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ABSTRACT

Designed as a avenue of communication for mathematics educators concerned with the views, ideas, and experiences of two-year college students and teachers, this journal contains articles on mathematics exposition and education, and regular features presenting book and software reviews and math problems. Volume 15 includes the following articles: "Two Mollweide Equations Detect Triangles," by David E. Dobbs; "Using Recursion To Solve a Probability Problem," by Thomas W. Shilgalis and James T. Parr; "Calculus to Algebra Connections in Partial Fraction Decomposition," by Joseph Wiener and Will Watkins; "Guidelines for the Academic Preparation of Mathematics Faculty at Two-Year Colleges: A Report of the Qualification Subcommittee of AMATYC"; "Fractals and College Algebra," by Kay Gura and Rowan Lindley; "Using Computer Technology as an Aid in Teaching the Introductory Course in Quantitative Methods," by Joseph F. Aieta, John C. Saber, and Steven J. Turner; "Summing Power Series by Constructing Appropriate Initial Value Problems," by Russell J. Hendel and John D. Vargas; "Simpson's Paradox and Major League Baseball's Hall of Fame," by Steven M. Day; "The Ubiquitous Reed-Solomon Codes," by Barry A. Cipra; "Predicting Grades in Basic Algebra," by Elsie Newman; "Why Do We Transform Data?" by David L. Farnsworth; and "Student's Perceptions of Myths about Mathematics," by Victor U. Odafe. (KP)

THE AMATYC REVIEW

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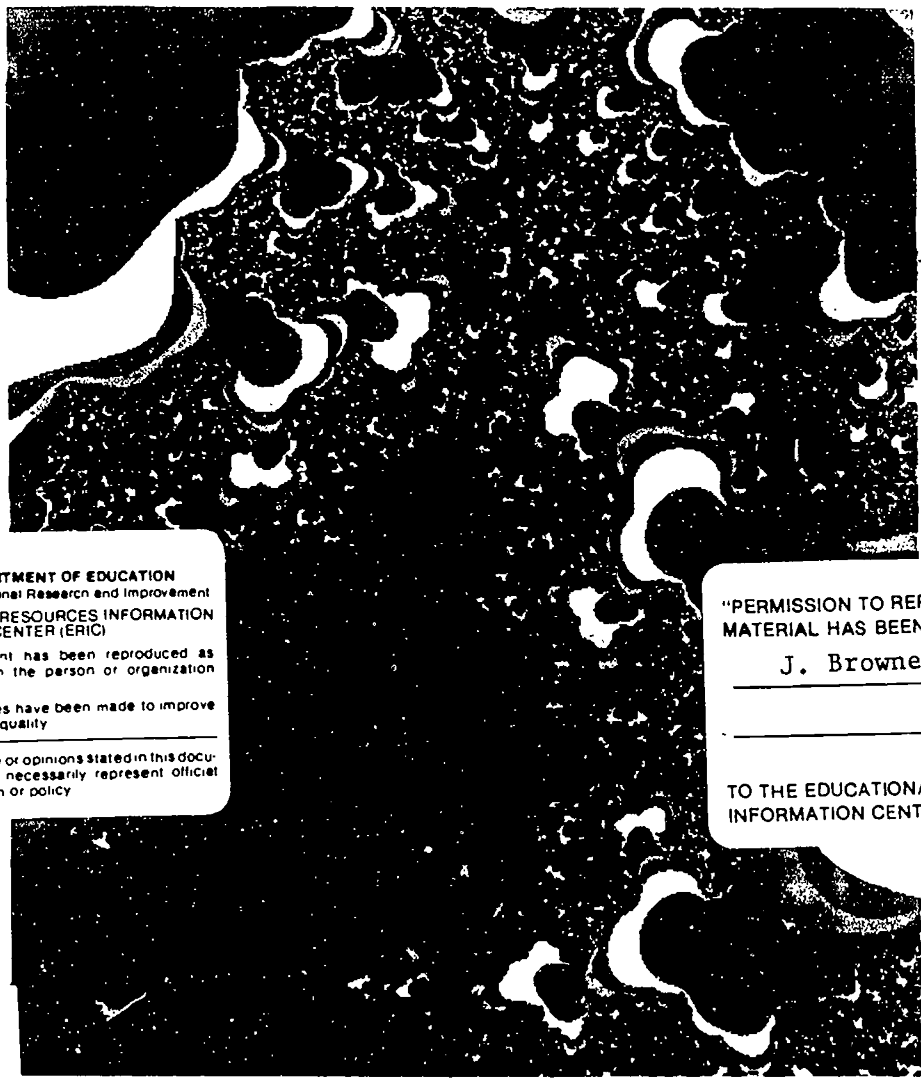

A glimpse of the Mandelbrot Set

ALSO IN THIS ISSUE

- Guidelines for Faculty Preparation — Final Report
- Web Puzzles
- "Dalmation Damation"

The Official Journal of the

American Mathematical Association of Two-Year Colleges

PURPOSE: AMATYC remains committed to its original purposes:

- To provide a national forum for the exchange of ideas
- To further develop and improve the mathematics education of students of two-year colleges
- To coordinate activities of affiliated organizations on a national level
- To promote the professional development and welfare of its members

The AMATYC Review provides an avenue of communication for all mathematics educators concerned with the views, ideas and experiences pertinent to two-year college teachers and students.

SUBMISSION OF MANUSCRIPTS: Manuscripts must be typed, doubled-spaced, on one side of 8-1/2" x 11" paper. They should not have been published before, nor should they be under consideration for publication elsewhere. Photographs should be black and white glossy prints. To provide for anonymous reviews, the author's name and affiliation should appear on a separate title page. The title should also appear on the first page of the exposition. Authors are advised to consult the *Publication Manual of the American Psychological Association.* A guideline for authors is available from the editor. Five copies of each manuscript should be submitted to Joseph Browne, Onondaga Comm. College, Syracuse, NY 13215.

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The AMATYC Review Fall 1993 Volume 15, Number 1

TABLE OF CONTENTS

EDITOR'S COMMENTS ....p. 4

LETTER TO THE EDITOR ....p. 4

ABOUT THE COVER ....p. 5

THE AMATYC REVIEW GUIDELINES FOR AUTHORS ....p. 6

VIEWPOINT

Time, Indeed, for a Change in Developmental Mathematics ....p. 8
by Jack W. Rotman

MATHEMATICAL EXPOSITION

Two Mollweide Equations Detect Triangles ....p. 13
by David E. Dobbs

Using Recursion to Solve a Probability Problem ....p. 20
by Thomas W. Shilgalis and James T. Parr

Calculus to Algebra Connections in Partial Fraction Decomposition ....p. 28
by Joseph Wiener and Will Watkins

SHORT COMMUNICATIONS

Web Puzzle ....p. 32
by Don St. Jean

The Prime Division Method of Finding the Greatest Common Factor ....p. 36
by Terence Brenner and Michael Shapira

Dalmation Darnation ....p. 38
by Kathy Sparling

MATHEMATICS EDUCATION

Guidelines for the Academic Preparation of Mathematics Faculty at Two-Year Colleges ....p. 40
A Report of the Qualifications Subcommittee of AMATYC

Fractals and College Algebra ....p. 51
by Kay Gura and Rowan Lindley

Using Computer Technology as an Aid in Teaching the Introductory Course in Quantitative Methods ....p. 57
by Joseph F. Aieta, John C. Saber, and Steven J. Turner

REGULAR FEATURES

The Chalkboard ....p. 64
Edited by Judy Cain and Joseph Browne

Software Reviews ....p. 67
Edited by Shao Mah

Book Reviews ....p. 71
Edited by John J. Edgell, Jr.

The Problem Section ....p. 74
Edited by Michael W. Ecker

Mathematics: An International View ....p. 82
Edited by Igor Malyshev and Joanne Rossi Becker

Editor's Comments

Thanks to several of you for writing or calling with your comments on the last issue (my first one). The encouraging remarks are appreciated, and the suggestions are being considered.

Lucky Larry Correction

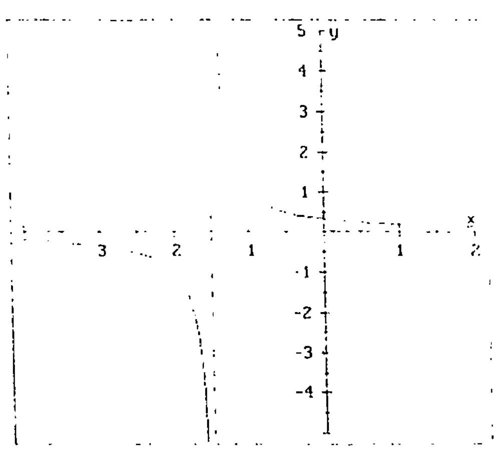
Several readers pointed out that in "Lucky Larry #3" Larry made one too many (or is it one too few) errors. The last line should read "$-\frac{3}{2} < x$." Enid M. Nagel added a few thoughts on how we should be teaching "Lucky Larry" and others. See her letter below.

Call for Papers

In this issue you will find an article by Gura and Lindley on the inclusion of fractals and chaos in a college algebra course. I would welcome a few more articles on including these topics in courses aimed at the first two years. Another area of considerable interest recently is collaborative learning. One aspect that seems to need more treatment is methods of collaborative assessment, with safeguards to prevent a loafer from passing when all the work is done by groupmates.

Letter to the Editor

To help Lucky Larry out of the dilemma of manipulating algebraic symbols willy nilly, I suggest that this problem [Lucky Larry #3, Spring, 1993] be solved using a graphing calculator or a computer graphing program. If Larry graphs $f(x) = \frac{1}{2x + 3}$, the result is a hyperbola [see graph]. Larry can then easily see the values of x for which this function is greater than zero, since the graph will be above the x-axis. Instead of repeatedly trying to instill algebraic manipulations into our students, let's help them look at problems differently using the technology that is available to us.



Enid M. Nagel
Cincinnati OH

About the Cover

A close up view of a boundary region of the Mandelbrot set reveals many examples of the concept of "self-similarity" as well as a near copy of the Mandelbrot set itself. The set may be defined as follows: Let c_0 be a complex number and define a sequence by $c_n = c_{n-1}^2 + c_{n-1}$. If the sequence is bounded, c_0 is in the Mandelbrot set. As might be expected, all points in a nearly round set near the origin are in the set. The boundary, however, is not a simple convex curve, but rather a fuzzy area with nearby points exhibiting vastly different behavior (which is referred to as chaos).

For most choices of c the arithmetic quickly becomes messy and is given to a computer. Indeed, such sets were not significantly investigated before the computer was available. This, in turn, leads to some interesting discussion on how a computer with approximate arithmetic can determine convergence or divergence of the sequence. Altering the criterion used can sometimes have profound effect on what points are included.

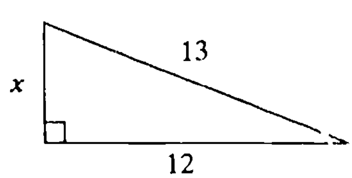
More spectacular pictures are generated when points for which the sequence diverges are colored differently according to how fast it diverges. A typical routine counts how many terms are needed to get more than two units from the origin. The cover picture was originally in 16 colors, with the black region (lower left) being the Mandelbrot set. The three-dimensional appearance is an illusion since we have merely selected a color for each point according to its behavior in the sequence.

There are now several software packages on the market for exploring chaos, the Mandelbrot set, and fractals (entities with fractional dimension). A fast computer, math co-processor, and high-resolution graphics display are highly desirable. The cover was produced by "Chaos: the Software" (Autodesk, Inc.) which was issued to accompany James Gleick's excellent introductory book, *Chaos: Making a New Science.*

Lucky Larry #6

(Lucky Larry is the infamous student who, in spite of making numerous errors, manages to get right answers. Send your samples of his work to the editor for inclusion in future issues.)

(Lucky Larry vs Pythagoras) The object was to find the missing side. Lucky Larry proceeded as follows:



$$x^2 = 13 + 12$$
$$= 25$$
$$x = 5$$

Submitted by Sharlene Cadwallader
Mt. San Antonio College
Walnut CA 91789

The AMATYC Review
Guidelines for Authors

The AMATYC Review is a semi-annual publication of the American Mathematical Association of Two-Year Colleges. Its purpose is to provide an avenue of communication for all mathematics educators concerned with the views, ideas, and experiences pertinent to two-year college teachers and students.

Subject Matter: The AMATYC Executive Board has identified the following priority areas:

a. Developmental mathematics
b. Technical mathematics
c. Mathematics content of the two-year college curriculum
d. Educational theory and practice
e. Academic computing
f. Research in mathematics education in the two-year college
g. Equal opportunity in mathematics
h. Problems, issues, and trends in two-year college mathematics

Perspective: No article should simply be a rewrite of a topic readily available from other sources. Preferably, articles should present traditional concepts with a new perspective, give extensions to such materials, or reach out just beyond our current teaching content for enrichment. Articles devoted more to methodology than to exposition may be either research-oriented or descriptive case studies. Authors are encouraged to offer personal opinions and suggestions. Letters to the editor may be used as a format to comment on previously published articles, to offer opinions on controversial topics, or to offer short expository notes.

Computers and Calculators: Technology-oriented articles may be grouped into two, not necessarily distinct, categories: technology used as a teaching aid and technology used as a mathematical tool. In either case, the major intent of an article should be to help teachers and students to learn about mathematics, not about the machine or software. References to technology should be as generic as possible (e.g. "using a computer algebra system we find..." rather than "using Derive's [specific command] on an IBM PS/2-55 yields [specific output]"). Program listings, specific commands, and sequences of button pushes are usually inappropriate, though short segments may be included when they are essential to understanding the (mathematical or pedagogical) point being made.

Regular Features: Authors are invited to submit reviews of books and computer software with which they are thoroughly familiar. The materials should be of exceptional interest (outstanding/poor) to our audience. The Problem Section depends on reader involvement, and the other features incorporate reader input on a regular basis. Please communicate directly with the columnists. (There is no need of five copies of anything submitted for the columns; one will do.)

Review Criteria: *The AMATYC Review* is a refereed publication. Each appropriate submission is reviewed by three mathematics educators: two members of the review panel and a member of the Editorial Board. The Editorial Board member will analyze all reviews and make a recommendation to the Editor on the acceptance or rejection of an article. The Editor makes final decisions on the publication of articles.

The following review criteria are used:

a. relevance to two-year college mathematics content or pedagogy
b. significance of topics
c. originality
d. accuracy of content
e. explicit, clear, logical, and concise writing style
f. appropriate length and format

In general, *The AMATYC Review* is seeking lively articles dealing with topics that will enrich the reader's mathematical background or help the reader improve his or her classroom teaching.

Manuscript Style: Articles may vary in length from less than one page up to about 15 pages (typed, double spaced, with wide margins). Brief, "to the point," articles are encouraged. Illustrations should preferably (but not necessarily) be camera ready and be placed on separate pages from the text. Graphics produced on dot matrix printers usually reproduce poorly and should be avoided.

The AMATYC Review uses the *Publication Manual of the American Psychological Association* (Washington DC) as its style reference. In particular, note that the author-date method is used for citations within the text, e.g.

Smith and Jones (1987) demonstrated that....

The reference list at the end of the article should include only the sources that were used in the preparation of the article. References should be arranged in alphabetical order by the surname of the first author, e.g.

Smith, John R., & Jones, David (1987). Sets with missing elements. *The AMATYC Review*, 7(2), 104-113.

Wallace, Donald P. (1992). *Digital arithmetic: counting on your fingers* (2nd ed.). Centerville: Farber and Sons.

To provide for anonymous reviews, the author's name and affiliation should appear (only) on a separate title page. The title should also appear on the first page of the exposition.

Submission: Send five copies of articles for possible publication to: *The AMATYC Review*, Joseph Browne, Editor, Onondaga Community College, Syracuse, NY, 13215.

> The method of "postulating" what we want has many advantages; they are the same as the advantages of theft over honest toil.
>
> Bertrand Russell

VIEWPOINT

Time, Indeed, for a Change in Developmental Mathematics

by

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Jack has been a professional mathematics educator since 1973 with the Mathematics Laboratory at Lansing Community College. Most of these years have been spent as a Course Coordinator for the range of the curriculum —from basic arithmetic to intermediate algebra. He is also the editor of the Developmental Mathematics Committee (AMATYC) newsletter, the "DMC REPORT". Jack has also been involved in the state affiliate (Mich-MATYC), and with the Michigan Department of Education.

In *The AMATYC Review*, volume 13 number 2 (Spring 1992), Edward Laughbaum shared a viewpoint that it was time for a change in "remedial" mathematics ("A Time for Change in Remedial Mathematics," pages 7-10). The following is a response to Laughbaum and presents an opposing view.

A Rebuttal

Laughbaum agrees with Lynn Arthur Steen that most mathematics remediation is a failure. I disagree. My own developmental program is quite successful by traditional standards; our students show mastery of specific skills such as those Laughbaum lists and our students are adequately prepared to succeed in the next mathematics course up to the college level and beyond. I know that many other colleges value their developmental mathematics program and show this commitment with the support that results in a successful program. At the same time, it is clear that there are developmental mathematics programs that do not achieve even minimal goals, and that we need to re-examine our goals and methods.

In his conclusion, Laughbaum implies that developmental mathematics curricula is turning off students, who are "leaving the study of mathematics at the rate of 50% per year," based on the National Research Council report. Our own figures at Lansing Community College estimate that 60% to 80% of those students completing a developmental mathematics course enroll in the subsequent mathematics course within one year. We also know that many of the remaining 20% to 40% have achieved as much mathematics as is needed for their academic goals. It is dangerous to apply general statistics to a specific environment without knowing whether it is appropriate; the 50% attrition rate cited by Laughbaum is based on general enrollment patterns and does not mean that any particular level of course work is turning off students. Many other factors influence course enrollment patterns, especially at colleges and universities. Although I doubt the accuracy of Laughbaum's negative view of "remedial" mathematics, I do have a basic agreement with the call for change.

The General Need for Change

At the most general levels, I agree entirely with Laughbaum's position that there is a major need for change in our most elementary college mathematics courses. At the beginning of the article, he states:

> As currently taught, our elementary mathematics courses are so burdened with symbolic manipulations that it is difficult for instructors to spend adequate time on fundamental concepts. (pg. 7)

This statement is quite true. However, Professor Laughbaum attributes the problem to the use of "curricula, texts, tests, and teaching habits [which] are all products of the precomputer age." Actually, this statement is false and diverts our attention from some basic issues in mathematics education. In the case of textbooks, most of the current popular worktexts are derivations based on programmed learning and behavioral theories generally applied since computers became commonplace. Some common hardcover texts <u>are</u> "old-fashioned" (precomputer age), but I will argue below that this is not a major problem. Beyond this, I don't know what would define a textbook as "postcomputer age." When we teach, many of us use some of the techniques developed in the last 20 years — computer tutoring and simulation, computer testing, small group work, and others; lectures are certainly still used, but most of us use some modern technology.

The problem is not that we are stuck in a "precomputer mode," but that we do not have a consensus on what "learning mathematics" means for students in developmental mathematics. There are professionals who really believe that "learning mathematics" means to show proficiency in about 100 specific skills per cours , while others believe that we need to build mathematical literacy and understanding. <u>We do not need a bandwagon, but a forum.</u>

Before discussing this needed forum, I want to explore my major disagreement with the proposals that Laughbaum made in his article and to discuss some of the forces that can be the "drivers" (forces) behind a curricular reform.

Technology Does Not Equal Learning

My major disagreement is with the proposal:

> To change the current pattern of instruction, I propose that teaching methods be changed to support implementation of the graphing calculator into the remedial sequence. (pg. 8)

Part of my concern here is with "teaching methods" as opposed to "teaching content and methods," and I also am concerned about the limited lifespan of such a specific type of technology. The graphing calculator is an amazing piece of technology, but it is just a piece of technology. Using a different technology does not insure that better learning will take place. If students are taught specific skills with a specific type of technology, there will be a tendency for their learning to be limited by those skills and technology. Let me list some reasons why using the graphing calculator will not contribute to mathematical literacy in developmental mathematics (such as beginning or intermediate algebra):

1. The technology may require the input of expressions and equations in specific forms, which sometimes conflict with standard notation still encountered in other disciplines. One example is the use of a "*" for a product, or the use of a "^" for exponentiation. While this example is relatively minor and may be eliminated with a future generation of calculators, the different notation may interfere with the student's ability to transfer what she/he learns.

2. The technology allows (and encourages) the input of expressions in a non-simplified form. This is not a problem in itself; however, if students complete our course without being able to transform equations and expressions into simpler forms, they will be handicapped when they seek to apply their mathematics in academia or business and industry. We do need to address content as well as methods.

3. If the technology is used to solve equations, at the exclusion of earlier technology (paper and pencil algorithms), students will be handicapped when they apply outside the classroom what they learned inside the classroom. People are shocked when students need to pull out their calculators to add 14 and 19; won't they be just as shocked if students need to pull out their calculator to solve $3x - 1 = -7$?

4. At the beginning levels of algebra, students struggle to understand and internalize mathematical concepts; an emphasis on a graphing calculator may promote confusion. Algebraic ideas have meanings and purposes independent of the technology, and it is the student's mastery of these basic concepts that the beginning course should focus on. Time spent on teaching the technology will detract from time spent on understanding concepts; once students master the basic concepts, technology can (and should be) used at higher levels.

5. As Laughbaum states, the use of graphing calculators "will force students . . . to study material at a more sophisticated level" (pg. 9). However, I believe that most students are not ready for all of this sophistication at the introductory level.

My own view is that graphing calculators should be avoided in a beginning algebra course, with a progression towards heavier usage as students proceed through a sequence of courses up to calculus. Whether or not one subscribes to this view of technology, there remains another basic issue to consider: what "driver(s)" do we want for our curriculum?

Other "Drivers" for Curricular Reform

A current danger in mathematics education is that our curricular reform is generally being driven by technology; the graphing calculator is one instance, and there are others. However, technology should not be the most basic driving force in changing mathematics education. The technology will change; the graphing calculator of today will be replaced by some new machine that does even more. Before this decade is over, we are likely to see a handheld calculator that has the capacity of a current personal computer, including the use of more powerful software such as DERIVE. Basing curricular reform on technology is too risky for my comfort and is too short-sighted for our profession; we should base our curriculum on principles that can be supported by the use of whatever technology is available.

There are other principles or drivers for curricular reform. One type of driver is a theory of learning mathematics; for example, if one subscribes to a constructivist view of learning, our curriculum would change in radical ways, perhaps to be based on algebra experiments and on a process approach. Another type of driver is access; for example, if one believes that college mathematics must be equally accessible to all people, who bring a variety of learning skills and needs to our courses, the curriculum also would be changed in substantial ways. Still another type of driver is "applications": if one believes that students learn what they can apply, this will require a wide variety of changes in our courses. Directives from a State are sometimes a driver for curricular reform, though these seldom represent what mathematics educators believe is best.

The various drivers — technology, learning theories, access, applications and others — may result in very similar changes in some ways, but very different changes in our curriculum in other ways. However, we — as a profession — should make a reasoned choice among the possible drivers so that we achieve what we need.

A Conclusion: What Do We Really Need?

The basic issue facing mathematics educators today is how to integrate the various forces attempting to drive our mathematics curriculum. The solution involves dialogue and consensus building. Institutions such as AMATYC provide a needed forum and structure for this work. As we work together, our theories and standards will converge, resulting in changes in our curriculum which will certainly integrate technology in many ways.

However, we need to be wary of bandwagons. Technology is a very attractive bandwagon, but it is still like other bandwagons: those who jump on will have to get off later and find out where else they could have gone. We have had other bandwagons in our profession: the "programmed texts," the "low-read texts," "learning by objective," and others. In each case, the bandwagon had some merit and some concepts from each are still being used successfully today.

Our paradigm should not be a bandwagon, but a task force whose job it is to define the problems and explore total solutions. The **Curriculum and Pedagogy Reform** project of AMATYC could be this task force, as long as all of us keep larger issues in mind.

> If only applied research had been done in the Stone Age, we would have wonderful stone axes but no-one would have discovered metals!
>
> G. P. Thomson [paraphrased]

14

MATHEMATICAL EXPOSITION

Two Mollweide Equations Detect Triangles

by
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Knoxville TN 37996–1300

David E. Dobbs is a professor of mathematics at the University of Tennessee (Knoxville). His earlier academic positions were at Rutgers University (New Brunswick) and the University of California (Los Angeles), the latter while he held a postdoctoral associateship from the Office of Naval Research. He is a member of the A.M.S. and the M.A.A. He is the author of four books and approximately 150 research articles on algebraic geometry, homological algebra, commutative algebra, category theory, algebraic number theory, and mathematics education.

A result usually attributed to Mollweide (1808) asserts that if a triangle Δ has sides of length a, b, c and corresponding angles of radian measure α, β, γ, then

$$\frac{\cos\frac{\alpha-\beta}{2}}{\sin\frac{\gamma}{2}} = \frac{a+b}{c}, \quad \frac{\cos\frac{\alpha-\gamma}{2}}{\sin\frac{\beta}{2}} = \frac{a+c}{b}, \quad \text{and} \quad \frac{\cos\frac{\beta-\gamma}{2}}{\sin\frac{\alpha}{2}} = \frac{b+c}{a}. \qquad (1)$$

Actually, these equations were published several times prior to 1808, with a variant of the third equation in (1) appearing as early as Newton's *Arithmetica Universalis*. In former days (before the advent of electronic hand-held calculators), (1) was employed mainly to check for calculation errors in finding the parts of Δ. Our main purpose here is to argue for a contemporary interest in (1), by explaining how (1) figures in a characterization of triangles.

It will be convenient to refer to an ordered list of positive real numbers a, b, c, α, β, γ, as **Mollweide data**; and to (1) as the **(associated) Mollweide equations.** It seems reasonable to ask about the converse of the Newton-Mollweide result: if Mollweide data $a,\ldots,\gamma$ satisfy (1), does there exist a triangle with sides a, b, c and corresponding angles α, β, γ? Our main result gives an affirmative answer to this question if we assume $\alpha + \beta + \gamma = \pi$. Indeed, Theorem 7 establishes that if Mollweide data $a,\ldots,\gamma$ satisfy $\alpha + \beta + \gamma = \pi$ and at least two of the associated Mollweide equations, then, up to congruence, there is a unique triangle with sides a, b, c and corresponding angles α, β, γ.

What we have just stated amounts to a characterization of triangles. In addition to the intrinsic interest of such a result, it is hoped that our methods will find use as classroom enrichment for the often taught, but rarely used, identities expressing sums and products of sines or cosines.

It will be convenient to say that a triangle Δ **realizes** Mollweide data $a,\ldots,\gamma$ (or that the data is **realized by** Δ) if Δ has sides a, b, c and corresponding angles α, β, γ. We begin our study of the realization problem with the following simple observation.

EXAMPLE 1. If Mollweide data $a,\ldots,\gamma$ satisfy $\alpha + \beta + \gamma = \pi$ and (at least) one of the associated Mollweide equations, there need not exist a triangle that realizes $a,\ldots,\gamma$.

Proof. It will be convenient in the examples to measure angles in degrees. Consider $a = 0.89$, $b = 0.11$, $c = \sqrt{2 - \sqrt{2}}$, $\alpha = 127.5°$, $\beta = 7.5°$, and $\gamma = 45°$. The sum of the radian (resp., degree) measures of α, β, γ is π (resp., 180°). Moreover, the first associated Mollweide equation in (1) is satisfied. (To see this, express sin(22.5°) by means of a standard half-angle formula. It is useful for our later work to notice that neither of the other two equations in (1) is satisfied.) However, no triangle realizes $a,\ldots,\gamma$ since the triangle inequality is violated: indeed, $a > 0.8754 \approx b + c$. ■

In view of Example 1, it is reasonable to ask what may be inferred from a single Mollweide equation. Proposition 2 gives a useful answer.

PROPOSITION 2. If Mollweide data $a,\ldots,\gamma$ satisfy $\alpha + \beta + \gamma = \pi$ and the first associated Mollweide equation in (1), then $a + b > c$.

Proof. By a standard identity, we can rewrite the Mollweide equation's denominator:

$$\sin \frac{\gamma}{2} = \sin\left(\frac{\pi}{2} - \frac{\alpha+\beta}{2}\right) = \cos\frac{\alpha+\beta}{2}.$$

Hence, $\frac{a+b}{c} = \frac{n}{d}$, where $n = \cos\frac{\alpha-\beta}{2}$ and $d = \cos\frac{\alpha+\beta}{2}$. Since cosine is an even function, we may assume that $\alpha - \beta \geq 0$; then

$$0 \leq \frac{\alpha-\beta}{2} < \frac{\alpha+\beta}{2} < \frac{\pi}{2}.$$

As cosine is a decreasing function over the interval $[0, \pi/2]$, we have

$$\cos\frac{\alpha-\beta}{2} > \cos\frac{\alpha+\beta}{2}; \text{ that is, } n > d.$$

Hence,

$$\frac{a+b}{c} = \frac{n}{d} > 1, \text{ and so } a + b > c. \quad ■$$

17

Consider Mollweide data $a,\ldots,\gamma$ satisfying $\alpha + \beta + \gamma = \pi$. According to Proposition 2, if (at least) two of the equations in (1) hold, then (at least) two of the associated triangle inequalities hold. However, two triangle inequalities do not ensure realizability. (Consider, for instance, $a = 3$, $b = 1.5$, $c = 1$: $a + b > c$ and $a + c > b$, but $b + c < a$.) Nevertheless, we shall see in Theorem 7 that two Mollweide equations, in conjunction with $\alpha + \beta + \gamma = \pi$, do ensure realizability. (In particular, the converse of Proposition 2 is false.) We pause next to indicate that two Mollweide equations do not ensure that $\alpha + \beta + \gamma = \pi$.

EXAMPLE 3. It is possible for Mollweide data $a,\ldots,\gamma$ to satisfy exactly two of the associated Mollweide equations. (In any such case, the Newton-Mollweide result assures that $a,\ldots,\gamma$ is not realized by any triangle; then, granting Theorem 7, it follows that $\alpha + \beta + \gamma \neq \pi$.)

Proof. We build an example that violates $\alpha + \beta + \gamma = \pi$. Take $\alpha = 40°$, $\beta = 70°$ and $\gamma = 80°$. Changing a, b, c by some proportionality constant does not affect (1), and so we may take $c = 1$. Assuming, as we may, that the last two equations in (1) are satisfied, we find that $\frac{a+1}{b} \approx 1.6383041$ and $\frac{b+1}{a} \approx 2.9126784$. Solving the associated system of two linear equations in two unknowns, we find that $a \approx 0.6994716$ and $b \approx 1.0373359$. Hence, $\frac{a+b}{c} \approx 1.7368075$. But $\dfrac{\cos\frac{\alpha-\beta}{2}}{\sin\frac{\gamma}{2}} \approx 1.5027138$; so, the first equation in (1) fails. ■

It is well known (and evident) that positive numbers a, b, c are the lengths of the sides of some triangle if and only if $a + b > c$, $a + c > b$, and $b + c > a$. Therefore, by invoking Proposition 2 and the Side-Side-Side congruence criterion of Euclidean geometry, we immediately infer the next result.

COROLLARY 4. If Mollweide data $a,\ldots,\gamma$ satisfy all three associated Mollweide equations, then there exists a triangle (unique up to congruence) with sides of length a, b, c.

The preceding result does not assert that the uniquely determined triangle realizes $a,\ldots,\gamma$. That issue is addressed in Proposition 6. First, we state a useful lemma.

LEMMA 5. Suppose that Mollweide data $a,\ldots,\gamma$ satisfy $\alpha + \beta + \gamma = \pi$. Then the associated Mollweide equations (which may or may not hold) in (1) may be rewritten equivalently as

$$\frac{\cos\frac{\alpha-\beta}{2}}{\cos\frac{\alpha+\beta}{2}} = \frac{a+b}{c}, \quad \frac{\cos\frac{\alpha-\gamma}{2}}{\cos\frac{\alpha+\gamma}{2}} = \frac{a+c}{b}, \quad \text{and} \quad \frac{\cos\frac{\beta-\gamma}{2}}{\cos\frac{\beta+\gamma}{2}} = \frac{b+c}{a}. \tag{2}$$

Proof. Rework the first sentence of the proof of Proposition 2. ■

PROPOSITION 6. Suppose that Mollweide data $a,\ldots,\gamma$ satisfy $\alpha + \beta + \gamma = \pi$ and all three associated Mollweide equations. Then the unique triangle Δ with sides a, b, c (whose existence is assured by Corollary 4) realizes $a,\ldots,\gamma$.

Proof. Suppose specifically that the last two equations in (2) hold. (For the proof of Theorem 7, it is important to fix attention here on some pair in (2).) The first of these asserts that

$$\frac{a+c}{b} = \frac{n}{d}, \text{ where } n = \cos\frac{\alpha-\gamma}{2} \text{ and } d = \cos\frac{\alpha+\gamma}{2}.$$

Since $\gamma = \pi - \alpha - \beta$, a standard identity yields that

$$n = \cos\left(\frac{2\alpha+\beta}{2} - \frac{\pi}{2}\right) = \sin\frac{2\alpha+\beta}{2}$$

and, similarly, $d = \sin\frac{\beta}{2}$. Rewrite the third Mollweide equation similarly. Hence

$$\frac{a+c}{b} = \frac{\sin\frac{2\alpha+\beta}{2}}{\sin\frac{\beta}{2}} \text{ and } \frac{b+c}{a} = \frac{\sin\frac{2\beta+\alpha}{2}}{\sin\frac{\alpha}{2}}.$$

As in the proof of Example 3, we may take $c = 1$. Then, motivated by that earlier proof, we solve the previous equations as a linear system in the "unknowns" a and b. The result is

$$a = \frac{\left(\sin\frac{\alpha}{2}\right)\left[\sin\frac{\beta}{2} + \sin\frac{2\alpha+\beta}{2}\right]}{-\sin\frac{\alpha}{2}\sin\frac{\beta}{2} + \sin\frac{2\alpha+\beta}{2}\sin\frac{2\beta+\alpha}{2}};$$

by interchanging α and β in the right-hand side, one obtains the solution for b. The solutions for a and b have a common denominator D, and D can be simplified, using the identities

$$\sin x \sin y = \frac{\cos(x-y) - \cos(x+y)}{2} \text{ and } \cos 3\theta = 4\cos^3\theta - 3\cos\theta.$$

After using the Pythagorean identity, $\sin^2\theta + \cos^2\theta = 1$, we find that

$$D = 2\sin^2\frac{\alpha+\beta}{2}\cos\frac{\alpha+\beta}{2}.$$

We proceed to show that α, β, γ are the angles of Δ (and thus that Δ realizes the given Mollweide data). By symmetry, it suffices to consider α. As cosine is a one-to-one function on the interval $(0, \pi)$, it is enough to show that $\cos\alpha$ has the value predicted by the Cosine Law when applied to Δ. In other words, we must prove that

$$\frac{b^2 + c^2 - a^2}{2bc} = \cos\alpha. \tag{3}$$

Using the above formulas for a and b, and writing $c = \frac{D}{D}$, we find, after some cancellation, that the left-hand side of (3) simplifies to

$$\frac{\left(\sin\frac{\beta}{2}\left[\sin\frac{\alpha}{2} + \sin\frac{2\beta+\alpha}{2}\right]\right)^2 + D^2 - \left(\sin\frac{\alpha}{2}\left[\sin\frac{\beta}{2} + \sin\frac{2\alpha+\beta}{2}\right]\right)^2}{2D\sin\frac{\beta}{2}\left[\sin\frac{\alpha}{2} + \sin\frac{2\beta+\alpha}{2}\right]}. \tag{4}$$

This can be simplified using the identity

$$\sin x + \sin y = 2 \sin \frac{x+y}{2} \cos \frac{x-y}{2}.$$

Then, after cancelling $\sin^2 \frac{\alpha+\beta}{2}$ and using the double angle formula $2 \sin \theta \cos \theta = \sin 2\theta$ five times, we may rewrite (4) as

$$\frac{\sin^2 \beta + \sin^2 (\alpha+\beta) - \sin^2 \alpha}{2 \sin \beta \sin (\alpha+\beta)}. \tag{5}$$

In order to establish (3), we must show that the expression in (5) can be simplified to $\cos \alpha$, equivalently that

$$\sin^2\beta + \sin^2(\alpha+\beta) - \sin^2\alpha = 2 \sin \beta \sin(\alpha+\beta) \cos \alpha.$$

Since $\sin(\alpha+\beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$, this is equivalent to showing that

$$\sin^2\beta + \sin^2\alpha \cos^2\beta + \cos^2\alpha \sin^2\beta - \sin^2\alpha = 2 \cos^2 \alpha \sin^2\beta. \tag{6}$$

Using the Pythagorean substitutions $\sin^2\alpha = 1 - \cos^2\alpha$ and $\cos^2\beta = 1 - \sin^2\beta$, we easily simplify the left-hand side of (6) to the right-hand side of (6). ■

We come, finally, to the titular result: if $\alpha + \beta + \gamma = \pi$, any one of the Mollweide equations is redundant (and the Mollweide data is uniquely realizable). Notice, via Example 3, that the hypothesis $\alpha + \beta + \gamma = \pi$ cannot be deleted.

