DOCUMENT RESUME

ABSTRACT
        This paper brings a teacher's perspective to bear on
recent policy initiatives requiring all high school graduates to
complete an Algebra One course. The paper is based on the teacher's
experience in teaching high school students who have not been
successful in mathematics, at Holt High School in Michigan. The
teacher is caught between a desire to provide students in a lower
track class with access to mathematics, which plays an important role
in providing opportunities for a college education, and students'
questions about why study algebra. This leads the teacher to rethink
his own understanding of algebra and his approach to teaching the
subject. The teacher becomes convinced of students' sense-making
abilities in mathematics, but is dismayed by the complexity of the
task of working with the class as a social unit. The paper finds that
the current algebra curriculum is deeply flawed and makes it
impossible for many students to study successfully. An alternative
view of the central objects of study in algebra is explored, one that
suggests connections between algebra and calculations with quantities
that are performed repeatedly in everyday work situations. (Contains
35 references. (JDD)

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Algebra for All Students?

Daniel Chazan

NCRTL

National
Center for Research
on Teacher Learning

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NATIONAL CENTER FOR RESEARCH ON TEACHER LEARNING

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Abstract

This paper brings a teacher's perspective to bear on recent policy initiatives requiring all high school graduates to complete an Algebra One course. The teacher in this paper is caught between a desire to provide students in a lower track class with access to mathematics, which plays an important role in providing opportunities for a college education, and students' questions about why study algebra. This predicament leads the teacher to rethink his own understand of algebra and his approach to teaching the subject.

When teaching algebra differently, he is convinced of students' sense making abilities in mathematics, but is dismayed by the complexity of the task of working with the class as a social unit. This complexity is illustrated with a controversy about defining mathematical terminology and the insights to which this controversy leads.

ALGEBRA FOR ALL STUDENTS?[1]

Daniel Chazan

I had assumed something I now learn is not true. I thought that everyone in high school took algebra and geometry. (Cohen, May 12, 1991)

Too many of us were forced to take algebra when the time and energy could have been devoted to subjects that truly were beneficial individually and nationally.

Algebra isn't essential too [sic] much of anything . . . Would millions of high school students trudge into their algebra classes if they weren't a gate through which they were forced to pass to enter college? . . . Let students who like it [algebra] take it." (McCarthy, April 20, 1991)

Mathematics is not just another science; it is the language through which all of science and much of management science is taught. The student who closes the door on high school algebra (and so on all of mathematics) closes the door on much more. (Roberts, April 27, 1991)

A 1992 analysis of 1,400 jobs by the New York Department of Education found that 78 percent of them required no algebra, and only 10 percent required more than a little. (Bracey, June 12, 1992)

Maybe American workers are not using algebra (and other math) in their jobs because that's the way their jobs are set up. . . . Maybe the fact that so few of our workers use math is a symptom of something wrong with the way our workplace is structured rather than something wrong with teaching algebra to our youngsters. (Shanker, June 21, 1992)

Even my own fields . . . do not require much in the way of algebra. I could have learned all I needed to know in a month or so. (Bracey, June 12, 1992)

Sounds reasonable, but I wonder if we'd want to limit education to what is "useful." . . . How many of them (students) use the language of William Shakespeare or Charles Dickens or Frederick Douglass in writing memos? (Shanker, June 21, 1992)

THE ALGEBRA POLICY DEBATE

The challenge that has always faced American education . . . is how to create both the social and cognitive means to enable a diverse citizenry to develop their ability. It is an astounding challenge: the complex and wrenching struggle to actualize the potential not only of the privileged. (Rose, 1989, p. 225).

The slogan "Algebra for All" is a call to arms. It calls forth the vision implied by Rose's challenge and struggle, the vision Cusick calls the egalitarian ideal, that schools will "provide each student with an opportunity for social, political, and economic equality" (1983, p. 1). In practice, however, differing assessments of the value of academic knowledge, difficulties negotiating across difference, societal inequities, adolescent struggles for self-definition, individual interests, and a commitment to compulsory education and the egalitarian ideal put public high schools in the business of

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getting all students to come, even those that do not want to, getting them to stay, and attend class regularly, even if they would rather be somewhere else, getting those that are repeatedly disorderly to try again to see if they can complete the required work. (p. 111).

The goal of this essay is to prompt reflection on the slogan, "Algebra for All," and its policy implications by examining teaching algebra in a lower track class.

Currently, in most United States public high schools, Algebra One is taught in tracked classes and is not compulsory. The majority of college-bound students take Algebra One in ninth grade or earlier, while those in vocational tracks or who will drop out may take Algebra One late in their high school careers or not at all.

The policies which lead to this situation are now in contention. For a host of political reasons, there are regular calls for increasing the mathematical preparation of high school students. The new California Mathematics Framework (1992) suggests that high schools stop tracking in mathematics classes. The quotations on the opening page were provoked by the adoption of Algebra One, "or its equivalent," as a graduation requirement in the Washington, DC, area. Finally, others suggest that the majority of students take algebra in the eighth grade (Usiskin, 1987).

As policy makers think about Algebra One, they confront the challenge Rose describes. On the one hand, making Algebra One a graduation requirement, as was done in Washington, DC, Louisiana, and Mississippi, ensures that all students, at least theoretically, have access to the opportunities for which a successful completion of the course is prerequisite. Yet, many students in such algebra classes are not adequately prepared and, absent changes in the course we teach, are likely to fail. At the same time, given current social and economic realities (e.g., limited access to college) and a student body of diverse interests, such a policy ensures that many students will be sitting through a course which seems remote from their real possibilities and interests.

