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ABSTRACT

The reform movement in mathematics education calls for students to engage in problem solving, to discuss and communicate their thinking, and to develop meaningful understandings. This paper reports a case study of a fourth-grade class in line with the reform call, emphasizing the process of change the teacher experienced and the support she needed. The teacher in this study taught mathematics using the Cognitively Guided Instruction approach. Observations, interviews, and student assessments were collected. Reported are four phases of teacher change: (1) the teacher's teaching and thinking at the beginning of the study; (2) learning, thinking, and stimulating the process of change; (3) learning from and helping individuals; and (4) building on children's thinking in instruction. Contains 25 references. (MKR)

Toward Instructional Reform in the Math Classroom: A Teacher's Process of Change

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The reform movement in mathematics education calls for students to engage in problem solving, to discuss and communicate their thinking, and to develop meaningful understandings (NCTM, 1989, 1991). Teachers should act as facilitators who help students construct their own knowledge. Yet we have few images and descriptions of classrooms that meet this vision (Cobb, Wood, & Yackel, 1990; Hiebert, & Wearne, 1993; Philipp, Flores, Sowder, Schappelle, in press; Schifter & Fosnot, 1993) and "we do not understand enough about the challenge of trying to create such classrooms" (Ball, 1993).

This paper reports a case study of a class in line with the reform call, emphasizing the process of change the teacher experienced and the support she needed. We also relate the instruction in the class to the learning process.

CGI Framework and Teacher Change in CGI

The teacher in this study taught mathematics using the Cognitively Guided Instruction approach (CGI). See Carpenter & Fennema (1992) for a summary of research on CGI. CGI encourages teachers to use research-based knowledge about children's mathematical thinking to make decisions in their classrooms. Activities are not prescribed for the teachers; instead they are helped to acquire knowledge of the structure of word problems and how children solve them. Each teacher decides how to use the knowledge in the class. Children are expected to be engaged in problem solving situations that are meaningful for them, usually through word problems. The children usually solve problems by a variety of strategies and they talk about their solutions.

Studies in the CGI project which used a variety of methodologies, such as case studies and the study of change of a group of teachers, examined: a) The teachers' growing knowledge of the content and children's thinking. b) The use of children's thinking in the class, including: providing opportunities to engage in mathematics thinking, teachers' listening to children's thinking and using the children's thinking to make instructional decisions. c) Teachers' beliefs that children can solve appropriate mathematical problems without direct instruction from the teacher (Fennema, Franke, Carpenter, & Carey, in press; Franke, Fennema, Carpenter, & Ansell, 1992; Franke, in progress). These studies showed that teachers can become knowledgeable about children's thinking and change their beliefs and teaching to take into account the children's thinking. However, not all teachers changed in the same way and more knowledge is needed of the teachers' journeys toward changing their math instruction. This study deepens the knowledge of teacher change by giving a detailed account of the process the teacher went through, her conflicts and dilemmas, her teacher-research and the solutions she found.

Teacher Change

McLaughlin (1991) studied staff development programs in the Change Agent Study. She identified four factors as crucial for successful implementation and continuation of local planned change efforts and staff development activities: institutional motivation, project implementation, institutional leadership, and certain teacher characteristics. Project success was related to the extent to which the teachers were included in the decisions about project strategies. Ambitious projects were more likely to elicit teachers' excitement and active participation. Well conducted staff training and support helped the change process. Assistance from individuals skilled in project methods was indispensable as teachers attempted to put plans into classroom practice. The most powerful individual teacher attribute was the teacher's sense of efficacy - a belief that the teacher can help even the most difficult or unmotivated students.

Loucks-Horsley and Steigelbauer (1991) see the indivdual as the key player in the change process. His or her needs must be the focus of help and support designed to facilitate change. The change process is a highly personal experience and too often it is ignored by the people who are responsible for trying to promote change. The change process is also developmental and the individuals involved in change go through stages in their perceptions and feelings about the innovation, as well as their skill and sophistication in using the innovation. Therefore, well designed staff development should diagnose and address the teachers needs and concerns. This must be done dynamically as needs and concerns change over time.

Teacher Reflection

One of the goals of this study was to help the teacher become more reflective about children's thinking in her class via an ongoing dialogue with the researcher as they collaborated in understanding the childrens' thinking. Reflection and teacher-research can be key components for a teacher's development (Cochran-Smith, & Lytle, 1993; Tabachnick & Zeichner, 1991.) Reflection involves inquiry through making the teaching situation problematic (Tom, 1985), a step needed for teachers to seek new solutions. This study differs from studies that emphasized general reflection; here the teacher's initial reflection was focused on how children in the class thought about math problems.

Schon (1983, 1987) argued against separating the professional knowledge of the research community in universities from practice. He called for more appreciation of the knowledge of practitioners, knowledge-in-action. Practitioners deal well with uncertainty, instability, uniqueness and value conflict in situations where research doesn't provide guidelines about how to act. Schon talks of "reflection-in-action," reflection that occurs while the person is conducting the action, and "reflection-on-action," reflection on the situation that occurs before or after the action. Schon considers the reflection-in and -on practice of practitioners as legitimate ways of doing research:

> When someone reflects-in-action, he becomes a researcher in the practice context. He is not dependent on the categories

of established theory and technique, but constructs a new theory of the unique case... He does not keep means and ends separate, but defines them interactively as he frames a problematic situation. He does not separate thinking from doing, ratiocinating his way to a decision which he must later convert to action. Because his experimenting is a kind of action, implementation is built into his inquiry. (Schon, 1983, p. 68)

Schon studied reflection-in-action in different professions and thinks there are ways to increase the rigor of research done via reflection-in-action:

The dilemma of rigor or relevance may be dissolved if we can develop an epistemology of practice which places technical problem solving within the broader context of reflective inquiry, shows how reflection-in-action may be rigorous in its own right, and links the art of practice in uncertainty and uniqueness to the scientist's art of research. (Schon, 1983, p. 69)

We studied the teacher's reflection and its relationship to her teaching and learned from her actions.

Method

All the data collection was done by the first author who was a participant-observer.

Sample

The sample consisted of one fourth grade teacher, Ms. Statz, and her class of 21 students. This was the teacher's third year of teaching, all in the fourth grade, and she implemented CGI from the first year. The school has a diverse population of racial and socio-economical backgrounds. The children in the school came from two different socio-economic neighborhoods and some were bused to the school. Ten children in the class received free lunch in the school. Six of the children in the class were African-American and 2 were Asian-American with limited English. Two students were classified as learning disabled. Nine of the students were girls. A third of the class was new to the school and the rest had some CGI math instruction in third grade. (All the childrens' names reported in this paper have been changed.)

Procedures

Observations. Thirty full mathematics lessons were observed by the researcher 3 to 4 times a week over a 5 month period. The lessons were audiotaped and parts were transcribed. Field notes were taken on teacher-student interactions, students' solution processes, class organization, and the teacher's knowledge of and efforts to build on children's thinking. Nine children were randomly selected as target students and their solution processes were documented regularly by observing them and asking them how they solved the problems. The other children were observed on a rotating basis.

Teacher's meetings with rsearcher. All 13 meetings were audiotaped and transcribed. The meetings included interviews devoted to the teacher's background in becoming a "reformed" teacher, her knowledge about the children's thinking, her decision making processes regarding content and classroom organization and their relation to children's thinking. The teacher was also asked about the changes she was experiencing in her thinking and practices. The meetings also included discussions of the researcher's interviews with the children and class observations on specific children's solution strategies. The meetings were usually held once a week for 30-40 minutes.