THEOREM 7. Suppose that Mollweide data $a, \ldots, \gamma$ satisfy $\alpha + \beta + \gamma = \pi$ and (at least) two of the associated Mollweide equations. Then the remaining Mollweide equation is satisfied. Moreover, there exists a triangle, unique up to congruence, that has sides a, b, c and corresponding angles α, β, γ.

Proof. In view of Proposition 6, it suffices to show that the last two equations in (2) jointly imply the first equation in (2). As in the proof of Proposition 6, we may take $c = 1$, and express a and b as fractions with common denominator D. Then

$$\frac{a+b}{c} = a+b = \frac{2 \sin \frac{\alpha}{2} \sin \frac{\beta}{2} + \sin \frac{\alpha}{2} \sin \frac{2\alpha+\beta}{2} + \sin \frac{\beta}{2} \sin \frac{2\beta+\alpha}{2}}{D}. \tag{7}$$

In view of the expression for D given in the proof of Proposition 6, establishing the remaining (first) Mollweide equation is equivalent to showing that the numerator of (7) simplifies to $2 \sin^2 \frac{\alpha+\beta}{2} \cos \frac{\alpha-\beta}{2}$. Using the earlier identity for $\sin x + \sin y$, we rewrite that numerator as

$$\sin \frac{\alpha}{2} \left[\sin \frac{\beta}{2} + \sin \frac{2\alpha+\beta}{2}\right] + \sin \frac{\beta}{2} \left[\sin \frac{\alpha}{2} + \sin \frac{2\beta+\alpha}{2}\right]$$

$$= \sin \frac{\alpha}{2} \left[2 \sin \frac{\alpha+\beta}{2} \cos \frac{\alpha}{2}\right] + \sin \frac{\beta}{2} \left[2 \sin \frac{\alpha+\beta}{2} \cos \frac{\beta}{2}\right].$$

By extracting a common factor of $2 \sin \frac{\alpha+\beta}{2}$ from these terms, we see that it suffices to show that

$$\sin \frac{\alpha}{2} \cos \frac{\alpha}{2} + \sin \frac{\beta}{2} \cos \frac{\beta}{2} = \sin \frac{\alpha+\beta}{2} \cos \frac{\alpha-\beta}{2} . \qquad (8)$$

According to the double angle formula, the left-hand side of (8) is $\frac{\sin \alpha + \sin \beta}{2}$. However, so is the right-hand side, thanks to the identity

$$\sin x \cos y = \frac{\sin (x+y) + \sin (x-y)}{2}.$$

The proof is complete. ■

REMARK 8. We close by presenting an alternate proof of Theorem 7. Unlike the above proof, it is not self-contained, but depends on the following fact (which has been proved in [1], [2, especially p. 44]): the Sine Law implies the Cosine Law. More precisely, if Mollweide data $a,\ldots,\gamma$ satisfy $\alpha+\beta+\gamma=\pi$ and

$$\frac{a}{\sin \alpha} = \frac{b}{\sin \beta} = \frac{c}{\sin \gamma}$$

then $a^2 = b^2 + c^2 - 2bc \cos \alpha$, with similar expressions for b^2 and c^2.

Suppose that Mollweide data $a,\ldots,\gamma$ satisfy $\alpha+\beta+\gamma=\pi$ and the first two equations in (1). Then, since

$$\sin \frac{\alpha+\beta}{2} = \sin\left(\frac{\pi}{2} - \frac{\gamma}{2}\right) = \cos \frac{\gamma}{2},$$

we have

$$\frac{\sin \alpha + \sin \beta}{\sin \gamma} = \frac{2 \sin \frac{\alpha+\beta}{2} \cos \frac{\alpha-\beta}{2}}{2 \sin \frac{\gamma}{2} \cos \frac{\gamma}{2}} = \frac{\cos \frac{\alpha-\beta}{2}}{\sin \frac{\gamma}{2}} = \frac{a+b}{c}$$

and, similarly, $\frac{\sin \alpha + \sin \gamma}{\sin \beta} = \frac{a+c}{b}$. It follows readily that

$$\frac{\sin \alpha + \sin \beta + \sin \gamma}{\sin \gamma} = \frac{a+b+c}{c} \quad \text{and} \quad \frac{\sin \alpha + \sin \beta + \sin \gamma}{\sin \beta} = \frac{a+b+c}{b}.$$

Hence, $\frac{c}{\sin \gamma} = \frac{b}{\sin \beta} = \frac{a+b+c}{\sin \alpha + \sin \beta + \sin \gamma}$ =, say, k. Then

$$a = k(\sin \alpha + \sin \beta + \sin \gamma) - b - c = k \sin \alpha$$

and so $\frac{a}{\sin \alpha} = \frac{b}{\sin \beta} = \frac{c}{\sin \gamma}$.

By the above-cited result from [1], [2], $a^2 = b^2 + c^2 - 2bc \cos \alpha$, with similar expressions for b^2 and c^2. As $0 < \alpha < \pi$ entails $\cos a > -1$, we have $a^2 < b^2 + c^2 + 2bc = (b + c)^2$, whence $a < b + c$. Similarly, $b < c + a$ and $c < a + b$. This (substitute for Proposition 2) yields (a substitute for Corollary 4, namely) that there exists a triangle Δ (unique up to congruence) with sides of length a, b, c. Finally, as in the proof of Proposition 6, we use the above upshot of [1], [2] and the fact that cosine is one-to-one on $(0, \pi)$ to conclude that the angles of Δ have radian measure α, β, γ. ■

References

[1] Burton, L.J. (1949). The laws of sines and cosines, *American Mathematical Monthly*, 56, 550 - 551.

[2] Dobbs, D.E. (1984). The sine law and the cosine law are equivalent, *Mathematics and Computer Education* , 18, 42-45.

Using Recursion to Solve a Probability Problem

by

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The concept of recursion is getting increased attention these days. In this article we show how it can be used to solve an interesting but fairly complicated probability problem. Consider the following scenario:

> At the end of basketball practice Andy, a 40 per cent free throw shooter, challenges Bob, a 70 per cent shooter, with a deal: the first one to make a free throw gets treated by the other when they stop at the fast-food stand on the way home.

Bob accepts the challenge and shows good sportsmanship by letting Andy go first. What is Andy's chance of winning? Although in theory the contest could go on forever, the question can be answered by using the tree diagram in Figure 1a, which shows that if both players miss their first attempts the contest effectively starts all over. Letting x be the probability that Andy wins, we see that

$$x = .4 + (.6)(.3)x, \qquad (1)$$

from which we obtain the solution $x = 40/82$, or approximately 0.4878.

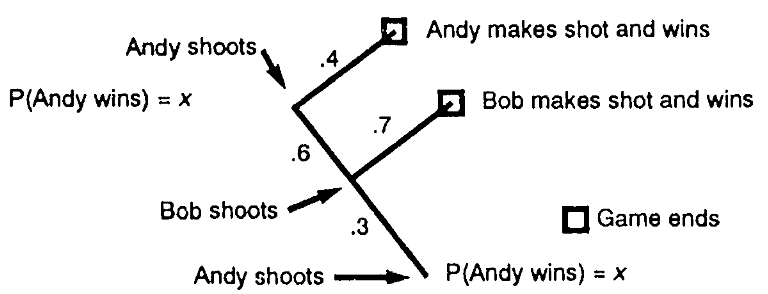

Figure 1a: Andy shoots first; x is the probability that Andy is the first to make a shot.

This proposition seems fair since each player's chance of winning is approximately 50 per cent. If Bob shoots first, however, Andy's chance of winning drops dramatically. Let y be the probability that Andy wins when Bob shoots first. Andy gets a chance to shoot only if Bob misses his first shot. This happens with probability 0.3, and if it happens the problem reduces to the case in which Andy shoots first (see figure 1b). That is, $y = 0.3x$. Thus $y = 12/82$, or approximately 0.1463.

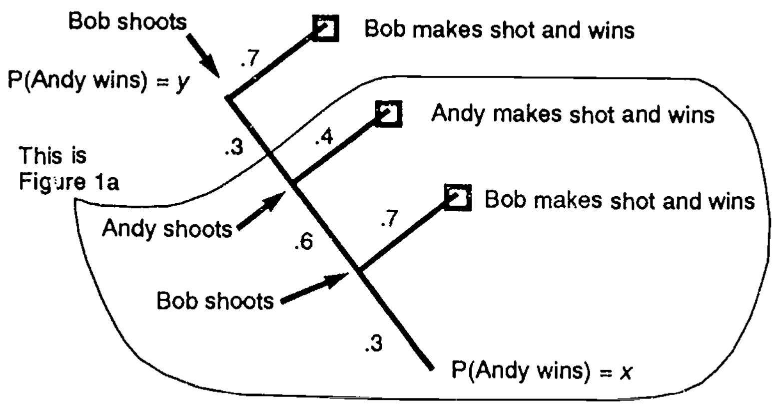

Figure 1b: Bob shoots first; y is the probability that Andy is the first to make a shot.

An extension

Equation (1) represents a form of recursion since we reach a point in the solution where the problem is reduced to the original situation. When both players miss their first shots, the game effectively starts over again. We can illustrate another form of recursion by complicating the game.

Suppose that, instead of just shooting until one player makes a free throw, the two players decide to award agreed upon numbers of points for made shots by each player and play until one of them reaches a certain total. With the percentages as given above (40 and 70), it seems only fair to give Andy more points for a "make" than Bob. Let us illustrate by arbitrarily deciding on three points for Andy and two for Bob. Let us further agree that the first player to reach ten points wins. The matter of who shoots first must also be decided upon.

These choices dictate that Andy must make four shots to win, while Bob must make five. It may turn out that these parameters give one player a decided advantage and we may want to change them. A computer program allows us to do just that, and a few sample outputs show some results which may or may not confirm our suspicions about the fairness of a given set of choices. Simulating 30 trials is an option in the program; some results from simulating 1000 trials are reported along with the theoretical probabilities in Table 2. A disk containing this computer program is available from the authors. (Send a blank disk and specify either PASCAL (3.5 in.) or Applesoft BASIC (5.25 in.).)

It is clear that the extension makes the problem much more difficult. Figure 2 shows that there are many ways in which the several possible final scores can be reached. The possible intermediate and final scores are shown, with Andy's score listed first in each case. Arrows pointing to the right indicate that Bob was the last player to make a shot, while the downward arrows indicate that Andy was the last to make a shot. Determining the probabilities of traveling the various paths through this maze is complicated not only by the multitude of paths but also by consideration of whose turn it is. For example, to determine the probability of moving from the score 6-4 to the score 6-6 we must know how the score 6-4 was reached. If 6-4 was reached from 3-4, it is now Bob's turn. But if 6-4 was reached from 6-2, it is Andy's turn.

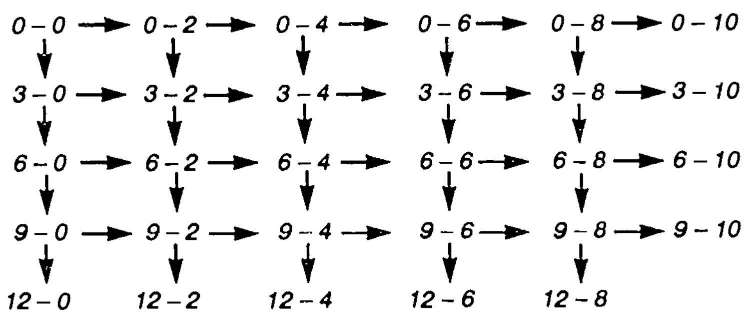

Figure 2: Possible scores when Andy gets 3 points for a made shot and Bob gets 2 points. Andy's score is given first. Andy's probabilities of winning from scores along the bottom row are all 1; from scores on the right edge they are all 0.

Using recursion

Such mind-boggling considerations deter us from an approach that involves so many cases. We therefore employ a method, involving recursion, which is both tractable and insightful. Suppose, for example, that we wish to compute Andy's chance of winning if the score is 6-4 and it is Andy's turn to shoot. Let x be this probability and refer to the tree diagram in Figure 3. With probability 0.4 the score will change to 9-4 and it will be Bob's turn, so it would be helpful to know Andy's chance of going on to win from that point; with probability 0.6 the score will remain 6-4 and it will be Bob's turn. From that point the score will change to 6-6 with probability 0.7, while with probability 0.3 the score will remain 6-4; in either case it will then be Andy's turn. It would also be helpful to know Andy's chance of going on to win from a score of 6-6 when it is his turn to shoot. The lowest branch in the tree in Figure 3 is a "back to where we were" point at which Andy's chance of going on to win is x. Here is the idea of recursion again. Using the standard notation for conditional probability "P(C|D)" for "the probability of event C, given event D," we can write

$$P(\text{A wins} \mid 6\text{-}4, \text{A's turn}) = (.4)P(\text{A wins} \mid 9\text{-}4, \text{B's turn}) + (.6)(.7)P(\text{A wins} \mid 6\text{-}6, \text{A's turn})$$

$$+ (.6)(.3)P(\text{A wins} \mid 6\text{-}4, \text{A's turn}), \qquad (2)$$

or $x = (.4)\mathrm{P}(\text{A wins} \mid \text{9-4, B's turn}) + (.6)(.7)\mathrm{P}(\text{A wins} \mid \text{6-6, A's turn}) + (.6)(.3)x$,

which can be written

$$x = \frac{(.4)\mathrm{P}(\text{A wins} \mid \text{9-4, B's turn}) + (.6)(.7)\mathrm{P}(\text{A wins} \mid \text{6-6, A's turn})}{1 - (.6)(.3)} \qquad (3)$$

We now note that the two probabilities in the numerator of (3) are the probabilities that Andy will go on to win from the two higher scores in Figure 2 which are adjacent to the present score of interest, 6-4. A similar observation holds at each of the twenty non-final scores in Figure 2. Thus, if we could compute Andy's chance of winning when the score is 9-8, taking into account whose turn it is, we could use this information (along with his probabilities of 1 along the bottom of the grid and 0 down the right side of the grid) to compute his chances of going on to win from scores of 9-6 and 6-8. We would then continue to work backward to compute Andy's chances of going on to win from any intermediate score, knowing only whose turn it is. The final computations would yield answers to the most interesting questions: what are Andy's chances of winning from the start? These chances depend, of course, on who shoots first.

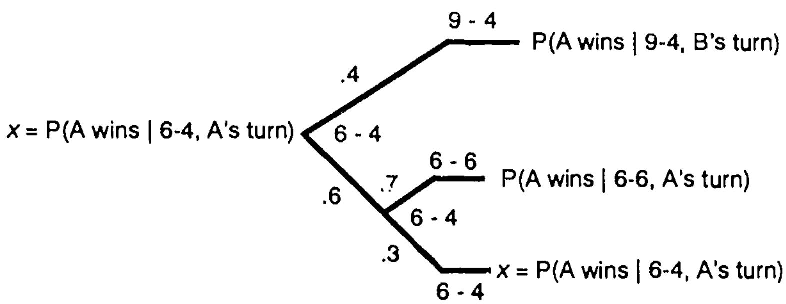

Figure 3: Recursive computation of x, the probability that Andy wins when the score is 6-4 and it is Andy's turn.

Figure 3 refers to the case in which the score is 6-4 and it is Andy's turn to shoot. If it is Bob's turn, the analysis is a bit simpler. Figure 4 shows that with probability 0.7 the score will change to 6-6, while with probability 0.3 the score will remain at 6-4. In either case it will then be Andy's turn. If we let y be Andy's chance of going on to win from a score of 6-4 when it is Bob's turn, we see that

$$y = (.7)\mathrm{P}(\text{A wins} \mid \text{6-6, A's turn}) + (.3)\mathrm{P}(\text{A wins} \mid \text{6-4, A's turn}) \qquad (4)$$

The first probability in (4) is a "later" one and will have been computed since we are working backward. The second probability in (4) was computed from equation (3). Thus the determination of y is straightforward. To compute x using equation (3), we refer to Table 1 and find that S(9,4), which is P(A wins | 9–4, B's turn), is

26

0.7760, and that R(6,6), which is P(A wins | 6-6, A's turn), is 0.3111. Thus, from equation (3), we get

$$x = \frac{(.4)(0.7760) + (.6)(.7)(0.3111)}{1 - (.6)(3)}$$

$$x = 0.5379$$

Note that this agrees with the value of R(6,4) in Table 1.

Equation (4) then gives y = (.7)(0.3111) + (.3)(0.5379) = 0.3791, the value of S(6,4) in Table 1.

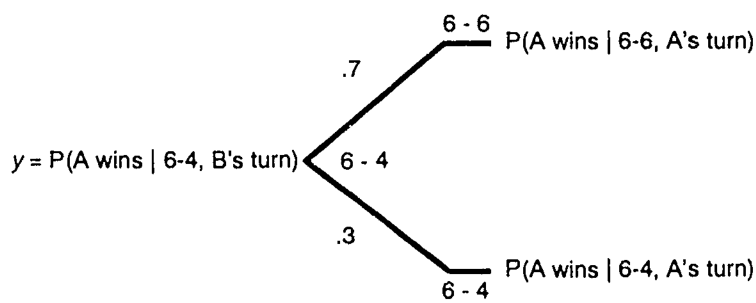

Figure 4: Computation of *y*, the probability that Andy wins when the score is 6-4 and it is Bob's turn.

Enter the computer

This analysis employs the concept of recursion in a backward direction. All that remains to make it practical is a suitable computer code. The details of programming are less important than the ideas involved, so we leave these to the interested reader. Sample output is given in Table 1, using the data of the discussion above. In the left column, for example, R(6,4) denotes Andy's chance of going on to win if the score is 6-4 and it is Andy's turn to shoot, while in the right column S(6,4) is Andy's chance of going on to win if the score is 6-4 and it is Bob's turn. We note also that inspection of the bottom row shows that the chosen parameters of this example work to Andy's disadvantage regardless of who shoots first, but, as our intuition should tell us, Andy's chance is better (0.3068 to 0.2104) if he shoots first. We note also that the first row in Table 1 gives the probabilities found in the "first one to make a shot wins" contest discussed at the beginning of this article. This is not surprising since, with the score at 9-8, the game will end with the next made shot.

Table 1: Sample computer output giving Andy's probabilities, R(*a*,*b*) and S(*a*,*b*), of going on to win from all possible scores (*a* for Andy, *b* for Bob). In the left column Andy shoots first; in the right column Bob shoots first.

```
ENTER ANDY'S CHANCE OF MAKING EACH SHOT.
? .4
ENTER BOB'S CHANCE OF MAKING EACH SHOT.
? .7
HOW MANY POINTS FOR A MAKE BY ANDY?
? 3
HOW MANY POINTS FOR A MAKE BY BOB?
? 2
HOW MANY POINTS NEEDED TO WIN?
? 10
```

R(9,8) = .4878	S(9,8) = .1463
R(9,6) = .7377	S(9,6) = .5628
R(9,4) = .8656	S(9,4) = .7760
R(9,2) = .9312	S(9,2) = .8853
R(9,0) = .9647	S(9,0) = .9412
R(6,8) = .0714	S(6,8) = .0214
R(6,6) = .3111	S(6,6) = .1433
R(6,4) = .5379	S(6,4) = .3791
R(6,2) = .7074	S(6,2) = .5887
R(6,0) = .8214	S(6,0) = .7416
R(3,8) = .0104	S(3,8) = .0031
R(3,6) = .0753	S(3,6) = .0299
R(3,4) = .2235	S(3,4) = .1197
R(3,2) = .4017	S(3,2) = .2769
R(3,0) = .5675	S(3,0) = .4514
R(0,8) = .0015	S(0,8) = .0004
R(0,6) = .0154	S(0,6) = .0056
R(0,4) = .0663	S(0,4) = .0306
R(0,2) = .1690	S(0,2) = .0971
R(0,0) = .3068	S(0,0) = .2104

Varying the parameters

The computer program gives us an opportunity to experiment with values of the parameters that could yield a nearly fair game. In real life we have little control over the probabilities of making a free throw. What we can control, given the players' chances of success on any given shot, are the points to be awarded for a make and the total points needed to win. Students enjoy this "what if" aspect of the problem. Some sample values of R(0,0) and S(0,0) are given in Table 2, where the values of 0.4 and 0.7 are used for Andy's and Bob's chances of making a shot. We note that, with three points given for a make by Andy and two for a make by Bob, a nearly fair (0.4902) game would be to play to 15 and let Andy go first. With seven points awarded to Andy and four to Bob (in reverse order to their shooting percentages), a nearly fair (0.4943) game would be to play to 28 and let Bob shoot first. Compare the results, however, if the game is played to 48 or 49. Note also the reasonably good agreement between the theoretical probabilities and Andy's numbers of wins in 1000 simulation trials.

Table 2: Shooters' percentages are constant; points awarded for makes and points needed to win are varied. The column headed by R(0,0) gives Andy's chance of winning from the start if he goes first. The column headed by S(0,0) gives Andy's chance of winning from the start if Bob goes first. Simulation results are reported in columns 7 and 9.

Andy's pct.	Bob's pct.	Andy's points	Bob's points	Needed to win	R(0,0)	Andy's wins in 1000 trials	S(0,0)	Andy's wins in 1000 trials
.4	.7	3	2	10	.3068	283	.2104	242
.4	.7	3	2	15	.4902	474	.4013	375
.4	.7	3	2	20	.3349	309	.2652	297
.4	.7	7	4	28	.5894	572	.4943	479
.4	.7	7	4	48	.5421	538	.4705	484
.4	.7	7	4	49	.6361	626	.5703	604

An interesting special case occurs when the two players are equally likely to make any given shot. Equal probabilities of 0.3, 0.5, and 0.8 are given in Table 3. We might suspect, other factors being equal as they are in Table 3, that the advantage to shooting first diminishes as the number of points needed to win increases. This does happen, but at varying rates which depend on the shooters' probabilities.

Table 3: Effects of the points needed to win on Andy's chance of winning when both shooters are equally proficient. R(0,0) = Andy's chance of winning from the start if he goes first. S(0,0) = Andy's chance of winning from the start if Bob goes first.

Andy's pct.	Bob's pct.	Andy's points	Bob's points	Needed to win	R(0,0)	S(0,0)	Difference
.3	.3	1	1	5	.5246	.4754	.0492
.3	.3	1	1	10	.5166	.4834	.0332
.3	.3	1	1	50	.5072	.4928	.0144
.5	.5	1	1	5	.5488	.4512	.0976
.5	.5	1	1	10	.5329	.4671	.0658
.5	.5	1	1	50	.5142	.4858	.0284
.8	.8	1	1	5	.6317	.3683	.2634
.8	.8	1	1	10	.5852	4148	.1704
.8	.8	1	1	50	.5361	.4639	.0722

Summary

In this presentation we have tried to show how the idea of recursion is useful in a certain type of probability problem. Recursion comes into play in two situations, first when attempting to find a player's probability of making the first shot when the contest might well go on forever, and again when using the probabilities of going on to win from later scores to compute the probabilities from earlier scores when a certain target score has been set (the first being a special case of the second). The latter is greatly facilitated by the employment of a computer, illustrating once again that many problems which are conceptually solvable are not practically solvable without appropriate technology.

Calculus to Algebra Connections in Partial Fraction Decomposition

by
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Will Watkins, Professor of Mathematics at the University of Texas Pan American, received his Ph.D. in Mathematics from the University of Wyoming. He has taught mathematics in all levels from junior high to university. His interests include topology, technology's impact on education, and mathematics education.

Partial Fraction Decomposition is an indispensable technique for integrating rational expressions. The Algebra to Calculus connection is a common one. Students imagine mathematics as an ordered string of concepts, moving from Algebra to Calculus. Mathematics is more of a web than a string. To overlook the interconnectedness is to miss much of the beauty and power of mathematics. There are also powerful Calculus to Algebra connections. Here we illustrate methods of Partial Fraction Decomposition that rely on Calculus and Algebra concepts. The first of these methods enhances the practicality of the so called Heaviside Method. Oliver Heaviside (1850-1925) was a British physicist and electrical engineer, who pioneered work in operational calculus "which was particularly useful for the analysis of ... transmitted telegraph waves." [1]

Determine the coefficients in the Partial Fraction Decomposition,

$$\frac{9x+1}{x(x+1)(x-1)^2} = \frac{A}{x} + \frac{B}{x+1} + \frac{C}{x-1} + \frac{D}{(x-1)^2}.$$

Students readily accept the strategy of multiplying both sides by the denominator of the left hand side.

$$9x + 1 = A(x+1)(x-1)^2 + Bx(x-1)^2 + Cx(x+1)(x-1) + Dx(x+1)$$

This is appealing because now the identity involves no denominators. Since the equation must hold for every value of x, students evaluate both sides at each root of the original denominator. For each unique root all but one term on the right hand side goes to zero. The non-zero term is the product of the partial fraction coefficient associated with the root of evaluation and each factor of the original denominator which is not zero when evaluated at the root in question. For example, choosing $x = -1$ yields

$$-8 = B(-1)(-2)^2 = -4B, \text{ so } B = 2.$$

This solution process involves multiplying by certain quantities and almost immediately dividing by that same quantity, requiring considerable writing and potential for errors. A few simple observations make these written calculations a simple mental exercise.

Heaviside observed that multiplying both sides by either x, $(x+1)$, or $(x-1)^2$ results in a new identity that can be evaluated at $x = 0$, $x = -1$, or $x = 1$, respectively, because that is no longer a root of any denominator. For example, multiplying by x yields,

$$\frac{9x+1}{(x+1)(x-1)^2} = A + \frac{Bx}{x+1} + \frac{Cx}{x-1} + \frac{Dx}{(x-1)^2},$$

which, when evaluated at $x = 0$ reveals that

$$\frac{1}{(0+1)(0-1)^2} = A + \frac{B\cdot 0}{0+1} + \frac{C\cdot 0}{0-1} + \frac{D\cdot 0}{(0-1)^2} \quad or \quad A = \frac{1}{1} = 1.$$

Similarly multiplying by $(x+1)$ and evaluating at $x = -1$, or multiplying by $(x-1)^2$ and evaluating at $x = 1$ yields

$$B = \frac{9(-1)+1}{-1\cdot(-1-1)^2} = \frac{-8}{-4} = 2 \quad or \quad D = \frac{9+1}{1\cdot(1+1)} = \frac{10}{2} = 5.$$

Heaviside's method successfully reduced the problem to finding C in the following equation,

$$\frac{9x+1}{x(x+1)(x-1)^2} = \frac{1}{x} + \frac{2}{x+1} + \frac{C}{x-1} + \frac{5}{(x-1)^2}.$$

Heaviside's method can be done mentally by covering the factor in question with a finger and evaluating the resulting expression at the root of the covered factor, thus obtaining the value of the partial fraction coefficient associated with that factor. Of course this only works for coefficients associated with the highest power of that factor. Calculus can help us find coefficients associated with lower powers of the factor.

Calculus students soon recognize how to calculate the limit as x goes to infinity of a rational expression. The limit equals the ratio of the leading coefficients when the degree of the numerator equals the degree of the denominator; the limit equals zero when the degree of the numerator is less than the degree of the denominator; and the limit is unbounded otherwise. Multiply both sides by x,

$$\frac{9x+1}{(x+1)(x-1)^2} = 1 + \frac{2x}{x+1} + \frac{Cx}{x-1} + \frac{5x}{(x-1)^2},$$

and let x go to infinity,

$$0 = 1 + 2 + C + 0 \textit{ from which } C = -3.$$

32

This last calculation, a direct application of limits, is considerably simpler than those resulting from methods in traditional texts [2].

These methods also apply in cases where the denominator has complex roots. For example, determine the coefficients in the Partial Fraction Decomposition,

$$\frac{3x+1}{(x-1)(x^2+1)^2} = \frac{A}{x-1} + \frac{Bx+C}{x^2+1} + \frac{Dx+E}{(x^2+1)^2}.$$

Multiplying both sides by $(x-1)$ and evaluating at $x = 1$, yields $A = 1$. The coefficients D and E can also be identified using Heaviside's method. Multiply both sides by $(x^2+1)^2$ and evaluate at i, the complex root of x^2+1,

$$\frac{3i+1}{i-1} = Di + E,$$

$$1 - 2i = E + Di,$$

from which $E = 1$ and $D = -2$. Now multiplying by x and letting x go to infinity, $0 = 1 + B$ or $B = -1$. The final coefficient C is determined by evaluating the original equation at any value that is not a root of the denominator. Zero is an excellent choice in this example,

$$-1 = -A + C + E.$$

Substituting $A = 1, E = 1$ and solving yields $C = -1$.

Another fascinating Calculus to Algebra connection involves derivatives. Consider the rational expression $P(x)/Q(x)$ in which the degree of $P(x)$ is strictly less than the degree of $Q(x)$. Remember that long division transforms any rational expression into the sum of a polynomial and a rational expression satisfying the degree condition on $P(x)$ and $Q(x)$. Suppose there exists a number c (either real or complex) so that $Q(c) = 0$ but $Q'(c) \neq 0$, which is another way of saying c is a simple root of $Q(x)$. Then the following represents a partial fraction expansion,

$$\frac{P(x)}{Q(x)} = \frac{A}{x-c} + r(x) \textit{ where } r(x) \text{ is some rational function.}$$

The rational function $r(x)$ is the rest of partial fraction decomposition. Multiplying by $(x - c)$ yields,

$$\frac{P(x)(x-c)}{Q(x)} = A + (x-c)r(x).$$

The limit of the right hand side, as x goes to c, is clearly A. We investigate the limit of the left hand side. Since both the numerator, $P(x)(x\text{-}c)$ and the denominator, $Q(x)$, equal zero when $x = c$, this is a candidate for L'Hôpital's rule. Hence

$$\lim_{x \to c} \frac{P(x)(x-c)}{Q(x)} = \lim_{x \to c} \frac{P'(x)(x-c) + P(x)}{Q'(x)} = \frac{P(c)}{Q'(c)} = A.$$

As an example consider

$$\frac{P(x)}{Q(x)} = \frac{4x + 12}{x^3 - 2x^2 - 3x} = \frac{A}{x} + \frac{B}{x + 1} + \frac{C}{x - 3}.$$

The following function becomes a formula for all the coefficients in the partial fraction decomposition,

$$F(x) = \frac{P(x)}{Q'(x)} = \frac{4x + 12}{3x^2 - 4x - 3},$$

yielding

$$A = F(0) = -4,\ B = F(-1) = 2,\ C = F(3) = 2.$$

References

[1] *Encyclopedia Americana*, 1991.

[2] Thomas, Jr., G. B. and Finney R. L. (1992). *Calculus and Analytic Geometry* (8th ed.). Reading, MA: Addisson–Wesley.

SHORT COMMUNICATIONS

Web Puzzle

by

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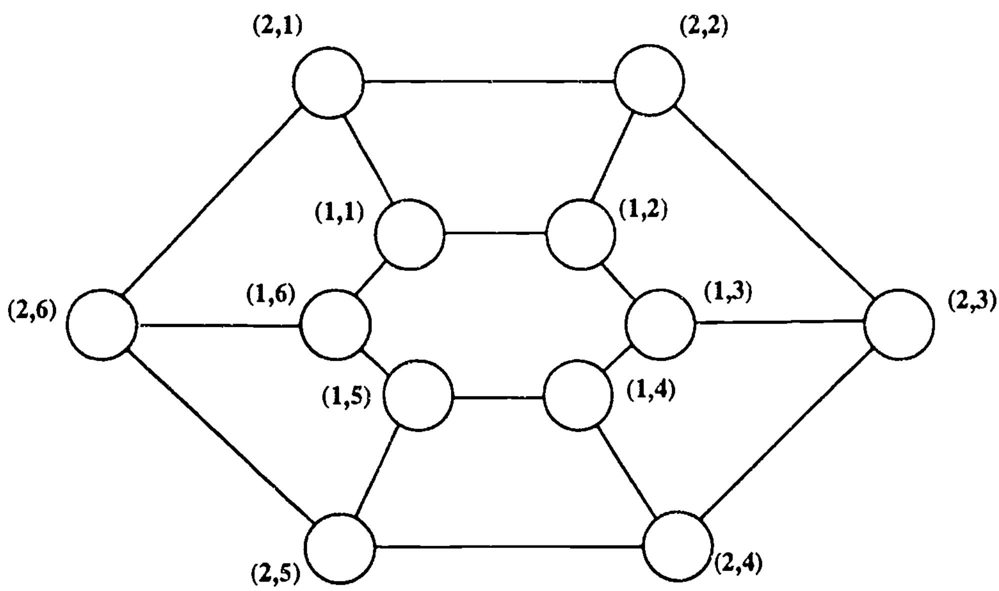
Web puzzles have the merit of great flexibility. Those who like to doodle on the backs of envelopes in their spare time can create and solve smaller ones, while serious hackers have virtually no upper bound on the size of the problem.

A Web, W(n,k), is defined as a set of n concentric k-gons in which we label the nodes (1,1) to (1,k) on the innermost figure, (2,1) to (2,k) on the next, and so on, then join node (x,y) to node (x + 1,y) for x = 1 to n - 1, y = 1 to k. Thus W(2,6) could be represented:



The object of the puzzle is to insert the integers from 1 to nk in the nodes so that any adjoined pair of numbers adds to a prime. More rigorously, if $C(x_i,y_i)$ is the integer at node (x_i,y_i) then we require that $C(x_1,y_1)$ and $C(x_2,y_2)$ add to a prime if

i) $x_1 = x_2$ and $|y_1 - y_2| = 1$ or $k - 1$,

or ii) $y_1 = y_2$ and $|x_2 - x_1| = 1$.

There are many solutions to W(2,6); you may like to derive one before continuing further.

