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ABSTRACT

This paper discusses a study that explored the mental models of fifth (n=35) and sixth (n=49) graders attending two urban public elementary schools in South Korea in word problem solving. Two levels of mental models (problem model and mathematical model) constructed in the process of word problem solving were investigated. Coding categories for both problem and mathematical models were developed to be used in think-aloud protocol analysis. Results indicate that the majority of students who constructed correct problem models rapidly classified the problems as they read them or restated the problems, focusing on specific words, and related these words to other statements in the problems. Students who misrepresented the problems varied in their problem models: incorrect relation of specific words to other information model, diagrammatic-incorrect location of information model, number-consideration model, and number-operator model. Performance levels in problem solving are discussed in relation to the problem model and mathematical model. Includes samples of solutions. The appendices contain the categories and sample behavior used to determine problem models and the categories used to determine mathematical models. Includes 36 references. (Author/MKR)

Mental Models in Word Problem Solving: An Analysis of Korean Elementary Students

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Abstract

Mental models of Korean sixth (n = 49) and fifth (n = 35) graders were analyzed in word problem solving. Two levels of mental models (problem model and mathematical model) constructed in the process of word problem solving were investigated. Coding categories for both problem and mathematical models were developed to be used in think-aloud protocol analysis. The results of the study indicated that the majority of students who constructed correct problem models classified the problems rapidly as they read the problems, or restated the problems focusing on the specific words and related them to other statements in the problems. Students who misrepresented the problems varied in their problem models (e.g., incorrect relation of specific words to other information model, diagrammatic-incorrect location of information model, number-consideration model, number-operator model). Performance levels in problem solving were discussed in relation to the problem and mathematical models.

Mental Models in Word Problem Solving: An Analysis of

Korean Elementary Students

Numerous studies have reported that the way knowledge is represented by students influence subsequent learning and problem solving (e.g., Glaser & Bassok, 1989; Hong & O'Neil, 1992). Studies investigating knowledge representation such as mental models (Wilson & Rutherford, 1989) or schemata (Rumelhart, 1980) have been concerned with the use of such knowledge structures in various problem-solving situations (e.g., Gentner & Stevens, 1983). Solving word problems involves a complex process that requires different skills such as understanding of the problem situation, selecting data needed to solve problem, choosing operation, translating the problem into a symbolic equation, and computing to solve the problem. Stern (1993) suggests that different levels of mental models (i.e., episodic situation model, problem model, and mathematical model) are constructed in the process of word problem solving.

When constructed properly, the first two situation models (i.e., episodic and problem model) will help problem solvers understand and represent the problem. While the episodic situation model (Reusser, 1990) guides the understanding of specific problem situation, the problem model (Riley & Greeno, 1988; Quintero, 1983) helps one represent the problem by selecting data or elements such as names, objects, or actions/operations, from the episodic story problem. This problem model includes only the relevant information for answering the problem. The mathematical model is derived from the problem model and helps problem solvers describe the problem mathematically, for example, to translate the problem representation into numerical equation and solve the problem.

In discussing the importance of language understanding in the problem solving, Riley and Greeno (1988) developed cognitive models that included two processes: (a) representing patterns of information in the meanings of terms in the text, that is, constructing semantic networks that represent propositions based on problem texts; and (b) constructing semantic models (problem models in Stern's terminology) that represent the problem situation that the text describes. van Dijk and Kintsch (1983) also defined a semantic model as a model of the problem situations, including symbols that correspond to the objects and sets that a child needs to reason about.

The results of the study by Riley and Greeno (1988) indicated that, in solving simple word problems, such as addition and subtraction that involved combinations and changes of sets, inferences are limited to properties of sets that exist in a semantic model. The semantic models of the situations used in their study corresponded to sets of wooden blocks that children put on pieces of green or yellow paper as they work on the problems. However, in solving more complex problems, relations between sets are represented internally and support more complex reasoning.

