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ABSTRACT

This paper reports on a pilot study of 34 sixth-grade students engaged in mathematical explorations that was carried out in the context of a national Logo project in Costa Rica. Students worked in pairs investigating a mathematical microworld written in Logo. The investigation had three phases: (1) open exploration of the microworld, during which the students recorded their observa and formulated hypotheses about how the program worked; (2) group discussion and sharing of hypotheses; and (3) additional guided discovery and problem solving. Through observations and collection of written worksheets, it was found that the students were successful in discovering certain functions of the microworld during the first phase, and their hypotheses were improved after discussion with their peers, guided by the instructor. The students were successful in applying their knowledge of the computer microworld in the problem-solving tasks during the third phase. Contains 12 references. (MKR)

Making Sense of a Mathematical Microworld: A Pilot Study from a Logo Project in Costa Rica

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MAKING SENSE OF A MATHEMATICAL MICROWORLD: A PILOT STUDY FROM A LOGO PROJECT IN COSTA RICA[1]

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A pilot study of sixth grade students engaged in mathematical explorations was carried out in the context of a national Logo project in Costa Rica. Thirty-four students, approximately 12 years old, worked in pairs investigating a mathematical microworld written in Logo. The investigation had three phases: (1) Open exploration of the microworld, during which the students recorded their observations and formulated hypotheses about how the program worked; (2) Group discussion and sharing of hypotheses; and (3) Additional guided discovery and problem-solving. The students were successful in discovering certain functions of the microworld during the first phase, but their hypotheses were improved after discussion with their peers, guided by the instructor. The students were successful in applying their knowledge of the computer microworld in the problem-solving tasks during the third phase.

Introduction

The purpose of this report is to describe work with Costa Rican students in the area of computer-based mathematical explorations, setting this work in the context of recent research on social and cognitive factors involved in the construction of mathematical knowledge, and on the role that new interactive technologies can play in the learning process. The report will describe the results of a pilot study in which a simple Logo microworld was introduced to upper-elementary students, who worked together in pairs to make sense of the environment, discussing and writing hypotheses and completing guided discovery worksheets. The students had never carried out mathematical explorations within the kind of collaborative social setting used in the pilot study; however, they made good progress overall in being able to use the microworld to create or match patterns, and in beginning to discriminate its distinctive features, both of which have been described as important aspects in building a mathematical understanding of a microworld (Hoyles & Noss, 1987).

Related work and theoretical framework

The research reported here was carried out in the context of related work in two important areas: the use of computer-based learning environments in mathematics instruction, and the role of social interaction in the construction of mathematical knowledge. A wide range of computer environments labeled "microworlds" have been created, and a body of research has accumulated on their use in the learning of mathematics and science (*e.g.*, diSessa, 1990; Edwards, 1992a; Hoyles &

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Noss, 1987, 1992; Thompson, 1987) Pea defines a microworld as "a structured environment that allows the learner to explore and manipulate a rule-governed universe, subject to specific assumptions and constraints" (Pea, 1987, p. 137). Students learn from microworlds by interacting with the mathematical or scientific concepts embodied in the computer programs, and by "debugging" their understanding of the rules which govern how objects behave in the environment (Edwards, 1992b). However, the learning which takes place depends not only on the design of the technological artifact but also on the social context, on the way in which the computer environment is introduced and on the activities structured around it.

The social nature of knowledge construction has been addressed both within the mathematics education community and more broadly (*cf.*, Resnick, Levine & Teasley, 1991). Cobb, Wood and Yaekel (1992), for example, have investigated in great detail how young learners co-construct understandings of mathematical ideas and operations while interacting in pairs and small groups. These children are taught in classroom settings where they are encouraged to work together to solve mathematical problems, and then to verbalize their solutions to the whole group. In such settings, the children engage in processes which include:

> ...resolving obstacles or contradictions that arise when they use their current concepts and procedures, accounting for a surprising outcome..., verbalizing their mathematical thinking, explaining or justifying a solution, resolving conflicting points of view, and constructing a consensual domain in which to talk about mathematics with others (Cobb, Wood & Yaekel, 1992, p. 158)

One goal of the pilot study reported here was to create a social setting for upper elementary Costa Rican students in which they had the opportunity to engage in the learning processes described above, but one in which the mathematical problem solving would take place while exploring computer microworlds.

