

I look forward to seeing you with the bairns.

Yours sincerely

Tricia.

Patricia R Watterson

Director CATO Project

24th February 1993

Dear Colleague,
I have visited more than half the teachers who attended last term's mathematics courses. So it seems a good time to set everyone a second classroom task to help in the move towards fulfilling the 5-14 guidelines.

First let me say how enjoyable my visits were and how revealing of children's current strategies and attitudes in mathematics. It was heartening that no child was 'turned off' and that all applied themselves with zest to the tasks I set them. Though some teachers informed me that the children never thought of activities as maths, only a text book task was put in the 'work' category!

I was delighted that so many of you had devised or utilised suitable games and hope that you will continue to see value in incorporating them into the maths schedule as legitimate tasks which help children to internalise and 'possess' their mathematics. To support you in this I include a handout I devised for a parents' talk which you can copy and distribute, if you want, to the parents of your pupils. Attached also is a set of games for 'small number know-how' to augment your own stock.

It is a good idea to introduce a new game every two weeks or so and gradually withdraw one that has been in use for a while to put in the 'resting' pile. Over a period of time you become better at choosing games that exactly fit a current learning difficulty and at creating variations on games that serve a long term purpose.

The task I would like you to undertake as a collaborative one this term is to tackle the 'finger counters', particulary those children who have a well ingrained strategy of always counting from one and in ones. I am not thinking of those children receiving learning support because it appeared to me that they were already being supported in this regard. I would like each of you to target some of your 'middling' children, in some cases it might even be one of your brighter children, so successful is this strategy with some of them. First decide your target child or children, no more than four of them. Quantity is not important, quality of success is.

Find ten minutes every day when you will home in on this problem by doing one of the following:
1. Note the child's name, date of birth and date of observation in a small notebook.
2. Devise an opportunity to engage the child in a counting activity where s/he can tell you how s/he got the answer.
3. Note the strategy. Is it?.....
By using the fingers on one or both hands: by using the fingers in a left to right order ; by using the fingers in a special patterned order ; by using the ceiling boards or tiles instead of fingers ; by tapping the side of the face with the hand or a finger and reciting the numbers in order ; by holding the open hands against the face and 'playing them in order' against the cheeks ; by 'reading' up the numbers alongside the chalkboard ; by counting along a desk number line or by some other method that I did not spot on my travels!
4. Talk to the child within a group about the ways that people 'get an answer' investing in the process a legitmacy proven by your genuine non condemnatory interest in the matter.

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5. Use this talk to look at other less tedious ways of reaching an answer. In particular use games and discussion to highlight the use of the 'easy' facts such as the doubles, making to ten and repeat patterning.
e.g. 8 add 7 is close to 7 add 7: 9 add 5 can be transformed to 10 add 4, if you know 9 add 4 you know 19 add 4, 29 add 4 etc...

6. Find some way of convincing the child that another method is just as certain. The reason they cling to the finger strategy or the counting on in ones is that it was devised by themselves and has been found always to work. Why should they change such a great method? Convincing them will depend on their character.
Are they anxious and cling to the method because they feel rushed doing the class maths? Let's give them a little space and more praise when seen to try a new method.
Are they bright and want always to be first and correct? Let's show them that other lesser mortals in the class are getting there first with a more efficient method.
Are they plain lazy and do any written work only under duress? Let's convince them that another method will give them less hassle!
7. Note what triggers success with such pupils and write up a statement about the procedure you used and how long it took for the child to begin trying out a new method and then how long until the new method became first choice.

I enclose an article that you might enjoy reading as you tackle the activity. Remember that the philosophy underpinning the 5 to 14 Maths is that children develop a problem solving attitude to mathematics that enables them to 'possess' it and so utilise and enjoy it.

Can I ask you to look out Anita Straker's Mathematics for Infants software and use that in addition to the maths programs in Fantasy Islands and Travelling Shops as part of your games schedule? The advantage of the software is that you are free to observe the children's methods and listen in to their working without being obtrusive. If you have found any other software that 'fits the bill' please make a note of its special contribution so that we can share the information with each other.

Do let me know how it is going. Please dip into the 'Talking Mathematics' pamphlet which I handed out on the course and try out some of the discussion points there with your pupils. Feel free to write to me with any queries or successes.

