DOCUMENT RESUME.

ED 373 969 SE 054 560

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TITLE	Calculators: A Learning Environment To Promote Number Sense.
PUB DATE	Apr 94
NOTE	16p.; For related papers, see SE 054 523 and SE 054 559. A paper presented at the Annual Meeting of the American Educational Research Association (New Orleans, LA, April 5-8, 1994).
PUB TYPE	Speeches/Conference Papers (150) -- Reports - Research/Technical (143)
EDRS PRICE	MF01/PC01 Plus Postage.
DESCRIPTORS	*Arithmetic; *Calculators; Elementary Education; *Elementary School Students; *Estimation (Mathematics); Foreign Countries; Interviews; Mathematics Achievement; *Mathematics Instruction; Number Systems
IDENTIFIERS	Australia (Melbourne); Calculators in Primary Mathematics Project; *Mental Computation; *Number Sense

ABSTRACT

The Calculators in Primary Mathematics Project in Australia was a long-term investigation into the effects of the introduction of calculators on the learning and teaching of primary mathematics. The Australian project commenced with children who were in kindergarten and grade 1 in 1990, moving up through the schools to grade 4 level by 1993. Children were given their own calculators to use when they wished, while teachers were provided with some systematic professional support. Over 60 teachers and 1,000 children participated in the project. This paper describes some critical features of project classrooms which supported the development of number sense and reports on the results of interviews with 4th-grade children (n=58), approximately half of whom had long-term experience with calculators. Children with long-term experience with calculators performed better on the 12 mental computation interview items overall, the 24 number knowledge items overall, and the 3 estimation items taken individually. Overall, their performance was better on 34 of the 39 items, with the greatest differences in performance in mental computation generally occurring on the most difficult items. Their pattern of use of standard algorithms, left-right methods, and invented methods for mental computation items did not vary greatly from that of the non-calculator children. Contains 39 references. (MKR)

Calculators: A Learning Environment to Promote Number Sense

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The Calculators in Primary Mathematics* *project was a long-term investigation into the effects of the introduction of calculators on the learning and teaching of primary mathematics. The project commenced at kindergarten and grade 1 level in six schools in 1990, moving up through the schools to grade 4 level by 1993. All children in the project were given their own calculator to use whenever they wished, while teachers were provided with systematic professional support. The purpose of introducing calculators was not to make children dependent on calculators, but rather to enhance that elusive quality "number sense", by providing children with a rich mathematical environment to explore. Over 60 teachers and 1000 children participated over the four years of the project. This paper describes some critical features of project classrooms which supported the development of number sense and reports on the results of interviews with 58 grade 4 children, approximately half of whom had long-term experience of calculators, while the other half did not. Children with long-term experience of calculators performed better on the 12 mental computation items overall, the 24 number knowledge items taken overall, and the 3 estimation items taken individually. Overall, their performance was better on 34 of the 39 items, with the greatest differences in performance in mental computation generally occurring on the most difficult items. Their pattern of use of standard algorithms, left-right methods and invented methods for mental computation items did not vary greatly from that of the non-calculator children.*

Paper presented at the

Annual Meeting of the American Research Association

New Orleans – April 1994

*The *Calculators in Primary Mathematics* project was funded by the Australian Research Council, Deakin University and the University of Melbourne. The project team consisted of Susie Groves, Jill Cheeseman, Terry Beeby, Graham Ferres (Deakin University); Ron Welsh, Kaye Stacey (Melbourne University); and Paul Carlin (Catholic Education Office).

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As early as 1978, a calculator experiment in a UK primary school found a rich variety of uses for calculators in primary mathematics teaching – far from undermining basic arithmetic, the calculator encouraged and helped children to develop their own number skills (Bell, Burkhardt, McIntosh & Moore, 1978, p.1). Since then, there has been widespread agreement amongst mathematics educators that calculators should be integrated into the core mathematics curriculum (National Council of Teachers of Mathematics, 1980, p.9; Cockcroft, 1982, p.109; Curriculum Development Centre and The Australian Association of Mathematics Teachers, 1987). Nevertheless, apart from the *Calculator-Aware Number* (CAN) project (Shuard, Walsh, Goodwin & Worcester, 1991) there is little evidence that such changes are commonly occurring (Curriculum Development Centre, 1986, p.18; Hembree & Dessart, 1986, p.83; Reys, 1989, p.173).

Recently, powerful attempts have been made to change this situation in Australia. The *National Statement on Mathematics for Australian Schools* (Australian Education Council, 1990) endorsed the 1987 national policy on calculator use, recommending that all students use calculators at all year levels (K-12) and that calculators be used both as instructional aids and as learning tools. In line with world-wide trends (see, for example, National Curriculum Council, 1988; National Council of Teachers of Mathematics, 1989; National Research Council, 1989), the national statement emphasises the development of number sense and mental computation, partly in recognition of the role of the calculator.

Calculators are also excellent vehicles for developing number sense, allowing students to explore number freely and construct their own network of mathematical relationships (Howden, 1989, p. 8; Wheatley & Shumway, 1992, p. 3). In this context, number sense, although difficult to define, refers to a well organised conceptual network, which enables a person to relate number and operation properties and use flexible, creative ways to solve number problems (Sowder, 1988, p. 181). Among characteristics of number sense given by Resnick (1989) are its non-algorithmic and complex nature; the fact that it yields multiple solutions; and the judgement and interpretation its use requires (p. 37).

