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ABSTRACT

This booklet summarizes findings from data on mathematics achievement gathered on the 1992 National Assessment of Educational Progress (NAEP) for grades 4, 8, and 12. Approximately 26,000 4th, 8th, and 12th graders in 1,500 public and private schools participated in the national assessment. Information included in the report are: trends in achievement between 1990 and 1992, distribution of overall mathematics proficiency organized by state for grades 4, 8, and 12, performance on and examples of constructed-response questions, national results for demographic subgroups on two regular constructed-response tasks and an extended-response task, example of an extended-response task with scoring guide and sample responses, the school context for learning mathematics, trends and trouble spots in mathematics instruction, school effectiveness, background questionnaires, and procedures and methods. Findings for the performance of students on extended constructed-response questions include: (1) Approximately one-third to two-thirds of the students provided incorrect responses; (2) Substantial percentages of students left their papers blank; (3) Most students who did seem to understand the problems had difficulty explaining their work; and (4) From 1 to 16 percent of the students provided extended responses to each one of the tasks. On regular constructed-response questions, the average percentage correct by grade level was 42% for grade 4, 53% for grade 8, and 40% for grade 12. (MKR)

NATIONAL CENTER FOR EDUCATION STATISTICS

America's Mathematics Problem: Raising Student Achievement

A Synthesis of Findings from NAEP's 1992 Mathematics Assessment

THE NATION'S REPORT CARD naep

Prepared by Educational Testing Service under contract with the National Center for Education Statistics
Office of Educational Research and Improvement
U.S. Department of Education

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The National Assessment Governing Board

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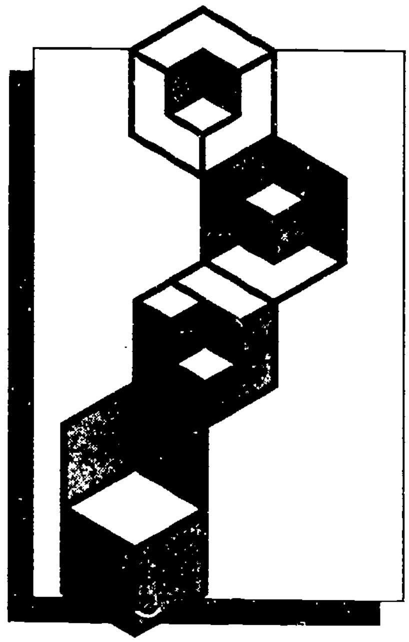

Adapted from:

- *The NAEP 1992 Mathematics Report Card*
- *Can Students Do Mathematical Problem Solving?*
- *How School Mathematics Functions*
- *Effective Schools in Mathematics*

Ina V.S. Mullis, Editor

Report No. 23-FR-03

October 1994

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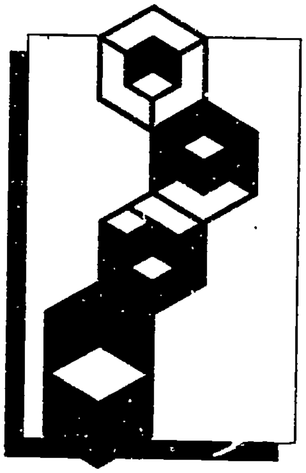


America's Mathematics Problem: *Raising Student Achievement*

A Synthesis of Findings from NAEP's 1992 Mathematics Assessment

The results from NAEP's 1992 mathematics assessment indicated that student performance is improving nationally and in some states, but that a considerable challenge remains. The proportions of students demonstrating success with more complex tasks continued to be low, particularly for those subpopulations of students historically considered to be "at risk."

Trends in Achievement Between 1990 and 1992

For the nation, there were statistically significant increases in average mathematics proficiency between 1990 and 1992 for fourth-, eighth-, and twelfth-grade students.[1]

DATAFILE

Trends in Average Mathematics Proficiency, Grades 4, 8, and 12

	GRADE 4	GRADE 8	GRADE 12
Assessment Years	Average Proficiency	Average Proficiency	Average Proficiency
1992	218>	268>	299>
1990	213	263	294

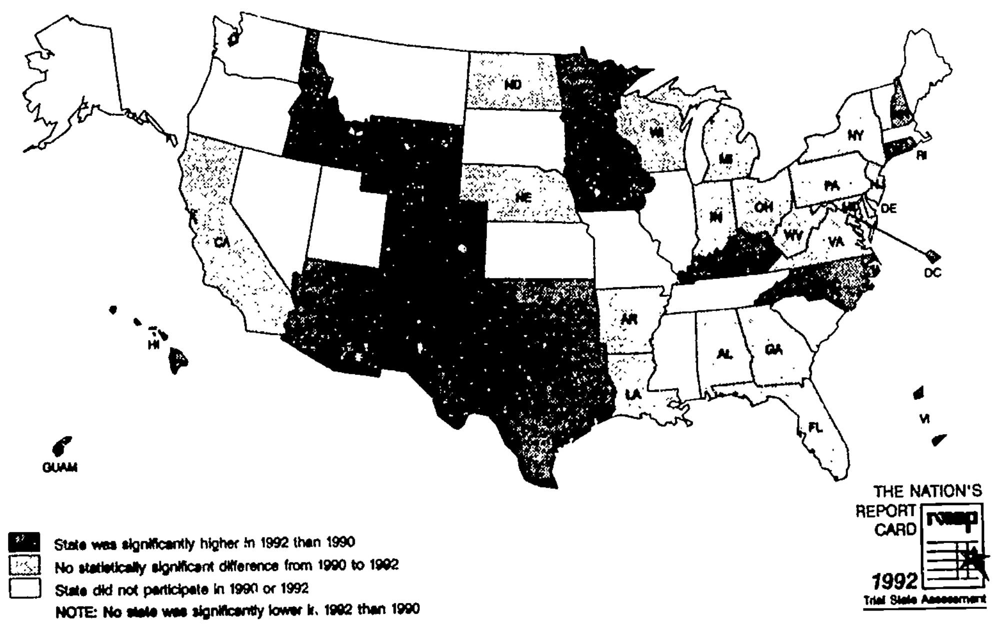
>The value for 1992 was significantly higher than the value for 1990 at about the 95 percent confidence level. Average proficiency was estimated using a scale of 0 to 500.

[1] At each grade, NAEP's nationally representative samples included students attending public and private schools.

As presented in Figure 1, 18 of 37 states and territories that participated in the grade 8 Trial State Assessment Program in both 1990 and 1992 showed significantly increased average mathematics proficiency for their public-school students.[2]

Figure 1

The NAEP Trial State Assessment – Comparisons of Overall Mathematics Proficiency at Grade 8, 1992 vs. 1990



The increases in mathematics proficiency between 1990 and 1992 for the nation and in many states did little to alter the relative standings of the various demographic groups.

- Average performance increased for White students at all three grades. The only other statistically significant gains in average mathematics proficiency by racial/ethnic group were found for Black and Hispanic students at grade 12. In 1992, Asian/Pacific Islander and White students had higher average mathematics proficiency than did Black students, with American Indian and Hispanic students performing somewhere in between.

[2] The NAEP 1990 and 1992 Trial State Assessment Programs included only students attending public schools.

- Average mathematics proficiency for both males and females increased at all three grades. In 1992, gender differences were not large, but males tended to outperform females students at grade 12.
- The national increase at grade 4 in overall average mathematics proficiency was exhibited in each region except the West. At grade 8, there were gains in the Central and West regions, and at grade 12 students in the Southeast showed improvement in average proficiency. The Southeast however, continued to trail behind the Northeast, Central, and West at all three grades assessed.
- Students attending schools in the top one-third of the performance distribution showed increased mathematics proficiency at all three grades. The only gain for students attending schools in the bottom one-third of the distribution occurred at grade 12.
- As shown in Figures 2 and 3, there was considerable variation in performance within a cross participating states and territories. Based on sophisticate. ests of statistical significance, the states with the highest average mathematics proficiency at grade 4 included Maine, Iowa, New Hampshire, Wisconsin, North Dakota, Minnesota, New Jersey, Connecticut, Massachusetts, and Nebraska. At grade 8, the top-performing states included Iowa, North Dakota, Minnes ta, Maine, New Hampshire, Wisconsin, and Nebraska.