Yours sincerely

Patricia R Watterson
Director CATO Project
School of Educational Studies
Jordanhill Campus
Faculty of Education Designate
University of Strathclyde

UNIVERSITY OF STRATHCLYDE
FACULTY OF EDUCATION
Jordanhill Campus

7 May 1993

Dear Colleague,

I look forward to seeing you next week when we meet on the final day of our course. I believe that Annie McDonald has spoken with you recently and suggested what you might bring with you. May I either add to the list or reiterate it.

The hope is that we will share experiences of the past year where we have focused on how children learn mathematics, the difficulties they have, the practical ways of helping them and how to plan, assess and record the mathematical learning we would like to take place in our classes. To that end I would like you to bring the notebook in which you recorded how certain children counted and what strategies you tried in order to shift their thinking. Please bring the 5-14 guideline in mathematics, the 'calendar' of small stages in mathematics which I extracted for you from my publication, Mathematics and the Slower Learner , your current teacher book for whatever maths text scheme you are using and a text book of one of the stages you are teaching. We are hoping that part of the day will be given over to planning and so these 'tools of the trade' will be helpful.

As we want to share the good things as well as the depressing things it would be helpful if you brought one child's maths jotter or work book to highlight a development that really pleased you which you think resulted from some small change you made in your approach as a result of the stimulus of the course. If you have taken photos of your class doing something mathematical, do bring those.

A range of games have been devised by the members of the course. Please bring a note of the ones that were particularly helpful for areas of common difficulty and the source. Were they from a commercial set, or an adaptation by you of a commercial set or ones published in a book or were they on the computer? Did you invent something original that you would like to share?

Some of you made up your own mathematical topics related to the children's interests and/or the local environment. Some of you had a good go at putting the work down in the format handed out on the course. If you did, don't be shy, bring an outline. It is always helpful to exchange such ideas.

Looking forward to seeing you,

Yours sincerely,

Tricia

Tricia Watterson
Director CATO Project
School of Educational Studies

76 Southbrae Drive
Glasgow G13 1PP
Telephone: 041-950 3000
Fax 041-950 3268

Dean: James McCall BSc MEd PhD

UNIVERSITY OF STRATHCLYDE
FACULTY OF EDUCATION
Jordanhill Campus

1 June 1993

Dear Colleague,

Thank you f. · your enthusiasm at our last meeting. I do hope you took enough away with you to keep you interested and some good ideas to try with your own class.

The two things we did not touch were the individual examples in your children's workbooks and a second look at putting your own forward plans for this term into the format that I distributed at our previous meeting to make it easy to keep a check on the requirements of the 5-14 document. I am very interested to know what you might have brought from your children's work books that surprised and delighted you and why you felt that way. I would be grateful if you could send me any example, or a photocopy, with a wee note about the circumstances.

Some of you told me that you had begun to formulate your plans in the 5-14 framework and were pleased with its simplicity and so I am sorry that we did not have time to share that with everyone. Others I know have been meeting together and sharing forward planning, working out mini topics, and have found that a trouble shared is a trouble halved. I am enclosing three sets of handouts. One is taken from the book **Talking About Mathematics** by Tom Brissenden, published by Basil Blackwell, and gives two teachers' ways of organising their maths classes. There are other chapters in the book on classroom management which you might find interesting. Mike Dunn and Joan McKinnon have offered to make sure that books I recommend will be ordered for your centres.

The second handout is from the book **Developing Mathematical Thinking** by **Ann Floyd**, published by Addison-Wesley. The extract is an example of a survey of children in the England and it is salutary to see how little maths 'sticks' across an ability range. Those of you with older children might be curious to see how your own children tackle the examples with the poorest response. Perhaps let them try it themselves first and then after you've assessed their first attempt, let them talk to you or a near age partner about how they worked them out. This will give you some insight into their difficulty. I thought the range of items were well thought out and was most surprised at the results.

The third handout is from the **Best of Mathematics in School** published by Longman for the Mathematical Association which we used on the course to find puzzles on number boards and dominoes. The extract is interesting because it displays three straight forward examples of a set presented to children aged 10 to 13 all over UK, including Scotland. Best of all it lists some of the discussion and reiterates the message that we are running too fast with too many of our children. Again, if you have 10 year olds or bright 9 year olds you might like to try the examples out and compare. I have photocopied the examples and repeated them on the last page so

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76 Southbrae Drive
Glasgow G13 1PP
Telephone 041-950 3000
Fax 041-950 3268

Dean: James McCall BSc MEd PhD

that you can cut them out and fold them over to simulate the way they were presented.