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The investigation focused on the extent and purpose of calculator use; changes in teachers' expectations of children's mathematical performance and consequent changes in the curriculum; long-term implications for numeracy; and changes in teachers' beliefs and teaching practice.

Among a number of papers on various aspects of the project, Groves (1993a; submitted) reported that children with long-term experience of calculators performed better than children without such experience on "real world" problems and computation items, made more appropriate choices of calculating device and were better able to interpret their answers when using calculators, especially where knowledge of decimal notation or large numbers was required.

This paper briefly describes the way in which calculators were used in project classrooms and attemprts to identify critical features which support the development of number sense. It also reports on the results of interviews with 58 grade 4 children, comparing the performance of children with long-term experience of calculator use with that of children without such experience on mental computation, number knowledge and estimation.

The Calculator Learning Environment

Plunkett (1979) contrasted the efficient, automatic, general, but essentially passively used, standard written algorithms with the fleeting, flexible, active, but limited, mental algorithms (pp. 2–3). He recommended the use of mental methods to develop children's understanding of number and regarded calculators as providing the opportunity to abandon standard algorithms in favour of children developing their own mental techniques (pp. 4–5).

In the United Kingdom, the guidelines for the *Calculator-Aware Number* (CAN) project required teachers to discontinue teaching the standard algorithms (Duffin, 1991, p. 42). The CAN project and other non-calculator projects with similar prohibitions on the teaching of standard algorithms (for example, Olivier, Murray & Human, 1990; Resnick, Lesgold & Bill, 1990; Kamii, Lewis & Livingston, 1993) have documented the diverse procedures which children invent under such conditions.

The *Calculators in Primary Mathematics* project, while sharing the belief that children can and should invent their own procedures, did not place any restrictions on ways in which calculators should be used – other than requiring that they be freely available to children – nor did it prohibit the teaching of algorithms. Despite the lack of prescription from the project and the extent to which curriculum decisions are made at the school level, the extensive data collected by the project from a variety of sources allows some features common to many classrooms to emerge.

Calculator use

The project did not supply teachers with activities or a program to follow. Rather, teachers were encouraged to share the activities they devised through regular school and network meetings and the project newsletter.

The presence of the calculator provided children with a flexible tool, which enabled them to explore number in a way which is not possible with the use of concrete materials alone.

Four major ways of using the calculator emerged:

- as an object for discovery – the calculator allowed children to freely explore number, as well as to explore the calculator itself;
- as a counting device – one of the most effective uses of the calculator with young children is as a counting device, enabling children to generate large amounts of data from which they can infer patterns and relationships;
- as a "number cruncher" – the presence of the calculator allows children to work with larger numbers and solve more realistic problems;
- as a recording device – calculators provide young children, who often find writing numerals a time-consuming and onerous task, with the opportunity to easily record (often very large) numbers and change them at will. Hiebert (1989) regards written symbols as important tools for thinking (p. 83). Calculators provide young children with the opportunity to record (and manipulate) a much wider range of numbers than is usually possible at the elementary school level.

Extensive classroom observation revealed that many young children were dealing with much larger numbers than would normally be expected, as well as, in many cases, negative numbers and, to a lesser extent, decimals.

Some examples from project classrooms

Some examples are given below to indicate both the nature of the classroom activities and the number sense being exhibited by young children in the project.

Number rolls: Use of the built-in constant function allowed counting by any chosen number, from any desired starting point. One kindergarten teacher initiated an activity, which she called *number rolls*, which became popular with many project teachers. Long strips of paper were used to vertically record counting on by a constant. Many children began by counting by 1's and continued to do so. Others, however, moved on to counting by numbers such as 5, 10 or 100. At least one child observed that counting by 9's usually leads to the units digit decreasing by one each time, while the tens digit increases by one.

Free exploration: In another example of children using the calculator as a counting device, this time as part of a free exploration segment within a lesson, a kindergarten child decided to count by 1's from 1 000 000. When challenged by another child to reach 1 000 100, he initially said that there was no such number, but as he got to 1 000 079 he began to think that perhaps there was. When he finally reached 1 000 102 he was thrilled to see that he had "gone right past it".

Number line-up: Many children, while using their calculators to count backwards, discovered negative numbers. These "underground numbers" were used and discussed freely in many classrooms. A grade 1 and 2 teacher devised an ordering activity *number line-up,* which involved a small group of children entering numbers of their choice into their calculators and then ordering themselves according to the numbers on their calculator displays. More and more children were added to the "line-up", with each new child needing to find their correct position. Children frequently included negative or very large numbers in this activity and exhibited a quite sophisticated knowledge of the number system.

Tree survey: Grade 2 children exhibited their knowledge of decimals while discussing how to make a pictograph of the results of a tree survey they had carried out. A group of 7 children had found 64 trees of a particular category and needed to cut out 64 pictures of trees to paste onto their chart. When asked by the teacher how many trees each child in the group would need to make, children used their calculators to find that $64 \div 7 = 9.1428571$. The teacher recorded the answer and asked what it meant. A child quickly replied: "It is nine and a bit. So if we made ten each we would have some left over – actually we would have 6 left over".

For more detailed discussion of classroom activities and the role of the calculator, see Stacey (1994); Groves, Cheeseman, Clarke and Hawkes (1991); Groves, Ferres, Bergfeld and Salter (1990) and the videotape*Young Children Using Calculators* (Groves & Cheeseman, 1993).