BRINGING A TEACHER'S PERSPECTIVE TO THE POLICY DISCUSSION

Trying to solve many common pedagogical problems leads to practical dilemmas. As the teacher considers alternative solutions to any particular problem, she cannot hope to arrive at the "right" alternative . . . This is because she brings many contradictory aims to each instance of her work, and the resolution of their dissonance cannot be neat or simple. . . . However, the teacher must do something about the problems she faces. (Lampert, 1985, p. 181).

For the last two years, I have chosen to do research on the teaching and learning of algebra by developing and teaching a low-track Algebra One course. While on the faculty at Michigan State University, Sandra Callis Betheli[2] and I have team taught at Holt High School[3] in a predominantly white, working class, rural/suburban township not far from campus. Our students are 10th, 11th, or 12th graders who have not been successful in mathematics.[4] Typically, 35 to 40 percent of these lower tracked students drop their Algebra One course during the year or fail.[5]

A generalized academic portrait of our students does not paint a promising picture. Even though some individuals are in higher tracked history or English courses, in general, it is accurate to describe our students, academically, as low achievers. On the whole, our students are not good at many of the behaviors expected in school. Many have trouble both reading and writing.[6] Many have trouble articulating their views orally. Most are not good at diagnosing their own understandings. They frequently over and underestimate what they know and as a result have trouble appreciating their own progress. Most have trouble "remembering" how to use abstract symbolism, like the long division sign, correctly. In academic classes, they are not usually active discussion participants. From an examination of their notebooks, on the whole, they are not good note takers. Finally, they tend not to be diligent about homework.

But, these "realities" are not the whole story. Intermittently, during class, many of our students have important and valuable insights and articulate them in class discussions. At such

times, I wonder about how to unleash this potential that our students have for making sense of the mathematics that we are studying on a more regular basis.

So, as teachers of students in a low-achieving track, the challenge we continually face looks different than the policy makers' challenge of "access." How do we help students who have diverse interests and talents, who are learning in a tracked, compulsory setting, who are often planning to work and are not college-intending, see value in a shared exploration of algebra? What is it that we really want them to learn when we think of "algebra"? On what grounds can we appeal, implicitly or explicitly, for their energies and effort? How can we help as many students as possible be successful (whatever we might mean by that)? Or should I, as Noddings (1992) suggests, reject "an ideology of control that forces all students to study a particular, narrowly prescribed curriculum devoid of content they might really care about" (p. xii)?

From my vantage point, it seems important that conversations about making algebra compulsory for all students (and other algebra reforms) keep in focus both the students and teachers whose lives will be affected by policy decisions and the Algebra One curriculum itself. Yet, many of these conversations seem to focus solely on students who are college-bound, ignore the difficulties faced by teachers, and refer only in passing to the nuts and bolts of the curriculum.

In this essay I will share experiences and thoughts which, I believe, should be integral to policy discussions aimed at improving algebra instruction. I believe that the current algebra curriculum is deeply flawed and must be challenged—that the way we teach algebra makes it the difficult subject that, some argue, is impossible for many students to study successfully. Sandy and I have been exploring an alternative view of the central objects of study in algebra, one that suggests a wealth of connections between algebra and calculations with quantities that are performed repeatedly in everyday work situations. One benefit of this view of algebra is that it helps us bridge our students interests and the curriculum; it also helps us identify strengths that our low achieving stu-

dents exhibit, in addition to their easily identified academic weaknesses. I will describe and analyze some of our students' strengths in the context of a first semester final exam problem.

Changing our view of the content has helped us identify students' strengths, but we have also changed the way we teach. We have tried to help our students view algebra, and mathematics more generally, not as a long list of recipes and problems to which they belong, but as an arena of human endeavor that can be understood. We have tried to change our role and our students' roles, so it becomes clear that we do not imagine them as passive recipients of incontestable knowledge, but rather that we would like to invite them to become active participants in a long tradition of mathematical inquiry, a tradition that insists on the presentation of evidence and reasoning to support claims. We have also tried to solicit their interests and questions and make these an important ingredient in shaping our curricula.

However, making such changes in classroom roles is very difficult. Changes in what is taught and how it is taught do not magically (and overnight) transform reluctant, resistant, and passive students into self-motivated inquirers. Participants in policy discussions must remember how difficult and multi-faceted the teacher's role is. In particular, policy makers must keep in mind some of the circumstances endemic in lower-track math classes that make it difficult for students and teachers to change their roles. I will illustrate the impact of some of these circumstances on reform-minded teaching with a story reconstructed from my teaching journal.

THE CURRICULUM, PART ONE: MANIPULATION OF SYMBOLS

"The way it [algebra] is usually taught it increases only the ability to memorize formulas by rote, nothing else." (Bracey, June 12, 1992)

Dewey (1902/1956) argues that when educators forget that a subject has two aspects "one for the scientist as a scientist; the other for the teacher as a teacher" (p. 22) a series of evils result. I will focus on two. The first is that "the lack of any organic connection to what the

child has already seen and felt and loved makes the material purely formal and symbolic." (p. 24) The traditional algebra curriculum is regularly criticized on this score (Fey, 1989; Thorpe, 1989). It does not adequately explain the nature of symbolic expressions and the purpose and goal of the manipulations of symbols.

For example, the first use of symbols encountered in a traditional text is the algebraic expression (e.g., $2x^2+3x+1$), which is thought of as a description of an unknown number. One does not know what number $2x^2+3x+1$ is until someone specifies what number x is. After practicing evaluating such expressions for particular values of x, the instructions for exercises involving manipulation of these expressions read "Simplify," "Expand," "Multiply," or "Factor." These cryptic instructions are meant to suggest the type of manipulation that the student must do to the expression, but they are not conceptually sufficient. These instructions do not adequately describe the desired final version of the expression, do not alert students to properties that are being preserved as the expression is rewritten, and, finally, do not embed rewriting expressions in a larger, purposeful context. It is difficult to understand how doing such exercises can be justified on bases other than the "exercise" of certain mental muscles.

plan to enter the military, feeling that in some ways the military may offer them more opportunities than the civilian economy. In addition, this argument holds four years of college as a desirable necessity. Such a vision leaves students who are not college-intending, or who question the value of "book learning," cold.

