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ABSTRACT

This paper uses a qualitative case study approach to discuss the training of special education teachers in mathematics instruction. The case focuses on a remedial mathematics intervention based on social competence theory with an adolescent with cerebral palsy. Two student teachers attempted to encourage mathematics instruction which focused on the real life needs of the individual student. This report is organized around the group discussion by the teachers in training, which addressed: behavioristic and psychodynamic patterns of human conduct; the aims of mathematics education; appropriate mathematical content for the student with disabilities; necessary special education teacher preparation; constructivist goals for mathematics in teacher education; and a cognitive clinical approach for mathematics in special education. The paper concludes that mathematics studies for the special education student teacher should include: (1) mathematical knowledge; (2) literature on mathematical learning of students with special needs; (3) discussion of old and new beliefs about mathematics learning; (4) observation of students having mathematical difficulties; (5) construction of a remedial program for a special needs student; and (6) record keeping. (Contains 45 references.) (DB)

MATHEMATICS AND QUALITY OF LIFE
A NEW THEME IN SPECIAL TEACHER EDUCATION

Olof Magne

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Students with special educational needs usually display poor retention of mathematics. This happens often for those who have sight, hearing or motor impairment and experience general learning difficulties and motivational, emotional or perceptual disadvantages. Some researchers use technical terms, like "Dysmathematika", others prefer less defect oriented expressions as "maths poor" children.

Is the low achievement a consequence of their impairment, or of wrong learning strategies, unwise special teaching traditions etc? Mathematics teaching has been subjected to many impulses for some time - not always favourable.

This presentation will discuss an alternative treatment of mathematical learning. It will speak in favour of open learning practices and principles by which the special teacher education may get a character of experimental design.

Future special teacher education should accept that mathematical literacy and competence are valuable parts of and essential for the quality of life for everybody and that its study consists of more than basic arithmetic. Individual needs must be emphasised more than in traditional teacher education. Mathematical activities ought to begin at an early age and continue as adult education. Younger children mainly acquire mathematical experiences by play. For older students mathematical learning should aim at social competence, and this is particularly important for disabled persons.

This indicates a need of new competences also for the teacher as well as new forms of curricula, learning methods and educational aids. The new approach was called *social mathematics* which would mean *mathematics tailor-made for the disabled individual's future life.*

Keywords: Behaviorism, constructivism, disability, dysmathematika, error pattern, individualisation, mathematical learning, social competence, special teacher education.

Mathematics as quantitative reasoning needed for the life of the individual

This paper will deal with Sven andUlrik, future secondary school teachers, and their observations of a handicapped student Bob and, later on their discussion with the group they belonged to during their teacher training. The verb ´train´ is not used pejoratively. The terms ´train´ and ´educate´ are chosen interchangeably because ´educate´ may have too wide meaning and ´train´ too narrow signification. The French may have a clearer word,´former´ (´formation´) as it has an in-between significance.

The paper is also aimed at examining the intricate web of cognitive processes underlying the training (or education) of mathematics teachers who wish to tutor disabled students as effectively as possible.

What I am going to take up is related to an actual event at the Malmö School of Education during the academic year 1992/1993.

I became involved because I was asked to act as an extra tutor to Sven and Ulrik who concentrated on mathematics teaching. The two of them had respectable academic qualifications.Ulrik had graduated with a dissertation in mathematics and Sven was a civil engineer. Their aim was to be mathematics teachers.

It then happened that both were allotted to a practice period in the class of sixteen year old students, in what in Sweden is called *gymnasium*, roughly corresponding to the British sixth form or, in the United States, college.

In this class one student, Bob, aroused the sympathy and interest of the two young mathematicians. Bob had to use a wheel-chair due to cerebral palsy; he could not move his legs. Apparently he was rather capable. His linguistic ability was above average. He was also well-informed in history, literature, philosophy and social science.

However, in mathematics Bob achieved extremely poorly. Sven and Ulrik brought up the scheme to diagnose Bob´s mathematical performance and suggest an adequate programme for him.

They used the following method of study and tuition. They followed Bob's work in mathematics and registered his progress in a diary particularly concerning his sense of responsibility, his power of concentration and independence in his classroom work. In addition, they tested Bob's performance with theoretical and practical tasks in mathematics.

An interview with him revealed that Bob found mathematics teaching rather devoid of meaning for him. He wanted realistic problems which he would use in everyday life and future employment.

Since Bob wanted mathematics to suit a future as a social worker Sven and Ulrik aimed at performing a generalised type of mainstreamed education for dysmathematical physically disabled students in order to observe and assess this alternative during one term. They wanted to reject the traditional view that the disabled person's disorder explained the unsuccessful mathematical learning. Instead they preferred to assume an alternative hypothesis that the learning methods/materials and the learning situations must be adapted to the learner's learning style.

The interview also disclosed one interesting fact that should have great importance from two aspects, firstly for the construction of Bob's learning programme, secondly for the further discussion of basic principles for the education of disabled adolescents.

Bob had been given individual special education in mathematics for about three years before entering the *gymnasium*. In this tutorship the special teacher had coached Bob intensively by using an imitation strategy which left very little initiative to Bob. Sven and Ulrik critised this approach observing that it could result in an attitude of inactivity in relation to mathematics studies.

The diagnosis mainly followed a procedure described by Magne and Thörn (1987) which has a wholistic character. Bob's schoolwork was registered by a diary method accompanied by interviews and observations.

When he was tested by Sven and Ulrik Bob displayed a pronounced incapacity for taking responsibility for his actions. Thus, he always wanted instructions before starting exercises. He did not like to study the texts independently. He often failed to perform his home work although he liked to get weekly projects.

The outcome was a solid study of Bob's mathematical knowledge, skills, interests and motivation. A remarkable thing was that Bob liked to learn mathematics but on a level that corresponded with his knowledge. It proved impossible, though, to suggest other than conventional arrangements for Bob concerning study methods and learning aids.

Later, Sven and Ulrik came up with the idea - or rather principle - that for Bob *mathematics should be what he needed for his future life*.

The principle raises a central issue. In the ensuing discussions together with the student teacher group *social competence* turns out to be a key-word. Social competence in mathematics means that an important part of mathematical knowledge is to reach a level of performance by which the individual attains optimal social emancipation.This conception of mathematics didactics will be thoroughly discussed later in report (see for instance p. 13).

Bob was firm about his coming career, namely to work in social service, preferably among disabled persons. For Bob mathematics should mean application to social service.

Sven and Ulrik tried to sell this idea to Bob's mathematics teacher who only reluctantly let himself be persuaded into presenting Bob with social service problems on rare occasions. He left this service to be done by Sven and Ulrik.

The teacher said that he lacked instructional aids for this purpose. Which was true! So the project finished with a feeling of mutual dejection. Sven and Ulrik looked upon the outcome as a failure. Thus, to Sven and Ulrik mathematics for a disabled student appeared to be a neglected field of study.

During the following check-up by Sven, Ulrik and their fellow students they all suggested that research is more or less lacking, what is not exactly true, at least in Sweden. But if you try to test this, I guess, you might reach a similar conclusion for your respective countries or school systems.

When they discussed the needs of the disabled students, the group of Sven and Ulrik agreed that the mathematical learning, as well as the teacher education, ought to be innovated.

Student teachers discuss

Now, let us look at the mathematical education of a prospective special teacher from the horizon of the student teachers themselves.

Essentially the following account is based on the discussion with Sven and Ulrik, but is expanded to a broader review of teaching and learning principles in the field of special education.

What would a student teacher think as he/she meets with the problem how to stimulate and tutor a disabled student to come up to optimal mathematical development?

At the Malmö School of Education, some years ago my colleagues and I conducted interviews with student teachers on their task as teachers of mathematics for disabled children. A few suggestions have emerged. This enables us to derive some conclusions how special educators may acquire their instructior 'dills.

Let us think of a problem conscious student teacher like Sven and Ulrik, as in the project I have just commented on. He/she will probably ask for answers to key questions that begin with words as *Why? Who? What? When?* and *How?*

Here are some possible questions:

(1) Concerning aims of the mathematics education

Why should the special teacher argue that a disabled person needs mathematics?

Who would have use of such mathematics?