This little puzzle soon becomes quite fiendish for higher values of n and k if one is restricted to pencil and paper. Computerized searches are an alternative, and the puzzle lends itself readily to such. Not counting rotations, reflections, or "inside-out" involutions there are four distinct solutions to W(2,8) and three to W(3,6).

How do we characterize n and k so that $W(n,k)$ is solvable? I do not know the complete answer to this and would welcome correspondence on any discoveries. Here are some problems to start your investigations.

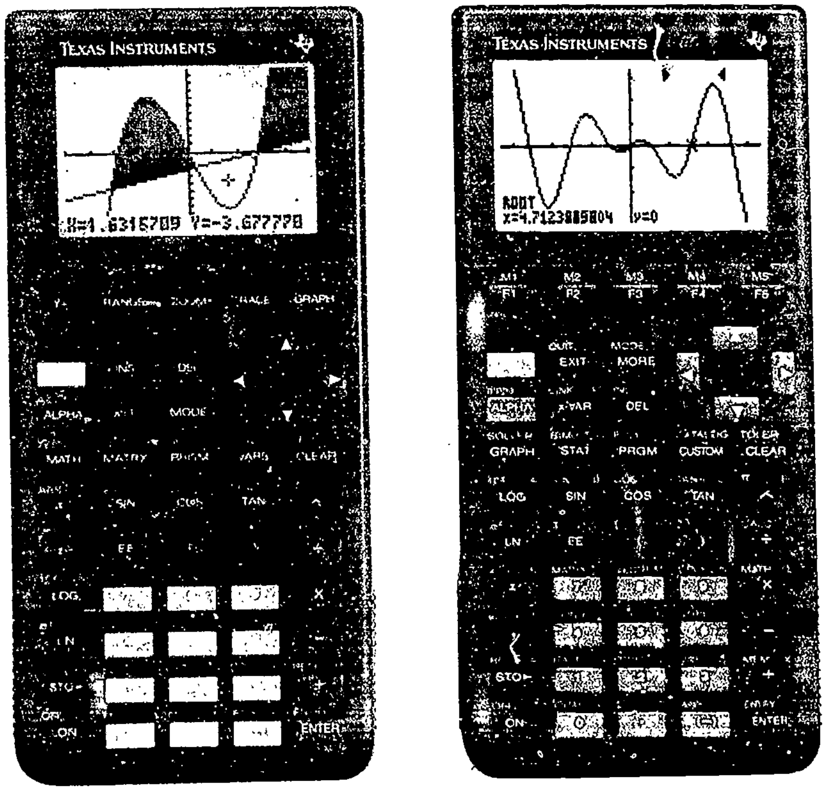
1. Prove (one sentence should suffice) that W(2,4) is not solvable.
2. Prove that k must be even for $W(n,k)$ to be solvable. (Note that this is a necessary, but not sufficient, condition for solvability.)
3. What is the least even k for which $W(1,k)$ is not solvable?

Moving on to Generalized Web puzzles, we define $GW(n,k,a,b)$ as above with the stipulation that the entries are the integers a, $a + b$,..., $a + (n - 1)\,b$. Once again, certain necessary conditions are apparent: b must be odd and relatively prime to a. Experimenting with different values will produce interesting results, but rules and theorems are scarce.

Finally, one solution for W(2,6) is: let nodes (1,1) to (1,6) and (2,1) to (2,6) equal, respectively, 5,6,1,10,9,8 and 2,11,12,7,4,3.

> Logic is the science of justifying your prejudices.
>
> N. J. Rose
>
> Logic is the art of going wrong with confidence.
>
> Morris Kline
>
> Logic is neither a science nor an art, but a dodge.
>
> Benjamin Jowett

First with the best...



The Prime Division Method of Finding the Greatest Common Factor

by Terence Brenner and Michael Shapira
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Terence Brenner received his Ph.D. from Yeshiva University in 1984. The title of his dissertation is "On the Spectra of the Schrodinger Operator."

Michael Shapira received his Ph.D. from Polytechnics University.

In this note we describe a method for finding the greatest common factor (GCF) that is a modification of a method for finding the least common multiple found in many texts such as Keedy and Bittinger (1987). It was brought to our attention that this method is taught in some parts of the world. We present it here for readers who are not familiar with it.

Algorithm: To find the GCF of numbers a and b, place them in columns 2 and 3 of an array. Then

1) Place in column 1 a prime, p, which divides at least one of the numbers.
2) Divide each of the numbers in columns 2 and 3 by p.
   If the division is exact, write the quotient in the next row (same column);
   If the division is not exact, copy the dividend in the next row and cross out p.
3) Repeat steps 1) and 2) until the numbers in columns 2 and 3 are relatively prime.
4) The GCF is the product of all the primes in column 1 which have not been crossed out.

For example, to find the GCF of 80 and 200, we could proceed as follows:

2	80	200
2	40	100
2	20	50
~~2~~	10	25
5	5	25
	1	5

$GCF(80,200) = 2 \cdot 2 \cdot 2 \cdot 5 = 40.$

The method can easily be extended to find the GCF of a larger set of numbers.

To find the GCF of 90, 72, and 54 we might write

$$
\begin{array}{r|rrr}
2 & 90 & 72 & 54 \\ \hline
\not{5} & 45 & 36 & 27 \\ \hline
3 & 9 & 36 & 27 \\ \hline
3 & 3 & 12 & 9 \\ \hline
 & 1 & 4 & 3
\end{array}
\qquad \text{GCF}\,(90, 72, 54) = 2 \cdot 3 \cdot 3 = 18.
$$

This method can also be used to find the GCF of polynomials, as follows:

$$
\begin{array}{l|ll}
x+1 & x^3 + 4x^2 + 5x + 2 & x^3 + 5x^2 + 8x + 4 \\ \hline
x+2 & x^2 + 3x + 2 & x^2 + 4x + 4 \\ \hline
 & x+1 & x+2
\end{array}
$$

so GCF $(x^3 + 4x^2 + 5x + 2, x^3 + 5x^2 + 8x + 4) = (x + 1)(x + 2) = x^2 + 3x = 2.$

How the method works is probably evident by now. If p^e and p^g are the largest powers of p which divide a and b respectively, we will have $\min\{e,g\}$ "successful" divisions by p, indicating that $p^{\min\{e,g\}}$ is a common factor. Repeating this for all prime divisors and then taking the product produces the GCF.

Naturally the method is easiest to use when dealing with numbers or polynomials having easily recognizable factors, but has the advantage of being able to reduce the size of the numbers even when a common factor is not known. Programming a computer to carry out the method is an interesting exercise.

Reference

Keedy, M. L. and Bittinger, M. L. (1987). *Basic Mathematics* (5th Ed). Reading MA: Addison-Wesley.

Lucky Larry #7

Evaluate $\lim_{x \to 0} \frac{\sin 2x}{\sin 3x}$. Larry "cancelled" the sin and the x, getting

$$\lim_{x \to 0} \frac{\cancel{\sin} 2\cancel{x}}{\cancel{\sin} 3\cancel{x}} = \lim_{x \to 0} \frac{2}{3} = \frac{2}{3}$$

Submitted by Philip G. Hogg
Punxsutawney PA

Dalmation Darnation

by
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(Editor's Note: This poem is also a statistics problem, written by a student to satisfy a requirement in a class taught by Barbara Illowsky. For more on the use of student written problems, see Illowsky's contribution to "The Chalkboard" in this issue.)

A greedy dog breeder named Spreckles
Bred puppies with numerous freckles
The Dalmatians he sought
Possessed spot upon spot
The more spots, he thought, the more shekels.

His competitors did not agree
That freckles would increase the fee.
They said, "Spots are quite nice
But they don't affect price;
One should breed for improved pedigree."

The breeders decided to prove
This strategy was a wrong move.
Breeding only for spots
Would wreak havoc, they thought.
His theory they want to disprove.

They proposed a contest to Spreckles
Comparing dog prices to freckles.
In records they looked up
One hundred one pups:
Dalmatians that fetched the most shekels.

They asked Mr. Spreckles to name
An average spot count he'd claim
To bring in big bucks.
Said Spreckles, "Well, shucks,
It's for one hundred one that I aim."

Said an amateur statistician
Who wanted to help with this mission.
"Twenty one for the sample
Standard deviation's ample;
We'll count freckles, as does my beautician."

They examined one hundred and one
Dalmatians that fetched a good sum.
They counted each spot,
Mark, freckle and dot
And tallied up every one.

Instead of one hundred one spots
They averaged ninety six dots.
Can they muzzle Spreckles'
Obsession with freckles
Based on all the dog data they've got?

MATHEMATICS EDUCATION

Guidelines for the Academic Preparation of Mathematics Faculty at Two–Year Colleges

Report of the Qualifications Subcommittee
A Subcommittee of the Education Committee
of the
American Mathematical Association of Two-Year Colleges

Qualifications Subcommittee:

Gregory D. Foley, Sam Houston State University, Chair, 1986-1992
Pansy Brunson, Community College of Western Kentucky University, 1987-1992
Robert L. Carson, Lower Columbia College, 1986-1987, 1990-1992
Sharon Douglas, Cochise College, 1990-1991
David Ellenbogen, St. Michael's College, 1986-1988
Michael E. Greenwood, Clark College, 1986-1992
Lou Hoelzle, Bucks County Community College, 1987-1988
Sue Parsons, Cerritos College, 1990-1992

Approved by the AMATYC Delegate Assembly on November 7, 1992 in Indianapolis, Indiana

Preface

The following document has evolved over a period of years through the work of the Qualifications Subcommittee of the Education Committee, an academic committee of the American Mathematical Association of Two-Year Colleges. The Subcommittee was formed at the AMATYC annual meeting in San Francisco in 1986, largely in response to concern about reports that unqualified persons were being hired by some two-year colleges to fill mathematics teaching positions. Although these positions were often of a part-time, emergency, or temporary nature, at some colleges, faculty from other departments were reportedly being transferred into permanent mathematics teaching positions without appropriate credentials or provisions for retraining. Because of these reports, the Education Committee decided to form the Qualifications Subcommittee to investigate this problem and to make recommendations or establish guidelines for its resolution.

Subsequently, the Subcommittee met, discussed the issues, formulated a framework for the report, and responded to several draft versions of the report. These deliberations led to an original form of the report that was organized according to five categories of instruction: (a) adult basic education, (b) continuing education, (c) developmental and precollege vocational and technical courses, (d) college-level vocational and technical courses, and (e) university-transfer courses. Guidelines were established for each of these five categories.

The Subcommittee sought input from the chairs of related AMATYC standing committees: Developmental Mathematics, Equal Opportunities in Mathematics, and Technical Mathematics. Between meetings, drafts were composed and circulated to Subcommittee members and interested others. Lines of communication were established and kept open with the Committee on Preparation for College Teaching of the Mathematical Association of America chaired first by Guido L. Weiss, and later by Bettye Anne Case. In late 1989, after the Baltimore AMATYC meeting, the finishing touches were put on the original version of the report. After approval by the Subcommittee and the Education Committee Executive Committee, the report was sent to the AMATYC Executive Board for its consideration in the spring of 1990.

The AMATYC Executive Board returned the report for revision with the recommendation that, in the best interest of the profession, there be one set of guidelines for all two-year college mathematics faculty rather than five sets of guidelines based on different categories of instruction. At the 1990 AMATYC meeting in Dallas, the relative advantages of one set of guidelines versus five sets of guidelines were debated in an open forum, at the Education Committee meeting, and at a meeting of the Subcommittee. A consensus was never reached.

A version of the report that took into account the views of both camps was reviewed at the Seattle meeting in 1991. It laid out one primary set of guidelines, but since several members of the Subcommittee strongly believed that it was not realistic in all circumstances to have one set of guidelines for all mathematics faculty, the report included some disclaimers.

The present report is a slight revision of that version, with fewer disclaimers. The most important recommendation of the report is that "hiring committees for mathematics positions at two-year colleges should consist primarily of full-time two-year college mathematics faculty." We have confidence in the professional judgment of current two-year college mathematics faculty and know that they are capable of making appropriate decisions based on any local constraints that may exist. As the title of this report suggests, it is intended to guide decisions, not control them.

This report has undergone a rigorous review process and has benefitted greatly from it. The report now presents, as much as is possible, a shared vision of what the academic preparation of two-year college mathematics faculty should be. I thank the Subcommittee members for their efforts, and we thank the many reviewers for their comments and suggestions. We hope this report will serve as a catalyst for further discussion of the important issues it addresses.

Gregory D. Foley
Subcommittee Chair

Statement of Purpose

This document is addressed to two-year college professionals involved in the staffing and evaluation of mathematics programs for their colleges, and to universities that have, or will develop, programs to prepare individuals to teach mathematics in two-year colleges. It is not intended to replace any regional, state, or local requirements or recommendations that may apply to hiring instructors, assigning them to classes, or evaluating their performance or qualifications. Rather, our goal is to provide guidelines that reflect the collective wisdom and expertise of mathematics educators throughout the United States and Canada regarding appropriate preparation for two-year college faculty involved in the teaching of mathematics, whether on a full- or part-time basis.

We strongly recommend that only properly qualified personnel be permitted to teach mathematics. Ill-prepared instructors can do much harm to students' knowledge of and beliefs about mathematics. Many two-year college students suffer from mathematics anxiety; this should not be reinforced or exacerbated through inappropriate mathematics instruction. **Individuals trained in other disciplines should not be permitted to teach mathematics unless they have received sufficient mathematical training as well.** Moreover, individuals hired to teach mathematics at one level should not be permitted to teach at another level unless they possess appropriate credentials.

We are guarding the gates of our profession. This is our responsibility as the leading professional mathematics organization that solely represents two-year colleges. Staffing practices and procedures vary greatly from college to college and from region to region. We wish to ensure the integrity of our profession and the quality of mathematics instruction at all two-year colleges.

Motivating Factors

Disturbing Trends

Reports such as *Everybody Counts: A Report to the Nation on the Future of Mathematics Education* (National Research Council, 1989) document deep-rooted problems concerning mathematics education in the United States. Among these problems is the need to teach meaningful mathematics to individuals from all social, economic, ethnic, and racial backgrounds. This is imperative if our nation is to maintain a leadership role in the world of the future. The mathematics community should especially strive to increase participation of groups that are underrepresented in mathematics.

Two-year colleges can play a major role in turning our country around in this regard. A study conducted during the 1985-1986 academic year revealed that, among two-year college students, "one-fourth are minority students, and more than one-half are women" (Albers, Anderson, & Loftsgaarden, 1987, p. 112). Steen et al. (1990) reported that, "One-third of the first and second year college students in the United States are enrolled in two-year colleges, including over two-thirds of Afro-American, Hispanic, and Native American students" (p. 13). Two-year colleges are critical to the national effort to recruit and retain minority students and women as majors in mathematics and mathematics-dependent fields. Two-year college mathematics teachers must be prepared to help and encourage students from these underrepresented groups.

Many two-year college mathematics instructors are nearing retirement age (Albers, Anderson, & Loftsgaarden, 1987). We must work hard at recruiting and preparing the next generation of two-year college faculty, and enable them to thrive as college mathematics teachers in our rapidly changing world.

Curriculum Reform Movements

The forces of curricular change have reached a relative maximum. The *Curriculum and Evaluation Standards for School Mathematics* (Commission on Standards for School Mathematics, 1989) and *Calculus for a New Century* (Steen, 1988) call for major changes in the content and methods of school and college mathematics. These and other related calls for reform (e.g., National Research Council, 1989, 1991) are due in part to the implications of the pervasiveness of computer technology in our society and in part to the sagging mathematics achievement of students. It is appropriate that we now reexamine the preparation of two-year college mathematics faculty.

Guiding Principles

Two questions have guided the preparation of this report: What are the characteristics of an effective mathematics teacher? How can these characteristics be fostered and extended through academic preparation and continuing education?

The growing body of research related to effective mathematics teaching (Grouws, Cooney, & Jones, 1988) indicates that effective mathematics teachers use their time wisely and efficiently, both in and out of class; they present well organized lessons; and they know their subject. Effective instructors are reflective; they think about their teaching before they teach, while they teach, and after they teach. They are creative, resourceful, and dedicated. They use a variety of methods and respond to the needs of the particular class and students they are teaching. Effective mathematics teachers are skilled questioners who encourage and challenge their students. They are clear and careful communicators who recognize the importance of language in mathematics, and mathematics as language. They model the behaviors they wish their students to exhibit, especially problem solving, exploration, and investigation.

Effective mathematics instructors know a great deal of mathematics and understand the interconnections among its various branches as well as applications to other disciplines. They are continually developing their knowledge and understanding of mathematics, of teaching, and of how students learn. They are independent learners who can adapt and contribute to changes in collegiate mathematics curriculum and instruction.

Effective mathematics instructors are active professionals. They read journals, attend professional meetings, and engage in other professional activities. Impagliazzo et al. (1985) further elaborated on the activities and characteristics of professionally active mathematics instructors in *The Two-Year College Teacher of Mathematics*. The present report outlines the academic preparation and continuing education necessary for a person to be an effective mathematics teacher at the two-year college level.

Organization of the Report

The remainder of this report is organized into four sections. The first concerns guidelines for the formal preparation of two-year college mathematics faculty. The second outlines important areas of mathematical and pedagogical content that should be included in such preparation. The third section discusses avenues other than formal education for continuing education. The final section briefly addresses the issues of part-time instructors and the desirability of diversity within a mathematics department. These sections are followed by a bibliography and an appendix that contains an outline for a course on college mathematics teaching. Such a course should be offered by universities that prepare two-year college mathematics instructors, and should be included in the academic preparation of these instructors.

Guidelines for Formal Preparation

Mathematics programs at two-year colleges reflect their diverse missions and particular needs. Mathematics instruction at a comprehensive community college may comprise adult basic education to prepare students for a high school equivalency examination; developmental and precollege vocational and technical courses designed to prepare students for college credit courses; courses for students in college-level vocational and technical programs; university-transfer courses through vector calculus, differential equations, and linear algebra; and continuing education courses that do not carry college credit. Other colleges may focus only on a subset of these types of instruction. Many two-year technical colleges, for example, focus on precollege and college-level vocational and technical courses.

Because of this diversity, the standard for the mathematical preparation of two-year faculty must be sufficiently robust to guarantee faculty flexibility. This standard is divided into three parts: minimal preparation, standard preparation, and continuing formal education.

Definitions

All full- and part-time faculty should possess at least the qualifications listed under *minimal preparation*. All full-time faculty should begin their careers with at least the qualifications listed under *standard preparation*. All faculty should continue their education beyond this entry level. The *continuing formal education* section provides some suggestions. Continuing education of a less formal nature is not only valuable but essential. Avenues for informal continuing education are discussed later in this report. Continuing formal education that requires full-time university enrollment is best undertaken after several years of teaching.

The terms *faculty* and *instructors* are used interchangeably to refer to persons who hold teaching positions. No particular level within a ranking system is implied by either of these terms.

Courses in physics, engineering, and other fields can contain significant mathematical sciences content. Although there is no simple, set formula for doing so, such courses should be taken into account by two-year college mathematics hiring committees when evaluating a candidate's transcripts. Similarly, such courses should be carefully considered by university personnel when making program admission decisions and advising students who hold or may seek two-year college mathematics teaching positions.

Minimal Preparation

All full- and part-time mathematics instructors at two-year colleges should possess at least a master's degree in mathematics or in a related field with at least 18 semester hours (27 quarter hours) in graduate-level mathematics. A master's degree in applied mathematics is an especially appropriate background for teaching technical mathematics. Course work in pedagogy is desirable.

Standard Preparation

All full-time mathematics instructors at two-year colleges should *begin* their careers with at least a master's degree in mathematics or in a related field with at least 30 semester hours (45 quarter hours) in graduate-level mathematics and have mathematics teaching experience at the secondary or collegiate level. The teaching experience may be fulfilled through a program of supervised teaching as a graduate student. Course work in pedagogy is desirable.

Continuing Formal Education

All mathematics instructors at two-year colleges should continue their education beyond the entry level. Appropriate continuing formal education would include graduate course work in mathematics and mathematics education beyond the level of the individual's previous study. Such advanced study may culminate in one of the following degrees: Doctor of Arts in mathematics, PhD or EdD in mathematics education, or PhD in mathematics. For mathematics instructors at two-year technical colleges, taking courses in technologies served by the two-year college mathematics curriculum is also appropriate. Advanced studies may result in a second master's degree.

Evaluating Credentials

A great deal of specialized knowledge and judgment is required to evaluate a candidate's credentials. **For this reason, hiring committees for mathematics positions at two-year colleges should consist primarily of full-time two-year college mathematics faculty.** All staffing decisions related to mathematics instruction—whether full- or part-time—should be made by content specialists.

The Course Content of a Preparatory Program

Mathematics Content

The core of the academic preparation of two-year college mathematics instructors is course work in the mathematical sciences. The mathematics course work for individuals preparing to be two-year college mathematics instructors should include courses chosen from several of the following areas. Graduate course work should fill gaps, broaden, and extend the undergraduate mathematics background of such individuals.

Discrete Mathematics
Computer Science
Mathematical Modeling and Applications
Calculus through Vector Calculus
Differential Equations
Real Analysis
Numerical Analysis
Complex Variables
Linear Algebra
Abstract Algebra
Probability
Statistics
History of Mathematics
Number Theory
Geometry
Topology
Combinatorics

Pedagogical Content

Course work in pedagogy is an important component in the academic preparation of two-year college mathematics instructors. Such course work should be chosen from the areas listed below. Courses in these areas should be offered by universities that prepare two-year college mathematics instructors.

Psychology of Learning Mathematics
Methods of Teaching Mathematics
Organizing and Developing Mathematics Curricula and Programs
Instructional Technology
Teaching Developmental Mathematics
Using Calculators and Computers to Enhance Mathematics Instruction
Measurement, Evaluation, and Testing
Teaching Mathematics to Adult Learners
Teaching Mathematics to Special-Needs Students
College Mathematics Teaching Seminar (see the Appendix)

Continuing Education

As noted earlier, effective mathematics instructors are active professionals. They read journals, attend professional meetings, and engage in other activities to continue their education. The American Mathematical Association of Two-Year Colleges (AMATYC). the Mathematical Association of America (MAA), the National Council of Teachers of Mathematics (NCTM), and other organizations sponsor conferences, offer minicourses and summer institutes, publish books and journals, and advertise other opportunities for continued professional growth. AMATYC, MAA, and NCTM workshops, minicourses, and institutes address many of the mathematical and pedagogical topics listed in the previous section. Participation in these activities is critically important in order for two-year college mathematics faculty to keep up-to-date in their field.

Closing Comments

Part-Time Faculty

Ideally, part-time instructors should possess the same level of preparation and commitment to quality teaching as full-time instructors. An MAA committee report entitled *Responses to the Challenge: Keys to Improving Instruction by Teaching Assistants and Part-Time Instructors* (Case, 1988) addresses this issue at length. We support the views of this report as they pertain to two-year college part-time mathematics faculty.

Variety of Expertise

A mathematics department should be composed of individuals who possess complementary strengths and areas of expertise. This is especially true within a comprehensive community college with a wide variety of degree programs. A mathematics department with experts or specialists in pedagogy, statistics, computing, applied mathematics, analysis, and history of mathematics is generally much stronger than one in which all members have similar academic backgrounds. This together with programmatic needs and candidate qualifications should be taken into account when seeking and hiring full- and part-time faculty.

Bibliography

Albers, D. J., Anderson, R. D., & Loftsgaarden, D. O. (Eds.). (1987). *Undergraduate programs in the mathematical and computer sciences: The 1985-1986 survey* [MAA Notes No. 7]. Washington, DC: Mathematical Association of America.

Case, B. A. (Ed.). (1988). *Responses to the challenge: Keys to improving instruction by teaching assistants and part-time instructors* [Report of the Committee on Teaching Assistants and Part-Time Instructors, MAA Notes No. 11]. Washington, DC: Mathematical Association of America.

Case, B. A. (Ed.). (1991). *Preparing for college teaching* [Preprint of portions of the report of the Committee on Preparation for College Teaching, which is due to appear in 1993]. Washington, DC: Mathematical Association of America.

Commission on Standards for School Mathematics. (1989). *Curriculum and evaluation standards for school mathematics.* Reston, VA: National Council of Teachers of Mathematics.

Committee on the Mathematical Education of Teachers (COMET). (1988). *Guidelines for the continuing mathematical education of teachers* [MAA Notes No. 10]. Washington, DC: Mathematical Association of America.

Davis, R. M. (Ed.). (1989). *A curriculum in flux: Mathematics at two-year colleges: A report of the Joint Subcommittee on Mathematics Curriculum at Two-Year Colleges.* Washington, DC: Mathematical Association of America.

Greenwood, M. E. (1976). *A report on the training of community college mathematics teachers.* Unpublished manuscript, University of Illinois, Urbana-Champaign, and Clark College, Vancouver, WA.

Grouws, D. A., Cooney, T. J., & Jones, D. (Eds.). (1988). *Perspectives on research on effective mathematics teaching.* Reston, VA: National Council of Teachers of Mathematics, and Hillsdale, NJ: Lawrence Erlbaum Associates.

Impagliazzo, J. (Chair), Ayers, S. W., Lindstrom, P., & Smith, J. B. (1985). *The two-year college teacher of mathematics.* American Mathematical Association of Two-Year Colleges.

Leitzel, J. R. C. (Ed.). (1991). *A call for change: Recommendations for the mathematical preparation of teachers of mathematics* [Report of the Committee on the Mathematical Education of Teachers (COMET)]. Washington, DC: Mathematical Association of America.

Long, C. T. (1985). *The academic training of two-year college mathematics faculty.* In D. J. Albers, S. B. Rodi, & A. E. Watkins (Eds.), *New directions in two-year college mathematics: Proceedings of the Sloan Foundation conference on two-year college mathematics* (pp. 393-403). New York: Springer-Verlag.

National Research Council. (1989). *Everybody counts: A report to the nation on the future of mathematics education.* Washington, DC: National Academy Press.

National Research Council. (1990). *Reshaping school mathematics: A philosophy and framework for curriculum.* Washington, DC: National Academy Press.

National Research Council. (1991). *Moving beyond myths: Revitalizing undergraduate mathematics.* Washington, DC: National Academy Press.

Steen, L. A. (Ed.). (1988). *Calculus for a new century: A pump, not a filter* [MAA Notes No. 8]. Washington, DC: Mathematical Association of America.

Steen, L. A. (Ed.). (1989). *Reshaping college mathematics: A project of the Committee on the Undergraduate Program in Mathematics* [MAA Notes No. 13]. Washington, DC: Mathematical Association of America.

Steen, L. A. (Chair), Goldstein, J. A., Jones, E. G., Lutzer, D., Treisman, P. U., & Tucker, A. C. (1990, November-December). Challenges for college mathematics: An agenda for the next decade [Center section insert]. *Focus: The Newsletter of the Mathematical Association of America,* 10(6).

Appendix

Outline for a Course in College Mathematics Teaching

Nature of the Course: The course should be a seminar focusing on timely and timeless issues faced by teachers of collegiate mathematics.

Participants: Enrollment should be open to all graduate students in mathematics and mathematics education.

Topics: Topics should be chosen chiefly from among those listed below:

1. *Teaching Issues:* Motivating ideas, motivating students, conveying the nature of mathematics, effective use of calculators and computers to convey mathematical ideas, learning theory, teaching for understanding, teaching problem solving, characteristics of effective mathematics teachers, individualized instruction, the use and grading of written assignments, teaching adult learners, testing and grading.
2. *Program Issues:* Curricular trends, textbook selection, course and program

development, course and program evaluation, student advising, placement of students.

3. *Other Issues:* Writing for publication, committee work, professional meetings, service. This discussion should include (a) organizations and publications, (b) types of institutions, and (c) finding and retaining jobs.

Activities: Practice presentations and lessons, discussions of issues, outside readings, sharing of obtained information, writing, computer demonstrations, hands-on computer and calculator activities, guest speakers, videotapes, and films.

Suggested Requirements:

1. Attendance at all meetings, participation in all activities including discussions of assigned readings (a bound collection of readings can be made available for purchase at a local outlet).
2. Term paper within the area of the impact of new technology on undergraduate mathematics education, or other appropriate topic: one draft plus a final manuscript.
3. A 10-15 minute conference-style presentation with handouts and prepared transparencies.
4. Presentation of a classroom-style lesson with a computer-demonstration, workshop, or other innovative format.
5. Preparation of the following documents: (a) a biographical sketch; (b) a chronological list of graduate courses with date, instructor, and institution; and (c) a full curriculum vitae.

Textbooks could be chosen from:

Albers, D. J., Rodi, S. B., & Watkins, A. E. (Eds.). (1985). *New directions in two-year college mathematics: Proceedings of the Sloan Foundation conference on two-year college mathematics.* New York: Springer-Verlag.

Davis, R. M. (Ed.). (1989). *A curriculum in flux: Mathematics at two-year colleges.* Washington, DC: Mathematical Association of America.

National Research Council. (1991). *Moving beyond myths: Revitalizing undergraduate mathematics.* Washington, DC: National Academy Press.

Ralston, A., & Young, G. S. (Eds.). (1983). *The future of college mathematics: Proceedings of a conference/workshop on the first two years of college mathematics.* New York: Springer-Verlag.

Schoenfeld, A. (Ed.). (1990). *A source book for college mathematics teaching.* Washington DC: Mathematical Association of America.

Steen, L. A. (Ed.). (1988). *Calculus for a new century: A pump, not a filter* [MAA Notes No. 8]. Washington, DC: Mathematical Association of America.

Steen, L. A. (Ed.). (1989). *Reshaping college mathematics: A project of the Committee on the Undergraduate Program in Mathematics* [MAA Notes No. 13]. Washington, DC: Mathematical Association of America.

Sterrett, A. (Ed.). (1990). *Using writing to teach mathematics* [MAA Notes No. 16]. Washington, DC: Mathematical Association of America.

Tucker, T. W. (Ed.). (1990). *Priming the calculus pump: Innovations and resources* [MAA Notes No. 17]. Washington, DC: Mathematical Association of America.

Zimmermann, W., & Cunningham, S. (Ed.). (1991). *Visualization in teaching and learning mathematics* [MAA Notes No. 19]. Washington, DC: Mathematical Association of America.

Fractals and College Algebra

by

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Many students in our nation's colleges take a course in college algebra to satisfy a mathematics requirement for an undergraduate bachelor's degree. Most of them are liberal arts students for whom mathematics is just one more hurdle that must be mounted in their quest for a diploma. Most will leave college without any further experience in mathematics. At Ramapo College of New Jersey, a four-year liberal arts state college of approximately 2700 full-time students, we wanted to expose these students to some interesting and beautiful mathematics that would enrich their experience and extend their vision beyond algebra.

During a recent semester, the authors decided to experiment with a module on fractals in two of the seven scheduled sections of college algebra. (Students intending to pursue the study of calculus elect the precalculus course instead.) Our college algebra course includes the study of complex numbers and quadratic, exponential, and logarithmic functions and is a terminal course for most students who take it. These topics are fundamental in the study of fractals at an elementary level. Beautiful fractal images can be generated by iteration of simple quadratic functions. Utilizing operations with complex numbers and function notation, students can explore complex-valued functions that result in Julia set fractals.

Fractal geometry is a relatively new field of study since Mandelbrot's work in the 1970's caught the attention of the mathematical community as he expanded on the work of Fatou and Julia done in the 1920's. Fractals appear in many ways in the real world from cloud formations and a cauliflower head to important applications discovered in recent years in such diverse fields as human physiology (Goldberger, 1990) and seismology (Levi, 1990). The visual, aesthetic appeal of fractals provides a context for the symbol manipulation of algebra and offers an enrichment and extension of standard topics.

In our college algebra course we used fractals to frame the standard topics, to reinforce and illustrate applications of the material, and to provide enrichment during the last two to three weeks of the regular course. One can pursue the study of fractals in a number of directions depending on the level of the course and the emphasis desired and A. Solomon (1989) suggests several alternatives. Our purpose was to give the students a glimpse into the world behind the beautiful images that are called fractals and provide the opportunity to use some of the mathematics learned in college algebra in a new context.

Content of the Fractals Module

Introduction

Some of the students had heard the word "fractal" and were curious about what a fractal is. To capitalize on students' interest and arouse curiosity about the mathematics behind the fractal images, we viewed pictures and slides from the beautiful illustrations in Peitgen (1986). A senior mathematics major who was doing some independent investigations on the computer with fractals presented a computer display of Julia sets and the Mandelbrot set using FRACTINT (1987). We then turned our attention to iterating some simple functions.

Iteration

Devaney (1990) calls the list of successive iterates of a point or number the **orbit** of that point. Students used calculators to iterate real-valued functions such as the square root function, the squaring function, and some linear functions and worked many problems to decide whether the orbit of a given value was unbounded, periodic, or convergent. An example of such a problem is:

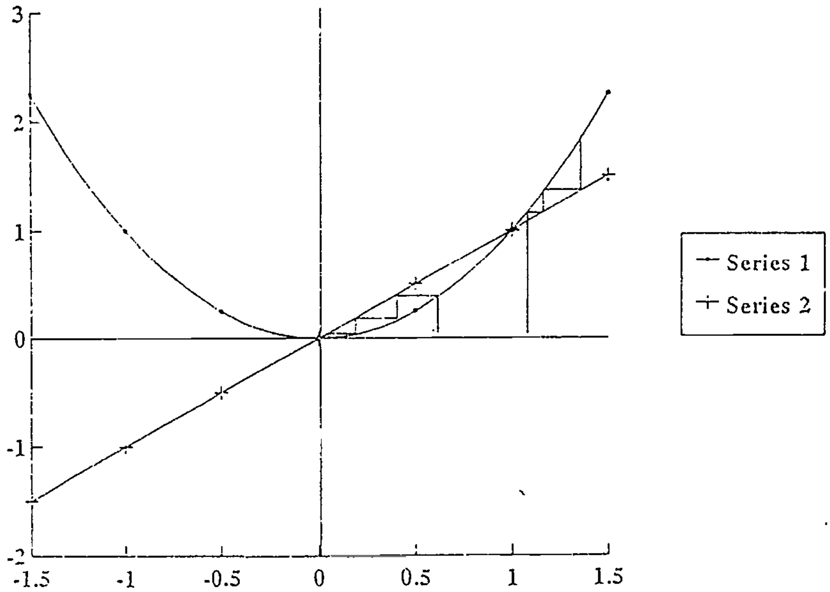
List the first 15 iterations of the following functions, beginning with the initial value zero. Decide whether the orbit is unbounded, periodic, or convergent.

1. $F(x) = x + 1$ *(unbounded)*
2. $F(x) = \frac{1}{2}(x + 1)$ *(converges to 1)*
3. $F(x) = x^2 - 1$ *(periodic between -1 and 0)*

In later exercises students were asked to find the orbits of other values for these same functions.

Graphical analysis

Graphical analysis, also referred to as a web diagram (Barnsley, 1988), fits naturally into the study of functions and graphing techniques in college algebra and provides a way of using a geometric procedure to follow the orbits that had been iterated for real-valued functions. After the class viewed part of the videotape by Devaney (1989), we outlined the graphical analysis procedure described in Devaney (1990) and the students traced the orbits of several different functions. The figure below illustrates how this technique works for the function $f(x) = x^2$. The line $y = x$ is used to trace the orbit by drawing a vertical line to the graph and then a horizontal line to the diagonal and repeating this process several times to see what happens.