Appeals to the utility of the knowledge and skills learned in Algebra One feel similarly alien when I think of our students.[7] Many of their job opportunities are in "low skill, low wage jobs" (Shanker, June 21, 1992) in fast food restaurants. These are not jobs where manipulation of algebraic symbols is required or utilized. Furthermore, this argument falls flat when our students do not see their parents solving equations or rewriting expressions. In fact, they are unlikely to see anyone around them use these algebraic manipulations. Thus, their experiences suggest that algebra is not useful.

Finally, it is difficult to see the analogy between an education in the enduring liberal arts and learning to manipulate expressions. How do the manipulative skills taught in the traditional curriculum give students a new perspective on, and insight to, our world? How do they relate to questions or problems with which students might become involved?

THE CURRICULUM, PART TWO: A MOTIVE FOR LEARNING

Dewey's second evil of poorly designed curricula is lack of "motive for the learning" (1902/1956, p. 25). Such motives are personal and individual. What ends of value to individual students might lead them to invest their energies in studying algebra? How do teachers help students with diverse interests and talents see value in a shared exploration of algebra? What general arguments can teachers make for algebra? Many traditional arguments do not seem applicable to teachers in our situation.

The argument that taking Algebra One will provide our students with access to opportunities otherwise unavailable to them, while well-intentioned, seems at times cruel or even preposterous. Our students are aware of the limitations on their opportunities regardless of whether or not they succeed in algebra. Many

THE CURRICULUM, PART THREE: AN ALGEBRA OF QUANTITIES[8]

In the light of previous experience, we must acknowledge the impossibility of determining, by direct measurement, most of the heights and distances we should like to know. It is this general fact which makes the science of mathematics necessary. For in renouncing the hope, in almost every case, of measuring great heights or distances directly, the human mind has had to attempt to determine them indirectly, and it is thus that philosophers were led to invent mathematics. (August Comte quoted in Michel Serres, 1982, p. 85)

In my experience, it is a very difficult task to help students in low track courses see a value in learning algebra. The argument that we have been exploring is that algebra provides a new insight into our world; but, in order for this argument to be credible, students must be able to see algebra in the world around them. As

teachers, we must then develop an understanding of algebra that will help us appreciate the algebraic thinking already done by our students, their working class parents, and other members of their community (even though it is not necessarily expressed in x's and y's and they might say that they do not use any algebra in their work). Such an understanding, like some approaches to literature (e.g., Sophie Haroutunian-Gordon's 1991 approach to Shakespeare), could legitimately lay claim to providing students with an enhanced appreciation for the world around us. Thus, we have been searching for a way to bring to our students' attention the algebraic thinking going on around them.

As a step in this direction, we have been exploring an approach that takes functions as the fundamental mathematical object studied in high school algebra (See Chazan, Schwartz, & Yerushalmy, unpubl.). For pedagogical reasons, this approach views functions primarily as mathematical theories about how the values of certain quantities depend on the values of other quantities, rather than as abstract correspondences between sets of objects (numbers or other mathematical objects). As we debate the validity of these theories about quantities we engage in applied mathematics and, as we compare and contrast different kinds of functions on the real numbers abstracted from their contexts we do theoretical mathematics.

The functions we study can be represented in verbal descriptions, graphs, sketches of graphs, tables of values, and algebraic expressions, like the one presented earlier. They can be presented in the context of the relationship between designated quantities, or they can be presented abstractly as the relationship between two sets of numbers. Thus, this approach views expressions not as numbers, but as representations of algorithms, or arithmetic procedures, that can be carried out and produce an output number for any input numbers. When an expression is presented in the context of quantities, it indicates exactly how the values of the dependent quantities depend on, and can be computed from, the value(s) of the independent quantity(ies).

Where can we find such algorithms, algorithms that are used repeatedly to compute quantities from measured or counted quantities? The quotation from Comte which opened this section suggests that there are quantities that cannot be measured or counted directly, because of their large size, but that people compute. I would add quantities that cannot be measured or counted directly because they are too small or because they are hypothetical, or theoretical, and do not exist as measurable qualities in our physical environment.

Such algorithms are found frequently in our cultural environment. The following examples come from our students' explorations of their hobbies or of the workplace of adults in their community. An approximation of the total height of a skyscraper can be computed if one knows the number of stories and an average height per story. A hypothetical average number of yards per carry can be created to compare the success of different running backs on a football team. Algorithms can be expressed also for changing the cost of an item from one currency to another, for figuring out the mortgage payments for loans, for determining the heights of tires from the codes on their sides, for determining the amount of dirt needed to fill cross-sectionally trapezoidal holes made to hold sewer pipes, for making predictions for different sized populations based on statistics collected from a small sample, for determining the total cost of repair jobs at a body shop, and so on. These algorithms are found in a wide range of blue-collar and white-collar occupations. Thus, "algebraic expressions" can be viewed as one representation of the algorithms which people use in everyday situations to find the measure of quantities that for some reason cannot be measured or counted directly.

How does this understanding of expressions change one's view of factoring $2x^2+3x+1$? A more explicit instruction than "Factor"—one that indicates the form of the desired expression—might read:

> Below is an expression written in the form ax^2+bx+c. Write it in the form $a(x - r_1)(x - r_2)$. Make sure that your final expression produces the same outputs as the original expression for any particular input.

