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AUTHOR Moore, Joyce L.; Schwartz, Daniel L.
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ABSTRACT

In an effort to create a mathematics learning environment in which the social nature of the classroom facilitates conceptual conjecture and justification, a visual representation has been designed that students can manipulate to make demonstrative proofs in the domain of proportional representation. This study explores student use, incorporation, and extension of this representation in classroom and face-to-face settings. Subjects were 49 high-ability sixth graders using the video-based mathematics series, the Jasper Adventure Series. The visualization helped students identify and articulate proportional relationships. Students used the representation in diverse and often innovative ways when they demonstrated their problem solutions to other students. While students were able to understand refutations of problems, they generally could not generate them, although the visual representation helped them generate contradictions in reasoning. Students began to create a culture in which demonstration became a social phenomenon. Results suggest a promising model of discourse that is an alternate to the procedural descriptions language in general use. Four figures illustrate the study. (Contains 5 references.) (SLD)

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Show how you know: A visual medium for demonstrative discourse

Joyce L. Moore and Daniel L. Schwartz
Vanderbilt University

1. Objective

Our objective is to create a mathematics learning environment in which the social nature of the classroom facilitates conceptual conjecture and justification. To provide a discourse medium we have designed a visual representation that students can manipulate to make demonstrative proofs in the domain of proportional reasoning. The visual nature of the representation provides reference for the complex relational structure of proportions, while its qualitative nature precludes the use of rote procedural explanations. In the current study, we explore student use, incorporation, and extension of this representation in classroom and face-to-face settings.

2. Theoretical perspective

The social dynamics of the classroom could provide a natural milieu for student conjecturing and justification (Moore, 1993). For example, in a discovery mathematics environment students may develop conjectures to explain or generalize specific patterns. If supported by classroom practices, students will want to convince others of their mathematical notions. Moreover, if given an appropriate means of discourse, the expression of their initial ideas and the ensuing interchange can lead to clarity, proof and deeper understanding. In a model like this, students uncover mathematical properties and relations and then formalize their understanding through the demands of effective communication and social demonstration (Lampert, 1988).

The discourse of conjectural justification can only play a role in mathematics classrooms if it is supported with an appropriate language of demonstration. Unfortunately, procedural language often dominates school mathematics discourse. For example, in pilot work this fall, we asked sixth-graders to construct business plans for setting up a booth at a fun fair. When we asked students why they reached specific conclusions or why they used specific mathematical procedures, the students invariably answered by repeating the steps of the procedures they had executed to reach their numerical conclusion. This should not be surprising given that procedural description is often the only medium of mathematical discourse available to students and teachers alike. One result of over-reliance on procedural discourse is that it becomes difficult for the student to distinguish between mistakes and impossibility. Short of an inductive proof, students have limited means to demonstrate the generality of a specific mathematical relationship -- they always work with numerical instances. So, for example, given the expression $a-b = b-a$ where $a<b$, a "procedural" student cannot demonstrate

that the equivalence is impossible. In each demonstration of the inequality, the possibility always exists that the student has simply made an arithmetic mistake.

The question then becomes one of finding an appropriate language that can supplant procedural descriptions. This can be a difficult problem because many math concepts are relational, making it difficult to ensure that other people understand what one is talking about. For example, if one is not sure whether a conversational partner understands the word "tapir", one can point to a tapir. However, if one is not sure whether a conversational partner understands the relationship of height to weight, one cannot simply point to a short and thin person. Because relational ideas do not have ready referents, and because students are just learning new relations and cannot rely on conventional meanings, mathematics discourse can suffer from a mutual knowledge problem (Clark & Marshall, 1981). A mutual knowledge problem occurs when conversants cannot be sure that their words mean the same thing for one another. One solution to the mutual knowledge problem is to provide visual reference for relational ideas (Schwartz, 1993). This often requires constructing abstract visualizations that make relationships manifest. A good example of such a visualization is a Cartesian graph which makes it much easier to anchor a discussion about linear relationships.