When would the special teacher introduce mathematical topics?

(2) Concerning mathematical contents for the disabled students

What would the special teacher like to include in the mathematics of the individual student?

(3) Concerning special teacher education

How should the special teacher acquire the necessary knowledge himself/ herself?

Behavioristic and psychodynamic patterns of human conduct: A position in a theoretical issue

Views of education are usually attributed to one or the other of two main patterns of human conduct: behavioristic/associationistic or psychodynamic ones. It is also presumed that, from views of education, there is a link to hypotheses of curriculum and, henceforth to educational methods and instructional aids.

Behaviorism, or associationism, dates back to ancient Greek philosophy. Aristotle was its originator.

Psychodynamic systems are from our time. Sigmund Freud is said to be their inventor.

Let us jump to education. According to behavioristic teaching systems the teacher is expected to follow prescribed recommendations of authorities, motivate the students so that the students acquire the desired responses. Learning is a transmission of 'knowledge' (or even transfer of training).

This leads to a curricular perspective in terms of 'frames'. A frame is known to be the mechanism through which the educative process in the classroom is steered from levels above the educative process. Conditions in the society are assumed to govern the educative process through objectives and organisation. The central Administration looks out into the society to perceive demands and aims, then looks at the schools and directs their actions in order to guide (or try to guide) the educative process by specified demands. Through allocation of money it hopes to control certain space and time conditions for the educative process in the classroom.

However, this behavioristic curricular system seems to show weak sides. Here is an, in itself, incomplete list of flaws.

(1) In reality this curriculum steers only imperfectly.
(2) Behavioristic belief in transport of knowledge is incomplete.
(3) The student can learn, the teacher cannot 'learn out'.
(4) Learning is not a formation of associations (or 'bonds').
(5) Learning is a social, although not a collective process.
(6) The learning process varies from person to person.

The psychodynamic approach has the distinctive property that feelings and needs are of primary importance in the life of the growing child. Some basic needs must be satisfied if the child would develop a healthy mind and personality, for instance need for love, security, attachment, success, belonging, dependence and independence, and a personality identity.

After Freud, the French and British cognitivists have introduced psychodynamic learning theories (Bartlett, Piaget). Piaget stressed that conceptual construction and creative action are the roots of childrens' knowledge. Piaget also maintained that learning is a private affair, a reconstruction without end of the learner's acting structures. Piaget used the term 'schème' for the perpetual temporal building-up of reconstructed units, added to the previously built configuration of changeable units of the mind.

At about the same time Bartlett suggested that remembering memory units takes place in terms of existing structures using the term 'schema'. A schema is thought to be an organised structure that integrates notions and expectations of some experienced event. Bartlett strictly emphasised that learning, as well as remembering, are active processes, where the learning always depends on a constant 'striving after meaning'.

In the educative approach the student is at the centre of learning.

This approach to the philosophy of learning and memory has been called *constructivism.*

Primarily, constructivism is a philosophical view of epistemology (on knowledge and the consciusness of knowledge).

Knowledge is not transferable from one person to another. Neither is knowledge a representation, or mapping, of the a 'real' world.

The student may conceive his/her impressions as direct pictures of the environment. Nevertheless, the knowledge of the individual should be assumed to be constructions of outside events. A clear apperception is created when specific notions are formed in a great many connections, thus constituting abstractions of many perceptual states of mind.

Knowledge has sometimes been said to be validated, in a biological sense, as the power to control the environmental factors, a global control of the relations between the individual and outside conditions, due to the vitality of the individual's knowledge structure (Maturana & Varela 1992). Apparently, knowledge can be defined as the individual's mastering the factual circumstances in his environment.

About this biological argument the constructivist reasons as follows. The human being is regarded as a self-organising system who constructs his own reality by means of all his perceptions and reactions, as a result of his contacts with the environment. Knowledge means mastering his environment, his lot of life. Thus, knowledge is a process by which the world literally comes into existence in the mind of the individual human being.

But what makes the individual perceive and understand the mind of a fellow-creature? The answer of some constructivists is: Knowledge is considered an 'unknowable reality'. This position suggests that the mental state of another individual can only be interpreted by analogy, that it is *created* thanks to a personal observation grid (which usually functions implicitly) and comparisons between apparent actions of the other individual, as results of external stimuli, which resemble similar input-output cases the subject himself has met with.

Other constructivists maintain that language communication makes it possible to make oneself at home in the world although no knowledge really can be carried across from me to others or from others to me.

Still another solution lies in the proposition of a 'consensual domain'about which Maturana says (1978, p. 47), ' A consensual domain is a fundamental biological relationship, and is established when two organisms interact recursively. Each organism acts in response to the other' (Quotation from Richards 1991, p 18).

Secondarily, constructivism is a view on learning, based upon child psychology and developmental psychology.

Along with this exists an application to mathematics learning. In this it is said that the learner constructs his/her knowledge in mathematics actively and personally. Janvier (1992) wrote,

* Firstly, the subject individually incorporates the new pieces to the previous assemblage;
* Secondly, there is a form of willingness to do so. It does not mean that the learner discovers knowledge: discovery learning should not be confused with constructivist learning' (p. 3).

The following practical consequences are important.

(1) Learning takes place as a social process in everybody's natural environment.

(2) Active learning is a necessary condition for lasting retention. Learning is characterised as a progressive reconstructuring of relations between self and world where world implies both tangible and representable reality. The student experiences reality subjectively and interprets knowledge as the abstraction of coordinated actions which are jointly targeted at survival. The role of the teacher is to stimulate the learning efforts of the student (catalyst effect).

(3) Constructivism has many faces. As Lerman (1989, 1990) points out, 'radical constructivism' is a relativist epistemology. According to this view, the ongoing acquisition of mathematical conceptions is one which only goes on in the mind of the student.

The constructivist process model in mathematics leads to another important approach.

Constructivist logic totally rejects the behaviorist learning model. According to available constructivist research it is unadvisable and, in fact, impossible to force all students to achieve the same goals in their education. (To some extent, this is the result of curricular intentions belonging to traditions of behavioristic type.)

It is possible to formulate a curriculum from the rule that the student is at the centre of mathematics education. In that case

* Firstly, the mathematical behaviour of the student is mapped out, and
* Secondly, those topic areas are priorited which the student should need first hand.

This way of looking at things is particularly fruitful in the education of students with special educational needs.

The attached fig. 1 illustrates the relations between student and school organisation. The graph has the student at the top. From the top position proceed arrows in various directions showing how to progress in the course of planning. For a student as Bob this concerns for instance

* The adjustment of educational policy as well as the financing of the education for the disabled student,
* The pedagogical methods (learning and teaching) etc.,
* The supply of instructional aids, and
* The individual learning itself.

It is a plausible hypothesis that this educational approach leads to higher efficiency in the learning of a student with special educational needs.

Question one: Why should the special teacher argue that a disabled person needs mathematics?

Mathematics - is it needed for children with special educational needs? Or to talk like ordinary people, will ´number work´ come in useful for ´maths poor children´?

Let us follow the discussion of the student teachers.

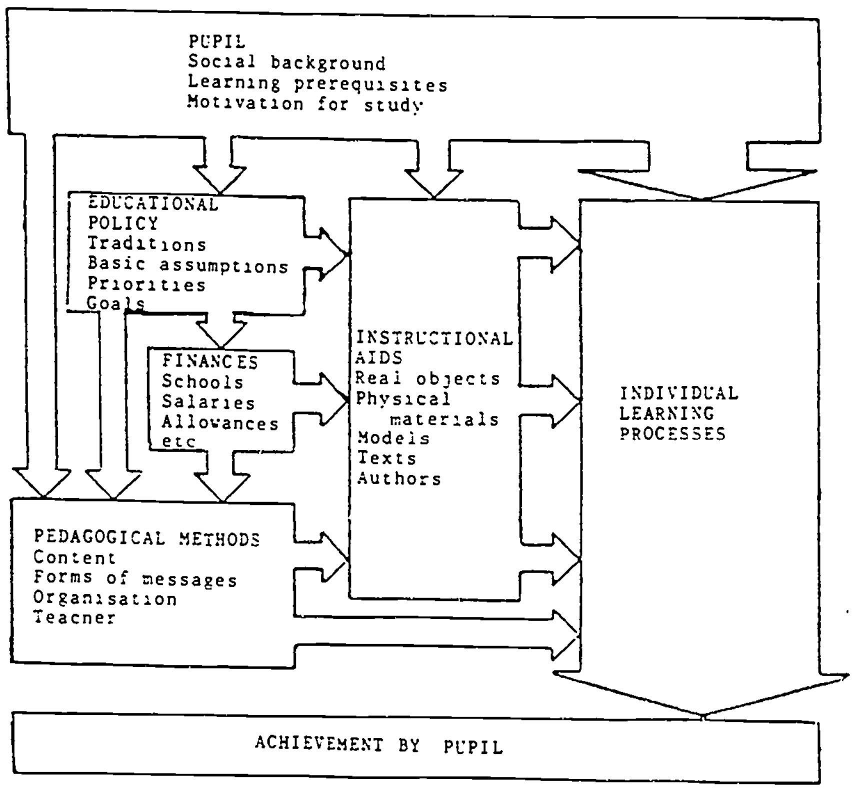

Figure 1. Student based educational philosophy (Magne 1978b, 1986).