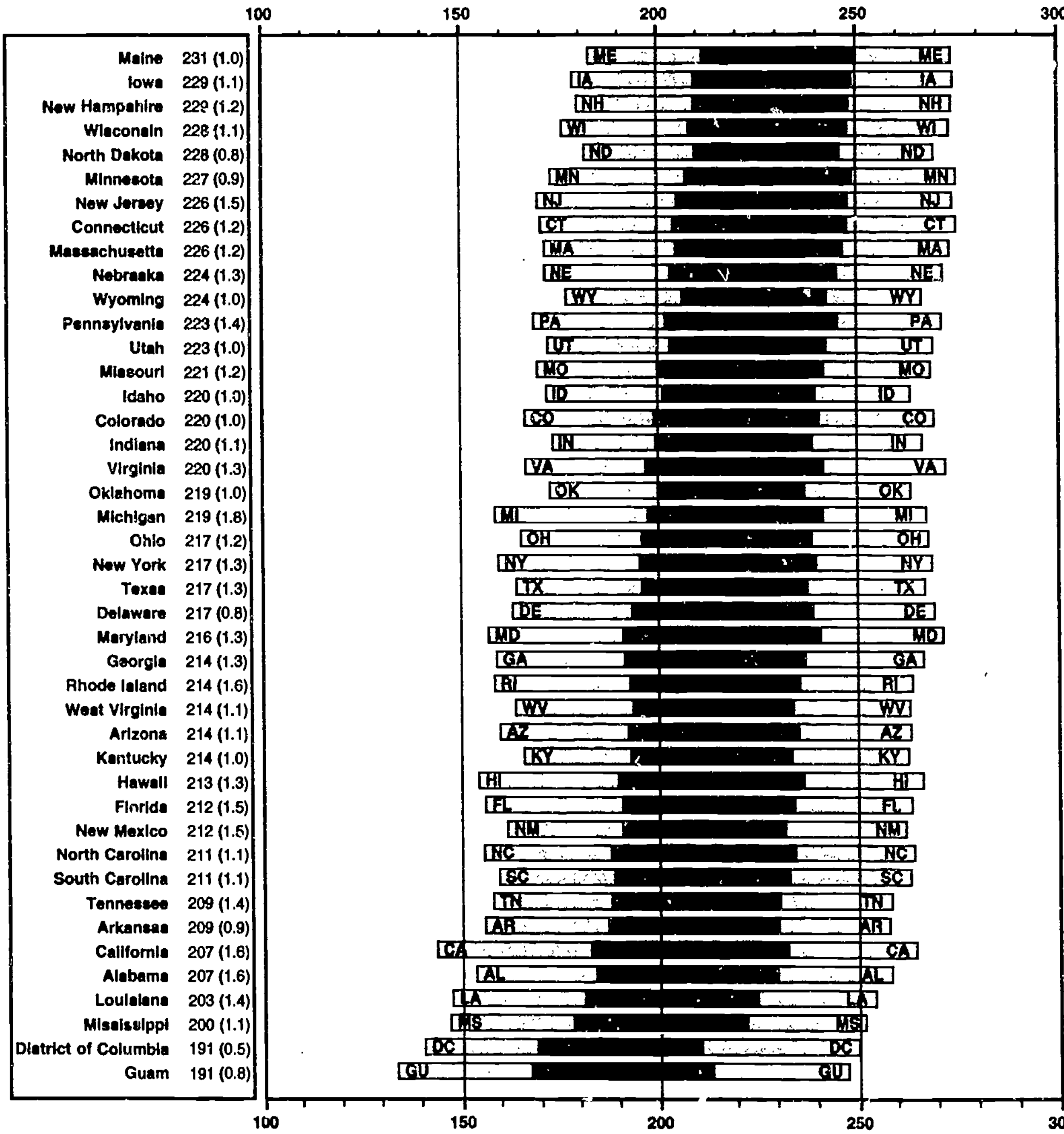
Mathematical Problem Solving

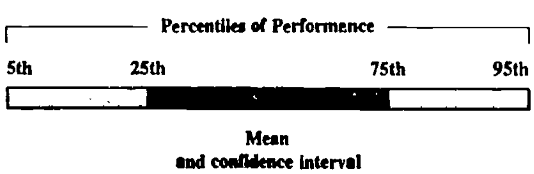
As part of the overall education reform initiative, there has been a concerted effort to improve mathematics education in the United States by emphasizing problem solving and application in real-life settings, rather than rote memorization. Beginning in the mid-1980s, the National Council of Teachers of Mathematics (NCTM) worked to develop *Curriculum and Evaluation Standards for School Mathematics*. Published in 1989, *The NCTM Standards* emphasize more balanced and dynamic curricular goals where students "do" mathematics — actively exploring, constructing, and justifying their ideas as they interact and work to solve problems.

The Mathematical Sciences Education Board (MSEB) also has developed a number of publications supporting the need for such reform, stressing the importance of active learning for students, and developing prototypes for assessing mathematics performance in ways that support instructional goals.

Figure 2

Distribution of Overall Mathematics Proficiency Organized by Average Proficiency, 1992 Grade 4

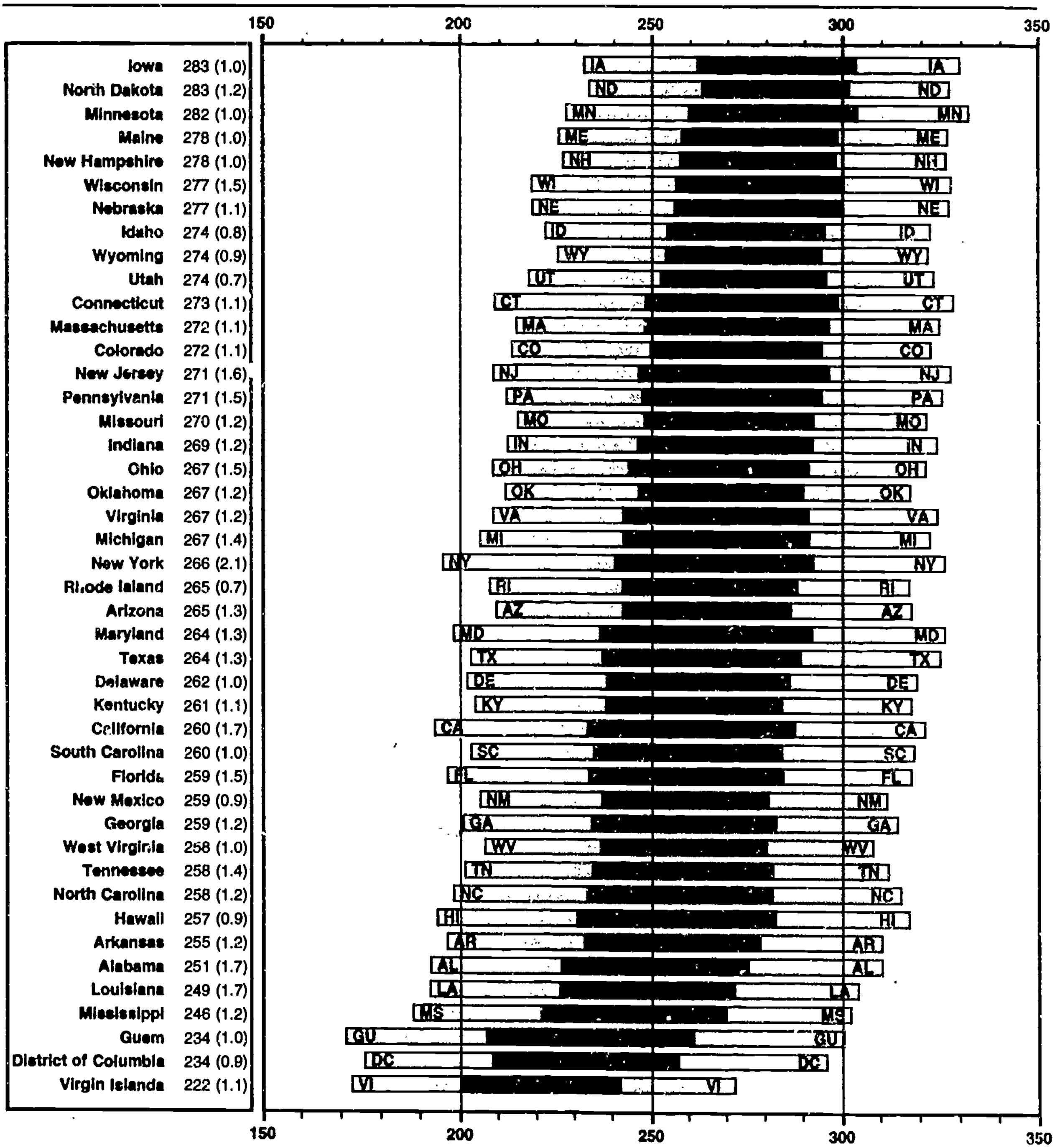


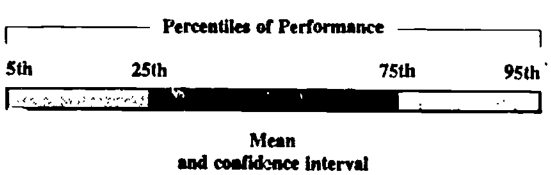


The center *darkest* box indicates a simultaneous confidence interval around the average mathematics proficiency for the state based on the Bonferroni procedure for multiple comparisons. Center boxes that do not overlap indicate significant differences between states in average mathematics proficiency. The *darker shaded* boxes indicate the ranges between the 25th and 75th percentiles of the mathematics proficiency distribution, and the *lighter shaded* boxes the ranges between the 5th to 25th percentiles and the 75th to 95th percentiles of the distribution.

Figure 3

Distribution of Overall Mathematics Proficiency Organized by Average Proficiency, 1992 Grade 8





The center *darkest* box indicates a simultaneous confidence interval around the average mathematics proficiency for the state based on the Bonferroni procedure for multiple comparisons. Center boxes that do not overlap indicate significant differences between states in average mathematics proficiency. The *darker shaded* boxes indicate the ranges between the 25th and 75th percentiles of the mathematics proficiency distribution, and the *lighter shaded* boxes the ranges between the 5th to 25th percentiles and the 75th to 95th percentiles of the distribution.

There is agreement by the NCTM, MSEB, and a number of national organizations, including The National Council on Education Standards and Testing, that the potential for educational improvement is enhanced if reform is systemic, simultaneously involving such areas as curriculum, instruction, assessment, and professional development. To reinforce reform efforts, assessments should embody the new instructional goals by providing thoughtful problem-solving situations and opportunities for students to explain their approaches.

In NAEP's 1992 mathematics assessment, about one-third of the questions and approximately one-half of the students' response time were devoted to questions asking students to construct their own responses. These questions were classified as regular constructed-response and extended constructed-response tasks. Regular constructed-response questions required students to provide a short answer giving a solution to the problem posed. The extended-response tasks were a new feature of the 1992 assessment. For these questions, students were allowed at least five minutes to complete tasks that required them to demonstrate — by writing, by giving examples, or by drawing diagrams — their mathematical reasoning and problem-solving abilities. For some of the constructed-response questions, NAEP provided students with protractors/rulers, calculators, or "manipulable" geometric shapes.

The procedures employed in constructing and scoring extended-response tasks showed that they could be successfully included in a large-scale national assessment and that they significantly contributed to understanding student proficiency in mathematics at each of the three grades assessed.

In general, the analysis of student papers showed that most made a conscientious effort to respond, but the performances exhibited left much to be desired. For the nation and across the states, there was a lower level of performance on both regular and extended constructed-response questions than on the multiple-choice items contained in the 1992 NAEP mathematics assessment. A summary of student performance on the constructed-response questions follows as do examples of these questions.

Performance on Constructed-Response Questions

On *regular* constructed-response questions, which required only a short constructed answer, the average percentage correct by grade level was 42 percent for grade 4, 53 percent for grade 8, and 40 percent for grade 12.

Similar performance was noted across the participating states and territories, with the average percentage correct ranging from 27 to 51 percent at grade 4 and from 30 to 63 percent at grade 8.