I enclose paper versions of the table game we played which you can put on to thin cardboard by putting it through the hand feed tray on your photocopier.

Since my return I have mulled over the ideas you shared and your willingness to stay in contact to receive and share any more teaching ideas. I have put in a proposal to our research committee to develop practical activities which might more clearly fit the needs of teachers in a range of composite large or very small classes. The idea would be to develop some short weekly activity which could start the ball rolling on a Monday with a short five minute whole class discussion and be picked up throughout the week by groups or pairs of children both in school and at home. Each week a new activity would be a new starting point to keep the involvement ticking over and to provide over the course of a year a wide compendium of activities in contexts easily fitted in to your regular topics or environment. The hard part will be finding 'spiral' activities. That is finding activities with a similar starting point but which can be taken up at different levels by a range of abilities and ages, in Gaelic and/or in English.

The notion came from comments by some of you that you enjoyed trying out my ideas but then things petered out because you had no stock of similar ideas to continue to work in this way. Others felt so liberated by the freedom to explore that they had to control their excitement and their urge to keep trying newer and harder things! Most fell somewhere in between. My hope is that we could produce something to suit everyone which could be dipped into to set an exploring theme for the week and over the year build up in each teacher's mind a feel for the same ideas surfacing again and again at different levels until confidence to 'do her own thing' bubbled over. All I would be seeking from you would be a willingness to try ideas out and return a comment form. I would share the task out amongst you all and share with you the activities that were most effective. I know some teachers want to take their interest in maths forward and they could use the exploration as their own starting point. If you are not interested or too busy when something drops through your mail box, you can simply ignore it or postpone it. If the research bid is successful the activities will not start until September.

I was 'skipped' back to my hotel in Barra by some children trying to ferret out of me the secret of the Strategy 31 game which I played in many of your classes. In case you did not work it out I have put the solution at the foot of the last handout.

I hope to send everyone a pamphlet on the Inservice Awards scheme within the coming week.

I look forward to hearing from you.

Patricia R Watterson
Director CATO Project
School of Educational Studies

Strategy 31 - An explanatory handout

1	2	3	4	5	6
1	2	3	4	5	6
1	2	3	4	5	6
1	2	3	4	5	6
1	2	3	4	5	6
1	2	3	4	5	6

Two players and two colours of counters.
Each player chooses a different number for each line.
Combined total is kept.

First player to reach 31 is the winner.

A winning gambit would be for **blue** to cover 3 in the first line. **Red** might cover 5 in the same line. The total so far would be **8**. **Blue** would then choose 2 in the second line ensuring an interim total for herself of **10**, another winning number in the sequence.

Strategy 31 Puzzle

It can be created on a number of grids with a range of targets. The one I used was six lots of six with a target a little below the answer to 6 times 6. You can try other grids yourself and choose suitable targets within the same parameters, that is a bit below 25 for a five by five grid etc.

If you place your counter on 24 your partner cannot get 31 because there is a distance of six plus one between 24 and 31. Your partner starting on a new line has only up to six to choose from and cannot make the seven difference.
Therefore to ensure that you do reach 24 first you must work out the same 'distancing' arrangement of six plus one for the earlier rows.

Gradually children notice that getting a 17 enables them to get the 'magic' 24 every time and then over a period it is noticed that 10 enables you to sure fire get 17. Eh voila! We have a pattern and if we are allowed to start, then three is the one to go for as that is the sure way to get 10, then 17, 24 and finally 31. Around 9 years old many children eventually spot the pattern though some bright 8 year olds have surprised me by offering other grids and knowing the starter number for their target. Often they know how to win but have not worked out the row number plus one generalisation.

Some children get the correct starting numbers by chance so don't get foxed too easily if you are trying it with them. Just hold those key numbers in your head if you really want to win and are kindly letting a child start first! I just used it as a way of catching interest so that I could observe the 'finger counters' and listen out for counting strategies without the children being alerted to that fact. I did not expect any primary three children to solve the puzzle.