Student assessments. Each child was interviewed at length on solution strategies for solving word problems in a variety of content areas at the beginning and the end of the study. The problems covered addition and subtraction wih multi-digit numbers, including Join Change Unknown problems, multiplication and division with basic facts numbers and with larger numbers (6x24, 42:3, 226:8 in 2nd interview), and fractions. The first interview the two Asian-American students, whose English was limited, wasdone with the help of a translator. Students kept journals of their math work which were examined regularly by the researcher.

Data analysis. The children's strategies in the interviews were coded and analyzed using procedures developed by Carpenter & Moser (1984). These data will not be reported explicitly in this paper An ongoing analysis was used to make observations and interview decisions. Themes consistent with CGI philosophy were marked (such as teacher's knowledge and beliefs, building on children's thinking, teacher-research and experimentation).

Results and Discussion

Ms. Statz reported that she changed her mathematics instruction much during the first year of her teaching, with help from the math resource person in the school, but for the second year and half of the third year she reported "a bit of increase but more of a plateau" (3/31 int. p. 14). The second half of the third year (during the study) was described by Ms. Statz as "a big jump." At the researcher's request, Ms. Statz drew a diagram of her change pattern (see figure 1). We describe how Ms. Statz "jumped" from being a good "reformed" teacher to an even more outstanding teacher whose increasing reflectiveness helped her acquire new knowledge and gain new insights into her teaching. Ms. Statz changed quite dramatically in her thinking and practices in the classroom during the few months of this study. We describe here four phases Ms. Statz went through during the study. Her reciprocal influences with her students will also be discussed and we will describe some of the interactions of the researcher with Ms. Statz and their role in triggering the changes in her thinking and teaching.

Ms. Statz's Background with CGI

In the first interview Ms. Statz (Dec 2) talked about how she became a CGI teacher. She was introduced to CGI ideas three years earlier, in her pre-service college education. She started using CGI from the first year of her teaching. The children's thinking in these classes was her most important motivation to start implementing CGI:

> When you see those kids, really really little kids... (first graders). Well, they blew us away. They wanted questions in the thousands and we were giving them questions in the tens. (Dec 2, int 1 p. 2.)

Ms. Statz's commitment to CGI ideas was also enhanced by her reflection on herself as a math problem solver as a result of her teacher education program:

During math (education methods) class, we were allowed to work together with partners. We were allowed to solve math any way that we wanted to. And it was the first time that I began to feel comfortable about math.

Thus Ms. Statz understood through her own experience the importance of each child/person developing individual solution processes in math that are meaningful for him or her.

Ms. Statz was open and whilling to try the new ideas of CGI, even though she felt some courage was needed for that: "I was ready to jump. I relate it to, jumping off a cliff. This is how I'm going to do it. And we'll see what happens." Ms. Statz tried to extend her knowledge of CGI in the earlier grades, even though less was known on children's thinking and on implementation of CGI in fourth grade.

<u>Support in the School Before the Study</u>

Ms. Statz felt she had support in the school from her first year of teaching which encouraged her to try to implement the new ideas she learned in college about CGI. The principal was supportive, as was the math coordinator, an expert herself in CGI teaching, all the fourth grade teachers were new teachers and most of them had been exposed to CGI. The teachers supported each other to try the new ideas, even though Ms. Statz was the only one who implemented CGI extensively. She gladly accepted help from the math coordinator and appreciated her support:

I think Ms. J. was in here more and I wanted her to come. She would say things like... 'Don't tell them that. Let them find it out.' Those kinds of things. (Dec 2 p. 4) She came back in regularly enough that she was able to carry me along, I think, that first year. (Dec 2 p. 6)

<u>General Classroom and Teacher Characteristics</u>

From the observations in the class it seems that Ms. Statz created a relaxed and pleasant atmosphere in her class. Her relationships with the children were warm and many times Ms.

Statz was seen walking in the hall hugging children from her class. She laughed a lot with the children and they liked to show her things and to talk to her. Together with the relaxed atmosphere, Ms. Statz created a learning environment in which she expected the children to work hard and to take responsibility for their learning. She was skillful in organizing the class in a variety of ways and the children were usually engaged in learning mathematics with few discipline problems.

Ms. Statz is very committed to work with a diverse student population. Her teacher education program placed special emphasis on promoting multi-cultural education and gave her a good background for working with a diverse population of children. The program also emphasized reflection and teacher-research.

Four Phases of Teacher Change

We will describe here the four phases that Ms. Statz went through in the five months of the study as she became more reflective and more knowledgeable about the students' thinking.

Ms. Statz went through an intensive period of growth in her understanding of children's thinking, in the decision making processes in relation to her classroom practices and her role as a teacher, and in her knowledge of the content area and CGI ideas. It is important to document this change, but also to try to understand the concerns of the teacher and her debates that inhibited or contributed to the change. We will try to examine what kind of support was helpful to the teacher.

Phase 1: Ms. Statz's teaching and thinking at the beginning of the study. At the beginning of the study (November 1992) Ms. Statz's class had many characteristics advocated for reform classrooms and by CGI philosophy. The students solved challenging word problems using a variety of strategies that they chose, the teacher gave the students opportunities to share and talk about their strategies with other children and with her, children recorded the ways they solved problems in writing in their journals and they invented their own story problems. Many times, after children solved problems individually or in centers, there were short discussions (5 to 10 minutes) and often four or five different solution strategies were presented by the children to one problem at the discussion time. Ms. Statz's beliefs were consistent with CGI ideas. She believed children need to construct their own knowledge. No textbook was being used.

When asked about her goals for the children, Ms. Statz mentioned many characteristics that are important for a CGI class and in the reform movement:

> Being able to write about math. And being able to verbalize what they're doing and thinking about math... Being able to feel comfortable enough about math to share what they're talking about. And to develop an appreciation for each other. That people solve math problems differently and that's okay. (Int. Dec. 4, p. 18).

When asked about what she would like her students to know from the problem solving point of view, she replied: "Probably that math is based on real life".

Ms. Statz often chose topics for word problems from the life of the classroom and the children. Sometimes the problems were related to a story the children read in class or to events in the children's lives (selling in the school store, holidays, number of names on a child's cast, figuring out how much pizza they need to order and how much would it cost, etc).

At this time Ms. Statz frequently had the children rotate among several math centers over a period of a few days. For example, on October 8th, the centers included : 1) Solving word problems. The teacher was with the children in this center. When the children solved individually, they told their strategies to the teacher and the other children. 2) "Secret symbols" center in which the children invent symbols for the numbers: 1, 10, 100. They created a number from the symbols and the other students tried to guess the number. 3) Write your own word problem - the children write the hardest word problems they can solve and solve it. 4) A number roll game. 5) Manipulative center - making the number 267 in twenty different ways using base 10 blocks.

Math lessons typically ended with a short discussion period in which children would report how they solved problems, read problems they wrote and showed how they solved them or discussed a strategy they developed to play a game.

Although Ms. Statz encouraged the children to discuss their math strategies, she seldom challenged a child to think of alternative solutions that might help him or her to progress. It seemed that there were a number of missed opprtunities to build on ideas that came up the whole-class discussion time. For example, on December 4, Dan showed the class in the disscussion time how he solved a word problem in which he needed to calculate 68 + 37. Dan drew 37 tallies and counted on one by one from 68, using the tallies to keep track. Ms. Statz then called the next child. She did not discuss the inefficient strategy or try to enhance his and other children's place value knowledge in ways that could have helped in solving this problem (such as adding the three tens and the seven ones).