The conceptual environment

Greeno (1991) likens the domain of numbers to a conceptual environment, where people with number sense know "the lay of the land" (p. 185). He further suggests that, contrary to common assumptions, children have significant implicit understandings of many concepts and can reason on the basis of these when there are appropriate supporting features in the environment (p. 207).

Wearne and Hiebert (1988), as part of their theory for the development of symbol competence, stress the importance of referents with rich associations for the student – usually non-written material, either in everyday use, such as money, or specially designed, such as Dienes blocks (p. 224). We believe that it is precisely by acting as such a referent that calculators can play a critical role in children's development of concepts, such as place value and decimal notation, by enabling children to experiment with and manipulate symbols in a way which is impossible to achieve with concrete materials. In other words, the calculator becomes one of the supporting features of the environment.

Interviews with teachers have revealed that many had changed their beliefs and practice, and now believed that their previous teaching practice had restricted the conceptual learning environment provided for children in their classrooms. For example, one teacher reported:

> I'm a lot happier to go where the children want ... it's a lot less teacher directed... You don't have to structure things so that you know the answer will be within their reach... If the children want to find out stuff, I say "go for it"... because I'm not so worried about them finding out things they won't understand anymore... Why impose artificial boundaries ... and say "you're not allowed to know any numbers beyond 1000"? It's so stupid and that's what we used to do ... I think I'm being a lot more open-ended with their activities... I'm putting more on them to do more finding out.
>
> (Grade 1 and 2 teacher, November, 1991)

The role of discussion

From a constructivist perspective, an important aspect of learning is communicating and constructing shared understanding (Davis, Maher, & Noddings, 1990, pp. 2–3; Wheatley, 1991; Greeno, 1989, p. 45). The presence of the calculator not only provides children with the opportunity to engage in mathematical investigation, it also enables them to share their discoveries with teachers and other children by providing an object which can become the focus of genuine mathematical discussion.

Many teachers in interviews specifically mentioned more sharing and discussion as a change in the mathematics teaching. For example:

> I've had to really encourage them to share what they've done. I think I've always done that in language. I haven't really done that very much at all in maths before. What I see in the use of calculators and how it's changed my maths teaching is that I think I'm teaching maths now more in a way that I've been teaching language for a while, and I've always taught maths much more formally .... It certainly encouraged me to talk to the children much more in maths, and discuss how did they do this, why did they do that, and getting them to justify what they're doing, which I guess previously I haven't done in maths. Much more discussion and sharing.
>
> (Kindergarten teacher, June, 1991)

For a more detailed discussion of teachers' perceptions of changes in their mathematics teaching, see Groves (1993b).

The Grade 4 Interviews

The "number sense" interview

Four different tools – a written test, a test of calculator use and two different interviews – were used over the three year period 1991 to 1993 at grade 3 and 4 levels to determine the long-term effect of calculator use on children's learning of number.

The written test was designed to test children's understanding of the number system; their performance on items related to basic numeracy, such as reading train time-tables; routine paper-and-pencil computations; and their choice of operations for word problems. The calculator use test required children to solve numeric and simple "real world" problems using a calculator. The first interview focused on different aspects of children's understanding of the number system, together with their choice of calculating device for various computational tasks and their solutions to "real world" problems based on division and multiplication.

The second interview, which focused on number sense, was designed to complement the two tests and the first interview described above. A draft version of the *Framework for considering number sense* produced by McIntosh, Reys and Reys (1992, p.4) was used to ensure that critical aspects of number sense were included in this interview if they had not already been covered elsewhere. Items focused on mental computation, knowledge of numbers (including ordering of numbers within and among number types, relationships between number types and place value) and estimation.

The interview contained a total of 12 mental computation items (5 additions, 4 subtractions and 3 multiplications), 3 estimation items, 5 items requiring children to place numbers on a number line, 11 items asking children to state whether a given decimal or fraction is bigger or smaller than another number (usually 1/2), and 8 items related to place value, set within the context of planning for a swimming carnival. Details of the interview items can be found in Tables 2 to 6. The numbering of the items refers to the order in which the questions were asked.

The mental computation items were presented in horizontal form in large type face on individual cards.

Estimation items were also presented on cards (see Figure 1), with an advertising flier for home delivered pizza also being shown to the children for item D3.

D1. I was given a huge jar containing about 1000 jellybeans.

If I share the jellybeans among 3 people, about how many does each person [illegible]

50 300 30 500 3000

Why? / How do you know?

Figure 1. Sample card for estimation item.

For the number line items, children were presented with a large number line and self-adhesive labels, in the form of arrows with numerals, to attach to each number line as appropriate. The numbers and the number lines used are shown in Table 4.

Decimal and fraction items were again presented on cards, with children being asked whether the numbers shown were "bigger or smaller than" 1/2 (or 1 or 2).

The place value items were set in the context of planning for a swimming carnival, with children being told that the numbers on the reply slip from one region had been smudged as a result of a staff-room accident with a cup of coffee (see Figure 2) and that it was important for planning reasons to make some sort of estimate of the numbers.

GRADE 3 AND 4 SWIMMING CARNIVAL

I need to know how many grade 3 and 4 children from your region are coming to the swimming carnival next week.

Please fill in the details below.

Region: Sunny Valley

Grade 3 children: 3

Grade 4 children: 4

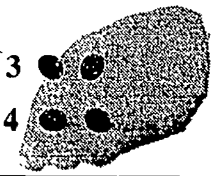

Figure 2. Card used for place value items based on swimming carnival.

This interview was specifically devised to fill gaps which became apparent after the testing and interview program commenced in 1991, and hence was only administered in 1992 and 1993.

