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ABSTRACT

Ten high-ability middle school students participated in a 13 week enrichment program on inferential statistics. Consistent with the NCTM Standards, classes emphasized problem solving in real world contexts and the communication of mathematical ideas. Students struggled with written assignments, yet worked effectively in pairs, small groups, and whole class discussions. Students also proved capable of teaching the class and developing their own tests. Significant improvement in student performance from pretest to posttest showed that instruction improved students' statistical understanding. Analysis of students' written work and protocols of classroom dialogues indicated that students had considerable experience with both chance and inferences in decision-making. Students were familiar with the language of probability although they struggled with the concept of equally likely outcomes. Students were comfortable with the use of sampling in their everyday lives but their understanding was limited to particular problem contexts and constrained by their inability to make spontaneous connections between statistical results and how those results might be used. Contains 25 references. (Author/MKR)

Statistics in Middle School: An Exploration of Students' Informal Knowledge

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Abstract

Ten high-ability middle schoolers participated in a thirteen week enrichment program on inferential statistics. Consistent with the NCTM Standards, classes emphasized problem solving in real world contexts and the communication of mathematical ideas. Students struggled with written assignments, yet worked effectively in pairs, small groups, and whole class discussions. Students also proved capable of teaching class and developing their own tests. Significant improvement in student performance from pretest to posttest showed that instruction improved students' statistical understanding. Analysis of students' written work and protocols of classroom dialogues indicated that students had considerable experience with both chance and inference in decision-making. Students were familiar with the language of probability although they struggled with the concept of equally likely outcomes. Students were comfortable with the use of sampling in their everyday lives but their understanding was limited by particular problem contexts and their inability to spontaneously make connections between statistical results and how those results might be used.

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As mandated by the National Council of Teachers of Mathematics' Curriculum and Evaluation Standards for School Mathematics (NCTM, 1989), statistics will play an increasingly important role in the K-12 mathematics curriculum. In the past, this subject matter has been reserved for college courses and occasionally high school courses. Consequently, little information (from research or practical experience) exists to guide the implementation of NCTM's Standards in statistics. Similarly, there is a paucity of research on the effectiveness of instruction in enhancing students' statistical understanding. (See Lajoie, Jacobs, & Lavigne, in preparation, for a review of the research on pre-college learning of statistics.) This study[1] provides both some initial recommendations for instruction and evidence that instruction can improve statistical understanding.

While numerous research efforts are currently in progress to investigate statistics learning at pre-college levels, as a group, these projects have been remarkably silent on the issue of children's statistical thinking. At the same time, researchers in other content areas have begun to see the value in building instruction on children's thinking (Hiebert & Carpenter, 1992). An understanding of children's thinking can provide teachers with a starting point for instruction and a framework for understanding their students' development of additional knowledge. This study investigated students' informal statistical knowledge -- knowledge which may stem from school or non-school environments, but exists before students formally study statistics. Precedence for this approach can be found in the research on out-of-school learning (Carraher, Schliemann, & Carraher, 1988; Lave, 1988; Resnick, 1987; Saxe, 1988; Scribner, 1984) and in several mathematics education programs which have been able to successfully build instructional programs based on students' informal knowledge (for example, see Carpenter & Fennema, 1992; Cobb et al., 1991; Mack, 1990).

In short, the primary goal of this study was to examine students' informal knowledge that may influence their learning of statistics. This informal knowledge was examined in the context of an enrichment program for high-ability sixth and seventh graders. Both the content and pedagogy of the enrichment classes were designed to provide students with an opportunity for extended thought and discussion about statistical problems. The secondary goal of this study was to explore whether instruction improved middle schoolers' statistical understanding and provide teachers with some preliminary activities that elicit and encourage discussion about statistical issues.

Content of the Enrichment Program

The enrichment program was designed to help students explore the conceptual components of inferential statistics (i.e., the use of the results from a sample to make a conclusion about a larger population). Based on the assumption that chance and sampling are the conceptual core of inferential statistics, the lessons focused on:

(1) chance and its role in everyday decisions

(2) sampling and the logic involved in identifying the situational factors that affect one's ability to draw population conclusions from sample results.