Ms. Statz's belief in accepting and encouraging a variety of strategies from children was so strong that she percieved her role as passive regarding helping children progress. Ms. Statz reflected later on her work at that phase: "I would just accept what was put on the board and that was all 'good', that's fine. Go have a seat. Next person." (March 31, p. 18) Ms. Statz did not stop to ask herself at that time if the children's strategies were adaquate, if they are progressing well and if there is a way she can encourage them to move forward.

Ms. Statz gave the children in her class opportunities to solve problems in the ways they chose and allowed them to talk about their solutions. Nevertheless, there were many indications that Ms. Statz was not aware of how the children in her class solved problems and did not have a solid knowledge of the children's solution strategies or how she should react to some of the strategies they used. It is not clear why Ms. Statz did not learn more from the children's descriptions. Sometimes her theoretical knowledge of strategies the children used was not complete, which made it difficult for her to interpret what the children told her in a way that was helpful to her. That she perceived her role in helping the children progress as passive may also have played a role: she did not see the need to really understand their strategies, since she did not intend to use the knowledge for instruction. We will document some of her knowledge and lack of knowledge on children's strategies when we discuss the second phase. Ms. Statz was not aware of her limited knowledge and was not bothered by it in this phase.

We also learned about Ms. Statz's knowledge of the children's thinking at the beginning of the study (Dec. 16) by asking her to classify the children in her class into three or four categories according to the strategies they use and to talk about her classification. Her list matched the interview data, except for five children. Ms. Statz gave reasons for classifying the least sophisticated strategies by talking about "Direct Modeling" strategies, but she had a hard time articulating the criteria for classifying the other groups of children. Thus, she had a general idea of the children who used higher and lower strategies, but she couldn't refer to specific ways that they solve problems.

In the following example, we see that Ms. Statz's attempt to help a child did not relate to the child's understanding and strategy. Later, as the teacher's knowledge of children's thinking began to grow she was able to respond to children in ways that were more closely tailored to their strategies.
In an observation prior to the formal beginning of the study (Oct 8), Ms. Statz tried to help a child, David, who had difficultiy solving a Join Change Unknown problem (17 + ? = 33) with an additive strategy (Counting Up). His method of keeping track of how many numbers were added was to go back and forth between each number sequence (fingers were not used). David said: "first is 18, second is 19, third is 20" and so forth until he got to "ninth is 26." At this point he got confused and stopped. His strategy was appropriate to the additive structure of the problem but his keeping track method was very difficult and made great demands on working memory. The teacher's first attempt to help David was to encourage him to still use an additive strategy, but did not address hisdifficulties with the keeing track process. The teacher suggested adding 10 to 17 and asked if that will be enough. Before she pursued this strategy further, another child,

Tom, said he solved the problem by subtracting 17 from 33. Ms. Statz suggested to David that he use a similar strategy and solve the problem by separating 17 blocks from 33. So, although the child modeled the problem according to its additive structure, he was requested to solve it by subtraction without checking if he understood the relationships.

A few days later the researcher asked Ms. Statz what she had n thinking during this episode and if she thought that David understood the strategy she suggested to him. She responded that he probably did not and that she does not always know how to respond to a child on the spot and that it was also the end of the lesson and she needed to finish the interaction with the child. Ms. Statz also remarked, though, that if she had to help David then she would know better how to help him, since in the meantime she saw another child solving the same problem by counting up and keeping track by drawing tallies and counting them and she could have suggested to David to use this method. This method would have been helpful to him by directly addressing his difficulties with the keeping track methods and allowing him to use the additive structure that he saw for the problem. I asked her if she thought of David when she saw the "new" strategy and she responded that she had not, and she just thought about it when I asked her. The discussion stimulated Ms. Statz to reflect on her new observation and to connect it to how it could help David.

We see that when Ms. Statz began to be more familiar with a greater variety of children's strategies it was clearer to her how to help these children. In this example we saw that Ms. Statz learned on her own from the children. This kind of learning was enhanced very much by our discussions of the children's thinking. That was the beginning of much reflection and growth in the teacher's knowledge of the children's thinking that later followed.

In summary, we can classify the teacher in this phase according to the classification of teacher change by Fennema, Franke, & Capenter (1992) as being in their Level 2. Ms. Statz fits this classification by: a) believing that children can solve problems without instruction; b) knowing that she needs to provide the children problems to solve and giving them the opportunities to solve the problems and share their strategies, but not using what she hears in an integrated way.

Phase 2 - Learning, thinking and stimulating the process of change. In this phase, Ms. Statz became aware of how little she knew about the children's strategies. She also began to realize that many of the students' strategies were very basic or wrong and sometimes did not show a good understanding. This phase saw the beginning of reflection, in which Ms. Statz formulated the questions for teacher-inquiry that guided her in further phases. She thought about her knowledge of the children's thinking, the students' ways of solving and her beleifs of her role as a teacher. Little action took place in this phase to solve the dilemmas. Rather, this phase was distinguished by "reflection-on action" (Schon, 1987), as dilemmas coalesced and promoted questions from Ms. Statz.

The change from phase 1, in which the teacher was satisfied with her work in the class, to this phase, in which questions and dilemmas formed, was triggered by the researcher's interviews with the children. Ms. Statz was able to sit in on some of the interviews at the beginning of the study and the researcher discussed with her the strategies the children used throughout the interviews. She was very intrigued, and often surprised, by the ways the children solved the problems

For example, one child wrote down every single number between 398 and 500 to solve a Join Change Unknown problem. ("Robin has 398 dollars. How many more dollars does Robin have to save to have 500 dollars to buy a new bike?") Surprised by this strategy, Ms. Statz went to check the child's journal and discovered that the child was using similar strategies there as well:

> The way she solved this is kind of strange... Can I go see what she's got in her journal? ...Yeah, she's doing similar things. (Dec 2, int, p. 16)

At that time, Ms. Statz became particularly concerned about seven children who used a standard subtraction algorithm incorrectly and in a rote manner in the interviews. The children used "buggy" procedures (Brown, & Burton, 1978) in which they systematically subtracted the small digit from the larger one. For example, one child wrote 245-178=133. The child "couldn't take" 8 from 5 so he "took" 5 from 8. The researcher asked Ms. Statz if she had seen the children solve in this way before in the classroom. She replied: "That's something that surprised me during these interviews." (Dec 2, p. 10). It was very important for Ms. Statz that the children use strategies that are meaningful to them and that they always solve problems with understanding. This belief is important in the CGI framework and in the reform movement in mathematics. Thus she was taken aback to find out how the children solved the problems:

> The borrowing with regrouping really disturbs me now. That's all that I've been thinking about now that we've been interviewing the kids and we see that they don't have it and they don't, they're not even coming up with a good way of explaining it. It doesn't make sense to them. (Dec 2, int, p.10)

Ms. Statz's increasing knowledge of the children's thinking sharpened her awareness of the diversity of the class. She had a strong belief, developed through her teacher education program, against grouping children by ability or even by type of errors. She was thus confronted with a challenge of accomodating a wide range of children without resorting to grouping.