Administration and sample

The "number sense" interview was conducted with a stratified random sample of over 10% of grade 4 children at the six project schools in 1992 and 1993 and a similar sample of grade 3 children in 1993. In each year, at each grade level used, three children were selected at random, subject to achieving gender balance, from each complete class at that grade level at each school, while in classes composed of two grade levels, one boy and one girl were randomly selected at each relevant grade level. This resulted in a sample of 27 grade 4 children in 1992, with 31 grade 4 and 27 grade 3 children in 1993.

This paper will only report results for the 1992 and 1993 grade 4 children. At the time of the interviews, the 1993 grade 4 children had taken part in the project for $3^1/_2$ years – i.e. since grade 1 – while the 1992 grade 4 children, who formed the control group, were the last cohort at their schools who had not taken part in the project, and had therefore had minimal exposure to calculators in their schooling.

Children were interviewed individually, with responses recorded on a detailed record sheet. Most items required children to explain the processes used in arriving at their answers. There was no time limit placed on responses, although there was some attempt made to complete interviews in approximately 25 minutes.

Results

All of the results below compare responses by the 1993 grade 4 project children (i.e. those with long-term calculator experience) with responses by the 1992 grade 4 control group (i.e. those with no such calculator experience).

Table 1 compares the scores obtained by the 1992 and 1993 grade 4 children on the 12 mental computation items and the remaining 24 non-estimation items, which have been grouped together under the heading of "number knowledge". Each item answered correctly was given a score of 1, with an incorrect answer or no answer being scored as 0. It can be seen from the table that the 1993 project children performed better than the 1992 control group on both sets of items, with the difference being statistically significant at the $p \leq 0.05$ level for the mental computation items.

Table 1
Comparison of Scores for 1992 and 1993 Grade 4 Children on Mental Computation and Number Knowledge Items

	1992 (n = 27)		1993 (n = 31)	
	Mean	Standard Deviation	Mean	Standard Deviation
Mental Computation	6.704	3.136	8.161	2.609
Number Knowledge	12.667	3.883	14.387	4.47

significant difference between 1993 and 1992 at $p \leq 0{\cdot}05$ level. (df = 56, t = 1.932)

Table 2 provides an item by item comparison of correct answers and methods used for the 12 mental computation items. For the purpose of this analysis, children's answers have been categorised as follows:

Standard: This category includes those responses where children appeared to actually visualise the standard paper-and-pencil algorithm (often pointing to imaginary figures in the air), as well as those where children merely indicated that they had carried out the operation working strictly from right to left in the manner of the standard algorithm.

Left-right: This category was used for responses which, while working from left to right, involved no reformulation of the numbers involved. For example, finding 6 x 25 as " 6 x 20 = 120 and 6 x 5 = 30, so 6 x 25 = 120 + 30 = 150 " was classified as "left-right".

Invented: This category was used for all coherent answers which did not fall into the above two categories. It included those methods which involved reformulation of numbers, such as finding 99 + 187 as " 99 + 1 = 100; 100 + 187 = 287; 287 – 1 = 286". But it also included several instances of children using "counting on" with fingers (such as counting by 25's six times for 6 x 25), as well the use of " 46 x 2 = 92 " for 46 + 46. While in one sense neither counting on nor using multiplication might be considered as "invented", these methods were devised by the children to answer the specific question asked and did not use the taught standard algorithms or a "left-right" approximation of these. For convenience of presentation of the data, for item C5 (24 x 10) only, it also included finding the answer by adding zero, which accounted for approximately half of the responses in this category.

Pass/Unknown: This category includes all passes on the item and the relatively rare cases of children being unable to explain their methods, as well as responses by children who said they guessed the answer (which was again relatively rare for mental computation items).

Table 2
Percentages of Correct Responses and Methods Used on Mental Computation Items

					Method Used			
No	Item	Year	n	Correct*	Standard#	Left-right#	Invented#	Pass/Unknown#@
A1	24 + 32	1992	27	93	41 (100)	48 (92)	11 (67)	0 (–)
		1993	31	94	29 (100)	58 (100)	10 (67)	4 (0)
A2	48 + 37	1992	27	63	37 (67)	48 (69)	15 (50)	0 (–)
		1993	31	87	32 (90)	48 (93)	13 (100)	6 (0)
A3	46 + 46	1992	27	74	44 (75)	48 (85)	4 (0)	4 (0)
		1993	31	90	29 (78)	52 (100)	16 (100)	3 (0)
A4	79 + 26	1992	27	67	37 (60)	37 (70)	22 (83)	4 (0)
		1993	31	84	32 (70)	48 (87)	16 (100)	3 (100)
A5	99 +187	1992	27	33	4 (0)	4 (100)	59 (50)	33 (0)
		1993	31	65	19 (83)	6 (50)	55 (82)	19 (0)
C1	87 – 34	1992	27	77	41 (100)	37 (80)	19 (40)	4 (0)
		1993	31	87	45 (100)	42 (85)	6 (100)	6 (0)
C2	62 – 48	1992	27	19	59 (19)	4 (0)	19 (40)	19 (0)
		1993	31	32	29 (55)	29 (10)	32 (50)	10 (0)
C3	103 – 98	1992	27	48	30 (50)	0 (–)	52 (57)	19 (20)
		1993	31	48	26 (50)	6 (0)	48 (73)	19 (0)
C4	100 – 52	1992	27	67	7 (0)	0 (–)	85 (78)	7 (0)
		1993	31	75	10 (67)	0 (–)	84 (85)	6 (0)
C5	24 x 10	1992	27	56	7 (50)	11 (100)	63† (59)	19 (20)
		1993	31	68	13 (75)	23 (100)	52† (69)	13 (0)
C6	44 x 5	1992	27	37	22 (67)	7 (100)	26 (57)	44 (0)
		1993	31	32	23 (71)	16 (80)	35 (9)	26 (0)
C7	6 x 25	1992	27	41	11 (33)	33 (44)	33 (67)	22 (0)
		1993	31	58	13 (75)	19 (83)	45 (64)	23 (14)

* Figures indicate percentage of correct responses for each item.