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[1] This study was part of the first author's master's thesis completed at the University of Wisconsin-Madison in 1993 under the direction of Dr. Susanne Lajoie.

A focus on inferential statistics was selected for two reasons. First, the issues involved with inferential statistics are consistent with those recommended for this age group by the NCTM Standards and the curriculum guidelines published by the American Statistical Association (1991). Second, while there is minimal research on students' understanding of any statistical issues, research on inferential statistics is particularly sparse. Furthermore, many of the current statistical projects emphasize descriptive statistics (especially data visualization) over inferential statistics (for example, see Day, Webb, Nabate, & Romberg, 1987; de Lange, Burrill, Romberg & van Reeuwijk, 1993; Hancock, Kaput, & Goldsmith, 1992).

Pedagogy of the Enrichment Program

The pedagogy of the enrichment program was designed to be consistent with the NCTM Standards. The Standards recommend a constructivist pedagogy which entails an entirely new view of mathematics learning. The Standards redefine "knowing math" as "doing math." This approach promotes an activity-based curriculum in which students act as problem solvers, and a new emphasis is placed on students' informal knowledge and their ability to communicate how they reached an answer. With the move toward more authentic activities and skills, there has been a similar movement toward more authentic assessment. Although the authentic assessment movement is still in its infancy, most agree that assessment should be incorporated into instruction and consistent with a notion of cognition that is socially shared, contextualized, and focused on problem solving (Lajoie, 1991).

In this study, the enrichment lessons emphasized problem solving in real world contexts and the communication of mathematical ideas. Conceptual problem solving was strongly stressed over computational skills. Similarly, students were required to explain how they arrived at solutions, rather than simply providing "correct" answers. Assessment was also modified to reflect these new beliefs about knowledge and learning.

Method

Ten high-ability middle schoolers in rural Wisconsin participated in a thirteen week enrichment program on inferential statistics. A researcher/teacher worked with the students for forty-five minutes per week during the students' regular mathematics class. The enrichment lessons were considered part of the students' regular curriculum as the students' performance in the enrichment program was incorporated into the students' semester grade in mathematics. The enrichment lessons were designed to promote statistical discussions, and activities were invented or adapted from existing problems described in textbooks and research articles. Performance measures included class participation, weekly homework assignments, and a comprehensive exam (administered as a pretest and posttest). All class dialogues were audio-taped, and protocol analysis provided the core of the analyses.

Results and Discussion

The results of this study contribute to the research on students' statistical thinking about chance and sampling, the impact of instruction on statistical understanding, and guidelines for the development of instruction to encourage statistical discussions. The following sections describe what was learned from the enrichment classes for each of these areas.

What did the Students Understand about Chance?

All students seemed to have a workable definition of chance. The students defined chance as follows:

Student (S): When you're not sure how something's going to end up.
S: Chance, you have a 50-50 chance.
S: No, no.
S: It could be more likely, equally likely.
S: The outcome could be anything.

Students recognized chance in their everyday lives, and were able to describe how chance affects the decisions they make. Students easily generated examples of chance events in their everyday lives such as taking a shot in basketball, the lottery, and telling someone a secret (i.e., you do not know whether they are going to tell it or not). On the other hand, students varied in their ability to actually compute probabilities. However, even when students were unable to compute exact probabilities, they were able to use the language of probability (e.g., more likely, less likely, impossible, etc.).

These observations question the standard textbook approach to introducing chance in terms of calculating probabilities for standard probability items such as coins and spinners. Perhaps students can and should discuss chance and its role in their everyday lives before learning how to compute probabilities. Most importantly, educators need to be careful not to underestimate the students' conceptual understanding of chance due to their numerical difficulties.

Despite their informal sense of chance and probability, most students had difficulty understanding chance events with equally likely outcomes. The concept of equally likely outcomes refers to chance events whose outcomes have the same probability of occurrence. For example, tossing a coin is a chance event. The two possible outcomes are heads and tails, and each outcome is equally likely. Equally likely outcomes are not limited to events with only two outcomes. For example, rolling a die is a chance event with six possible outcomes, each of which is equally likely.