Effects of wording (cue or key words) in construction of appropriate problem models are examined by a number of studies (Bilsky, Blachman, Chi, Mui, & Winter, 1986; Carey, 1991; Hudson, 1983; Judd & Bilsky, 1989; Karrison & Carroll, 1991). Carey (1991) and Riley, Greeno, and Heller (1983) suggested that initially young children begin problem solving with direct modeling of the action (words) in the problem, and become more flexible in their choice of solution strategies. Judd and Bilsky (1989) found in their study with mentally retarded and nonretarded individuals that cue words facilitated the performance of both groups, presuming that the cue words affected problem representation. More operation errors were also observed for problems without cue words. First- and second-grade children also performed better on the more explicitly worded arithmetic problems compared to the performance on the basic wordings. (De Corte, Verschaffel, & De Win, 1985).

However, emphasizing the use of cue words in problem representation by providing instruction with a simple list of cue words (e.g., all, together, total, for addition or multiplication; left, more, longer, taller, for subtraction; divide equally, in each group, for division) could be misleading when the word problems are not simple and straightforward because some words overlap two operations. In the study of Narayanan (1983) it was found that accuracy of performance was negatively affected by the use of misleading cue words.

Comparing the major solution steps used by many children with the ones used by the computer-generated solution, Briars and Larkin (1984) stated that children translate the words of the problem, not into a complete representation of all relevant information, but into a simple representation containing just the numbers from the problem statement and some operator believed appropriate for computing them. The process of abstracting the operators from key words is prone to error especially when problems require more than one operation. Briars and Larkin (1984) suggested teaching problem solving by encouraging children first to act problems out and later to develop mathematical knowledge for building rich problem representation.

Diagrammatic or pictorial problem representations that represent the objects and quantities in problems also help students construct appropriate problem models (Hong & O'Neil, 1992; Moyer, Sowder, Threadgill-sowder, & Moyer, 1984; Quintero, 1983; Tamburino, 1982). When compared performance of children provided with text (verbal) presentations and with presentations of drawings of collections of objects with associated quantities, Moyer, Sowder, Threadgill-sowder, and Moyer (1984) found that third to seventh grade children did better for problems presented with drawings than for problems presented verbally. Hong and O'Neil (1992) investigated the effects of mental models strategy in teaching and learning statistical hypothesis testing. In their study, mental models analysis was conducted using both experts and less-than-experts, and the results of the mental models analysis were incorporated in the instruction of the domain. Compared to subjects in the control group, subjects who were provided with material in diagrammatic presentation represented problems more diagrammatically and performed better in their problem solving. Evidence for the usefulness of diagrammatic representation in constructing problem models was also obtained with young children between five and eight years old in solving simple word problems. Children who learned to construct appropriate diagrams performed better (Tamburino, 1982).

The notion of domain-specific extensive learning experiences of the experts in acquisition of problem-solving expertise has been discussed by numerous cognitive psychologists (e.g., Anderson, 1993; Goldman, Mertz, & Pelligrino, 1989; Nesher, 1986; Sweller, 1989). A significant factor for obtaining problem-solving expertise is a readily accessible knowledge base of declarative knowledge (e.g., basic math facts in mathematics) along with highly proceduralized (automatized) computational skills that may be determined by the amount of practice and experiences (Anderson, 1993; Goldman, Mertz, & Pellegrino, 1989; Suydam & Dessart, 1980).

Studies related to experts' and novices' behavior in simple math problem solving indicated that adults used memory look-up strategy, resulting in quick response (Groen & Parkman, 1972). When children were given math problems, they attempted to retrieve an answer (a basic math fact) from memory first and then try to solve the problem by computation when retrieval failed (Siegler & Shrager, 1983). It was also evidenced that speed of retrieving basic math facts of normally achieving children (approximately 9 to 10 years of ages) and adults are essentially similar to each other (e.g., Ashcraft, Fierman, & Bartolotta, 1984; Miller, Perlmutter, & Keating, 1984).

In the present study, the notion that different levels of mental models are constructed in the process of word problem solving is employed. The investigator reserves the term problem model for a student's problem representations that may or may not be correct. Students' problem models will be investigated to determine whether students understood or represented the problems correctly or not (e.g., selecting data needed to solve problem, using diagrammatic representation). The term mathematical model is reserved for a student's mathematical representation of the problem: For example, the way the problem representation is expressed in quantity and symbols (e.g., number sentence), and various computation models (e.g., retrieval of math facts, computation steps).

The present study analyzed mental models of Korean students of sixth and fifth graders in word problem solving by examining their mental states and mental processes as they solve problems. Specifically, it was the purpose of this study to investigate students' problem and mathematical models and to determine how they affect students' performance in problem solving.