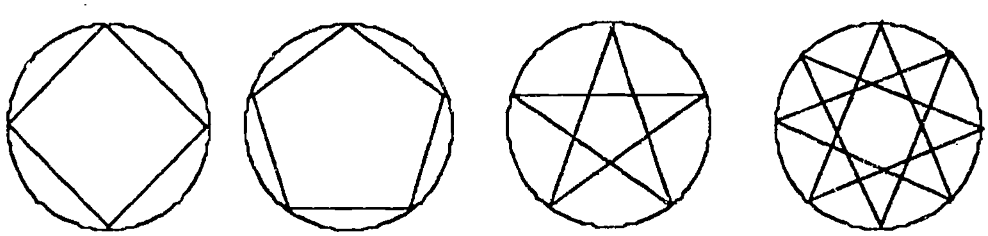
The context of the project

The Mathematical Explorations in Logo study was carried out as one part of the Omar Dengo Foundation's Computers in Elementary Education Project. This project, ongoing since 1988, has placed computer laboratories (a network of 20 IBM PS/2 computers plus printer) into more than 160 Costa Rican schools, primarily in rural and marginal urban settings. The study reported here thus took place within the context of an established and successful Logo education project. The students and teachers were comfortable with computers, and all students, from kindergarten through sixth grade, were accustomed to spending at least 80 minutes a week on Logo activities. However, although the students had an adequate level of programming knowledge for creating their Logo projects, there was no explicit attempt to relate mathematics to programming. Furthermore, as in many places, Costa Rican students were often taught mathematics in a somewhat rote and decontextualized fashion. Collaboration and discussion did not typically occur in their regular mathematics classes. Thus, a new goal was set, that of extending the use of Logo into the teaching and learning of mathematics through the creation of Logo-based microworlds, and of utilizing these microworlds in a social setting which encouraged collaboration, discussion and verbalization of mathematical discoveries.

The CUERDAS microworld

The microworld, called "CUERDAS" ("Strings"[2]) is illustrated in Figure 1. (This microworld is an adaptation of a Logo procedure which has been used within the Logo community for some time). The procedure CUERDAS takes two inputs, and creates either polygons or "star" figures by connecting equidistant points on the circumference of a circle. The first input gives the number of evenly-spaced points around the perimeter of the circle, and the second input is used to "count off" the distance between successively-connected points. The figure created depends on a fairly simple mathematical pattern, specifically, whether the ratio of the first to the second input results in a whole number or not. As seen in Figure 1, whole number ratios create simple polygons; ratios involving fractions result in stars.

These and other mathematical regularities or "rules" are not immediately obvious, yet learners can discover them through experimentation with the microworld. The first task set for the students in the study was to find out the purpose of each input in the CUERDAS procedure. Further guided explorations focused on which inputs create polygons and which create stars, and on the fact that pairs of inputs having a common ratio result the same figure (for example, both CUERDAS 6 2 and CUERDAS 3 1 make an equilateral triangle).



CUERDAS 4 1   CUERDAS 5 1   CUERDAS 5 2   CUERDAS 8 3

FIGURE 1: The microworld CUERDAS with four sets of inputs

[2]The microworld was called "CUERDAS" or "Strings" because a concrete version of this activity involves creating "string pictures" by physically connecting pins or nails on the circumference of a hoop with lengths of string.

Methodology

The study took place in an urban school in a poor neighborhood in San José, the capital city of Costa Rica. A class of 34 sixth-grade students (approximate age, 12 years) participated in the session, which was set up to pilot-test the software. Sixteen boys and 18 girls took part in the pilot study during an 80 minute afternoon session in the school's computer lab. The students' regular computer teacher was present, as well as the investigator and a specialist in computer education from the Omar Dengo Foundation, who served as the instructor during the pilot session.

Data were collected through observations of the students as they worked in pairs and participated in group discussion, and by collecting the written worksheets they completed during the session. There were two kinds of worksheets: a "data sheet" and a follow-up set of questions and problems to solve using the microworld. For the students, the "data sheets" were used as a way to gather information so that they could write a hypothesis about how the microworld functioned. For the investigator, the data sheets and the problem sets served as written records of the students' work during the session, and were used in conjunction with written notes to analyze the students' learning as they interacted with the microworld and with each other.

Procedure:

The session began with all students gathered in a circle, where the instructor spent about 10 minutes introducing the activity. She stated that the children would be working that day as scientists or detectives, gathering data in order to come up with a hypothesis for how the computer program worked. Data sheets were distributed to pairs of students; each data sheet had two columns, one headed "What I Tried" and the other "What I Found Out." The students were instructed to record each "experiment" as they entered various inputs into the computer, and to sketch or describe the result. At the bottom of the sheet the students were asked to write out their hypothesis by answering the question: "How does the program work? My hypothesis:__________." The purpose of the data sheet was both to help students remember and record their explorations in a systematic way, and also to allow them to reflect on the results of their experiments with the microworld.

The students then moved to the computers in self-selected pairs, and set to work, one pair per computer, with the computers arranged side-by-side around the perimeter of the room. The students were able to talk freely with their partners and neighboring pairs of students. They spent approximately 30 minutes exploring the microworld and filling in their data sheets. Near the end of this time period, they were asked to spend five more minutes working and then to write down their final hypotheses. They were then gathered into a small group, and asked to describe or read out their hypotheses. A discussion followed, during which the students were encouraged to question each other and to clarify their hypotheses to the group (the instructor did not evaluate the hypotheses;

instead, the process of deciding on the best hypothesis was carried out by the group). Finally, the students were given a second worksheet, which asked specific questions about the function of each input to the procedure (offering, in essence, an opportunity to correct or refine their hypotheses) and presented a set of problems and challenges to solve using the program (i.e., further applications and explorations of the microworld). The students spent an additional 20 minutes filling out this worksheet, working until the end of the 80 minute session.