At first they look up what is said about mathematics in the National Curriculum. They find something like two basic aims:

(1) To give the pupil a foundation for higher formal mathematical studies at universities, colleges etc.

(2) To furnish the pupil with general information of quantity and space for occupation, communication, and leisure activities in everyday common life.

Good! But how can this be put into practice in special education?

The student teachers discussing this are inclined to guess that a majority of special teachers look at mathematics learning as the *Ugly Duckling* of special education. Common belief may be that a lot of other subjects are more important.

Nevertheless, a mathematics teacher must find it necessary to uphold a good and sensible balance between various school subjects.

Literature on mathematical didactics could be supposed to throw additional light upon the question why mathematics might be studied, by Bob for instance. But no! There is little information in this Literature.

There is no lack of publications on the mathematical training of sight or hearing impaired or physically and mentally disabled or emotionally disturbed children. Most writers confine themselves to a narrow range of topics, usually within the following three themes:

* Calculation with small natural numbers,
* Aphasic phenomena, and
* Anxiety in mathematics.

Typically, international conferences seldom contain more than one or two contributions on mathematical behaviour of disabled person although the grand total of presentations very well may go beyond one thousand.

There are exceptions. There is a rather small group of international experts who have taken an interest in a field that has been represented by words as arithmasthenia (Ranschburg, Hungary, 1905), acalculia (Henschen, Sweden, 1920, 1925), dyscalculia (Gerstmann, Germany, 1924, 1940) and dysmathematika (Magne, Sweden, 1988).

For a disabled person it is a question of power, about quality of life.

Preliminary studies indicate a need of a partly changed foundation of the mathematics instruction and learning for the disabled students. The school mathematics is too much dominated by mechanical drill. It consists too much of arithmetic. Traditional mathematics for the disabled following the drill method may cause a risk for a student achievement that might be called *consumerism*. (Donovan 1990, 1992).

What does it mean to be a consumer, rather than an active master of knowledge?

It means to be stuffed with skills and habits with passive competence in routines, but without flexible liberty of choice; to get the attitude that just one answer belongs to every task, and that it is the authority of the teacher which vouches for the truth of this answer; to get into the habit that the student gets the correct answer by keeping to a marked track, and this would just be the purpose of education; finally to have been guided to receive information without questioning the purpose of the instruction.

It is sometimes said that an intellectually disabled person is virtually unable to pursue reasoning in mathematics. The teachers mediates the correct reactions in mathematics.

A more modern view gains ground. The traditional view is looked upon as obsolete by many researchers.

Traditional mathematics has apparently proved unsuccessful for many disabled students (Magne 1991). An alternative would be to change the approach and concentrate on *social aspects* of the subject.

In what way can mathematics add to the power? Paolo Freire (1985) and others have maintained that literacy and mathematical competence are valuable parts of and essential for the quality of life of the individual (cp. Mellin-Olsen 1987). If such knowledge capital can be enlarged there is also an increase of the power of the individual to be master of his/her life.

Paolo Freire is an inspired pioneer for educational programmes in order to raise the oppressed and to strengthen international consciousness offering disabled and deprived persons opportunity to achieve social maturity and independent life in our society.

Freire claimed that mathematics was one motor for development towards independence. Social maths would very well be a purposeful answer for such deprived students..

On one hand, education can preserve established injustices and barriers and contribute to producing rigid, timid and alienated subordinates. A social incompetence and a lack of linguistic and mathematical communication may lead to isolation and cultural deprivation.

On the other hand, education may build up active consciousness and self-confidence also for isolated and deprived human beings, through socialisation and by a setting a good example. For persons with such labels it is important that their learning goes beyond mechanical drill or training, as ends in themselves, or results in shallow consumer routines.

Basicly, it depends on what the meaning of mathematics is. Has the administration fixed why and what the student must learn? Or is it you and me who have power to say what is mathematics, based on your needs, interests and ability to think?

Bob and many other students may want to learn mathematics in its application to social services. This may be called *social mathematics*. It can be a purposeful answer for students with special educational needs. However, let us wait just a moment for a more distinct explication of the term social mathematics

A three-parted answer to the Why-question could be that

(1) Mathematical knowledge is part of, or a component of, human survival.

(2) All students, also students with special educational needs, should learn mathematics according to their individual needs, interests and power of thinking.

(3) Future mathematical programmes should harmonise with the specific background of the individual, make allowance for social aspects of the subject matter and the *social competence* of the student.

Question two: Who could have use of mathematics according to the discussion on question one?

Prospective teachers must learn about how children with special educational needs learn. They should go through a learning process about their pupils´ learning, but also get an indirect as well as a direct teaching experience.

The special teacher may look upon himself/herself as a detective in the Wimsey or Poirot tradition to deduce from seemingly meaningless or insignificant clues what is going on in the child's mind. As he or she becomes better at this observation and detection technique their efficiency as teacher will improve. With the help of persistent, devoted assessment founded in modern educational philosophy the special teacher is able to prescribe a programme of decent validity.

Research people seem to agree about the fact that there are great individual variations in the mathematical behaviour among those who are said to have special educational needs.

Learning is something that usually takes place in a conscious mind due to active volition. For all human beings quantitative thinking is part of cognitive invention and creative action. This is something appurtenant to the mathematical thinker but also to the disabled child. In many cases, however, the disabled child stops on the lower levels of cognition (cp. Piaget's preoperational thought and concret operation).

At school leaving time (about 16) mildly intellectually disabled are reported to display an average retardation in mathematics of four years, hearing impaired about three years, sight impaired 2-3 years and neurologically impaired, with great variations, from average to more than ten years lag of attainment. For many kinds of impairment, disability, handicap or separate syndromes no information is available. Traditional mathematics usually is unsuitable for all these students. Little is known about optimal learning conditions.

Blind children are at a disadvantage because they have a handicap to visualise. They may be inhibited in their efforts to imagine numbers and form conception.

Deaf children may be linguistically impaired. This can lead to a restricted vocabulary and, secondarily, to limited understanding of verbal reasoning.

Emotionally disturbed children sometimes display a disordered state of consciousness, and their reasoning would be confused. They may have difficulties to establish orderly structures in their problem solving.

Physical and neurological impairment often inhibits the growth of form perception and geometric notions but also communication as with Bob.

Most intellectually disabled have great difficulty to grasp the structure of the decimal system. Due to reasoning disability their mastery of mathematical expressions usually is very low (formal problem solving). Simple visualisation and geometric skills can be attained up to a certain standard. Addition is better mastered than the other arithmetical operations. Least skill is found in division. Desk calculators or computers should always be used instead of pencil-and-paper computation.

Comparisons beween high achievers and low achievers in mathematics display many similarities in their behaviour patterns:

* The first thing common to all pupils is that they do both correct and incorrect mathematics because they think.
* Secondly, it is obvious that in all pupils many inadequate reactions are caused by oversight, that is an accidental lack of concentration.
* A third important observation is that all pupils work with an apparent striving after meaning (Bartlett's term).

