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ABSTRACT

This study investigated how 8 secondary school students coped with the task of choosing their own problems for a 9-week course in an alternative school in Ithaca, New York. The students chose a theme to be investigated before the class started, collected real data on the theme, and if they felt the need for it, analyzed it using Function Probe, a software package. Observation, interview, and journal data from the students and teacher and written work of the students were compiled. Preliminary analysis of the results showed that working with real data made a difference in the level of excitement about data collection, helped the students deal with new issues in mathematics, and helped students feel more control over what they were learning. Contains 22 references. (MKR)

High School Students' Mathematical Problem Posing: An Exploratory Study in the Classroom

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The separation between educators and learners in assessment mirrors the separation between them in the design of the educational experience. For a learner to perceive education as something of value in their lives, they must contribute to its design.

(Tim Best, Cristopher Pacione, John Rheinfrank and Kate Welker, 1990)

Introduction

This exploratory study investigated how students coped with the task of choosing their own problem for a 9 week long course in an alternative school in Ithaca, NY. Students chose a theme to be investigated before the class started as they interacted with the teacher. They then collected "real data" (Hancock & Kaput, 1990) on the theme and analyzed it using a software package (Function Probe©, Confrey, 1989) if they felt the need for it. Students were already familiar with Function Probe from a previous course taught for the same class (although with a different pedagogical approach). Function Probe has three windows - graph, table and calculator - which are connected through "send" commands that can send data from one window to another. Every window has algebraic facilities.

Theoretical Framework

Problem posing by students has recently gained greater attention among mathematics educators (Silver & Leung 1992, Leung & Silver 1992, Silver & Cai, 1993). Although it has been a subject of investigation for more than a decade in the U.S.A. (Kilpatrick, 1987; Brown & Walters, 1983), it has never gained greater enthusiasm. Even constructivist mathematics educators (Confrey, 1990, Confrey & Smith, 1991, Schoenfeld, A, Smith III. J, and Arcavi, A., 1989) who believe that most things (or all) are constructed by students seem to have overlooked this area of investigation. This lack of attention to problem posing may indicate that, for most mathematics educators, such an approach is simply not feasible. It may also reflect skepticism about its effectiveness.

Recent studies and curriculum proposals in problem posing (NCTM, 1989, Leung & Silver 1992) have focused on having students generate problems with pre-specified results or that utilize specific arithmetic operations. Although studies focusing on this kind of task may become useful if problem

[1] I would like to acknowledge the comments and criticism made by Dr. Ole Skovsmose, University of Aalborg, Denmark, on earlier version of this paper, even though he is not responsible for the content of this version.

posing becomes a trend in mathematics education, it does not answer the question of how open problems might be designed by students. In this exploratory study, students faced a different kind of challenge. They were asked to identify situations or questions that they wanted to investigate and to apply mathematics to their study. It was also expected that students could also learn new mathematics as they were dealing with a given situation. In other words, students (interacting with the teacher) were to identify a realm of interest of their own, to conceptualize a problem related to it, to propose a way of approaching the problem, and then try to employ mathematical tools to solve the problem/situation they had posed for themselves.

Although this study took place in the U.S.A., its theoretical inspiration is derived from two Brazilian mathematics education traditions: ethnomathematics (D'Ambrosio, 1985, Borba, 1987, 1990, 1992) and "Modelagem" (Queiroga, 1990, Nobre 1989), which can be translated as "modeling". In ethnomathematics, several studies have been realized outside the school setting investigating how the generation of problems could express a culture of a given socio-cultural group (Borba, 1987). In "modelagem", students in a school setting have "chosen" - with the agreement of the teacher - a theme to be studied for several consecutive classes. Both pedagogical approaches, which have become increasingly intertwined, have their roots in the ideas of Freire (1992) and Dewey (1938); especially the one where learners participate in curriculum design.

Problem posing by students, in this framework, therefore has importance beyond the goal of improving instruction. Problem posing is seen as a way of democratizing the classroom, since students also take part in the decisions about their own education. Also, it is expected that, as students pose their own long term problems, traces of their culture will surface, because one's interests are embedded in cultural values. If more research supports this contention, it may offer an effective approach for working with diversity in the classroom, since cultural diversity could be expressed in the choice of the problem.

In this framework, the teacher will have a crucial role. The teacher will have to decide whether the theme chosen by the students has potential for a significant educational experience (Dewey, 1938). If she/he is convinced the theme is not suitable for an investigation, she/he will have to convince the students of her/his point of view. Moreover, a teacher will have to be prepared to study themes identified by students with which she/he is not familiar.