Students demonstrated an incomplete understanding of chance events with equally likely outcomes. For these events, students did not focus on the probability of occurrence of the outcomes. Rather, they concentrated solely on the number of possible outcomes. For example, students were asked to generate examples of chance events with more than two possible outcomes, all of which were equally likely. Responses included "getting on a horse, and getting bucked off, being fine, or not even getting on", "a hockey game one team could win or the other could win or it could be a tie," and "consequences if you get in trouble (office referral, call home, time after class)." Each of these responses correctly identified more than two possible outcomes, but none of the outcomes were equally likely. On the other hand, students had little trouble with unequally likely outcomes. Students easily distinguished more likely outcomes from less likely outcomes. For example, students understood that having cereal for breakfast is more likely than having cereal for lunch.

The students' difficulties with equally likely outcomes are not surprising given that few of these events exist in their everyday lives. Most everyday events with equally likely outcomes fall into one of these two categories: (1) standard probability games or items such as coins, dice, or spinners; (2) events with a percent indicator such as a 50% chance of rain tomorrow, or player X has a 50% chance of making a free throw. Nonetheless, the concept of equally likely outcomes is a fundamental statistical concept. Given the students' difficulties, instruction needs to pay special attention to this topic.

<u>What Did the Students Understand about Sampling?</u>

Students easily identified the use and importance of sampling in their everyday lives. In particular, students were especially familiar with su.veys. Perhaps this familiarity was due to the recent survey unit in their regular mathematics classes, and/or the abundance of nationally-publicized surveys related to the presidential election happening concurrent with the enrichment class. Students also showed remarkable sophistication in their ability to apply basic logic to real world statistical situations, and this ability improved over the semester. In the following example, students discussed how to select a sample for an opinion poll to determine the favorite fast food restaurant of all the sixth and seventh-graders in Wisconsin:

Instructor (I): How do you decide who to put in your sample?
S: Anybody.
S: No, just sixth and seventh-graders ...
S: You could also um -- maybe go to every county and maybe ask three middle schools and go around the state asking.
I: Ok, how come you want to go to different places in the state?
S: So you have a basic idea of everything.
S: So you have a random sample.
S: And also maybe up north they don't have a Kentucky Fried Chicken in their little town, but in Madison you probably do because it's a larger city.
S: And also cause some people just aren't as rich and they don't go out to eat ever.
I: Ok, so that's good. So different parts of the state might have different fast food places. Different income levels might go to different fast food places. Anything else? What else would you consider when you are making your sample? ... What about males and females? Would it make a difference if I asked just males?
S: Yeah.
S: Probably.
S: Cause maybe a place that offers little things -- women like or maybe girls like that more because sometimes they're not as hungry. [laughter]
S: I don't know that that's all true, maybe girls are more hungry.
S: Like if some are greasy or something. Some potatoes or something.
S: Some people just don't care whether its greasy or not.
S: If someone is on a diet, they don't want grease. ...
S: Vegetarians.

In this example, students recognized the need for care in selecting a sample and were able to identify several pertinent considerations for this particular research question (i.e., geographical location, family income level, gender, and dietary preferences). The importance of the use of logic in statistics cannot be over-emphasized. Given the practical impossibility of random sampling, logic and general reasoning skills are essential for making the conceptual leap from the sample to the population. However, while all students were able to participate in the statistical discussions, many consistently had trouble in two areas. First, students had difficulty incorporating the use of results into their discussions. Second, students were inconsistent in their ability to identify what constituted a representative sample. This inconsistency seemed to be linked to the context of the research question -- opinion questions were different than other questions. These two difficulties are discussed in more detail in the following sections.

Use of results. Students did not consider the use of sample results when discussing statistical issues. Inferential statistics allows individuals to use conclusions to make informed decisions. Students need to consider the implications of these decisions to understand the importance of a representative sample. For example, if a state is trying to determine educational funding based on standardized test scores, it is essential that the sample be representative of the entire state. If decisions were made according to the test scores of a small, unrepresentative sample, the financial implications could be disastrous.

