DOCUMENT RESUME

ED 374 996 SE 055 235

TITLE	Active, Meaningful Mathematics Learning: A Guidebook.
INSTITUTION	Midwest Consortium for Mathematics and Science Education, Oak Brook, IL.; North Central Regional Educational Lab., Oak Brook, IL.
PUB DATE	[94]
NOTE	19p.
AVAILABLE FROM	North Central Regional Educational Laboratory, 1900 Spring Road, Suite 300, Oak Brook, IL 60521-1480.
PUB TYPE	Guides - Non-Classroom Use (055)
EDRS PRICE	MF01/PC01 Plus Postage.
DESCRIPTORS	Constructivism (Learning); *Educational Change; Elementary Secondary Education; *Mathematics Education; *Program Evaluation; Student Evaluation; *Teacher Education Programs; Technology
IDENTIFIERS	*NCTM Curriculum and Evaluation Standards; NCTM Professional Teaching Standards; *Reform Efforts

ABSTRACT

The National Council of Teachers of Mathematics' (NCTM) Curriculum and Evaluation Standards and Professional Standards for Teaching Mathematics grew out of the Council's vision for developing a confident command of mathematics in all students. This booklet provides a brief overview of the NCTM Standards--the important components, the underlying beliefs, and the changes that need to be made in school mathematics. Also included are examples of successful mathematics programs that mesh with the NCTM Standards and a checklist for comparing mathematics activities in schools with the NCTM Standards. Components considered are: student learning, content and process, curriculum and instruction, assessment, and technology. Annotations including contact information are provided for: the Algebra Project, Cognitively Guided Instruction, College Preparatory Mathematics Program, Comprehensive School Mathematics Program, Core-Plus Mathematics Project, Michigan Mathematics Inservice Project, Middle Grades Mathematics Project, and The University of Chicago School Mathematics Project. (MKR)

NCREL

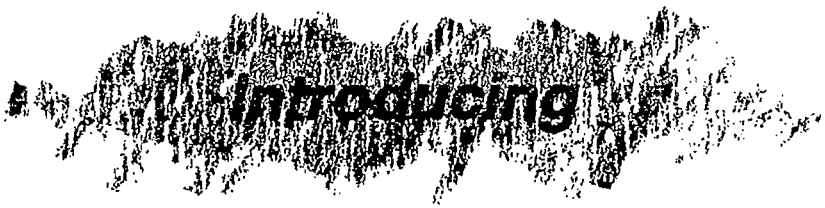
North Central Regional Educational Laboratory

NCREL, a not-for-profit organization, works to improve education in Illinois, Indiana, Iowa, Michigan, Minnesota, Ohio, and Wisconsin. It also operates the Midwest Consortium for Mathematics and Science Education.

Active, Meaningful Mathematics Learning:

A Guidebook

MATHEMATICS



The Importance of Mathematics Education

Mathematics is not just the language of science, it is a key to opportunity. Mathematics contributes in direct and fundamental ways to business, finance, health, defense, and many other fields. It opens doors to careers, enables informed decisions, and provides knowledge needed to participate in a technological society. Students who wish to succeed in school and in life must gain the power of mathematics.

Mathematics, of course, is not the **only** key to opportunity in today's world. Reading and communicating are also critical skills in learning and in life. But the importance of mathematics in many areas of life is greatly increasing.

Mathematics is a dynamic cultural invention that grows and changes with the needs and interests of society. The growth of technology—especially the impact of computers—and the expansion of mathematics itself have combined in the past 25 years to extend the scope and applications of mathematical science. These forces have changed the nature and role of mathematics, and our approach to mathematics must reflect these changes if we are to prepare our students for tomorrow's world.

Introduction

Reformers of mathematics have called for a change in school mathematics since the beginning of this century, yet the teaching of mathematics has changed very little. Today's reformers, supported by research findings, believe that excessive emphasis on the mechanics of mathematics inhibits meaningful learning and leads to widespread misconceptions about the strengths and limitations of mathematical methods.

The National Council of Teachers of Mathematics' (NCTM) *Curriculum and Evaluation Standards for School Mathematics* (1989) and *Professional Standards for Teaching Mathematics* (1991) grew out of the Council's vision for developing a confident command of mathematics in *all* students.