* Fourthly, it is evident that lowachievers and underachievers, just as other pupils, construct their knowledge and do their problem solving or do their calculation in accordance with ´schemata´ or strategies they create themselves from earlier experiences.
* Fifthly, shortcomings can be looked upon as individual attempts to reason logically as a result of the interplay of mathematical contents and the pupil´s reasoning power.

But there are also important divergences between error patterns of highachievers and lowachievers. The following three observations are particularly striking:

* Most shortcomings of lowachievers are caused by lacking or insufficient logic. Shortcomings of highachievers are mostly due to oversight.
* Lowachievers not only make systematic errors but continue with one and the same type of error for long periods of time, if not given appropriate intensive intervention. Highachievers seldom persist in habitual error patterns and easily give up such deviations from logical structure.
* Remediation is quite effective for highachievers whereas it is a very time-consuming and laborious affair for mathematical lowachievers, as well as underachievers.

Mathematical behaviour is located to various regions of the brain. Important functions are found in connection with the visual cortex (the occipital lobe) and the sensory and motor centres (the parietal lobe) of the left hemisphere. Form and geometry have at least one centre in the right hemisphere, near centres for music, aestetic regulations etc. Localisation of these functions in the right hemisphere is uncertain. Due to brain damages in the regions just mentioned, several syndromes of mathematical dysfunction are known (Magne 1991). There are some authors who seem to deny the existence of dysmathematics, for instance Miles and Miles (1992). This seems to be a unwarranted view.

The consequences for learning are obvious. Visualisation and manipulation methods must be important elements in the mathematical cognition. Therefore, visual and manipulation instructional aids would be useful for all acquisition processes.

But it is also important to inspire the student with confidence, hope, courage and self-respect. The way to attain this goes through the student's invention of his/her own mathematical knowledge. The learner must learn actively by strenuous work. As has been said before, it is not the teacher who "learns" the student. The student must learn by his/her own efforts.

Equally erroneous is the belief that the student must imitate an adult person or should be pushed by a step-by-step piloting method away from the difficulties of a problem or learning situation. A student will soon be so accustomed to indulgent help and support of a teacher or an assisitant that he/she then, gradually, grows inactive and dispirited.

Let us take the case of Anneli. Her syndrome is a mental handicap. She is, at the age of 12, educated in a regular class.

Her teacher has used a lot of sensible ideas and equipment to get her to learn addition tables, I mean number facts as 2+3, 3+5 etc. In a way this is successful. Anneli knows quite well that 2+3 can be represented by two sets of objects like two apples and three plums. The union of these sets could by Anneli be called 'five fruits'. She talks about such objects without need to put things in front of her. She is motivated to say sums by heart like 2+3=5 and so on. She goes on and on repeating these expressions and hopes to be fluent one day. But so far, all this training does not improve upon her knowing of the tables.

The interpretation of this failure may be the following.

Anneli 'trains' in a mechanical way, not because of the teacher but her own motivation and strategy. Her many thousands of repetitions do not come farther than to her eyes and lips. Her sense organs signal to her sensory brain and motor brain. What she is not doing is to perceive. Perceiving is a voluntary, organising act.

Neither is she using her brain to actively compare the various addition facts with each other in order to find hidden equalities or interesting structures, as the observation that 2+3=3+2 or that 2+3=2+(2+1) or that adding 9 to a number is equivalent to n+(10-1).

The result is: no apprehension, no construction, no abstraction, no memory.

The fault is not her teacher's. She has done her best to foster a sense of thinking.

Anneli has a perception and thinking handicap. She can look. She can talk. But she has shortcomings in her perceptive brain cells and a strange confusion in the way how to organise her recollections or experiences.

Similar educational shortcomings may be found in other children with or without special educational needs. They exist, for instance, among sight, hearing, motor or neurologically impaired or emotionally disordered children but among average children too.

What could Anneli do? It seems first of all necessary to quit the formal way to learn mathematics. Next thing would be to take leave of the type of maths learning which she has used.

For Anneli and many of her equals an alternative might be to change the approach to learning fundamentally and concentrate on 'social' aspects of mathematics. Anneli must be at the centre of her particular learning field.

Recently a *factor-interplay 'model '* has been proposed (see Magne & Thörn 1987, Magne 1991) as a rationale for remedial work in mathematics.

Through the individual's learning runs the connecting thought that at first the learner should build up experiences. Next step would be to create invariances and to construct insights, generalised representations and abstract ideas. Then, the assimilated topic structures should be used to acquire reasoning patterns. A fourth learning phase may be characterised as skill acquisition or rather accommodation of the new patterns to old schemata.

The factor-interplay model implies that the educator simultaneously considers *content of mathematics* and the *pupil's reactions*.. It would be wrong to base the teaching on the mathematical subject matter only or on the mental deviations of the pupil alone. The interplay between the subject´s contents and the pupil´s quantative reactions must determine the shaping of the programme.

Three answers to question two may be the following ones.

(1) Mathematics learning/teaching for disabled has two necessary dimensions: topic and behaviour (the factor-interplay model). Mathematical disability appears as complex and multi-factored.

(2) *Low achievement* is characterised as a sub-average knowledge and achievement in mathematics of a person with general low capacity to learn. *Specific learning difficulty* (or underachievement) is defined as a sub-average knowledge and achievement in mathematics only, despite average or higher attainment in other school subjects. In both cases individual intervention is justified.

(3) Intervention should heed individual learning prerequisites.

Question three: What would the special teacher include in the mathematics of disabled individuals?

In a subsequent discussion of a student teacher group the "What"- question is treated as a central issue. In these talks *social competence* turns out to be the key-word. Let us confine ourselves to mental handicap.

Social competence is a name for the extent of independence in a persons´s daily life.. Over and above primary ADL-training, social competence means power of practical attainment of a social-adaptive character. It is an aptness to construct and command relations between self and environment (see for instance Doll 1953).

As has been said earlier, an important objective of the mathematical knowledge is to reach optimal social emancipation (see also Kylén 1974, Nyborg 1986).

Nyborg speaks about fundamental ´concept systems´. He argues that ´concepts´ may be both verbal and non-verbal. He defines a concept as a form of knowledge or knowing which is incorporated and retained in the mind of the individual and used by the mind to ´grasp´ or understand relations between various events or objects (p. 8). A concept system is a set of abstraction units that is organised into differentiated totalities, composed of principals, subordinates and collateral elements which are usually kept together with the aid of symbol combinations (page 11). The ´concept´, in Nyborg´s meaning, may rather be interpreted psychologically than logically.

Nyborg has thoroughly explored the learning procedures and conditions of learning in mentally handicapped and also described how concept systems begin, are created and consolidated or decomposed in socialisation processes.

With regard to the social aspect of mathematical learning it is fruitful to exemplify fundamental concept systems (according to Nyborg´s terminology). Obviously, it is a case of open systems, formed under influence of their social usefulness:

form (ex. round, square)
position (ex. horisontal, vertical; seated, erect)
quantity (ex. large, small; four; increase - reduce amount)
size - height - width etc.
placement - space - distance; location - coordinates
direction
temperature
mass - weight etc.
time
value (incl. money)
speed - velocity rate etc.

Bob and Anneli seemed to be examples of social competence on quite different levels, according to the discussion of the student teachers.

Bob would have a programme aiming at professional knowledge within the range of his social interests. His problem is that no available textbook suited him. He should himself take a hand in his planning. In theory it might be possible to construct a study course for Bob centered round welfare service, labour market conditions, adminstrative work and so on although in practice it proved difficult to do so effectively. Exercises ought to concentrate on analysing practical problems instead of formal mathematical subject matter. Among mathematical topics, elementary statistics and probability could be used, preferably population studies, as well as practical use of formulas in algebra. A low-cost calculator should be the instrument for habitual computations. Computers are necessary, particularly in practical problem solving tasks. Rote learning of mental pencil-and-paper type, as algorthmic drill, should be totally avoided.

This could be summarised with the catch-word: individualised do-and-think learning instead of collective step-by-step drill.

What is needed for Anneli? Perhaps a non-formal, social and practical way of learning.

For Anneli, mathematics literally means survival learning, to achieve independence in her future life.

Social competence consists of social main areas as in the following list.

