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ABSTRACT

The elements of statistical formulae that cause people to perceive them as difficult were studied, working from theoretical bases that involve cognitive load, motivational perception, and impasse driven learning. In a pilot study conducted to identify difficulties that novices perceive while learning statistical formulae, 22 students in an introductory statistics class participated as subjects. As novices, on the first day of class, students were asked to rate 60 formulae from statistics texts for confidence in their ability to learn to compute, apply, and understand each formula. Results suggest that something other than the difficulty of the concept influences student attitude. A protocol analysis was conducted for three adults with no training in statistics. Results indicate that confidence declines as the number of unfamiliar elements in a formula increases. Perceived difficulty increased as the amount of information to be processed increased, supporting the idea of cognitive overload. Five tables and two graphs present study findings. (Contains 13 references.) (SLD)

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## Identifying and Alleviating Problems Novices Have While Learning Formulae

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## **Identifying and Alleviating Problems Novices Have While Learning Formulae**

### **Introduction**

In this study I investigate what elements of statistical formulae cause people to perceive said formulae as difficult. The perception of difficulty is important because it affects how and what people study as well as the amount of time they devote to study. By understanding how people perceive formulae we can design our instruction to accommodate these perceptions.

I approach this investigation from three major theoretical bases. The first theoretical base will concern cognitive load. Are students intimidated by certain formulae because the formulae contain too much information to be held in working memory? The second theoretical base will concern motivational perceptions. Are there aspects of certain formulae which cause students to perceive certain formulae as more difficult and thereby stimulate negative emotions that interfere with the efficient use of working memory. The third theoretical base involves impasse driven learning. When learners reach a cognitive impasse, are they able to work around it. If learners can work around certain impasses and not others, how do the impasses they can work around and those they cannot work around differ?

### *Cognitive Load*

According to current information processing theory (Anderson 1987, 1990), learning takes place in working memory. Working memory is similar to short term memory and might be described as conscious attention. While conscious attention is necessary for learning, it is not sufficient. For a new concept to be learned, i.e., stored in long term memory, it must be held in working memory long enough for it to have

meaning. Since the capacity of working memory is limited, the amount of information that can be learned at any one time is likewise limited.

Much has been written in expert-novice literature about the enhanced working memory capacity of experts (Chi, Glasser, Farr 1988,). This enhanced working memory appears to be more a function of experience than of general intelligence. As such, the enhanced working memory of experts is domain specific. DeGroot's (1966) classic study of differences between expert and novice Chess players illustrates these points. Through the use of protocol analysis deGroot analyzed in detail the depth that experts and novices searched the Chess board, the heuristics behind attack and defense strategies of both groups, and other aspects of the reasoning process. Although the experts made the right moves more often than the novices, there was nothing in the protocol analysis to indicate why.

Next deGroot exposed players to a slide of the twentieth move of a Chess master's game. These were all games with which the players were unfamiliar. After five seconds the slide was removed and the players were asked to reproduce the positions of the Chess pieces on a Chess board. The Chess masters could reproduce nearly all of the pieces with little or no errors. The novice performance was much poorer (Posner 1988 p. xxx). However, when the experiment was repeated with the Chess pieces placed on the board in a random manner, the experts did no better than the novices. Furthermore, the experts were upset over the chaotic board positions (Anderson 1990 p. 280).

Given the enhanced working memory capacity of experts, the obvious question is how do experts exceed the normal limitations of working memory? One primary strategy for overcoming the limitations of working memory is to link new information with old information. By linking bits of information together, the expert is cognitively able to move backward, forward, and laterally along chains of inference that are too long to be held in working memory. This cognitive strategy is known as "chunking."

Chunking requires a knowledge base with which new information can be linked. This has two implications for the educator. First, the educator must understand the cognitive background of the individual learner: Second, the educator must find ways to link new information to information the learners already possess. These implications present difficulties when educators attempt to introduce learners to an alien domain like statistics. Novices frequently cannot chunk the new information because it is so different they cannot link it to any prior knowledge. This means the novice must hold the entire inference chain of a new concept in working memory long enough for the new concept to become meaningful. If the inference chain is short enough to be held in working memory, it can be learned by the novice: Otherwise, the novice may learn an incomplete or incorrect chain of inference. This problem is exacerbated if the novice is also required to remember the meaning of special symbols often used in a formal language like statistics. Most of the literature asserts that ultimately, working memory limitations are overcome by gradually acquiring domain specific inference rules. While this knowledge is necessary for the construction of adequate expository text, it is not sufficient. For the instructor, one primary requirement in knowing how different types of information impact working memory.