An orbit of a dynamical system is **stable** if it has the property that if you change the initial input slightly the resulting orbit behaves similarly. If nearby orbits have vastly different behavior, the orbit is said to be **unstable**, as exhibited by the squaring function with values 1 and -1 (Devaney, 1990). Students traced the orbits of .5 and then .6 and 1 followed by 1.1 for $f(x) = x^2$ to see that a small difference in the starting point may make a big difference in the orbit. This sensitive dependence on initial conditions, as illustrated by the previous example, is what is known as **chaos**. A short excerpt from James Gleick (1987) provided a focus for discussing this "butterfly effect" named by Edward Lorenz.

Graphical analysis provided valuable practice for students in understanding more about the meaning of graphs and how to interpret function values, an area that students often need to strengthen. Long orbits are sometimes difficult to trace with graphical analysis, however, and since it is useful only for real-valued functions, we returned to iteration, this time with complex-valued functions.

Complex numbers and iteration

Since the students had studied operations with complex numbers, we introduced complex-valued functions using a simple exercise such as:

Evaluate the function $F(z) = z^2 - 1$ for

$z = 1 + i$

$z = -1 + 2i$

$z = -4 - 4i$

In each case plot z and $F(z)$ in the complex plane.

This provides review and practice with function evaluation and operations with complex numbers as it traces the orbit of $1 + i$ through the first few iterates of F since $F(1 + i) = -1 + 2i$ and $F(-1 + 2i) = -4 - 4i$.

The quadratic family $F(z) = z^2 + c$ contains virtually all of the behaviors of a typical dynamical system in a simple and accessible setting. By choosing different values of c, these quadratic families produce an incredible variety of Julia sets - some fatty clouds, others skinny brambles of a bush, sea-horse tails and the shape of a rabbit.

We limited our work in this area to examining Julia sets generated by certain quadratic iterations since trigonometric functions, and therefore complex-valued exponential functions, were not within the scope of the course. Students were asked first to consider the simplest case $F(z) = z^2$, where $c = 0$. They examined the orbit for points where $|z| < 1$ which converge to 0 making it a fixed point under iteration, the orbits for points with $|z| > 1$ which are unbounded (escape to infinity), and the orbits for points with $|z| = 1$ which do neither, i. e. the points are at a distance of 1 from 0 and they stay there. Students then examined the orbits of several other complex-valued quadratic functions and discovered that, for example, the orbits of the function $F(z) = z^2 - 1$ behave similarly to the real case: either the orbits "escape" to infinity or else tend to oscillate between 0 and 1.

One method for picturing Julia sets is to color points white if their orbits escape and black if they do not. The boundary between them is the Julia set. Another interesting way to picture the phenomena is to use different colors to paint escaping orbits depending on how quickly the orbit escapes, i. e. the number of iterations that occur before the points exceed a specified absolute value. In this way the computer generates the stunning pictures that represent fractals.

The calculation of iterations for complex functions is laborious for students to undertake by hand, but a simple program can be written (Vojack, 1992) for the programmable calculator to calculate how many iterations are needed for $F(z)$ to increase beyond a chosen value. The students can produce a Julia set on graph paper by choosing z as the center of a square and coloring it according to the magnitude of that number.

It was Mandelbrot who first looked at complex numbers instead of real numbers and followed the iterative process in the complex plane rather than on the real line. The fact that Mandelbrot is a living, practicing mathematician is of great interest to students whose perception is that all of mathematics was developed and made into a neat package in the 17th century. At this point, we discussed the fact that Mandelbrot coined the word "fractal" and used it to describe geometric shapes having two key characteristics: self-similarity and fractional dimension (Peitgen, 1986). The famous Mandelbrot set is obtained by fixing the initial value of z at 0 and varying c and then looking at the orbit of 0 under the function $F(z) = z^2 + c$ for each different c-value (Douady, 1986). In studying Julia sets and Mandelbrot sets, notions of metric spaces and connectedness arise and are beyond the scope of a college algebra course. The interested reader may refer to Barnsley (1988), Devaney and Keen (1989), Lauwerier (1991), and Peitgen and Richter (1986). The students especially enjoyed viewing part of the Robert Devaney video "Chaos, Fractals, and Dynamics" (Devaney, 1989) showing Julia set fractals generated by other functions such as exponential functions.

Geometric iteration

We then turned our attention to fractals generated by iterating certain geometric constructions. Using grid paper, students generated the Sierpinski triangle by iterating the procedure of removing the middle triangle formed by joining the midpoints of the sides of the starting triangle. Then they generated the Sierpinski carpet. In each case they examined the number patterns generated by counting the number of triangles/squares at each stage and the area of the regions at each stage, leading them to the conclusion that the area of the Sierpinski triangle and the Sierpenski carpet is zero while the length of the boundary is infinite.

Since we did not have access to computers for this course, students used pencil and paper to generate the geometric figures. The Koch curve, first introduced by Helge von Koch in 1904, was presented as the classic prototype for a coastline-type fractal, a family of fractals based on the repetition of a geometrical procedure. Students generated the Koch curve and snowflake as well as some original variations where they defined their own recursive process. At each stage they calculated the length and noted the scaling factor. As in the previous geometric iterations, the boundary of the Koch snowflake was infinite while the area was finite.

Dimension

The concept of fractal dimension provides an application of the logarithm and a means of comparing fractals. A number representing fractal dimension can be attached to clouds, coastlines, etc. and allows us to compare sets in the real world with fractals generated in the laboratory. The concept of fractal dimension is an attempt to quantify a subjective feeling which we have about how densely the fractal occupies the space in which it lies (Barnsley, 1988).

If we select an arbitrarily small measuring unit a as the yardstick, and measure the length of the Koch curve by using the yardstick N times, the total length is Na. The fractional dimension can be written

$$D = \frac{\log N}{\log \left(\frac{1}{a}\right)}$$

where a is the size of the unit measure and N is the number of times the measure is used (Lauweirer, 1991). It is not difficult to show that the length increases as the size of the unit measure decreases.

The article by Kern (1990) suggests a more detailed development for studying dimension. We first reviewed examples of dimension 1, dimension 2, and dimension 3 in the Euclidean sense and then discussed what fractional dimension might mean. With some guidance, students then calculated the dimension of the Koch snowflake as about 1.34 and suggested a possible meaning for fractal dimension as a measure of the relative complexity of the object, the wiggliness of a curve or the roughness of a surface. Students were able to calculate the dimension for a few carefully constructed fractals.

Conclusion

Fractals provide a meaningful way to integrate the seemingly disparate topics of college algebra and to appeal to the aesthetic interest of liberal arts students. We used Devaney (1989) as the centerpiece of qur module and supplemented with other sources to provide what we conceived as a brief encounter with this new world of fractals. While we barely scratched the surface of the possibilities in this field, we enjoyed the opportunity to introduce our students to some new and exciting mathematics. It was stimulating for us and for them. Students had a better opinion of their own mathematical knowledge as they realized that they could study such contemporary mathematics with applications in other fields that are still being discovered. They could begin to believe that the mathematics they studied - even in college algebra - is useful and important.

References

Barnsley, M. (1988). *Fractals Everywhere*. Boston: Harcourt Brace Jovanovich.

Devaney, R. L. (1990). *Chaos, Fractals, and Dynamics: Computer Experiments in Mathematics*. New York: Addison-Wesley Publishing Company.

Devaney, R. L. (1989). *Chaos, Fractals, and Dynamics: Computer Experiments in Mathematics*, Video, The Science Television Company. New York.

Devaney, R. L. and Keen, L. (1989). *Chaos and Fractals: The Mathematics Behind the Computer Graphics*. Providence, RI: American Mathematical Society.

Douady, A. (1986). Julia Sets and the Mandelbrot Set. *The Beauty of Fractals* by H. O. Peitgen and P. H. Richter. New York: Springer-Verlag. 161-175.

FRACTINT. 1987. Shareware.

Gleick, J. (1987). *Chaos: Making a New Science*. New York: Viking Press.

Goldberger, A. L., Rigney, D. L., and West, B. J. (1990, October). Chaos and Fractals in Human Physiology. *Scientific American*, 262(2), 42 - 49.

Kern, J. F. and Mauk, C. C. (1990, March). Exploring Fractals - A Problem - Solving Adventure Using Mathematics and Logo. *Mathematics Teacher*, 179 - 185.

Lauwerier, H. (1991). *Fractals: Endlessly Repeated Geometrical Figures*. Princeton, New Jersey: Princeton University Press.

Levi, B. G. (1990, November). Are Fractures Fractal or Quakes Chaotic? *Physics Today*, 17-19.

Peitgen, H. O. and Richter, P. H. (1986). *The Beauty of Fractals: Images of Complex Dynamical Systems*. New York: Springer-Verlag.

Solomon, A. (1989, Fall). A Fractal Outline of a Fractal Course. *The AMATYC Review*, 11(1), 35 - 41.

Vojack, R. (1992, Spring). Julia Set Experiment. Paper presented at Precalculus Conference at Rutgers University, New Jersey.

From an actual college catalog:

"The purpose of this course is the deepening and development of difficulties underlying the contemporary theory of..."

Using Computer Technology as an Aid in Teaching the Introductory Course in Quantitative Methods

by

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I. Executive Summary

In the fall of 1991, Babson College ran thirteen sections of an undergraduate required course in Quantitative Methods, QTM1100, which included three tracks: Track 0, consisting of students judged to have weak algebra skills and having the lowest math SAT and Math Achievement test scores; track 2, consisting of students who took a calculus course in high school and having the highest math SAT and Math Achievement test scores; and track 1, containing all other students. A controlled experiment was performed with classes in the fall term of 1991 based upon decision criteria established with historical data (Miller, 1990). An IBM Corporation software product, the Math Exploration Toolkit (MET), was utilized by four of the seven instructors and an investigation based on a comparison of final exam grades was conducted. Results indicate that the main reasons students have failed, dropped, or done poorly in the course *in the past* are more likely related to the students' inability to perform algebraic manipulations than to a low aptitude for understanding the concepts.

II. Background

Over the past few years, many colleges and universities across the country have intensified efforts to modernize and improve mathematics instruction, particularly in required quantitative courses for entering students (Douglas, 1986; Ferrini-Mundy & Graham, 1991; Gordon & Hallett, 1991; Johnson & Lamagna, 1991; Lopez & Mathews, 1990). At Babson College, the effort has resulted in an increased focus on business applications and computer assisted techniques (Prichett, 1988). This paper reports on how these activities appear to have enhanced the learning experience and increased student achievement in a first year required course in quantitative methods.

III. Course Content and Organization

QTM1100 is a required, first year quantitative methods course with primary focus on business applications. The course is designed for tomorrow's managers whose needs include (1) development of problem-solving skills (2) understanding the role that quantitative modeling plays in decision making at the operational and strategic policy levels and (3) exposure to the power and convenience of computer assisted problem solving.

Many of our students come to Babson with inadequate basic algebra skills. In response, we have developed an environment with a strong emphasis on applications that helps motivate students to rectify their algebra deficiencies and to make good judgments relating to the selection and use of technology. The following organization of topics is sequenced in a manner which facilitates a gradual development of the students' level of mathematical sophistication.

1. Review of basic algebra and graphing.
2. Determination of linear cost functions and break-even analysis in marketing, accounting, manufacturing, and other applied contexts.
3. Systems of two or more linear inequalities, including graphical solutions.
4. Linear Programming: problem formulation, graphical solution procedures for two variable problems, computer assisted techniques and interpretation of computer output for n-variable problems.
5. Overview of the difference between optimization under assumptions of linearity, and optimization when a constraint or an objective function is non-linear.
6. Introduction to differential calculus: limits, difference quotients, simple and function power rules, product and quotient rules, and optimization applications.
7. Use of exponential and logarithmic functions in solving equations and estimating exponential growth function parameters.
8. Mathematics of finance including ordinary annuities and their connection with mortgages, sinking funds, and cash-flow analysis.
9. Derivatives of exponential and logarithmic functions including applications in marketing such as response functions.
10. Antiderivatives and the indefinite integral.
11. The fundamental theorem of calculus, the area between curves, and applications of integral calculus in economics.
12. Asymptotic integrals, numerical integration, and the importance of these concepts in statistical applications.

There are several advantages in this approach:

1. The pertinent parts of algebra are reviewed in the context of applications as they are used in the course.
2. The student is continually presented with applications of the topics and so can make important connections between theory and practice.
3. Students are more receptive to learning the theoretical basis of a topic once its relevance to a management education is more obvious.

IV. Student Placement

In the fall of 1991, Babson College ran thirteen sections of QTM1100. Entering students were separated into three tracks using the results of each student's Math Achievement test, last high school math grade, and Math SAT. These were: Track 0, consisting of students judged to have weak algebra skills; track 2, consisting of students who took a calculus course in high school; and track 1, containing all other students. Four of the seven instructors utilized the Math Exploration Toolkit (MET) software from IBM in their sections.

V. The Experiment

An investigation was conducted which was based on a comparison of final exam grades. The final exam contained ten questions, the first eight of which were the same for all sections, independent of track. The following were permitted in all sections for use in taking the final exam: calculator, one 8.5" x 11" note sheet, and a table of derivatives and integrals provided by instructor. Additionally, both section 7 (track 0), and section 9 (consisting of students we estimated to be the best of the track 1 students prior to course commencement) took their exam in their regular assigned classroom where the MET computer software was available for use. Almost all students, independent of assigned section, were still working on their exams at the end of the two-hour exam period. A common grading scheme was distributed to all instructors.

A total of 355 student records were analyzed in this study. Section sizes ran from 11 students to 38 students with the smallest sections being held in a networked computer lab environment. The most significant of our discoveries occurred when we factored out instructor, class size, and time of day effects. This was possible as a result of analyzing the results for instructor five (one of the authors), who taught two sections under the following conditions:

Section A: Seventeen students;
Instructor use of MET as an electronic chalkboard;
Student use of MET during the lectures and on exams;
Track 0.

Section B: Eighteen students;
No instructor use of MET software;
No student use of MET software;
Track 1;
Class meeting time followed that of Section A.

The results of this analysis are shown in Table I. It is important to note that section B students had the benefit of possible instructor learning curve effects since they were the second class given the same lecture topic on any given class day. Lastly, any possible differences in grading have been minimized since the same instructor graded both sections.

TABLE I

ANALYSIS OF INSTRUCTOR 5

Question	p-value	Conclusion
1	.626	Median grade the same for both sections
2	.119	Median grade the same for both sections
3	.024	Median grade for section B is less than for section A
4	.193	Median grade the same for both sections
5	.632	Median grade the same for both sections
6	.004	Median grade for section A is less than for section B
7	.358	Median grade the same for both sections
8	.404	Median grade the same for both sections

In questions 1, 2, 4, 5, 7, and 8 the use of MET offered no advantage except for the possible elimination of algebraic requirements. The two sections taught by instructor five performed at the same median level in each of these questions. One explanation for this is that the track 1 students were penalized for mistakes in algebra that the track 0 students did not make because they utilized the MET software. A second explanation is that neither group made significant algebraic mistakes, but instead, understood the material tested at the same conceptual level. An algebra skills pre-test given the first day of class indicates that the second explanation is much more likely than the first. This leads us to believe that the use of the MET software allowed the track 0 students to concentrate better on the concepts being taught with less distraction caused by problems with algebra. We feel the evidence leans strongly in the direction that the reason many of our students have done poorly in the past was due to an inability to perform algebraic manipulations rather than due to a lower aptitude to understand the concepts being taught. Data is currently being collected from additional QTM1100 classes.

The MET software could calculate the derivatives of the functions appearing in question 3 directly. The track 0 students would be expected to do better than track 1 students on this question since they had the MET software at their disposal. The results are a confirmation that they knew how to use the MET software effectively for this type of problem.

All students used calculators, rather than computer software, as an aid in solving problems related to compound interest and ordinary annuity formulas. Evidently, on problem 6 the track 0 students could not correctly classify or execute the necessary manual calculations as effectively as the track 1 students.

VI. Integration of Technology

We have found that the use of electronic media by instructors and the integration of computer assisted techniques in our quantitative courses has allowed us to focus more on applications, problem formulation, selection of solution techniques, and interpretation and presentation (communication) of numerical and graphical results. Over the past few years, we have revised the curriculum to be consistent with this approach and be more reflective of a course which might well be entitled "Foundations of Quantitative Methods in Management" (Prichett, 1988). This has led to the de-emphasis of several traditional calculus topics, such as techniques of integration, and to more emphasis on problem-solving techniques which were sometimes omitted in the past.

The current organization and the use of technology allows us to shift the emphasis on certain topics and activities.

Specifically:

1. Student use of computer algebra system software in the assessment and remediation of algebra skill deficiencies;
2. More active learning and cooperative learning among students;
3. Problem formulation and identification of possible solution techniques;
4. Analysis of results as they relate to the applied situation being modeled;
5. Expansion of the scope of the course to address recent innovations such as the biweekly mortgage (Gordon & Saber, 1989);
6. Development of algorithms for optimization in cases such as quantity discounts based on order size in the retail marketing sector;
7. Earlier introduction of numerical integration techniques.

VII. Observations

We have found that MET can be a valuable tool when used as an electronic chalkboard by an instructor and/or when used as a computer algebra system and graphics utility by students. Several other software products such as TrueBasic, Derive, MACSYMA, Mathematica, and Maple might serve these needs equally well depending on the characteristics of the students and the program of study where they would be utilized. Here at Babson, MET has proven to be relatively easy for our students to learn and has the attractive feature of requiring students to specify solution steps, especially in optimization problems, rather than simply generating a solution with a single keystroke. In our earliest experiments with the software in 1989 and 1990 students received less encouragement or instruction in the software. We found it interesting to observe different patterns of usage depending upon student attitudes towards technology and their confidence levels in mathematics. By the time of the second term examination all students in networked labs were using MET and several commented that they wish they had started using MET sooner.

We are also experimenting with MET "defers" (macros) which students can utilize in or out of class. These macros can be used as "interactive slide shows" in class and have the advantage of allowing the student to "re-play" key portions of the demonstration at their convenience. We are also utilizing network software wherein the instructor's microcomputer is connected to an LCD plate on an overhead projection device and the instructor is able to capture and transmit the contents of a screen to all nodes on the network and to a large screen. The instructor can easily demonstrate typical student difficulties and appropriate corrections to the entire class. This feature appears to increase class interactivity and to foster a more efficient discussion between the instructor and the students. Weak students seem to find it less intimidating than writing on the chalkboard.

VIII. Need for Additional Research

Although there appears to be a trend towards acceptance of technology by mathematics teachers, there are considerable differences of opinion about the role that calculators and computers should play in the curriculum (Karian, 1989; Kemeny, 1991; NCTM, 1989; Palmiter, 1991; Whitney & Urquhart, 1990). Some instructors argue in favor of teaching students advanced programming techniques along with the most powerful computer algebra system software available. Even those teachers that are the most enthusiastic about integrating technology into their teaching are quick to point out that technology is not a panacea for the problems in mathematics education. Some instructors of experimental sections reported very limited success accompanied by too heavy a price in that many students regarded the computer more as an additional burden than as a facilitator. A major challenge faced by teachers is how to change the attitudes of many students who express little confidence in their ability to succeed in mathematics. There seems to be less reporting in the professional literature regarding change in courses and teaching strategies for weak or average mathematics students than in courses for mathematics, engineering, or science majors.

Some of the issues that continue to intrigue us are the following: First, some students resist using a computer algebra system such as MET even though the software is easy to assimilate and utilize. Second, we need to investigate the possibility that the use of computers hinders the performance of some students. Third, the instructor needs to ensure that students do not regard the computer as just another means to support rote learning as opposed to truly extending their intellect. Based upon our experience and the experience of others we believe that appropriate use of technology will continue to be a major focus of innovative teaching techniques in the 1990's and beyond (Barrett & Goebel, 1990; Countryman & Wilson, 1991; Demana & Waits, 1990; Karian, 1989; Prichett, 1988; Ralston, 1991; Robertson, 1991; Swadener & Blumbaugh, 1990).

References

Barrett, G., & Goebel, J. (1990). The Impact of Graphing Calculators on the Teaching and Learning of Mathematics. *Teaching And Learning Mathematics in the 1990s.* Reston: National Council of Teachers of Mathematics.

Countryman, J., & Wilson, E. (1991). Living in a Mathematical World. *The College Board Review*, No. 160.

Demana, F., & Waits, B. K. (1990). Enhancing Mathematics Teaching and Learning Through Technology. *Teaching And Learning Mathematics in the 1990s.* Reston: National Council of Teachers of Mathematics.

Douglas, R. (ed.). (1986). *Toward a Lean and Lively Calculus (M.A.A. Notes No. 6).* Washington: Mathematical Association of America.

Ferrini-Mundy, J., & Graham, K. (1991). An Overview of the Calculus Reform Effort: Issues for Learning, Teaching, and Curriculum Development. *American Mathematical Monthly,* 98(7), Aug-Sep.

Gordon, S., & Hallett, D. (1991). Calculus Reform and Its Implications for Two-Year Colleges. *The AMATYC Review,* 12(2), Spring.

Gordon, S., & Saber, J. (1989). Biweekly Mortgages: Do They Really Decrease Costs? *Babson College Teaching Note Series*, TN-89-4.

Johnson, D. L., & Lamagna, E. A. (1991). The Calculus Companion: Computational Environment for Exploring Mathematics. *Computer Algebra Systems in Education Newsletter*, Colby College, No. 9.

Karian, Z. (1989). The Use of Computer Algebra Systems in the Undergraduate Mathematics Curriculum. *FOCUS*, 9(2), Feb.

Kemeny, J. G. (1991). Software for the Classroom. *Mathematics and Computer Education*, 25(1), Winter.

Lopez, R., & Mathews, J. (1990). Using a Computer Algebra System to Solve for Maxima and Minima. *The College Mathematics Journal*, 21(5), Nov.

Miller, A. P. (1990). *Abstracts, Priming the Calculus Pump: Innovations and Resources (M.A.A. Notes No. 17)*. Washington: Mathematical Association of America.

National Council of Teachers of Mathematics. (1989). *Curriculum and Evaluation Standards for School Mathematics*. Reston: Author.

Palmiter, J. R. (1991). Effects of Computer Algebra Systems on Concept and Skill Acquisition in Calculus. *Journal for Research in Mathematics Education*, 22(2), Mar.

Prichett, G. D. (1988). Calculus in the Undergraduate Business Curriculum. In *Calculus for a New Century (M.A.A. Notes No. 8)*. Washington: Mathematical Association of America.

Ralston, A. (1991). Calculators for Teaching and Testing Mathematics. *The College Board Review*, No. 160.

Robertson, D. (1991). A Program for the Math Anxious at the University of Minnesota. *The AMATYC Review*, 13(1), Fall.

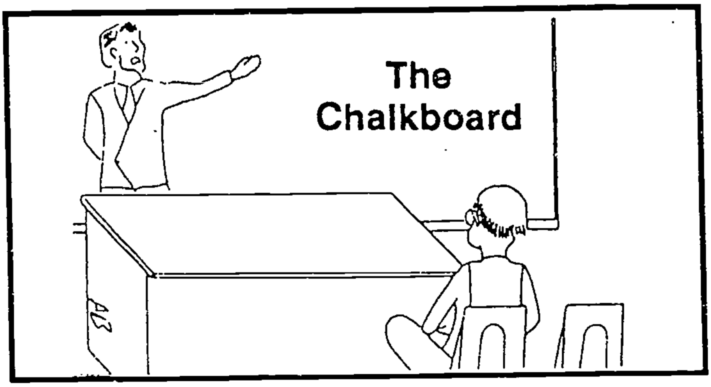
Swadener, M, & Blumbaugh, W. (1990). Teaching Mathematics with a Vision: Integrating Computers into Instruction. In *Teaching and Learning Mathematics in the 1990s*. Reston: National Council of Teachers of Mathematics.

Whitney, R. E., & Urquhart, N. S. (1990). Microcomputers in the Mathematical Sciences: Effects on Courses, students, and instructors. *Academic Computing*, March.

Proof by Circular Reference

The proof of Theorem A is omitted because it is equivalent to Theorem B which will be proved later. When that day comes we are short of time, so we omit the proof of Theorem B, noting that it is equivalent to Theorem A which is already known.

REGULAR FEATURES



Edited by

Judy Cain and Joseph Browne

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This column is intended as an idea exchange. We hope to facilitate an open exchange of ideas on classroom management, teaching techniques, tips for helping students get past the usual stumbling blocks, techniques for improving student participation, etc. We know there are lots of good ideas out there, and this is your chance to share them. Please send your contributions to Judy Cain.

Collaborative Learning, Writing, and Math

Having a student write word problems generally enhances the student's understanding of a concept. Instead of offering suggestions and grading the student's word problem myself, I expand the assignment and turn it into a collaborative project.

Working in pairs, with specific written instructions, each student writes his or her own word problem, with a minimum length of half a single-spaced typewritten page. The partners then trade problems, and each must edit and solve the other's problem. Each is responsible for assuring that the partner's problem is "perfectly" written. The partners' goal is to produce problems.

The students learn a great deal when editing each other's work. They must be able to solve the problem, and they must correct grammar and be sure that the English flows. When necessary, they must be able to explain to their partners why the logic of a problem is wrong and how to correct it, which requires a thorough understanding of the subject.

The class must be well-prepared for this assignment. I keep a collection of high-quality problems from past classes and hand them out (with authors' permission) as practice problems, take-home quizzes, and exam questions.

Submitted by Barbara Illowsky, DeAnza College, Cupertino CA 95014

(Editors' Note: Copies of the instructions and samples of student problems are available from Professor Illowsky. The half-page minimum length mentioned is for a statistics class; you might want to adjust this for other courses. For an outstanding example by one of Professor Illowsky's students, see "Dalmation Darnation" by Kathy Sparling elsewhere in this issue.)

What's This Stuff Good For?

All mathematics teachers have been asked about the usefulness of the mathematics being studied. While not every topic has a readily available real-world application, many do. Here are a few of my favorites.

Braking distance D (in feet) of a car traveling V mph is given by the equation $D = .044V^2 + 1.1V$ (a good example because the coefficients are not "nice" ones). You might ask your students to find the braking distance if a car is traveling 25 mph (a good calculator exercise). Or if a child runs into the street 100 feet in front of a car and the driver reacts instantly and brakes, what is the maximum rate the car can be traveling in order to avoid hitting the child? (This question uses the quadratic formula and gives a realistic reason for testing the reasonableness of the two answers obtained.)

A realistic example of a piecewise function can be found in the Tax Rate Schedules in your income tax booklet. Have students write the tables as functions and then sketch the graphs. This can lead to such questions as the domain of the function; it also provides an introduction to continuity of functions.

Another favorite example that students relate to is the Drunk Driver Problem. The probability that a person will have an accident while driving at a given blood alcohol level b is given by the function $P(b) = e^{21.5b}$. You might ask at what blood alcohol level the probability of an accident is 50%. Another question (with a surprising answer) is the following: A person is legally drunk in many states if the blood alcohol level is 0.10. What is the probability of an accident at that level? Try graphing the function for some added insight.

While it does require a little extra effort on our part to find these more realistic applications, the benefit to the student makes it all worthwhile.

Submitted by Martha Clutter, Piedmont Virginia Comm. College, Charlottesville VA 22902

Pick a Number, Any Number

There is often a need in math classes for students to roll a die, flip a coin, or draw a card from a deck. While very few students have a die or a deck of cards, virtually every student has a calculator. The random number key can simulate random rolls, flips, and draws.

Most inexpensive scientific calculators have a random number key which, when pressed, displays a "random" decimal number, usually consisting of three decimal digits. The FIX mode on the calculator controls the number of decimal digits on the display; if the calculator is "FIX'ed" to display zero decimal digits, only integers will be shown. In all the following examples, set FIX at zero.

For a coin flip, press RAN =. This will produce either 0 or 1 (heads or tails).

To roll a die, press RAN x 7 =. This will produce 0, 1, 2, 3, 4, 5, 6, or 7. If 0 or 7 is "rolled," ignore and "roll" again, because after rounding, the probability of rolling 0 or 7 is half that of any other digit; the probabilities of "rolling" 1, 2, 3, 4, 5, and 6 are all the same.

To draw a card, let 1 = Ace, 2 = 2, . . . , 11 = Jack, 12 = Queen, and 13 = King. Press RAN x 14 =. "Draw" again if 0 or 14 is displayed. If a suit is needed, let 1 = spades, 2 = hearts, etc, and press RAN x 5 =; ignore 0 or 5 and "draw" again.

Submitted by Ronald Hatton, Sacramento City College, Sacramento CA 95822

Lucky Larry #8

Reduce $\frac{18x+9}{12x+6}$. Larry "cancels" term-by-term as follows:

$$\frac{\overset{3}{\cancel{18x}}+\overset{3}{\cancel{9}}}{\underset{2}{\cancel{12x}}+\underset{2}{\cancel{6}}}=\frac{6}{4}=\frac{3}{2}$$

Notice that this error produces correct answers whenever the numerator and denominator are simple multiples of the same polynomial.

Submitted by Dona Boccio
Queensborough Community College
Bayside NY 11364

> Basic research is what I am doing when I don't know what I am doing.
>
> Werner von Braun

Software Reviews

Edited by Shao Mah

Title: Midshipman Differential Equations Program
Author: J.L. Buchanan
Address: Mathematics Department
U.S. Naval Academy
Annapolis MD 21402
Cost: No charge; software available by sending formatted 5 1/4 diskettes to Prof. J.L. Buchanan
Hardware Requirements: IBM compatible computer with at least 512K of memory and a CGA, EGA, VGA or Hercules board but a color monitor with EGA or VGA graphics board is recommended.

MDEP (Midshipman Differential Equations Program) was developed through the U.S. Naval Academy. It provides a useful tool in solving ordinary differential equations (ODE). In addition, it can be used to solve a system of linear equations, calculate a numerical integration, find the sum of an exponential series and plot graphics for user provided functions.

The program helps students to solve the differential equations or systems of equations by the use of MDEP's four numerical methods so that various requirements of accuracy and speed of calculation can be accommodated.

There are three screens available: function, graphics and data. On each of the screens MDEP provides pull-down menus so that students can easily find the useful information and quickly master MDEP without going to the reference manual.

There are some drawbacks. The maximum order and dimension of the differential equations which can be solved by MDEP are limited to 4 and 8, respectively. Boundary value problems can not be solved by using MDEP.

This software is suitable for college math labs. Students interested in solving differential equations in mathematical, engineering and scientific fields should find MDEP to be a useful tool.

Reviewed by: Liming Dai and Yi Sun, Department of Mechanical Engineering, University of Calgary, Calgary, Alberta, Canada.

Title: MATLAB Software for Numerical Methods
Author: Dr. John Mathews
Distributor: Prentice-Hall, Inc.
Englewood Cliffs New Jersey 07632
Computer: Disks are available for both IBM and Macintosh.
Price: Software and "MATLAB Guidebook for Numerical Methods" are free to instructors who adopt the textbook "Numerical Methods for Mathematics, Science, and Engineering" by John H. Mathews.

The author has taken the algorithms from his numerical analysis text and written them for execution in the MATLAB environment. This software requires a copy of the MATLAB software package for execution. The software includes programs for nonlinear equations, systems of linear equations, interpolation, curve fitting, integration, optimization, differential equations, partial differential equations, eigenvalues, and eigenvectors.

The software consists of ten subdirectories labeled Chap_2 through Chap_11. Each subdirectory corresponds to a chapter in the guidebook that accompanies the software. For example, Chap_2 contains algorithms for the solution of nonlinear equations. In the subdirectory Chap_2 one finds programs such as A2_5 (this corresponds to Algorithm 2.5 in the guidebook), which is the Newton-Raphson Algorithm. The user would edit the program A2_5 by entering the desired function, its derivative, initial approximation to the zero, tolerances, and the maximum number of iterations.

The software and accompanying guidebook are well-documented, with examples clearly indicating how to handle input and how to obtain output in both numerical and graphical formats. I have tested all of the programs on the disk and found them to work well on the examples I used. Users of this software should have had some previous experience programming and be familiar with the MATLAB software package.

Students in their first two years of college mathematics are usually introduced to algorithms for solving nonlinear equations, systems of linear equations, and systems of differential equations. In addition, they see algorithms for least-squares approximations, and finding eigenvalues and associated eigenvectors. With this software package students could implement these algorithms, in a realistic way, to solve non-trivial applications. I have used the software for demonstrations and student assignments in my calculus courses.

Reviewed by: Dr. Kurtis D. Fink, Department of Mathematics and Statistics, Northwest Missouri State University, Maryville, Missouri 64468

Titles: Expert Algebra Tutor version 2.3, Expert Arithmetic Tutor version 1.0
Author: Dr. Sergei Ovchinnikov, San Francisco State University
Distributor: TUSOFT, P.O. Box 9979, Berkeley, CA 94709, tel. 510-527-5177
Computer: IBM PC or PS and compatibles, any graphic adapter, 512K memory, DOS 2.3 or higher
Language: C, Borland C compiler, MetaWindow graphics library from Metagraphics Software Corporation
Price: $399.00 single-user version, $999.00 network or multi-user version $1,250 site license, $5.00 prepaid for demonstration disk

Expert Algebra Tutor contains 52 lessons (seven 5.25"or four 3.5" disks) covering Numbers and Operations, Linear Equations and Inequalities in One Variable, Linear Equations in Two Variables and Systems of Linear Equations, Quadratic Equations and Inequalities, Roots and Radicals, Operations on Polynomials and Factoring Polynomials, Rational Expressions, and Exponential and Logarithmic Functions. Expert Arithmetic Tutor contains 47 lessons (six 5.25"or three 3.5" disks) covering Whole Numbers, Common Fractions, Decimal Fractions, Ratio and Proportion, Percent, Signed Numbers, Algebraic Expressions, and Linear Equations in One Variable. Thus these two programs cover most of the material found in pre-algebra and algebra courses in two-year colleges.

The programs are very easy to use, especially, when loaded on a hard drive to avoid diskette swapping. The user does not need written instructions on how to use the programs. A window at the bottom of the screen tells the student which keys to press, including a function key for help, and will not accept other input. The author makes good use of single-key strokes like Enter, F1 for help, Home, End, and arrow keys for selection. These are all common and known to an IBM user.

When loaded, the Tutor displays the title of the program and gives the student an option to enter his/her name. Later, the program uses this name in its personalized messages. Next, a Table of Contents appears on the screen allowing the students to select a topic to work on. The Tutor describes the work to be done, notes help features and suggests that the problems be worked out with a pencil and paper.

Each lesson has a problem session and a remedial session. To start with the student is offered a series of multiple choice questions organized in sets of three. The student uses paper and pencil to solve the problems and then selects among options presented on the screen. At first, I did not like the multiple-choice approach, but then it became clear to me that the choices represent the results of most likely errors. The Tutor uses this procedure to determine sources of misconceptions and establishes individual tutoring strategies. If the student is doing fine, the program proceeds toward problems of increasing difficulty choosing among its various tutoring approaches depending on the student's responses. If the student stumbles, the Tutor interrupts for a remedial session. It is during the remedial session that real learning takes place. The Tutor explains problems which proved difficult for the student in a lucid step-by-step exposition clarifying the solution method. The Tutor chooses among its two thousand tutoring strategies implemented in its rule-based expert system and adopts its program to each student's needs. If the student is completely off, the program shuts down and tells the student to review the material in the book.