In the current study we explored the use of classroom conjecture and justification in the context of proportional reasoning. Proportions involve the coordination of multiple relationships between quantities, presenting a demanding domain in which to test our beliefs that conceptual justification can be facilitated through classroom discourse. Proportions are particularly difficult to discuss in a demonstrative fashion because the numerous relations and terms often lack clear referents. For example, in common usage the very term "proportion" is sometimes used to refer to the relation a/b and sometimes to the relation a/b = c/d (Ohlsson, 1988).

We have created a visualization with spatial properties that represent relationships, providing students with a visual referent for ratio. In Figure 1 the representation is being used to demonstrate that a rate of 10 gallons per 1 minute will fill a 60 gallon tank faster than a rate of 10 gallons per 2 minutes. The representation can also be used for part-whole proportions. In these cases the total box represents the whole, and the shaded and unshaded areas represent the parts. The representation was designed to meet five criteria: (1) It had to be comprehensible to middle-school students and had to involve a minimum of new symbolic conventions. (2) It had to support conjectures about mathematical relationships without explicit reference to numbers. This is an important attribute if students are to avoid reliance on procedural explanations. (3) It had to make it easy to refer to relationships. This is accomplished by making it possible for students to refer to size and relative size.

(4) It had to be highly flexible so it could be used for multiple types of problems such as rate and part-whole. (5) It had to be extensible so students could construct original demonstrations.

In the current study, we had three goals. The first was to explore instructional techniques and classroom practices that foster demonstrative discourse. The second was to examine how students would interact with the visualization in a climate of demonstration. In particular, we were interested in how they incorporated and extended the representation. The final goal was to see if students would incorporate the representation into their own work.

3. Method

Forty-nine high-ability sixth-grade mathematics students in two classes participated. Prior to introducing the representation, the students spent one week working on a complex video-based mathematics problem from The Jasper Adventure Series (Cognition and Technology Group at Vanderbilt, 1992). Embedded in the larger video problem were two sub-problems that required extrapolating a ratio to a second ratio. In one sub-problem, students used the flow rate of a hose to determine how long it would take to fill-up a tank of a specific size. In the other sub-problem, students determined the expected level of participation at a school given a sample participation rate (found from a survey). The Jasper adventure created a context for our instruction with the visual representations. One class worked with the visualizations for flow problems and one class for sampling problems.

Our instructional technique followed three principals: (1) Discourse is in the context of demonstration. This is to establish discourse norms and to prevent fallback to procedural descriptions. Thus, even when students are first taught to use the representation it is in the context of demonstrating mathematical relationships. (2) Discourse is based on equivalences or lack of equivalences. This creates the logic of a proof-oriented discourse, as opposed to simple correct vs. incorrect judgments. Thus, reasoning centers around proving or judging equivalences. For example, the instructor presented the conflicting assertions $3/4 + 2/2 = 5/6$ and $3/4 \times 2/2 = 6/8$. Students had to prove which of the assertions was correct. (3) Discourse is classwide but focussed on student to student communication. Much of the classroom discourse involved presentation and justification of student work to other students. This is to make sure that the social catalyst for clarity and justification are established. Moreover, it ensures that students develop a common body of referential language. Thus, for each problem students can work individually or in small groups, but it is necessary that classwide discourse and demonstration take place after each problem.

In addition to videotaping the classroom instruction, structured interviews were conducted with half of the students in each class. During this interview the students were requested to justify conjectures in the domain in which they were not instructed. Thus, a rate student would work with part-whole problems and vice versa.

Finally, we collected pre- and post-test samples of student work on proportion problems. At the end of each problem was the statement, "Show how you know." We included these measures to see whether students would interpret this question as a demonstrative question more frequently after instruction.

4. Results

Below we develop several points relevant to the promise of a visual medium for supporting conjectures and justifications within the classroom.