\# Figures indicate percentage of all responses using given method.
Figures in parentheses indicate percentage of correct responses for given method.
Note: Percentages are rounded and do not necessarily add to 100%.

@ Includes responses by children who said they guessed the answer

Significant difference at $p \leq 0{\cdot}05$ level using a χ^2 test on frequencies.

† Includes 7 children in 1992 and 8 children in 1993 who "added zero".

In Table 2, the figures in parentheses, following the percentage of responses using each method, indicate the percentage of children using that method who obtained a correct answer.

It can be seen from Table 2 that the 1993 children performed better on 11 of the 12 mental computation items, although the difference was only statistically significant at the $p \leq 0.05$ level for items A2 and A5, and some differences (such as on items A1 and C3) were negligible.

In general, in each of the categories of addition and subtraction, the greatest differences in performance occurred for the most difficult items – A2, A3, A4 and A5 all showed substantial improvements for addition, C2 for subtraction and C7 for multiplication. The only item which did not follow this pattern was C6 (44 x 5), which was difficult but was also the only item on which the 1993 children performed worse. Although the overall performance on C6 was only slightly worse, it was disappointing that the decrease was due to considerably less success by 1993 children in the use of invented methods. We have no explanation for this.

It is interesting to note that for both addition and subtraction, the items with the highest success rate, after the items A1 and C1 which required no bridging, were A3 (46 + 46) and C4 (100 – 52), both of which made use of knowledge of doubling. Item C4 showed by far the highest use of invented methods over all of the mental computation items for both years, with all except three children who successfully used an invented method using a version of "100 – 50 = 50; 50 – 2 = 48".

The patterns of use of different methods did not vary greatly between 1992 and 1993. The major exceptions to this were items A5 and C2. For item A5 (99 + 187), the item which showed the greatest improvement, the improvement shown by the 1993 children could be accounted for equally by the improved success rate for the invented methods, and the shift of responses from the unsuccessful pass/unknown category to a highly successful use of the standard algorithm. This result was unexpected. For item C2 (62 – 48), there was a shift away from the standard algorithm, coupled with an increase in success rate for those using it.

While the patterns of use of different methods did not vary greatly, the improved performance in 1993 was largely due to a general improvement in the success rate for almost all methods on almost all items. Apart from a few negligible negative differences, each representing no more than a difference of one child, the only decrease in success rate was for invented methods for item C5, which was discussed above.

These results concerning patterns of use of different methods are not unexpected in view of results from an intervention study at grade 7 reported by Markovits and Sowder (1994), where, for items of a corresponding type to all of those used here except for C7, there was little change in the methods used before and after the intervention program (pp. 14–16). For item C7 here, so few children used standard algorithms in either year that there was little scope for change.

For approximately half of the responses classified as standard, children were observed mentally applying paper-and-pencil algorithms, often pointing at imaginary numbers and mumbling statements such as "put down 1 and carry 1" – a phenomenon which has been noted in other studies (see, for example, Reys, Reys & Penafiel, 1991; Koyama, 1993).

Table 3 provides an item by item comparison of correct answers and methods used for the three estimation items. For the purpose of this analysis, children's answers have been categorised as follows:

Estimation: This category includes all attempts to obtain the answer by methods other than direct calculation or guessing. For example, the following responses for D1 were all categorised as estimation: "about 336 or 337", "1000 ÷ 3 is nearly 300", elimination – "not 3000, bigger than 1000; 500 only shares between 2 people; has to be 300", "300 + 300 + 300 = 900; close to 1000". Examples for D2 were: "90 + 90 + 90 = $3; 19 + 3 + 6 + 6 = $34" and "$19.90 is close to $20; $6.90 is close to $7; so about $34".

Calculation: Calculation refers to any method which attempts to obtain an exact answer. For example, for item D2 children sometimes attempted to count by 20's to 1000, keeping a tally on their fingers of the number of 20's counted. No children attempted to use the standard division algorithm for either D1 or D2.

Guess: This category was used for all responses which were described by children as a guess or could not be explained.

Table 3
Percentages of Correct Responses and Methods Used on Estimation Items

					Method Used			
No	Item	Year	n	Correct*	Estimation#	Calculation#	Guess#	Pass#
D1	1000 jellybeans, 3 people	1992	27	74	89 (79)	0 (–)	11 (33)	0 (–)
	50, 300, 30, 500 or 3000?	1993	31	87	84 (96)	0 (–)	16 (40)	0 (–)
D2	1000 jellybeans, 20 people	1992	27	67	56 (87)	11 (33)	33 (44)	0 (–)
	200, 50, 2000, 500 or 20?	1993	31	77	61 (89)	6 (100)	32 (50)	0 (–)
D3	Pizza order:	1992	27	44	48 (54)	26 (57)	19 (20)	7 (0)
	$19.90, $6.90 & $6.90	1993	31	61	42 (70)	58 (56)	0 (–)	0 (–)

* Figures indicate percentage of correct responses for each item.
\# Figures indicate percentage of all responses using given method.
Figures in parentheses indicate percentage of correct responses for given method.
Note: Percentages are rounded and do not necessarily add to 100%.