The help facility is available throughout the program and gives the method, property, and an example of a problem analogous to the one the student is attempting. All messages, examples, and help facilities are actually windows that pop up on the display, and disappear when not needed.

The student can interrupt a lesson at any time by pressing the ESC key and returning to it later. The Tutor remembers where the students left off and will continue the lesson at that point. The program also keeps an individual file of the student's progress. This file can be viewed on the screen, printed on any printer, or imported by another program.

The author makes excellent use of colors, windows and pleasing screen design when the color graphics monitor is used. On a monochrome display, he accomplishes the same effect through the use of bold print instead of color. There is little use of graphics with this program, but there is no apparent need for more. The program does use the number line for inequalities and the rectangular coordinate system for linear functions. I am curious to know how the program will handle more complex graphics such as conic sections and other functions. The author has informed me that a college algebra version of the program that includes these topics will be available in 1993.

The main weakness of the program is the lack of word problems, although it is hard to imagine how the present version of the software could handle applications. I still think that future releases of the program will be more attractive if they include word problems.

The programs are completely self-explanatory, and the student does not need to study the manuals before using this software. The documentation provided with the programs describes technical requirements and gives detailed instructions how to install programs on a hard disk or a network. It also outlines main features of the software and lists the topics covered in the tutorials.

The Tutor is designed to be used as a supplement to classroom instruction, and is recommended as a computer companion to any secondary school or undergraduate algebra class. It can also be used by itself in a remedial mathematics laboratory.

Reviewed by: Anya M. Kroth, West Valley College, 14000 Fruitvale Ave., Saratoga CA 95070

Send Reviews to: Shao Mah, Editor, Software Reviews
The AMATYC Review, Red Deer College, Red Deer, AB, Canada T4N 5H5

Book Reviews

Edited by John J. Edgell, Jr.

Editor's Note: For a number of years the "Book Review Section" of *The AMATYC Review* has primarily served in the capacity of publishing reviews of classroom texts. Priority was given to those reviews which seemed to address the primary concerns of the AMATYC organization such as successful remediation program designs, effective pedagogy, women and minorities, and practical ways of implementing technological advances. On rare occasions reviews of candidates for the "state of the art" references have been published. While published reviews of classroom texts certainly fulfills a service to organizational members, there is also a need to help filter the vast array of volumes which might serve as outstanding references, such as those which most of our clients would want in their personal library or to have available in institutional libraries. Consequently, this section, "Book Reviews", is shifting priorities somewhat in seeking substantial reviews of works on ideas, topics, issues which are so well presented as to be outstanding enough as to recommend for such libraries. In conjunction with such reviews one would not only expect evaluatory remarks about the qualities of such works, but perhaps comparative remarks as well. So, please look over your resources, old or new, which you consider to be particularly outstanding resources, or standards, or statements indicative of the state of the art and write a creditable review of such. Also, major influential professional mathematical organizations sometimes invest a tremendous amount of energy in getting the best available minds together, conduct extensive research over important issues, and make conclusive recommendations. Substantial reviews of such are always important contributions. With respect to standard classroom texts, although being aware of the most recent n-th edition is relatively important, let's reserve our meager available space for those texts which are substantially outstanding with real alternatives to the standard fare.

This change in emphasis seems to be a more mature step and *The AMATYC Review* needs and solicits your expertise with respect to such reviews.

Send Reviews to: Dr. John J. Edgell, Jr., Editor, Book Reviews, *The AMATYC Review*, Mathematics Department, Southwest Texas State University, San Marcos TX 78666.

THE CONCEPT OF FUNCTION: ASPECTS OF EPISTEMOLOGY AND PEDAGOGY, edited by Guershon Harel and Ed Dubinsky, Mathematical Association of America, Washington, D. C., 1992. 317 pp., ISBN 0-88385-081-8.

The notion of a function is the connecting thread for unifying undergraduate mathematics. Whether discrete or continuous, the function (mapping) idea permeates virtually all of mathematics. Unfortunately, as various articles in this collection indicate, but certainly not surprising, some teachers and many, many students of mathematics have only a vague understanding of this relationship. Apparently the past tradition of determining a discrete subset (usually small) and hastily sketching such with the usual smoothing techniques hasn't really communicated well with preservice teachers or others.

Among various suggestions in the articles towards remediation of this functional illiteracy, applications of computers are emphasized. Computers were presented as pedagogically important instruments from at least two important perspectives. First, computers have the capacity to generate graphs. The graphs are generated discretely, which can be spread over a generous time frame with nice distance intervals, or alternately, the graphs can be generated with small intervals in time and distance which tends to communicate continuity. And, the person in charge of the computer can exercise control in such dimensions, thus possibly enhancing an understanding of the functional relationship. A second seemingly important pedagogical attribute of the computer is that the computer languages tend to be functionally oriented. One such language that was emphasized was "C".

Unfortunately, a peculiar phenomenon analogous to the following scenario may develop. Computer scientists, in their ongoing effort to make computers user friendly, increasingly tend to emphasize machine communication via the English language. But apparently national exams demonstrate that the majority of the users are illiterate with respect to technical applications of the English language. Analogously, suppose with the emphasis upon understanding functions via graphs, on the assumption that a "picture is worth a thousand words," we discover that a significant part of society is similarly illiterate with respect to communicating with or interpreting pictures.

In conclusion, mathematics teachers at all levels with an interest in this issue will find nuggets of value.

Reviewed by Charles Ashbacher, Kirkwood Community College, Cedar Rapids IA 52402.

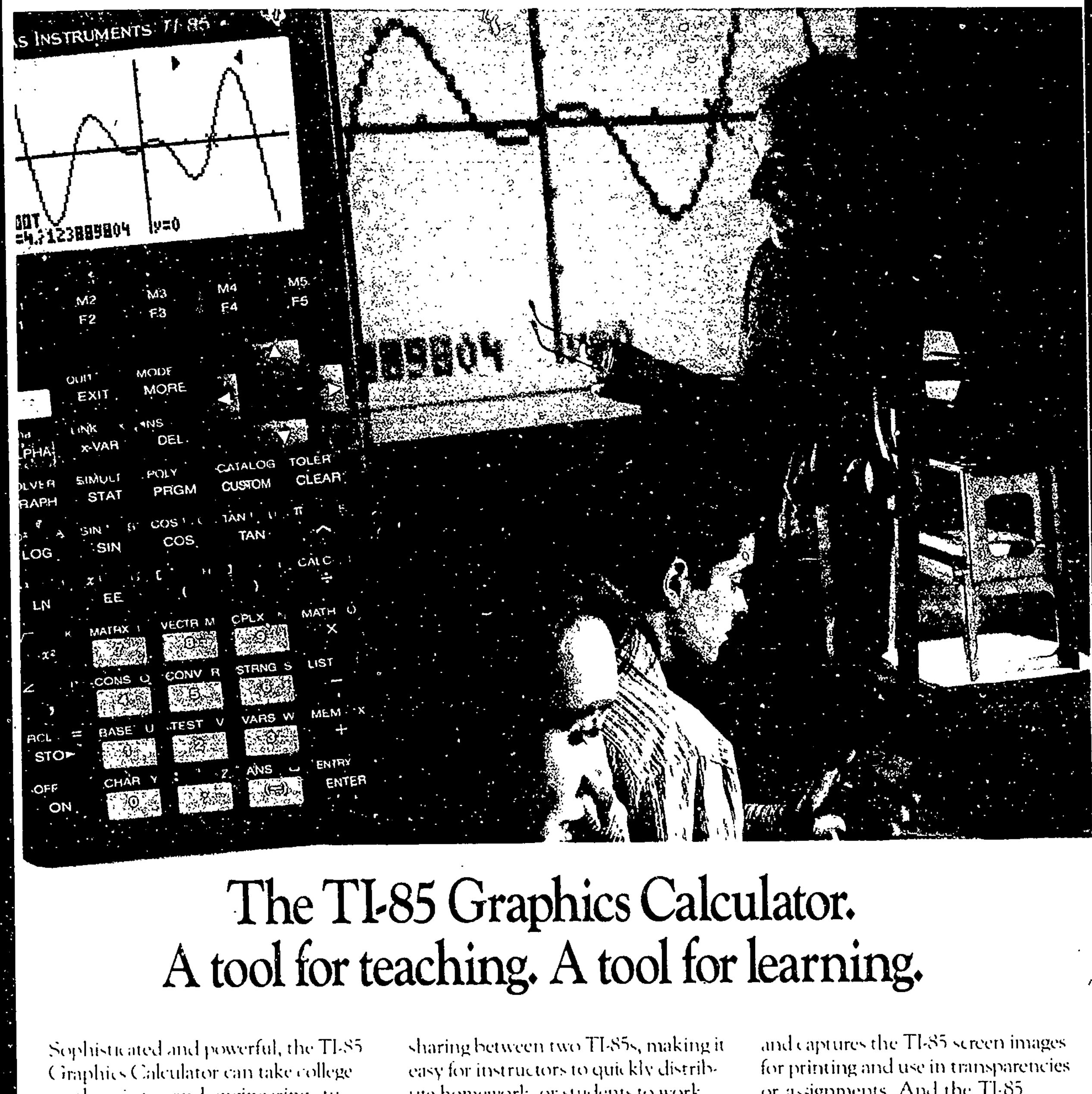
FRACTALS FOR THE CLASSROOM, PART ONE: INTRODUCTION TO FRACTALS AND CHAOS. By Heinz-Otto Peitgen, Hartmut Jurgens, and Dietmar Saupe, Springer-Verlag, New York, NY, 1992. 432 pgs., ISBN 0-387-97041-X.

This introduction to fractals reference is not intended or oriented towards being implemented as a classroom text. The ideas and materials are organized to function as a resource on fractals and chaos theory for mathematicians and teac[illegible]s of mathematics. Ideas are not chopped up, sequenced and interrupted with developmental exercises for student use. With the dramatic increase in widespread interest and applications of these ideas, there is clearly a need for such a cohesive body of information.

The resource embodies the traditional examples such as Julia sets, Sierpenski's Gasket, Koch's Curve and Brownian Motion. These and other topics are developed with a mature mathematical audience in mind. Along with a mathematical maturity associated with having accomplished an undergraduate degree with a major or minor in mathematics, one would be expected to have a working knowledge of applications of series and limits. Although one could benefit from the presentation without a computer background, any seemingly serious realistic involvement would be enhanced with computer implementation. A nice feature is the inclusion of complete BASIC programs that graph examples within the development. None-the-less, the emphasis of the development of the ideas of fractals and chaos are founded upon substantial mathematics with computer programs in an auxiliary role.

This is an excellent foundational resource for mathematics teachers, particularly those wanting to integrate such ideas into their presentations. This is the first (part) of a sequence of resources on these ideas and on the basis of the quality of this part, one is left with eager anticipation.

Reviewed by Charles Ashbacher, Kirkwood Community College, Cedar Rapids IA 52402.



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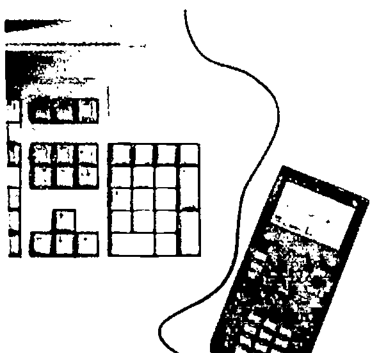

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TEXAS INSTRUMENTS

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The AMATYC Review Problem Section seeks lively and interesting problems and their solutions from all areas of mathematics. Particularly favored are teasers, explorations, and challenges of an elementary or intermediate level that have applicability to the lives of two-year college math faculty and their students. We welcome computer-related submissions, but bear in mind that programs should supplement, not supplant, the mathematics.

When submitting material for this department, please note that we have separate editors for problems and for solutions. Send new **problem proposals**, preferably typed or printed neatly with separate items on separate pages, to the Problem Editor. Include a solution, if you have one, and any relevant comments, history, generalizations, special cases, observations, and/or improvements. Enclose a mailing label or self-addressed envelope if you'd like to be assured a reply. All **solutions** to other's proposals, *except Quickies*, should be sent directly to the Solutions Editor.

Dr. Michael W. Ecker (Pennsylvania State University—Wilkes Barre)
Dr. Robert E. Stong (University of Virginia)

Correspondence

My colleague Ken Boback pointed out that Problem X-4 last issue was a generalization of his own Problem P-3 from several years ago (proposal published Spring 1989 and solution published Spring 1990). Another Ken, Ken Seydel of Skyline College, wrote to mention some corollaries that he realized from Problem W-3 (see solutions this issue): Every Pythagorean triangle has even area, and the hypotenuse of every primitive Pythagorean triangle is odd. He added: "The teasers (quickies) are just the thing to attract new people into problem-solving. Keep up the good work."

Thanks to both Kens for their insights and positive words.

Quickies

Quickies are math teasers that typically take just a few minutes to an hour. Solutions usually follow the next issue. All correspondence to this department should go to the Problem Editor, not the Solutions Editor.

New Quickies

Quickie #10: Proposed by the Problem Editor, based on teaching Calculus I.

Consider two planets with no air resistance and different constant gravitational accelerations g and G. Suppose a projectile is launched straight up with the same initial constant speed on both planets. Let $s(t)$ and $S(t)$ respectively denote the projectile positions at time t on the two planets. Prove that the maximum heights achieved on the planets are inversely proportional to the gravitational accelerations: $\frac{s_{max}}{S_{max}} = \frac{G}{g}$.

Quickie #11: Proposed by the Problem Editor, based on teaching Calculus I.

Adjoin the origin to the graph of the relation $\frac{x}{y} + \frac{y}{x} = 4$. Prove that this full graph may be regarded as the union of two functions that intersect at the origin at a 60° angle.

Comments on Old Quickies

Quickie #4. Revisited with another solution by Steve Plett: What's the smallest possible value for the sum of a positive real number and its reciprocal?

This is the third time we're giving solutions for this one, suggesting that the topic of inequalities is a gold mine to be explored. This time Steve writes

$$x + \frac{1}{x} = \frac{x^2+1}{x} = \frac{2x + x^2 - 2x + 1}{x} = 2 + \frac{(x-1)^2}{x} \geq 2 \text{ since } x > 0.$$

Quickie #5 Comment on $a + \frac{1}{a} = b + \frac{1}{b}$ if and only if $a = b$ or $a = \frac{1}{b}$. Frank Flanigan of San Jose State University generalized the result to certain rings. I hope he will publish the details of his result in this publication. (Alas, this column is not the appropriate place.)

Quickie #6 Solution revisited: Joseph A. Melita of Macon College gave an alternative example of an irrational number a raised to an irrational power b producing a rational value, using $a = \sqrt{2}$ and $b = \log_a 3$. The crux is to show the latter is irrational, which is proven by contradiction.

Quickie #8 asked why the structure wasn't a vector space. Steve Plett responded, noting non-closure, such as .5(0, -1) being undefined. However, he added that the upper half-plane would work since positive reals have inverses and powers.

Quickie #9 Proposed by Ken Seydel. Prove that there exist non-real complex numbers a and b such that a^b is real.

We got numerous solutions to this, most formal and ignoring the issue of the multi-valuedness of a^b. Charles Ashbacher emulated a classical argument and suggested choosing a nonzero real number r such that r^i is nonreal. Then

$$(r^i)^i = r^{i \cdot i} = r^{-1} = \frac{1}{r}, \text{ which is real.}$$

Most solvers explicitly used Euler's formula $e^{i\theta} = \cos\theta + i\sin\theta$ with $\theta = \frac{\pi}{2}$ and raised both sides to the power i to obtain $i^i = (e^{\frac{i\pi}{2}})^i = e^{-\frac{\pi}{2}}$.

Solvers using this approach included Bella Wiener (University of Texas - Pan American) and Wesley W. Tom (Chaffey College).

New Problems

Set Y Problems are due for ordinary consideration April 15, 1994. Our Solutions Editor requests that you please not wait until the last minute if you wish to be listed or considered. Timing Change: Please note also that this date is two weeks earlier in the annual cycle than has been used in the past. Of course, regardless of deadline, no problem is ever closed permanently, and new insights to old problems - even Quickies - are always welcome.

Problem Y-1. Proposed by Michael H. Andreoli, Miami Dade Community College (North), Miami FL.

Find a one-to-one, onto function $f:[0,1] \rightarrow (0,1)$.

Problem Y-2. Proposed as private communication by Jack Goldberg (Teaneck NJ, in 1985) to Michael W. Ecker (Problem Editor), Pennsylvania State University, Lehman, PA, who now passes it on for others to enjoy.

Consider a real number x with integer part I and decimal part $.a_1a_2...a_n$. If we multiply x by 10^n we obtain an integer, of course. Under what condition(s) is there a positive integer $m < 10^n$ such that mx is integral?

Problem Y-3. Proposed by Jim Africh, College of DuPage, Glen Ellyn IL.

In triangle ABC, let D be on side AB such that $\frac{AD}{DB} = \frac{2}{3}$, E on side BC such that $\frac{BE}{EC} = \frac{1}{5}$, AE and CD meet at F, and AC and BF meet at G. Find $\frac{BF}{FG}$.

Problem Y-4. Proposed by Michael H. Andreoli, Miami Dade Community College (North), Miami FL.

Find a closed-form expression (without using a symbolic math program such as Derive) for the antiderivative of $\frac{1}{1+x^4}$.

Problem Y-5. Proposed by the Solution Editor, University of Virginia.

Let p, q, and r be fixed nonnegative integers and let $C(m,n)$ equal, as usual, the binomial coefficient representing the number of unordered selections of n items from m. What is the value of the determinant of the $(r + 1)$ by $(r + 1)$ matrix $[a_{ij}] = [C(p + i + j - 2, q + j + 1)]$?

Note: This is essentially a square array in the middle of the Pascal triangle:

$$\begin{pmatrix} C(p+0, q+0) & C(p+1, q+1) & \cdots & C(p+r+0, q+r) \\ C(p+1, q+0) & C(p+2, q+1) & \cdots & C(p+r+1, q+r) \\ & & \cdots & \\ & & \cdots & \\ C(p+r, q+0) & C(p+r+1, q+1) & \cdots & C(p+2r, q+r) \end{pmatrix}$$

Problem Y-6. Proposed by Steve Plett, Fullerton College, Fullerton, CA.

What is the smallest number of people needed so that the probability that at least three of them share a common birthday is at least 50%? More generally, what is the probability that at least three of m people share a birthday?

Set W Solutions

Root Sums

Problem W-1. Proposed by Jim Africh, College of DuPage, Glen Ellyn, IL.

Solve for x: $\sqrt[3]{1+\sqrt{x}} + \sqrt[3]{1-\sqrt{x}} = \sqrt[3]{5}$

Solutions by Mangho Ahuja, Southeast Missouri State University, Cape Girardeau, MO; Charles Ashbacher, Kirkwood Community College, Cedar Rapids, IA; Nicholas G. Belloit, Florida Community College at Jacksonville, Jacksonville, FL; Robert Bernstein, Mohawk Valley Community College, Utica, NY; Kenneth G. Boback, Pennsylvania State University, Wilkes-Barre Campus, Lehman, PA; Joseph E. Chance, University of Texas-Pan American, Edinburg, TX; Jim Culliver and Davis Finley, Community College of Southern Nevada, North Las Vegas, NV; Joseph A. Melita, Macon College, Macon, GA; Darrell Minor, Columbus State Community College, Columbus, OH; A. Muhundan, Manatee Community College, Bradenton, FL; Leonard L. Palmer, Southeast Missouri State University, Cape Girardeau, MO; Stephen Plett, Fullerton College, Fullerton, CA; Ken Seydel, Skyline College, San Bruno, CA; Grant Stallard, Manatee Community College, Bradenton, FL; Susan Timar, Centennial College, Scarborough, Ont, Canada; Bella Wiener, University of Texas-Pan American, Edinburg, TX; and the proposer.

Cubing the equation gives

$$2 + 3\sqrt[3]{1-x}\left[\sqrt[3]{1+\sqrt{x}} + \sqrt[3]{1-\sqrt{x}}\right] = 5$$

$$\sqrt[3]{1-x}\left[\sqrt[3]{5}\right] = 1$$

$$\text{so } 1 - x = \frac{1}{5} \text{ or } x = \frac{4}{5}.$$

An Average Inequality

Problem W-2. Proposed by Juan Bosco Romero Marquez, Avilla, Spain.

Prove that, if $w \geq 1$ and $0 < b < c$, then $\dfrac{wb + c}{w + 1} \leq \dfrac{b + c}{2} \leq \dfrac{b + wc}{w + 1}$

Solutions by Jim Africh, College of DuPage, Glen Ellyn, IL; Mangho Ahuja, Southeast Missouri State University, Cape Girardeau, MO; Charles Ashbacher, Kirkwood Community College, Cedar Rapids, IA; Nicholas G. Belloit, Florida Community College at Jacksonville, Jacksonville, FL; Robert Bernstein, Mohawk Valley Community College, Utica, NY; Kenneth G. Boback, Pennsylvania State University, Wilkes-Barre Campus, Lehman, PA; Joseph E. Chance, University of Texas-Pan American, Edinburg, TX; Frank Flannigan, San Jose State University, San Jose, CA; Bill Fox, Moberly Area Community College, Moberly, MO; Joseph A. Melita, Macon College, Macon, GA; Darrell Minor, Columbus State Community College, Columbus, OH; A. Muhundan, Manatee Community College, Bradenton, FL; Leonard L. Palmer, Southeast Missouri State University, Cape Girardeau, MO; Stephen Plett, Fullerton College, Fullerton, CA; Ken Seydel, Skyline College, San Bruno, CA; Grant Stallard, Manatee Community

College, Bradenton, FL; Susan Timar, Centennial College, Scarborough, Ont, Canada; Bella Wiener, University of Texas-Pan American, Edinburg, TX; and the proposer.

From $0 \le (c - b)(w - 1)$, $wb + c \le wc + b$. Dividing by $w + 1$ gives the outer inequality. Since $\frac{b+c}{2}$ is the average of the outer terms, it lies between them.

Problem Editor's Comments: This result is made plausible by thinking of w as a weight.

Side Issue

Problem W-3. Proposed by the Problem Editor, Penn State U, Lehman, PA.

Look at the first few Pythagorean triples: (3,4,5); (5,12,13); (8,15,17); (7,24,25); (9,40,41). Now prove: a) Every primitive Pythagorean triple contains one and only one element divisible by 3. b) Same, but divisible by 5.

Solutions by Mangho Ahuja, Southeast Missouri State University, Cape Girardeau, MO, Charles Ashbacher, Kirkwood Community College, Cedar Rapids, IA; Robert Bernstein, Mohawk Valley Community College, Utica, NY; Joseph A. Melita, Macon College, Macon, GA; Darrell Minor, Columbus State Community College, Columbus, OH; Leonard L. Palmer, Southeast Missouri State University, Cape Girardeau, MO; Stephen Plett, Fullerton College, Fullerton, CA; Susan Timar, Centennial College, Scarborough, Ont, Canada, and the proposer.

Working modulo 3, we find the squares are 0 and 1. The only sums of two squares which give squares are then $0 + 0 = 0$ and $0 + 1 = 1$. For the primitive Pythagorean triple a2 + b2 = c2, not all terms can be divisible by 3, eliminating the first form. Thus exactly one of a and b is divisible by 3, and c is not. Working modulo 5 the squares are 0, 1, and 4. The sums of two squares which give squares are: $0 + 0 = 0$, $0 + 1 = 1$, $0 + 4 = 4$, and $1 + 4 = 0$. Since the first form is excluded by primitivity, exactly one side is divisible by 5.

Twice A Triangle

Problem W-4. Proposed by Louis I. Alpert, Bronx Community College (part of the City University of New York), Bronx, NY.

If primitives u, v generate Pythagorean triples $u^2 - v^2$, $2uv$, $u^2 + v^2$, find the primitives m, n in terms of u, v that generate the triple twice that generated by u and v.

Solutions by Mangho Ahuja, Southeast Missouri State University, Cape Girardeau, MO; Charles Ashbacher, Kirkwood Community College, Cedar Rapids, IA; Robert Bernstein, Mohawk Valley Community College, Utica, NY; Joseph A. Melita, Macon College, Macon, GA; Leonard L. Palmer, Southeast Missouri State University, Cape Girardeau, MO; Stephen Plett, Fullerton College, Fullerton, CA; Ken Seydel, Skyline College, San Bruno, CA; Susan Timar, Centennial College, Scarborough, Ont, Canada; Bella Wiener, University of Texas-Pan American, Edinburg, TX; and the proposer.

Taking $m = u + v$ and $n = u - v$ gives $m^2 - n^2 = 2(2uv)$, $2mn = 2(u^2 - v^2)$, and $m^2 + n^2 = 2(u^2 + v^2)$.

Nonlinear Homomorphisms

Problem W-5. Proposed by the Problem Editor.

Find all functions defined on the set of reals for which $f(a + b) = f(a) + f(b)$.

Solved by the proposer. Partial solutions were submitted by Charles Ashbacher, Kirkwood Community College, Cedar Rapids, IA; and Stephen Plett, Fullerton College, Fullerton, CA.

This problem is classic. From $f(0) = f(0 + 0) = f(0) + f(0)$, $f(0) = 0$. Then $f(0) = f(x + (-x)) = f(x) + f(-x)$ gives $f(-x) = -f(x)$. Induction gives $f(nx) = nf(x)$ for n a positive integer, and for n an integer by using $f(-x) = -f(x)$. Then for m and n integral, $mf\left(\frac{n}{m}\cdot x\right) = f(nx) = nf(x)$ gives $f(rx) = r\,f(x)$ for r rational. Thus f is a linear function over the field of rationals. Choosing a basis for the real numbers as rational vector space (a so-called Hamel basis), one sees that there are many such functions. In the presence of continuity or many weaker conditions, $f(x) = cx$ for a constant c.

Problem Editor's Comment: There are many interesting continuity equivalents, but one needs only continuity at a single point to ensure $f(x) = cx$.

NOTE: William F. Fox, Moberly Area Community College, Moberly, MO, suggests the paper "The Linear Functional Equation" by G.S. Young in *American Mathematical Monthly*, 65(1958), 37-38, which was reprinted in the MAA collection "Selected Papers on Calculus."

Digital Digits

Problem W-6. Proposed by Stephen Plett, Fullerton College, Fullerton, CA.

Upon looking at my digital wristwatch, I noted that all of the digits were different. Was this unusual? What's the exact probability that a digital timepiece will display distinct digits?

Solutions by Dennis Allison and Mike Dellens, Austin Community College, Austin, TX; Charles Ashbacher, Kirkwood Community College, Cedar Rapids, IA; Robert Bernstein, Mohawk Valley Community College, Utica, NY; and the proposer.

Assume a 12-hour clock which displays leading zeros in the minute and second, but not in the hour. There are $12 \times 60 \times 60 = 43{,}200$ possible displays. For each choice of hour, one can choose the tens digit of the minute, the tens digit of the second, the unit digit of the minute, and the unit digit of the second in the order given. For hours 1 - 5, the numbers of choices are 5,4,7, and 6. For hours 6 - 9, they are 6,5,7, and 6. For hours 10 and 12, they are 4,3,6, and 5.

The number of times with distinct digits is then

$$5(5 \times 4 \times 7 \times 6) + 4(6 \times 5 \times 7 \times 6) + 2(4 \times 3 \times 6 \times 5) = 9960$$

giving the probability $\frac{9960}{43200} = \frac{83}{360}$.

Late Solution. A solution to Problem V-3 from Mark de Saint-Rat, Miami University, Hamilton, OH was received after the last column was sent to be printed.

Mathematics: An International View

Edited by
Igor Malyshev and Joanne Rossi Becker
San Jose State University, One Washington Square, San Jose CA 95192

This column is an avenue for discussion of the first two years of college mathematics with a global perspective. Because mathematics is an international culture, it is important to follow the development of mathematics education throughout the world. We feel that by sharing information from other countries with systems of education different from ours, we will gain an enriched understanding of, and a better perspective on, our own system. A completely different viewpoint on a topic can serve to stimulate our imaginations and help us find new solutions to a problem which will work in our context. As we move toward dramatic changes in both the curriculum and instruction in mathematics called for in many reports, reflection on practices used in other countries can only help our efforts.

For this column, we would like to publish two types of items: material with an international origin or context that you could use directly with your students; or information about the content and structure of coursework in other countries. We welcome submissions of 3-4 pages or suggestions of mathematicians from other countries from whom we could solicit relevant material.

(**Editor's Note:** Professor Malyshev is on sabbatical leave this semester at the University of Kiev. The "International View" will return next issue with an extended report on that university. In the meantime, those with potential contributions to this column should correspond with co-editor Becker.)

Lucky Larry #9

Reduce $\frac{x^2-1}{x-1}$. Using the time honored technique of "term-by-term division," Larry got

$$\frac{x^2}{x} + \frac{-1}{-1}$$

$$x + 1.$$

Submitted by Sharlene Cadwallader
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Rev F, 1993

89

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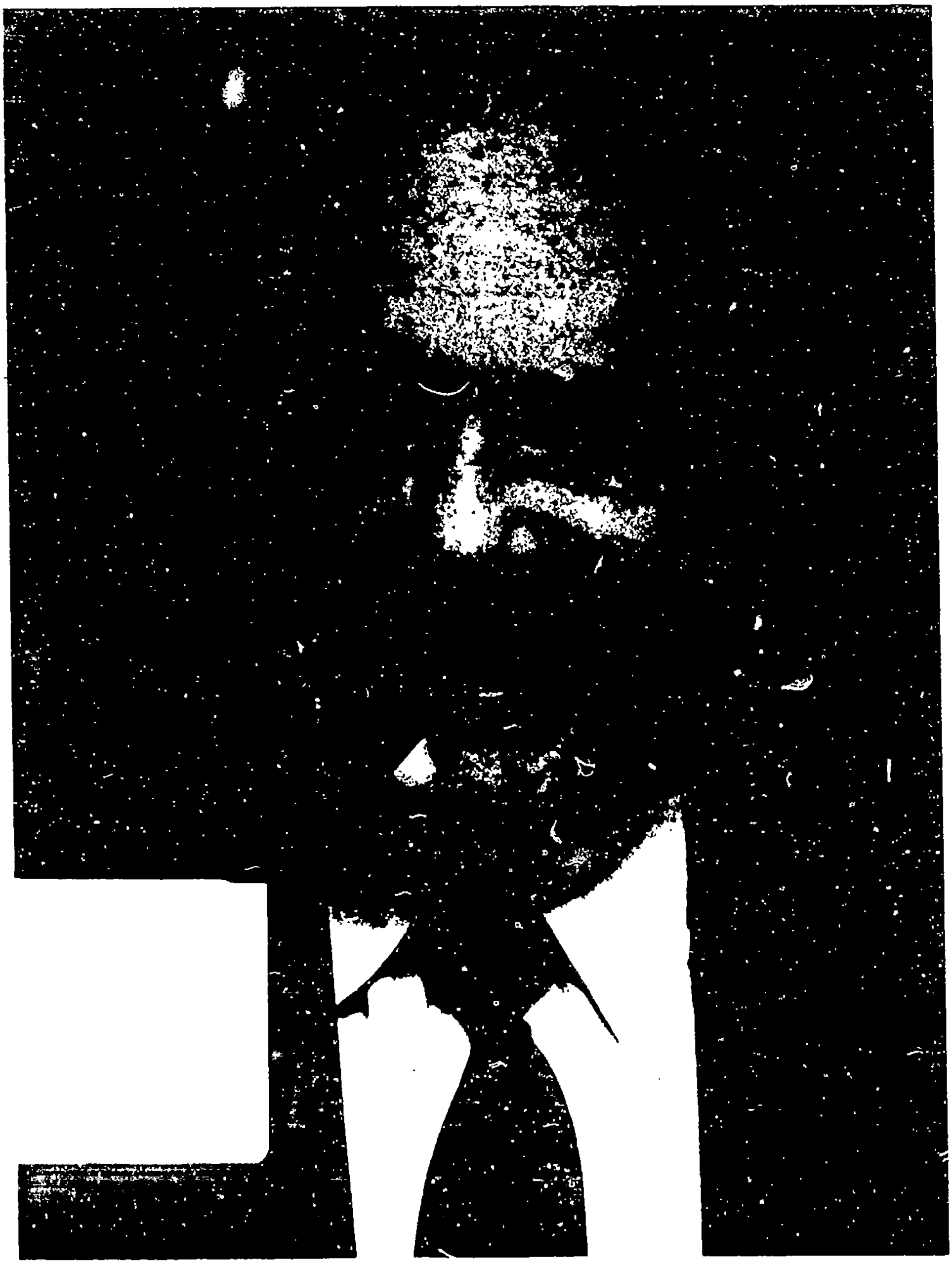

HERBERT I. GROSS
Founding President of AMATYC

The Official Journal of the
American Mathematical Association of Two-Year Colleges

PURPOSE: AMATYC remains committed to its original purposes:

- To provide a national forum for the exchange of ideas
- To further develop and improve the mathematics education of students of two-year colleges
- To coordinate activities of affiliated organizations on a national level
- To promote the professional development and welfare of its members

The AMATYC Review provides an avenue of communication for all mathematics educators concerned with the views, ideas and experiences pertinent to two-year college teachers and students.

SUBMISSION OF MANUSCRIPTS: Manuscripts must be typed, doubled-spaced, on one side of 8-1/2" x 11" paper. They should not have been published before, nor should they be under consideration for publication elsewhere. Photographs should be black and white glossy prints. To provide for anonymous reviews, the author's name and affiliation should appear on a separate title page. The title should also appear on the first page of the exposition. Authors are advised to consult the *Publication Manual of the American Psychological Association.* A guideline for authors is available from the editor and is also printed in the Fall 1993 issue. Five copies of each manuscript should be submitted to Joseph Browne, Onondaga Community College, Syracuse, NY 13215.