It is unclear why students rarely discussed what decisions would be affected by the results. Perhaps the research questions discussed in the enrichment program were not completely understandable to the students. Although these questions were real world examples, the rationale for how the results would be used was often assumed or not emphasized. The students may not have had enough life experiences to make these connections for themselves. For example, students discussed sampling for state achievement testing, opinion polls to determine soda preferences, and methods for identifying defective products in a factory. These situations are realistic situations, and the results of these projects would certainly have an effect on some aspect of society. Students easily related to the scenarios, often asking pertinent, clarifying questions. However, their questions rarely addressed how the results would be used. While the implications of the results of these projects may be obvious to adults, the students rarely made the connections spontaneously. On the other hand, students were generally capable of discussing the use of results when they were specifically asked about the rationale and potential consequences of a research question.

Consequently, instruction needs to encourage logical thinking by using the students' perspective when developing real world scenarios. Contexts that adults may consider as relevant and realistic may not be contexts with which students have enough familiarity. Without a complete understanding of the statistical situation (including how the results will be used), students cannot fully appreciate the importance of representative sampling. It is crucial to pick scenarios for which students can relate to the questions and how the results will be used. Similarly, it is necessary to help students learn to value the use of results when discussing statistical issues. Initially, implications of results may need to be specified more clearly than would be necessary with an adult audience.

Opinion questions. Students treated opinion questions differently than other research questions. In particular, they had a different conception of representative sampling for opinion questions. For example, students were able to recognize that for a sample of M&M's to be representative of the population, the sample proportions of each color needed to be as close to the population proportions as possible. However, when discussing an opinion question, students suggested that the sample should not be representative of the population. Rather, there should be an equal chance for each option (e.g., 50-50, 33-33-33, etc.). The following excerpt portrays a discussion about selecting a sample for an opinion poll to answer one of the student's research questions: "Who do you think is going to win the presidential election?"

I: How would I pick a random sample?
S: A random sample. Well actually it can't be random cause you might get more Republicans, but you do want an equal portion of Republicans and Democrats.
I: Ok, that's a good question. Do I want an equal portion of Republicans and Democrats? What if there are 60% Republicans and 40% Democrats. When I take a random sample, what do I want?
S: We want half and half.
S: Equal. We want it equal. ...
S: It depends on how many Republicans there are in it cause if there are one Republican and two Democrats, it's unfair for the Democrats cause some people will vote like Clinton and then other people will vote for Brown. And then every Republican will vote for Bush.
[Students were then asked to compare their suggestions to a previous discussion about how to get a random sample of M&Ms in order to estimate what proportion of a small bag would be green, red, etc. In this discussion, students indicated that the sample proportions should reflect the population proportions which are not equal for each color.]
I: Why is it that when I want to take a random sample of the population of all voters, that if there are 60% that are Republicans and 40% that are Democrats, why do I want 50-50?
S: Cause that's an opinion. There, you have to have an equal chance.
S: So it's equal, that each person gets a chance.
S: A 50 -50 chance that each
S: person will win.
S: It depends on how many -- 33% 33% 33%.
I: But is there a 50-50 chance that each person will win?
S [all together]: No! ...
I: Why is it when I have the M&Ms, and I get a sample, that it's a random sample even though there's not an equal chance for all of those colors?
S: Because a random sample is either when you don't know ... or you don't care how many of each different thing but you do care -- you want to know the thing, but before you do all the counting and stuff like that -- you don't -- you just take a random sample, but something like deciding for the president and stuff like, it has to be a different thing. I can't explain why, but I just know that it has to be.

Students held a particularly strong belief that opinion questions are different than other types of questions. In this situation, the students dismissed the goal of having the sample represent the population in favor of giving each presidential choice an equal chance. It is unclear exactly why students considered opinion questions differently than other questions. Perhaps their sense of fairness played a role. Nonetheless, it is important to recognize this distinction and further explore its origins.

Consequently, instruction should present a variety of contexts so that students will not have a limited conception of sampling. Educators should be especially careful with opinion questions. For example, student opinion polls are often used to introduce surveys and sampling. Although this activity has an intuitive appeal for students, when used alone, it may artificially limit the students' conception of representative sampling. The results from this study suggest that opinion polls should be combined with other types of surveys during instruction on sampling.