Item D3 (pizza order) presented a number of difficulties. It was intended that the item ask children to estimate the order for a "Pizza Deal" at $19.90, lasagne at $6.90 and spaghetti also at $6.90. However, due to some confusion with the advertising fliers, some children used $6.30 as the price of spaghetti. This was not considered to be a serious discrepancy, as the difficulty of the item appears to be the same in both cases. A more serious difficulty was caused by the fact that children frequently started to calculate the answers immediately, often even before hearing the question. For children who estimated the answer, responses greater than $31 and less than or equal to $34 were scored as correct. However, for children who calculated their answer, only those answers which gave the exact price were scored as correct.

It can be seen in Table 3 that the 1993 children performed better on all three estimation items, although none of the differences were statistically significant. The largest difference occurred for item D3 (pizza order), where the improvement could partly be attributed to better estimation skills, but more so to the disappearance of the highly unsuccessful categories of guess and pass, in favour of an increase in the moderately successful category of calculation. For the two jellybean items (D1 and D2), the pattern of use of estimation remained largely unchanged, while the success rate within each category of method used increased.

Table 4 shows the percentage of correct responses for each of the number line items. In case children had not had previous experience with number lines, children were first given practice placing 10, 5 and 12 on a number line marked at 0 and 20. Items B2a and B2b were presented consecutively, using the same number line, as were items B3a and B3b. In both cases, if children moved their first answer, their final answers were recorded.

Among the original hypotheses of the *Calculators in Primary Mathematics* project was the expectation that project children would successfully deal with larger numbers and acquire certain concepts (such as place value, negative numbers and understanding of decimal notation) at an earlier age than other children. While there was very little difference between the results for the 1992 and 1993 children on the two relatively easy items B1 and B2, the 1993 children performed considerably better on the more difficult decimal item (B2b) and the two negative number items (B3a and B3b). However, contrary to expectations, there was no difference for item B4, which proved to be one of the most difficult items for both groups of children.

Table 4
Percentages of Correct Responses for Number Line Items

No	Number	Number line	1992 (n = 27)*	1993 (n = 31)*
B1	10	0 … 100	85	90
B2a	0.5	0 … 1	70	68
B2b	0.25	0 … 1	11	26
B3a	− 3	0 … 5	59	77
B3b	− 2	0 … 5	52	71
B4	143	0 … 10 000	26	26

* Figures indicate percentage of correct responses for each item.

Table 5 shows the percentage of correct responses and reasonable explanations given for each of the decimal and fraction items. It can be seen from the table that the 1993 children performed better on all five of the single decimal items (E1 to E5), and two of the three fraction items (E7 and E8), although the differences were not statistically significant and some differences (such as on items E2 and E7) were negligible. The 1993 children performed better on only one (E9b) of the three items which asked them to compare 0.47 + 0.89 with 1/2, 1 and 2 .

Table 5
Percentages of Correct Responses and Reasonable Explanations Given for Decimal and Fraction Items

No	Number	< or >	1992 (n = 27) Correct*	1992 (n = 27) Explanation#	1993 (n = 31) Correct*	1993 (n = 31) Explanation#
E1	0.4	1/2	74	33	84	52
E2	0.48	1/2	44	11	48	23
E3	0.53	1/2	78	11	87	29
E4	0.2897	1/2	22	7	23	10
E5	0.0003	1/2	74	22	87	19
E6	1/3	1/2	56	19	48	48
E7	4/5	1/2	74	11	74	52
E8	3/11	1/2	33	19	45	32
E9a	0.47 + 0.89	1/2	74	33	71	23
E9b	0.47 + 0.89	1	63	26	77	16
E9c	0.47 + 0.89	2	52	19	45	16

* Figures indicate percentage of correct responses for each item.
\# Figures indicate percentage of all respondents giving reasonable explanation
Significant difference at $p \leq 0{\cdot}02$ level using a χ^2 test on frequencies.
Significant difference at $p \leq 0{\cdot}01$ level using a χ^2 test on frequencies.

For these decimal and fraction items, in particular, it is not enough to consider percentages of correct responses – the reasons why the responses were chosen are of critical importance. Resnick, Nesher, Leonard, Magone, Omanson and Peled (1989) attempt to identify children's (possibly erroneous) rules for comparing decimals and fractions. Items E1, E3 and E5, which all show above 70% of correct responses, would all produce a correct response by either the correct rule or by what Resnick *et al* call the "whole number rule", which in this case would mean treating the decimal part of the number as if it were a whole number and comparing it with 5. For the remaining two single decimal items the "whole number rule" would give an incorrect answer, and the percentage of correct responses is correspondingly much lower. For four of the five items, a higher percentage of 1993 children were able to give reasonable explanations for their answers, while there was a slight reduction for item E6, corresponding to a difference of one child only. None of these differences were statistically significant.

Although 1993 children performed worse on item E6 in terms of correct responses, they performed significantly better in terms of the percentage giving reasonable explanations, with all children who gave a correct answer in 1993 also giving a reasonable explanation. In 1992, approximately a third of the children giving a correct answer could also give a reasonable explanation, with the remainder either guessing or giving explanations such as: "1/3 is smaller than 1/5 which is a half" – note the persistence of the association between 1/2 and 5 (i.e. 0.5), even when decimals are not involved.