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TABLE OF CONTENTS

EDITOR'S COMMENTS ....p. 4

ABOUT THE COVER ....p. 4
Herbert I. Gross

LETTERS TO THE EDITOR ....p. 5

SPECIAL FEATURE ....p. 8
Two-Year College Mathematics Pioneers: Herb Gross

MATHEMATICAL EXPOSITION
Summing Power Series by Constructing Appropriate Initial Value Problems ....p. 18
by Russell J. Hendel and John D. Vargas

Simpson's Paradox and Major League Baseball's Hall of Fame ....p. 26
by Steven M. Day

The Ubiquitous Reed-Solomon Codes ....p. 36
by Barry A. Cipra

SHORT COMMUNICATIONS
A Short Proof Linking the Hyperbolic and Exponential Functions ....p. 40
by Michael J. Seery

L'Hôpital's Rule Via Linearization ....p. 42
by David E. Dobbs

MATHEMATICS EDUCATION
Predicting Grades in Basic Algebra ....p. 47
by Elsie Newman

Why Do We Transform Data? ....p. 54
by David L. Farnsworth

Students' Perceptions of Myths About Mathematics ....p. 60
by Victor U. Odafe

REGULAR FEATURES
The Chalkboard ....p. 69
Edited by Judy Cain and Joseph Browne

Software Reviews ....p. 73
Edited by Shao Mah

The Problem Section ....p. 78
Edited by Michael W. Ecker

Advertiser's Index ....p. 91

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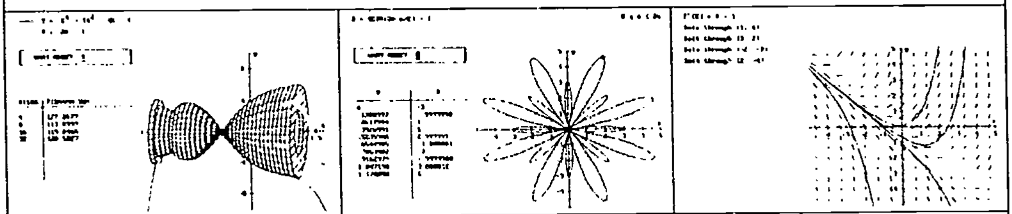

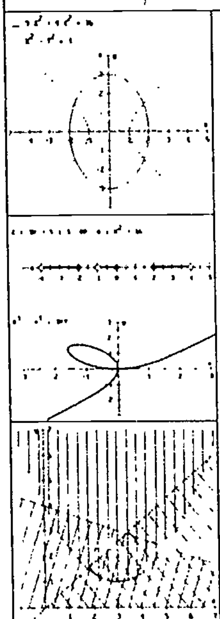

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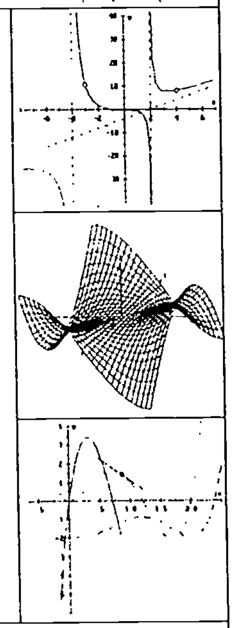

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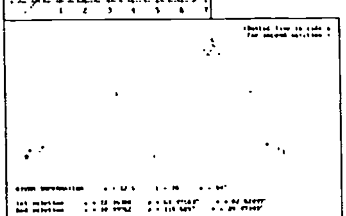

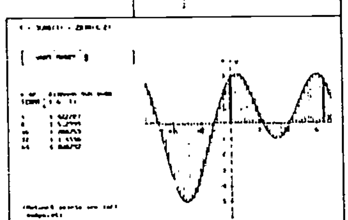

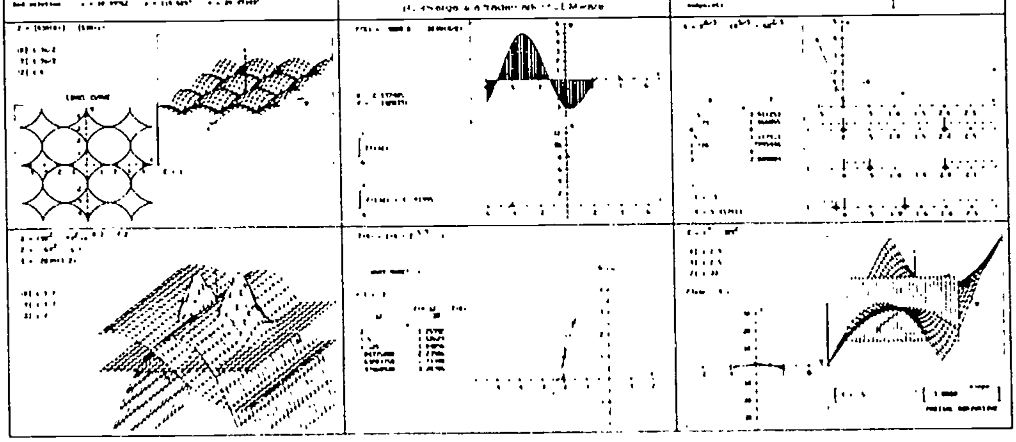

Editor's Comments

In this issue two of our columns, Book Reviews and Mathematics: An International View, are missing. The reason is, quite simply, that the editors of those sections have not been receiving quality submissions from readers. To those editors and me, this is a problem; but to you it may be an opportunity. Submitting material to a column is relatively easy. The scope and size are well defined, and if your subject is interesting, you'll get plenty of editorial help. Feel free to contact the section editor to discuss your ideas before writing them up.

In the Book Review section we are trying to move away from reviewing standard textbooks. An exception could be made for a text which offers a real alternative to what the current market is offering (e.g. *For All Practical Purposes* when it was new). We seek more reviews of books of general interest (e.g. John Briggs' *Fractals: The Patterns of Chaos* or the Taylor and Taylor biography *George Polya: Master of Discovery*), non-text resources for instructors (e.g. MAA's *A Century of Calculus*), and important reports (e.g. NCTM *Standards*). We would like to see somewhat longer reviews (2-4 typed pages). When appropriate, they should attempt to "place" the book among the others in its field. Toward this end it might sometimes be best to have a review cover a group of books. Our editor is John J. Edgell, Jr., Mathematics Department, Southwest Texas State University, San Marcos TX 78666.

Mathematics: An International View aims to improve our global view of mathematics in the first two years of college. By looking at the way things are done elsewhere, we often gain insight on how we wish to do our own teaching. If you have experience at a college in another part of the world, consider sharing it with our readers. Editors are Igor Malyshev and Joanne Rossi Becker, Mathematics Department, San Jose State University, One Washington Square, San Jose CA 95192.

About the Cover

We have all encountered situations which led us to say, "Someone ought to do something about that." Many times in his career Herb Gross has been that "someone." Whether founding organizations like AMATYC and NYSMATYC, creating videotape self-study series, initiating prison education programs, participating in the creation of new colleges, or chairing an Anti-Noise Commission in his community, Herb has never been put off because "we've never done it that way before." He sees the need, and then goes to work.

Those who have met Herb, know that talking to people is both his greatest love and greatest talent. He is inspiring and entertaining. Herb plans to retire this year, but you can be sure that he'll continue to speak at banquets, convocations, and commencements all over the country. The feature on Herb in this issue was to be an interview, but by the time we sat down he was already expounding on a favorite topic. This "interviewer" did little more than some editing. So take a little time and enjoy the words of one of the least intimidating, yet most effective, activists of the past few decades.

Letters to the Editor:

Change is tough! For many years I taught paper and pencil logarithmic and trigonometric computational techniques and exploited many useful connections. I no longer do so because scientific calculators are simply *better tools* to do the same job. Wiener's and Watkin's partial fraction article (Fall, 1993) was interesting and useful — yesterday!

Derive® (in my pocket sized HP-95 computer) does every decomposition illustrated in their article in *one step*. Today, we need to examine other issues.

(1) Is it true *today* that "Partial Fraction Decomposition is an indispensable technique for integrating rational expressions?"

(2) If so, are there better *tools* available *today* for Partial Fraction Decomposition than the Heaviside Method?

Today, we need articles dealing with these issues. We far too often teach freshman calculus as a history course.

Bert K. Waits
The Ohio State University

(In the context of using partial fraction decompositions as a tool, *I am in full agreement with the spirit of Prof. Waits' remarks. The article was included because it attempted to show that certain mathematical insights can be gained from working through partial fraction decompositions, at least a few times, which could not be gained from a tool which hands us the result. Ed.)*

Martha Cutter's Drunk Driver Problem ("The Chalkboard," Fall, 1993) is a good source of interesting questions. However, we need to practice good mathematics: for growth problems, especially exponential growth, there is often a *limit to growth.* The author's model, $P(b) = e^{21.5b}$, does not show a limit to the risk. We know that the limit to the risk of an accident is 100%. In practical terms, the limit to the risk is likely to be lower (based on the logic that the blood alcohol level reaches a toxic level which prevents the person from having an automobile accident because they are not conscious).

If we assume that the limit of the risk is 100%, the better model is

$$P(b) = \frac{100}{1 + 99e^{-22.9b}}.$$

We should *at least* tell students when the model is valid [i.e. the domain]. In this case, if we restrict the blood alcohol levels to $0 \leq b \leq 0.2$, the simple model provides a good estimate. A physiologist could tell us if this limitation is reasonable.

Applications of exponential functions to growth problems often ignore the limits of growth Perhaps this is a minor flaw with the Drunk Driver Problem, but I think we would be well served to deal with the issue.

Jack Rotman
Lansing (MI) Community College

Student Lucky Larry (#7, Fall, 1993, p. 37) may be clever, not misinformed. He could be taking advantage of the fact that $y = \sin x$ is well approximated by $y = x$ near $x = 0$ as indicated by the well known $\lim_{x \to 0} \frac{\sin x}{x} = 1$. He may have used this to replace "sin $2x$" by "$2x$" and "sin $3x$" by "$3x$", giving $\frac{2x}{3x}$ or $\frac{2}{3}$.

The technique of replacing $\sin x$ by x, $\cos x$ by 1, and $\tan x$ by x, near $x = 0$ shortens many limit problems, brings in the concept of approximations, and gives correct answers with minimal caution in its application.

David L. Farnsworth
Rochester Institute of Technology

1994 Kellogg Institute

The Kellogg Institute for the Training and Certification of Developmental Educators has announced dates and application procedures for its 1994 training program. The 1994 Institute will be held from June 24 through July 22 on the campus of Appalachian State University in Boone, NC.

The 1994 Kellogg Institute will train faculty, counselors, and administrators from developmental and learning assistance programs in the most current techniques for promoting learning improvement. The Institute program consists of a summer session followed by a fall term practicum project on the home campuses of participants. The 1994 summer program will focus on the use of learning styles and their implications for instruction, the process of designing and implementing developmental evaluation activities, adult development theories and alternative ways of knowing, the helping relationship, as well as the use of computers for management, data collection, and instructional purposes. Faculty for the summer program have included James Anderson, Barbara Bonham, Hunter Boylan, Elaine Burns, Nancy Carriuolo, Frank Christ, Chuck Claxton, Darrell Clowes, Phoebe Helm, Gene Kerstiens, Berniece McCarthy, Bill Moore, Ed Morante, John Roueche, J. Otis Smith, Milton "Bunk" Spann, and Anita George.

Institute fees are $750 plus $455 for room and board. A graduate credit fee for the three-hour practicum will also be charged. Up to six (6) hours of graduate credit may also be obtained for participation in the summer program.

Applications and additional information about the Institute may be obtained by contacting Ms. Elaini Bingham, Director of the Kellogg Institute, or Margaret Mock, Administrative Assistant, National Center for Developmental Education, Appalachian State University, Boone, NC 28608; (704) 262-3057. Early application is encouraged to ensure a space in the Institute. The application deadline is March 15, 1994.


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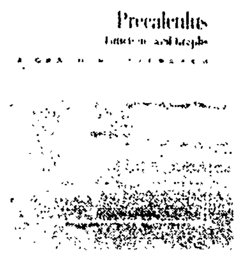
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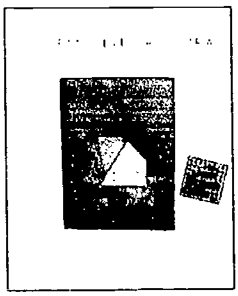

SPECIAL FEATURE

Two-Year College Mathematics Pioneers:
Herb Gross

Herb Gross was the founding president of AMATYC and, earlier, NYSMATYC. He is also a charter faculty member of two community colleges, Corning C.C. in New York and Bunker Hill C.C. in Massachusetts. In addition to this last position, which he still holds, Herb spends summers preparing educational materials for and teaching in prisons in North Carolina. He has written books and made many teaching videotapes. Among his awards are Educator of the Year (twice) from Volunteers of America, the Arkansas Traveler's Proclamation, and the Manual Carballo Governor's Award for Excellence in Public Service from Massachusetts. He is a popular keynote speaker at conferences and has given many commencement addresses. A graduate of Brandeis University, Herb did his graduate study at MIT. What follows was originally intended to be an interview, but those who know Herb know that he needs no questions. We met on November 20, 1993, shortly after his banquet address at the AMATYC Convention in Boston. These are some of his thoughts.

Getting Started in a Community College, and Getting a Community College Started

In 1958 I got a call from Bill Perry, who was the founding President of Corning Community College. At that time I had never heard the phrase "community college." If he had said "Corning Junior College," I would have said: "Hey, I'm an MIT guy. What do I want with Corning Junior College?" But he said "community college," which I thought was a term he had made up. As I've pointed out to many people, you have to have an acute sense of hearing to hear whether that's two-year or four-year, public or private, voc-tech or liberal arts. All you hear is "a college that exists for the community."

Corning Community College, 1958.

I remember an incident that happened when I recommended that we start a glass technology program. The dean and the president said that no one else in the country was doing this. I said: "First of all, if I wanted to go to a place that was doing what others were doing, I could have gone there. We came here to start a fresh institution. Secondly, doesn't 'community college' mean 'for the community?' You name me another place that is to the glass industry what Corning Glass is to Corning, New York, and I'll reconsider what we should be doing." There are community colleges which by the nature of where they are located, should be one in a million. Like the community college in Southern New Jersey which started a program for gambling table operators. Sure, some people may think gambling is immoral or corrupt, but then they should ban the gambling. Once gambling is there as a major industry, let the people make a living that way.

It's very important to make a living. There's a joke in my family about the time I was considering becoming a rabbi. They were sure it would lead to a national movement for the separation of church and clergy. As religious as I am, there is one commandment that I find actually scary. It's the one that says to love your neighbor as yourself, because it implies something awful if you don't love yourself. It's assumed that you love yourself. Not narcissistically, of course, but to feel good about yourself. Obviously, to feel good about yourself it's necessary that you be making a living, and to be proud of the living that you're making.

One of the speakers [at the opening session] got off on this kick about how wonderful it was if a kid could transfer to MIT. That's OK. I'll accept that kind of credit. At Corning we had a student who went on to be Phi Beta Kappa at Colgate. His problem was that he was shy and socially immature, and his family thought it would be OK if he came to Corning first, and then went on to Colgate. I recall some panel at which a speaker said, "You can't take credit for him because he was already bright when he came to you." I said, "Ma'am, would you at least concede that we didn't hurt him a hell of a lot while he was with us." I'm proud of that. But what I'm really proud of is the people who didn't get that far, but who [previously] thought they were nothing.

I remember we started Corning with just six faculty members and a hundred students. Now the question is "How can you have committees with just six faculty members?" So it was decided that everything would be a committee of the whole. And we wanted equality; there were six committees so each faculty member could be chair of one committee. I love telling people that at an open-enrollment college, I became chair of the Admissions Committee. They thought I couldn't do any harm there. A very interesting thing happened. I was interviewing a kid, and remember, I came from an MIT background where the norm is that the lower 80% don't go to college, and the lower 98% don't go to MIT. But in the military I learned that whenever you interview someone, let them start out positively. So I started with a question he obviously could answer with a "Yes." I could see from his papers that he wasn't a great student, so I asked, "Were you in the upper half of your graduating class?" figuring there is no way that can't be true. He looked at me with a twinkle in his eye and said, "No, sir, but I was in the half that made the upper half possible." That's when it dawned on me that there's a hell of a lot of difference between being intelligent and being book-smart. That changed a bunch of my attitudes.

Return to Boston

When I finally looked for another position, MIT hired me [in 1968]. I made 88 videotapes, black and white, talking head, made when television wasn't nearly as sophisticated as it is now. And in 25 years over fifty thousand engineers and scientists have learned from them. I recently got a copy of a letter showing that with all the other stuff out there, Boeing picked those MIT tapes for a training program they were running. When that project was over, I tried to get MIT to sponsor a set of arithmetic tapes, and let that be MIT's contribution to people who will never get to MIT. The only response I got was, "It's not part of our charter." Since I couldn't make those tapes, I left MIT; gladly, in fact, because I had a chance to found another community college, in the inner city. I came to realize that all those years in Corning were like the minor leagues. I discovered that the big difference wasn't between the two-year college and the four-year college, but between suburban and urban. See, Corning Community College [in the 60's] might as well have called itself "little Cornell," and they could have written on their letterhead "a subsidiary of Corning Glass Works." I went to this urban community college and found kids who were *really* disadvantaged.

One thing that the universities have not done is to validate what we're doing at the community college. I wish MIT had said, "It's out of our domain, we don't do this stuff, but what we will do is give you an endorsement." If the nation saw that MIT thought developmental mathematics was important, they would pay more attention to it. So we have to take the initiative and prove on our own that this is important.

Senior lecturer at Center for Advanced Engineering Study, MIT, 1968.

The Role of the Community College

How do Herb Gross the MIT Lecturer and Herb Gross the Community College Professor justify that they deserve the same pay? I think we have to carve out our own turf. I have an analogy in mind. I think that the marriage of the university and the community college is that the university should be responsible for the state of the art. As much as I love Bunker Hill Community College, I have an intuitive feeling that if we ever find a cure for AIDS, it's going to be at the Harvard Medical Center. In that sense, that elite part belongs to the university. We go to them for that. They should be coming to us for ideas in pedagogy, because they're too busy doing research to have to worry about the pedagogy. In fact, we should make it blatantly clear to the country that our contribution to higher education is that we siphon off the masses who without us would be infiltrating the universities for their piece of the pie. They would divert effort from all the great research that is going on in order to teach them. We facilitate their work.

There's so much unrest going on right now. Take the unrest in Los Angeles. If it weren't for the community colleges, where would the masses be clamoring for their upward mobility? They'd say, "We belong at the university." Now what the universities can say is: "We believe you, but the gaps in your educational background put you where our people don't know how to teach you. So go see these guys for a couple of years. It's a farm system. We'll red-shirt you for a couple of years. Then you can come succeed with us as juniors." That's the dream I've always had.

Let me go back to "Love thy neighbor as thyself." What about the guy who is a frustrated automobile mechanic, and he's frustrated because he believes that if somebody had cared about him and had given him the money, he could have gone to the university and become a mechanical engineer, designing cars instead of repairing them. He's kind of a surly guy. Going back to that commandment, nobody did anything for him, so why should he do anything for anyone else? Now a community college opens up and he goes there. The way some people would like to tell the story, he would start a new company and put Ford out of business. That's a great story if it happens, but the Japanese already did that. I look at the other end of it. It turns out he can't do calculus, he doesn't understand chemistry, and physics blows his mind altogether. So after a year he thanks all the people who tried to help him and leaves. The difference is that he will no longer play the "What if" game. Now he knows he wasn't destined to be an engineer. It's his choice, now, not somebody else's. Because of that he may become a much happier automobile mechanic. And if he's happier about himself, now he's going to contribute to the community.

Attrition and Access

Now we don't count that as a community college success story; it shows up in attrition, just another guy who didn't make it. The words may not sound nice because of other uses we have for them, but I think we should distinguish between hard-core attrition and soft-core attrition. Hard-core attrition is when a kid wants to stay in school and we won't let him. Soft-core attrition refers to when a student is eligible to stay in school but elects to drop out. I believe that the only attrition figures that should be used against the community college is the hard-core kind.

The soft-core attrition is a learning experience; people make a value judgment. I think our track record is fantastic if we only count the hard-core, because we don't have much hard-core attrition. Let's say that 40% don't make it, for whatever reason. What we really have is a statement that of all these students who have no access [to higher education] except through us, 60% do make it. I was on a panel where someone put down that figure. "Sure, they graduate," he said, "but they end up as shift manager at McDonald's." My reply was: "What were they going to be before they graduated from us? They couldn't even be counter people in many cases." For that kid, shift manager is upward mobility. It's more than that. Once he becomes a shift manager and sees that he can handle it, that glass ceiling that always kept him down is shattered. There are all kinds of [community college] students going to the universities and making the dean's list as juniors who couldn't have survived their first freshman semester at the university. We provide the access.

Community College, Junior College, and Two-Year College

When I was at Corning I started getting some strange ideas. I'm the first generation in my family born in America. I remember my grandmother had trouble with the English language. When she got hold of a new word, she would use it in as many different contexts as possible. So, once she learned the word "haddock" it not only became a fish, it became the thing you took two aspirin for, "I have a terrible haddock," and it was the place you put the old furniture, "up in the haddock." I found that people in the community college had the inverse problem; they were taking three different terms and thinking they were synonyms. They were "community college," "junior college," and "two-year college." Even now I find people interchanging them. "Junior colleges" are what most of the places in upstate New York were at that time [the 1950's]. When Corning started, 70% of the students were in liberal arts. We were hoping for the day when it would be 50-50. Kids who wanted to work with their hands didn't need college at that time. The junior college, as I understand it, is what was happening in California and the Midwest as early as the 1920's, when the universities found out that certain bright students couldn't come to college because they had to work on the farm. It was the university-parallel track.

In a sense, the veterans of World War II were the first non-traditional students. Here were professors who thought that all freshman were 18 years old, took five courses and didn't have to work part-time. This assumption still drives financial aid. Now we have a 38 year old single parent with three children and a 30 hour a week job, and we tell them that if they don't take four courses, they don't qualify for full financial aid. There's a certain sickness to that. Our presidents should be out lobbying the legislature to explain how our welfare costs would go down if we were a little more liberal in letting these people get educated. These GI's came back. How do you tell a guy who's been in a foxhole for four years that it's improper to chew gum in class? These guys were tough. It wasn't that they were nasty; they just demanded answers to certain questions. The faculty wasn't prepared to answer them, so they called them "unteachable." We have great patience as teachers as long as we don't run out of explanations. I only call you a stupid SOB when I run out of explanations and you still don't understand. These teachers were frustrated.

Guest lecturer for local second grade, 1972.

I still think [community colleges] should be funded as a social agency and not have to worry about making money. You don't say to Welfare, "Go out and find 500 new welfare recipients, or we can't give you any money." You want to keep them *not* finding any more. Maybe one year there would be 5000 students at the community college, and the next year 700. Who cares?

Is it reverse snobbery for us to say to Harvard: "We're taking in kids who are much worse than yours [on SAT's, etc.], but don't worry. With our superior way of teaching, in two calendar years they'll be caught up to your guys." My feeling was that the price for open enrollment was that you might have to go an extra year or so to make up for the shortcomings which caused the university not to accept you. I had a student say to me, "I don't know why I should take your basic math course since UMass won't give me credit for it." I said: "It's simple. UMass wouldn't take you in if you didn't take this math course, because they assume that by the fourth grade you should know this stuff. We help you just by your getting in there. Now, if you have to pay one extra year to get a lifetime education benefit, and that's all you have to pay, be thankful."

Affirmative Action

It was Harry Truman who inspired the enabling legislation to create the comprehensive community college. It was the first, non-color version of affirmative action. To me the philosophy of affirmative action wasn't that if you came from a minority, you were entitled to a job. It was that if, because of being a member of a disadvantaged minority, you had an aptitude that had never been tapped, you deserved an extra chance. Here's a sports analogy. Suppose I came from a school where Jews weren't allowed to play football. I'm six foot four, weigh 280, and run the hundred in 9.8 seconds. I try out for the football team in college, and kids half my size are knocking the [bleep] out of me. The coach says:

"It's because he doesn't understand the game. Even though we will cut kids who play better than him now, we'll keep him because a year from now he should be great." It wasn't meant to mean that 97 pound poetry majors could play football. So, at that time there was a large bunch of guys who, if it wasn't for the war, might have done OK in college, or might have become excellent mechanics. Most of the guys on the GI Bill weren't scholars; they were newly-weds, and they had to make a living. They found out that if they went to school, the GI Bill would pay them enough to raise a family.

I predicted that when President Bush first started his Educational Summit, it couldn't succeed; not *wouldn't* succeed, it *couldn't*. Why? Because he was trying to solve a coaching problem, and all he invited were the best players and the best general managers. What the hell does a governor know about teaching? What do the people at the prestigious universities know about teaching? There may have been one or two community college people or inner-city people at that summit, but it was too few and they were drowned out by the ones who, from the point of view of what they were trying to solve, knew absolutely nothing.

The Origins of NYSMATYC and AMATYC

In the 60's there was a group, AMTNYS [Association of Mathematics Teachers of New York State], which tried to represent everyone K-12, college, and the university people who were interested in school and college mathematics. Their meetings never had a section for community colleges. People from the community colleges had to pick from sessions which were really about a different situation than we experienced. When it was decided that there should be a community college strand, they brought in Rosenberg from Cornell. He was a nice guy, but he just didn't know what [our job] was like. He was talking from the heart, but he had a different base of information than the people he was talking to. He was talking about the [CUPM] Math 0 course. I kept waiting for the older, more respected community college people to get up and put this guy in his place, but they never did. Finally I got up and said, "What you call Math 0, we call Math 897." I waited for the others to tell me to please sit down and hear this guy out. But then, one by one, they chimed in the same way. That must have been 1965.

I got together with [John] Vadney from Fulton-Montgomery and a few others — let's see, there was Abe Weinstein from Nassau, and Phil Cheifetz [Nassau], Ray McCartney [Suffolk], Fred Misner [Ulster], and Don Cohen [SUNY Cobleskill] — and said: "We know what we're about. Let's organize a strand for two-year colleges with some continuity to it." I got the best possible people for the next meeting, working under the adage that you only get one chance to make a first impression. The meeting was in Syracuse in May of 1966. The adrenaline flow was so strong at that AMTNYS meeting that we got together late that night and caucused. We set up a framework for a new organization [NYSMATYC] which was officially formed the next year, again in Syracuse. The only reason they made me president was that I sounded like a downstater, but I was teaching at an upstate college. That was the day that we came to the realization that *we* knew best what was good for the community colleges. And that was the day that I became a rebel.

Al Washington mentioned that I was saying, "Let's go national," right from the beginning. Chronologically, in 1974 we did form a national organization. That would seem like about the right number of years for the idea to germinate. So, if I wanted to make up revisionist history, I could say that's what happened. But really, when I said, "Let's go national," it was just my euphoria. At that time I didn't know whether we were the first state-wide two-year college math association. Someone else could have had the idea first. But it was spontaneous. We didn't say, "Look what they're doing in California or Virginia." We just did it, and it was so good we thought, "wouldn't it be wonderful if everybody did this?" More realistically, we weren't ready to go national in 1967. We had people at Nassau who didn't trust the people in Corning. And the people in Buffalo said they could never get to New York. We had our own problems to deal with.

What really started AMATYC was the feeling: "What are we going to do? Meet once a year, and that's it? Let's have a newsletter." Then [Frank] Avenoso, John Walter, and George Miller said they would do it at Nassau, and it was planned. What wasn't planned was that the newsletter was so stuffed with vital information that it became *The NYSMATYC Journal*. In 1968 I went back to MIT. I never noticed that the "NYS" was dropped from the name of the journal. I didn't know that those guys had gone national with the journal. I kept reading it, and thought it was nice that other people from around the country were contributing articles.

I got a call from George Miller in '73 or '74. George said there was a problem. The way I interpreted it was that someone was muscling in on NYSMATYC. Somebody was telling him that our journal was no good. He may have been telling me the truth, but what I was *hearing* was that Prindle, Weber, and Schmidt wanted to put *The NYSMATYC Journal* out of business. It still hadn't dawned on me that this had become a national journal, and was competing with the Prindle, Weber, and Schmidt journal [*The Two-Year College Mathematics Journal*, which was later transferred to the MAA and is now called *The College Mathematics Journal; The MATYC Journal*, which was AMATYC's official journal for a few years, still exists as *Mathematics and Computer Education*]. I was busy at MIT, and not following this stuff at all. If I had been aware that this was a battle between two national journals, I would have said: "I don't have time for this. You guys are big boys. You've outgrown NYSMATYC. If you can't run with the big dogs, stay on the porch." If they had been cutting into NYSMATYC, I would have fought them tooth and nail. But if NYSMATYC wanted national limelight, then they had to take on all comers. But I didn't know that. In fact, even after I came to the Essex [Hotel in NYC, site of the 1974 NYSMATYC conference and AMATYC organizational meeting], it wasn't until the second day that it occurred to me, when I was talking to, I think it was, Paul Prindle, that he said: "Herb, what are you getting excited about? We aren't cutting into New York. New York wants to be national." Then I felt embarrassed, not because anybody had deceived me, but because I had misinterpreted. Here I am fighting a battle, and I hadn't done my homework.

But in the course of that battle, I see that we haven't grown, nationally, from those days when people were talking about Math 0. I was [bleep]. When they asked me to talk after a meeting, I told them, in my usual emotional way, exactly how I felt. If I can't talk in a way to inspire people, why talk at all? I tell my students, "You can call this a course, but it's really 45 after-dinner speeches." That's when Joe Cicero [then of Clayton State College, GA], whom I had never met up to that time, said: "Listen to what this man is saying. Let's stand up and be counted." It was a ground-swell. One person after another said, "Let's form a national organization."

I am not a leader in the sense of being an administrator. I am a leader in terms of motivating people. Actually, AMATYC is a thousand per-cent better off without me at the helm. I am the perfect example of the difference between a leader and a manager. I am the world's worst manager. When an organization gets big, you need a manager. What you need a person like me for is to come in every x years to get people fired up, or to be available as a patriarch for young people to learn from. I was very happy when I was no longer president of AMATYC, because, on top of everything else, I was at a community college that had no concept of greatness. At Bunker Hill, I got no secretarial help the year I was president, I was going through financial problems, my house was my silent partner. AMATYC had no budget, because there was no real organization yet. My wife was developing asthma, and I was thinking about relocating to a place with a dryer climate. I probably gave AMATYC its worst year of leadership ever. But, it was exciting! On my little old typewriter I was sending out memos every so often; nothing substantive, but something to get people thinking. That's how I see that AMATYC finally came into being.

I am so proud of AMATYC. As I understand it, we now sit on every important committee in education. That's the first step. Now we have access, but I don't believe we have yet established ourselves as equal partners in many cases. In too many places we back off.

Herb with wife, Louise, 1993.

Some Personal Reflections

My hobby, my avocation, and my livelihood are all the same thing. In that sense, I never get away from my work. Of course there's a financial side to life. But I believe that if I were independently wealthy, 95% of what I would do would be the same. It's that other five percent that's tiring me out. If I were independently wealthy, I would be all over the country, talking to people. I'd use that as vacation, too, taking my wife with me. Even in my early days when the kids were young, whenever I had a speaking engagement, I took the entire family with me because one image I didn't want to have was that of the guy who tries to fix the whole world but neglects his own family. My kids were very athletic. I wanted them to see that a short, fat, bald guy who could talk to people could get a standing ovation. My feeling is this: take whatever amount of money you think you are being underpaid by, multiply by whatever factor you want to magnify it, and that's what it's worth to be able to say, "I do for a living the same thing I'd do for a hobby if I were independently wealthy." I feel like the guy who was hired to work as janitor in the girls' dormitory. He never came in to pick up his paycheck. One day they asked him, "Sven, why don't you ever collect your pay?" He said, "You mean I get paid, too!" I feel I lucked out to become part of the community college movement. It was the only place, back in the fifties, where a populist like myself could work in higher education; or maybe in education at all. That's what I have to come to grips with: was I really a community college guy, or was the community college the only game in town for a populist like me?

MATHEMATICAL EXPOSITION

Summing Power Series by Constructing Appropriate Initial Value Problems

by

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Introduction

The mathematical counterpart to the proverbial question, "Which came first, the chicken or the egg?", is, "Which is primary, the function or the equation?" One mathematical viewpoint regards functions as primary. According to this viewpoint a real world situation that was modeled, for example, by $z' = Az$, A a matrix, would be approached by first solving the equation explicitly in terms of exponential functions and then deriving properties of the model from the functional solution.

Frequently however, equations yield information without having to be solved. For example, using direction fields, a differential equation may directly yield graphical information about the phenomena it is modelling even though a closed form solution has not been found. Similarly, even if a closed form solution of a differential equation is available its phase portrait alone will often yield information about the asymptotic behavior of its solution. As an example, it is possible to determine whether the singular point of a linear equation, $z' = Az(\det(A) \neq 0)$ is stable, asymptotically stable, or weakly stable depending on whether the eigenvalues of A have no real positive part, negative real part, or are purely imaginary respectively.

Another area where differential equations can yield information is in finding closed forms of functions represented by power series. According to Knopp (1952, 1956), by a closed formula for a convergent power series

$$\sum_{n=0}^{\infty} a_n x^n$$

we mean a function $f(x)$ derived from the familiar elementary functions – power, root, exponential, rational, logarithmic, trigonometric, hyperbolic, and cyclometric – such that

$$f(x) = \sum_{n=0}^{\infty} a_n x^n$$

for all x in some appropriate region. Although closed formulae may often be found by conventional algebraic manipulations, substitutions, and the use of differential and integral operators, skillful use of the equations that the functions represented by these power series satisfy often provides a more elegant approach.

This note reviews several instances from the current literature where a function represented by power series is determined in closed form by means of an associated differential equation. This method of using equations is an elegant, succinct alternative to those methods involving only differential and integral operators, term by term manipulations, or traditional algebraic derivations. The method applies to equations whose terms are rational, (inverse) trigonometric, or exponential functions as well as to simple variants of these functions. The examples can easily be introduced to complement the more traditional examples and techniques in upper level calculus or first semester differential equation courses.

Examples

Example 1. (Marcus, 1991, p. 448, prob. 41) Obtain a closed form expression for

$$1 - \frac{x}{3} + \frac{x^2}{5} - \frac{x^3}{7} + \dots .$$

Solution. Let $y(x) = 1 - \frac{x}{3} + \frac{x^2}{5} - \frac{x^3}{7} + \dots$. Then by the ratio test $y(x)$ is analytic for $|x| < 1$. The uniform convergence allows term by term differentiation from which it is easy to see that $y(x)$ is a solution of the initial value problem

$$2xy' + y = \frac{1}{1+x}, \; y(0) = 1.$$

This is a first order, nonhomogeneous linear differential equation. Standard techniques show $\sqrt{|x|}$ to be an integrating factor.

If $x \in (0,1)$ then $\sqrt{|x|} = \sqrt{x}$ and routine manipulations show the solution of the equation to be $y(x) = \dfrac{\arctan\sqrt{x}}{\sqrt{x}}$.

If $x \in (-1,0)$ then $\sqrt{|x|} = \sqrt{-x}$ and routine manipulations show the solution to be

$$y(x) = \frac{1}{2\sqrt{-x}} \ln \left| \frac{1+\sqrt{-x}}{1-\sqrt{-x}} \right| .$$

Finally, we note that at $x = 0$ the initial value conditions require $y = 1$ while at $x = 1$ Abel's limit theorem (Knopp, 1956, p. 169), applied to the solution $y(x) = \dfrac{\arctan\sqrt{x}}{\sqrt{x}}$ yields Gregory's formula.

The other problems in (Marcus, 1991) may be treated in a similar manner. Such an approach also provides a new source of problems dealing with first order linear differential equations. Many current differential equation textbooks do not contain equations of the form, $P(x)y' + y = Q(x)$ with $P(x)$ linear and $Q(x)$ rational, in the problem sets of the relevant sections.

Example 2. (Stone, 1991) Find the Maclaurin series expansion for $\sec^4 \emptyset$, for $\emptyset \in (-\pi/2, \pi/2)$.

Solution. Note, that the convergence of the series for $\cos\emptyset$ on the entire real line implies that its reciprocal, $\sec\emptyset$, is analytic in the given open interval. It follows that, $\sec^4\emptyset$, a power of $\sec\emptyset$, is also analytic in the given open interval. Furthermore, since $\cos\emptyset$ is even, its reciprocal, $\sec\emptyset$ is also even, and therefore all odd powers in the expansions of both $\sec\emptyset$ and $\sec^4\emptyset$ vanish. It is well known that

$$\sec\emptyset = E_0 - \frac{E_2}{2!}\emptyset^2 + \frac{E_4}{4!}\emptyset^4 - \frac{E_6}{6!}\emptyset^6 + \ldots$$

where the E_i are the Euler numbers whose values may be computed using the recursive relation

$$\sum_{k=0}^{n} \binom{2n}{2k} E_{2n-2k} = 0, \quad E_0 = 1.$$

These facts will aid us in finding the expansion of $\sec^4\emptyset$.