Kosslyn's research (1980) revealed that working memory was necessary to transform a visual image to a mental representation. This implies that seeing something requires working memory. The visual factors that impact working memory are the amount of information contained in the visual image and the amount of space that the information occupied. While it took more time for Kosslyn's subjects to search information spread over a larger area, searching a large area required less working memory than identical information presented in a smaller space.

Larkin and Simon (1987) study strengthened and then went beyond Kosslyn's view of the working memory demands of a visual search. According to Larkin and Simon, the working memory demands of learning from a diagram were affected by 1) the amount of

information presented, 2) the area over which the information was spread, 3) the complexity of the information, and 4) whether the information was implicit or explicit.

Information presented in a diagram was easier to access than information in an expository text. This meant that it took longer to search text than it did to search a diagram. Consequently, subjects learning from text had to hold information in working memory longer than those learning from diagrams. Furthermore, information about computations required more working memory than information about objects, e.g., more working memory was required to hold  $a/b$  in working memory than was required to hold  $abc$  in working memory.

Where Larkin and Simon used physics problems for their study, Sweller, Chandler, Tierney, and Cooper (1990) conducted similar research using coordinate geometry problems. For this research three groups were used. The first group received instruction in the form of a conventional expository text. The second group received a conventional worked out example. The conventional worked out example had a diagram at the top of the page and an explanation at the bottom. The third group received a modified worked out example. The modified worked out example had the text written on the diagram next to what the text explained.

Subjects in the third group (modified worked out example) finished the problem quicker and made fewer problems than subjects in the other two groups. Subjects in the first group (conventional expository text) took longer and made more mistakes than subjects in the other two groups. The conclusion was that the length of time a subject had to hold information in working memory while searching for other relevant information had a negative impact on performance.

### *Fear and Loathing*

The previous section of this paper assumes that learners take a reasoned rational approach to learning statistical formulae. The assumption is that learners will understand

the information they are given, be able to locate relevant information, process the information, chunk the information, and move on. This may be an incorrect assumption for some learners. If a learner cannot understand the information presented, locate relevant information, process the information, or chunk the information, then what happens?

According to Mandler(1989),

" ...on a majority of occasions, visceral arousal follows the occurrence of some perceptual or cognitive discrepancy or the interruption or blocking of some ongoing action. Such discrepancies and interruptions depend to a large extent on the organization of mental representation of thought and action.... these discrepancies occur when the expectations of some schema are violated (p. 8)."

Mandler goes on to say that the arousal caused by the blockage pre-empts other information held in working memory. The reduced working memory capacity prevents the full utilization of the cognitive apparatus. The thought process may become simplified (stereotyped and canalized) thus causing the learner to revert to simpler modes of problem solving (p. 9).

This visceral reaction is intensified when people feel they should know something but do not (Weiner 1993). Anyone who reaches graduate school has had at least two courses in Algebra (high school and college). They feel that they should understand formulae. When they encounter a formula they do not understand they feel a sense of personal failure. To use Weiner's terminology, they feel they have sinned. These intense negative emotions waste enormous amounts of working memory. This inefficient use of working memory inhibits the ability to concentrate which intensifies the feelings of failure. Eventually the learner is overcome by frustration and guilt. If these reactions are intense enough or frequent enough, they can become a conditioned response, i.e. learned helplessness. In this case the conditioning must be overcome before learning can occur.

Ideally, the educator will not present the learner with more information than the learner can hold in working memory or assume prior knowledge the students do not have.

In fact, this frequently happens (Reif & Larkin 1991, Larkin & Raniard 1984). Experts (teachers) often assume prior knowledge on the part of the novices (learners) which does not exist. The teacher's assumption that students should have the relevant prior knowledge gets passed to the students. If the students accept this assumption then one could expect them to feel that this lack of knowledge is due to a flawed sinful nature on their part. The feeling of having sinned causes feelings of shame and guilt. The feelings of shame and guilt prevent the students from asking for help because asking for help would require an admission of their sinful nature. Since no one asks for help, the teacher assumes that he or she is teaching and the students are learning. The truth is not known until it is time for an evaluation. Once the evaluation reveals the sinful nature of the class, feelings of shame and guilt get replaced with feelings of anger and hostility. If the teacher pontificates, these feelings of anger and hostility are exacerbated. Once a class reaches this point, it is difficult for the teacher to teach or the learner to learn.