Ms. Statz also began to reflect on her teaching. She began to reexamine her belief that the teacher should just accept whatever strategies the children choose to solve problems. She recognized the need to be more active and to help children progress, especially those who used incorrect strategies or still counted by ones to solve problems in the hundreds. She started to think about how she could help children without actually telling them how to solve problems. She thought about how to help with specific concepts, especially for the children who used the subtraction algorithm incorrectly. She talked with the researcher about the fact that the children do not connect their algorithms to their knowledge of working with manipulatives (base ten blocks). Ms. Statz struggled to find ways of teaching that are congruent with CGI ideas and ideas of constructivism. She believed in letting the children construct their knowledge. Now, for the first time, she was debating if she should explicitly connect the algorithms to the manipulatives to make sure all the children understand them:

> I was even struggling with the idea of getting the overhead projector and doing it for the whole class. Here are the base ten blocks, here is my marker and this is what we are doing. But maybe we should try it, let them construct it themselves first? (Int. Dec. 4, p. 11).
>
> I am struggling with how to go about doing that. Give them lots of take away problems and try to go around to each person individually? That is the hard part. That's what I'm trying to figure out, how to do that. (Int. Dec. 4, p. 13)

The researcher and Ms. Statz talked about alternative algorithms and children's invented procedures. We disscussed the reasons for using or not using standard algorithms. The researcher suggested that children can discover connections between working with the manipulatives and ways to write down their steps (invented algorithms) and that the teacher can facilitate this by asking children to write down in numbers what they did with the blocks. She also showed Ms. Statz how you can explicitely connect the standard addition and subtraction algorithm to the steps with the manipulatives step by step (Fuson 1990). Ms. Statz chose the more structured way out of her deep concern that the children in that time of the year in the fourth grade should have a good understanding of addition and subtraction. Ms. Statz also wanted to move on to other content.

Ms. Statz discussions with the researcher about the children's strategies also led her to think of trying to give children more challenging problems and to move towards multiplication and division problems with large numbers.

In summary, in this phase Ms. Statz became aware of her lack of knowledge of children's strategies. As she learned more about what they did, many questions and dilemmas arose regarding her teaching. This phase was an intense period, characterized by much reflection-on-action and thinking. Ms. Statz reported that she tought about the issues "even in the shower." She was faced with conflicts, since she raised doubts about her knowledge, the children's knowledge and her ways of teaching, but as yet she saw no solutions. Ms. Statz developed a strong desire to learn more about the children's thinking. With all its' intensity, this phase was very short, about two weeks (Dec. 2 to Dec. 18).

Phase 3 - Learning from and helping individuals. Ms. Statz felt a need to spend much more time with individual children to assess their thinking and to try to help them progress with their solution strategies. She began to spend time working with individual children at their desks. Previously, these sessions had usually lasted no more than a minute - Now they often ran for more than 10 minutes to a child or a pair of children:

> What I am noticing in these last couple of weeks is that I am spending less time with all kids and more time with particular kids. (Jan 6, int, p. 4)

Spending more time with individual children and thinking about ways to help the children progress in their thinking were very fruitful for Ms. Statz. She started hearing what the children said. Her growing understanding and knowledge of what the children did also helped her to better match the instruction to their needs and to think more of specific content areas in relation to the children's thinking. She helped the children individually:

> I definitely think I am noticing more than I noticed before. And not only just noticing it but knowing where to take it and how to push further and how to question more. (Int. Jan 6, p. 4) It seems the times that I am working on it (on subtraction with borrowing) have been real good. (Int. Jan 6, p. 3) I have been thinking about reaching the kids that have the 'bugs.' I have been thinking about pushing the kids that need more higher level thinking. (Jan 6, int, p.6)

Ms. Statz stopped the use of centers almost completely since she wanted to work with children whom she felt needed more attention from her on a particular day or week:

I feel I need more time with each kid... I see that they have more needs that I need to be working with them one on one and I don't feel I am getting enough time with them when they are all around the room four days a week, and then one day with me (Jan 6, p. 9)

Another reason for not using the centers was Ms. Statz's desire to give the children more challenging problems. She felt that if she designed activities for centers they needed to be easier so the children could work independently.

After a few we s of intensive learning about childrens' thinking she was asked if she felt confident that she knew what the children were doing and if she understood their strategies:

I would say that I pretty much could pinpoint something about each person and say where they are and how they would go about solving a problem. (Feb. 10, int. p. 9)

The work with individual children was very important and beneficial for Ms. Statz and the children. However, Ms. Statz became frustrated that she did not have enough time to do this kind of in-depth work with all the children:

I think I'm spending more quality time with them too. But sometimes I feel like I'm spending too much time and neglecting the rest of the classroom. I've felt like that, more frustrated almost, this year because I've needed more time with each kid. (Feb 10, Int. P. 5)

Ms. Statz was concerned that she would not be able to work intensively with all children. She was also concerned that the children she was not working with would need her help and would be less engaged in their work and that discipline problems would arise. (The researcher's observervation was that the children continued to work well while Ms. Statz was working with individuals, but were somewhat less engaged.) Thus, Ms. Statz started thinking of new ways to organize her class that would enable her to spend much time with individuals but ensure that all children were working and progressing.

Ms. Statz's desire to work closely with each individual student triggered new cycles of reflection. she raised questions about how to organize the class so that she could spend time with individuals but also ensure that all the children were working progressing. She also began to think about new ways to organize the class that would help her learn from individual children.

The researcher suggested to Ms. Statz that the children work in pairs so they could discuss strategies and help each other while she worked with individual children. She was worried that some children would not understand their partner's strategies and that she would have a hard time working with some individuals if the other children in their pairs were working at a different level. (These concerns came up in the context of children working closely together in a group, even though the children were reporting their thinking regularly at the whole class sharing time.) Since the researcher's suggestion did not address Ms. Statz's concerns, she did not accept it and continued to look for a solution. Ms. Statz thought intensively about ways to reorganize her class and she started to examine many of her practices. That was rather stressful for her:

> I think I am more frustrated now about teaching math. I think math is harder for me. I am more exhausted at the end of the math time, now, because I am then spending more time thinking about it. Although the things that I think are coming out of it are good, I am seeing what needs to be done. I am spending more time with kids who need specific things. I think it is making a difference...Now there are all these other questions that are on top of it. (Jan 6 p. 22).

Two days after our discussion (Jan 8) Ms. Statz came up with an ingenious solution that took into account her concerns. She asked the children to "choose partners who solve problems in a similar way to you." Each pair got a sheet with the problems and a space for two strategies. Each child could come up with his or her strategy or the pair could come up with two strategies together. The results of this experiment were very pleasing to Ms. Statz. The children were fairly accurate in choosing partners who were solving problems in a similar way. They helped each other, discussed their strategies and taught each other different strategies. Ms. Statz had time to work with individuals while the other children were working nicely with their partners. Having ten pairs to get to instead of 20 individuals made the class more manageable for Ms. Statz. Some children helped resolve their partners' "bugs" by challenging them that their strategy was incorrect or didn't make sense. A few children gave meaningful explanations to their partners why their strategy was incorrect or why the standard algorithm is correct. The children continued to use a variety of strategies, as well as the addition and subtraction algorithms that were on Ms. Statz's agenda at that time. Some of the children's interactions were shared with the rest of the class.

Ms. Statz reacted to the working in pairs:

> When they're working with a partner. Then if I can't come to them, they are able to get through it better...Some of the talk (in the whole class discussion) was done already in the group. (Jan 20, int, p. 3)

Ms. Statz started to see solutions and to feel better about her experimentation.