This booklet introduces you to the NCTM standards—the important components, the underlying beliefs, and the changes that need to be made in school mathematics. You also will find examples of successful mathematics programs that mesh with the NCTM standards and a checklist for comparing the mathematics activities in your school with the NCTM standards.

If you are interested in more information about mathematics reform in schools, please contact:

Midwest Consortium for Mathematics and Science Education
North Central Regional Educational Laboratory
1900 Spring Road, Suite 300
Oak Brook, Illinois 60521-1480
1-800-356-2735

NCTM Standards

The NCTM standards describe a mathematics curriculum that is conceptually oriented, actively involves students in using mathematics, and develops students' mathematical thinking and reasoning abilities. The standards emphasize students' ability to solve problems, reason, communicate mathematical ideas, make connections within mathematics, and link mathematics to other subjects.

The driving force behind the standards is a vision of mathematics in which all students have the opportunity to learn serious and challenging mathematics. The vision also encompasses the type of instruction that

will achieve this goal. Classrooms should be places where interesting problems are explored using important mathematical concepts. Students should actively make conjectures and discuss ideas, and teachers should pose appropriate problems, prompt and facilitate students' thinking, and probe for ideas.

The standards answer the following important questions:

- What should students know and be able to do in mathematics?
- How should we design and orchestrate instruction to enable all students to know and use mathematics?
- How should we measure students' progress as they are learning?

It is important to understand that the standards are statements about what is valued; they are not hurdles to be overcome or distant targets that few students can be expected to reach. The standards provide direction and set national goals for student progress in mathematics, while leaving room for local decision making about how to achieve those goals.

Student Learning

Research indicates that students learn mathematics well only when they construct their own mathematical understanding. That is, they gain a better understanding of mathematical content when they modify or even rework their prior beliefs and behavior, rather than simply record and store what they are told. To understand what they learn, students must enact for themselves verbs that permeate the mathematics curriculum, such as examine, represent, transform, solve, communicate. Students' mathematical understanding progresses from developing concrete models

of situations using tools such as diagrams and manipulatives, to learning how to think in increasingly abstract, symbolic ways. The students' acts of construction and invention build their mathematical power and understanding, enabling them to solve problems that they have never seen before. This progress occurs most readily when students work in groups, engage in discussion, make presentations, and in other ways take charge of their own learning.

When students explore mathematics on their own, they construct strategies quite different from those presented in textbooks. Students need an environment for learning mathematics that leaves room for trial and error. Students must be encouraged to express their approaches both orally and in writing. They must learn and be encouraged to work cooperatively in groups to solve problems and learn to argue convincingly for their approaches when they are not accepted by the cooperative group or the class as a whole.

The central findings of the reforms, grounded in research, is that all students—not just a talented few—are capable of learning mathematics. It is expected that students will learn everything from arithmetic to the foundations of calculus more readily when the techniques used are more open, more inquiring, more practical, more "hands-on," more collaborative, and more in tune with the students' perceptions of the world. This approach contrasts sharply with the old lecture-and-memorize, find-the-right-answer approach that has traditionally dominated mathematics classrooms.

Content and Process

Mathematics is a skill that people use to solve problems, communicate, and reason. The NCTM curriculum standards emphasize developing all students' ability to use mathematics in solving problems, reasoning,

and communicating, while helping students to value mathematics and feel confident in their ability to use mathematics.

The mathematics curricula at all levels should introduce more of the breadth and power of the mathematical sciences. These curricula should have a broad range of content. Students should have experience with data gathering, probability and statistics, geometry and spatial thinking, the study of ratio and proportions, and algebraic thinking.

Many exemplary programs embody the NCTM standards at both the elementary and secondary levels. For example, at the elementary level, the *Comprehensive School Mathematics Program (CSMP)* introduces K-6 students to a curriculum that emphasizes content such as geometry, measurement, patterns, relations, numeration, probability, statistics, logic, algorithmic thinking, and applications.

Students leaving secondary school after studying a core of broadly useful mathematics should be able to understand chance, change and variability, and data and experiments; to interpret graphs; and to grasp the nature of mathematics sufficiently to support the mathematics education in the nation's schools.

One program that would help students in secondary schools achieve these abilities is the *Core-Plus Mathematics Project (CPMP)* curriculum, which features multiple strands of algebra/functions, geometry/trigonometry, statistics/probability, and discrete mathematics connected by the fundamental themes of data, representation, shape, change, and chance.