On *extended* constructed-response tasks, which required students to solve problems requiring a greater depth of understanding and then explain, at some length, specific features of their solutions, the average percentage of students producing satisfactory or better responses was 16 percent at grade 4, 8 percent at grade 8, and 9 percent at grade 12. Similar performance was noted across the participating states and territories, with the average percentage providing satisfactory or better responses ranging from 7 to 22 percent for grade 4 and from 0 to 13 percent for grade 8.

- Approximately one-third to two-thirds of the students provided incorrect responses to these extended questions, indicating little evidence of understanding the mathematics concepts involved or even the question being asked.
- Substantial percentages of students, sometimes as many as one-fifth, simply left their papers blank.
- Most students who did seem to understand the problems had difficulty explaining their work.
- It is encouraging, however, that some students — from 1 to 16 percent — provided *extended* responses to each one of the tasks.

Examples of Short Constructed-Response Questions

The following questions were given to eighth graders as part of the national and state assessments, and to twelfth graders as part of the national assessment. (State assessments were not conducted at grade 12.) Students were provided with hand-held calculators to use in solving the two short constructed-response questions.

To calculate the number of whole packages of paper (reams) that Raymond would have to purchase, students were required to calculate the number of pages needed (1792) and round that number up to the nearest multiple of 500 (2000). For that new number, they then had to determine what multiple it was of 500 and respond that Raymond needed to purchase 4 packages of paper. Fifty-two percent of the eighth graders and 72 percent of the twelfth graders completed the question correctly.

Example 1

Numbers and Operations

Overall Percent Correct*

Grade 8 — 52 (1.4)

Grade 12 — 72 (1.4)

Raymond must buy enough paper to print 28 copies of a report that contains 64 sheets of paper. Paper is only available in packages of 500 sheets. How many whole packages of paper will he need to buy to do the printing?

Answer: 4

Did you use the calculator on this question?

(Yes) No

*The standard errors of the estimated percentages appear in parentheses.

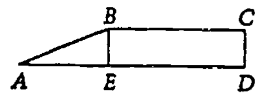
In the second example, students were asked to determine the area of a trapezoid in square inches. They needed to disassemble the given information and use the area relationship for a rectangle to determine that the altitude of the trapezoid was four units. Then they could proceed to use the area formula for a trapezoid or use the area formula for a triangle to find the area of triangle ABE and add that area to that for the rectangle BCDE to get 80 square inches. In either approach, the mathematical computations were not difficult, but students were required to carry out a series of sequential calculations in order to obtain the desired answer. This problem was quite difficult for students at both grades 8 and 12. Only 10 percent of the eighth graders and 23 percent of the twelfth graders provided a correct response.

Example 2
Measurement

Overall Percent Correct*

Grade 8 — 10 (0.9)

Grade 12 — 23 (1.6)



The area of rectangle *BCDE* shown above is 60 square inches. If the length of *AE* is 10 inches and the length of *ED* is 15 inches, what is the area of trapezoid *ABCD*, in square inches ?

Answer: 80

Did you use the calculator on this question?

Yes (No)

*The standard errors of the estimated percentages appear in parentheses.

The national results for demographic subgroups for "Raymond's Report" and "Area of a Trapezoid" for grades 8 and 12 are found in Table 1, and the state results for both questions at grade 8 are found in Table 2. In general, students had more success solving the problem of "Raymond's Report" than they did in finding the "Area of a Trapezoid." For example, across the states, from 18 to 71 percent of the eighth graders were able to find the correct answer to "Raymond's Report." In contrast, performance ranged from 1 to 16 percent correct in finding the trapezoid's area (even when the solution could have been determined by adding the area of the rectangle to the area of the triangle). In general, the level of correct responses suggests that students have little grasp of how to integrate and sequence information to solve a problem.

Table 1

National Results for Demographic Subgroups for the Regular Constructed-Response Tasks, "Raymond's Report" and "Area of Trapezoid"

	GRADE 8					
	Raymond's Report			Area of Trapezoid		
	Correct	Incorrect	No Response	Correct	Incorrect	No Response
Nation	52 (1.4)	46 (1.4)	3 (0.4)	10 (0.9)	81 (1.3)	9 (0.8)
Northeast	58 (3.5)	38 (3.5)	4 (0.7)	9 (1.4)	80 (1.7)	11 (1.5)
Southeast	42 (2.9)	54 (3.1)	4 (1.1)	9 (1.9)	82 (2.8)	9 (1.3)
Central	61 (3.3)	39 (3.4)	0 (0.2)	10 (1.9)	84 (2.1)	5 (0.8)
West	43 (1.9)	49 (1.6)	3 (1.0)	10 (2.0)	79 (3.0)	11 (2.3)
White	62 (1.7)	36 (1.7)	2 (0.4)	12 (1.2)	80 (1.4)	8 (0.7)
Black	20 (3.2)	74 (3.5)	6 (1.1)	2 (1.1)	84 (3.8)	14 (3.3)
Hispanic	30 (3.7)	65 (3.8)	5 (1.4)	3 (1.3)	88 (2.3)	9 (1.9)
Male	51 (2.3)	46 (2.4)	3 (0.6)	10 (1.4)	79 (2.0)	12 (1.4)
Female	52 (1.9)	45 (1.9)	2 (0.5)	9 (1.2)	84 (1.5)	7 (0.9)
Advantaged Urban	66 (5.3)	34 (5.3)	0 (0.0)	19 (3.8)	70 (4.0)	11 (4.9)
Disadvantaged Urban	25 (4.5)	68 (4.8)	6 (1.7)	4 (1.3)	82 (2.8)	14 (3.1)
Extreme Rural	55 (8.2)	45 (8.1)	0 (0.4)	9 (3.3)	83 (4.8)	8 (2.7)
Other	52 (1.6)	44 (1.6)	3 (0.6)	9 (1.2)	83 (1.5)	9 (0.7)
Public	50 (1.5)	47 (1.5)	3 (0.5)	9 (1.0)	82 (1.4)	9 (0.9)
Catholic and Other Private	62 (3.2)	35 (3.2)	3 (0.6)	12 (2.0)	78 (2.5)	9 (1.6)

	GRADE 12					
	Raymond's Report			Area of Trapezoid		
	Correct	Incorrect	No Response	Correct	Incorrect	No Response
Nation	72 (1.4)	25 (1.4)	2 (0.5)	23 (1.6)	67 (1.6)	10 (0.9)
Northeast	75 (2.2)	23 (2.3)	2 (0.9)	26 (2.8)	64 (3.4)	10 (1.6)
Southeast	68 (2.8)	28 (2.7)	3 (1.1)	16 (2.3)	77 (2.2)	7 (1.5)
Central	78 (1.9)	21 (2.2)	2 (0.9)	26 (3.2)	64 (2.1)	10 (2.4)
West	69 (3.5)	29 (3.7)	2 (0.6)	24 (4.2)	65 (4.3)	11 (1.4)
White	78 (1.3)	20 (1.3)	1 (0.3)	27 (2.1)	63 (1.8)	10 (1.1)
Black	51 (4.2)	44 (4.7)	5 (1.5)	8 (1.9)	81 (3.0)	12 (2.6)
Hispanic	62 (5.9)	34 (6.2)	4 (2.4)	14 (2.3)	80 (2.5)	6 (1.8)
Male	74 (2.0)	23 (2.0)	4 (0.9)	24 (1.7)	65 (1.8)	10 (1.2)
Female	71 (1.9)	28 (1.9)	1 (0.3)	22 (2.4)	68 (2.2)	9 (1.3)
Advantaged Urban	79 (3.6)	21 (3.7)	0 (0.2)	46 (4.8)	50 (4.6)	4 (1.4)
Disadvantaged Urban	62 (3.4)	33 (3.8)	5 (1.4)	10 (2.1)	79 (2.7)	11 (2.3)
Extreme Rural	73 (3.8)	25 (3.5)	2 (1.0)	25 (4.0)	68 (2.7)	8 (2.4)
Other	73 (1.5)	25 (1.6)	2 (0.6)	21 (2.6)	68 (1.9)	11 (1.0)
Public	72 (1.5)	26 (1.5)	2 (0.5)	21 (1.9)	70 (1.8)	10 (0.9)
Catholic and Other Private	79 (2.4)	20 (2.4)	2 (0.8)	42 (3.4)	49 (3.5)	9 (1.8)

The standard errors of the estimated percentages appear in parentheses. It can be said with about 95 percent certainty that for each population of interest, the value for the whole population is within plus or minus two standard errors of the estimate for the sample. In comparing two estimates, one must use the standard error of the difference (see Appendix for details). When the proportion of students is either 0 percent of 100 percent, the standard error is inestimable. However, percentages 99.5 percent and greater were rounded to 100 percent and percentages 0.5 percent or less ere rounded to 0 percent. Percentages may not total 100 percent due to rounding error.