Sharpening the Referential Focus. A primary goal of using visual representations was to provide reference for the relational ideas inherent in conjectural discourse. In pilot interviews we found that the visualization helped sixth-grade students identify and articulate proportional relationships. In addition, we found a framing of 'what stays the same and what changes' between two representations useful in moving students toward discriminating quantities from relationships among quantities. In the following example, Mike has drawn a bar chart to represent the number of students who will and won't buy a ticket in a sample, and he has extrapolated that relationship in a second bar chart of the population. Below, the interviewer asks Mike what is variant and invariant in his sample and population representations. Mike initially struggles to articulate what remains invariant.

Int: what changed?
Mike: the length [of a bar representing the number of students who will buy a ticket]
Int: what didn't?
Mike: the amount that would buy a ticket

The interviewer pointed out to Mike that he has indicated on his representation that the number of students who will buy a ticket in the sample and population is different, and then again asks:

Int: what changed?
Mike: the amount of people [who will buy a ticket]
Int: what didn't?
Mike: the fraction of how many people are going to buy and how many aren't

Notice in the first interchange that Mike has answered one question with a property of the representation and another in terms of the quantities being represented. In the second interchange, he focuses solely on the quantities being represented, and successfully discriminates between the magnitude of quantities and the relationship between quantities. In the current study, we employed the device of contrasting variant and invariant properties of the representation in the classroom.

Flexibility and originality under syntactic constraint. A critical component of visual demonstration is having a visual "grammar" that allows students to create original yet coherent demonstrations. Although it may be a result of the high math aptitude of these students, they wielded the representation with unanticipated originality and rigor. When students demonstrated their solutions to the other students, their approaches were diverse and often innovative. For example, students were asked to show whether x will be greater than, less than, or equal in the two equations 45/60 = x/360 and 90/120 = x/360, although the question was presented as in Figure 2. Four students or pairs of students drew their solutions on the board, and then explained their solutions to the class. Three of these solutions are illustrated in Figure 2. The first of these involves stretching the representation vertically, rather than horizontally as was presented in class (Figure 2a). Notice that the ratio of the shaded area to the unshaded area remains the same in the two representations; however, the size of the two areas gets larger. In a horizontal representation, the areas also get larger, but for any slice of the representation the height of the two areas remain constant (e.g., Figure 1). A horizontal representation represents an additive model of extrapolation, while a vertical representation is multiplicative. In that sense, the vertical representation created by this student is more sophisticated than the one presented in class.

The second representation involved rescaling the representation (Figure 2b). In this case, the representation for 45/60 and 90/120 are drawn the same size, but with different scales. Because the ratios are equivalent, the shaded areas are the same size. Again, this is a manipulation of the representation which was not presented during instruction. The third demonstration involved a representation for 90/120 which is then extrapolated to 270/360 (Figure 2c). That result can then be compared to the result of extrapolating 45/60 to the population.

Impossibility and refutation. An important skill in proof-based discourse, as well as other forms of discourse, is the ability to enter and think within the space of an opposing position. In proof discourse, this often takes the form of refutation. An important question is whether the students could learn to refute alternative positions. In the course of three days, we found that the students could understand refutations but could not generate them. On the preceding problem, a fourth student attempted to show that 90/120 was in fact smaller than 45/60. As she attempted to construct a visualization, she found that her assertion could not be defended under the constraints of the visualization. We believe that it is compelling for a student to generate a contradiction in their reasoning, and have that contradiction visually available to them. To promote this type of reasoning the instructor generated negative proofs showing the implications of common misprocedures, an example is presented in Figure 3.

Students create a culture of shared demonstration. An important question was whether the students would begin to develop a culture in which demonstration became a social phenomenon. We found some evidence of this. For example, on one question students were asked to show whether a rate of 20 gal./1 min. would fill up a 60 gallon tank faster than a rate of 10 gal./1 min. Again, there was a range of student responses, several are shown in Figure 4. Of particular interest are the students who drew and used Figure 4b. This pair incorporated an element of demonstration that we had not considered. The two students agreed to make a check mark in a one minute segment of each of the representations every five seconds, and then see which of the representations "filled up" faster. They solicited class participation by asking them to count out the five second intervals that stood for single minutes.