Did Instruction Improve Students' Statistical Understanding?

The students' overall performance was measured by a fifty point test that was administered during the fourth and thirteenth weeks. A directional Matched Pair Wilcoxon showed a significant improvement in student performance from the pretest ($M=33.6$) to the posttest ($M=38$; $T=54$, $p<.05$). Furthermore, all but one student improved and student improvement was distributed throughout the test. This distribution suggests that participation in the enrichment program increased the students' general level of statistical reasoning rather than one or more particular skill(s). It is important to note that test scoring was consistent with the class philosophy of rewarding correct explanations as well as correct answers. Rating scales were constructed for scoring solutions to open-ended problems. Two raters scored all tests at both time periods, and any discrepancies were discussed until an acceptable score was determined. Interrater reliability was calculated for each question with an intraclass correlation coefficient. The pretest mean coefficient was .86 and the posttest mean coefficient was .92.

In addition to their conceptual gains, students generally enjoyed the course, as indicated by verbal anecdotes from the students' regular teachers and the students' individual written course evaluations. In the written evaluations, student responses on a five point scale (1 = not like the course at all; 5 = liked the course a lot) ranged from 2 to 5 with a mean of 3.9 ($SD=.88$).

What Instructional Activities were Useful for Encouraging Statistical Discussions?

Classroom activities were designed to stimulate discussion about statistical issues. Discussion allowed students to explore a variety of strategies, confront misconceptions, and improve their ability to communicate mathematically. Discussion also provided the instructor with a wealth of information about what students understood and topics with which they were struggling. In general, students were surprisingly articulate during classroom discussions. They worked well with open-ended questions, effectively shared their strategies, and even enjoyed the fact that some questions did not have a single answer. On the other hand, students struggled with open-ended written assignments. The quality of homework and test essays was far inferior to the reasoning demonstrated in oral exchanges. Consequently, it is important not to underestimate students' conceptual understanding by reviewing only their written work. This finding supports the idea that multiple mediums of assessment provide a more complete picture of student reasoning (Collins, Hawkins, & Frederiksen, 1991). Two activities, in particular, promoted discussion and allowed students to learn from each other: a student-generated test and a student-directed class. The following two sections explain each of these activities in more detail and provide suggestions for improved implementation.

Student-Generated Test. Having students generate a test has been suggested as an authentic means of assessing student knowledge (de Lange, Burrill, Romberg, & van Reeuwijk, 1993). At the end of the enrichment program, students split themselves into two teams (by gender), and developed their own test for the course. Each team was required to develop questions to present to the other team in a competition the following week. Minimal instructions were provided, and during test construction, the instructor offered guidance only when the students were totally off-task.

This activity was only mediocre in helping students review for their final exam. However, it provided the instructor with useful information that may have been difficult to acquire with traditional assessment measures. For example, the two teams held different conceptions of what makes a problem difficult. Given that each team was trying to stump the other team, the content and format of the questions reflected what the team considered to be difficult. More specifically, the boys' team concentrated solely on problems involving numbers (e.g., probabilities and ratios). When the boys considered a problem too easy, they made the numbers larger and the mathematical operations more difficult. In contrast, the girls' team was systematic about their test construction, and included all of the major conceptual topics. Their questions involved definitions and applications of concepts, and incorporated a variety of question formats. When the girls considered a problem too easy, they searched for more difficult concepts or a more elaborate question format.

Test construction has great potential to provide educators with a unique glimpse at students' understanding of concepts and the links between those concepts. However, it is unclear how to implement this activity most effectively. The results of this study suggest that the activity needs to be more structured and the following three issues should be considered for improved implementation.

First, the students needed more instructions about the content their tests should cover. Given that the program only met once a week, thirteen weeks was too long to expect the students to remember even the majority of the content in any detail. Instructions need to remind students of general content areas, without limiting their creativity or ability to select what they consider important information. Although one of the benefits of this activity is that students are forced to identify relevant information, it needs to be reasonable for them to consider all of their options. It is important to remember, however, that instructions can have substantial impact with unintentional results. For example, when the girls' team was told that they had to include answers with their questions, they changed the format of their subsequent questions from essay to objective (e.g., true/false, multiple choice, and matching). Therefore, while instructions should provide structure, they also need to promote the desired format of questions.