SOURCE: National Assessment of Educational Progress (NAEP), 1992 Mathematics Assessment

Table 2

Percentages of Correct Responses to Regular Constructed-Response Questions "Raymond's Report" and "Area of Trapezoid," with Calculator Available

	GRADE 8 – 1992					
	Raymond's Report			Area of Trapezoid		
Public Schools	Correct	Incorrect	No Response	Correct	Incorrect	No Response
Nation	50 (1.5)	47 (1.5)	3 (0.5)	9 (1.0)	82 (1.4)	9 (0.9)
Northeast	58 (4.1)	39 (4.1)	3 (0.8)	7 (1.9)	81 (2.2)	12 (1.7)
Southeast	38 (2.5)	58 (2.9)	4 (1.2)	9 (2.1)	82 (3.0)	9 (1.4)
Central	60 (3.7)	39 (3.7)	0 (0.2)	10 (2.3)	85 (2.3)	5 (1.0)
West	47 (2.0)	49 (1.7)	4 (1.1)	9 (2.0)	79 (3.2)	11 (2.5)
States						
Alabama	41 (2.3)	56 (2.3)	3 (0.7)	4 (0.7)	89 (1.5)	7 (1.3)
Arizona	53 (2.2)	44 (2.2)	3 (0.7)	9 (1.3)	82 (1.8)	9 (1.2)
Arkansas	43 (1.9)	54 (2.0)	3 (0.8)	5 (0.8)	89 (1.2)	6 (0.9)
California	48 (2.3)	47 (2.2)	5 (1.0)	10 (1.3)	76 (1.5)	14 (1.4)
Colorado	56 (1.9)	41 (1.9)	3 (0.8)	9 (1.4)	81 (1.5)	10 (1.2)
Connecticut	58 (2.2)	40 (2.3)	2 (0.6)	12 (1.4)	81 (1.8)	7 (1.1)
Delaware	54 (2.7)	43 (2.3)	2 (0.9)	6 (1.3)	85 (1.7)	9 (1.3)
District of Columbia	30 (2.8)	62 (2.7)	8 (1.2)	3 (1.1)	87 (2.2)	10 (1.9)
Florida	52 (2.1)	44 (2.1)	4 (1.0)	6 (0.9)	85 (1.6)	9 (1.4)
Georgia	45 (2.3)	52 (2.6)	3 (0.9)	5 (1.0)	90 (1.3)	4 (0.8)
Hawaii	48 (2.1)	48 (2.1)	4 (0.8)	8 (1.2)	79 (1.9)	13 (1.5)
Idaho	58 (1.7)	39 (1.6)	2 (0.6)	13 (1.6)	79 (1.8)	8 (1.0)
Indiana	55 (2.0)	43 (2.0)	2 (0.7)	9 (1.3)	84 (1.6)	7 (1.1)
Iowa	71 (2.2)	28 (2.3)	2 (0.5)	13 (1.5)	82 (1.5)	5 (1.0)
Kentucky	54 (2.3)	43 (2.2)	3 (0.7)	7 (1.1)	87 (1.4)	6 (0.9)
Louisiana	42 (2.5)	54 (2.4)	5 (1.1)	4 (1.1)	90 (1.6)	7 (1.5)
Maine	67 (2.3)	32 (2.3)	2 (0.4)	12 (1.5)	80 (1.9)	8 (1.3)
Maryland	54 (2.4)	42 (2.2)	4 (1.0)	9 (1.5)	81 (2.0)	10 (1.4)
Massachusetts	59 (2.6)	39 (2.5)	3 (0.8)	9 (1.1)	82 (1.4)	8 (1.1)
Michigan	55 (2.1)	43 (2.2)	2 (0.7)	10 (1.4)	81 (1.6)	8 (1.2)
Minnesota	66 (1.7)	32 (1.7)	2 (0.6)	15 (1.7)	78 (1.9)	7 (1.0)
Mississippi	35 (2.1)	61 (2.2)	4 (0.9)	3 (0.7)	89 (1.1)	8 (1.0)
Missouri	53 (1.9)	46 (1.9)	1 (0.5)	8 (1.2)	85 (1.6)	7 (1.2)
Nebraska	58 (2.3)	41 (2.3)	2 (0.6)	12 (1.5)	83 (1.8)	4 (0.8)
New Hampshire	63 (2.1)	36 (2.1)	1 (0.5)	13 (1.5)	78 (1.9)	8 (1.3)
New Jersey	59 (3.2)	37 (3.1)	3 (1.0)	9 (1.7)	84 (2.3)	7 (1.2)
New Mexico	46 (2.2)	52 (2.2)	2 (0.6)	7 (1.1)	85 (1.6)	7 (1.2)
New York	55 (2.4)	43 (2.4)	2 (0.8)	12 (1.7)	84 (1.8)	4 (0.9)
North Carolina	48 (1.9)	50 (1.8)	2 (0.6)	5 (0.9)	90 (1.2)	6 (0.8)
North Dakota	67 (2.0)	32 (2.0)	1 (0.5)	16 (1.8)	79 (1.9)	5 (1.0)
Ohio	58 (2.3)	41 (2.2)	1 (0.4)	6 (1.0)	87 (1.4)	6 (1.0)
Oklahoma	59 (2.5)	39 (2.5)	2 (0.5)	8 (1.2)	85 (1.4)	6 (1.2)
Pennsylvania	56 (2.5)	41 (2.6)	2 (0.7)	10 (1.3)	84 (1.9)	7 (1.4)
Rhode Island	54 (2.2)	45 (2.0)	2 (0.6)	6 (1.0)	88 (1.4)	6 (0.8)
South Carolina	44 (2.5)	54 (2.4)	3 (0.7)	8 (1.1)	86 (1.4)	6 (0.9)
Tennessee	44 (2.4)	52 (2.4)	4 (0.7)	5 (1.0)	88 (1.4)	7 (1.0)
Texas	51 (2.3)	45 (2.2)	4 (0.7)	9 (1.2)	83 (1.6)	8 (1.2)
Utah	61 (1.8)	38 (1.8)	1 (0.4)	10 (1.3)	83 (1.4)	7 (1.1)
Virginia	57 (2.1)	42 (2.0)	2 (0.6)	8 (1.1)	86 (1.3)	6 (1.1)
West Virginia	52 (2.2)	46 (2.1)	2 (0.5)	5 (0.8)	88 (1.3)	8 (1.1)
Wisconsin	65 (1.7)	34 (1.5)	1 (0.4)	11 (1.3)	83 (1.3)	6 (0.9)
Wyoming	61 (2.2)	37 (2.3)	2 (0.8)	9 (1.1)	86 (1.3)	6 (0.8)
Territories						
Guam	29 (2.8)	64 (2.7)	7 (1.3)	4 (1.0)	87 (1.8)	9 (1.7)
Virgin Islands	18 (1.8)	68 (2.2)	14 (2.0)	1 (0.5)	82 (2.1)	17 (2.2)

The standard errors of the estimated percentages appear in parentheses. It can be said with 95 percent certainty that for each population of interest, the value for the whole population is within plus or minus two standard errors of the estimate for the sample. In comparing two estimates, one must use the standard error of the difference (see Appendix for details).

SOURCE: National Assessment of Educational Progress (NAEP), 1992 Mathematics Assessment

Example of Extended-Response Task: Radio Stations

An example of an extended constructed-response task given to eighth graders is shown in Example 3. In "Radio Stations," students were asked to apply measurement and geometry knowledge to diagram the intersection of signal transmissions from two radio stations.

An important aspect of mathematical power is the need to use logic and diagrams to make sense of a situation and to communicate this reasoning. However, the results indicate that many students do not recognize that diagrams can be effective analytical and communications tools. Even though a variety of diagrams or explanations could be used to help explain the intersection of the broadcast areas of the two radio stations and no particular approach was preferred, only 5 percent of the eighth graders were able to read and interpret the question and translate this information to develop a labelled model that represented the situation.

Students with incorrect responses provided no evidence that they were able to make sense of the problem, often copying a piece of information from the problem or submitting a meaningless drawing (or both). Forty-five percent of the eighth graders nationally provided such responses and another 16 percent did not answer the question at all.

Although this meant the majority appeared to be essentially at a loss as to the nature of this task, about one-third did seem to have some understanding of the information presented in relation to the task required. Approximately 22 percent received minimal credit and another 13 percent received partial credit. The difficulty with these responses was that they were incomplete, at best understandable only to those familiar with the problem. These sketchy solutions appeared in spite of directions explicitly telling students what to diagram and to be sure to label the distances and the part of the highway where both stations could be received.

Example 3

Extended Constructed-Response Task

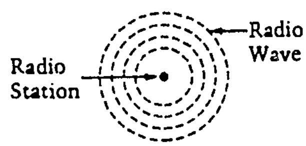
Grade 8 Question: Radio Stations

The Task

This question requires you to show your work and explain your reasoning. You may use drawings, words, and numbers in your explanation. Your answer should be clear enough so that another person could read it and understand your thinking. It is important that you show <u>all</u> your work.

Radio station KMAT in Math City is 200 miles from radio station KGEO in Geometry City. Highway 7, a straight road, connects the two cities.

KMAT broadcasts can be received up to 150 miles in all directions from the station and KGEO broadcasts can be received up to 125 miles in all directions. Radio waves travel from each radio station through the air, as represented below.

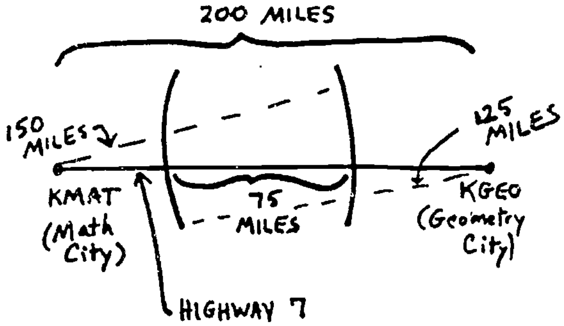


On the next page, draw a diagram that shows the following.

- Highway 7
- The location of the two radio stations
- The part of Highway 7 where both radio stations can be received

Be sure to label the distances along the highway and the length in miles of the part of the highway where both stations can be received.

Possible Solution



There is a 75-mile part of Highway 7 that is within both broadcast areas. It starts 75 miles outside Math City and ends 150 miles outside Math City.

Students need to assimilate and translate semantic information in order to draw a diagram that graphically depicts the location of the radio stations and Highway 7 accurately in terms of given boundary conditions. A graphical approach to this task should enable students to determine the length of the overlapping portion of Highway 7, along which both radio stations can be received. Any satisfactory response must clearly illustrate an overlapping region, whereas, in addition, any extended response must clearly identify the overlap and correctly determine its length to be 75 miles.

National Results, Scoring Guide, and Sample Responses

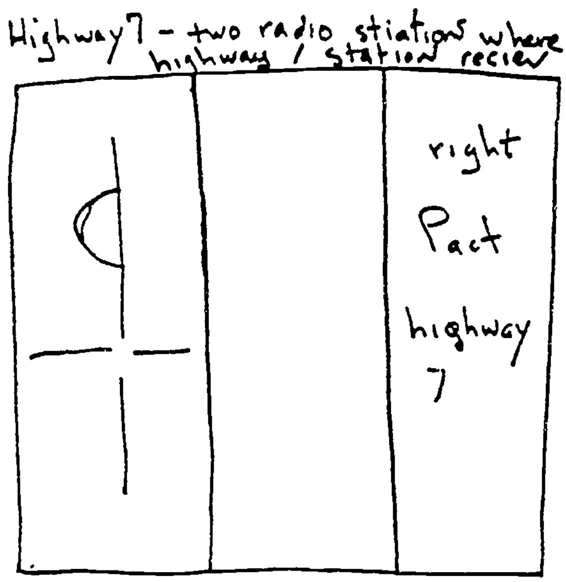
NO RESPONSE

*National Percentage** = 16 (1.1)

INCORRECT

National Percentage = 45 (1.6)

Scoring Guide — The work is completely incorrect or irrelevant, or the response states, "I don't know."