The "function is primary" approach would solve this problem using the known expansion of $\cos\emptyset$ (about 0) and then deriving the coefficients of the expansion of $\sec\emptyset$ from the infinite set of coefficient equations associated with the power series expansion of the left side of the equation $\cos\emptyset \sec\emptyset = 1$. The expansion of $\sec^4\emptyset$ would then be obtained from the expansion of $\sec\emptyset$ by exploiting the known (Cauchy) formulae relating the coefficients of power series and their products. The deficiency of this approach (as the reader can verify) lies in the excessive computation needed.

However, the "equation is primary" approach would first seek an initial value problem that the given function satisfies. It is easily verified that $y(\emptyset) = \sec\emptyset$ is a solution to the initial value problem

$$(D(y'))^2 + y^2 = \sec^4\varnothing,$$
$$y(0) = 1.$$

The technique of repeatedly differentiating the differential equation results in the following identities:

$$D^1(y(0))\,[D^2(y(0)) + D^0(y(0))] = \frac{1}{2}\,D^1(\sec^4(0)),$$

$$D^2(y(0))\,[D^2(y(0)) + D^0(y(0))] + D^1(y(0))\,[D^3(y(0)) + D^1(y(0))] = \frac{1}{2}\,D^2(\sec^4(0)),$$

$$D^3(y(0))[D^2(y(0)) + D^0(y(0))] + 2D^2(y(0))[D^3(y(0)) + D^1(y(0))] +$$
$$D^1(y(0))[D^4(y(0)) + D^2(y(0))] = \frac{1}{2}\,D^3(\sec^4(0)),$$

etc.

Several patterns emerge from the preceding formulae: the powers of D on the right side are ascending, the numerical coefficients of the terms on the left side form Pascal's triangle, etc. A simple inductive argument can then be used to generalize these formulae to yield the following basic recursive relation on $D^n(\sec^4\varnothing)$

$$\sum_{k=0}^{n-1}\binom{n-1}{k} D^{n-k}(y(0))\,[D^{k+2}(y(0)) + D^k(y(0))] = \frac{1}{2}\,D^n(\sec^4(0)).$$

This basic recursive relation combined with the explicit expansion of $\sec\varnothing$ allows us to calculate the expansion of $\sec^4\varnothing$:

$$\sec^4\varnothing = 1 + 2\varnothing^2 + \frac{7}{3}\varnothing^4 + \dots .$$

For example, to calculate the coefficient of $\varnothing^2$ we would first compute that $\{D^i(y(0)): i = 0,1,2,3\} = \{1,0,1,0\}$, and then, letting $n = 2$ in the basic recursive relation we have $1(1+1) + 0(0+0) = \frac{1}{2}D^2(\sec^4(0))$ from which the value 2 is readily calculated.

Example 3. (Levine & Wagner, ©*College Math Journal*, 1991) Evaluate

$$\sum_{k=1}^{\infty} (-1)^k \frac{\pi^{6k}}{(6k)!}$$

Solution. Let $y(x) = \Sigma(-1)^k \frac{x^{6k}}{(6k)!}$. Since $y(x)$ is analytic everywhere, we may denote its j-th derivative with respect to x by $D^{(j)}(y(x))$. Then it is easily verified that $y(x)$ is a solution of the initial value problem

$$(D^6-1)(y(x)) = 1; \; y(0) = 0, \; D^{(j)}y(0) = 0, \; j = 1,2,3,4,5,$$

for all real x.

Since the coefficients of the linear differential equation are constant, there are several available methods of solution. Here, the nature of the initial value conditions suggests use of the Laplace transformation. It is then found with little difficulty that

$$y(x) = 1 - \frac{1}{3}\cos x - \frac{2}{3}\cos\frac{x}{2}\cosh\frac{\sqrt{3}x}{2}.$$

Since this solution is guaranteed to be unique, we have a closed form for the series representation of $y(x)$.

In particular, $y(\pi) = \frac{4}{3}$ provides the desired evaluation of the proposed series.

This example may clearly be generalized to deal with series of the form

$$s(a,b,c,d) = \sum_k (-1)^k \frac{d^{\frac{ak+b}{c}}}{(ak+b)!},$$

with appropriate restrictions on a,b,c, and d. Since $ak + b$ must be nonnegative we require $a \geq 0$ and $|b| \leq a$. There is no loss of generality in letting $c = 1$ since for real $c > 0$, $s(a,b,c,d) = s(a,b,1,d)^{1/c}$. With these restrictions on a,b,c, the parameter d may be any real number. $s(a,b,1,d)$ is also well defined for complex d provided care is taken on the selection of roots.

Example 4. (Apostle, 1961) Evaluate

$$\sum_{n=1}^{\infty} \frac{n^2}{n!}.$$

Solution. Define a function

$$y(x) = \sum_{n=1}^{\infty} \frac{n^2 x^n}{n!}.$$

A standard technique for finding the differential equation that a given function satisfies is the method of operators and annihilators (Marcus, 1991, pp. 210–225).

It is well known that $(D-1)$ annihilates

$$\sum_{n=0}^{\infty} \frac{x^n}{n!}.$$

From the equation

$$(D-1)\left(\sum_{n=1}^{\infty} \frac{nx^n}{n!}\right) = \sum_{n=0}^{\infty} \frac{x^n}{n!},$$

it follows that the operator $(D-1)^2$ annihilates

$$\sum_{n=1}^{\infty} \frac{nx^n}{n!}.$$

Similar reasoning shows that $(D-1)^3$ annihilates $y(x)$. Furthermore, since

$$y(x) = x + \frac{4x^2}{2} + \frac{9x^3}{6} + \dots .$$

we see that $y(x)$ satisfies the initial value problem

$$(D-1)^3(y) = 0;$$
$$y(0) = 0, y(1) = 1, y(2) = 4.$$

Using standard techniques it then follows that a solution is

$$y(x) = (x + x^2)e^x.$$

In particular the sum in the problem equals $y(1) = 2e$.

This result can be generalized to evaluate sums such as

$$\sum_{n=1}^{\infty} \frac{n^3}{n!}, \qquad \sum_{n=1}^{\infty} \frac{n^2}{2^n n!}, \qquad \sum_{n=1}^{\infty} \frac{(2n)^2 2^n}{(2n)!}.$$

Indeed, for given $r \geq 1$ define the function

$$y_r(x) = \sum_{r=1}^{\infty} \frac{n^r x^n}{n!}.$$

Then the above three sums are $y_3(1)$, $y_2(0.5)$, and $\dfrac{y_2(\sqrt{2}) + y_2(-\sqrt{2})}{2}$. In the spirit of this note the reader is encouraged to evaluate these sums by finding a differential equation that each satisfies.

Summary

The techniques used in the examples presented in this paper include many of the conventional methods (separation of variables, exact equations via integrating factors, characteristic equations, Laplace transforms, etc.) used in first semester differential equation courses. It is hoped that instructors will introduce similar approaches in their courses.

References

Apostle, T. (1961). *Calculus, Volume 1*, 1st edition. (p. 427). New York: Blaisdell Publishing Company. Example 23a, b.

Knopp, K. (Translated by Frederich Bagemihl, 1952). *Elements of the theory of functions.* (pp. 106–137). New York: Dover.

Knopp, K. (Translated by F. Bagemihl, 1956). *Infinite sequences and series.* (pp. 163–232). New York: Dover.

E. Levine (Proposer) and R. Wagner (Solver). (1991). Solution to Problem 434. A rational alternating sum. *The College Mathematics Journal,* 22(4), 347.

Marcus, D. (1991). *Differential equations an introduction.* Dubuque, Iowa: W.C. Brown.

Stone, A.P. (1991). Power series expansions for trigonometric functions via solutions to initial value problems. *Mathematics Magazine,* 64(4), 247-251.

Lucky Larry #10

Students were instructed to find the equation of the parabola passing through (0,3), (1,0), and (3,0). Lucky Larry didn't bother with standard forms, but proceeded as follows:

The x-intercepts are $y = 0$, $x = 1$, and $x = 3$.

$$0 = y \qquad (x-1) = 0 \qquad (x-3) = 0$$

$$0 = y = (x-1)(x-3)$$

$$y = x^2 - 4x + 3$$

Submitted by Mike Majeske
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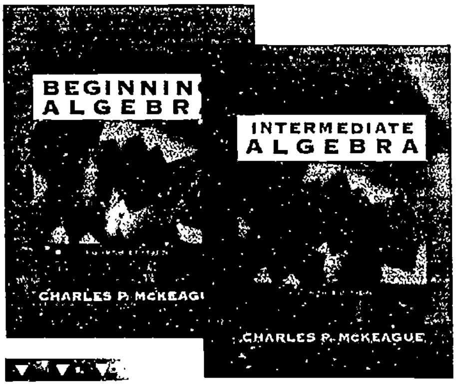
The average classroom teacher wants a book that's up to date, but not so innovative that you have to throw away last year's lecture notes.

Herb Gross

(Herb Gross quotation., found in this issue are from his banquet address or other remarks made at the AMATYC Convention in Boston, 1993.)



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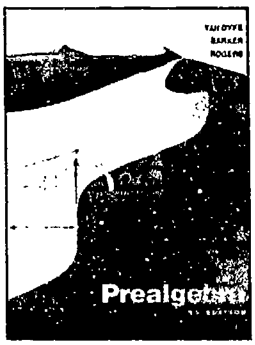

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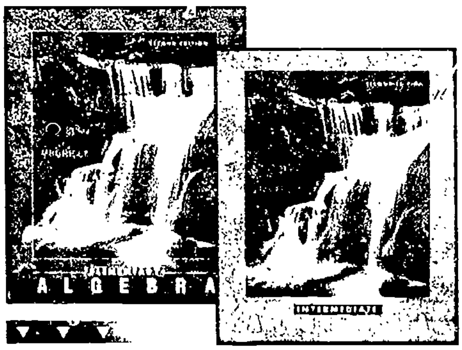

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Simpson's Paradox and Major League Baseball's Hall of Fame

by

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Jim and Ron are two former Major League Baseball players who played for the same team during two consecutive World Series. For the first Series, Jim's batting average was .400 while Ron's was .379. For the following Series, Jim's batting average was .167 while Ron's was .143. Thus Jim apparently outperformed his teammate, Ron, at batting for each of the two World Series. Yet, when hits and at bats for the two Series were tallied, Ron had the better overall batting average: .333 to .273 (table 1). How can this be?

Table 1. Simpson's Paradox: The Reversal Occurs when Real Data from Two Consecutive World Series Are Combined.

	Series 1			Series 2			Combined		
	AB	H	Avg.	AB	H	Avg.	AB	H	Avg.
Ron	29	11	.379	7	1	.143	36	12	.333
Jim	10	4	.400	12	2	.167	22	6	.273

AB = At Bats, H = Hits.

This is an example of what Blyth (1972) called Simpson's Paradox. Simpson (1951) described, "the dangers of amalgamating 2x2 tables," and pointed out that statisticians had long been aware of these dangers. In its statistical form, the "paradox" can occur in virtually any two by two stratification of data. Simpson (1951) gave a wonderful, hypothetical example involving a baby who separated an ordinary deck of playing cards into two groups; in each group, the ratio of red face cards to face cards was lower than the ratio of red plain cards to plain cards, while in the deck as a whole those ratios were, of course, equal (table 2). Blyth (1972) gave a hypothetical example involving rates of recovery of two groups of patients given two different medications; medication A seemed in each group to give the better recovery rate, but when the data from the two groups were combined, medication B was better (table 3). Cohen (1986) gave hypothetical and real life

Table 2. Simpson's Playing Card Example.

	Group 1		Group 2		Combined	
	Red	Black	Red	Black	Red	Black
Face	4	3	2	3	6	6
Plain	8	5	12	15	20	20

Note that 4:7 < 8:13 and 2:5 < 12:27 but 6:12 = 20:40.

examples involving age specific rates of mortality in two different countries; for each age group, country A had the higher mortality rate, but when all ages were combined, country B's mortality rate came out higher (table 4). Mitchem (1989) and Beckenbach (1979) gave hypothetical examples involving batting averages of baseball players for two seasons. Other examples, real and hypothetical, may be found in Shapiro (1982), Paik (1985), and Wagner (1982).

Table 3. Blythe's Medication Example.

	Group 1		Group 2		Combined	
	R	D	R	D	R	D
Treatment A	1000	10000	100	9	1100	10009
Treatment B	9	100	10000	1000	10009	1100

R = Recovered, D = Died. For each group, treatment A has a better recovery rate, yet overall, treatment B is far superior.

In this paper, we review the mathematics of Simpson's Paradox and use a graphical method for constructing examples of it which was given by Goddard (1991) to examine the frequency with which the paradox can occur in a special, symmetrical case. Next we uncover some real life examples of the paradox from the annals of Major League Baseball. Finally we pose some problems which, upon reflection, will be seen to involve this paradox directly.

Table 4. Cohen's Hypothetical Death Rate Example

	Young		Old		Combined	
	At Risk	Deaths	At Risk	Deaths	At Risk	Deaths
Country A	90	25	10	4	100	29
Country B	40	10	60	20	100	30

Unravelling the Paradox

Mathematically, Simpson's Paradox consists of real numbers, a_i, m_i, b_i, n_i, i = 1 to k, with the following properties:

$$0 \le a_i \le m_i,\ 0 \le b_i \le n_i$$

$$\frac{a_i}{m_i} > \frac{b_i}{n_i} \text{ for } i = 1 \text{ to } k \tag{1}$$

but,

$$\frac{a}{m} \le \frac{b}{n}$$

where letters without subscripts represent sums over i of corresponding letters with subscripts.

Restricting attention to the case $k = 2$, we may get an idea of what is happening by considering a somewhat extreme example.

Table 5. A Somewhat Extreme Example of the Symmetric Case.

	Season 1			Season 2			Combined		
	AB	H	Avg.	AB	H	Avg.	AB	H	Avg.
Player A	100	60	.600	900	100	.111	1000	160	.160
Player B	900	450	.500	100	10	.100	1000	460	.460

In table 5, player B's average of .500 for the first season is not as good as player A's .600, but it is weighted much more heavily. A necessary condition for player B to come out on top in the aggregate average is for one of B's season averages to be better than player A's aggregate average; then, if that season is weighted heavily enough, B's aggregate average can be higher than A's.

The reversal of inequalities in the paradox can be extreme. As Blyth (1972) pointed out, it is possible to have

$$\frac{a_1}{m_1} > \frac{b_1}{n_1} \approx 0$$

and

$$1 \approx \frac{a_2}{m_2} > \frac{b_2}{n_2}$$

but

$$0 \approx \frac{a}{m} < \frac{b}{n} \approx 1.$$

To see how these extremes might be approached, consider that for any positive integer N we have

$$\frac{N}{N^2} > \frac{1}{N+1} \approx 0$$

and

$$1 \approx \frac{N}{N+1} > \frac{N^2 - N}{N^2}$$

but

$$0 \approx \frac{2N}{N^2 + N + 1} < \frac{N^2 - N + 1}{N^2 + N + 1} \approx 1.$$

With sufficiently large N, we can make the approximations above as precise as we wish. In fact, if we allow $b_1 = 0$ and $a_2 = m_2$, the first two sets of approximations can be replaced with equalities.

$$\frac{1}{N} > \frac{0}{1} = 0$$

and

$$1 = \frac{1}{1} > \frac{N-1}{N}$$

but

$$0 \approx \frac{2}{N+1} < \frac{N-1}{N+1} \approx 1.$$

Note that in the foregoing analysis, $m_1 = n_2$ and $m_2 = n_1$. This is also the case in table 5 where $m_1 = n_2 = 100$, $m_2 = n_1 = 900$ and the total number of at bats is the same for each player. This symmetrical case is especially intriguing, and we shall consider it in some detail shortly.

The Poverty of Examples

Although the literature is full of hypothetical and real life examples of Simpson's Paradox, it is certainly true that one's intuition is correct far more often than not; in general, Simpson's Paradox is quite rare. Just how rare it is may be explored by way of a picture given in Goddard (1991). We develop that picture, then consider the frequency of examples of the paradox, and consider, for a special case, extreme values of that frequency.

Fixing $k = 2$ in (1) and rearranging some of the inequalities there we obtain

$$a_1 > \frac{b_1 m_1}{n_1}, \qquad a_2 > \frac{b_2 m_2}{n_2}$$

and (2)

$$a = a_1 + a_2 \le \frac{bm}{n}.$$

Fixing b_1, b_2, n_1, n_2, m_1 *and* m_2, we allow a_1 and a_2 to vary and graph the three lines

$$a_1 = \frac{m_1 b_1}{n_1}, \quad a_2 = \frac{m_2 b_2}{n_2}$$

$$a_1 + a_2 = \frac{mb}{n} \qquad (3)$$

obtaining a triangle of pairs (a_1,a_2) defined by (2) which yield examples of the paradox (figure 1).

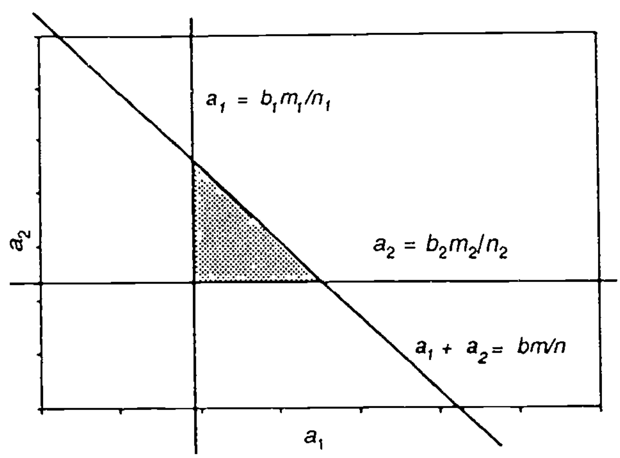

Figure 1: Goddard's Triangle. Here, all parameters are fixed except a_1 and a_2.

Under the assumptions (1) we have a rectangle from which a_1 and a_2 must come (A player, for example, cannot have more hits than "at bats")

$$0 \le a_1 \le m_1, \quad 0 \le a_2 \le m_2. \qquad (4)$$

Two interesting questions present themselves. First, how large can the triangle of figure 1 be? In general, there is no limit to how large it can be, but if we fix some of the parameters and allow others to vary, then the size is limited. The second, more interesting, question concerns the ratio of the area of the triangle ((2), figure 1) to the area of the rectangle (4). How great can this ratio be? There is a complication to consider and many parameters. We will consider these two questions in detail for a special, symmetrical case wherein $m_1 = n_2$ and $m_2 = n_1$.

In the case $m_1 = n_2$, $m_2 = n_1$, our equations (3) become

$$a_1 = \frac{b_1 n_2}{n_1} \qquad a_2 = \frac{b_2 n_1}{n_2} \qquad (5)$$

$$a_1 + a_2 = b_1 + b_2.$$

To have any triangle (2), the diagonal line in figure 1 must lie above the intersection of the other two lines. This means we must have

$$\frac{b_1 n_2}{n_1} + \frac{b_2 n_1}{n_2} < b_1 + b_2. \qquad (6)$$

To produce an example of figure 1, we fix all parameters except a_1 and a_2. If we now allow $m_1 = n_2$ to vary, the area of the triangle of figure 1 will vary also, and subject to (6) we can obtain bounds on n_2 between which the triangle will have some area. If we multiply the inequality (6) by n_2 and rearrange the terms, we obtain

$$b_1 n_2^2 - (b_1 + b_2) n_1 n_2 + b_2 n_1^2 < 0$$

which is quadratic in n_2. The solutions to the corresponding quadratic equation are

$$n_2 = n_1 \text{ and } n_2 = \frac{b_2 n_1}{b_1}.$$

Hence if we assume that $b_2 > b_1$, inequality (6) implies

$$n_1 \le n_2 \le \frac{b_2 n_1}{b_1}. \qquad (7)$$

How big can the triangle be? Its base is equal to its height and equal to

$$b_1 + b_2 - \frac{b_1 n_2}{n_1} - \frac{b_2 n_1}{n_2}.$$

The area is then

$$A = \frac{1}{2}\left[b_1 + b_2 - \frac{b_1 n_2}{n_1} - \frac{b_2 n_1}{n_2}\right]^2.$$

Within the limits on n_2 given by (7), the partial derivative of this area function, A, with respect to n_2 is zero and the area of the triangle is maximized when

$$n_2 = n_1 \sqrt{\frac{b_2}{b_1}}.$$

Note that this is the geometric mean of the limits on n_2 in (7).

The question of the ratio of the area of the triangle in figure 1 to that of the rectangle (4) is complicated by the fact that part of the triangle may lie outside the rectangle. To avoid this complication and ensure that the triangle will lie entirely within the rectangle throughout the range (7) of values of n_2 it is sufficient to require that

$$b_2 \le n_1. \qquad (8)$$

We ask, then, what is the maximum value of

$$\frac{\frac{1}{2}\left[b_1 + b_2 - \frac{b_1 n_2}{n_1} - \frac{b_2 n_1}{n_2}\right]^2}{n_1 n_2} \tag{9}$$

as n_2 ranges between the limits (7)? (This ratio is not correct if part of the triangle lies outside the rectangle, of course; a suitable portion of the triangle's area would have to be subtracted in that case. We require (8), and thus avoid this issue.)

Taking the partial derivative of (9) with respect to n_2 and setting it equal to zero leads one to seek solutions to a fourth degree polynomial in n_2. Fortunately, two of the solutions are the limits on n_2 imposed by (7) and we can divide the fourth degree equation by

$$(n_2 - n_1)\left(n_2 - \frac{b_2 n_1}{b_1}\right)$$

to obtain a quadratic equation in n_2. The positive solution to the resulting equation is

$$n_2 = \frac{-n_1(b_1 + b_2) + n_1\sqrt{(b_1 + b_2)^2 + 12b_1 b_2}}{2b_1}. \tag{10}$$

As an example, consider $n_1 = 200$ and $b_2 = 180$ (figure 2). The values given by (10) are then $n_{2max} = 225.88$, 233.79 and 242.44 for $b_1 = 140$, 130 and 120 respectively, with corresponding ratios .00006958, .0001756, and .0003769. The maximum value of the ratio increases as b_1 approaches zero and/or as b_2 approaches n_1. For the special, symmetrical case under consideration, wherein we assume (8) so that the triangle lies entirely within the rectangle (4) throughout the limits (7), the limit of the ratio (9) is 2/27. For this symmetric case, if we use the ratio of the area of the triangle in figure 1 to that of the area of the rectangle of all allowable pairs (a_1,a_2) as a measure of the likelihood of the occurrence of Simpson's Paradox, we see that that likelihood is quite small.

Major League Baseball's Hall of Fame

The paradox is rare. Major League Baseball has a rich and varied history, though, and it is a history brimming with statistics. With diligent searching, examples can be found. Former major leaguers Jim and Ron of the introduction are Jim Lefebvre and Ron Fairly, who played for the Los Angeles Dodgers in the 1965 and 1966 World Series. I began my search for Simpson's Paradox with World Series because several players' statistics are conveniently displayed together in that section of Reichler (1988). Soon, however, I turned my attention to full seasons of some of the greatest baseball players of all time. Here are three notable discoveries which the reader may easily verify by consulting Reichler (1988 or later edition).

In each of his first three years in major league baseball, upstart Lou Gehrig,

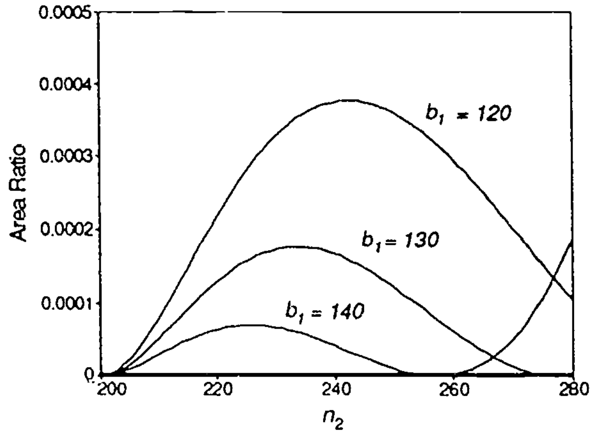

Figure 2: The ratio of Goddard's triangle to the rectangle of possible pairs (a_1, a_2) as a function of n_2. Here, $n_1 = m_2 = 200$ and $b_2 = 180$.

playing for the New York Yankees, had a better batting average than his teammate, the veteran Babe Ruth. When the data for the three years were combined, however, Ruth's average was superior, perhaps pointing out to the young Gehrig that to beat the Sultan of Swat, it wouldn't be enough just to beat him one year at a time (table 6).

Table 6. Yankee Teammates Ruth and Gehrig, 1923–1925.

	1923		1924		1925		Combined	
	AB	H	AB	H	AB	H	AB	H
Ruth	522	205	529	200	359	104	1410	509
Gehrig	26	11	12	6	437	129	475	146

Table 7. Hall of Famers Babe Ruth and Rogers Hornsby.

	1934		1935		Combined	
	AB	H	AB	H	AB	H
Ruth	365	105	72	13	437	118
Hornsby	23	7	24	5	47	12

Ruth was at it again in 1934 and 1935. Fellow Hall of Famer Rogers Hornsby beat him in each of those years, but again Ruth was better when the years were combined (table 7).

In 1941 and 1942, Stan Musial did battle with Joe DiMaggio. Stan "The Man" came out on top each year, but "Joltin'" Joe's average was better for the two years combined (table 8).

Table 8. Joe DiMaggio and Stan Musial.

	1941		1942		Combined	
	AB	H	AB	H	AB	H
Musial	47	20	467	147	514	167
DiMaggio	541	193	610	186	1151	379

The Paradox in Other Forms

We conclude with some problems. The first is simply the paradox as defined in (1) with $k = 50$. The others illustrate how the paradox may arise in fields other than statistics.

1. Two basketball players, Grace and Asina, play the same position on the same team. They each play every game, though not necessarily the same number of minutes. They play 50 games and at the end of the 50th game each has taken the same total number of shots over the course of the 50 games. Grace has a better shooting average than Asina during every one of the games, but Asina has the better average over the entire 50 games combined. How can this happen?

2. Let $\boldsymbol{a}_1$, $\boldsymbol{a}_2$, $\boldsymbol{b}_1$, and $\boldsymbol{b}_2$ be vectors in standard position each with tip in quadrant I. The angles of these vectors are α_1, α_2, β_1 and β_2 respectively. If $\alpha_1 > \beta_1$ and $\alpha_2 > \beta_2$, then how does the angle of $\boldsymbol{a}_1 + \boldsymbol{a}_2$ compare with that of $\boldsymbol{b}_1 + \boldsymbol{b}_2$?

3. Let z_1, z_2, w_1 and w_2 be numbers in the complex plane with real and imaginary parts all positive. If $\arg(z_1) > \arg(w_1)$ and $\arg(z_2) > \arg(w_2)$, then how does $\arg(z_1 + z_2)$ compare with $\arg(w_1 + w_2)$?

4. If $\boldsymbol{A}$ and $\boldsymbol{B}$ are two matrices with entries from $\mathbb{R}^+$ such that $\det\boldsymbol{A} > 0$ and $\det\boldsymbol{B} > 0$, what can be said about $\det(\boldsymbol{A} + \boldsymbol{B})$?

5. Consider the following "ordering" on $\mathbb{R}^+ \times \mathbb{R}^+$:

$$Def: \quad (a,b) <^* (c,d) \quad if \quad \frac{b}{a} < \frac{d}{c}.$$

What properties does this ordering have? Assuming addition is defined in the usual way for vectors, how does this ordering relate to the addition?

Summary

It seems reasonable to surmise that if a hypothesis is supported by two independent trials, then it will be supported when the data from those trials are combined. We have seen how Simpson's Paradox confounds this idea. It does so rarely, but in any forum, from playing cards to Major League Baseball's Hall of Fame.

References

Beckenbach, E. F. (1979). Baseball statistics. *The Mathematics Teacher,* 72, 351-352.

Blyth, C. R. (1972). On Simpson's paradox and the sure-thing principle. *Journal of the American Statistical Association,* 67, 364-366.

Cohen, J. E. (1986). An uncertainty principle in demography and the unisex issue. *The American Statistician,* 40, 32-39.

Gardner, M. (1976). On the fabric of inductive logic and some probability paradoxes. *Scientific American,* 234, 119-124.

Goddard, M. J. (1991). Constructing some categorical anomalies. *The American Statistician,* 45, 129-134.

Mitchem, J. (1989). Paradoxes in averages. *The Mathematics Teacher,* 82, 250-253.

Paik, M. (1985). A graphic representation of a three-way contingency table: Simpson's paradox and correlation. *The American Statistician,* 39, 53-54.

Reichler, J. L. (Ed.). (1988). *The Baseball Encyclopedia* (7th ed.). New York: MacMillan.

Shapiro, S. H. (1982). Collapsing contingency tables - A geometric approach. *The American Statistician,* 36, 43-46.

Simpson, E. H. (1951). The interpretation of interaction in contingency tables. *Journal of the Royal Statistical Society,* Ser. B, 13, 238-241.

Wagner, C. H. (1982). Simpson's paradox in real life. *The American Statistician,* 36, 46-47.

Lucky Larry # 11

While solving a radical equation, Lucky Larry used the student method of "squaring parts." For him, of course, it worked.

$$\sqrt{x+3} + \sqrt{2x+7} = 1$$

$$(\sqrt{x+3})^2 + (\sqrt{2x+7})^2 = 1^2$$

$$x + 3 + 2x + 7 = 1$$

$$3x = -9$$

$$x = -3, \text{ which checks!}$$

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The Ubiquitous Reed–Solomon Codes

by

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In this so-called Age of Information, no one need be reminded of the importance not only of speed but also of accuracy in the storage, retrieval, and transmission of data. It's more than a question of Garbage In, Garbage Out. Machines *do* make errors, and their non-man-made mistakes can turn otherwise flawless programming into worthless, even dangerous, trash. Just as architects design buildings that will remain standing even through an earthquake, their computer counterparts have come up with sophisticated techniques capable of counteracting the digital manifestations of Murphy's Law.

What many might be unaware of, though, is the significance, in all this modern technology, of a five-page paper that appeared in 1960 in the *Journal of the Society for Industrial and Applied Mathematics*. The paper, "Polynomial Codes over Certain Finite Fields," by Irving S. Reed and Gustave Solomon, then staff members at MIT's Lincoln Laboratory, introduced ideas that form the core of current error-correcting techniques for everything from computer hard disk drives to CD players.

Reed-Solomon codes (plus a lot of engineering wizardry, of course) made possible the stunning pictures of the outer planets sent back by Voyager II. They make it possible to scratch a compact disc and still enjoy the music. And in the not-too-distant future, they will enable the profit mongers of cable television to squeeze more than 500 channels into their systems, making a vast wasteland vaster yet.

"When you talk about CD players and digital audio tape and now digital television, and various other digital imaging systems that are coming – all of those need Reed-Solomon [codes] as an integral part of the system," says Robert McEliece, a coding theorist in the electrical engineering department at Caltech.

Why? Because digital information, virtually by definition, consists of strings of "bits" – 0s and 1s – and a physical device, no matter how capably manufactured, may occasionally confuse the two. Voyager II, for example, was transmitting data at incredibly low power – barely a whisper – over tens of millions of miles. Disk drives pack data so densely that a read/write head can (almost) be excused if it can't tell where one bit stops and the next one (or zero) begins. Careful engineering can reduce the error rate to what may sound like a negligible level – the industry standard for hard disk drives is 1 in 10 billion – but given the volume of information processing done these days, that "negligible" level is an invitation to daily disaster. Error-correcting codes are a kind of safety net – mathematical insurance against the vagaries of an imperfect material world.

The key to error correcting is redundancy. Indeed, the simplest error-correcting code is simply to repeat everything several times. If, for example, you anticipate no more than one error to occur in transmission, then repeating each bit three times and using "majority vote" at the receiving end will guarantee that the message is heard correctly (e.g., 111 000 011 111 will be correctly heard as 1011). In general, n errors can be compensated for by repeating things $2n + 1$ times.

But that kind of brute-force error correction would defeat the purpose of high-speed, high-density information processing. One would prefer an approach that adds only a few extra bits to a given message. Of course, as Mick Jagger reminds us, you can't always get what you want – but if you try, sometimes, you just might find you get what you need. The success of Reed-Solomon codes bears that out.

In 1960, the theory of error-correcting codes was only about a decade old. The basic theory of reliable digital communication had been set forth by Claude Shannon in the late 1940s. At the same time, Richard Hamming introduced an elegant approach to single-error correction and double-error detection. Through the 1950s, a number of researchers began experimenting with a variety of error-correcting codes. But with their SIAM journal paper, McEliece says, Reed and Solomon "hit the jackpot."

The payoff was a coding system based on groups of bits – such as bytes – rather than individual 0s and 1s. That feature makes Reed-Solomon codes particularly good at dealing with "bursts" of errors: Six consecutive bit errors, for example, can affect at most two bytes. Thus, even a double-error-correction version of a Reed-Solomon code can provide a comfortable safety factor. (Current implementations of Reed-Solomon codes in CD technology are able to cope with error bursts as long as 4000 consecutive bits.)

Mathematically, Reed-Solomon codes are based on the arithmetic of finite fields. Indeed, the 1960 paper begins by defining a code as "a mapping from a vector space of dimension m over a finite field K into a vector space of higher dimension over the same field." Starting from a "message" $(a_0, a_1, \dots, a_{m-1})$, where each a_k is an element of the field K, a Reed-Solomon code produces $(P(0), P(g), P(g^2), \dots, P(g^{N-1}))$, where N is the number of elements in K, g is a generator of the (cyclic) group of nonzero elements in K, and $P(x)$ is the polynomial $a_0 + a_1x + \dots + a_{m-1}x^{m-1}$. If N is greater than m, then the values of P overdetermine the polynomial, and the properties of finite fields guarantee that the coefficients of P – i.e., the original message – can be recovered from any m of the values.

Conceptually, the Reed-Solomon code specifies a polynomial by "plotting" a large number of points. And just as the eye can recognize and correct for a couple of "bad" points in what is otherwise clearly a smooth parabola, the Reed-Solomon code can spot incorrect values of P and still recover the original message. A modicum of combinatorial reasoning (and a bit of linear algebra) establishes that this approach can cope with up to s errors, as long as m, the message length, is strictly less than $N - 2s$.

In today's byte-sized world, for example. it might make sense to let K be the field of degree 8 over Z_2, so that each element of K corresponds to a single byte (in computerese, there are four bits to a nibble and two nibbles to a byte). In that case, $N = 2^8 = 256$, and hence messages up to 251 bytes long can be recovered even if two errors occur in transmitting the values $P(0), P(g), \ldots, P(g^{255})$. That's a lc better than the 1255 bytes required by the say-everything-five-times approach.

Despite their advantages, Reed-Solomon codes did not go into use immediately – they had to wait for the hardware technology to catch up. "In 1960, there was no such thing as fast digital electronics" – at least not by today's standards, says McEliece. The Reed-Solomon paper "suggested" some nice ways to process data, but nobody knew if it was practical or not, and in 1960 it probably wasn't practical."