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Thus, $2(x+1)(x+1/2)$ is the factored form of $2x^2+3x+1$, while $2(x+3)(x+1)$ is an inappropriate answer; $2(x+1)(x+1/2)$ produces the same outputs as $2x^2+3x+1$ for any particular input. Yet, such instructions do not address issues of purpose. Why is writing expressions in different forms useful to people that our students might know?

One such rationale is to suggest that rewriting expressions is a way of seeing whether two different algorithms compute the same quantity or are equivalent, in the sense of always generating the same output for the same input. The following example comes from an examination of the work of a Michigan tax accountant.[9]

Frequently, this accountant needs to recover the tax paid by his client from a receipt for the total bill (where the tax is not listed). The traditional method is to:

- Take the total bill, divide it by 1.04 to find out what the original sales price was, then subtract the original sales price from the total bill to find the tax.

$$X - \frac{X}{1.04}$$

This method requires the entry of the total bill twice, as well as the use of two operations. The accountant had generated an alternative, shorter method, but was unsure whether it would always work. His method was to:

- Take the total bill and divide by 26 to find the tax directly.

$$\frac{X}{26}$$

The question which puzzled him was: Are these two methods—for calculating the amount of tax paid when you know the total bill—equivalent (Assuming 4% sales tax)? An algebraic simplification suggests that the two methods are equivalent and could be used in-terchangeably, thus indicating that simplifying expressions can be found in our cultural environment and might be of value to some adults in their job.

THE CURRICULUM, PART FOUR: TEACHING FOR UNDERSTANDING

We have supplemented this approach to the content of our algebra course with teaching methods that propose that algebra is a subject to be understood. However, such methods run counter to traditional practice with its emphasis on symbolic manipulation.

Textbooks, teachers, students, and the public at large have set expectations about how algebra is taught. In a traditional class, each day of instruction focuses on a particular type of manipulation (e.g., factoring trinomials, Dolciani 1970/1973, pp. 272-273).[10] At home, students practice that manipulation and come back to class to discuss the exercises and have them graded. The teacher, or the textbook's key, is the arbiter that decides whether an exercise is correct or incorrect. A typical lesson would involve review of homework on the previous manipulation (either by students putting exercises on the board, the teacher listing the correct answers, or the teacher collecting and grading the homework), teacher presentation of a new skill, guided practice on that skill in class, and then a homework assignment.[11] This structure of the class period fits this approach to the curriculum that considers algebra as a series of skills to be mastered.

Sandy and I have been exploring alternative methods. We structure the class period differently and seek different relationships between teacher, student, and knowledge than the ones portrayed above. Like others who stress "teaching for understanding" (Ball, in press; Lampert, 1990; Peterson, Fenema, & Carpenter, 1991; Wood, Cobb, & Yackel, 1991), we want our students playing an active role in constructing knowledge. In class, we spend a large amount of time exploring a small number of problems, instead of a small amount of time on each of a large number of exercises. We seek to understand what a problem is asking and why that might be important or useful to know. Students work on problems, like deciding whether the

two methods for figuring out tax are equivalent, individually or in pairs, then we, teachers and students, discuss their results as a whole group. In these discussions, we try to get our students to articulate their ideas. We try to take students' ideas seriously and help them develop and extend their ideas.

As Lampert (1990) suggests, based on Lakatos's (1976) *Proofs and Refutations*, it requires both courage and modesty to air one's ideas publicly for criticism. So, these kinds of expectations might seem to be a poor match with students who have not been successful in mathematics and who are not deeply engaged in school learning. Yet, as a teacher, I believe that I will be more effective ʳ th more students if I have students explain hoᵥ they are thinking. The better they can articulate their understandings and the more I can understand their mathematical thinking, the more they will learn and the better I will be able to aid in this process.[12]

For example, as we begin a new topic, we try to find out how our students describe what they see. We want to hear what terms they use to describe the objects that we are going to study and to see what is salient for them. From such discussions, we learn which mathematical terminology students already use; which should be easy to introduce, because it is consistent with a distinction they are already making; and which will require some preparation, because they do not see as salient a distinction that the terminology highlights. After we have learned how our students talk about the topic, we might introduce the terms from the mathematics lexicon that we deem to be central.

STRENGTHS STUDENTS EXHIBIT WITH THIS APPROACH TO ALGEBRA

"There are 26 sheep and 10 goats on a ship. How old is the captain?" The odds are roughly three out of four that you (a school child) would produce a numerical answer to the problem by combining the given numbers (Reusser, 1986). In fact, the more you'd been exposed to mathematics in school, the more likely you would be to produce "answers," by combining numbers, to problems that did not even ask any questions! . . . Children making their way through school are increasingly habituated to working routine, stereotyped exercises—to

the point where they no ː ᵥnger ask that the situations described in the exercises be meaningful. . . . This lack of demand for meaningfulness was documented on the Third National Assessment of Education Progress (Carpenter, Lindquist, Matthews, & Silver, 1983), where a plurality of the students working an "applied" problem wrote that the number of buses required to transport a group of soldiers to an army base was "31 remainder 12." (Schoenfeld, 1989, pp. 99-100)

Unlike the students described above, the students I've worked with treat some of the problems we explore as meaningful and even, in some cases, critique mathematical models that do not make sense to them. One example of such potential, in contrast to the quotations from the literature on students' performance on "word problems," is the strength that our students exhibited on an extra credit, first semester, final exam problem about calculating the break-even point for a business. Each student was interviewed on this problem for about eight to ten minutes. They then had the option of working further on the problem and submitting any additional findings along with the rest of their test.