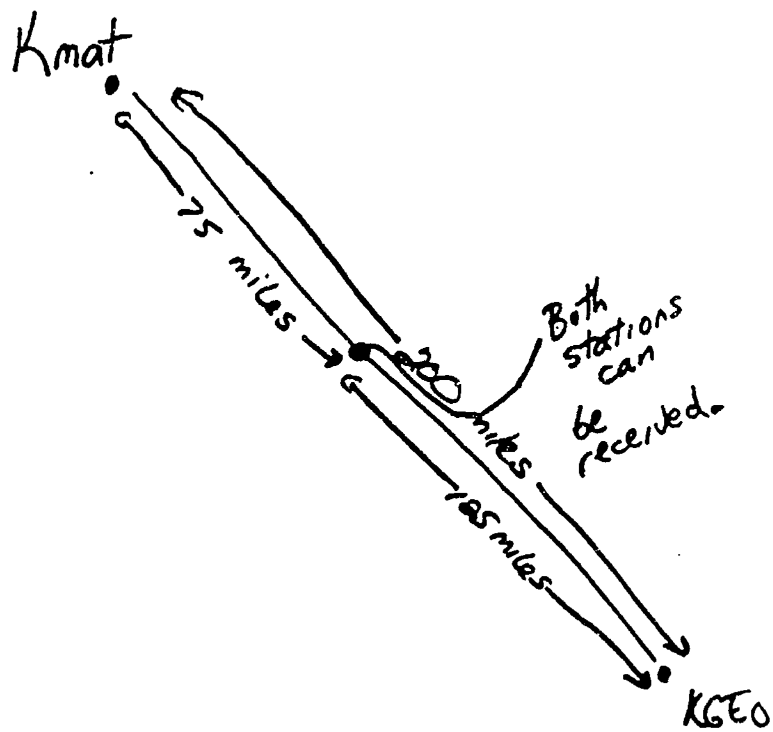


This INCORRECT response does not relate the information given in the problem in a manner that conveys either a meaningful problem solving approach or an adequate solution.

*The standard errors of the estimated percentages appear in parentheses.

National Percentage = 22 (1.2)

Scoring Guide — Diagram with only cities, Highway 7, and 200 miles labeled; or a diagram that shows some, but not all, of the given distances: 125, 150, or 200 miles. Minimal responses do not recognize that the common broadcast area is a *length* along the highway.

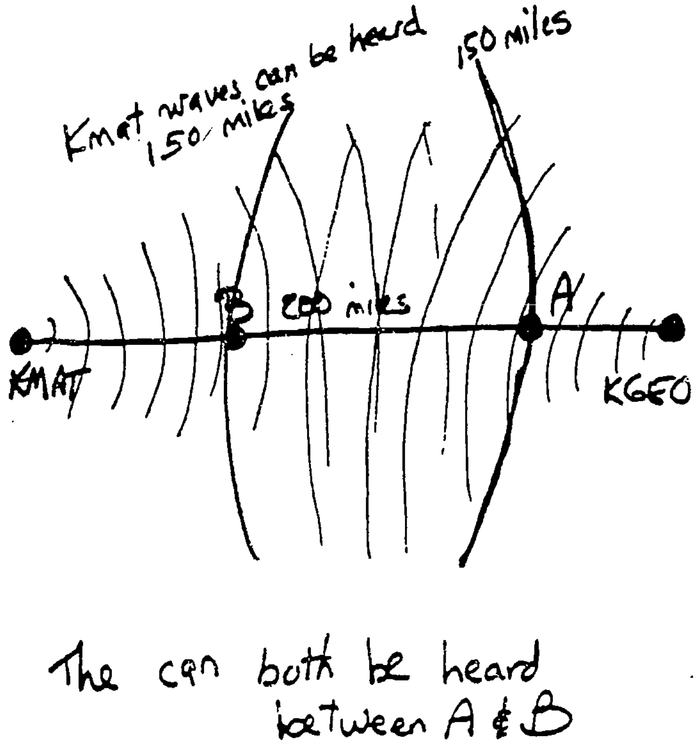


This MINIMAL response correctly depicts two pieces of information (radio stations KMAT and KGEO are 200 miles apart and station KGEO can broadcast 125 miles and shows rudimentary understanding. It does not show the common broadcast area as a length along the highway.

PARTIAL

National Percentage = 13 (0.9)

Scoring Guide — Diagram with cities, Highway 7, and 200 miles labeled *and* identification of common broadcast area as a length along (or not on) the highway. Two or more of the radio wave distances (250, 125, and 75) are insufficiently labeled.

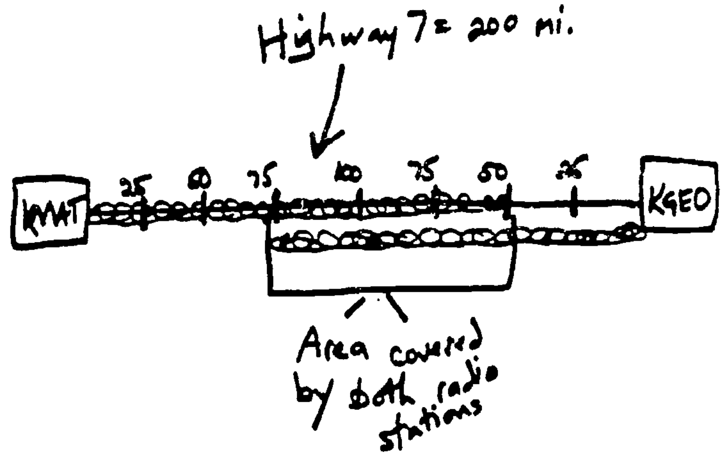


This PARTIAL response indicates considerable understanding of the task relative to the given information. The diagram shows the radio stations to be 200 miles apart and that KMAT can broadcast 150 miles. Additionlly, the diagram shows a part of the highway (from A to B) along which both radio stations can be heard. However, the response does not show the broadcast range of station KGEO and does not indicate the length of the commo.. :roadcast area.

SATISFACTORY

National Percentage = 4 (0.5)

Scoring Guide — Diagram with cities, Highway 7, 200 miles, and all radio wave distances labeled *and* identification of common broadcast area on Highway 7 as a length. At the same time, omits or incorrectly computes length of the highway along which both radio stations can be received.

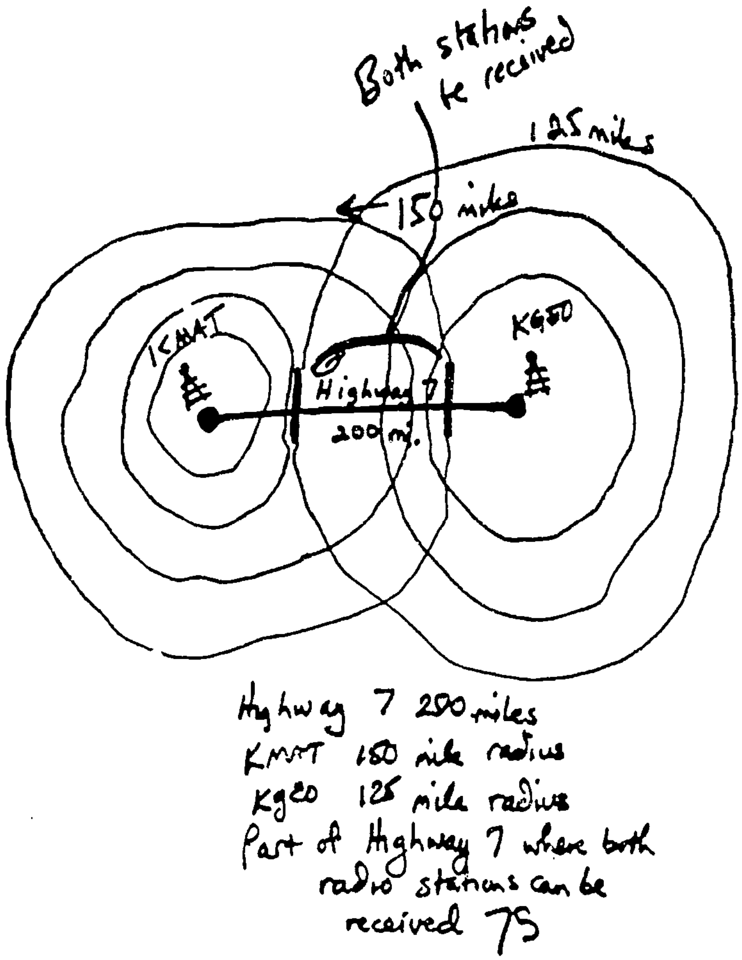


This SATISFACTORY diagram shows a good understanding of the problem. Although the student correctly labeled the common area along Highway 7 where the two stations could be heard, the length in miles of this region was not indicated.

EXTENDED

National Percentage = 1 (0.3)

Scoring Guide — An accurate, well-labeled diagram (as described in the "Satisfactory" category) clearly indicating that the portion of Highway 7 along which both radio stations can be received is 75 miles in length.



This is a solid EXTENDED response. The diagram is accurate and well-labeled. Additionally, below the diagram a statement correctly concludes that the length of the part of Highway 7 along which both radio stations can be heard is 75 miles.