Methods

Subjects

The subjects were children in two age groups of both sexes, attending sixth (n = 49) and fifth (n = 35) grade in two urban public elementary schools in Korea. The participating schools were considered to be typical schools based on school demographics. Two classes from each school (one sixth grade and one fifth grade) participated in the study. About 50% and 40% of students from each class of sixth and fifth graders, respectively, were randomly selected from those present on the day the investigation was conducted. Students assignment to each class at the beginning of each school year had been made based on the students average achievement levels collected at the end of school year in the major subject matter areas (i.e., Korean language, social studies, arithmetic, and science). This practice had been employed in order to insure that each class consisted of students with various achievement levels but which were equal in averages across classes. This system insured that classes involved in the study consisted of students with various achievement levels.

Materials

The materials were a set of seven elementary arithmetic word problems intended to deal with addition, subtraction, multiplication, and division. The information in each problem included the numerical values of three quantities (whole numbers). To find the answer, which was a whole number, two of the four arithmetic operations should be applied to the three given numbers. Problems that involved addition were the combination type with the total set unknown and problems that involved subtraction were the change type with the result unknown (Riley, Greeno, & Heller, 1983). Problems that involved multiplication or division were the allocation type (Anghileri & Johnson, 1988). Sample of the problems are presented in Table 1.

Insert Table 1 about here

All seven problems could be solved using two steps. However, depending on the subject's problem model and mathematical model (e.g., number sentence) for the particular problem, the particular problem could be solved in three steps. For example, the first problem in Table 1 can be solved in the sequence of "multiplication, multiplication, and addition," or in the sequence of "addition and multiplication."

Original word problems in English were translated by two bilinguals in Korean and English (i.e., there were two sets of translation) and the current investigator compared and finalized the development of Korean version. The names (e.g., John) in the problem texts were changed to common Korean name. Extra care was exercised in translation so that reading problem statements in either version would induce the same types of problem representation and the same order of computation process.

Procedure

Each student participated in the experimental session individually because of the mode of experiment, i.e., think aloud protocol. Due to the intensity of think aloud problem solving, students were allowed to solve problems at their own pace. The session lasted approximately ten minutes. A student was pulled out of his or her class to attend the session and relayed to the next student when his or her session was over. Care was taken not to diffuse the content of the problems.

Students were informed of the general purpose of the experiment and of the protocol procedure. Instruction for the protocol procedure was given by a tape recorder. In addition, the investigator explained that the problem-solving session was not a test and would not be used for grading. Each problem was presented separately on a worksheet. The subjects were instructed to utilize the problem solving worksheet in their attempts to solve each problem; for example, for computation or for problem representation. The problems were given to each child in the same order.

The investigator attended the entire experimental session and reminded the subjects to talk when they lapsed into silence. The subject's protocol for the entire session was tape-recorded. The experiment was conducted near the end of the first semester over a period of roughly two weeks.

Results

Coding Categories and Scoring

For the protocol analysis, coding categories were developed based on the two levels of mental models that may be constructed in the process of problem solving. Because this study was to determine the subject's mental models (i.e., problem model and mathematical model) and how they affected students' problem solving, coding categories were developed to describe each student's problem solving process that led to a solution. In developing the coding categories, the typology of processes and their relative occurrences were considered. No attempt was made to deal with all the information in the think-aloud protocols (Ericson & Simon, 1984; Hong & O'Neil, 1992).

Before developing categories, all participants' protocols were analyzed to enhance the coding categories along with the literature reviews on the problem-solving heuristics and process steps (Abbott & Wells, 1985; Baroody, 1993; Phillips, Uprichard, Soriano, 1984; Polya, 1945). Two major categories and subcategories that are relevant to the current study are presented in Appendix A (problem models) and Appendix B (mathematics models) along with sample behaviors for each subcategory of problem models. Analysis of metacognitive activities such as monitoring, self-checking, or validating activities, were not included in the study. Each subject's protocols on seven problems were analyzed and mapped onto the categories.