The problem was patterned on similar problems we had explored with materials from the Michigan Department of Commerce. The materials were about calculating break-even points and were created to help potential start up businesses assess their viability. Like the problems presented in the those materials, inventory is ignored; there is an implicit assumption that the number of shoes bought in a month is equal to the number sold. The problem was presented in the following form:

Ms. G is considering investing in a shoe store. The average price of a pair of shoes in the store is $35. The average cost for buying the shoes from the manufacturer and paying the commission to the sales staff is $18. The rent for the store is $900 a month. The combination of insurance, utilities, and all other fixed costs comes out to about $400 a month.

- Write a rule for the revenue per month.

- Write a rule for the costs per month.

- Write a rule for the profit/loss per month.

- Calculate the break even point. How many shoes do they have to sell in a month to break even? Explain your calculations.

In contrast to Schoenfeld's description of the acceptance of unsolvable or meaningless problems, many of the students treated this problem as a real problem involving the business of a particular individual. They referred directly to "her," or "Ms. G," in describing their solutions and the course of action they felt their solutions indicated.[13]

One student went farther; she felt that the problem as written did not model the situation appropriately. She brought to bear considerations which were not represented in the problem.[14] Connie indicated that one of the store owner's concerns had to be pricing her shoes "so people won't go elsewhere for their shoes." She did not see this concern recognized in the problem. Thus, for her, in order to model the situation realistically, the average price "per" pair of shoes should have been a second independent variable. She also indicated a concern for the season in which the problem occurred, because that would effect sales. Her focus on these contextually important issues made it hard for her to work successfully towards finding a break-even point for the problem as posed.

To illustrate the "algebraic" strengths other students demonstrated, I will focus on the work of one student, Norris. Norris is comfortable enough with algebraic symbols to be able to start by writing an expression for the revenue per month. He suggests 35x. Yet, his understanding does not seem to be mechanical:

> Because 35, 35 per shoe, and then you don't know how many shoes she's going to sell, so, you just put 35x, wouldn't that be the same as saying 35 times whatever?

When pressed to explain what he is using x to stand for he is able to articulate clearly that it stands for "How many shoes she's going to sell."

With the aid of the context, Norris recognized a difference between per quantities, in this case the average selling price of a pair of shoes ($35 per pair) and the cost to buy each pair of shoes ($18 per pair), and the fixed quantities in the situation. Thus, when calculating the costs, he says:

N: Your rent is 900 a month, and insurance and everything is 400 and then . . . or wait, I'm not supposed to add the shoes on though.

DC: What do you mean you're not supposed to add the shoes on, I don't get that.

N: Well, I mean 'cause . . . she don't know how many shoes she's going to buy, you know? So . . . you'd be, let's see. That'd just be 13 . . . 1300, let's see, it would be 1300x wouldn't it? No, 'cause, and then x would stand for . . . no. It would be 1300 plus 18x. [Writes: 18x+1300] All right. Because the 18 is how much shoes cost but she don't know how many she's getting so the x would stand for how many shoes she's buying, plus, the 1300 would be, with the insurance and utilities and the rent.

Though he does seem to have a good grasp of the situation and he has written the revenue and cost functions, Norris, like all of his classmates, does not then solve the problem abstractly, by writing an equation and solving it symbolically. Though some of the students followed an algorithm of their own devising to generate directly the numbers of pairs of shoes to sell in order to break-even, most, like Norris, use a guessing and testing method:

DC: So now how are you going to figure out how many shoes she needs to sell a month in order to break even?

N: Hmm. Oh, let's see. Well, you don't know . . . you'd probably have to, I can't think of how you would do it except for the, just kind of guess and put a number into these 'cause I can't think of any rules to figure out, you know, you'd have to use the trial and error, I'd have to use the trial and error method.

The use of such a method indicates an appreciation of the way in which the number of pairs of shoes sold in a month can be viewed as a variable quantity and the understanding that the break-even point can be found by the imagining of the results of selling a particular number of pairs of

shoes. He understands that he can make up a number that stands for the numbers of pairs of shoes that might be sold in a month and then evaluate the profit or loss of the company.

Finally, in contrast to the National Assessment of Educational Progress result reported above, those students who did not use a guess and test method, but found a number directly, were able to interpret the meaning of their result in the situation. Thus, Christopher showed how he would interpret the result of 76.470588 which he found with a calculator.

C: It means she has to sell 76, almost, 77 pairs of shoes.

DC: Okay. So if she sells 76, is she going to break even?

C: No. She needs to sell, like a little more, like 77.

DC Okay, if she sells 77, is she going to break even?

C: No, [sic] she'll make a little profit.

The students reveal impressive strengths in treating the problem as a meaningful one that can be modeled with expressions. Yet, they do not use the full range of algebraic tools that could be applied to the problem. Specifically, they do not solve the equation symbolically.[15]

In general, situations, not abstract problems, seem to be contexts where our students display the sense they make of mathematics. The challenge for us as teachers is to capitalize on those strengths as we attempt to help our students become full members of the mathematics "club," able to employ mathematical symbolism and techniques in both situated problems and abstract ones.

DIFFICULTIES ASSOCIATED WITH TEACHING IN A LOWER TRACK

The students . . . have not done well in math in the past, so they are recognized as students who don't do well in math. Most of them don't feel confident in math. Math is conceived as a difficult subject for them. They take it because they have to. . . . Many of these kids can't concentrate because of what else is going on in their lives. (A teacher from Cleveland quoted in Bruckerhoff, 1991, pp. 167-169)

My account so far has stressed the strengths that our students bring to class and the ways in which the current curriculum is flawed and might be remedied. Such an account might suggest that simply changing the curriculum would lead students to become successful, motivated inquirers, but anyone familiar with the day to day realities of teaching students in a low achieving track knows that simply changing the curriculum is not sufficient.