Table 3

National Results for Demographic Subgroups for the Extended-Response Task, "Radio Stations"

	GRADE 8						
	No Response	Incorrect	Minimal	Partial	Satisfactory	Extended	Satisfactory or Better
Nation	16 (1.1)	45 (1.6)	22 (1.2)	13 (0.9)	4 (0.5)	1 (0.3)	5 (0.6)
Northeast	15 (1.7)	42 (3.5)	22 (2.0)	15 (2.5)	5 (1.2)	1 (0.4)	6 (1.1)
Southeast	18 (2.1)	50 (2.8)	17 (1.7)	12 (1.4)	3 (0.8)	0 (0.3)	3 (0.8)
Central	12 (2.2)	42 (2.0)	26 (2.8)	14 (2.4)	5 (1.3)	1 (0.2)	6 (1.3)
West	17 (2.5)	43 (3.5)	23 (2.5)	10 (1.2)	4 (1.1)	2 (0.9)	6 (1.4)
White	11 (1.2)	40 (2.0)	26 (1.6)	16 (1.2)	5 (0.8)	2 (0.4)	7 (0.9)
Black	32 (4.1)	55 (4.2)	8 (2.1)	4 (1.6)	1 (0.6)	0 (0.0)	1 (0.6)
Hispanic	26 (2.5)	58 (2.9)	11 (2.0)	4 (1.3)	1 (0.6)	0 (0.0)	1 (0.6)
Male	17 (1.2)	46 (2.1)	19 (1.8)	13 (1.3)	4 (0.7)	1 (0.3)	4 (0.8)
Female	14 (1.7)	43 (2.0)	24 (1.7)	12 (1.3)	4 (0.8)	2 (0.5)	6 (1.0)
Advantaged Urban	4 (1.7)	32 (3.2)	30 (3.3)	24 (3.1)	7 (1.8)	3 (1.0)	10 (1.5)
Disadvantaged Urban	38 (4.6)	49 (4.7)	7 (1.7)	4 (1.0)	2 (1.5)	0 (0.0)	2 (1.2)
Extreme Rural	15 (4.9)	39 (7.3)	30 (5.6)	14 (4.8)	2 (1.0)	0 (0.5)	2 (1.2)
Other	15 (1.5)	47 (2.2)	21 (1.4)	12 (0.9)	4 (0.7)	1 (0.4)	5 (0.8)
Public	17 (1.2)	45 (1.8)	21 (1.4)	12 (1.1)	4 (0.6)	1 (0.3)	5 (0.7)
Catholic and Other Private	8 (1.6)	42 (2.7)	25 (2.3)	18 (2.2)	6 (1.3)	1 (0.4)	7 (1.4,

The standard errors of the estimated percentages appear in parentheses. It can be said with about 95 percent certainty that for each population of interest, the value for the whole population is within plus or minus two standard errors of the estimate for the sample. In comparing two estimates, one must use the standard error of the difference (see Appendix for details). When the proportion of students is either 0 percent or 100 percent, the standard error is inestimable. However, percentages 99.5 percent and greater were rounded to 100 percent and percentages 0.5 percent or less were rounded to 0 percent. Percentages may not total 100 percent due to rounding error.

SOURCE: National Assessment of Educational Progress (NAEP), 1992 Mathematics Assessment

Across the categories of students by region, race/ethnicity, gender, type of community, and type of school, a majority of only one subgroup provided at least minimal responses: advantaged urban students. From 32 percent (advantaged urban) to 58 percent (Hispanic) of the students by subgroup provided meaningless information (see Table 3).

As shown in Table 4, the percentages of success for public-school eighth graders in the jurisdictions participating in the Trial State Assessment Program were similar to those for the nation. However, in two states, Iowa and Minnesota, at least 10 percent of the students were estimated to have provided satisfactory or better diagrams. For five states, Iowa, Minnesota, Nebraska, New Hampshire, and North Dakota, the majority of the students were estimated to have provided minimal or better responses.

Table 4

Percentages for Responses to Extended-Response Question, "Radio Stations"

	GRADE 8 – 1992						
Public Schools	No Reponse	Incorrect	Minimal	Partial	Satisfactory	Extended	Satisfactory or Better
Nation	17 (1.2)	45 (1.8)	21 (1.4)	12 (1.1)	4 (0.6)	1 (0.3)	5 (0.7)
Northeast	17 (2.0)	40 (4.2)	22 (2.6)	16 (3.1)	4 (1.5)	1 (0.5)	5 (1.6)
Southeast	19 (2.4)	52 (2.7)	15 (1.8)	11 (1.4)	2 (0.7)	0 (0.3)	3 (0.7)
Central	14 (2.6)	43 (2.5)	25 (3.3)	13 (2.9)	4 (1.3)	1 (0.2)	5 (1.4)
West	17 (2.8)	44 (4.0)	23 (2.7)	9 (1.4)	4 (1.2)	3 (1.0)	7 (1.5)
States							
Alabama	22 (2.0)	53 (2.4)	14 (1.5)	8 (1.1)	2 (0.6)	1 (0.4)	3 (0.8)
Arizona	16 (1.7)	48 (2.3)	20 (1.8)	13 (1.6)	2 (0.7)	1 (0.4)	3 (0.9)
Arkansas	13 (1.8)	57 (2.6)	18 (1.8)	8 (1.2)	2 (0.6)	1 (0.3)	3 (0.7)
California	19 (1.7)	44 (2.3)	19 (2.1)	13 (2.1)	3 (0.9)	1 (0.5)	4 (1.1)
Colorado	11 (1.2)	43 (2.0)	23 (1.6)	16 (1.7)	5 (0.9)	2 (0.6)	7 (1.1)
Connecticut	10 (1.2)	44 (2.3)	23 (2.0)	16 (1.7)	6 (1.0)	2 (0.4)	7 (1.2)
Delaware	16 (2.0)	51 (2.4)	17 (1.5)	12 (1.8)	3 (0.9)	1 (0.5)	4 (1.0)
District of Columbia	30 (2.1)	57 (2.3)	8 (1.0)	3 (1.1)	1 (0.5)	1 (0.5)	2 (0.7)
Florida	22 (1.8)	46 (2.2)	19 (1.9)	10 (1.3)	3 (0.9)	1 (0.3)	4 (0.9)
Georgia	19 (1.7)	49 (2.1)	19 (1.6)	7 (1.1)	4 (0.9)	1 (0.5)	5 (0.9)
Hawaii	23 (1.8)	47 (2.3)	15 (1.6)	11 (1.3)	3 (0.8)	1 (0.4)	3 (0.8)
Idaho	11 (1.1)	46 (2.2)	21 (2.0)	15 (1.2)	5 (1.0)	2 (0.6)	7 (1.2)
Indiana	7 (1.1)	49 (2.6)	25 (2.1)	15 (1.9)	3 (0.7)	1 (0.6)	5 (1.0)
Iowa	6 (0.9)	35 (1.8)	28 (1.9)	21 (2.0)	6 (0.9)	4 (1.1)	10 (1.2)
Kentucky	13 (1.4)	52 (2.3)	22 (1.7)	10 (1.1)	2 (0.5)	1 (0.4)	3 (0.6)
Louisiana	24 (2.2)	54 (2.1)	15 (1.4)	6 (1.0)	1 (0.4)	0 (0.2)	1 (0.5)
Maine	7 (0.9)	46 (2.1)	21 (1.9)	18 (1.8)	6 (0.9)	2 (0.6)	8 (1.1)
Maryland	15 (1.7)	48 (2.1)	20 (1.8)	13 (1.6)	3 (0.9)	1 (0.5)	5 (1.1)
Massachusetts	12 (1.2)	45 (2.8)	22 (2.4)	14 (1.9)	5 (0.9)	2 (0.7)	7 (1.0)
Michigan	15 (1.6)	46 (1.9)	21 (1.9)	12 (1.1)	5 (0.9)	2 (0.6)	7 (1.1)
Minnesota	6 (1.1)	41 (2.3)	20 (1.9)	21 (2.4)	7 (1.0)	4 (1.0)	11 (1.6)
Mississippi	20 (1.9)	57 (2.2)	14 (1.7)	6 (1.2)	2 (0.6)	0 (0.2)	2 (0.6)
Missouri	10 (1.2)	47 (2.1)	23 (1.9)	12 (1.6)	5 (0.9)	2 (0.7)	8 (1.2)
Nebraska	6 (1.4)	44 (2.8)	23 (2.0)	20 (3.0)	6 (1.0)	1 (0.6)	7 (1.1)
New Hampshire	8 (1.1)	42 (2.1)	24 (1.8)	18 (1.5)	5 (1.0)	3 (0.9)	8 (1.3)
New Jersey	12 (1.6)	46 (2.3)	22 (2.0)	14 (1.8)	5 (0.9)	1 (0.5)	6 (1.0)
New Mexico	17 (1.7)	51 (2.2)	19 (1.6)	11 (1.4)	3 (0.7)	1 (0.3)	3 (0.8)
New York	15 (2.1)	44 (2.4)	22 (1.6)	11 (1.5)	5 (0.9)	2 (0.7)	7 (1.2)
North Carolina	12 (1.4)	54 (2.2)	21 (1.8)	10 (1.5)	2 (0.5)	1 (0.4)	3 (0.7)
North Dakota	6 (1.2)	43 (2.5)	25 (2.0)	18 (1.8)	5 (0.9)	3 (1.2)	8 (1.5)
Ohio	12 (1.3)	47 (2.1)	25 (2.1)	12 (1.8)	5 (1.0)	0 (0.2)	5 (1.0)
Oklahoma	10 (1.4)	46 (2.0)	23 (1.6)	16 (1.7)	3 (0.8)	2 (0.8)	5 (1.1)
Pennsylvania	12 (1.2)	44 (2.1)	22 (1.7)	14 (1.4)	6 (1.1)	3 (0.7)	8 (1.5)
Rhode Island	10 (1.1)	49 (3.5)	25 (2.9)	12 (1.6)	2 (0.8)	2 (0.6)	5 (1.0)
South Carolina	12 (1.4)	55 (2.1)	21 (1.7)	10 (1.2)	2 (0.6)	1 (0.4)	3 (0.6)
Tennessee	16 (1.5)	51 (2.4)	22 (1.7)	10 (1.6)	1 (0.5)	0 (0.2)	2 (0.6)
Texas	16 (1.9)	45 (2.2)	21 (1.5)	12 (1.4)	3 (0.7)	3 (1.1)	5 (1.3)
Utah	10 (1.4)	46 (2.5)	20 (1.7)	16 (1.5)	7 (1.2)	2 (0.4)	8 (1.3)
Virginia	13 (1.5)	48 (2.5)	22 (1.7)	12 (1.4)	4 (0.8)	1 (0.5)	5 (1.0)
West Virginia	13 (1.5)	53 (2.4)	21 (1.7)	10 (1.2)	3 (0.6)	0 (0.3)	3 (0.6)
Wisconsin	9 (1.9)	43 (1.8)	23 (1.8)	18 (1.9)	4 (1.0)	4 (1.0)	8 (1.2)
Wyoming	8 (1.2)	45 (2.4)	22 (1.7)	18 (1.6)	6 (0.9)	3 (0.7)	8 (1.1)
Territories							
Guam	45 (2.4)	40 (2.4)	10 (1.9)	3 (1.0)	2 (0.8)	1 (0.5)	3 (0.8)
Virgin Islands	47 (3.4)	47 (3.3)	4 (1.1)	1 (0.7)	0 (0.3)	0 (0.0)	0 (0.3)

The standard errors of the estimated percentages appear in parentheses. It can be said with 95 percent certainty that for each population of interest, the value for the whole population is within plus or minus two standard errors of the estimate for the sample. In comparing two estimates, one must use the standard error of the difference (see Appendix for details).