In scoring the student's performance, a partial credit system was employed. The maximum score for each problem and the total maximum score were 10 points and 70 points, respectively. The total problem-solving score was computed by aggregating scores over seven problems. Weights were given to each problem based on the types of errors students made. For example, a solution with correct problem models but with erroneous mathematical models (i.e., "slips" in constructing methematical models; cf. computational slips, Norman, 1981) were given eight points; a solution with minor computation errors but with correct mental models were given nine points; a solution with wrong mental models but with correct answers which resulted from the number-consideration or number-operator models were given two points. Also considered in the partial credit system was the multi-step process, that is, when part of the process steps was correct, partial credit was given. In addition, the speed of responses was recorded.

Effects of Mental Models on Problem-Solving

The majority of students constructed correct problem models in the process of solving word problems: 77.2% of the responses (82.5% and 69.8% for sixth and fifth graders, respectively) indicated that students used one of five correct problem models suggested in the present study; 22.8% of the responses (17.5% and 30.3% for sixth and fifth graders, respectively) indicated that students incorrectly represented problems using one of six incorrect problem models suggested in the present study (see Appendix A). Numbers and percentages of responses which used correct and incorrect problem models in word problem-solving are provided for each subcategory (see Table 2).

Insert Table 2 about here

Most students who represented problems correctly either classified the problems rapidly according to solution procedures as they read the problems, or restated/reread the problems focusing on specific words and related them to other statements in the problems (96.7% of the responses). A few students organized or made a list of the critical information in the problems or used diagrammatic problem representation in the process of problem solving (2.7% of the responses). An example of response that employed diagrammatic problem representation is presented in Figure 1. Number-

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Insert Figure 1 about here

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operator models were useful only for the simple and straightforward problems: Students could solve the straightforward problems using the numbers from the problem text and operators selected using key words (e.g., gave). Interestingly, there was only one response which used more than one problem models. In solving one of the problems one sixth grader used both the diagrammtic problem model and the information organization model. However, the diagrammatic model was used as the primary model.

Among those who correctly related the specific words to other information, 42.6% of the responses (51.6% and 28.3% for sixth and fifth graders, respectively) were made by restating the problems in their own words while 57.4% of the responses (48.4% and 71.3% for sixth and fifth graders, respectively) were made by rereading the problems.

Students who misrepresented the problems varied in their problem models. About 20.9% of the responses that used incorrect problem models missed the critical information (e.g., cue words) in solving problems not because of lack of ability in building correct problem models, but because of "slips" in representing the problems (cf. computational slips, Norman, 1981). The protocol analysis indicated that these students were impulsive in answering the questions or less careful in reading the problems.

While some students tried to relate key words to other statements in the problems without success (27.6% of the responses), some were incapable of understanding the problems even after reading the problems several times (26.1% of the responses). These students usually gave up solving the problems. Even though it is very possible that students may employ diagrammatic representation with incorrect location of the information or with incomplete information in the diagram that leads to incorrect solution, the findings indicated that there was not the case in the current study. This could be partly because the word problems in the study were not extremely complex for the current subjects in terms of incorporating critical information in diagram formulation.

Some students manipulated all numbers in the problem statements and tried all combinations of operations (i.e., number-consideration model) until the size of the numbers look reasonable (7.5% of the responses). Even though number-operator models were useful for solving the straightforward problems, students (16.4% of the responses) were not able to represent the problems by properly using numbers and operators simply selected from the key words without relating relevant information in the problems. Examples of number-consideration model and number-operation model are presented in Figure 2 and Figure 3, respectively.

Insert Figure 2 about here

Insert Figure 3 about here

In order to judge if some incorrect responses could be interpreted as incorrect number-operator model use, missing critical information model use, or other incorrect problem model, the particular student's responses to all problems were examined to determine the response pattern and performance level. For example, the response to problem 2 (the first problem in Table 1) should be solved using either "(9 x 10) + (9 x 4)" or " 9x(10+4)" equation (only process steps are considered, i.e., both complete horizontal equation form and subproblem-computation form were considered correct as long as the student used proper process steps). The student's response such as "9 x 10 + 4" was interpreted as either "missing information model" use or "number-operator model" use, by examining the student's response pattern and performance level. For instance, if the responses to other problems were correct and the above response was considered to be a mistake, the response was mapped onto the "missing information model" category.