The Study

This exploratory study took place in an alternative school in Ithaca, NY, USA in the school year 1990/91. The school had about 300 students from 6th to 12th grade. Classes were small in size, rarely exceeding 25 students. A nine week long (one quarter) course was offered. Class met twice a week; once for three and half hours and another for one and half hours. As researcher/teacher, I went to the school at the end of the previous year and advertised the course for all the high school students. The course was advertised as a course in which they would have to choose a problem, with some help from the teacher, and work with the theme/issue/problem for a period of nine weeks. It was expected that the participants would agree on one theme to make it easier for the teacher to handle the course. The students were also told that computers would be available for their investigation if they wanted to use them. Eight students signed up for the course and seven completed it. According to a mathematics teacher who knew the students, the eight students represented a range of mathematical competences.

This study focused on two issues: 1) It explored the possibilities of students working with a problem posed by them for a long period of time; 2) It explored the application in the formal school classroom of the approach summarized in the previous section. Previous research, based on the ethnomathematical approach to problem posing, has been mostly conducted in educational settings outside the formal school classroom.

The data collected included video-tapes of the classes taught and of conversations between the students and the teacher/researcher. It also included students' journals of written reflections about the themes that arose throughout the course and other activities developed by the students on their own initiative or in response to questions raised by the teacher. The video-tapes and students' written journals, together with the teacher's own written journal notes and the written work undertaken by students in the course of their projects, provided an opportunity for "data triangulation" (Lincoln & Guba, 1985) which is still an ongoing part of the data analysis in progress.

Although the curriculum was open to contributions from the students, the teacher/researcher had his own agenda: improving students' understanding of mathematical functions. As they collected data about the themes they had chosen, the teacher expected to help them to model variations in data using different functions and to improve their understanding - in the context of their investigations - of different coefficients of different functions.

Results

Since analysis is still underway, the results presented here are only preliminary. Five weeks before the course was officially scheduled to begin, I met with the students who signed up for the course to discuss which themes they wanted to investigate. After a period of brainstorming, four themes arose that elicited varying levels of excitement from the students. These themes were: a) Roller coasters and amusement parks; b) McDonalds and its economical impact in the world; c) the great depression in the USA and; d) the distance of clouds from the earth.

Although these themes had no strong opposition from any member of the group, neither did any of the themes really generate enthusiasm among them. As the discussion continued, one student mentioned inflation as a theme, pointing to how they might not be able to attend college if inflation continued at the current rates. This student wanted to know how inflation grows and how "it affects our lives". Another student was interested in investigating the relationship between inflation and homelessness.

The teacher settled on the theme of "inflation" to be studied due to five criteria: i) the interest demonstrated by some of the students and the possibilities for expanding the theme in different directions such as education or homelessness, which could accommodate different interests among students; ii) absence of strong resistance from any students to work with the theme; iii) although the mathematics developed by them was going to be influenced by the way they organize their project, the theme was potentially rich to explore themes related to the conceptual field of multiplicative structures and functions; iv) the theme could facilitate interdisciplinary practice and the study of how the students might construct "different mathematics"; v) I, as the teacher/researcher, was myself interested in the theme. These criteria are suggested by the theoretical framework previously discussed, which includes the relevance of taking into account students' and teachers' participation in curriculum design as a means of expressing different cultural traces of the students and teacher.

As the course began, five weeks later, most of the students continued to be interested in inflation, although the issue of housing prices arose as a dominant issue. Housing prices eventually became the theme of investigation for two groups of three students. A dissenting group formed by two students, later reduced to one student, convinced the teacher that they had the right to work with a theme other than housing, since the course meant to involve students in the curriculum design. Moreover, they had already chosen an alternative theme: fractals. The three groups had quite a different dynamic, as might be expected.

Group 1

The following questions guided the investigation of Group 1: "How do politics, seasons and other events influence the housing market?", "How do housing prices vary by geographic region?" and "How are house values assessed?" (Handout #3). It took them two to three weeks to formulate these questions, especially because different members of the group were absent on different occasions.

The work developed by this group after they chose their questions had two different phases: the first, data collection, was marked by enthusiasm and quite an intensive exchange between students and the teacher; the second one was marked by dispersion and lack of interaction with the teacher. The group used the software to store their data, but continually avoided contact with the teacher and could not make any more indepth analysis from the data.