In addition to the books mentioned in this letter the Report of the PRIME project, masterminded by Hilary Shuard, was the other document that I recommended to you. It has been purchased and has been available from the cluster groups, I understand from Mike Dunn. It is the material to use if you want to support your own mini workshops with colleagues. It has been designed to be used by groups of teachers working together on their own inservice. The handouts I gave you with the snowman example came from there. It has sections on problem solving, classroom organisation and dealing with parents and a great deal of it you will find acceptable.

Challenges that raise questions give opportunities to practise skills already acquired and so build confidence.
I hope to send an interesting challenge most weeks.
Please read them to your children. They can be started in school and taken home to be talked about. If anything interesting happens in your class through a challenge pop the child's work or comment in the post to me - name, age, class and school - or drop a note in my mailbox.
Tricia Watterson
Challenge 1 – Comments for Teachers–
With primary 1 we just want to know the words they use for size and exactly what each word means to them in practical terms. Is long an across or up and down word. Do they make the 'worm' straight across the desk, diagonally or quite wiggly? Can they imagine a shape before they form it? Have they names for closed shapes? Is their picture an accurate representation of what they did? When asked to tell what it shows do they use size words appropriately? Did any of them take the 'worm' and measure it against themselves?

Primary 2 – all of the above and how did they tackle the ambiguous question? Too big for what? Or did they use comparative words and point out that the 'worm' was bound to fit on when curled round? Did any of them think the sizehad changed? Did you ask to see? Did any of them show some idea of measuring by laying rods or blocks alongside, inside or all the way round the 'worm'?

Primary 3 – all the above. Did any of them show a distinct developmental change compared with primary 2 in how they tackled the measuring? What was the distinction? Did they mainly go for the width of the closed shape with a ruler or by chance open it out?
Primary 4 – all of the above AND were there any surprising questions? Tricia Watterson
Comments for teachers relating to P5/6/7 activities.
Look at the notes for the younger children just in case you have a child with no concept of size. Did any child make a rectangle with no area by simply folding the tape in half? If so how did s/he explain that it was a rectangle? And, was the child challenged by other pupils? Did the children compare with each other and try to find a single 'right answer'? In Desperate Journey there is a computer investigation, page 21, along similar lines based on choosing of size when building a croft. Try that as a follow up if you have sparked an interest. It gives support & a model for following through a problem.

Challenge 2 – Sometimes sheep catch cold too.
P1/2/3 We can get 2 sheep on our wee trailer. We have 5 sheep to bring down to the vet. How many trips do we have to make? Draw me a picture of your answer. Write in numbers, too.

P3/4 Our trailer is bigger. It holds 10 sheep. We have 42 sheep to bring to the vet. How many trips do we make?

P5/6/7 We run the vans to the market. They hold 36 sheep each. We have to carry 1128 sheep. Write your guess first. You can use a calculator. How many vans will we be running?

Challenge 2 – Comments for teachers. What I want to know is how the children go about this one. Do they go successfully thro' the 3 stages –
1. Translate it into a maths calculation. 2. Perform it somehow. 3. Translate it back into a context.
The problem is arithmetically too difficult for P1/2 but they don't know this. How do they do it as a common sense problem? How do they record it if they bring numerals into their picture?
Do the older children make a reasonable guess? Do they get the correct arithmetic answer but the wrong practical one?

Challenge 3 - Dark nights, lights in windows.
P1/2/3 Draw a little house with a door and some windows. Next to it draw a bigger house with one more window. Next to that draw a house the same size with one less window. What kind of house comes next? Can you go onalong the street keeping to your pattern. Write down how many windows are on each house. Write the number of each house on its door. Write or tell about your pattern.

P4/5 Following the same instructions draw two different streets of no more than 12 houses in each. The rent for each house is doubled for every window and trebled for each storey. If the window rent is two pounds what is the rent for each house? Each street?

Challenge 3. P6/7 Dark nights, lights in windows.
In your two streets the rents are fixed by a mad accountant. It starts at two pounds for the first window and doubles for each following window down the street until big house changes to small house. Then it halves until small house changes to big house when it doubles again. There is a shorthand way of writing these doubling numbers. Findout how to do it. The number of storeys makes no difference to the rent in your streets. Is there a repeating pattern? Make a table of the rents for each street. You can use a calculator to check your working.
Tricia Watterson Strathclyde University

Challenge 3 - Teacher Notes What did P1/2/3 think made a STREET? Was there a wide variation in the patterns? Were some unable to make a pattern of their own? Were there any surprises? If they enjoyed this challenge check out Fantasy Islands where houses are used to develop maths extensively.