Most students who had correct problem models also constructed mathematical models properly. Excluding simple computation errors, only 3.1% of the responses (3.2% and 2.9% for sixth and fifth graders, respectively) showed mistakenly constructed incorrect number sentences even though their think-aloud protocols indicated that they had correct problem models. A total of 19 (of 588 responses) simple miscomputation errors were made.

Not counting the students who gave up solving problems, 41.7% of the responses (48% and 32.6% for sixth and fifth graders, respectively) constructed complete horizontal number sentences in the process of problem solving. Among the students who wrote number sentences in computational forms, most students (91.5% of the responses) described the problems mathematically in vertical form. However, a few described in horizontal or mixed forms, 2.1% and 6.4%, respectively.

Not one student estimated in the process of problem solving. This might have resulted because the students have not used estimation strategy in their classrooms or homes, or because the problems in the present study contained only whole numbers which may not require estimation in the current subjects.

Almost all students had basic math facts (i.e., simple one- or two-digit operations such as 9 - 3 = 6, 9 x 10 = 90, 10/5 = 2) stored in long-term memory and retrieved them directly in the process of problem solving: Only two of 566 responses excluding give-ups (22 give-ups all together) did not directly retrieve basic math facts. All students participated in the study applied standard computational method (for sixth or fifth graders level) in their computation of the answers. That is, no students used, for example, counting strategy using fingers or any unusual methods.

The results of the analysis on both problem models and mathematical models indicated that students had little or no difficulty in constructing proper mathematical models, that is, the students in the study could perform very well in writing number sentences either in complete or in computational form and computing answers with only a few slips in computation. However, for those students who did not solve the problems successfully, difficulties arose from the construction of problem models in the process of problem solving.

Performance scores and response rates for each problem and total scores summed over the seven problems are shown in Table 3 and Table 4, respectively. For both sixth

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Insert Table 3 about here

Insert Table 4 about here

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and fifth graders problem 3 (the second problem in Table 1) was the most difficult problem and they also took longer time to complete their problem solving. The problem 3 required addition and division operations. Analysis of the structure of problem 3 indicated that while the problem should be solved by addition first and then division, the text of the problem started with the statement that required division operation. In addition, in the very statement there was a cue word "all" that suggested addition or multiplication. However, using simple representation containing just the numbers from the statement and operator abstracted from the cue word would not be appropriate for solving the problem 3. For the reasons described above students had difficulties constructing correct problem models resulting in relatively poor performance. Eleven students (7 and 4 for sixth and fifth graders, respectively) gave up solving the problem 3 (see Table 4).

Sixth graders performed better on all word problems than fifth graders: Total performance scores of sixth graders were significantly higher than those of fifth graders, $t(55.37) = 2.66$, $p < .01$, using separate variance estimate. However, considering that the problems were developed for the sixth graders, fifth graders did perform well.

Discussion

Most Korean sixth graders in the study constructed problem models properly in the process of problem solving, while the fifth graders performed relatively well considering that the difficulty level of selected word problems were intended for sixth graders. The students who represented problems correctly either had rapid problem classification models indicating that the students were familiar with the types of problems and automatic categorization of the problem were possible, or restated the problems in their own words focusing on specific words and relating them to other information. While some students reread (instead of restated) the problems, they were able to relate the critical information in the problem statements to understand the structure of the problems.

The response rates and performance levels varied due to variation in the construction of problem and mathematical models and in speed in computation. However, most students who could not solve the problem successfully had difficulties in constructing correct problem models while having little or no difficulty in constructing number sentences and computation. The protocol analysis also indicated that almost all students in the study used direct retrieval of basic math facts in their computation. Simple computation errors usually occurred when the students could not recall the basic facts from the memory, for example, 5 + 3 = 7.

Even though only a few students employed information organization models or diagrammatic problem representation models to promote understanding of the problems, based on their good performance it could be predicted that the students who had difficulties building proper problem models, such as the ones who used number-consideration models or number-operator models, could benefit from using information organization or diagrammatic model to understand the word problems. The relative small numbers of diagrammatic or information organization models used in problem solving in this study may be an indication that most high performers did not use those two problem models because they were at the near expert or mastery level in solving word problems provided in this study.

When drawing a diagram in the process of understanding the problem, students used external problem representations, that is, drawing diagrams or figures on the worksheet, as well as internal problem representations (i.e., mental images). For example, some of the diagrams they drew did not have numbers embedded in them. However, the critical information missing in the diagrams might be located and accessed through their mental representation when solving problems.