Second, the instructor needs to consider the mix of personalities and skills before forming development teams. In the enrichment program, the two teams differed drastically in their group dynamics and how they attacked their goal. Consequently, the test produced by each team also varied in format and quality.

Third, test construction needs to become an instructional activity that is incorporated into the curriculum. Students need time to develop test construction skills through modeling and practice -- a single test construction activity is not sufficient. Frederiksen and Collins (1989) suggest the use of libraries of exemplars as a means of helping students understand the assessment criteria so that they can improve their performance. Perhaps examples of "ideal" tests would have helped students better understand the goals of test construction.

<u>Student-directed class.</u> In one lesson, students individually solved the following proportional reasoning problem:

> On a farm there is a fishing pool. The owner wants to know how many fish there are in the pool. She took out 200 fish and marked each of them with a colored sign. She put the marked fish back in the pool and let them get mixed with the others. On the second day, she took out 250 fish in a random manner, and found that, among them, 25 were marked. What is the approximate number of fish in the pool?
> (adapted from Fischbein & Gazit, 1984, pp. 4-5)

Given that the ten students provided seven different answers, each student was given the chance to explain and defend his or her answer. The students taught each other, and the instructor merely supervised the turn-taking.

Students effectively and non-destructively discussed other students' answers. Without knowledge of the correct answer, the students came to a conclusion on their own. Not only did they determine the correct answer of 2000, but they also argued for the most efficient solution strategy. The initial strategies basically fell into two categories: additive or multiplicative. By the end of the lesson, all students were supporting a multiplicative strategy, and were able to apply this approach to similar problems. Not only were the students capable of teaching each other, they seemed to enjoy the opportunity. This class was probably the most memorable class for the students. They often referred to it throughout the semester, and when evaluating the course (eight weeks after the fishing pool lesson), students remembered the exact number of fish in the pool.

While the students' presentations were effective, it was clear that the students' ability to discuss mathematical strategies was less sophisticated than their ability to discuss chance or the logical components of statistical issues. This discrepancy is understandable given that traditional mathematics instruction emphasizes well-defined skills and knowledge and does not require students to develop mathematical communication skills (Resnick, 1988). However, it is becoming increasingly important for students to be able to communicate mathematically. Providing students with the opportunity to teach each other may prove to be an enjoyable and effective method of developing these communication skills.

Conclusions

This study combined instruction with an examination of students' statistical thinking. In addition to the initial glimpse at students' statistical understanding, this study provides hope that instruction can aid statistical understanding. This conclusion is promising for mathematics educators who are now faced with the task of implementing statistics in the curriculum for grades K-12 (NCTM Standards, 1989). Although the study's conclusions are limited by the small sample and restricted ability range, the clinical methodology used provided a rich pool of information that allowed an examination of students' thinking at a very detailed level. Many researchers argue that clinical methodology is a necessary starting point for designing and evaluating instruction (Confrey, 1990). The results of this study suggest that students arrive with a great deal of informal knowledge about statistical issues. It will become important to build on this knowledge if statistical instruction is to be effective.

As statistics becomes a standard component of pre-college curriculums, educators need to carefully consider the role of statistics in the mathematics curriculum. Students in the enrichment program had difficulty understanding why the conceptual components of statistics were classified as mathematics at all. For example, when reviewing the enrichment program, students identified the program as mathematics because "we added, we divided, multiplied, we did fractions." In reality, only one of the thirteen lessons emphasized numbers and operations on those numbers. The students' narrow definition reflects society's tendency to view mathematics as numbers and numerical manipulations rather than a conceptual field. This limited view has been destructive as it has encouraged practices such as the use of memorized algorithms without understanding. As mathematics moves from an elitist domain to an essential life skill for everyday citizens, this conception of mathematics needs to change (National Commission on Excellence in Education, 1983; National Research Council, 1989; Paulos, 1988). Perhaps statistics, with its dual emphasis on logic and numerical evidence, can provide a useful vehicle for expanding the students' understanding of mathematics.

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