P4/5 Was every child able to create two distinct streets keeping to the rules? Did they argue about what was meant by DIFFERENT? How many differences were possible? I hope the rent collecting stimulated a lot of patterned arithmetic. Did any child create a clear pictorial record of the street rent?

Challenge 3 Teacher note. P6/7
Was this too difficult a challenge or did they rise to it? Did they use pattern to cut down on the arithmetic or try to plod through? My purpose this term wasto encourage children to draw a picture as a first step in a maths problem, then numbers and finally return to the picture to check the sense of their numbers and operators. I wanted them to feel confident that they had a good solution to talk about.

Challenge 4
Dark nights, lights in windows (Primary 1, 2 and 3)
Draw a little house with door and some windows. Next to it draw a bigger house with one more window. Next to that draw a house the same size with one less window. What kind of window comes next?
Can you go along the street keeping to your pattern? Write down how many windows are on each house. Write the number of each house on its door. Write or tell about your pattern.
Primary 4 and 5
Following the same instructions draw two different streets of no more than 12 houses in each. the rent for each house is doubled for every window and trebled for each storey. If the window rent is two pounds, what is the rent for each house? Each street?
(Primary 6 and 7)
Following the same instructions draw two different streets of no more than 12 houses in each. In your two streets the rents are fixed by a mad accountant. It starts at two pounds for the first window and doubles for each following window down the street until big house changes to small house. Then it halves until small house changes to big house when it doubles again. There is a shorthand way of writing these doubling numbers. Find out how to do it. The number of storeys makes no difference to the rents for each street. You can use a calculator to check your working.

Children's Questionnaire

Side 1

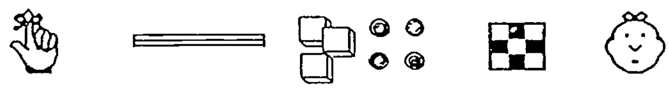
I sent puzzles to you through the computer.

1. If you remember doing a puzzle, draw a ☺ in its box.
2. If you did some of it at home, draw a
3. If you were happy about it, draw a ✓
4. If you found it too hard, draw a ✗

Name	Class	Age

snake	sheep	houses	shapes

What do you use to count?
Draw a circle round the answers.



fingers | ruler, number line | cubes, counters | tiles | head

If you like to use your fingers put a big ✓ under that picture.

If it hurts to use your head, then put a ✗ under the head.

Side 2

The Way You Count

one.

two.

three.

four.

five.

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six.

I am trying to find out how you count and I wonder if you will help me by doing what is on this sheet. You may ask the teacher to read it to you if that makes it easier for you.

Primary 1
1. Count out loud from 1 and see how far you can go before you get muddled.
2. Tell your teacher the number between two and four.
3. Tell your teacher the number between seven and nine.
4. Tell your teacher the number that comes before ten.
5. Write down number six.
6. What number comes after twenty?

Primary 2
1. What is the highest number you can count to?
2. Tell your teacher the number between fifteen and seventeen.
3. Tell your teacher the number between thirty six and thirty eight.
4. Tell your teacher the number after one hundred.
5. Write down the numbers sixteen and sixty one.
6. Tell your teacher the highest number between twenty and thirty.

Primary 3
1. What is the highest number you can count to?
2. What is the next number to that?
3. Tell your teacher what is the lowest number between ninety and a hundred.
4. Tell your teacher the answer to
   9+6, 19+6, 29+6, 39+6, 49+6
5. Write down the numbers seventeen and seventy one.
6. What is the difference between them?

Primary 4
1. What is the highest number you can count to?
2. What is the next number to that?
3. What number is half way between one hundred and two hundred?
4. What number comes just before one thousand?
5. What is the biggest number you can write with the figures one, two and three?
6. Tell your teacher the answer to
   56 -7, 46 -7, 36 -7, 26 -7, 16 -7

I am trying to find out how you count and I wonder if you will help me by doing what is on this sheet. You may ask the teacher to read it to you if that makes it easier for you.

Primary 5

1. What is the highest number you can count to?
2. What is the next number to that?
3. What number comes half way between one thousand and two thousand?
4. What number comes just before one million?
5. Tell your teacher the answer to
   3 x 9, 30 x 9, 300 x 9, 3000 x 9
6. Which is bigger, a third or a half? Show me why.