This curriculum is consistent with the vision of school mathematics described in the NCTM standards. It is designed to make core topics accessible to all students. Differences in student performance and interest are

accommodated by the depth and level of abstraction to which common topics are pursued and the nature and degree of difficulty of each application.

A curriculum developed around these principles holds promise for making mathematics accessible to a diverse student population. Developing mathematics each year along multiple strands nurtures the differing strengths and talents of students and simultaneously helps them to develop diverse mathematical insights.

Curriculum/Instruction

Even if the mathematics curriculum is changed to cover the full spectrum of the mathematical sciences, this change in emphasis is not enough. The role of the teacher is critical to advancing reform. The teacher has a key role in setting mathematical goals and creating a classroom environment in which these goals are pursued.

Change in teaching and learning depends heavily on the teacher. If mathematics instruction is to engage all students in solving problems dynamically, then teachers need to create an environment that encourages students to explore, develop, test, discuss, and apply ideas. They need to develop classrooms into communities of mathematical inquiry.

Teachers should strive to foster student-centered, mathematical communities in the classroom, where the teacher considers the students' prior knowledge and builds upon that knowledge in meaningful ways. Problem solving should be emphasized often, with problems placed in real-world contexts. In a classroom that is a student-centered, mathematical community, the teacher's role changes from dispenser of knowledge to facilitator of communication, discussion, critical thinking, and generation of alternative solutions to problems.

The most common approach to teaching mathematics today—requiring students to memorize rules and do routine exercises largely geared toward standardized tests—at best imparts "lower-order" thinking skills that make students into robot-like number manipulators. It is a rote approach to teaching mathematics that keeps students from making sense and meaning out of mathematics.

Assessment

Student assessment should be integrated into instruction. Teachers need to discover how students are thinking about and interpreting mathematics to guide their instructional decision making. We must design assessments that communicate the nature of the subject but also allow students to see mathematics used in real-life contexts. Because assessment is so pervasive and has such a powerful impact on the lives of both students and teachers, it is very important for assessment to fulfill both the purpose of the test and curricular objectives.

For years, assessment has focused entirely on discrete mathematical facts and isolated skills. When teachers focus their assessment on such low-level skills and facts, they are giving students an impoverished conception of mathematics, which limits students' ability to value and have confidence in mathematics. Very little has been done to assess the students' ability to explore and conjecture in a mathematical setting—to address the distance between the classroom and the "real world." The NCTM standards state what we should value in mathematics teaching and learning; tests are also statements of the knowledge and performance that we value, and mathematics reform needs to act on aligning testing practice with the standards.

Traditionally, student assessment was used to assign grades and to rank students, but more powerful assessment procedures serve as a vehicle for tapping meaningful learning. Such assessment *informs*, *improves*, and actually *promotes* learning. The teacher's job is to promote students' abilities to their fullest, and assessment becomes a tool for reaching this goal.

Assessment strategies should be designed to give meaningful information about:

- Students' ability to use mathematics in complex situations
- Students' work on extended, project-like problems
- Students' use of computation and other procedures in real-life contexts
- The extent of students' understanding or lack of understanding about central concepts and generalizations
- Students' ability to discern and define problems
- Students' ability to formulate and implement problem-solving strategies when presented with a problem
- Changes in students' work in mathematics over a period of time

To measure the development of students' mathematical power, teachers need to use multiple assessment methods: essays, projects, journals, oral presentations, group projects, portfolios, homework, and so on. Only broad-based assessment can reflect fairly the important, higher-order objectives of the mathematics curricula.

Technology

If technology, increased applications, and the impact of computers have extended greatly the scope of mathematical sciences, then the schools' mathematics curriculum should include tools that are available in the real world. Schools need to give sufficient recognition to the use of computers and calculators in mathematics courses.

Research shows that the appropriate use of technology enhances students' mathematical understanding and improves problem-solving abilities. The calculator and computer affect not only the approaches to content, but also the importance of that content.

Mathematics learning can become more active and dynamic with the use of technology. Using calculators for computation enables students to explore a wider variety of examples, witness the dynamic nature of mathematical processes, engage in realistic applications using typical data, and focus on important concepts rather than routine calculations. Although calculators and computers will not necessarily cause students to think for themselves, they can provide an environment in which student-generated mathematical ideas can thrive.