### *Impasse Learning*

In contrast to the previous section of this paper, VanLehn's concept of impasse driven learning (1990) postulates that when a student encounters the blockage of some ongoing cognitive activity (an impasse), the learner will attempt to work around the blockage. To use information processing terminology, when a person can no longer use an algorithm to solve a problem, he or she reverts to the use of weak methods, i.e. heuristics, to work around the blockage.

On encountering a new situation, people attempt to match the situation to similar patterns stored in memory. These patterns are called fetch patterns. Fetch patterns fetch facts and rules from declarative knowledge. The rules they fetch match "the most specific patterns consistent with positive training instances (p. 152)." Fetch patterns tend to be overly specific, i.e. they do not lend themselves to generalizations. If applicable fetch patterns cannot be located then one is at an impasse.



Once rules have been fetched, they are used as predicates for deciding which rule to execute. VanLehn refers to these rules as test patterns (p.89). "Test patterns are the maximally specific patterns consistent with the examples (p.154)". There are two situations where test patterns can cause an impasse. As with fetch patterns, overly specific test patterns cause an impasse. Unlike fetch patterns, test patterns can also be overly general. An overly general test pattern cannot discriminate well enough between relevant facts and rules to lead to a conclusion. If no conclusion can be reached then one is at an impasse.

As mentioned previously, when an impasse is encountered the learner reverts to heuristics in order to work around the impasse. If an solution is found, whether correct or incorrect, then the solution is stored in memory as a rule. Using computer terminology, VanLehn calls the incorrect procedures "bugs." Once bugs are stored in memory they become rules which are applied to other similar situations. In this manner the bugs spawn more bugs. Buggy procedures are easy to correct once they are identified. Once one is aware of the existence of bugs they are easy to identify, locate, and correct. The ultimate solution for the educator is to identify potential bugs and design instruction to eliminate them before they manifest.

VanLehn never discusses the visceral reactions to cognitive blockage mentioned by Mandler. Apparently motivation is not an issue with impasse driven learning. When cognitive activity is blocked, the student simply works around the blockage.

### Pilot Study

A pilot study was conducted to identify difficulties that novices perceive while learning statistical formulae. If the perceived difficulties are due to having to hold too much information in working memory at one time then longer formulae should be perceived as more difficult than shorter formulae. Furthermore, formulae which required learners to hold computations in working memory should be perceived as more difficult than formulae

which do not require such an activity. Formulae that present a more difficult concept should also be rated as more difficult than those presenting a simpler concept. If the perceived difficulty is due to visceral reactions caused by cognitive blockages which result from lack of adequate information, then formulae with more of these elements should be perceived as more difficult. If subjects can successfully work around any cognitive impasse encountered in a formulae, then there should be little difference in the perceived difficulty of said formulae. If subjects cannot successfully work around impasses, can factors that prevent this be identified?

### *Method*

#### *Subjects*

The subjects were twenty two students in an introductory statistics class. The data was collected before the first day of class.

#### *Procedures*

Subjects were given sixty formulae taken from introductory statistics books. The formulae covered the mean, the median, variance, standard deviation, regression and correlation. Subjects used a seven point Likert scale to rate their confidence in their ability to learn to compute, apply, and understand each formula. A score of one on the Likert scale meant "not certain" a score of seven meant "absolutely certain."

#### *Results*

After collecting the data, the mean scores for each aspect (compute, understand, apply) of each formula were correlated. The correlations were all over .95. Apparently the subjects saw little difference between learning to compute apply and understand the formulae. For this reason, the rest of the analysis was only done on the mean scores for compute.

	Compute	Apply
APPLY	0.963	
UNDERSTAND	0.969	0.981

**Table One. Pearson correlation matrix for mean scores on the students confidence in their ability to learn to compute, understand and apply statistical formulae.**

Next, the mean scores were used to rank the formulae. A ttest between the highest and fifth highest scores (absolutely certain) gave a non-significant p value. A ttest between the lowest and fifth lowest (not certain) scores gave a  $p < .001$ .