* SHG-area: Helping oneself, e.g. being capable of moving, successful with hygiene and health.
* SHD-area: Taking care of one´s clothing.
* SHE-area: Providing meals for oneself.
* L-area: Transporting oneself.
* O-area: Occupying oneself and having an employment.
* C-area: Communicating with others by language.

* SD-area:	Taking the responsibilty for oneself and others.
* S-area:	Collaborating in social activities with other persons.

I also refer to the Magne-Thörn list of main behaviour areas associated with elementary mathematics education (Magne & Thörn 1987):

P-area:	Linguistic representation and problem solving
N-area:	Numeration and notation conceptions
G-area:	Form perception and space representation, body awareness, money, geometry, measuring and units
ASMD-area:	The four operations
F-area:	Functions, algebra, equations
B-area:	Descriptive statistics, probability.

Here are some reflections that emerge from the discussion in a student teacher group.

Important social activities can be found among the main areas of mathematical behaviour suggested by Magne & Thörn (1987), especially within the P-, N-, G-, and ASMD-areas. In addition it might be useful for a disabled person to have experience about probability at gambling, at pools or in a lottery (B-area).

To prepare a meal often means mathematical components as proportionality (P- and N-areas), measuring (G-area), cost of ingredients (G-area), space of time and chronological order (G-area) and sometimes rough estimate (ASMD-area).

The *transportation area* is a matter of outright motor movement capacity, at first. When the child grows up, he or she walks to a nearby playmate. The need arises to orientate oneself with the help of various reference points, to have a body representation and to interpret the surroundings geometrically (G-area). To move about freely in the home district requires to use buses, and the transportation is then complicated by the necessity to buy and use tickets (N-area, ASMD-area). For journeys to more distant places it is important to make plans and consider, for instance, time, means of conveyance, sleeping accommodation and meals (thus an all-round mathematical problem solving).

The *occupation area* is simple only during the first years of life. Already a two-year old child uses toys, tools and expresses himself/ herself with the aid of abstract words and phrases (number words, quantity nouns and adjectives), used when the child plays, constructs, models, draws and cuts (G-area). Money comes in early. Older children do odd jobs deserving payment (G-area). The children discover number words and digits and their meanings (N-area). Those who get a job meet with economic situations (P-area). This also is the case with homework, for instance to cook according to a recipe (P- and N-areas). Maybe the child wants to compare contents and prices of two prepacks (ASMD-area). If the child wants to know about a two per cent discount of the price, then the N-area is involved.

Linguistic contacts, among other things, lead up to quantity terms and verbal puzzles; later on to more sophisticated problem solving (P-area). Telephone routines imply language but also concern the technical consequences of usage and sometimes payment (G-area and ASMD-area). This is the case, as well, when a boy or a girl buys a magazine, some sweets or an apple (P- and G-areas).

To take responsibility for oneself or others is an area full of complex activities. First of all, a lot of money and management of money is brought in here (N- and G-areas). The young person is entrusted with money. He or she must check up on change. They do some shopping (P-area). They buy for other persons or get themselves articles of clothing. They have freedom of movement in daytime or more permanently.

Collaboration in social activities includes mathematical tasks of intricate nature. This can occur in simple competitive plays (N-area), in sports (G-area), party games (B-area), in school childrens´ games and otherwise in team occupations of young persons (P-area). Later civil obligations demand for mathematical reasoning and achievement.

Many of these activities or group duties can be anticipated at school. Anneli can be offered an optional learning programme integrated with other school subjects where the development of her *social competence* should/would be particularly emphasised.

Donovan´s example "At the Newsagent´s Buying a Paper"

A practical example from disability didactics is the project which Donovan made with Australian teenagers in a special school for intellectually disabled (Donovan 1989).

The experiment began with a video in which the students viewed a newsagency through the eyes of a person visiting it, looking around inside it, buying a newspaper and then leaving. Afterwards the students continue with a conversation about the video. They relate own visits to similar shops. They tell about experiences what shops, stores or supermarkets look like. They are asked to describe in their own words and constructions what they remember, what signs they see, whether they often visit a news agency, if they have somebody to accompany them and what they buy.

The main task consists of a real visit to a newsagency. Even this time they get information in advance. The students take the opportunity to buy something. The teacher observes their way of handling money, and particularly change.

Finally, the task is followed up by giving the students a chance to make shopping on their own, at first in the now well-known shop, later in other shopping places. They sometimes go shopping at the request of their teacher, sometimes on their own initiative. Of much importance here is the student´s point of view, interest and responsibility, to be brought face to face with problem situations concerning contents and purposes. Situations should deal with real money and real needs.

The teacher students should accept Donovan´s view that cleverness with real money tends to make the person independent in his/her social life. Such a skill is not sufficient for being independent, however, but part of being confident of his/her own power. The way towards emancipation implies that the educator considers the context between the tutor setting a good example and the socialisation of the student. When working out programmes for independent citizenship, it is also necessary to realise that the guidance takes place with the bakground of ´power relations´.

There is a risk that the teacher produces barriers that hinder the students to be independent. On the other hand, the teacher has the privilege to reduce insecurity and raise the students´ competency of seizing power of life. The mathematics teacher should always put this question to himself/herself, ´How can mathematics teaching be directed at better understanding and transforming reality towards a more human existance for the handicapped?´

Question four: When would the special teacher introduce mathematical topics?

In the subsequent discussion the student teacher group may wonder when mathematics should begin and how long it ought to continue.

There are various opinions about this. Some want to begin early. Others will postpone the start. It seems that at least three reasons imply that mathematics tuition ought to start at an early age and continue during adult age.

The first reason is that some reactions of a quantative and spatial nature already exist during the first two years of life, at least for average children.

The second reason is that even innate reaction patterns which get no stimulation tend to be inhibited or never develop in the child´s behaviour repertoire.

The third reason is that acquired knowledge tends to fade away if it is not upheld or renewed from time to time.

Some studies have been conducted with a successful play method for mathematical learning in preschool and special education (see Magne 1990, 1992). As for the method, it is built on the theory of Magne and Thörn (1987)

The method presupposes that during the first time of learning (age one to ten years) the disabled children mainly acquire their mathematics experiences by *play activities* . The programmes shall be so designed that the activities are arranged according to a predesigned structure. Due to this rationale, the basis for all mathematical education primarily is the active learning of the student, in relation to the subject matter, and only secondarily a subject theory of mathematics.

It follows that the teacher should rely on the student´s actual, social experiences. The language ought to be enriched also in the mathematical domain. The disabled students must improve their form, body and space conception; it must be correlated with the experience of real objects. Number conception should always be associated with concrete, practical experiences, never start from skills of talking, reading or writing number words or digits, or performing simple operations with the four arithmetic rules.

The results of the experiments indicate that systematic and structured play activity is effective on age levels from about three to about ten years of age for the learning of mathematics:

Take time to play

Playing is the source of eternal youth.

But playing may not to be enough.

For adolescents and grown-up persons it will be necessary to find other activities than play. A growing boy or girl gets more earnest interests and needs, even if playful occupations persist and are of life-long importance.

It is the life and quality of life that come to the front. Vocational training, employment, trade, wages, possessions, sex, entertainment, residence, family, and journeys form the centre of needs and interests.

The education must make allowance for these different motives.

The basic learning process seems to be similar for most human beings. But learning objectives, learning strategies and types of retention usually vary a lot. An intellectually disabled learns slowly and advances a much shorter distance into mathematical knowledge than the average student or the gifted.

The curriculum for the Swedish education for the mentally handicapped (*LSÄ* 90) declares for instance that the school must ´prepare the students for facing demands that the community calls for´ (p. 76), that is to say, also expectations on the handicapped and their learning of specified topics ought to be adapted to the resources or prerequisites of the individual.

Specific selection of the subject matter must be made particularly for each pupil, even rejection of various topics. The principle of selection seems to be to bring in socially fitting learning elements.

These recommendations are necessary to follow also for adult education:

Take time to think

Thought is the source of power.

Question five: How should the special teacher acquire the necessary knowledge himself/herself

What does the special teacher education in mathematics look like as it is represented by the discussions of Sven, Ulrik and their fellow-students?

The esteem, space and form of special education mathematics usually is low among teacher trainers, mainly due to low priority. An example: In Swedish special teacher education mathematics and its didactics often have, at most, three per cent of the total teaching time. I suppose it may have similar length of time in many countries.