> My whole week was fine. Math was especially good. I felt good about trying different things. I felt good about the partners. I thought it was a good way to reach more kids. I like to see them working together more. They're talking with each other more about how they're solving their math problems when they're working. (Jan 20, int. p. 1)

Ms. Statz asked the children to work with partners a few more times, choosing new partners each time. The exposure to different partners enabled the students to see a variety of strategies, even though they tried to choose partners who "solve simil `y." The success of these sessions gave Ms. Statz more confidence about the benefit of working in pairs. She was no longer concerned about the children's ability to understand each other's strategies and suggested that they work with <u>partners "who solve problems in different ways than you"</u>.

To increase the likelihood the children would cooperate and discuss their strategies in the pair and to help them progress to more sophisticated strategies, Ms. Statz told the children that she would call both children from the pair to the board during the discussion time and ask each one to explain his or her partner's strategy. It seems that the children were engaged more and discussed their strategies more in this way. Most children were able to explain the other person's strategy. When a child had difficulties, the partner explained the strategy. More children were called to the board in this way during the discussion time and that helped to solve another issue that concerned Ms. Statz: giving opportunities to more children to share strategies.

An example of how the children helped each other in pairs and how Ms. Statz interacted with them can be seen in the folowing example from Jan. 8. Two boys worked on a problem in which they needed to solve 315-188. They both used incorrect written solutions ("bugs"), even though they were both very flexible in their ability to manipulate and calculate these numbers mentally using invented procedures. Tom traded a 10 to the ones column correctly, but then instead of taking 8 from 0 tens he took 0 from 8, getting 287 for an answer. Bob wrote 188-315 vertically and got the answer 205. (Not clear what he did). Bob imediately recognized that the solutions were incorrect. He started estimating, saying that 315-100 is 215 and they still need to take away more. They decided to check their work with base-10 blocks. Ms. Statz joined them then. They traded well (starting from hundreds) and got the correct answer: 127. Ms. Statz asked: "Bob, what does that have to do with what you did on paper? Tom, you too." Tom corrected his difficulty in writing by crossing the 3 hundreds to 2 hundreds and 10 tens. He then was not sure how to continue. Ms. Statz asked Tom to explain to Bob "because Bob is confused." Bob resorted to his sure mental ways. He subtracted a 100, and got 215. He then took away 80 by subtracting 20 to get 195 and 60 more to get 135. He then took away 8 more and got 127. Ms. Statz left them at this point, being aware (as she told the researcher later) that the boys still didn't know to connect their mental knowledge with the written algorithm well. She did not have enough time to return to help them with this difficulty. Several days later, Tom was paired with Mary and she gave him a very meaningful explanation why the standard algorithm works. That day he showed how he used the standard algorithm in the whole class discussion and was able to explain it meaningfully. Mrs. Statz was surprised and asked him how he figured it out. Tom answered that Mary taught him.

Choosing problems. Ms. Statz started thinking of what problems she should choose to help children progress:

> I wouldn't say it is any easier (to come up with the problems for the children). In fact, maybe more challenging to decide what type of problems to use...I think I am thinking more of the problems. (Jan 6, int. p. 10)
>
> The problems that I am giving them and the kids who are doing the problems that I am writing specifically for them. (Jan 6, int. p.12)

In summary, Ms. Statz learned much from the children about their solution strategies and their difficulties and how to help them by working with them individually. The new need to spend much time with individuals stimulated her to think of how to change the class organization. She began to experiment with the organization, an activity that continued in the fourth phase. New questions and dilemmas were formed and she sought for solutions. Ms. Statz reflected much about her thinking and teaching. This reflection was triggered by concentrating on the children's thinking. Much Reflection-in-action while working with children and reflection-on-action occured.

This phase fits Fennema, Franke, & Capenter's level 3 classification (1992) of teacher change. In this level Ms. Statz a) She started using her new knowledge gained from listening to the students in instruction. b) the lesson was still driven by the content and global notions about children's thinking, and the individual children's thinking were just starting to be taken in account in designing the lesson.

Phase 4 - Building on children's thinking in instruction. In this phase Ms. Statz began examining all her teaching practices more deeply. The struggles turned into experimentation and solutions and Ms. Statz started using the knowledge she gained from working with the children individually to guide her whole class time in math. One issue that concerned Ms. Statz was to improve the whole class discussion. Ms. Statz was concerned that the children were not attentive in the whole class discussion. She wanted to increase the number of children who shared their strategies on the board, yet she felt that the sharing time was not going well. The researcher hypothesized that the children did not comprehend some of the strategies that the other children were sharing, and that maybe the strategies were presented too quickly and with too little discussion (Jan 5). She suggested that the children or the teacher repeat strategies of the children who report, explain them a few times, that other children ask the presenter questions, and that the teacher start writing the strategies on the board. The researcher also suggested that children try to solve the problem in the same way as the child who was presenting. Ms. Statz was concerned about children who would not be able to model the more sophisticated strategies. She thought about how to guarantee that more children would understand the strategies presented and found ways to improve the discussion. She started to get the students more actively involved in the discussions: she would typically stop the child who reported and ask the class or specific children what they think the child's next step would be. Ms. Statz asked many other questions such as "can you tell what she did?" "How is her strategy different than somebody else's?" "How can we make this strategy easier? clearer?" The children were much more attentive during the discussions and Ms. Statz increased the discussion time considerably, from 5-10 minutes typically up to two consecutive lessons of 45 minutes of discussion on solutions the students found to one set of problems. We measured how long the discussion time lasted in the observations. We divided the period into two parts, the first one from October 8 to January 19 (16 observations) and the second from January 21, when the discussion started getting longer, to the end of the study (18 observations). The average number of minutes in the first period was 7.9 and in the second period 21.8. Discussion times were significantly longer in the latter period ($P=0.0023$). We used the Mann-Whitney test (Hayes, 1973) due to the presence of a small number of unusually long discussions.

Ms. Statz also decided to begin the whole class discussion at the start of some lessons, when the children are more alert. This emphasis stressed to the children that the discussion is important and valued.

Ms. Statz started to use the whole-class discussions to help individual children and the class as a whole to progress in their strategies. This marked a change in goals and was another solution she found to build on individual children's solution strategies besides working with them individually:

> I guess I never used the sharing strategies as a time to move the kids along. That was just a time for the kids to be able to talk about their strategies... And maybe that's why they're being more focused on it (now). Because I'm including more of them in the discussion. (Feb 10, int, p. 1) I have given the kids more time to discuss their strategies. And I have used the information that I am getting from their strategies to move other kids. In the past I would just have Kanisha show the class that problem and that would be all but not use it as a teaching moment, to teach the rest of the class about renaming fractions or whatever it was. (March 31, int. p. 7).

Ms. Statz remembered individual children's strategies or difficulties she discovered working with the children individually that she addressed in the whole-class discussion.