### *Discussion*

While there was no significant difference in the scores of the lower ranked (easier) formulae, there were significant differences in the scores of the higher ranked (harder) formulae. One might assume that formulae presenting more difficult concepts would be considered more difficult to learn than those presenting easier concepts. To a certain extent this was true. Formulae for correlation were ranked higher (more difficult) than formulae for the mean. However, one formulae for the median was ranked in the lowest 5 formulae (least difficult) while two formulae for the median were ranked in the highest five formulae (most difficult). Clearly there was something beyond the difficulty of the concept causing the student to perceive the formulae as difficult.

### Protocol Analysis

#### *Method*

In order to gain a greater focus on the factors causing the students to perceive certain formulae as difficult to learn to learn, a protocol analysis was conducted.

#### *Subjects*

The subjects for the protocol analysis were not in college at the time. The first subject was a grade school teacher. The second subject was an Attorney, the third

subject was a middle school teacher with an Masters degree in education. None of them had any training in the use of statistics.

*Procedure*

Like the class, the subjects in the protocol analysis used a seven point Likert scale to rate their confidence in their ability to learn to compute each formulae. In addition these subjects were asked to identify and count the number of things in each formula that made the formula difficult. The first subject in the protocol analysis identified four variables that she thought might make the formulae difficult to learn. These variables were, 1) unfamiliar objects (Greek letters, subscript, capital letters, and words), 2) familiar objects (numbers and lower case English letters), 3) uncommon mathematical operators (square root, brackets), and 4) common mathematical operators (addition, multiplication, subtraction, division, and squaring). As she worked through each formula she counted and recorded the number of variables in each category of each formula. As she was finishing with the formulae, she realized that she had not identified the inferred multipliers (bx). Rather than start over, a fifth category for inferred multipliers was added. Subjects two and three added no new categories of variables and included the inferred multipliers in category of common mathematical operators.

In order to identify the subject's score most representative of the class, the scores of the subjects were correlated with the mean scores of the class. The results are shown in the following table.

PEARSON CORRELATION MATRIX

	CLASS MEANS	SUBJECT 1	SUBJECT 2	SUBJECT 3
SUBJECT ONE	0.737	1.000		
SUBJECT TWO	-0.076	0.116	1.000	
SUBJECT THREE	0.506	0.655	0.060	1.000

**Table 2. Correlation matrix for protocol analysis subjects and class means.**

Next the scores of the subjects were plotted against the class mean scores and a lowess line was added. The results are in graph 1.

### *Discussion*

None of the subjects in the protocol analysis seemed to be affected by the length of a formula. They seemed to chunk the relevant information and move on. The problems occurred when they encountered information which they did not understand. On several occasions subject one said, "I can't learn to do this. I don't want to learn to do this."

Subject three reacted even more violently. She repeatedly apologized for "not being good at math." Several times she covered a formula with her hands and physically recoiled from it. Before we finished, her reactions became too intense to continue. She asked if she could take the formulae home and return them later. It was several weeks before she finished ranking the formulae and categorizing the elements in each formula. She indicated that she considered the task extremely difficult.

By contrast, subject two took a reasoned rational approach to ranking the formulae and categorizing the elements. When encountering something that he did not understand subject two would say, "If you tell me what that means then I can work this formula."

The protocol also revealed the reason two formulae for the median were perceived as difficult. When encountering a formulae like B5. The subjects asked questions like, "Is that M times e d or M times e times d, or is Med on thing?" Subject three was upset over the inclusion of words in some of the formulae. Subject one considered Med, L, cum, and fmed unfamiliar objects. All subjects in the protocol analysis were confused by the presence of uppercase letters in formulae. Greek letters distressed subjects one and three. Subject three recognized the upper case Sigma used in many of the formulae but repeatedly apologized for not knowing what it meant.

$$Med = L + \frac{(.05N - cum f)}{f_{med}}$$

**Formula B5.**

**Analysis of Pilot Data Using Information Gained From the Protocol Analysis**

Since subject one appeared to be most representative of the class, her counts of the unfamiliar objects, familiar objects, uncommon mathematical operators, common mathematical operators, and inferred multipliers were used as independent variables in a multiple regression analysis. The class mean scores were used as the dependent variable. The results are shown in the following table.