Our district aspires to have Algebra One taught to ninth graders at the junior high school. Most of the high school students we work with have failed mathematics before and many need to pass Algebra One in order to complete their mathematics requirement and graduate. Most have failed the mathematics portion of the state certification test, given at the beginning of tenth grade, required to receive a state-certified diploma.

By definition, those taking Algebra One in tenth, eleventh, or twelfth grade are behind.[16] Students arrive in Algebra One at the high school by three common paths—either from a preparatory course at the junior high school, a preparatory course at the high school, or a failure in algebra. All these paths have in common an evaluation by the school that these students are not at "grade level" in mathematics; most include failure along the way. Thus, mathematics has become a "charged" subject; it carries a heavy burden of negative experiences. Many of our students bring an anger to math class that they do not bring to the Spanish classes that Sandy also teaches.

Many students have internalized these evaluations of their peers' mathematical achievement and potential and do not think of themselves or their peers as able to be successful in mathematics. They expect little from a senior coming from the Practical Math class at the high school, or students labeled as "special ed" or even "voc ed." Correspondingly, they expect more of a sophomore who took pre-algebra at the junior high school. Other social categories also play

an important role in shaping students' expectations of their fellow students. "Smokers" and those cultivating a "hard reputation" are not expected to invest their efforts in schooling, whereas "jocks" and "preppies" are expected to be concerned about their grades.

Yet, these previous academic experiences, peer expectations, and the poverty of the current curriculum are not the only reasons that students do not invest their full efforts in studying. Like the students in Cleveland, there is much in their lives that distracts them from schooling. Many of our students are poor. Some lead troubled and violent lives. They frequently test the limits of both school and society. As a middle-class professional, there are aspects of their lives that I do not share and about which I am just beginning to learn. Some of what I see and learn is frightening and disheartening.

In school, our students frequently run afoul of the school's regulations. For example, on the second day of my first year teaching at Holt, three of my female students had received suspensions ranging from five to ten days long for fighting. During the first three weeks, each day at least two students were not in class because they were serving suspensions. While this stretch of time was extreme and though absences are not endemic in our school, students in lower track classes miss class regularly for suspensions, many given for fighting.

Violence is also a part of many students' lives outside of school. In an admittedly extreme case, one student in our class was stabbed to death on a Sunday night early in the school year. But while this example is out of the ordinary, it reflects an underlying reality of regular violent episodes. More than one quarter of the students this past year had formal encounters with law enforcement agencies. One of the students was incarcerated for a large portion of the year and came to class from jail on a release program.

These "realities" are important to bear in mind as we think about policies affecting algebra instruction. Teaching a low track mathematics course is usually avoided by mathematics teachers with seniority. Simply changing the curriculum is not sufficient. Current tracking systems create concentrations of students who are having difficulty academically and outside of school. New policies must address this issue, while being sensitive to the differing mathematical achievement and interests of students.

A STORY FROM OUR TEACHING

Teaching for understanding is difficult in any context. However, in some contexts, teachers can take for granted that students will feel safe and will value school learning. Teaching in a low-track algebra class is a yet more difficult task. As our students look around the class, they see classmates with a wide range of abilities, experiences, and knowledge, who are the academic lower class of the school.

While the students have certain school experiences in common, their lives and aspirations can be quite different. In our case, some class members live in wealthier parts of town, while others live in trailer parks. Some are in vocational training; some are labeled as "special education students;" some work outside of school; some will pursue a two-year college education; some are in trouble with law enforcement agencies; and some are involved with drugs. Some of the students are more involved in school learning and others are not as engaged.

What they have in common is having been unsuccessful in mathematics in the past. Many think that algebra, unlike "Practical Math" or other arithmetic courses, is adult and difficult and that to pass is an achievement. So, they are concerned about their chances of being successful, and, in some cases, skeptical about the chances of their classmates. Students who feel more successful want to be tracked away from those they consider less interested in learning and less likely to succeed. Others fear that they cannot succeed, or are even too frightened, to try on a regular basis.

Rather than conclude on the cautionary note of the previous section, I would like to conclude on a more ambiguous, complex, and conflicting note by presenting a story from our teaching. I am not presenting this story as an example of exemplary practice of any sort; I'm sure our practice can be criticized from many perspectives. Instead, I offer this story as a way of

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integrating all that has come before. In this story, I see an attempt to teach algebra as a subject to be understood; I see students who would like to study the mathematics before the class and those who would not; I see students' social categories and experiences outside of school shaping our interactions around mathematical content; and I see combinations of sophisticated understandings of the need for and purposes of mathematical conventions alongside a desire simply to be told what is correct.

I will describe a mathematical controversy that sparked one student's frustration and, in a chain reaction, led us to investigate students' perceptions of the "pace" of the class.

The mathematical controversy— What is a mirror image?

Early in December, we were starting to examine the graphs of functions systematically. We were eventually interested in concentrating on linear functions, but wanted to develop a sense of the range of different polynomial functions. We also wanted students to practice using terminology we were developing to describe graphs. We asked each student to make a poster of the graph of a function (given an algebraic expression—or rule as the students called them—and using a computer program which would produce the graph) and then to introduce that graph to the class. In the course of these introductions, the student who came to school on the release program used the term "mirror image" as a descriptor for graphs of functions. In retrospect, it was not clear whether he intended that the term should be used to describe any parabola at all, or only those like the graph of $f(x)=x^2-9$ (See Figure 1), which are symmetric around the y axis. In such a case, if the y axis is centered in your scale, then when you fold along the y axis, the quadrants match up, as well as the pieces of the graph.
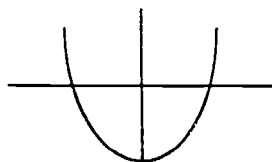

Figure 1—A graph of $f(x)=x^2-9$.