Group 2

In contrast with Group 1, Group 2 got off to a rapid start. As one member went to visit a real state agency, another obtained some data through the phone and a third member of the group, Mayra, went to the County Library to look for bibliographic references. According to Mayra's description in her journal, this moment was very important for her since she felt that she had control over her learning. These specific activities developed by the students were done outside the class schedule, although such activities were done during regular class hours as well. Group 2 at first decided to focus on the local housing market but later decided to switch to studying how affordable housing would be for different socioeconomic groups.

This group had no problems asking questions of the teacher. They raised issues such as "now, Marcelo we have this graph but we don't know what to do with this". When that occurred, the I proposed that they should try to use prototypic functions (Confrey & Smith, 1991), such as $y=x^2$, $y=|x|$ and $y=a^x$, to model the points they had plotted into the graph window, using the table window as a path from the "raw data" to the graph. Next I led them into a discussion about whether a given prototypic function should be translated and/or reflected and/or stretched in order to better fit the data they have collected. This discussion was also mediated by Function Probe, in particular by icons in the graph window which allow students to transform graphs without having to be concerned with equations initially. In approaching this problem, the students also faced problems that were in the "computer context" (Borba, 1993). They faced scale problems such as one in which they had prices plotted that ranged from US$20,000 to US$100,000 in the y-axis and months represented by 1, 2 , 3 ... on the x axis. As they tried to plot the prototypic function $y=x$, nothing appeared in the screen, since with the scale they had on each axis, $y=x$ was approximated by $y=0$ in the portion of the graph displayed on the screen.

Finally, the work of Group 2 led them to engage in discussions that might be considered "less mathematical" if a narrow view of mathematics is taken. They debated whether the mathematics used, or any mathematics, would be efficient enough to make some kind of prediction about the future variation of housing prices, since if the Gulf War had started the prices would have behaved differently than if the war had not started.

Group 3

As mentioned before, the two members of this group argued that they wanted to work with fractals instead of housing prices. Soon after the course started, one of the members of the group dropped the class due to personal reasons. We initiated the study of fractals studying a text that was found to be too difficult for the one student remaining. To overcome the difficulties of the text, I prepared handouts to put the student in touch with some of the more well-known fractals and to help him to "discover" the formula $M^D=C$, where M is the magnification factor, D the dimension and C the number of copies of the self similar figure after being magnified. The "discovery" of the notion of fractal dimension, embedded in the above formula, fascinated Mark, the student that was working with this problem. I tried to lead Mark in the task of using fractals to study the variation of housing prices through long periods of time (Mandenbrolt actually studied the variation of commodities prices over the years) but he did not show any enthusiasm about this study.

Discussion

As argued by many authors, such as Hancock & Kaput (1990) and Borba (1990), working with real data made a difference. For Group 1 and Group 2, it made a difference in the level of excitement about data collection. For Group 2, real data also helped them deal with new issues in mathematics, such as transformations of graphs to fit data points, and helped them to be critical about use of mathematics. It should be added that, besides "working with real data", it seemed important for some of the students to look for real data and decide which part of this data would be useful and at the same time feasible for them to work with.

Mark did not work with what is usually called real data; but both groups and Mark had a say in the curriculum design of this course, which seemed to have positive effects in every student. It should be said, however, that Group 1 did not seem to go very far, and more investigation is necessary to know which were the causes of their lack of engagement in the task. It could be the case that they were not "really" interested in the theme to begin with, and that therefore the main assumption of this paper holds; however if this is the case, it means that the teacher/researcher failed to recognize this lack of interest at the time the theme was being chosen.

It should be noted that the effects of computer use in the approach of problem posing can be highly beneficial, in the sense that it facilitated the manipulation of "real data" and generation of graphs. Also, the use of computers generated new "mini-problems", such as the "scale problem" that helped them to better understand how computers "manipulate" mathematics as well as it raised conceptual issues related to graphs and the "a" coefficient of linear functions (y=ax+b) .

Conclusion

It can be said that this study supports the notion that problem posing in the way discussed in this paper is feasible. In this study, some students experienced a part of the process of mathematization which is absent from mathematics classes and, in the process, learned some new mathematics topics. Some students felt control over what they were learning, which is seen as result of students' participation in curriculum design. Therefore, problem posing, inspired by ethnomathematical studies done outside the classroom, may be feasible in regular classrooms not only as means of improving instruction, but as means of democratizing the classroom through empowerment of students.