DEP VAR: MEAN SCORES COMPUTE N: 60 MULTIPLE R: 0.723 SQUARED MULTIPLE R: 0.523

ADJUSTED SQUARED MULTIPLE R: .479 STANDARD ERROR OF ESTIMATE: 0.331

VARIABLE	COEFFICIENT	STD ERROR	STD COEF	TOLERANCE	T	P(2 TAIL)
CONSTANT	4.910	0.110	0.000	.		44.662 0.000
COMMON OBJECTS	-0.004	0.026	-0.022	0.455		-0.159 0.874
COMMON OPERATORS	-0.046	0.025	-0.271	0.413		-1.855 0.069
UNFAMILIAR OPERATORS	0.062	0.040	0.228	0.423		1.575 0.121
UNFAMILIAR OBJECTS	-0.050	0.023	-0.360	0.322		-2.175 0.034
INFERRED MULTIPLIERS	-0.112	0.039	-0.457	0.344		-2.854 0.006

**ANALYSIS OF VARIANCE**

SOURCE	SUM-OF-SQUARES	DF	MEAN-SQUARE	F-RATIO	P
REGRESSION	6.497	5	1.299	11.851	0.000
RESIDUAL	5.921	54	0.110		

**Table 3. Multiple Regression with subject one's count of common, objects common operators, unfamiliar objects, and inferred multipliers as independent variables. The dependent variable is the class mean score for its ability to learn to compute the formulae.**

Using some exploratory data analysis techniques, the number of unfamiliar objects, familiar objects, and common mathematical operators were reduced to the .3 power.

Using this transformed data increased the squared multiple r from .523 to .721.

DEP VAR: MEAN SCORES COMPUTE N: 60 MULTIPLE R: 0.849 SQUARED MULTIPLE R: 0.721  
ADJUSTED SQUARED MULTIPLE R: .695 STANDARD ERROR OF ESTIMATE: 0.253

VARIABLE	COEFFICIENT	STD ERROR	STD COEF	TOLERANCE	T	P(2 TAIL)
CONSTANT	6.127	0.265	0.000	.		23.090 0.000
COMMON OBJECTS	-0.473	0.159	-0.282	0.571		-2.970 0.004
COMMON OPERATORS	0.000	0.000	0.107	0.568		1.119 0.268
UNFAMILIAR OPERATORS	0.067	0.029	0.245	0.449		2.287 0.026
UNFAMILIAR OBJECTS	-0.860	0.108	-0.842	0.459		-7.934 0.000
INFERRED MULTIPLIERS	-0.032	0.026	-0.130	0.464		-1.234 0.222

ANALYSIS OF VARIANCE					
SOURCE	SUM-OF-SQUARES	DF	MEAN-SQUARE	F-RATIO	P
REGRESSION	8.953	5	1.791	27.906	0.000
RESIDUAL	3.465	54	0.064		

**Table 4. Multiple Regression with transformed data.**

Stepwise regression (table 5) with the transformed data eliminated the common mathematical operators and reduced the squared multiple squared r to 0.715.

DEP VAR: MEAN SCORES COMPUTE N: 60 MULTIPLE R: 0.845 SQUARED MULTIPLE R: 0.715  
ADJUSTED SQUARED MULTIPLE R: .694 STANDARD ERROR OF ESTIMATE: 0.254

VARIABLE	COEFFICIENT	STD ERROR	STD COEF	TOLERANCE	T	P(2 TAIL)
CONSTANT	6.026	0.250	0.000	.		24.079 0.000
UNFAMILIAR OBJECTS	-0.825	0.104	-0.807	0.502		-7.934 0.000
COMMON OBJECTS	-0.416	0.151	-0.248	0.637		-2.751 0.008
UNFAMILIAR OPERATORS	0.079	0.028	0.287	0.511		2.848 0.006
INFERRED MULTIPLIERS	-0.045	0.023	-0.184	0.584		-1.951 0.056

ANALYSIS OF VARIANCE					
SOURCE	SUM-OF-SQUARES	DF	MEAN-SQUARE	F-RATIO	P
REGRESSION	8.873	4	2.218	34.413	0.000
RESIDUAL	3.545	55	0.064		