Subsequent use of visual proof. Finally, we wanted to see if students would re-interpret the meaning of the expression "Show how you know." Our analyses of pre- and posttest performance indicate that students incorporated visual proof into their problem-solving repertoire. After instruction, students were more likely to respond to the request "show how you know" with a visualization, rather than a procedural explanation. On the pretest, only 2% of the students spontaneously generated a visualization in the solution to any problem, while on the posttest this increased to 41%. While this result may be due to perceived task demands, the students had nonetheless learned that demonstration is a legitimate form of evidence in mathematical discourse.

5. Implications

In the current study, we have sketched evidence of how a visual representation was effectively used by students to develop conjectural justifications. This suggests a promising alternative model of discourse to the current procedural descriptions language used in many textbooks and classrooms. Although we have yet to employ performance measures, the use of demonstrative reasoning may yield deeper student understanding. As students attempt to articulate their ideas in the forum of visual demonstration, they discover new meanings and relationships.

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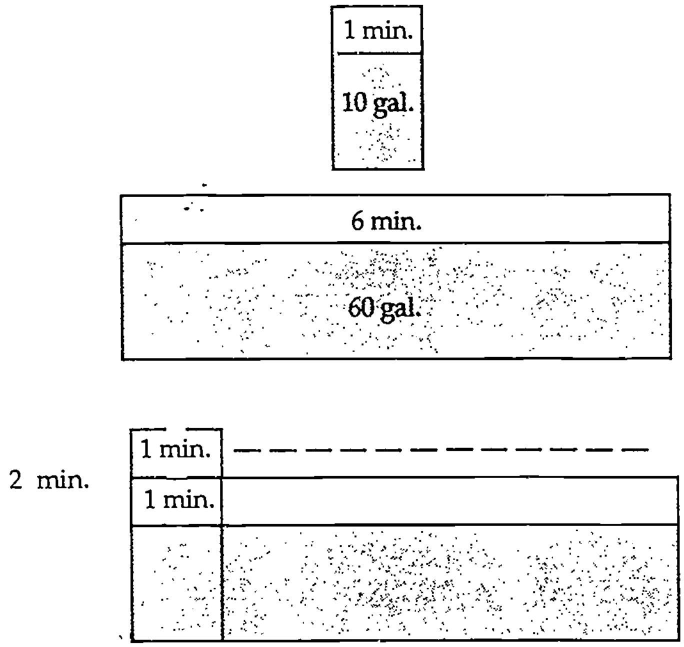

Figure 1. An example of a visual proof to demonstrate that 10 gal./2 min. is a slower rate than 10 gal./1 min.

One sample says 45 out of 60 kids will use the funfair booth. A second sample says that 90 out of 120 kids will use the booth. Does one of the samples say that more of the 360 kids at the school will use the booth?