Access to mathematics instruction via telecommunications equalizes educational resources for student populations that are underserved due to geographic remoteness, economic disadvantage, or student disability. Instructional technology can facilitate teacher and student access to a broad, worldwide inventory of instructional software, video resources, live interactive teleconferences, and research information.

Even today, as we struggle to adapt an "old" curriculum to today's technology, new technology is being developed. This technology will revolutionize the way in which mathematics is practiced and learned.

The ultimate goal of mathematics education is to enable students to think critically and solve problems with confidence and with an understanding of the value of mathematics.

What Can You Do?

You can help reform mathematics education by doing the following:

- Use the NCTM checklist below and on the next page of this booklet to think about particular aspects of your mathematics programs, considering whether your students are learning mathematics in a student-centered, mathematical community where each student is involved in investigating, reasoning, communicating, and developing self-confidence in mathematics. The goal is to assess the quality of specific parts of your mathematics programs and think of ways to bolster strengths and address weaknesses.
- Review the examples of *Successful Mathematics Practices* listed and, if interested in the program, contact the developer for assistance.

Checklist for Assessing Mathematics Programs

Note: The following checklist items are program dimensions that range along a continuum. They are intended to help you start thinking about the degree to which your programs are creating and sustaining student-centered communities of mathematical inquiry.

Vision

☐ Mathematics education is important for all people.

- ☐ Quality mathematics education is accessible to all students.
- ☐ Mathematics is viewed as an active, constructive process.

Curriculum and Instruction

- ☐ Curriculum has a broad range of content.
- ☐ Curriculum is based on themes and sequence.
- ☐ All students are included in all aspects of the curriculum.
- ☐ Instruction is conducted within a variety of contexts. Connections are deliberate so that students can make sense of mathematics.
- ☐ Instruction is based on problem situations found in the real world.
- ☐ Instruction is student-centered, actively involving students in their own learning.
- ☐ Instruction is experience-centered, with direct observation and manipulation of phenomena used as a means of building mathematics concepts.
- ☐ The teacher operates as "facilitator" of communication, not as "dispenser" of knowledge.
- ☐ The teacher builds upon students' prior knowledge.
- ☐ Appropriate technology is used to enhance students' mathematics comprehension.
- ☐ Curriculum is based on themes and sequence.

Assessment

- ☐ The goal of assessment is to promote and improve instruction and learning.
- ☐ Multiple assessment tools are used.

Resources

National Council of Teachers of Mathematics, Commission on Standards for School Mathematics. (1989). *Curriculum and Evaluation Standards for School Mathematics.* Reston, VA: The Council.

National Council of Teachers of Mathematics, Commission on Teaching Standards for School Mathematics. (1991). *Professional Standards for Teaching Mathematics.* Reston, VA: The Council.

National Research Council. (1989). *Everybody Counts: A Report to the Nation on the Future of Mathematics Education.* Washington, DC: National Academy Press.

Additional Resources

The Algebra Project

The Project is designed to move students (specifically, underrepresented students) through "gateway" mathematics courses by using teaching and learning processes that assist students in the construction of authentic mathematical knowledge. The key concepts, which form the underlying conceptual framework of algebra, are given direct, intuitive meaning for students by engaging them in a "curriculum development process." In this process, the conceptual, qualitative meaning of symbols used in algebra are grounded in the students' experiences following a proven five-step process.

For information, contact:

Bob Moses
Algebra Project
99 Bishop Allen Drive
Cambridge, MA 02139
(614) 491-0200

Cognitively Guided Instruction (CGI)

CGI is a model of curriculum development designed to increase learners' understanding by considering teachers, learners, and mathematics. The underlying assumption is that classroom instruction is determined by the decisions that teachers make, which are directly influenced by teachers' knowledge and beliefs. The major tenets of CGI are that (1) instruction must be based on what each learner knows; (2) instruction should take into account how children's mathematical ideas develop naturally; and (3) children must be mentally active as they learn mathematics. All decisions about what and how to teach should depend on what the child already knows and thinks.