The shift to help individual children in the whole-class discussion started about the middle of January. Especially striking was the remarkably different approach Ms. Statz used to help individual children in the whole-class discussion on January 21. One day earlier the researcher pointed out to Ms. Statz that there were still a number of children who solved large number word problems with hundreds by counting by 1's. We brainstormed possible ways to help these children develop strategies that would be based more on using tens and hundreds and develop a better understanding of place value. The next day Ms. Statz opened the lesson with 55 minutes of class discussion of the problems the children solved in pairs the previous day. She called on many of the children about whom we had spoken the previous day. We will describe the whole-class discussion Ms. Statz led for the folowing problem: "Ellen has 287 books. How many more books would she need to have 400 books?" First she called on Anne, who typically would solve a problem like that by writing down all the numbers between 287 and 400 and counting them by 1's. Anne began to solve the problem (with help of her partner) by adding 200 to 287 to get 487. The teacher stopped her and asked the class what was the problem Anne was facing at that point. The children said she had 87 too many. Ms. Statz asked a few children what they would do to continue and then asked Anne if that was what she did. She helped Anne and the rest of the class to do the calculation 200-87, which was needed for the next step, by counting down by 10's: "What is 200-10?" The teacher counted down together with the children: "190, 180, 170..." and she used her fingers to count how many 10's they were counting. To help Anne to take away the last 7, she asked her what is 120-5, and then took away 2 more.

Then Ms. Statz called on another child. Jared was hesitant to share his strategy with the class and he said "it will take me years." He drew tally marks and wrote next to each single number: 288, 289, 290, 291,.. The teacher stopped him when he got to 310 (in his notebook he drew tallies all the way to 400) and asked him what he needed to get from 300 to 400. Jared wasn't able to respond 100, but said 10 10's and also 2 50's. Other children suggested one 100 and Jared was able to add the 100 to the 13 tallies he made from 288 to 300. (At this point Jared said that the strategy is really easy.)

A third child started showing his solution of 287 + 100 (written vertically). The teacher stopped him and asked Anne again what is 287 plus 100. Then she asked a few of the children we had discussed the previous day a series of problems in which 100 is added (387+100, 487+100... 987+100). The child who solved the problem showed the next step: 387+10. Again, Ms. Statz returned to Jared and asked him to solve the problem. When he had difficulty she asked another child from the group she was trying to help that day. Similarly the next steps were done (397+3 and 100+10+3).

The fourth child who came to the board solved this problem by a subtraction algorithm (400 - 287 written vertically). That child and other children explained why he used the particular steps in a meaningful way (can't take 7 from 0, have no tens in the tens column to borrow from, need to borrow one hundred from the 400, that's 10 10's...). Another child, Mary, explained that she looked at 400 as 40 tens; if you take 1 ten, 39 tens are left, so she just writes 39 tens and a 10 on top.

In this example we see that Ms. Statz used the whole-class discussion to help specific children construct place value ideas and more advanced strategies. She had an agenda on that day to help specific children, building on her knowledge of their strategies. In this episode the children presented four different solution strategies, there was a discussion of a subtraction algorithm, and the children used counting by 10's and 100's and a few more ideas. Since then Ms. Statz continued to use the whole-class discussion to help individual children on a regular basis.

The building of children's strategies in the whole-class discussion didn't occur before. For example, in the eight lessons observed prior to January 21 Ms. Statz rarely attempted to build on children's thinking in the whole-class discussion.

Ms. Statz started reading more about children's strategies and word problem structure (from the CGI readings). She gave the children more challenging problems in multiplication and division with large numbers and encouraged many invented strategies. Some of the invented strategies children used in division can be seen in Steinberg, Carpenter, & Fennema (1994). The standard multiplication algorithm was not taught, but Ms. Statz encouraged children to use multiplication ideas and to break numbers into hundreds, tens and ones which helped children develop multiplication strategies.

The following example from February 18 shows some of the strategies the children were using with large numbers in multiplication and how Ms. Statz built on the strategies. On that day we were videotaping the class, which caused some excitement for the teacher and children. Ms. Statz chose a problem that was related to customs the children were making for a large school production about different cultures. The example will show the discussion to the following problem the children solved: "Mrs. Smith (one of the mothers) bought 13 bags of pom-poms for our costumes. Each bag contained 38 pom-pams. How many pom-poms did she buy?" That was the first time that a 2 digit by 2 digit multiplication problem was introduced in class. The children had solved 2 and 3 digit by one digit multiplication problems before. The teacher called the children to the board in pairs and each one had to explain their partner's strategy.

Ms. Statz first called Mary and Donna to the board. Mary solved 13X38 by using the standard algorithm that she had learned outside of class. (At other times when she or another child used the standard multiplication algorithm, the teacher made them explain why the algorithm works. After thinking about it a few times, they gave meaningful explanations of the mechanics of the algorithm). Mary explained then how Donna solved the problem. Donna wrote 38 vertically 13 times and added them. Mary: "She put down 13 38's." Ms. Statz asked the class: "Does that look familiar to anybody? How many of you did that?" (Many kids raised their hands.) Mary said (referring to the ones column): "she asked me how much is 13X8 (she added the ones column). I wrote a little problem and solved it and got 104." (Mary solved it using the standard algorithm. Donna used Mary here as her "calculator." She needed to calculate 13X8 and 13X3 (for the tens column) and didn't want to do it in her tedious way.) Donna then explained how she added all the numbers. Ms. Statz asked Donna: "if Mary wasn't you partner, how would you figure it out?" Donna showed how she used "touch points" to count on and use the dots "drawn" around the number as a keeping track device. She started counting the 8's, first by using patterns: 8, 16. Then she touched back and forth around each 8, with her movements "drawing" 8 dots (4 on each side of the 8), and said the numbers 17, 18, 19, etc. (On the right side of the 8: 17 down, 18 up, 19 down, 20 up. Then on the left side: 21 down, 22 up, 23 down, 24 up.) She did this counting very fast, moving from 8 to 8. Ms. Statz stopped her and asked: "do you think you would have come to the same number?" Donna said, "yes".

Ms. Statz (who will be marked as T for teacher in this discussion) called Mark and Ron to the board. She said: let's try something new. It is a lot of numbers to write. How else can we do it?" Ron explained Mark's strategy: "Mark wrote 38X13. He had a problem with that so he said 30X13=390. (Mark writes that.) Teacher asked him how he got 390. Mark explained: "I wasn't sure what is 30X13 so I did 30 13 times (and added it). Ron: "He added all the 8's 13 times and got 104. He added the 104 and 390 and got 494." (Mark wrote it on the board.) Ms. Statz thanked them and asked them to sit down. She said: "Mark and Ron, actually I think it was Mark who decided to break up just the 38. That was good, except you still need to multiply each number by 13 and that is a lot to multiply. Can you break that 13 the same way that he broke the 38? (asking the whole class). Mark, how can you? (he doesn't answer). 13 you break up to...?" A child answered 3 and 10. T: "O.K. lets' try it: 30X10. Class, is that an easy one to do?" Child: 300. T: "We know that because why?" Child: "When you times by 10, you add a 0." T: "O.K. we figured that out after many times we have done it. What else do we need to multiply by 30? If we are breaking up the 13, what is the next thing we need to multiply by 30?" Kanisha: "3." T: "Do we know what is 30X3?" Class: 90. T: "Nice. 30, 60, 90. What else do we need to multiply everything by?" Child: "8." T: "Yes. We haven't done anything with the 8 yet. We need to multiply the 8 by 10 and 3. 8X10?" Class: 80. T: "What is left for us to do?" Class: 8X3=24. T: "Let's add up. Do you think we will come up with the same number? (that the other children got)." (Teacher adds using the standard algorithm in addition.) T: "Not only did I break up the 38 into 30 and 8, I also broke the 13 into 10 and 3. You have more steps to add then, but you don't have to write it 13 times. It is a good way to do that. Double-digit multiplication." One child said: "You can do it with 3-digit." T: "Exactly, you can do it with triple-digit, you can keep going."