Both the answers and the explanations given by the 1993 children for items E9a to E9c were disappointing. The high rate of correct answers for items E9a and E9b, compared with the low rate of reasonable explanations and much lower rate of correct answers for E9c, suggest again that children were using some version of the "whole number rule". However, interviewers noted that there was also some confusion among children on these items, which is supported by the results for E9b and E9c in 1993, where more children thought that $0.47 + 0.89$ was bigger than 1 than thought it was bigger than 1/2.

Table 6

Percentages of Correct Responses for Place Value Items

No	Swimming Carnival Items (3 * * grade 3 children , 2 * * grade 4 children)	1992 (n = 27)*	1993 (n = 31)*
F1	What is largest possible number of grade 3's?	67	77
F2	What is smallest possible number of grade 4's?	74	77
F3	Books of tickets in 100's. How many books for grade 3?	52	74
F4	Charge $1 per child. About how much money from grade 4?	78	90
F5	Largest number of grade 3 & 4's altogether?	15	16
F6	Smallest number of grade 3 & 4's altogether?	63	77
F7a	Largest possible difference between grade 3's & 4's?	19	42
F7b	Smallest possible difference between grade 3's & 4's?	26	26

* Figures indicate percentage of correct responses for each item.

Table 6 shows the percentage of correct responses for each of the place value items. For item F4, all answers between $200 and $300 were scored as correct. It can be seen from the table that the 1993 children performed better on all items, although in some cases (items F2, F5 and F7b) the differences are negligible and none of the differences are statistically significant.

Conclusion

The *Calculators in Primary Mathematics* project was based on the premise that calculators, as well as acting as a computational device, are highly versatile teaching aids which have the potential to radically transform mathematics teaching by allowing children to experiment with numbers and construct their own meanings.

Classroom observations show that calculators have been widely used for exploration, as counting devices and as recording devices, as well as for "number crunching". Their use provides a rich mathematical environment for children to explore and promotes the development of number sense by removing previous restrictions on the types of numbers children use, by exposing children to written symbols which they can easily record and manipulate, and by facilitating sharing and discussion.

This paper and others (Groves, 1993a; submitted) show that children with long-term experience of calculators performed better than children without such experience on a range of computation and estimation tasks and some "real world" problems; exhibited better knowledge of number, particularly place value, decimals and negative numbers; made more appropriate choices of calculating device; and were better able to interpret their answers when using calculators, especially where knowledge of decimal notation or large numbers was required.

These results confirm the anecdotal evidence from project classrooms and support the assertion that the presence of calculators provides a learning environment to promote number sense.

References

Australian Education Council (1990). *A national statement on mathematics for Australian schools*. Melbourne: Curriculum Corporation (Australia).

Bell, A., Burkhardt, H., McIntosh, A. & Moore, G. (1978). *A calculator experiment in primary school*. Nottingham: Shell Centre for Mathematical Education.

Cockcroft, W. H.(1982). *Mathematics counts*. London: Her Majesty's Stationery Office.

Curriculum Development Centre (1986). *Report on the UNESCO pilot project on the application of calculators to mathematics teaching in Australia*. Canberra: CDC.

Curriculum Development Centre and The Australian Association of Mathematics Teachers (1987). *A national statement on the use of calculators for mathematics in Australian schools.* Canberra: CDC.

Davis, R. B., Maher, C. A. & Noddings, N. (Eds.) (1990). *Constructivist views on the teaching and learning of mathematics. JRME Monograph 4.* Reston, VA: National Council of Teachers of Mathematics.

Duffin, J. (1991). Written algorithms. *Mathematics in School, 20* (4), 41–43.

Greeno, J. G. (1989). Some conjectures about number sense. In J. T. Sowder & B. P. Schappelle (Eds.). *Establishing foundations for research on number sense and related topics: Report of a conference* (pp. 43–56). San Diego: San Diego State University Centre for Research in Mathematics and Science Education.

Greeno, J. G. (1991). Number sense as situated knowing in a conceptual domain. *Journal for Research in Mathematics Education, 22,* 170–218.

Groves, S., Ferres, G., Bergfeld, S. & Salter, S. (1990). Calculators in the infant school: The Victoria College Calculator Project. In Clements, M.A. (Ken) (Ed). *Whither Mathematics?* (Proceedings of the 27th Annual Conference of the Mathematical Association of Victoria, pp. 244-250). Melbourne: Mathematical Association of Victoria.

Groves, S., Cheeseman, J., Clarke, C. & Hawkes, J. (1991). Using calculators with young children. In O'Reilly, J. & Wettenhall, S. (Eds). *Mathematics: IDEAS.* (Proceedings of the 28th Annual Conference of the Mathematical Association of Victoria, pp. 325-333). Melbourne: Mathematical Association of Victoria.

Groves, S. (1993a). The effect of calculator use on third graders' solutions of real world division and multiplication problems. In I. Hirabayashi, N. Nohda, K. Shigematsu & Fou-Lai Lin (Eds.), *Proceedings of the Seventeenth International Conference for the Psychology of Mathematics.* (Vol. II, pp. 9–16). Tsukuba, Ibaraki, Japan: University of Tsukuba.

Groves, S. (1993b, November). *Calculators as an agent for change: Teachers' perceptions and practice.* Paper presented at the Annual Conference of the Australian Association for Research in Education, Freemantle, WA.