SOURCE: National Assessment of Educational Progress (NAEP), 1992 Mathematics Assessment

The School Context for Learning Mathematics

There were very large performance differences within grade level between the achievement of students in the highest-performing one-third of the schools compared to the lowest-performing one-third of the schools. These are not small or extreme segments of the school population. For example, one-third of the twelfth graders in the nation represents more than one million students. It is a matter of concern, then, to find that the twelfth graders in the lower-performing third of the schools had lower average mathematics achievement than the eighth graders in the higher-performing one-third of the schools for that grade.

Because it is considerably more difficult to implement reform in schools with large numbers of disadvantaged students,[3] it is of further concern to find the extent to which economically disadvantaged and minority students are concentrated in the lower-performing schools. Very few students in the top-performing one-third of schools (0 to 2 percent) were attending schools in disadvantaged urban communities, and these students showed very little racial/ethnic diversity (84 to 88 percent White students across the three grades assessed by NAEP). At grades 4 and 8, about half the students in the lower-performing one-third of the schools were in schools where the majority of the students were participants in the subsidized lunch program.

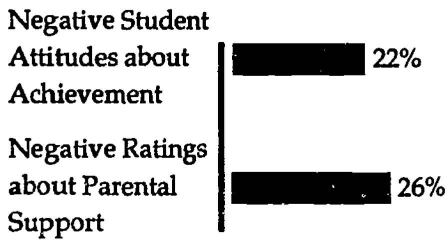
- *Students in the top-performing schools have a greater opportunity to learn by virtue of being in school more often and with less transiency.* In the top-performing schools there was less absenteeism, class cutting, tardiness, and transiency. The continuity of instructional approach that can be adopted in these schools simply is not possible in the lower-performing schools. By grade 12, all but 1 percent of the students in bottom-performing schools were in schools where principals considered absenteeism at least a minor problem.

- *In general, students in the top-performing schools have considerably more support for academic achievement than those in lower-performing schools.* Considerable research has found schools with positive school climates to be more effective than those without any central purpose

[3] O'Day, J.A. & Smith, M.S. "Systemic Reform and Educational Opportunity." In S.H. Furhman, editor, Designing Coherent Policy: Improving the System (San Francisco, CA: Jossey-Bass, 1993).

DATAFILE

Percentages of 12th Graders in Bottom One-Third Performing Schools with Negative Ratings about Academic Interest and Parental Support



or goals, and that such schools can work for even the most disadvantaged students.[4] For the twelfth graders in the lowest-performing third of schools, 22 percent were in schools receiving negative ratings for students' attitudes towards achievement and 26 percent in schools receiving negative ratings for parental support.

[4] Hill, P.T., Foster, G.E., & Gendler, T., *High Schools with Character* (Santa Monica, CA: The Rand Corporation, 1990).

Trends and Trouble Spots in Mathematics Instruction

Although the two-year period between 1990 and 1992 is too short to establish trends, some signs of movement were noted toward reform in school mathematics. These changes in instructional context tend to support the gains in average achievement noted across the same time period.

- *Students reported taking more advanced coursework.* At grade 8, the percentages of students taking pre-algebra increased (from 20 to 28 percent) while the percentages enrolled in a general eighth-grade mathematics class decreased (from 61 to 49 percent). Also, there was a decrease from 18 to 14 percent between 1990 and 1992 in the percentage of twelfth graders reporting three or fewer mathematics courses during high school.

- *Teachers reported moving toward a more broadly based curriculum.* Although most students (90 percent at grade 4 and 76 percent at grade 8) were receiving heavy emphasis in numbers and operations, there were increases between 1990 and 1992 in the percentages of fourth graders receiving moderate emphasis in geometry and introductory algebra concepts, and more eighth graders were receiving at least a moderate emphasis in measurement and geometry.

- *Students and teachers reported more access to and use of calculators and computers.* In 1992, more fourth graders had access to school-owned calculators than in 1990 (59 compared to 44 percent). More eighth graders were permitted unrestricted classroom use of calculators, both generally and in testing situations. There were increases in computer access and use at grade 4.

- *Students were doing more daily problem solving from their textbooks.* Teachers at grades 4 and 8 as well as students at all three grades reported an increase in daily problem solving from textbooks. In 1992, teachers reported that 76 percent of the fourth graders and 83 percent of the eighth graders solved problems from textbooks on a daily basis. As a related finding, at grade 8, teachers reported assigning somewhat more mathematics homework in 1992 than in 1990, with fewer students (28 compared to 41 percent) given only 15 minutes of mathematics homework per night.

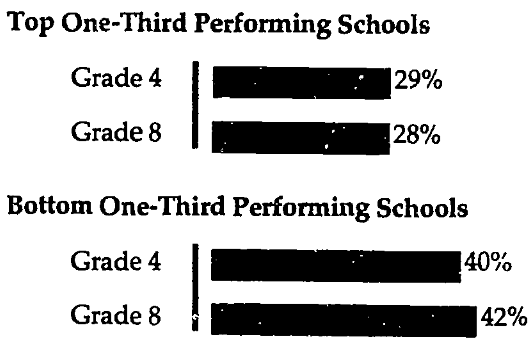
- *Students reported more positive attitudes about the value of mathematics.* More students in 1992 than in 1990 reported understanding the utility of mathematics to solve everyday problems and as part of job-related skills.

Still, not all the reports from teachers and students can be viewed positively.

- *Teachers reported discrepancies in resource availability and expectations between top- and bottom-performing schools.* Teachers reported that more students in bottom- than in top-performing one-third schools were in classrooms with only some or none of the necessary resources — 40 compared to 29 percent at grade 4, and 42 compared to 28 percent at grade 8. In 1992, 25 percent of the eighth graders in top one-third schools were expected to do 45 minutes of mathematics homework per night, compared to 16 percent in bottom one-third schools. The rather pervasive increase in calculator use at grade 8 was not observed in bottom-third schools.

DATAFILE

Percentages of Fourth- and Eighth-Grade Students in Schools with only Some or None of the Necessary Resources



- *Teachers and students reported very little change in the frequency with which students were asked to engage in extended problem-solving activity.* Teachers reported no increases between 1990 and 1992 in the percentages of students receiving heavy instructional emphasis in reasoning and communication for either grade 4 or grade 8. Only about half the students or fewer were receiving heavy instructional emphasis in these two areas emphasized in the *NCTM Standards*.[5] Teachers also reported that about half their students were never or hardly ever assessed using projects, portfolios, or presentations. The percentage of students reporting that they were never or hardly ever asked to write reports or do mathematics projects increased significantly between 1990 and 1992 (from 70 to 77 percent at grade 8 and from 71 to 82 percent at grade 12). As a related finding, 62 percent of the grade 8 students and 68 percent at grade 12 reported that they never or hardly ever were asked to write a few sentences about how they solved a mathematics problem.

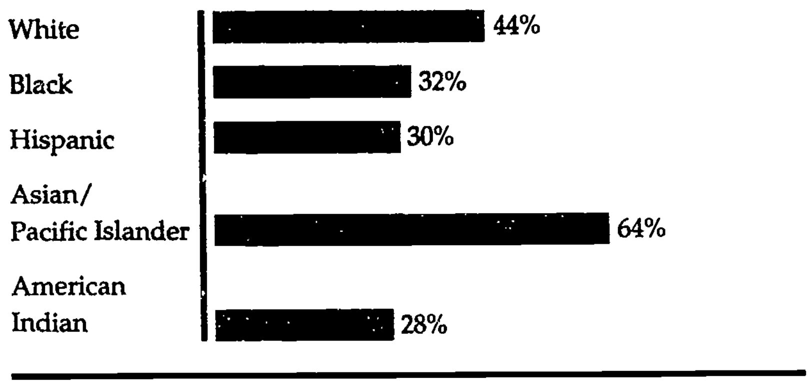
- *Despite a direct and powerful relationship between taking advanced mathematics courses and higher achievement, students reported an extremely low degree of mathematics coursework.* About half of the eighth graders (49 percent) were taking eighth-grade mathematics, while those with higher average proficiency were enrolled in pre-algebra (28 percent) or algebra (20 percent). Eighth graders planning to proceed to more advanced coursework (geometry or algebra I) in ninth grade also had higher average proficiency. However, 21 percent did not know what mathematics course they would take in grade 9.

  Only 42 percent of the twelfth graders reported taking eight semesters of mathematics coursework during their high school years. Twenty-three percent of the twelfth graders (36 percent in bottom-third schools) reported never studying geometry, which has become a "gatekeeper" course for access to higher education since most colleges are requiring this course prior to entrance. There were large discrepancies in the amounts of mathematics coursework reported among various subpopulations.

[5] National Council of Teachers of Mathematics, *Curriculum and Evaluation Standards for School Mathematics* (Reston, VA: National Council of Teachers of Mathematics, 1989).