About a week later, it became clear that people were using the term differently and students wanted to know how the term "is really used." I indicated that "a mirror image graph" is not a commonly accepted mathematical term. I suggested that an adjective that is widely used in mathematics is "symmetric," but that this term requires that one state that the line (or point) around which a shape is symmetric. (For example, the graph in Figure 1 is symmetric around the "y" axis) We examined the lines of symmetry of a square. Some students then asked for clarification about how to use the term "symmetrical" with respect to their graphs. Dave, the student who had introduced the term, asked whether according to mathematicians the quadrants had to be symmetrical around the line of symmetry of a graph of a function. I indicated that this was not the way the term was typically used. Mathematicians would be willing to use the word symmetric to describe the graph separate from the coordinate system on which it appears.

This issue resurfaced towards the end of April. We spent the first part of the period on a task that asked students to sketch a graph directly from an expression without creating a table of values. Students were trying unsuccessfully to draw the graphs of linear functions given either their "x" or "y" intercepts and their slope. They did not seem to realize that though the intercepts are sometimes described as numbers, they also provide information about points on the graph or entries in the table. Thus, if one knows how to read a linear expression (or rule) and extract its intercepts, or one intercept and the slope, one can make an accurate sketch of the graph without resorting to a computer or graphing calculator. The frustration level in the class was high.

During the second half of the period, we moved to a different task. Students were presenting graphs of quadratic functions that they had made earlier in the week. Again, the term "mirror image" was being used in different ways. I was confused and asked how they were defining the term at this point. Steve, a quiet, earnest student who did well all year, but considered himself above many of the other students, got very angry. He asked "Why don't you just tell us? I want

to know what is scientifically correct!" He seemed to be upset that we would continue to spend time on a distinction made by a student, particularly one he looked down on.

I was taken aback by his anger. In response, I again defined the word "symmetric" geometrically and again emphasized the importance of the line of symmetry in this definition. From my point of view, there was not any scientific definition of "mirror image graph" in this context. It was important that students realize that one can describe some characteristics of a graph (e.g., its symmetries) regardless of the coordinate system on which it appears. I then asked the student, Dave, who had introduced the term "mirror image," to indicate how he would like to define "a mirror image graph." He suggested that he would retain "mirror image" as a description of only those graphs which are symmetric around the "y" axis. I interpreted this choice as a desire to highlight a special relationship between some graphs and their coordinate system. AJ indicated that he thought this decision was not a good one because those graphs are simply a special case. For example, parabolas symmetric around the "y" axis are simply special parabolas because of their placement on the coordinate system.

Where the controversy led
Before we could explore AJ's comment and its relationship to Dave's choice, the class erupted with other comments, comments that revealed both students' questions about our pedagogy and their skepticism about their peers' abilities. Two students asked why we were allowing students to define words. A special education teacher who was in the back of the room[17] indicated that defining terms is one thing that mathematicians do. The students' immediate response was that students are not mathematicians.

Seeking to return the discussion to the particular mathematical question, I suggested that we summarize the controversial issue on the board and in our journals. In particular, I wanted to make sure that everyone saw the difference between parabolas that Dave would consider mirror image graphs and ones he would not. After documenting what the initial question was, Dave's definition, and a description of symmetry around a line, we asked students to use the last five minutes of class to write a short comment about both the difficulty they had with the activity on intercepts and their views on defining "mirror image."

Five comments stood out. Mary, who rarely participated in class, wrote "I think a mirror image is a better name." It was useful to her because it meant something to her. She seemed to be expressing some anger about Steve's need to know what was "scientifically correct" and a desire to affirm Dave's right to develop terminology. Dave wrote that he felt that regardless of our performance on the intercepts it was time to move on to new content. "I think we should start a new topic, new stuff." Steve wrote that he still was dissatisfied with the "rate at which we are going" and linked that rate to "the effort of some students (majority!)," without specifying who he meant. He wanted us to concentrate our efforts only on "the ones who want to learn." Stacy wrote that graphing was hard for her and therefore boring. Finally, Connie wrote that she did not understand what we had been doing and that she thought she was not the only one. She indicated frustration when she wrote, "But when I finally start to understand and then we learn something different." She asked if she could do extra work.

In thinking about these comments, we wanted somehow to balance some students' desire or need to move on—regardless of how well the class understood what it was doing—and other students' difficulties and confusions with the material. So, the following day I led a conversation in which students expressed their views about how the course was being orchestrated. The six students who spoke (including Dave and Steve) wanted to move more quickly and to split up into two groups by "ability" (since there were two teachers). However, as I listened, I was struck that I was hearing from the same people over and over and that every position seemed unanimous. No one seemed able to raise a disagreement with a position articulated by another student. In fact, as I listened I realized that from their journal writings I knew particular students (like Stacy and Connie) disagreed strongly with the articulated positions. We felt that many of the students in

the class were silenced (Belenky, Clinchy, Goldberger, & Tarule, 1986); we would not be able to take their perspectives into account in our planning.

Small group meetings—students' experience of the curriculum[18]

As a result, during the following week, Sandy and I met—in groups of three—with students who had been silent. We wanted to hear what they had to say and to exhort them to exercise their power to determine the course of the class by disagreeing (when necessary) and sharing their views during whole class discussions. In these small groups the students were serious and articulate. They were reflective about their own learning. For us, these small group discussions raised issues that had not come up in class before. They provided a window into some of the emotional and social issues that were central to students' view of our class.

Sandy met with three female students. They talked about the pace being too fast and said the environment was not a safe one for them. They indicated that they felt unsafe in a classroom with a fellow student who was coming to school from jail. They also indicated that they were being harassed by a male student. He was winking at them and "hitting on them" in other ways.