Many students wrote complete number sentences even if the investigator told them to solve the problem as if they were solving problems alone. This may indicate that the participating classroom teachers might have emphasized writing number sentences in order to understand the problems. Drawing diagrams or writing equations are important activities, especially for the low performers, since it helps the student construct proper problem representation (see also Hembree, 1992)

Interestingly, not one student used estimation in their process of problem solving. The closest remark about estimation made by a student was "since this problem requires multiplication, the numbers will be large." However, if the problems involved decimals or fractions, the estimation process could have been observed.

The problem types and difficulty levels of the problems and their relationships with problem-solving performance were not closely investigated in this study. The relationship and effects of types and difficulty levels of problem on problem solving performance should be further investigated.

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Table 1

Sample Problems That Include the Numeric Values of Three Quantities

Operations	Problem
Addition and multiplication	There were 9 boxes and each of the boxes contained 10 marbles. Terry added 4 marbles to each of the boxes. How many marbles are there all together in the 9 boxes?
Addition and division	Tommy and Judy decided to put all of their candy into 3 equal piles. Tommy had 25 pieces of candy and Judy had 38 pieces of candy. How many pieces of candy were put into each of piles?
Division and Division	Cathy had 105 pieces of ribbon. She divided the pieces of ribbon equally into 5 piles. Bobby took one of the piles of ribbon and divided it into 3 equal piles. How many pieces of ribbon were in each of the piles which Bobby made?

Table 2

Numbers and Percentages of Responses that Used Correct and Incorrect Problem Models

Problem models	Sixth		Fifth		Both	
	Number	Percent[a]	Number	Percent	Number	Percent
Correct problem models[b]						
Classification	112	39.6	76	44.4	188	41.4
Relation	159	56.2	92	53.8	251	55.3
Organization	3	1.1	1	.6	4	.9
Diagrammatic	6	2.1	2	1.2	8	1.8
Number-operator	3	1.1	0	.0	3	.7
Total	283	99.9	171	100.0	454	100.1
Incorrect problem models[b]						
Missing	10	16.7	18	24.3	28	20.9
Relation	23	38.3	14	18.9	37	27.6
Inability	15	25.0	20	27.0	35	26.1
Diagrammatic	0	.0	0	.0	0	.0
Number-consideration	5	8.3	7	9.5	12	9.0
Number-operator	7	11.7	15	20.3	22	16.4
Total	60	100.0	74	100.0	134	100.0

[a]Percentages were computed within each correct and incorrect problem models and for each grade. [b]Description of the problem models are provided in Appendix A.

Table 3

Performance Scores for Both Sixth and Fifth Graders

Problems[c]	Sixth graders[a]		Fifth graders[b]	
	Mean	SD	Mean	SD
Problem 1	9.63	1.60	8.66	2.82
Problem 2	7.10	3.86	6.71	4.06
Problem 3	6.37	4.64	5.69	4.66
Problem 4	9.65	1.48	8.77	2.74
Problem 5	7.78	3.24	6.37	3.65
Problem 6	9.88	.48	8.57	3.55
Problem 7	8.78	3.10	6.51	4.64
Total score[d]	59.18	10.26	51.29	15.28

[a] n = 49. [b] n = 35. [c] Maximum score for each problem = 10.
[d] Maximum total score = 70.

Table 4

Response Rates in Seconds for Each Problem for Both Sixth and Fifth Graders[a]

Problems	Sixth graders Mean	SD	n	Fifth graders Mean	SD	n
Problem 1	52.79	19.51	49	45.24	16.56	35
Problem 2	57.11	26.86	49	59.88	23.33	34
Problem 3	65.94	24.22	42	72.03	34.88	31
Problem 4	49.00	17.01	48	55.36	28.17	34
Problem 5	55.57	26.70	49	55.85	25.64	35
Problem 6	40.30	12.77	49	44.18	21.67	34
Problem 7	66.27	41.04	47	70.66	29.79	30
Total speed	391.56	102.15		403.93	111.29	

[a]Responses from the students who gave up were excluded in computing the speed.