**Table 5. Stepwise Regression with transformed data.**

### *Discussion*

The results of the multiple regression show that as the number of unfamiliar objects increased by 1.3 the class's confidence in its ability to learn to compute a formula decreased by .86. Using stepwise regression the coefficient changed to -.825. Since the students' confidence in their ability to learn to compute a formulae was based on a seven point scale, these results indicate that as the number of unfamiliar objects increase beyond one, the students' confidence in their ability to learn to compute a formula should decrease dramatically. In graph 2 a scatter plot of the transformed unfamiliar objects plotted against the class mean scores illustrates this. Even though the change was less dramatic, the unfamiliar mathematical operators also contributed to the model. Many formulae contained no unfamiliar mathematical operators and those that had them did not contain very many. For this reason the significance of the unfamiliar mathematical operators may be under estimated. In any case, the significance of the unfamiliar objects and the unfamiliar mathematical operators is consistent with the visceral reaction that Mandler predicts. The further contribution to the model made by the common objects and the inferred multipliers is consistent with the concept of cognitive overload. Specifically, the perceived difficulty of a task increases as the amount of information to be processed increases. It is surprising that the common mathematical operators did not contribute to the model. It is possible that the subjects had over learned the concepts behind the use of the common mathematical operators and were thus able to chunk this information and move on. However, it seems the same would be true of the inferred multipliers. The marginal contribution of the inferred multipliers may be explained by the confusion that arose when the subjects attempted to determine whether a string of letters was a word, an abbreviation, or several unknowns.



## General Discussion

Even though the subjects were instructed to rate their confidence in their ability to learn to compute the formulae, they still perceived formulae with more unfamiliar objects as more difficult than those with less. Furthermore, small increases in the number of unfamiliar objects in a formula caused dramatic decreases in the subjects' confidence in their ability to learn to compute the formula. From the protocol analysis it seems that subjects rapidly reach a point where further information processing, and therefore further learning ceases. This implies that instructors and texts should 1) when possible, avoid formulae with unnecessary unfamiliar objects in them, 2) not introduce more than one unfamiliar object at a time, and 3) introduce any unfamiliar objects prior to their use. This may seem obvious to some, but many instructors and statistics texts introduce formulae, complete with unfamiliar symbols, and then give the explanation. This requires the learner to hold decontextualized information as well as all possible links to the information in working memory and still process the information. If the symbols are introduced prior to their use, then that information can be proceduralized. Since the use of procedural knowledge requires little working memory, more working memory will be available for processing information, i.e., understanding the formula.

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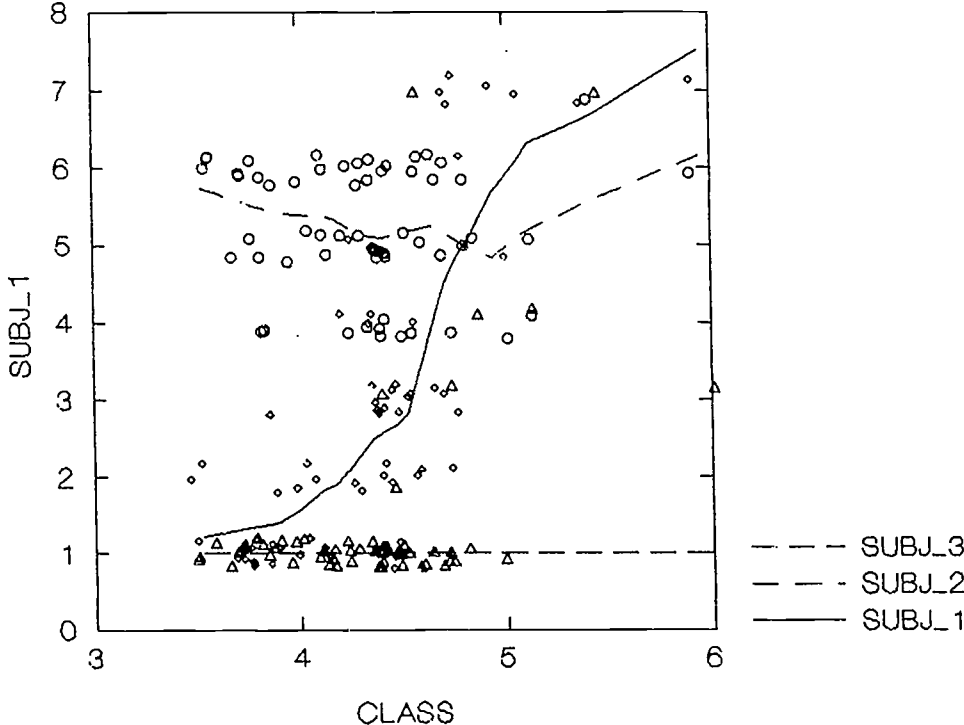
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Graph 1. Subject scores plotted against class scores



Graph 2. Transformed unfamiliar objects plotted against class scores