I met with three male students who also felt that they "need more time" to understand the kinds of things that we study in class. Christopher indicated that two students in particular made him feel stupid any time that he said something. George indicated that he has never made an unsolicited comment in any class. He did not think it was an important thing to do. When I indicated that I would not be able to take his views into account unless he spoke up, he said he had never thought of that.

Finally, in a third mixed-gender group there was considerable disagreement. Todd wanted to split the class in two because he thought he would be in the top group. When asked how he would feel if he ended up in the lower group, he said that even though he would feel badly about being in a lower group, he would still advocate such an arrangement. Christina indicated that she preferred not to be split into two groups. She preferred to be assigned to pairs

where students could help each other. She indicated that some of the people who think they know a lot, do not really. Marty indicated a strong preference for programmed instruction. He also wanted to be "put on the spot" in class. He felt that the danger of "looking dumb in public" would motivate him.

These small group conversations made it clear that, the earlier public unanimity notwithstanding, there were widely differing perceptions; there would be no simple solutions that would please everyone. Many students wanted us to change aspects of our instruction. Yet, the tone of the conversations was serious, constructive, and pleasant. It was a pleasure to have serious, articulate interactions with our students and to hear them reflect on their own learning. These small group discussions were an opportunity for us to learn how students were experiencing the class and to solicit their input; they also allowed us to encourage students who had not been participating.[19]

CONCLUSION

Let's face it. For most students the current school approach to algebra is an unmitigated disaster . . . . On this the mathematics education community and its critics agree: first-year algebra in is present form is not essential for a quality mathematics education. This is not to say that algebra is not essential. (Steen, 1992, p. 12)

Algebra policies are contested because they have implications for access to college. While we need to act to address inequalities in our society that limit access to college, I think it is wrongheaded to force students to take a class where almost half of the students fail. As a teacher, I think it is unfair to create such classes for teachers to teach. Finally, I think it is not sensible to hold out college as the only avenue to successful adulthood. Decisions about such policies need to keep in mind the students and teachers whose lives will be affected.

Whether the decision is made that all students must take algebra or not, I would like to learn how to teach algebra more successfully. I agree with Lynn Steen. The current algebra curriculum is in desperate need of reform; a new curriculum must be created

that helps the public and students understand why members of the mathematics education community consider algebra essential, or at least so important. This curriculum must be intended for a broad range of people—those who are not college-intending, as well as those who are.

If we really want a broad range of people to appreciate algebraic ideas, we are also going to have to rethink how we teach and how we structure math courses. If we continue to track, we will need to learn to serve lower track students better. If we discontinue tracking, we will need to learn to serve the needs of a wide range of students at the same time. At the very least, in these new settings we must be able to serve formerly lower-track students better than they have been in the past.

This is a daunting task. Even in the best of circumstances, with an appropriately reformed curriculum and drastic changes in how we teach algebra, I suspect that the job of teaching algebra to students who have not been successful in mathematics will remain a difficult challenge for those teachers willing to take it on.

Notes

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[2] Sandy teaches math and Spanish at Holt High School. Many thanks for your patience in working with me and for your comments on this paper.

[3] Holt High School is a Professional Development School, working in collaboration with Michigan State University. For the last few years, the faculty and administration have been restructuring and rethinking many of the school's programs. I was initially attracted to the school because the mathematics department expressed a strong commitment to revamping its algebra curriculum.

The high school has also been consistently named as a Michigan Exemplary High School. There is low student absenteeism, a lengthy student handbook of regulations, and a strict discipline code (which is honored, for the most part).

[4] While my description will, of necessity, be particular, I believe that it has wider ramifications. For example, the school in which I work bears little resemblance to the urban schools in which many of my colleagues work. In fact, the setting may seem bucolic to some. Yet, as I get to know our students and when I read about the feelings and behavior of mathematics students in urban settings (Bruckerhoff, 1991), without glossing over the differences, there are important similarities. This feeling of similarity suggests to me that the disparity I feel between the current curriculum and our students is one that is shared by those who teach Algebra One to students in lower track classes in a range of settings. It is this disparity which I believe is central for policy considerations.

[5] This figure is high, but probably common in most settings where non-college-intending students take algebra.

[6] Each year that I have taught, approximately one fourth of the students were labeled by the school as special education students.

[7] This argument is often made in Michigan, particularly by large corporations like General Motors.

[8] In response to critiques like the one outlined above, many projects are examining alternative approaches to the curriculum. For example, the Computer-Intensive Algebra materials (1991) emphasize situations and the use of the computer. I am indebted to Michal Yerushalmy and Judah Schwartz for introducing me to the perspective outlined below.

[9] My thanks to Nancy Alexander for bringing this example to my attention.

[10] I refer to this text because it has been so widely used.

[11] Fey (1981) documents such patterns of instructional style as dominant in secondary school mathematics.

[12] For example, I could not help Connie (see below) learn to solve word problems if I did not appreciate why she felt the problem's model of the situation was lacking.

[13] The name G is the name of a member of the school community.

[14] Her comments are reminiscent of comments Gilligan (1982) reports in response to traditional moral reasoning dilemmas.

[15] I wonder whether students would solve a different problem by working with the equation, whether something about this problem makes solving it symbolically unnecessary.

[16] Some of our students seem, on the surface, not to be aware of this.

[17] Eight of our students that year were labeled as special education students. As a result, a student advocate (a special education teacher) attended our class periodically.

[18] This term is used by Erickson and Schultz (1991). We are investigating this in greater detail now by hiring students as researchers of their own experience of our class.

[19] We have continued this practice and continue to wonder how we can help our students talk as seriously and articulately in a whole group setting.

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