Figure Captions

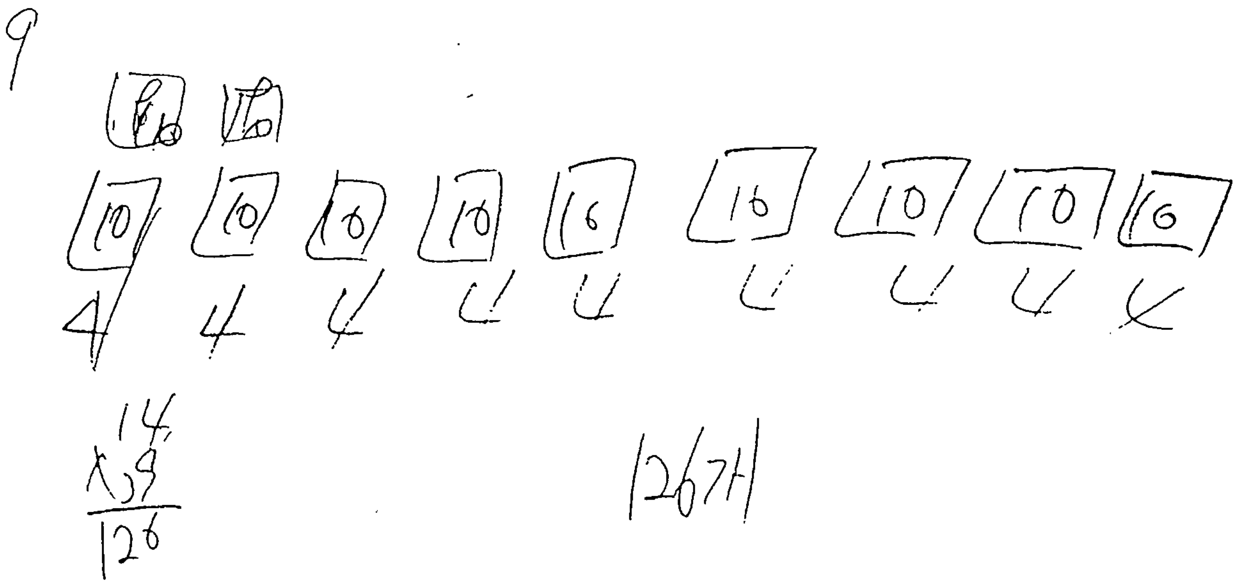
Figure 1. An example of a diagrammatic problem model constructed by a sixth grader.

Figure 2. An example of a number-consideration model constructed by a sixth grader.

Figure 3. An example of a number-operator model constructed by a fifth grader.

There were 9 boxes and each of the boxes contained 10 marbles. Terry added 4 marbles to each of the boxes. How many marbles are there all together in the 9 boxes?

상자가 9개가 있는데, 각각의 상자에는 구슬이 10개씩 들어 있다. 인수는 각각의 상자에 4개의 구슬을 더 집어 넣었다. 그러면, 9개의 상자 속에 있는 구슬은 모두 몇 개가 되는가?



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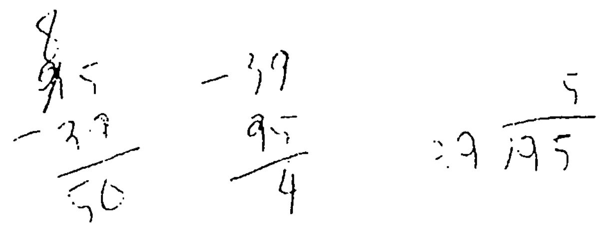
Kim's teacher graded 23 homework papers at school. that night he graded 16 of the papers before he ate dinner. He started with a total of 95 homework papers to grade. How many papers did Kim's teacher nave left to grade?

철수 선생님은 학교에서 23장의 숙제를 채점했다. 그날 밤 선생님은 저
녁식사 하기 전에 숙제 16장을 채점했다. 선생님은 전체 95장의 숙제를 채점해야
한다면, 몇 장의 시험지를 더 채점해야 하는가? (23+16) ~~÷95~~ 56 답: 56장



The subject tried three operations (two subtractions and one division) using the given numbers. The subject's think-aloud protocol indicated that he tried until the answer looked like a reasonable size of number. Even though the final answer was correct, the subject received 2 points (of 10) on his performance.