DATAFILE

Percentages of 12th Graders Reporting at Least Eight Semesters of Mathematics Course-Taking in Grades 9 through 12



Another Perspective on School Effectiveness

In addition to examining the characteristics of schools with the highest mathematics achievement, it is also informative to examine factors associated with schools that are most effective in maximizing learning beyond the students' home background and the socioeconomic levels of the communities in which the schools are located. From a value-added perspective, schools with a more difficult educational task can be just as effective as those with fewer hurdles to overcome, and NAEP used hierarchical analysis methods to examine this issue. Unfortunately, the state of the art in educational measurement is not nearly sophisticated enough to provide definitive answers or to completely untangle the relationships among the myriad inputs to students' learning. No matter how sophisticated the approach, the adjustments can never be complete, and such is the case in attempts to examine the effects of socioeconomic status. (Also, please note that the teacher questionnaire data were not included in the analyses for technical reasons.)

Nevertheless, two different analyses were conducted after average mathematics achievement was adjusted for variations in students' home background and the socioeconomic levels of the communities in which their schools were located. In one of these analyses, achievement was predicted based on school socioeconomic indicators. Schools with higher performance than their counterparts with similar socioeconomic characteristics were considered to be the most effective, and schools that did not perform as well were considered to be the least effective. An analysis of differences between the 15 percent most and least effective schools according to the NAEP background data showed that in the most effective schools, especially at the upper grades:

- students watched less television
- students were tested a moderate amount in mathematics class
- students were taking advanced mathematics courses
- students and parents had more positive attitudes toward academic achievement
- there were fewer school problems
- students read more for schoolwork each day (either at school or at home)
- students worked mathematics problems from textbooks more frequently
- more students went on to four-year colleges or universities

In another analysis, variables and sets of variables were examined to determine the major predictors of school effectiveness, with the following results:

- Important classroom factors included moderate testing in mathematics class, doing problems from textbooks more frequently, and more frequent calculator use.
- Important school characteristics were a positive school climate and limited problems. A stable student body, as opposed to a transient one, also was important.
- The largest effects were related to having more students taking more advanced courses. The number of advanced mathematics courses taken was the most powerful predictor of students' mathematics performance after adjusting for variations in home background.
- School-level socioeconomic indicators, such as community type and percentage of students participating in the subsidized lunch program, also were powerful predictors and were correlated with classroom, school, and course-taking factors.

Background Questionnaires

The background-data collection effort associated with NAEP's 1992 mathematics assessment involved asking students, their mathematics teachers, and the school principals to complete questionnaires. Students at all three grades completed questionnaires about demographic characteristics and their educational experiences in mathematics, including instructional activities, courses taken, use of specialized resources such as calculators, and their views about the value of the subject matter.

The teachers of the fourth and eighth graders who participated in the assessment provided information about their own background and training. The fourth and eighth graders' teachers also provided information, classroom by classroom, about students' ability levels, instructional time, homework assignments, frequency of instructional activities, and the emphasis given to various mathematics content areas.

School administrators at all three grades completed questionnaires about a variety of school characteristics, including absenteeism, policies about tracking, special priorities and school-wide programs, availability of resources, and school-wide problems. The student, teacher, and school background questionnaires were completed both as part of the national and Trial State Assessments.

In formulating the extensive set of background questions associated with NAEP's 1992 mathematics assessment, three goals were kept in mind:

- to provide an educational context for understanding data on student achievement
- to identify differences in access to instruction and distribution of services for various types of students
- to track changes in policy-relevant variables across time

The *1992 NAEP Background Questionnaire Framework* focused on five major educational policy areas.[6] These areas are broad, encompassing domains inclusive enough to address the needs, interests, and concerns of a variety of NAEP audiences. The five areas included: instructional content, instructional practices and experiences, teacher characteristics, school conditions and context, and conditions outside of school that affect learning and instruction.

[6] *National Assessment of Educational Progress, 1992 Background Questionnaire Framework* (Princeton, NJ: National Assessment of Educational Progress, Educational Testing Service, 1992).

Procedures and Methods

As with all NAEP assessments, the schools and students participating in the 1990 and 1992 mathematics assessments were selected through scientifically designed stratified random sampling procedures. Approximately 26,000 fourth, eighth, and twelfth graders in 1,500 public and private schools across the country participated in the national assessment. For each jurisdiction participating in the Trial State Assessment Program, approximately 2,500 students were sampled from approximately 100 public schools for each grade. Thus, a total of approximately 220,000 fourth- and eighth-grade students attending nearly 9,000 public schools participated in the 1992 Trial State Assessments. The 44 participating jurisdictions were:

Alabama	Louisiana	Ohio
Arizona	Maine	Oklahoma
Arkansas	Maryland	Pennsylvania
California	Massachusetts	Rhode Island
Colorado	Michigan	South Carolina
Connecticut	Minnesota	Tennessee
Delaware	Mississippi	Texas
District of Columbia	Missouri	Utah
Florida	Nebraska	Virginia
Georgia	New Hampshire	West Virginia
Hawaii	New Jersey	Wisconsin
Idaho	New Mexico	Wyoming
Indiana	New York	
Iowa	North Carolina	Guam
Kentucky	North Dakota	Virgin Islands*

*The Virgin Islands participated in the testing portion of the 1992 Trial State Assessment Program. However, in accordance with the legislation providing for participants to review and give permission for release of their results, the Virgin Islands chose not to release their results at grade 4 in the 1992 NAEP reports.

Trend results from a comparable assessment conducted in 1990 are available for the nation and for the 37 states and territories that participated in both the 1990 and 1992 programs at grade 8. NAEP's Trial State Assessment Program began in 1990 with mathematics at grade 8 and expanded in 1992 to include both grades 4 and 8, as well as reading at grade 4. The states that do not have trend data at grade 8 include: Maine, Massachusetts, Mississippi, Missouri, South Carolina, Tennessee, and Utah.

All NAEP data are collected by trained administrators. Data for the national assessment were collected by a field staff managed by Westat, Inc. However, in accordance with the NAEP legislation, data collection for the Trial State Assessment Program was the responsibility of each participating jurisdiction. Uniformity of procedures across states was achieved through training and quality control monitoring by Westat, Inc. Westat staff trained nearly 10,000 state assessment administrators using a video presentation accompanied by a scripted trainer's guide and practice exercises. Quality control was provided by unannounced, random monitoring of half the sessions in each state. The results of the monitoring indicated a high degree of quality and uniformity across sessions.

The materials, including approximately 4 million written responses constructed by students in 1992, were scored by National Computer Systems in Iowa City, Iowa, using scoring rubrics developed by the NAEP Mathematics Test Development Committee and staff at Educational Testing Service (ETS). The scoring rubrics were developed prior to the assessment, revised on the basis of field-test results, and modified a final time following an examination of samples of student responses obtained in the actual assessment. To evaluate the scoring reliability, 25 percent of the papers for each question were scored by two different scorers. The percentage of exact agreement, averaged across the questions and papers, was 94 percent.

ETS analyzed the assessment results to determine the percentage of students responding correctly to each multiple-choice and regular constructed-response question and the percentage of students responding in each of the categories for extended problem-solving tasks. Item response theory (IRT) methods were used to summarize results for each of the mathematics content areas, which included Numbers and Operations; Measurement; Geometry; Data Analysis, Statistics, and Probability; and Algebra and Functions. New for the 1992 NAEP assessment, a partial-credit scaling procedure employing a specialized IRT method was used to account for students' responses scored according to the five-point scoring guides used with the extended problem-solving tasks. An overall composite scale was developed by weighting each content area according to its importance in the framework.[7] Average proficiency on the composite scale, which ranges from 0 to 500, is the statistic primarily used in this report to compare overall mathematics performance for various groups of students. Unless

[7] *Mathematics Objectives*, 1990 Assessment (Princeton, NJ: National Assessment of Educational Progress, Educational Testing Service, 1988).

otherwise noted, all changes or differences discussed in this report are statistically significant at the .05 level of significance. This means that the observed differences are unlikely to be due to chance or to sampling variability.

Throughout the development and conduct of the 1992 assessment, the National Center for Education Statistics (NCES) and its contractors worked closely with the Trial State Assessment NETWORK, which includes representatives from all interested states. Federal funding permitted state education personnel to meet with staff members from NCES, the contractors, the National Assessment Governing Board (NAGB), and the Council of Chief State School Officers (CCSSO) at NETWORK meetings regularly held to review NAEP materials and procedures. Further details about the methods and procedures used in NAEP's 1992 mathematics assessment of the nation and states are provided in the *Technical Report of the 1992 Trial State Assessment in Mathematics* and *The NAEP 1992 Technical Report* (forthcoming).

A Note on Interpretations

The NAEP background questionnaires make it possible to examine the relationships between student proficiency and a wide variety of background factors, usually by relating performance to one or several variables at a time. The selection of background questions included in the NAEP mathematics assessment was guided by the *NAEP 1992 Background Questionnaire Framework*, derived by considering the wide body of available research about factors influencing student learning and the particular purposes of NAEP data collection. Because of their basis in research, the NAEP survey results often help to confirm our understanding of how school and home factors relate to achievement. Although the effects of schooling and instruction are of prime concern, these analyses do not reveal the underlying causes of the relationships between background factors and performance. The NAEP assessment results are most useful when they are considered in light of other knowledge about the education system, such as trends in instructional reform, changes in the school-age population, and societal demands and expectations.

This synthesis of findings from NAEP's 1992 Mathematics Assessments was adapted from the following NAEP reports:

Mullis, I.V.S., Dossey, J.A., Owen, E.H., & Phillips, G.W., *NAEP 1992 Mathematics Report Card for the Nation and States* (Wash`ngton, DC: National Center for Education Statistics, 1993).

Dossey, J.A., Mullis, I.V.S., & Jones, C.O., *Can Students Do Mathematical Problem Solving*? (Washington, DC: National Center for Education Statistics, 1993).

Dossey, J.A., Mullis, I.V.S., & Gorman, S., *How School Mathematics Functions* (Washington, DC: National Center for Education Statistics, 1994).

Mullis, I.V.S., Jenkins, F., & Johnson, E.G., *Effective Schools in Mathematics* (Washington, DC: National Center for Education Statistics, 1994).

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