However, if the social value of mathematics is primarily meant to be important for survival and betterment of the individual being, mathematics learning must be assessed much higher.

Mathematics programmes in special teacher education are aimed at one and only one goal: get the special teacher ready to teach mathematics well and efficiently.

Firstly, the special teachers must have got sufficient insights in mathematics before their their teacher education.

Secondly, the student teachers will have to explore how the disabled learns. They must discuss the philosophy of teaching, know some psychology, get indirect and direct teaching experience and, finally, try individual systematic interventions in classes so that learning can actually take place.

An important thing to observe is that the teachers´ trainers also must have knowledge, of the same kind, and certainly on an advanced level. The trainers must go through a learning process about learning for disabled, as well as teaching mathematics, showing the student teachers efficient methods, how learning is done by pupils with special educational needs.

In their teacher training, Sven and Ulrik had met with Bob. We have followed their observation studies and heard reflections that emerged out of their attempts of educational problem solving.

In our Swedish teacher training project problem solving was looked upon as a central concept.

It is much talk today about *problem oriented learning* and its companion *open learning* . In both cases the learner has a choice, not absolutely but freedom to manoeuvre. The learner shall get more control than in average training. A third, quite forceful trend is the movement to promote *autonomy in learning*, or learner strategies for learner autonomy.

In *open learning* it should be the learner who decides over the pace he/she is going to work at. Open learning means that the student teachers have much power over where, when and how they study. They may learn at home, in a library, in the office or elsewhere. Lectures are kept to a minimum although the lecturer is far from unnecessary. To mention a few examples: introductory materials is always to be presented in conventional face-to-face courses, including study guidance, aims and curricular specifications, options of literature, laboratories, experimental devices, tuition etc., directions for individual checking-up on objectives, tutor-marked assignments, and examinations etc. etc.

Race (1992, p. 17) recommends,

´The human side of open learning needs people who:

* can be counsellors, fine-tuning the learning programme to different needs and capabilities of learners
* can select the learning modules which will be most beneficial
* can assess learners´ work on the modules
* can encourage and motivate learners
* can deal with individual problems on an individual basis.

Thus, the open learning system presupposes that the learner has the feeling "I´m getting there under my own steam" ´.

The instructors in open learning might be said to be trainers. A better term is tutor: they would be resources, rather than ´transporters´ of information. To transmit knowledge is respectable, but how interested are the student teachers as ´receivers´? Teacher students have reached the age when they have interests and capacities which are typical of the independent, self-confident man/woman.

Adults being confronted with teacher training often feel alienated when they start their new education. These students feel bad at language since their new textbooks use a style and terminology of a very unfamiliar type. They feel insecure to write in the new style. They may even find it unpleasant with research facts and concepts.

The tutor for student teachers should be able to understand the problems of the persons who begin the various courses of their teacher education. This teacher must be able to respond to the problems of the students when they feel vulnerable or confused, when they say, ´I fail in everything!´ or ask, ´Do I need all these details?´ or ´Can I never get to the thing I want to learn about?´

Also in a *problem oriented course* the tutor-trainer will have a lot of responsibility of a new kind.

Problem oriented learning means just what it says: Problems of your future profession are starting-points for your learning. One main topic could start from the question, ´How to organise the mathematical learning for an underachieveing boy of 16?´ Just as in the case of Bob. Around this core theme various sub-themes will be brought together, partly it will be ´need-to-know-material´, partly ´nice-to-know-material´.

Problem orientation in teacher education is challenging. It makes individual assignments attractive. Various interests will be guaranteed.

Those that Race calls high-fliers benefit by being able to work at their own faster pace, particularly through initial parts of their courses. Low-fliers can spend extra time by working through modules more than once and put in extra reading and effort on topics where, from the start, they had shaky knowledge.

Problem solving techniques might be particularly appreciated among student teachers who would like to play a role of pathfinders rather than being interested in a sort of run-of-the-mill routine. But even getting somebody into something new and exciting can help an ordinary thinking-in-the-grooves teacher to become a small-scale innovator.

A danger with problem oriented learning is, for instance, that the learner may overlook useful areas or disregard formal skills. Another risk is that learners weak in language (could be peoplelearning in a second language or a person unsufficiently mastering the language in a foreign-language textbook) may spend too much energy and time on unnessary trifles. Over-anxious learners might not be able to keep time and pace and be left behind, or show too little confidence to dare the exams.

Furthermore, a substantial danger is that the actual learning will be haphazard and superficial, dominated by idle talk. The problem oriented studies must be kept up on a high performance level. The examinations should also guarantee good competence.

As a tutor the lecturer/trainer must assume a lot of responsibility for each student. With good planning a good tutor can make all the difference to the student teacher's voyage. A helpful beginning is to think about the individual learners. Many are keen to start. Others need help to work on their own. Most of them have good study qualifications, a few have limited opportunities to study or need help in learning from textbooks or otherwise.

In a scheme of this kind, the time for lecturing can go down from say, 25 to 30 hrs. a week during periods of face-to-face courses to not more than ten hours or even less. The remaining 15 to 20 hrs. will be used for group sessions or individual work, laboratory experiments or workshops, visits or work in clinics and institutions etc. Every student teacher partly follows a personal programme with a problem case as the core of the assignments, partly a common comprehensive training course for all. It would be desired that the individual problem programmes will be subject of investigation and discussion for a group of students.

The project of Sven and Ulrik may demonstrate the usefulness of the problem oriented approach. Our experiences are positive. They can be summarised as follows:

* Problem solving, as a training method, gives the student teacher better opportunities to develop socially.
* The method activates the students and calls for commitment, originality, responsibility, and cooperation.
* For the lecturer, as for students, it is a venture into the unknown where the lecturer is a guide, stage manager and mentor.
* The student teachers gradually become confident to make their own decisions. The tuition is never looked upon as supervision. The students never feel defence attitudes.
* Problem oriented learning turns to good account as a training method.
* The method is recommended for teachers in all sections of mental health education, including teachers for children with special educational needs.

A similar line is to develop the learner's knowing how to act *autonomously* within the structures of learning prescribed by whatever compulsory arrangements they participate in.

Particularly in foreign language education the *learner's strategies for learner autonomy* has come into focus. Interesting suggestions are to be found in Wenden and Rubin (1987) and Wenden (1991). But autonomous learning is also desirable in mathematics. The aim is to shift to action in classroom programmes, for instance in a number of key themes, as teacher education, teaching strategies for learning, the needs to change the attitudes and beliefs of teachers and students, and to assess the impact of curricula and various syllabuses on students' knowledge.

Underlying this view is the conviction that teacher education is an essential element in educational innovation. The teacher is looked upon as the main change factor. The potential value of learning aids will depend on the teacher. However efficient the new materials or techniques may be, they will be used inadequately if there are no teachers or if the teachers have the wrong attitudes or beliefs to the learner's capacity of self-tuition. Still, educational change is human change. Change very much depends on the student's own activity. This means that Wenden may overestimate the role of the teacher. To learn is a private affair due to the specific needs and characteristic learning efforts of each learner, and these needs will vary.

Strategy for learner autonomy is usually more based on the student's own initiatives than both open learning and problem oriented learning. Wenden (1991, p. 15) stresse the value of the independence of the "intelligent" learner, by the help of his/her tutor, "They have acquired the learning strategies, the knowledge about learning, and the attitudes that enable them to use these skills and this knowledge independently of the teacher. Therefore they are autonomous."

As a consequence neither student, nor teacher should look on learning as just one ready-made procedure. It is just the other way round: autonomous learning aims at creating various strategies, each one intended to serve the individual learner. Naturally, there is no endless amount of such individual strategies. This is, however, not the essential thing. The important thing is that every student will feel that he/she is master of his/her own knowledge and can meet his/her own thirst for knowledge.

Manning (1991) has described experiments on classroom practices of "cognitive self-instruction", following similar principles.

Some constructivistic goals for mathematics in special teacher education

Once again it may be stressed that mathematical programmes in special teacher education has the aim to get the special teacher ready to teach mathematics well and efficiently.