Groves, S. (submitted). *The effect of calculator use on third and fourth graders' computation and choice of calculating device.*

Groves, S. & Cheeseman, J. (1993). *Young Children Using Calculators.* [Videotape]. Burwood: Video Production Unit, Deakin University.

Hembree, A. & Dessart, D. (1986). Effects of hand–held calculators in pre–college mathematics education: A meta–analysis. *Journal for Research in Mathematics Education, 17,* 83–99.

Hiebert, J. (1989). Reflections after the conference on number sense. In J. T. Sowder & B. P. Schappelle (Eds.), *Establishing foundations for research on number sense and related topics: Report of a conference* (pp. 82–84). San Diego: San Diego University Centre for Research in Mathematics and Science Education.

Howden, H, (1989). Teaching number sense. *Arithmetic Teacher, 36* (6), 6–11.

Kamii, C., Lewis, B. A. & Livingston, S. J. (1993). Primary arithmetic: Children inventing their own procedures. *Arithmetic Teacher, 41* (4), 200–2031.

Koyama, M. (1993). Research into relationship between the computational estimation ability and strategy and the mental computation ability: Analysis of a survey of the fourth, fifth, and sixth graders in Japan. In I. Hirabayashi, N. Nohda, K. Shigematsu & Fou-Lai Lin (Eds.), *Proceedings of the Seventeenth International Conference for the Psychology of Mathematics.* (Vol. II, pp. 65-72). Tsukuba, Ibaraki, Japan: University of Tsukuba.

Markovits, Z. & Sowder, J. (1994). Developing number sense: An intervention study in grade 7. *Journal for Research in Mathematics Education, 25,* 4–29.

McIntosh, A., Reys, B. & Reys, R. (1992). A proposed framework for examining basic number sense. *For the Learning of Mathematics, 12* (3), 2–8, 44.

National Council of Teachers of Mathematics (1980). *An agenda for action – Recommendations for school mathematics of the 1980s.* Reston, VA: NCTM.

National Council of Teachers of Mathematics (1989). *Curriculum and evaluation standards for school mathematics.* Reston, VA: NCTM.

National Curriculum Council (1988). *Mathematics in the national curriculum.* York, UK: National Curriculum Council.

National Research Council (1989). *Everybody counts: A report to the nation on the future of mathematics education.* Washington, DC: National Academy Press.

Olivier, A., Murray, H. & Human, P. (1990). Building on young children's informal arithmetic knowledge. In G. Booker, P. Cobb & T. N. de Mendicuti (Eds.), *Proceedings of the Fourteenth PME Conference with the North American Chapter Twelfth PME-NA Conference* (Vol. III, pp.297–304). Oaxtepec, Mexico.

Plunkett, S. (1979). Decomposition and all that rot. *Mathematics in School, 8* (3), 2–5.

Resnick, L. B. (1989). Defining, assessing and teaching number sense. In J. T. Sowder & B. P. Schappelle (Eds.), *Establishing foundations for research on number sense and related topics: Report of a conference* (pp. 35–39). San Diego: San Diego University Centre for Research in Mathematics and Science Education.

Resnick, L. B., Nesher, P., Leonard, F., Magone, M., Omanson, S. & Peled, I. (1989). Conceptual bases of arithmetic errors: The case of decimal fractions. *Journal for Research in Mathematics Education, 20,* 8–27.

Resnick, L. B., Lesgold, S. & Bill, V. (1990). From protoquantities to number sense. In G. Booker, P. Cobb & T. N. de Mendicuti (Eds.), *Proceedings of the Fourteenth PME Conference with the North American Chapter Twelfth PME-NA Conference* (Vol. III, pp. 305–311). Oaxtepec, Mexico.

Reys, B. J. (1989). The calculator as a tool for instruction and learning. In P. R. Trafton & A. P. Shulte (Eds.), *New directions for elementary school mathematics* (1989 yearbook of the National Council of Teachers of Mathematics, pp. 168–173). Reston, VA: NCTM.

Reys, B. J. , Reys, R. E. & Penafiel, A. F. (1991). Estimation performance and strategy use of Mexican 5th and 8th grade student sample. *Educational Studies in Mathematics, 22,* 353–375.

Shuard, H., Walsh, A., Goodwin, J. & Worcester, V. (1991). *Calculators, children and mathematics*. London: Simon & Schuster.

Sowder, J. T. (1988). Mental computation and number comparison: Their roles in the development of number sense and computational estimation. In J. Hiebert & M. Behr (Eds.), *Number Concepts and Operations in the Middle Grades, Volume 2* (pp. 182–197). Reston, VA: National Council of Teachers of Mathematics.

Stacey, K. (1994, April). *Calculator s in Primary Mathematics: An analysis of classroom activities.* Paper presented at the American Educational Association Research Association 1994 Annual Meeting, New Orleans.

Wearne, D. & Hiebert, J. (1988). Constructing and using meaning for mathematical symbols: The case of decimal fractions. In J. Hiebert & M. Behr (Eds.). *Number concepts and operations in the middle grades* (pp. 220-235). Hillsdale, NJ: Lawrence Erblaum.

Wheatley, G.H. (1991). Constructivist perspectives on science and mathematics learning. *Science Education, 75* (1), 9–21.

Wheatley, G.H. & Shumway, R. (1992). The potential for calculators to transform elementary school mathematics. In J. T. Fey & C. R. Hirsch (Eds.). *Calculators in mathematics education.* (1992 yearbook, pp. 1–8). Reston, VA: NCTM.