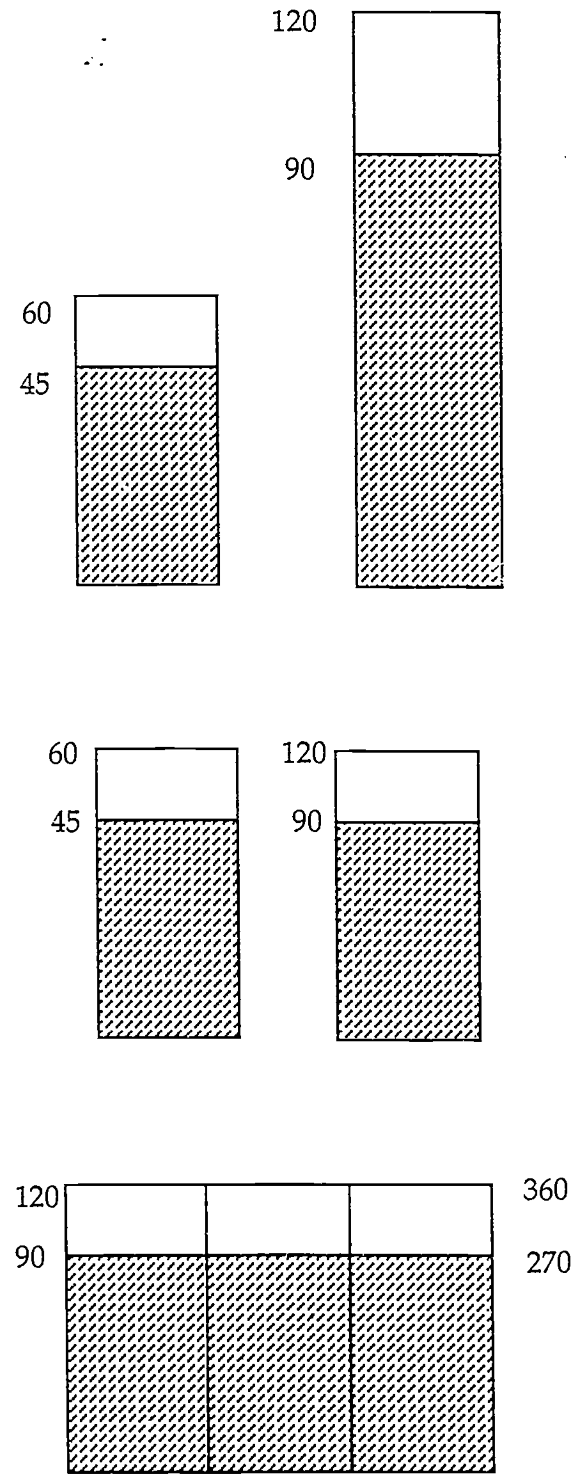

Figure 2.

Students have been asked to demonstrate which is impossible:
(a) 45/60 x 6 = 45/60 (b) 45/60 x 6/6 = 45/60.

After an impasse in the students' ability to make the demonstration, the instructor shows that 45/60 x 6 is not proportional to 45/60.

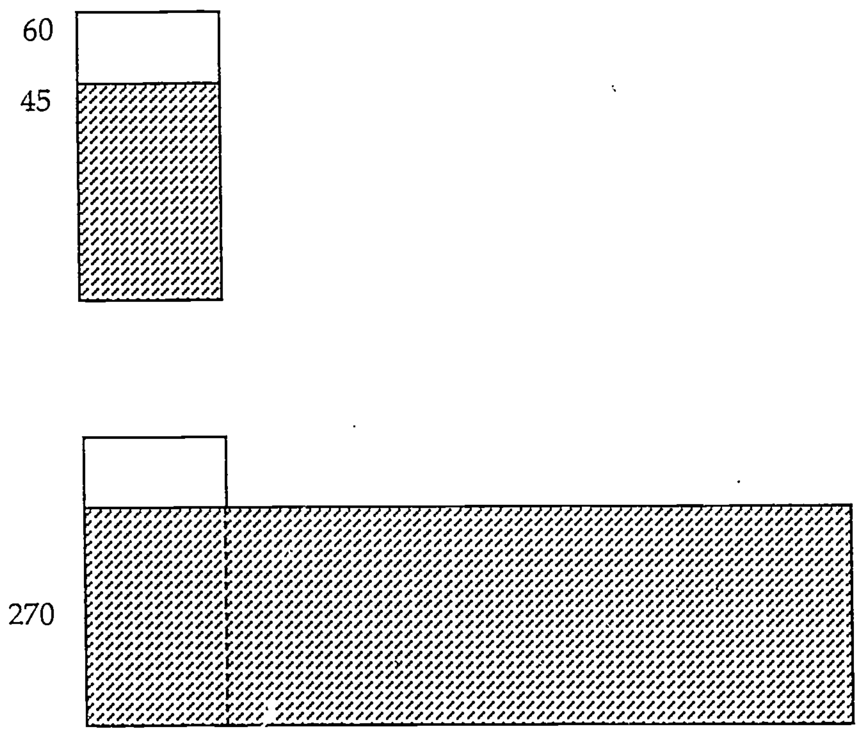

Figure 3.