It is possible to formulate constructivistic objectives for the education of a special teacher. This can be exemplified with a mixture of ideas of various constructivists.

(1) Mathematical subject matter

As a minimum of knowledge, the student teacher is recommended to have got a final certificate in a matriculation examination or equivalent standard. What has been termed a social competence study course ought to be optional during the special teacher training.

(2) Role of the teacher.

a. Giving examples how the teacher's instruction can be structured.

b. Discussing why constructivism is interpreted as a claim on the commitment of the teacher to promote the active learning of the pupil.

c. Demonstrating how the teacher could individualise the learning procedures for each pupil in the most favourable degree as to choice of topics, level of difficulty, learning methods, groupings and material.

d. Together with other normative decisions about human values this may be called the 'catalyst effect' of tuition.

(3) Intervention in the pupil´s learning process.

a. Providing the student teacher with competence to observe pupils and adequately interpret their observations.

b. Introducing organisational strategies in a mainstreamed class.

c. Suggesting methods how to prepare differentiated or individualised pupil programmes.

d. Presenting the student teachers with experiences of instructional aids (visual, non-print, text, computerised etc.) to inspire their pupils to assimilate new percepts and to accommodate these elements into the pupils´ totality of existing schemata.

e. Inviting the student teachers to discuss how processes of active perceiving and learning can give lasting retention of mathematical knowledge.

f. Making the student teachers aware of the impact of social context (cooperation, group work and group discussion, open-ended questions and social competence for survival).

g. Stressing the importance of affective positive and negative effects (motivation, satisfaction, anxiety, disgust).

(4) Innovative factors in the teacher´s instruction and tuition.

a. Causing the student teachers to change their belief and notions about the nature of mathematical learning, instruction and knowledge.

b. Influencing the student teachers to be confident of the need to use alternative individual syllabi for special educational purposes.

c. Discussing why the learner´s conception of knowledge may be central in the process of mathematical thinking.

d. Experimenting with the pupils´ learning strategies in order to make the student teachers sensible to pupils´ attitudes to different learning styles.

Teacher education should be treated as a continuing process of problem solving. A student teacher needs to meet a sequence of active learning situations where nothing is seen as definite truth.

A cognitive-clinical approach for special education in mathematics

As observed by many researchers, a *cognitive-clinical method* is indicated for work with disabled students. The cognitive-clinical method may be used in a class with many students as well as in a small group or in individual tuition.

For Bob an individualised programme seems favourable. The same applies to Anneli.

The consequence is that, in addition to constructivistic objectives, the teacher education should build upon a qualitative analysis whereby the student teacher ought to develop beliefs or belief systems which might make it easy to meet teaching situations with dysmathematical students.

It seems well-advised to attach great importance to individual *case studies*. Naturally, literature studies must precede teaching practice. In connection with exercises to instruct disabled students, the tutor can demonstrate how disability symptoms manifest themselves in the learning of mathematics. The student teacher should also meet with such students, do independent assessment work and attempt to formulate a programme.

The student teacher will get valuable information by following an experienced teacher, psychologist and physician who can demonstrate the background and the performance of individuals with various disadvantages.

Traditional didactics in the mathematical learning of disabled students appear to be based on two beliefs that may very well be delusions, namely

(1) compliance with and adaptation to traditions in the regular school systems, and

(2) recognizing primarily the demands of mathematics - in fact, more often arithmetic than real mathematical thinking - and sometimes neglecting the personal and social aims.

Constructivism finds it appropriate to replace old beliefs, based on preconceived curricular stipulations, by new beliefs where individual programme planning substitutes the collective curricular approach.

A fundamental example of new belief concerns the view of the function of special education.

While earlier the education of a handicapped person was looked upon as a case of *normalisation* and equalisation - or a transformation to normality - , a new belief would be *personalisation* (Magne 1982), namely to adapt educational goals to the needs, ability, interests, and social interactions of the individual. This changed view has an effect on terminology. Traditional terms are remediation and rehabilitation but new terms might be: guidance and guided learning. Below, the term *guided learning* will mainly be used.

A second example of new belief is the wish to proceed from an atomistic and piecemeal diagnosis to a more wholistic assessment of the student's reactions to mathematical contents and values.

A further example of new belief is a changed basis of guided learning of mathematics. Three types of views have dominated.

The oldest rationale can be described as the *content deviation model*. It dates back to old associationistic philosophy, long before the beginning of this century. The content deviation model focuses on formal mathematical concepts. In simplified form the model discribes difficulty to teach mathematics as a function of content complexity. This means that a topic of high complexity would be difficult to teach, and consequently more or less impossible to learn for some disabled students.

In about 1940 the *behaviour deviation model* appeared. It relates to variations in the learning styles of the learners. The interest concerning the diagnostic work was directed to the individual's performance, for instance physical, mental, social, emotional etc. behaviour categories. Those who refer to this model often consider mathematical contents one-dimensionally. This means that they enter just one mathematics "dimension" into each of the topics. Usually, they look upon calculation ("arithmetic") as the only essential, or necessary, topic area of mathematics.

Recently, the *factor-interplay model* was presented in several works at about the same time (Uprichard et al. 1975, Nyborg 1986, and Magne & Thörn 1987). This view combines the older presumptions, at the same time adding new, dynamic qualities to the body of knowledge, for instance stressing the flexible structure of the mathematical concept systems, as well as the multiform creativity in the learner's reactions. Functionally, the mathematical learning is represented by a complex vector space with many axes. The main vectors (or factors) may be illustrated as sets in subspaces of mathematical contents (stimulus dimensions) and pupil's ability etc. (response dimensions). An example is demonstrated in Fig. 2.

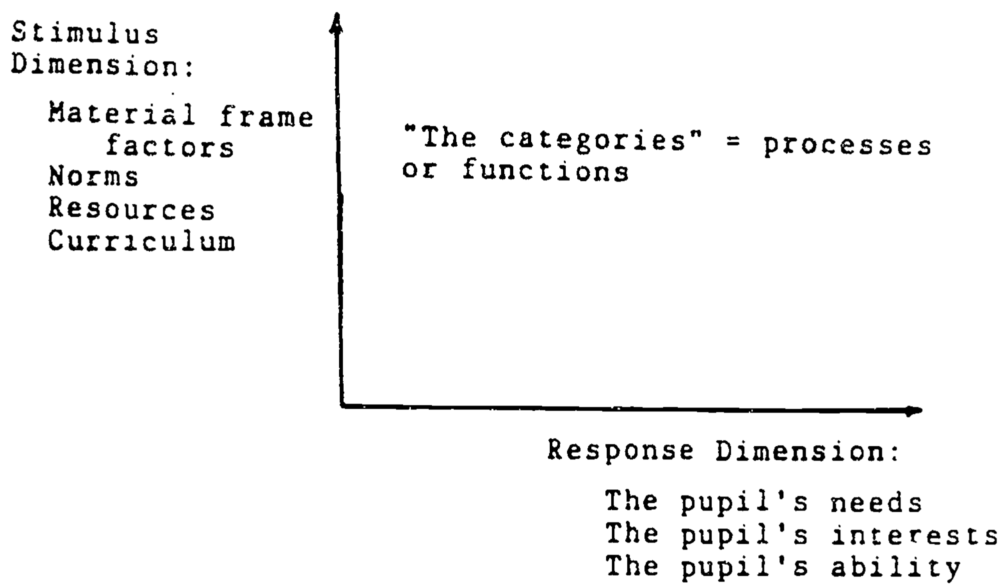

Figure 2. Graph displaying learning processes in mathematics education (the factor-interplay model)

Fig. 3 presents a flow chart to illustrate an example of application of the factor-interplay model, displaying consecutive steps of constructing a programme. It should be noted that diagnosis is an assessment that simultaneously contains the student's mathematical behaviour and his/her needs, abilities, interests, and social interactions. The programme refers to the systematic guided learning in relation to the two kinds of diagnoses and aims, in order to bring about the desired aims of learning.

During diagnosis of specific achievement tasks (phase 1) and a wider behaviour diagnosis (phase 2) the student's learning problems are being assessed so the teacher can identify mathematical behaviour compared with general behaviour.

The determination of objectives and programmes (phases 3 and 4) contains aims and prescriptions in order to systematically specify topics for the guided learning.