Danny wanted to show another way. He added 3 38's and got 114. Danny: "Then I times it by 2." T: "How come?" Danny: "It is quicker." He solved it by the standard algorithm. T: "So how many 38's do you have so far? He answered: "6". T to class: "How many 38's does he need?" Class: "13." Danny writes down how many 38's he has so far. Danny: "I timed it by 2 again and got 456." T: "If he times it by 2 again, how many times has he used so far?" Class: 12. T: "What does he need to do now?" Class: "Add 1 more 38" (which he added by a standard addition algorithm).

In this example, we saw four solution strategies to the problem. Ms. Statz tried to build on a strategy developed by a child and helped the class see how they can extend the child's strategy. At that time Ms. Statz tried to encourage the children to use multiplication procedures and to move away from the repeated addition strategies. It is interesting to note that Mark started to break up numbers for hundreds, tens and ones for 3 by 1 digit multiplication problems on Deceber 14, when Ms. Statz helped him. They than shared this idea with the group, who had difficulty understanding how to break up the numbers. Ms. Statz helped them break up a few numbers. Even though Mark was using this strategy for quite some time, it did not spread much to other children. It could be that it didn't spread because Ms. Statz didn't make a point to stress Mark's strategy for other children before. Another possibility is that many of the children still did not remember number facts and could not easily multiply hundreds and tens such as 40X30, 40X3, etc. In one of our discussions, we talked about the importance of developing these skills as a benchmark for moving on to multiplication strategies, but Ms. Statz did not choose this topic as a high priority and didn't deal with it much.

The children learned the new concepts (multiplication and division with large numbers, and fractions) with much understanding and they used many invented strategies. The fact the the children did not have misconceptions about the concepts when they started learning, like they had in subtraction, made the learning in the CGI spirit much easier.

Ms. Statz changed the way she taught fractions and adopted CGI ideas about how to introduce them, although they were developed with smaller children in mind (Baker, Carpenter, Fennema, & Franke, 1992). She started by giving partitive and measurment division problems that have fractions as the answers, without formal teaching. When the children represented the fractions in their drawings, she taught them their names and symbols. The children also got problems in which they needed to add or multiply fractions. Although they had no formal instruction, all students were very successful in solving the problems and in drawing representations to model them. The researcher and Ms. Statz discussed the children's strategies and the difficulty level of the problems.

The teacher's growing understanding of the children's thinking influenced her decisions about what content and what word problems to choose for the class. She started to think about specific children who could benefit from working on particular problems. Sometimes she chose problems that would help her diagnose how certain children solve problems:

> I'm looking at what the kids are doing and I think I'm changing the lesson because of what I'm seeing the kids do more. I think that's forcing me to change the problems. I'm spending more time on certain problems, like borrowing problems, because there was a definite need there. (March 10, p. 2)

Ms. Statz started building on the children's strategies in the whole class disscussion in new ways. Sometimes the class was able to suggest new ideas in an integrated way that was greater than the sum of each child's idea. Ms. Statz told the researcher with much excitement, the following example from her teaching on March 30. On that day the children wrote and solved their own word problems. Kanisha wrote a problem similar to the kind of problems they solved in the class at that time: "There were 20 cakes and there were 7 kids. How much cake will each kid get?" Kanisha gave each child 2 whole cakes and then started giving half cakes. She wrote 1-7 in each half (See figure 2). Then she gave each child fourths and wrote the numbers 1-7 in each. A half and a fourth were left in the last circle. She forgot the fourth and divided the half into 7 pieces (had hard time dividing them equally) and again wrote 1-7 on each. At this point Ms. Statz helped her individually to call each part by its fractional name. She reminded her to divide the fourth and Kanisha did that too. Ms. Statz helped her see that since the half was divided into 7 pieces, each piece was 1/14: "We said, if there are 7 slices in one-half, how many pieces will be in a whole cake?" Kanisha knew it was 14 and similarly she found that each of the 7 pieces in the quarter was 1/28. So Kanisha gave as an answer for her problem: 2 + 1/2 + 1/4 + 1/14 + 1/28 (These kind of strategies are not common in standard teaching of fractions.)

Ms. Statz accepted the interesting strategy and was very excited by Kanisha's achievement: "The fact that Kanisha did that was fabulous. I thought it was really good. But then what came out of it (in the whole class disscussion)... was really cool." (March 31 p. 6) In the whole class discussion the teacher asked Kanisha to share her work and then she asked the children what the result will be in one fractional name. Ms. Statz helped the children a little by asking them how many twenty-eights are in one-fourteenth. They continued to find out how many twenty-eighths were in a fourth and a half and they found the answer was 2 and 24/28. Ms. Statz coudn't rest for long on her achievement of building on the child's strategy. Another girl said she saw an easier way to solve the problem. She divided the 6 cakes that were left into 7 pieces each and got 2 and 6/7. Ms. Statz asked the children how they could know if the two numbers were the same. Ms. Statz had a hard time seeing on the spot how to help the children answer this question meaningfully and she resorted to formal techniques the child who brought the question learned on her own. She asked the girl "what numbers can go into both of these (24 and 28)?" and they found out it is four and found that 24/28 is the same as 6/7.

From this example we can see that Ms. Statz's whole-class discussions went a very long way from just having children present their solutions without any disscussion. She helped Kanisha complete her strategy and did not try to direct her to other methods, but she also used the problem and the strategies the children brought as a direct link to new challenging problems. The class found new ways of solving that went beyond each child's effort. Ms. Statz reacted to and built on the children's strategies spontaneously.

Ms. Statz researched more questions that we will just mention here. She asked herself if she was equitable enough in calling on children from different thinking levels and backgrounds. She debated if she should move into new content areas like fractions, or continue to work simultaneously on old topicss that still gave the children trouble (she did the latter). She examined her ways of teaching certain topics, especially fractions. She debated if she should drill her students to memorize their number facts in multiplication. She experimented with cooperative groups of four children solving challenging problems as a group (sometimes dividing work among them). She also tried to see if giving one problem at a time and discussing it was a better model to presenting a few problems and discussing at the end. She was worried how to help the children with the transition to fifth grade in which they will probably study math in a more traditional class.

In summary, in this phase Ms. Statz's knowledge of children's thinking continued to grow. She found ways to build on the children's strategies in whole-class discussions and not just individually. She continued to reflect much about the children's progress and her growing understanding of it. She continued to experiment and do teacher-research with the class organization and the content and problems she chose to improve the processes in the class. In this phase the change of the whole-class discussions was especially striking.

This phase fits Fennema, Franke, & Capenter's level 4 classification (1992) of teacher change. Ms. Statz a) believed that what she learned from children who talked about their thinking would inform her decision making both in interacting with the students and in designing the curriculum. b) listened to the children and adapted her instruction to what she learned. Instruction was driven by the teacher's knowledge about individuals.

Ms. Statz's change seemed to be reflected in children's solution strategies (as seen from the interviews with the children at the end of the study, not reported here, and from the solutions children used in class.) Some children used more creative strategies in the new topics like multiplication and fractions, than in the addition and subtraction, in which some children had already misconceptions they needed to overcome.