One hose can fill 10 gallons in 1 minute.
Another hose can fill 20 gallons in 1 minute.
Will one fill 60 gallons faster?

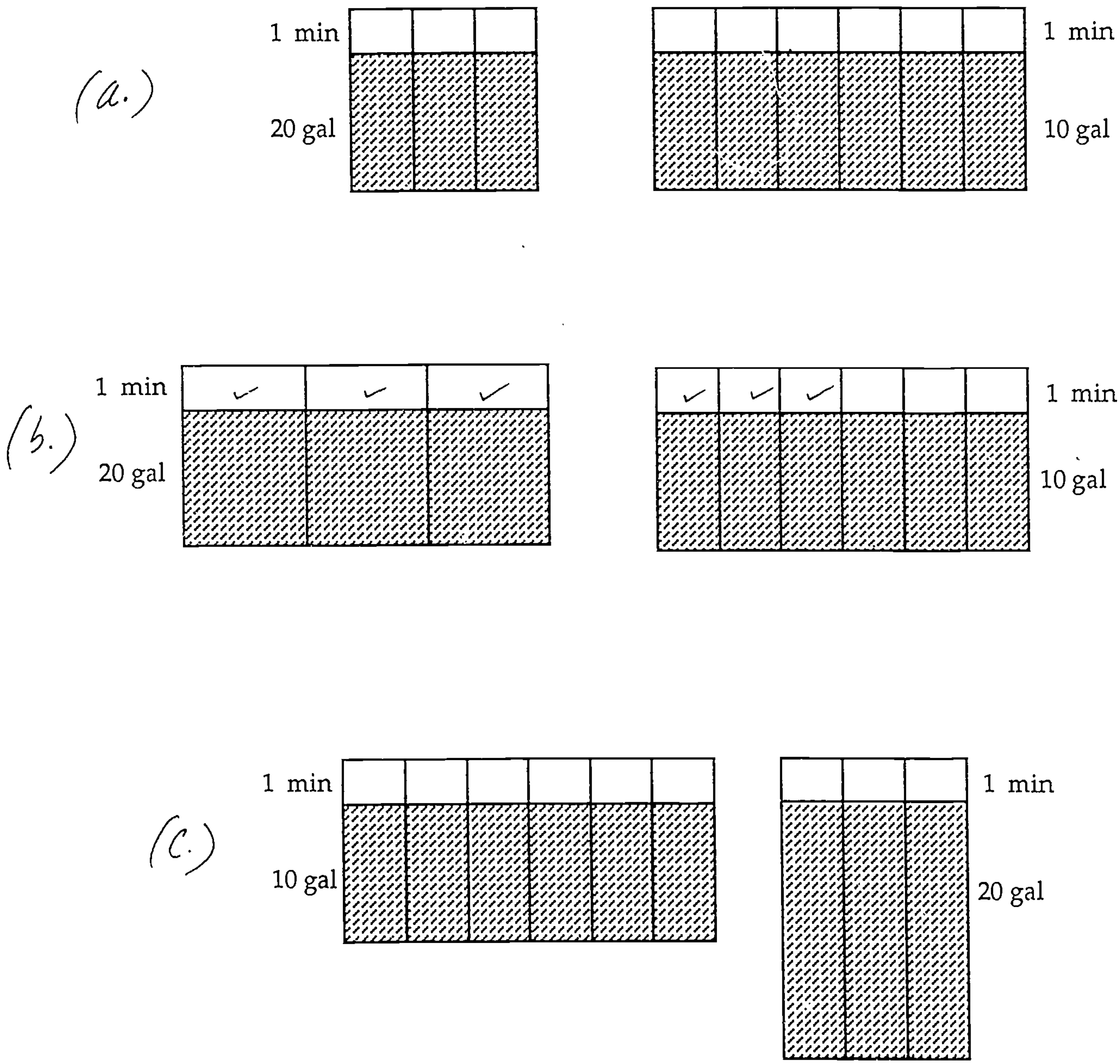

Figure 4.