However, more studies are needed in order to study the effects of this approach in larger classes. It should be also said that the data analyzed so far corroborates Skovsmose (in press) who argues for more systematization in courses employing long term projects such as the ones carried out by students in this study. In other words, the fact that students are involved with the choice of the theme and with the decisions during the investigation does not mean that the teacher should not try to summarize the activities made and the mathematics developed at the end of a project as a way of inviting students to reflect on their previous activities. In the class studied, the only summarizing activity was a final presentation by each group which proved to be insufficient to provoke more indepth reflections by the students regarding the mathematics developed in the course. Also, according to Skovsmose (in press), students should be encouraged to have, at the end of the task, a final public product. For instance, in this case, students might have tried to publish their findings about housing prices in a local newspaper.

Finally it should be said that there was a major drawback to the design of this study. Since this was not designed to be an ethnographic study, and since my interaction with the students was too short-term, no claims can be made about relationships between the culture of these students and mathematics education. Such a study would be very complex, since in complex societies like the USA, each student belongs to several cultural groups at the same time; nonetheless, it should be undertaken in the future, if we want to further explore the relationship between problem posing and culture as a means of acknowledging and valuing diversity in the classroom of a complex society.

Bibliography

Borba, M. C. (1993) - Students' understanding of transformations of functions using multi-representational software - Doctoral Dissertation, Cornell University, Ithaca, New York, EUA.

Borba, M. C. (1992) - Teaching mathematics: ethnomathematics, the voice of sociocultural groups. The Clearing House, 65 (3), Jan/Feb, pp. 34-135.

Borba, M. C. (1990) - Ethnomathematics and education. For the Learning of Mathematics, 10 (1), pp. 39-43.

Borba (1990) - Students Input in the Design of Mathematics Curriculum:

A Project-oriented Approach. Umpublished manuscript, São Paulo, Brazil: UNESP-Rio Claro.

Borba, M. C. (1987) - Um estudo de etnomatemática: Sua incorporação na elaboração de uma proposta pedagógica para o "Núcleo-Escola" da Vila Nogueira-São Quirino. Unpublished master thesis, UNESP, Rio Claro, São Paulo, Brasil.

Brown, S. and Walter, M. (1983) - The art of problem posing. Philadelphia, Pa: Franklin Institute press.

Confrey, J. (1990) - A review of the research on student conceptions in mathematics, science, and programming. In Courtney Cazden (Ed.), Review of Research in Education, 16. Washington, DC, USA: American Educational Research Association, pp 3-56.

Confrey, J. (1989) - Function Probe© [computer program]. Santa Barbara, CA, USA: Intellimation Library for the Macintosh.

Confrey, J. and Smith, E. (1991) - A Framework for functions: Prototypes, multiple representations, and transformations. Proceedings of the Thirteenth Annual Meeting of the Psychology of Mathematics Education, Vol. I, October 16-19, Blacksburg, VA, USA, pp. 57-63.

D'Ambrosio, U. (1985a) - Ethnomathematics and its place in the history of pedagogy of mathematics. For the Learning of Mathematics: An International Journal of Mathematics Education.

Dewey, J. (1938) - Experience and education. New York: The Mac Millan.

Hancock, C. and Kaput, J. (1990) - Computerized tools and the process of data modeling. A paper presented at PME-Mexico, July.

Freire, P. (1992) - Pedagogia da esperança. Rio de Janeiro: Paz e Terra.

Kilpatrick, J. (1987) - Problem formulating: where do good problems come from? In A. H. Schoenfeld (Ed.), Cognitive science and mathematics education. Hillsdale, NJ: Lawrence Erlbaum Associates.

Leung, S. and Silver, E. (1992) - Mathematical problem posing by prospective elementary school teachers: an examination of the influence of discovery and invention formats. A Paper presented at AERA, San Francisco, April.

NCTM (1989) - curriculum and evaluation standards for school mathematics. Reston, VA:NCTM.

Nobre, S. (1989) - Aspectos sociais e culturais no desenho curricular da matemática. Unpublished master thesis, UNESP, Rio Claro, Sao Paulo, Brasil.

A. Queiroga, M. (1990) - Considerações sobre a modelagem matemática e a educação matemática. Umpublished master thesis, UNESP-Rio Claro, São Paulo.

Schoenfeld, A, Smith III, J, and Arcavi, A. (1989) - Learning. Unpublished manuscript, University of California at Berkeley, Berkeley, CA, USA.

Silver, E. and Leung, S. (1992) - Mathematical problem posing and problem solving: an invstigation of processes, products and relationships. A Paper presented at AERA, San Francisco, April.

Silver, E. and Cai, J. (1993) - Mathematical problem posing and problem solving by middle school students. A paper presented at the Annual Meeting of AERA, Atlanta, GA.

Skovsmose, O. (in press) - Towards a philosophy of critical mathematics education.