Discussion

The most interesting aspect of this study is the dramatic change in Ms. Statz's thinking and practices in a period of only a few months. The extent of the change is especially striking given the fact that, at the beginning of the study, Ms. Statz already used many of the reform movement ideas in her math teaching and believed that children should construct their own knowledge. The primary driving force behind the process of change was Ms. Statz's increasing knowledge of children's thinking. As she began to learn more about how the children solved problems, Ms. Statz posed many questions. She started to conduct her own investigations (teacher-research) into building on children's thinking, choosing tasks, thinking of the math content, and organizing the classroom. When surprising information did not match the way she perceived her class and what the children had learned, conflicts and dilemmas emerged that needed to be resolved. Ms. Statz engaged in cycles of reflection and research. When some questions were answered, new questions arose.

Ms. Statz's focus on understanding children's solution strategies triggered questions regarding class organization and content. Initially these questions focused on how to maximize the teacher's opportunities to listen to children and to help them individually. Later, as she gained knowledge about children's thinking, she researched other ways to build on their thinking in whole-class settings. Thus, learning about children's thinking could not be separated from creating the class learning enviroment and social-norms that support it, such as working cooperatively, learning from other children, and not having the teacher as the only arbitrator of knowledge (Cobb et al., 1990).

Teacher change is very difficult to achieve even when well planned staff development programs have been implemented (Richardson, 1990). This study shows that focusing the teachers' reflection on children's thinking can be very meaningful in the change process. We also think it is important that the teacher's learning was in the context of her own class, in accord with the findings of Cobb, Wood, and Yackel (1990) and Shifter and Fosnot (1993).

A puzzling finding is why Ms. Statz was not learning much from the children at the beginning of the study, even though she gave them opportunities to solve problems any way they wished and to talk about their strategies. It seems that the change in Ms. Statz's perception of her role as a teacher, from passive to active, gave her motivation to really learn what the children were doing so that she could use that knowledge to help them advance. She realized that, even as a constructivist teacher, she could have goals that called for the children to progress. The goals were tailored to each child's knowledge and ways of solving, so that they still allowed children to construct their knowledge.

The researcher-teacher collaboration facilitated Ms. Statz's first efforts of inquiry. Later in the study, she was able to raise questions and research them on her own. The researcher-participant, consistent with the CGI approach for teacher development, did not give Ms. Statz ready activities, but perceieved her as a professional who makes decisions. One of the most important professional practices, according to Schon (1983), is the ability to find the problems to work on. Schon argued that this ability should be developed by practitioners and not just by university researchers. Ms. Statz showed that she could form and research important questions. She reflected on the process she went through collaborating with the researcher:

You allowed me to voice my concerns. And you were somebody to listen to the things that I had problems with. You gave suggestions. Yet you also said: 'It's up to you. Do it your way. Try it your way. It's up to you with your class.' I guess I learned to stop asking for advice and I learned to start thinking on my own. Because I knew you would say, 'What do you think?' So then I was already doing some of the thinking and trying it out on you more. (March 31, p. 12.)

Implementation of CGI is based on the organized body of knowledge on children's thinking drawn from research. Prior to this study, most of the research knowledge on children's thinking and classroom use within the CGI framework was on the younger grades (K-3). Not much was known on children's thinking or implementaion of CGI in the fourth grade. Thus Ms. Statz's accomplishments are doubly impressive, since she had to extend her knowledge of children's thinking to include topics in the fourth grade. This study shows that teachers who learn how to interview children on their thinking processes and have a good theoretical understanding may be able to discover unique strategies and how children think in new domains that are not yet known from the research literature.

We also learned that it is possible to start implementing CGI in fourth grade, even for children who were not exposed to such ideas before. CGI worked well even in a very heterogeneous class. The teacher had to establish the norms of working in math in this class, emphasizing solution processes, children solving in a variety of ways, child en talking and writing to describe their thinknig processes, are challenging problems that sometimes take a long time to solve, and avoiding extensive drill on computaional skills. We also learned, that problems arose from the fact that the children had different backgrounds. The teacher had to spend much energy helping children who used wrong procedures or worked with lack of understanding. More energy and time were spent in the class working on subtraction than on fractions. Thus, when a program like CGI that promotes understanding and childrens' ability to justify their mathematical solutions is begun at a later point in elementary school, time and patience will be required to unlearn misconceptions.

Conclusion

What can we, as math educators learn from Ms. Statz's experience? Although Ms. Statz made remarkable changes during the course of the study, we saw that the process of change was very difficult for her. It was hard emotionally to go through phases that had uncertainty, conflicts and dilemmas, with no obvious solutions for problematic situations. It is important that teachers get support when they try to reform their classrooms. We learned much from working with Ms. Statz about the concerns and questions that she had to overcome in the process of change and about what support was helpful for her. One form of support that might be very important is a team of teachers within a school or from a few schools who talk to each other about their concerns, their childrens' strategies and how their classes look. The atmosphere needs to be supportive and not judgmental. Teachers must realize that it is important to take risks: Ms. Statz was open to trying new ideas and she was whiling to try to teach in new ways, even though she was not sure what would result from these experiments.

How can we help teachers change in ways that do not demand the intensive one-to-one interactions and classroom presence of this study? Cobb et al. (1990) found that what they learned from one teacher enabled them to design better group workshops and refine the ways they approached teachers to better meet their needs and concerns. We feel similarly about this case study. An important lesson of this study is the role that a teacher's reflection about her students' strategies can play in facilitating change. Other studies have tried to increase teachers' reflection in group settings, using techniques such as teacher writing, disccussion, assignments of interviewing children, transcribing their own teaching segments (Shifter, & Fosnot, 1993), or the use of case methods (Richert, 1991; Shulman, 1992). It is very important that the teacher have opportunities to learn in the context of his or her class and be able to interview students from the class to learn about their thinking, even if the facilitator is not situated within the teacher's school and does not visit the class. The support for the teacher should take into account the teacher's concerns and beliefs (Loucks-Horsley and Steingelbauer, 1991) and help the teacher form and research questions to find solutions with which the teacher is comfortable. It is important to help teachers develop a sense that they are really autonomous to make decisions for their class.

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Figure 1

Ms. Statz's chart of her change over three years

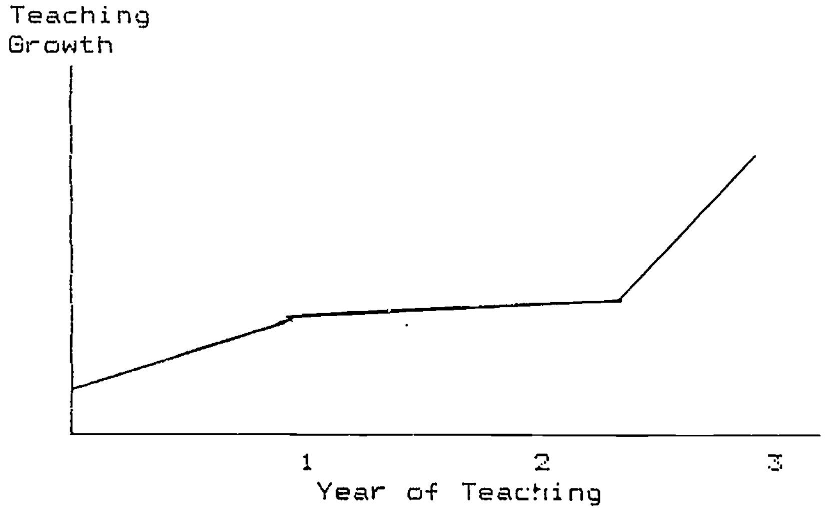

Figure 2

Kanisha's strategy for solving 20 cakes for 7 children