Results

The students quickly understood the goal of the activity and worked actively during the allotted time with their partners, experimenting with the microworld, recording their results, discussing possible hypotheses about how the program worked, and calling over the instructors or classmates to show them particularly interesting figures on the screen. During the last five minutes of the initial period of exploration, the instructor reminded the students that they needed to come up with a written hypothesis. This prompt resulted in a final burst of activity and conversation among the students. After writing their hypotheses, the students were gathered into a circle again and asked to read their hypotheses, one at a time. The group then discussed various hypotheses, some of which they decided were incorrect or only partially-correct. The correct hypothesis was demonstrated by one pair of students who had discovered it during their initial exploration of the microworld. The students then returned in their pairs to the computers, and most of them confirmed the correct hypothesis by entering additional examples, and then went on to complete the problems and explorations on the second worksheet.

An analysis of the written worksheets revealed that almost all of the students (14 out of 17 pairs, or 82%) were able to correctly describe the function of the first input to the procedure in their initial written hypothesis. However, only one pair of students (6%) was able to correctly determine the precise function of the second input within the time allowed for this task. The other pairs gave a variety of hypotheses. In general, these hypotheses stated that the second input had something to do with the shape of the figure inside the circle, but were not specific about the precise mathematical relationship. For example, two hypotheses taken from the written worksheets were: "El segundo # sirve para hacer la figura que ésta inscrita" ("The second number serves to make the inscribed figure"); and "El segundo es el numero de figura" ("The second is the number of the figure"). As can be seen, the students' hypotheses were vague and difficult to interpret. For the most part, the students were not able to discover the specific function of the second input, at least not within the amount of time they were allowed for the initial exploration of the environment.

As noted above, after the students shared their hypotheses, the pair of students who had discovered the correct functioning of both inputs demonstrated their solution on the computer. The other students at this point watched and carefully questioned the successful pair. After this demonstration and discussion, the majority of the students (13 out of 17 pairs, c: 76%) were able to accurately describe in writing the function of the second input. For example, one pair wrote, in answer to the question "Qué significa la segunda entrada?" (What does the second input mean?), "El espacio entre vertice y vertice para ser la figura dentro la circunferéncia" ("The space between vertex and vertex to make the figure inside the circumference").

Most of the children successfully completed the remainder of the second worksheet, which consisted of challenges to create specific figures using the microworld, or to discover general "rules" such as how to create stars rather than polygons. The students worked very productively during this third phase. By the end of the session, the children were able to use the CUERDAS procedure with confidence both to create shapes specified on the worksheet and to make new designs of their own. This facility in using the microworld to solve problems is evidence of an understanding of the program which apparently surpassed what the students could put into words. Although not all of the students could initially state in precise language how the microworld functioned, all could correctly select inputs in order to create specific figures, that is, they could successfully use the microworld in solving problems and creating interesting patterns of their own.

Discussion

According to one model of mathematical understanding (Hoyles & Noss, 1987), a learning sequence which reflects increasing mathematical power includes the following "stages:" Using, Discriminating, Generalizing and Synthesizing. "Using" indicates that the learner knows what to do with a mathematical operation or procedure, while a child who is at the "discriminating" stage can describe features of the procedure which make it distinct, and knows how it works. At the "generalizing" stage, a learner can test for mathematical regularities, and when "synthesizing" can make connections with other mathematical topics. The children working with the CUERDAS microworld were able both to use the procedure and to discriminate at certain aspects of its features, writing hypotheses which reflected their understanding of how the procedure worked. They were able to reach this level of understanding by experimenting with the microworld and talking about the results with their partners, by sharing their hypotheses with the whole group, and then returning to the computer for further exploration and problem-solving.

It is important to note that these children had never been asked to carry out an activity like the one described above, either in their Logo class or as part of their regular mathematics instruction. The initial description of the purpose of the project, the introduction of the idea of being a detective or scientist, was very important in helping these students to understand the idea of a mathematical exploration. Yet, aside from helping the children to start the program and answering a few questions, the teacher and investigator did not intervene during the session. The students consulted among themselves, argued, debated, questioned, looked at other screens and formulated their written answers together. This kind of activity, in which students engage in authentic debate and conversation, contrasts sharply with the traditional "recitation script" in which the teacher provides information to the whole class and then tests the students' recall through individual question and answer (Tharp and Gallimore, 1988). In the work described here, the students focused on the computer as a "conversation piece", one which embodied a mathematical puzzle that they worked together to solve. The teacher's role became one of coaching and guiding the students to make their discoveries explicit and to express them in words.

This pilot study, although limited in scope, contributes to the growing body of research showing that mathematically-rich computational environments can provide a basis for learning which goes beyond memorization and rote procedural knowledge. The students in this study acted, during the pilot session, as members of a community of fellow explorers, sharing a common goal of making sense of a new mathematical phenomenon, embodied in a computer microworld. In future work with these students and others, it is hoped that this pedagogical context will become a familiar and accepted way to support deep and significant learning through the exploration of computer environments.

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