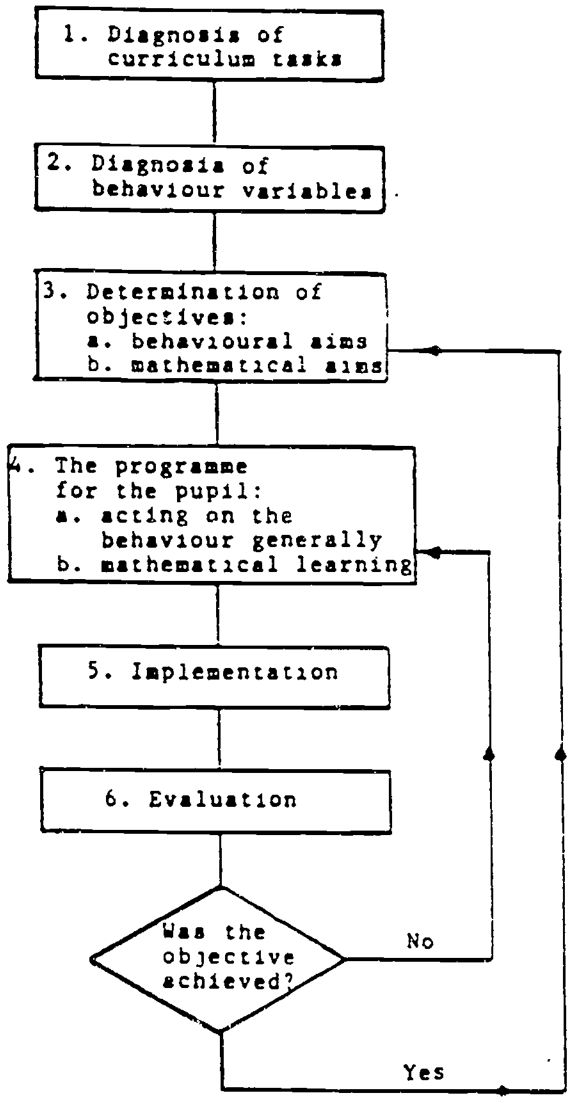

Figure 3. Planning according to the factor-interplay model.

Implementation (phase 5) and evaluation (phase 6) relate to the student's actual learning in an individual setting. In phase 6, as the student has fulfilled one aim of phase 3, work should start on another aim. If the aim is not achieved the phase 4 prescription should be revised.

The new special didactics in mathematics is described by many authors, for instance Chancerel (1986), Grissemann & Weber (1989), Gruszczyk-Kolcy´nska (1992, 1993), Lobeck 1992), Magne (1991, 1992), Magne, Bengtsson & Carleke (1972), Magne & Ohlin (1992), Milz (1993), Schöniger (1989).

For the special teacher education the general goals seems to be a suitable basis (pp. 22-23). Thus, it is necessary to get a sufficient broad knowledge of the subject matter. The student teacher must also study and have good information of various biological, psychological and social conditions of disability. The cognitive-clinical method should be trained at visits to classes with disabled students. Finally, the student teacher must plan by himself/herself and take active part of the tuition of individual students.

The practice may consist of

* preventive measures in a regular class or a special class,
* peripatetic work in cooperation with teachers responsible for a class, and
* individual tuition in or outside a class, here called guided learning (see p. 24).

The guided learning is based on the prevalent constructivistic hypothesis that mathematical learning depends on active thinking.Learning is a result of individual constructions of perceived impressions from conscious, purposeful acting. The role of teaching is to initiate and support the student's reasoning efforts. In a first learning phase, the student should aim at creating experiences of a practical nature, usually in social activities and exploiting concrete objects.

In severe cases, the work is marked by great intensity of effort by both student and teacher. Manipulative devises are explored when needed in order to develop the practical first-hand experiences and out of these build up the more abstract and - later- conceptual experiences. Laboratory material is often used in combination with and support of the usual everyday activities. Play is a common form of learning method. Language habits are strongly accentuated as instruments for learning.

To summarise. Mathematics studies of the special student teacher is suggested to contain the following elements.

(1) Mathematical knowledge.
(2) Literature on mathematical learning and performance of students with special educational needs.
(3) Discussion of old and new beliefs in the field of mathematics learning.
(4) Guided visits to classes with "math poor" students:
   (a) Demonstration of individual goals for mathematics learning.
   (b) Discussion of individual goals for mathematics learning.
   (c) Consideration how to vary organisation and method of guided learning.

(d) Use of differential instructional aids.

(5) Construction of a programme for a student with special educational needs:

(a) Anamnesis: What the teacher, student and other persons have to say about the complaint. The history must be guaranteed full discretion. In many cases the student teacher cannot be allowed access to all personal information. The interviewed person is encouraged to relate his/her history in his/her own words. First and foremost it is important to obtain a definite idea of how the problem has developed. Was the onset sudden or gradual? If sudden, can the person give exact time when it started? Has the complaint been getting steadily worse - or better - since the onset? Have there been particular severe incidents? Crises? Daily or weekly etc. variations? Particular mathematical topics? Known causes?

(b) Assessment: Information from tests, enquiries, observation. Sometimes tests or similar instruments prove useful. For some students a diagnostic programme can be recommended. Social or emotional and physical circumstances should be treated in its proper light. What can you find out from the "sum book" of the student? Is it possible to write a short survey of the findings?

(c) Programme: The evaluation should lead up to recommendations for the student's guided learning. The programme must be realistic which means that the suggested activities are well-balanced, bearing in mind the potential resources for the class. Are there enough time for teaching, suitable textbooks, manipulatives etc.? The activities should be well-balanced from a didactic aspects: Not to much drill or other rote learning, balance between topic areas, organisational propositions, differential exercises, a good supply of various problem types etc.

(d) Follow-up: After some time the effects of the programme elements should be evaluated. The outcome is expected to be positive according to the student and other persons (teacher, parent).

(6) Record: For various purposes the teacher may be expected to report on the student's progress.

(a) Demonstration of usual forms and procedures.

(b) Considerations on use and usefulness of statements concerning teachers' instruction, students' learning and learning results.

(c) Exercises in essay-writing and composition of reports.

Concluding words

It is important to look at teacher training as a continuing process of problem solving. Teacher education must be looked upon as a sequence of active learning situations where nothing is seen as definite truth.

An example: Mostly, it seems to be a good idea to foster as much reflection as possible. The special teacher should say to himself/herself that new knowledge is best created by sensible use of reflection. But this attitude leaves no space for the sensible view that some learning in mathematics is automatic due to repetition. In fact, it is a great help to have automatic knowledge in some situations, for instance to automatically know the result of the addition 2+3=5 or of the multiplication six by nine is fifty-four.

Another critical issue today is that many traditional skills or procedures in school mathematics are obsolete, not the least in elementary school arithmetic. Future use of mathematics has not always need of school arithmetic of yesterday.

The teacher should give to the disabled citizen of tomorrow at least an inkling of a more useful but also more imaginative mathematics and less of meaningless drill through textbook calculations.

Concluding this survey, everybody should find for himself or herself a mathematics that is what he or she needs for his/her future life. It should be a mathematics full of life and joy, literally meaning survival learning to secure independence in his/her future life.

We should remember our school days as the part of our life when we learned interesting things about ourselves in the world where we live.

I am reminded of the words of a great author who once said, referring to the beauty of good teaching, "Not for glory and least of all for profit, but to make something of the human spirit - something which did not exist before".

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Abstract card

Magne, O. Mathematics and quality of life. A new theme in special teacher education. Didakometry (Malmö, Sweden: School of Education), No. 75, 1994.

Students with special educational needs usually display poor retention of mathematics. This happens often for those who have sight, hearing or motor impairment and experience general learning difficulties or motivational and perceptual disadvantages. For such low performance, some researchers use technical terms like "Dysmathematika", others prefer less defect oriented expressions as "maths poor" children.

Future teacher education should accept that individual needs should be more stressed than in traditional special teacher education. Disabled students should aim at social competence in their mathematical learning. This indicates a need of new competences also for the teacher.

Keywords: Behaviorism, constructivism, disability, dysmathematika, error pattern, individualisation, mathematical learning, social competence, special teacher education.

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