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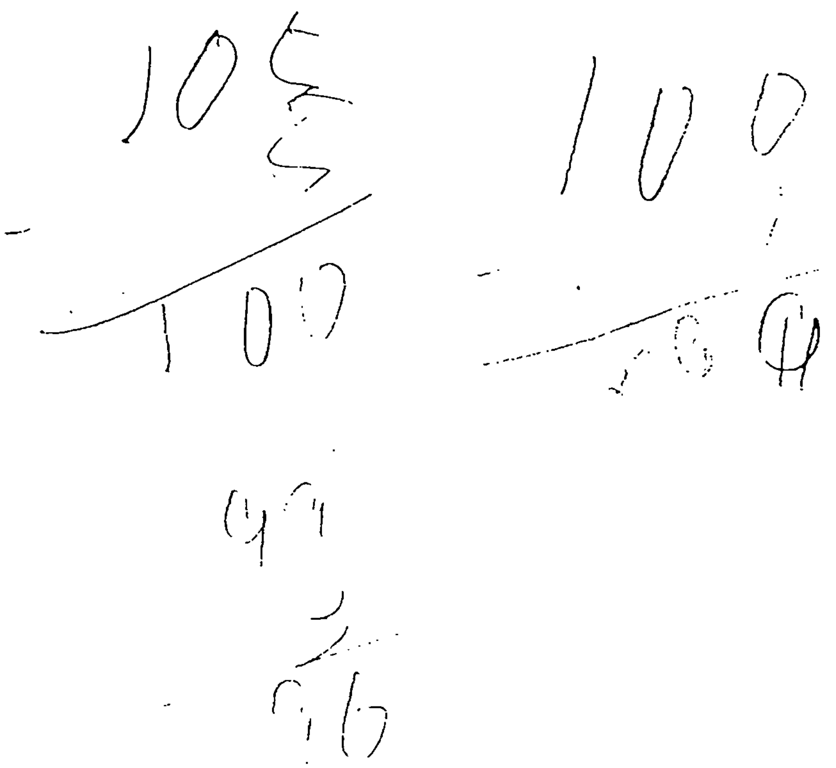
Cathy had 105 pieces of ribbon. She divided the pieces of ribbon equally into 5 piles. Bobby took one of the piles of ribbon and divided it into 3 equal piles. How many pieces of ribbon were in each of the piles which Bobby made?

희영은 105개의 리본을 가지고 있다가 5개의 주머니 속에 리본을 똑같이 나누었다. 원일이가 리본주머니를 1개 가져서, 그것을 똑같이 3개의 뭉치로 나누면, 한 뭉치에는 몇개의 리본이 있는가?



The subject used all the given numbers from the problem statement and an operator (subtraction) that looked reasonable to her.

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Appendix A

Categories to Determine Problem Models and Sample Behavior

Problem Models	Sample Behaviors
Correct problem models	
Rapid problem classification model	Reads and classifies the problem at once according to operations needed
Correct relation of specific words to other information model	Restates or rereads the problem using specific words and relates them to other statements in the problem
Information selection/ organization model	Selects and organizes critical information in the problem
Diagrammatic-correct location of information model	Draws a diagram or figure of the problem (including mental picture)
Number-operator model (for straight problem)	Solves the problem part by part using cue words and numbers
Incorrect Problem Models	
Missing critical information model	Restates or rereads the problem without paying attention to the critical information
Incorrect relation of specific words to other information model	Restates the problem using specific words, but incorrectly relates to other statements in the problem
Inability to determine structure and relation model	Reads several times without understanding the problem structure and relation of the elements (usually gives up solving problem)
Diagrammatic-incorrect location of information model	Draws a diagram of the problem without including all information provided or locate the information incorrectly
Number-consideration model	Tries all possible combination of operation until the result looks reasonable or decides operation based on the size of numbers
Number-operator model	Simply represents the problem containing numbers and operators that looks reasonable without relating all relevant information

Appendix B

Categories to Determine Mathematical Models

Mathematical Representation

Complete number sentence model followed by computational model

Subproblem-computational model: In horizontal form
In vertical form
In mixed horizontal-vertical form

Estimation

Computation

Basic math facts retrieval

Computation process: Standard computational rule application
Other model [strategy] (e.g., counting strategy)