For information, contact:

Tom Carpenter or Elizabeth Fennema
National Center for Research in Mathematical Sciences Education
University of Wisconsin-Madison
1025 W. Johnson Street
Madison, WI 53706
(608) 263-7582

College Preparatory Mathematics Program (CPMP)

A comprehensive, university-high school collaboration that intervenes to help potentially successful but underrepresented (minority and female) and at-risk students to succeed in precollege mathematics supported by the involvement of within-school counselors and other significant school-university support mechanisms. Cooperative small-group work is the primary mode of learning.

For information, contact:

Dr. John Baldwin or Dr. Roberta Dees
Department of Mathematics, Statistics, and Computers
University of Illinois at Chicago
208 S.E.O. M/C 249
Chicago, IL 60680
(312) 413-3748

Comprehensive School Mathematics Program (CSMP)

An exciting and powerful K-6 elementary mathematics program that focuses on problem solving and concept development. Its unique approach allows even very young children to grasp mathematical concepts and ideas. Using a variety of situational teaching methods; graphic, nonverbal "languages"; and colorful and unusual manipulatives—even fantasy stories, CSMP activates the imagination of young children and engages them in a fascinating exploration of mathematics, from developing basic skills to solving complex problems. This comprehensive curriculum has proven to be effective with all types of students at all ability levels.

For information, contact:

Clare Heidema
Mid-continent Regional Educational
Laboratory (McREL)
2550 S. Parker Road, Suite 500
Aurora, CO 80014
(303) 337-0990

Core-Plus Mathematics Project (CPMP)

A complete three-year high school mathematics curriculum for all students, together with a fourth-year option focusing on the transition to college mathematics, are under development. Each year, the curriculum

features multiple, connected strands of algebra/functions, geometry/trigonometry, statistics/probability, and discrete mathematics. The curriculum emphasizes mathematical modeling and modeling concepts of data collection, representation, prediction, cause-and-effect, and simulation.

For information, contact:

Mary Morgan
CPMP Project Manager
Western Michigan University
Kalamazoo, MI 49008
(616) 387-4562

Michigan Mathematics Inservice Project (M2IP)

The project uses a model for K-6 mathematics teaching comprising a set of ten (10) mathematics teaching principles based upon research on the teaching and learning of mathematics, mathematics education, and the teaching experiences of lead teachers. In addition t content modules, the project focuses on the development of problem-solving abilities, mathematical assessment, and mathematics equity. The concepts of the latter modules are reinforced throughout the other modules.

For information, contact:

Dr. Robert Laing or Dr. Ruth Meyer
Western Michigan University
Kalamazoo, MI 49008
(616) 387-4525

Middle Grades Mathematics Project (MGMP)

A broad-based research, curriculum development, and teacher enhancement project dedicated to the improvement of mathematics educaion at the middle school level, grades 5-8. Important mathematical ideas that were previously neglected in the 6-8 grade curriculum

are introduced. The topics are presented in five curriculum units that reflect (and inform) the NCTM standards using an instructional model that promotes an atmosphere of problem solving and inquiry in the classroom. Manipulative materials and calculators are used throughout the units.

For information, contact:

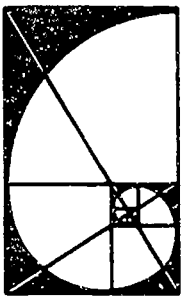
Dr. Glenda Lappan
101 Wills House
Michigan State University
East Lansing, MI 48824
(517) 335-2870

The University of Chicago School Mathematics Project (UCSMP)

The project focuses on upgrading the mathematics experience of the average K-12 student using materials created by UCSMP that fully implement the NCTM standards. The elementary curriculum gradually moves from manipulatives to abstract concepts, integrating math with other subjects. This transition is accomplished by two workshop series: *MathTools for Teachers* (K-3) and *Mathematics Specialist Program* (4-6). The six-year, 7-12 curriculum with an abundance of applications includes algebra, some discrete mathematics in all courses, and statistics and probability integrated into the study of algebra and functions. At all levels, UCSMP materials emphasize problem solving, take advantage of the latest technology, and relate mathematics to the real world.

For information, contact:

Carol Siegel
UCSMP
5835 S. Kimbark Ave.
Chicago, IL 60637
(312) 702-9770



MSC

Midwest Consortium
for Mathematics and
Science Education

North Central Regional Educational Laboratory
1900 Spring Road, Suite 300
Oak Brook, IL 60521-1480
1-800-356-2735