Primary 6

1. What is the highest number you can count to?
2. What is the next number to that?
3. What number comes half way between four thousand and five thousand?
4. What number comes just before two million?
5. Tell your teacher the answer to
   27 000 divided by 9, 2 700 divided by 9, 270 divided by 9,
6. Which is bigger, a third or a quarter? Show me why.

Primary 7

1. What is the highest number you can count to?
2. What is the next number to that?
3. What number comes half way between one million and two million?
4. What number comes just before two million one hundred thousand?
5. Tell your teacher the answer to
   a third of 270, a third of 27 000, a third of 2 700 .
6. Which is bigger, two thirds or three quarters? Show me why.

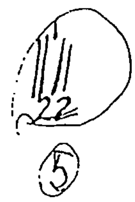
Ten Faces

When visiting with primary 1 children I played a game of **copy my bridge across the islands.** This I made with two parallel rods and one crossbar. We talked about the number pattern of five and then, when they made a copy, about the pattern of ten. Many young primary 1 children found the copying difficult. After that they were asked to make up a story for me to copy.

On this page you can see a chimney stack and a house shape. Elsewhere in the report you will find cupboards and cowsheds made with two sets of five rods. Each child had to think of a different story and arrangement from the others sitting nearby. We then played with our hands and some children were genuinely surprised to find that they had a 2, 2 and 1 pattern there as well. So we played making **ten faces** trying to find sets of 2, 2 and 1, twice, to make a face as you can see.

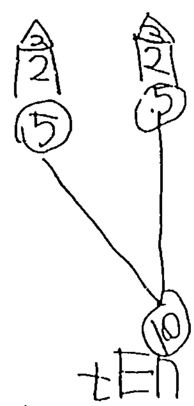
2 ears and 2 eyes, 1 mouth
then
2 nostrils, 2 eyebrows and
1 nose

a chimney stack



2 ears, 2 eyes, 1 mouth
then
2 glasses, 2 eyebrows
and 1 nose

A man singing a pop song.



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A house of five and another
house of five making a
little street of ten

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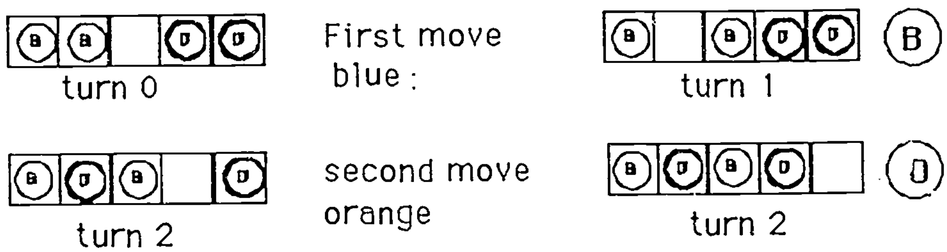
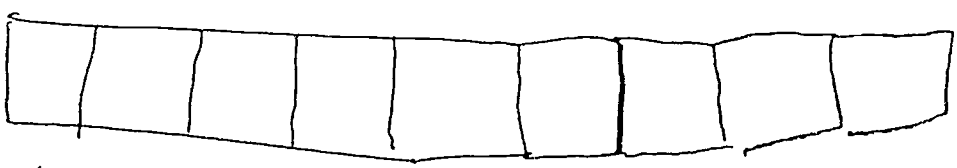
Leapfrog

When visiting with primary 3, 4 and five I tried out the Leapfrog game by giving strips of grid paper and two colours of counters. The counters are frogs trying to cross the road. They jump in turn a box at a time until no more spaces are available then the other coloured frog takes a turn. A frog may jump over another if there is a space but may not land on top of another.

Some children became interested enough to keep making longer roads for better games. Here is an example that amazed me of a very young pair of children who wrote out their 'turns' on a scrap of paper once they thought they had found the pattern. And so they had. Notice the mark they had drawn down the centre. Unfortunately, I had to leave before they finished writing their story about it. They said that they could go on forever!





B O B O B
1 2 2 2 1

O B O B O B O
1 2 3 3 3 2 1

Sarah 8
P4

B O B O B O B O B
1 2 3 4 4 4 3 2 1

Caroline 7
P3

Leap frog

We've found a pattern in Leap frog.

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