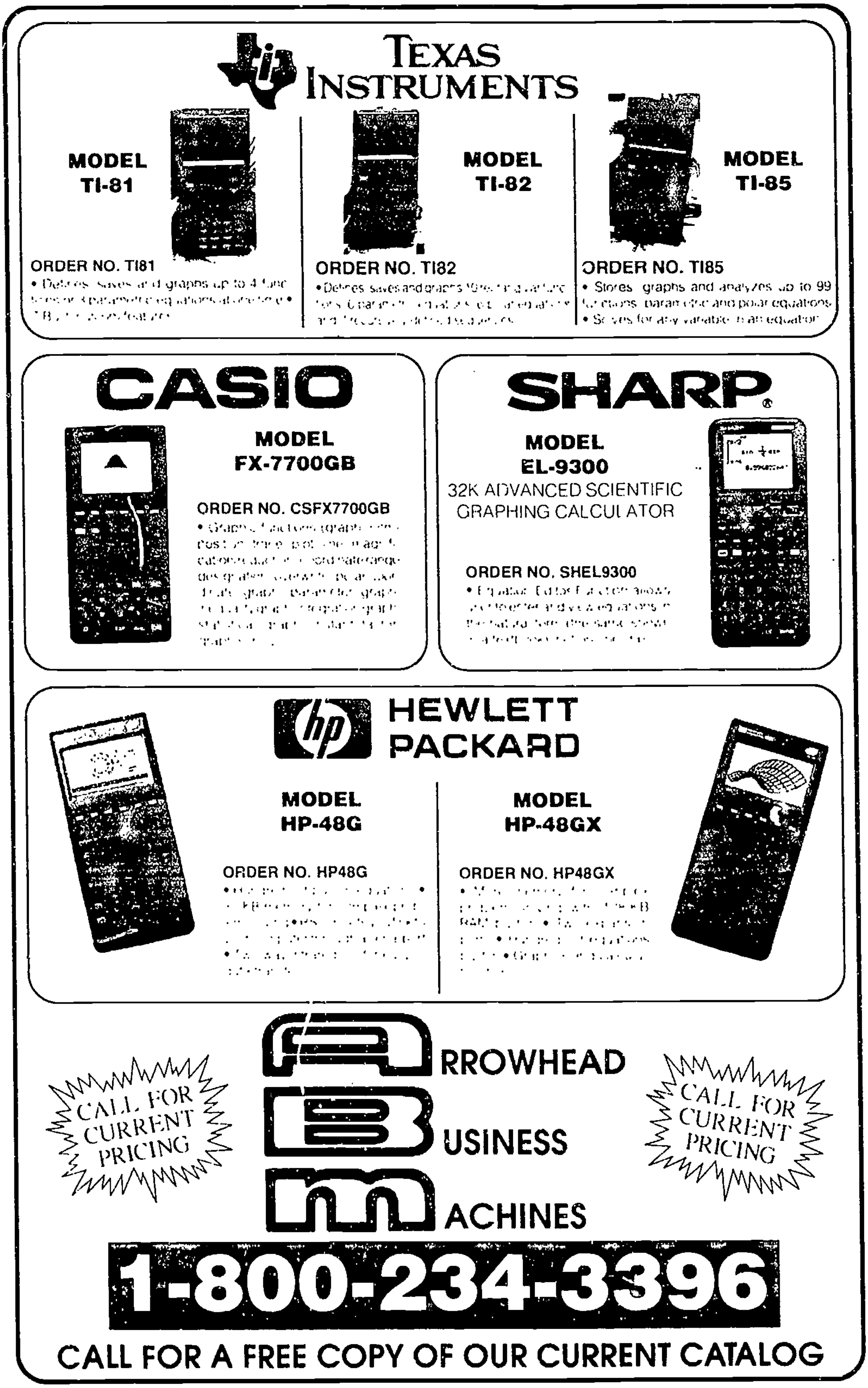
But technology did catch up, and numerous researchers began to work on implementing the codes. One of the key individuals was Elwyn Berlekamp, a professor of electrical engineering at the University of California at Berkeley, who invented an efficient algorithm for decoding the Reed-Solomon code. Berlekamp's algorithm was used by Voyager II and is the basis for decoding in CD players. Many other bells and whistles (some of fundamental theoretic significance) have also been added. Compact discs, for example, use a version called cross-interleaved Reed-Solomon code, or CIRC.

Reed, now a professor of electrical engineering at the University of Southern California, is still working on problems in coding theory. Solomon, recently retired from the Hughes Aircraft Company, consults for the Jet Propulsion Laboratory. Reed was among the first to recognize the significance of abstract algebra as the basis for error-correcting codes.

"In hindsight it seems obvious," he told *SIAM News*. However, he added, "coding theory was not a subject when we published that paper." The two authors knew they had a nice result; th y didn't know what impact the paper would have.

Three decades later, the impact is clear. The vast array of applications, both current and pending, has settled the question of the practicality and significance of Reed-Solomon codes. "It's clear they're practical, because everybody's using them now," says Berlekamp. Billions of dollars in modern technology depend on ideas that stem from Reed and Solomon's original work. In short, says McEliece, "it's been an extraordinary influential paper."

> I went through college in two terms: Truman's and Eisenhower's.
>
> Herb Gross



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SHORT COMMUNICATIONS

A Short Proof Linking the Hyperbolic and Exponential Functions

by

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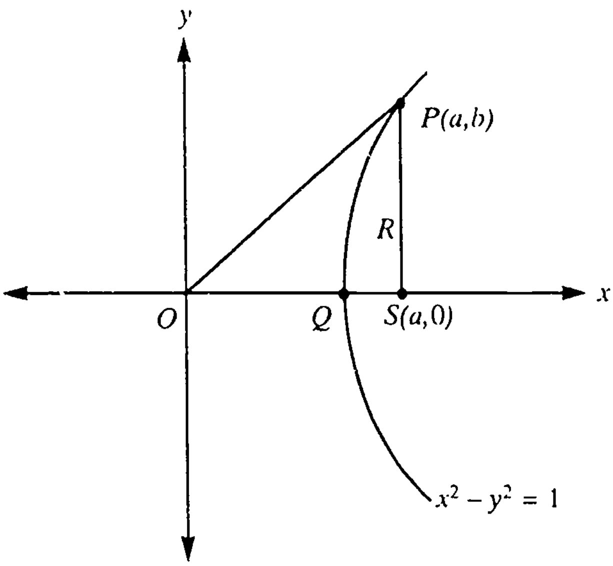
Rather than define the hyperbolic sine, cosine, etc. in terms of exponential functions (which seems nothing short of mystical to the student), we may choose to state these relationships as a theorem and prove it using more fundamental concepts. Submitted here is an original proof that is more concise than others I have seen.

Let $P(a,b)$ be a point on the right-half of the unit hyperbola $x^2 - y^2 = 1$ whose vertex lies at $Q(1,0)$. From P, drop a perpendicular to the horizontal axis and label the point of intersection $S(a,0)$. Define the hyperbolic cosine and hyperbolic sine functions as follows:

$$\cosh(t) = OS = a \text{ and } \sinh(t) = SP = b.$$



Define the measure of the hyperbolic angle (t) to be twice the area of the (shaded) sector OPQ. (This is analogous to the unit circle concept of angle measure in radians. Since the area of a circular sector is $r^2\theta/2 = \theta/2$, the measure of the angle is twice the area of the sector.)

So we have,

$$t = 2(\text{area of triangle } OPS - \text{area of region } R)$$

$$= 2\left((1/2)ab - \int_1^a \sqrt{x^2-1}\, dx \right)$$

$$= 2\left((1/2)ab - (1/2)\left(x\sqrt{x^2-1} - \ln\left| x + \sqrt{x^2-1} \right|\right)\Big|_1^a \right)$$

$$= ab - a\sqrt{a^2-1} + \ln\left| a + \sqrt{a^2-1} \right|$$

$$= \ln\left| a + \sqrt{a^2-1} \right|.$$ (Since (a,b) lies on the hyperbola, $b^2 = a^2 - 1$.)

Therefore, $e^t = a + \sqrt{a^2-1}$.

By isolating the radical, squaring both sides and solving for a, we obtain

$$\cosh(t) = a = (e^t + e^{-t})/2.$$

Substituting the above result into $b = \sqrt{a^2-1}$, we obtain

$$\sinh(t) = b = (e^t - e^{-t})/2.$$

This completes the proof.

The doctor told me, "The best thing for you, Herb, would be to lose forty pounds." I said, "Doctor, I don't deserve the best."

A student told me that the probability of dying was 1.2. Another student corrected him saying, "A probability can't be bigger than 1; it's always between negative one and positive one."

Herb Gross

L'Hôpital's Rule Via Linearization

by

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The placement of L'Hôpital's Rule in a calculus syllabus is problematic. In view of the Rule's extensive applications, an early placement may seem desirable. However, a number of recent texts introduce the Rule during the second semester: see, for instance, (Stewart, 1991, p. 407) and (Swokowski, 1991, p. 493). Both citations are for a proof of the general "0/0" case of L'Hôpital's Rule which depends on Cauchy's generalization of the Mean Value Theorem; in such texts, the Cauchy result is typically developed by an appeal to Rolle's Theorem. **The main purpose of this note** is to present a more accessible proof of the most natural "0/0" case of L'Hôpital's Rule. Not only can this proof be given during the first semester's discussion of rules for differentiation, but its early introduction reinforces the topics of differentiability and approximation by differentials and linear functions.

The general "0/0" case of L'Hôpital's Rule is the following assertion. Let f and g be functions defined on an open interval (a,b), each of which is differentiable except possibly at c in (a,b). Suppose that both $\lim_{x \to c} f(x)$ and $\lim_{x \to c} g(x)$ are 0. Suppose also that $g'(x) \neq 0$ whenever $x \neq c$ in (a,b) and that $\lim_{x \to c} \frac{f'(x)}{g'(x)} = L$ (either a real number, ∞, or $-\infty$). Then $\lim_{x \to c} \frac{f(x)}{g(x)} = L$.

In fashioning a proof of the above assertion, there is no loss of generality in requiring f and g to be continuous at c, and so we make this assumption from now on. (In detail, we are replacing f with the function f^* given by $f^*(c) = \lim_{x \to c} f(x) = 0$ and $f^*(x) = f(x)$ if x is in the domain of f and $x \neq c$; a similar replacement of g by a function g^* is employed. These replacements do not affect either of the above limits since $f^{*\prime}(x) = f'(x)$ and $g^{*\prime}(x) = g'(x)$ whenever $x \neq c$.)

To get into the spirit of a proof using approximations, consider the following argument that appeals to Taylor's Theorem (with Remainder). Since we assumed that $f(c) = 0$, we may use Taylor's Theorem to write

$$f(x) = f'(c)(x-c) + (x-c)^2 H(x)$$

where H is a bounded function. Similarly, we have

$$g(x) = g'(c)(x-c) + (x-c)^2 K(x)$$

for a suitable bounded function K. Consider the limit process as $x \to c$. As $x \neq c$ during this limit process, we may cancel $x - c$, obtaining

$$\frac{f(x)}{g(x)} = \frac{f'(c) + (x-c)H(x)}{g'(c) + (x-c)K(x)}.$$

As H, K are bounded, $\lim_{x \to c} (x-c)H(x) = 0 = \lim_{x \to c} (x-c)K(x)$. Applying standard limit theorems to the last equation, we have

$$\lim_{x \to c} \frac{f(x)}{g(x)} = \frac{f'(c)+0}{g'(c)+0} = \frac{f'(c)}{g'(c)}.$$

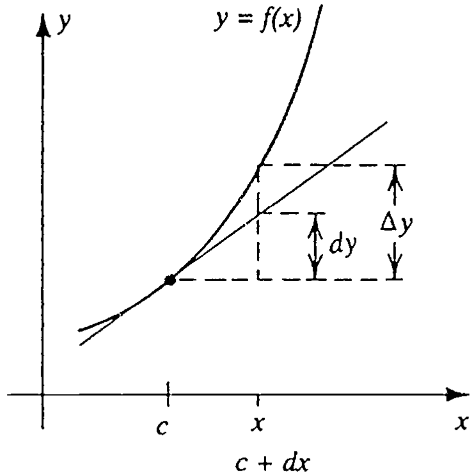
Provided that $\frac{f'}{g'}$ is continuous at c, we have $\frac{f'(c)}{g'(c)} = L$, and the proof is complete.

The preceding argument is far from optimal. In order to invoke Taylor's Theorem with Remainder, we needed extra hypotheses, such as existence and continuity of $f''(x)$ and $g''(x)$ in an open interval containing c, as well as the condition $g'(c) \neq 0$. (With these hypotheses, the rest of the proof is valid, as we then have continuity of f' and g' and, thus, continuity of $\frac{f'}{g'}$.) Nevertheless, various aspects of the above argument will next be used to devise the promised, more accessible proof. We emphasize that the following proof can be presented to a class *without* first giving the above proof that used Taylor's Theorem with Remainder.

We promised a proof of the most natural "0/0" case. By this, we mean, in addition to the harmless assumptions that $f(c) = 0 = g(c)$, that we shall assume $g'(c) \neq 0$ and both f' and g' are continuous (at and near c). Using the customary notation in the introduction of differentials, let

$$F = \frac{\Delta y - dy}{dx}, \text{ where } dx = x - c,\ \Delta y = f(x) - f(c), \text{ and } dy = f'(c)\,dx.$$

Several of these quantities are illustrated in the figure; notice that F can be interpreted as the difference between the slope of the secant line and the slope of the tangent line.



By the very definition of derivative (and a standard limit theorem),

$$\lim_{x \to c} F = \lim_{x \to c} \frac{\Delta y}{dx} - \lim_{x \to c} \frac{dy}{dx} = f'(c) - f'(c) = 0.$$

Hence, $f(x) = \Delta y = dy + (dx)F = f'(c)\,(x - c) + F(x)\,(x - c)$. Similarly,

$$g(x) = g'(c)\,(x - c) + G(x)\,(x - c), \text{ where } \lim_{x \to c} G = 0.$$

Precisely as in the preceding proof, we appeal to limit theorems and find

$$\lim_{x \to c} \frac{f(x)}{g(x)} = \frac{f'(c) + 0}{g'(c) + 0} = \frac{f'(c)}{g'(c)} = L,$$

to complete the proof.

The device of introducing F and G in the above proof should fit well in a classroom discussion of linear (or differential) approximations. We close with the following additional pedagogic remarks. Functions such as F (and G) recur in a careful proof of the Chain Rule in a first course. Moreover, prior experience with F and G, as in the above proof, prepares students for the definition (typically, in the second year of calculus) of differentiability for functions of several variables.

References

Stewart, J. (1991). *Calculus* (2nd ed.). Pacific Grove, CA: Brooks/Cole.

Swokowski, E.W. (1991). *Calculus* (5th ed.). Boston: PWS-Kent.

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Predicting Grades in Basic Algebra

by

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Elsie Newman is an instructor of Developmental Mathematics at Owens Technical College, Toledo, Ohio. Her degrees include a Bachelor of Science from Bowling Green State University and a Master of Education from University of Toledo. She is listed in Who's Who in American Education and belongs to Phi Kappa Phi, Kappa Delta Pi, NCTM, and Ohio Association of Developmental Education.

Abstract

The effective placement of students into college courses is of prime importance to both students and educators. The methods and tools vary depending upon the experiences and preferences of the placement officials. This study was conducted at Owens Technical College in northwestern Ohio to determine if performance in a Basic Algebra course could be associated with or predicted by any of sixteen variables including ACT and ASSET scores. The correlations and regressions from the statistical analysis indicated that high school grade point average had more strength as a predictor of the Basic Algebra grade. Significantly better results were obtained using a combination of several variables. Although the results of this study are most useful at Owens College, the need for closer scrutinization of placement procedures at all technical and two-year colleges is also implied.

Introduction

Choosing an effective method of placing college students into courses is a concern of many educators. Not only is the success of the student at risk, but the morale and workload of teachers is to a great extent also dependent upon these decisions. And ultimately, the reputation of the institution can even be influenced by the effectiveness of these procedures.

The current literature suggests a variety of methods for assessing and placing students into college courses. At North Harris County College, Texas, the low correlations between ACT and SAT scores and the grades of the students successfully completing the mathematics courses into which they were placed (.07 and .03) lead to the conclusion that neither the ACT nor the SAT were effective predictive tools for placement (Reap, 1980). At The University of Toledo Community and Technical College, ACT scores as well as a local diagnostic test, high school grade point average, and other variables were analyzed for predictive value. The conclusion was that no one variable could effectively predict success in either a developmental or technical mathematics course (Eshenroder, 1987). Another study by the Research and Statistical Services Department of the American College Testing Program examined the predictive validity of ACT scores and high school grades in English and mathematics in 233 institutions. The multiple regression analysis of the study yielded consistently higher median correlations from both ACT scores and high school grades than did either variable used individually (Noble & Sawyer, 1989). The ASSET program, another placement tool, was correlated with English composition grades at Riverside Community College. The resulting canonical correlation of .14 did not indicate a close relationship (Hughes & Nelson, 1991).

Other studies demonstrate the reliance on several factors for more effective student placement. At an accredited two-year technical college in Louisiana, the pre-enrollment variables found to be significant predictors of initial achievement measured in terms of GPA were race, gender, diploma type, student expected GPA, and entrance exam scores (Taube & Taube, 1990). In Grayslake, Illinois at the College of Lake County, nine factors proposed as influencing learning by Walberg (1984) were used as independent variables with GPA as the dependent variable. Having the most powerful effect on GPA was prior achievement measured by using self-reported ratings by students. Other important factors were time, motivation, social context of the classroom, and age (Johnson & Walberg, 1989). Additional studies at Rockland Community College (Blustein, et al., 1986) and at the City University of New York (Gerardi, 1990), concluded that both attitudinal and cognitive factors were predictors of GPA.

These findings appear to be varied, and as several researchers warned, making generalizations from one study to other situations is dangerous (Eshenroder, 1987; Noble, 1989; McDonald, 1989). Also, the literature indicates that two-year or community colleges which normally enroll a larger number of nontraditional students should consider different factors for placement purposes than are used in four-year colleges. Lastly, the importance of using more than one factor for predicting success seems to be substantiated by the research available.

Method

The current study was conducted at Owens Technical College, a two-year institution in northwestern Ohio. The college offers over 50 degree majors in business, health, public service, industrial technologies, and engineering technologies on the Toledo campus. Data was collected from 470 students enrolled in Basic Algebra, a Developmental Department course, for the Fall Semester, 1991. Sixteen variables were hypothesized as being possible predictors of the student's final grade in Basic Algebra. The following list includes the variables and their abbreviations.

RMTHGRD = Basic Algebra grade

HSGPA = high school grade point average

CGPA = college grade point average

CHATTP = college hours attempted

CHEARN = college hours earned
RANK = high school rank
LANG = ASSET Writing Skills score
READ = ASSET Reading Skills score
NUM = ASSET Numerical Skills score
EALG = ASSET Elementary Algebra score
DOBY = date of birth
HSGRADYR = high school graduation year
ACTMATH = ACT Mathematics score
ACTENG = ACT English Usage score
ACTSOC = ACT Social Studies score
ACTNAT = ACT Natural Science score
ACTCOMP = ACT Composite score

All the information was available from existing student records, and the Statistical Package for the Social Sciences (SPSSX) was used for the data analysis.

Four of the independent variables, ASSET scores in Writing Skills, Reading Skills, Numerical Skills, and Elementary Algebra, were chosen since they are obtained during registration of all new students at Owens Technical College. The ASSET program was developed by ACT for specific use in advising and course placement in two-year academic institutions. The ASSET scores are the primary tool for placement into English and mathematics at Owens College, and the raw scores from the test (Form B) are converted to scale scores before interpreting. The ranges of scale scores for each of the four ASSET tests are as follows: Writing Skills, 23 to 54; Reading Skills, 23 to 53; Numerical Skills, 23 to 55; and Elementary Algebra, 23 to 55. To be placed into Basic Algebra (Math 105), a Developmental Department course at Owens College, a student normally has a Numerical Skills scale score between 43 and 55 and an Elementary Algebra scale score between 23 and 42. Scale scores which differ from these in either category may place students in either Principles of Math (Math 100) or in higher level mathematics courses depending upon the technology of each individual student.

The 64 students who officially withdrew from the class and the one who received a grade of Incomplete were not included in this study. It would be unfair to categorize all these as failures since many factors are involved in withdrawal or incomplete work. At the time a student withdraws from a class, the average up to that point may fall anywhere in the range from A to F, and yet this average may not reflect what the student would have earned had the withdraw not occurred. Making any assumptions about these grades would have biased the results of the correlations.

Results

The first step in the analysis was to determine the Pearson product-moment correlations between all the variables.

Table 1: Summary Statistics for the 17 variables used in Simple Regressions

	Mean	Standard Deviation	Correlation with RMTHGRD
RMTHGRD	3.664	1.473	1.0000
HSGPA	2.398	.577	.3544**
CGPA	2.876	1.001	.2408*
CHATTP	12.162	19.388	.1149
CHEARN	11.339	14.975	.0715
RANK	48.226	23.948	–.3380**
LANG	41.550	7.367	.1552**
READ	42.075	7.278	.0794
NUM	41.708	5.109	.1972**
EALG	31.880	5.965	.1891**
DOBY	64.181	8.560	–.0892
HSGRADYR	83.172	8.188	–.1083*
ACTMATH	15.299	4.477	.1289
ACTENG	17.482	4.424	.1165
ACTSOC	17.424	5.830	.0607
ACTNAT	18.996	3.936	.2124**
ACTCOMP	17.435	3.526	.1551

* - SIGNIF. LE .05   ** - SIGNIF. LE .01   (2 - TAILED)

The variable most highly associated with the Basic Algebra grade was high school grade point average with a correlation coefficient of .3544. The relationship between high school rank and the Basic Algebra grade was slightly less with a value of –.3380 (negative because a higher number indicates a "lower" rank). Other lower correlations were college grade point average (.2408), ACT Natural Sciences (.2124), ASSET Numerical Skills (.1972), and ASSET Elementary Algebra (.1891). The level of significance for each of these was either .01 or .05. Since all the correlations were relatively low, using any one of them for predicting Basic Algebra grades or for placement and counseling could pose problems.

To determine if combinations of independent variables might better predict the dependent variable, multiple regressions were used. Table 2 shows the mean and standard deviation as well as the correlations for all the variables used in the regressions. Since it is necessary to have complete data concerning all 17 variables for each case to calculate the multiple regressions, the number of cases reduced to only 38. As Table 2 shows, these statistics differ slightly from those shown in Table 1.

Table 2: Summary Statistics for the 17 variables used in Multiple Regressions

	Mean	Standard Deviation	Correlation with RMTHGRD
RMTHGRD	4.395	1.079	1.000
HSGPA	2.604	.500	.429
CGPA	2.857	1.001	.311
CHATTP	15.053	23.661	.162
CHEARN	18.886	23.640	.138
RANK	38.109	17.717	−.219
LANG	40.447	10.727	.094
READ	38.737	9.457	.029
NUM	40.763	6.051	−.080
EALG	33.079	6.118	.040
DOBY	64.211	6.019	−.155
HSGRADYR	82.500	6.242	−.171
ACTMATH	14.263	4.625	.044
ACTENG	16.237	4.321	.269
ACTSOC	16.079	6.411	.105
ACTNAT	19.447	4.694	.098
ACTCOMP	16.605	3.803	.158

Number of cases = 38

The first regression was stepwise with the result of only one variable, high school grade point average, being a predictor of the Basic Algebra grade. The multiple *r* correlation coefficient was .429 accounting for 18.4 percent of the variability in grades while no other variables were found to add significantly to the equation. Next, a backward elimination regression was used to decide which other variables might be used with high school grade point average to predict Basic Algebra grades. It was found that high grade point average again accounted for 18.4 percent of the variability with a multiple *r* value of .429. However, when college grade point average was included, the variability increased to 23 percent and the multiple *r* value increased to .480. With all the variables in the regression equation, the multiple correlation was .735 accounting for 54 percent of the variability.

The last regression used was forced entry which yielded results only slightly different from those above. The first variable entered was the Elementary Algebra ASSET score with a multiple *r* value of .040 and a variability of .2 percent. The next entry, high school rank, increased the multiple *r* value to .219 and the variability to 4.8 percent. Other changes in multiple *r* (listed first) and variability (listed second) occurring with respect to the variables being entered were as follows: ASSET Writing Skills score, .236, 5.6 percent; ASSET Numerical Skills score, .261, 6.8 percent; high school graduation year, .382, 14.6 percent; college grade point average, .429, 18.4 percent; ACT Mathematics Usage score, .430, 18.5 percent; and high school grade point average, .596, 35.5 percent. In other words, these eight variables in combination accounted for over 35 percent of the variability in Basic Algebra grades and the multiple correlation coefficient was .596. These different methods of regression demonstrate the increased predictability when several variables are used together.

To complete the analysis of the data, the chi-square test was used to determine if ASSET Numerical Skills scores, ASSET Elementary Algebra scores, ACT Composite scores, ACT Mathematics Usage scores, ACT Natural Sciences scores, or high school grade point average were significant in predicting Basic Algebra grades. Of the six variables, only two were significant. ACT Composite score was significant at the .05 level (chi-square of 39.49716 and significance level of .0242) and high school grade point average was found to be extremely significant (chi-square of 64.72057 and significance level of .0000).

Based on the single correlations between the 16 independent variables and the dependent variable, high school grade point average was associated the most with grades in Basic Algebra. Although this was a low correlation, high school grade point average consistently appeared in the multiple regressions as well as the chi-square test as having more strength as a predictor of the dependent variable. It should be remembered, however, that the strength increased with the inclusion of several other variables as noted above.

Implications

The utilization of this information is most beneficial at the specific location of this study. A recommendation based on these results would be more emphasis placed on high school grade point average, less emphasis on ASSET scores, and less reliance on one variable for placement into Basic Algebra at this technical college. More generally, the theory that as many as 50 variables must be examined to explain any social phenomenon (Hodges, 1981) coupled with the literature cited earlier and the results of this study would lead to the conclusion that to be most effective in placing students into college courses, many factors need to be examined. For further study, it would seem appropriate to examine other variables including those that measure attitude and self-concept. Although such information was not available in this study, both have been found to be important in the academic success of nontraditional students as was evidenced in the literature.

References

Blustein, D. L., Judd, T. P., Kron, J., Viniar, B., Padilla, E., Wedemeyer, R., & Williams, D. (1986). Identifying predictors of academic performances of community college students. *Journal of College Student Personnel*, 27(3), 242-249.

Eshenroder, A. (1987). *Predictors of success in college developmental and technical mathematics courses.* A Dissertation presented to The University of Toledo.

Gerardi, S. (1990). Academic self-concept as a predictor of academic success among minority and low-socioeconomic status students. *Journal of College Student Development*, 31(5), 402-406.

Hodges, D. L. (1981). *Frustration: Or why LCC's system of testing and placement does not work better.* (ERIC Document Reproduction Service No. ED 221 234).

Hughes, R. E., & Nelson, C. (1991). Placement scores and placement practices: An empirical analysis. *Community College Review,* 19(1), 42-46.

Johnson, M. L., & Walberg, H. J. (1989). Factors influencing grade point averages at a community college. *Community College Review,* 16(4), 50-60.

McDonald, A. D. (1989). Issues in assessment and placement for mathematics. *Journal of Developmental Education,* 13(2), 20-23.

Noble, J. P., & Sawyer, R. L. (1989). Predicting grades in college freshman English and mathematics courses. *Journal of College Student Development,* 30(4), 345-353.

Reap, M. C. (1980). *Evaluation of the effectiveness of the developmental studies program.* (ERIC Document Reproduction Service No. ED 197 798).

Taube, S. R., & Taube, P. M. (1990). Pre- and post-enrollment factors associated with achievement in technical postsecondary schools. *Community/Junior College Quarterly of Research and Practice,* 14(2), 93-99.

Walberg, H. J. (1984). Improving the productivity of America's schools. *Educational Leadership,* 41(8), 19-30.

Why Do We Transform Data?

by

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R·I·T

David Farnsworth received his MA and PhD in Mathematics from the University of Texas at Austin. His BS is in Physics from Union College. About ten years ago he became interested in statistics. Currently, he is working in the areas of nonparametric statistics, exploratory data analysis, and, of course, transformation of observations.

Introduction

Sometimes, we ask our students to change units. For example, data may be converted from cubic inches to liters or from dollars to yen. On other occasions they may be asked to code yes-no data to one-zero values for data entry purposes or for future calculations. These are examples of transformations. Such modifications of observations are useful and often required in practice.

Here, we discuss a typology of seven reasons for transforming data, important aspects of analytic formulations of transformations, and potential problems or dangers in linking calculations and transformations on a data set. These topics in data handling arise in statistics courses as well as in many elementary mathematics courses such as survey courses that contain material on probability and statistics.

Transformation of observations is an excellent topic for open discussion in those classes. Once the students understand the basic idea of altering data, they can supply their own examples and reasons for transforming. A discussion of functional forms and cautionary warnings may be incorporated.

Our seven purposes for transforming observations describe experience inside and outside class. In practice we might quote more than one reason for transforming a particular set of observations. Nonetheless, this classification can help clarify classroom discussions. Some classes may create somewhat different looking reasons for transforming. If a typology of reasons is created for or by a very elementary class or early in a course, our seventh reason of preparation for a statistical analysis may not come up. It could be added later in the course or omitted.

Purposes for Transforming Data

1. To Change Units Because of Common Usage

Sometimes, there is no particular intrinsic reason for selecting one scale over another. Degrees Fahrenheit and degrees Celsius are equally good units, as are dollars and Japanese yen. A temperature in degrees Celsius may be transformed into degrees Fahrenheit, and a price in dollars may be transformed into yen just for common usage.

For example, many banks in New York State display the time and temperature outside their offices. A particular bank's temperature sensor may be calibrated in the Celsius scale, but the bank's customers are more familiar with the Fahrenheit scale. So, a conversion or transformation to the familiar scale is made for presentation. If the reading in degrees Celsius is represented by C, the display will show degrees Fahrenheit $F = 32 + (9/5)C$.

2. To Alter Observations for Their End Use

Occasionally, the units and the way that something is measured are determined by the situation, but the ideas or information contained in the data would be better revealed if the observations are transformed. For example, a company manufactures square photographic paper and measures the length of a side of the sheets. The areas may be important for pricing, so the transformation is squaring.

For a second example, take the times that are required for five people to run a mile: 5.12, 5.71, 6.21, 5.54, and 5.67 minutes. We want the average speeds for these individuals over this run. Since we have no mechanism for measuring average speed, we find the corresponding average speeds in miles per hour over the mile run by dividing each time into 60 giving 11.72, 10.51, 9.66, 10.83, and 10.58 miles per hour. Observations can be presented as times or as speeds, but they probably would be gathered as elapsed times. In auto racing the elapsed time to complete a lap or a race is used similarly to compute average speed.

3. To Compare Observations from Different Situations

Sometimes, measurements that are widely separated in time or space can be compared. The situations surrounding these observations may be different, but direct comparisons can be made by altering some or all of the measurements.

For a classroom example, say that two brothers studied chemical engineering at the same college and obtained similar grades. The younger brother graduated in 1990 and started his first job in Houston, Texas, with the same company that hired the older brother upon graduation ten years earlier. The younger brother's initial annual salary was $31,500, and the older brother's was $22,000. Which of them received the better starting salary?

Clearly, a dollar in 1980 could purchase more than a dollar could in 1990. For instance, the cost for the older brother to visit his doctor's office had doubled from $20 to $40 in the 1980's. It is possible to transform salaries in order to do a comparison from one year to another. The transformation is multiplication using the annual Consumer Price Indices that are published by the Bureau of Labor Statistics of the U.S. Department of Labor. These indices are the average annual inflation rate for goods and services purchased by consumers. Table 1 shows the rates of change for each year from 1981 through 1990. The 10.3 for 1981 means that someone who had identical credentials as the older brother and starting the same job in 1981 might expect to be paid $\$22,000(1 + 0.103) = \$24,266$ in order to keep up with expenses. The $22,000 transformed into 1981 dollars is $24,266.

Table 1. Annual percent change in prices of goods and services. From (Hoffman, 1992, p.135)

Year	1981	1982	1983	1984	1985	1986	1987	1988	1989	1990
Percent Change	10.3	6.2	3.2	4.3	3.6	1.9	3.6	4.1	4.8	5.4

Let us return to the comparison of the two brother's salaries. A way to compare them is to convert the 1980 initial $22,000 salary into 1990 dollars, which is $22,000 (1.103) (1.062) (1.032) ... (1.054) = $22,000 (1.586) = $34,892. The older brother did better by $34,892 – $31,500 = $3,392 in 1990 dollars.

The beginning salaries are in dollars, but the $22,000 and $31,500 are separated by a decade of inflation and have considerably different values. That difference can be gauged by transforming one salary to the other year's scale.

4. To Enter and Manipulate Data More Easily

This type of transformation is often called coding. Characteristics such as gender and religious affiliation can be assigned numbers or a code. For example, in a survey the replies to a gender question are MALE and FEMALE. The code or transformation could be to change each MALE response to a one (1) and each FEMALE to a zero (0). Ones and zeros may be easier to enter in a spreadsheet and at the same time be ready for arithmetic manipulation.

Another type of code replaces numbers with more manageable numbers. Consider a large data set containing year dates between 1980 and 1989. A practical code involves deleting the leading 198 or, equivalently, subtracting 1980 from each date. Only the integers 0 through 9 are required to identify the year. In this case only one fourth of the original digits have to be recorded - saving many key strokes in data entry and probably reducing the number of errors.

Coding can save recording time and speed up calculations. There is no loss of information with this type of transformation since the original observation can be uniquely identified by the transformed value.

5. To Streamline a Data Set for Inspection

On occasion we may not care about every digit in a number. For instance, speedometer readings to three decimal places, such as 32.174 MPH, would probably be considered more precise than necessary. The trailing 174 has little use and may be distracting. The transformation in this example is truncation to an integer. This reason for transforming observations is similar to that for coding. A distinction is that here some information is lost, whereas in coding no information is lost in the transformation.

Another common example is the generalization of peoples' ages. A person may be referred to as in her twenties or being twenty-something. This truncating of ages simplifies speech and may conceal some lack of knowledge about the person's true age. A whole set of age data may be reduced to 20s, 30s, 40s, and so forth. Unlike the speedometer example, this really does involve the loss of potentially important information.

6. To Transact Secret Business or Transmit Proprietary Information

Information can be transmitted in an altered form to preserve confidentiality or guard against theft. Scrambling and descrambling devices might be used on television or other communications hookups where eavesdropping on the signal would cause financial loss. In wartime the breaking or deciphering of an enemy's code or cryptography has changed the course of history.

A very simple example of a transformation used to deceive is a cipher alphabet which substitutes letters for other letters. These transformations are called codes, but their purpose is to deceive or protect. They are unlike the codes described above that are meant to simplify.

7. To Perform a Statistical Analysis

Observations are sometimes manipulated with a transformation before a statistical analysis is performed. One such transformation involves altering data so that they look more nearly bell-shaped or normal. The specific transformation may depend upon the shape of the observations' histogram or upon some theoretical considerations.

Applications of exponential growth are likely to be encountered in mathematics and statistics classes. It can be difficult to understand the details of a graph of exponential-growth observations, and the analysis of the nontransformed data can be beyond the scope of courses in the first few years of a college curriculum. The variables for this example are the number (N) of bacteria in a colony and time (t) in hours. We expect our data points $\{t_i, N_i\}$with $i = 1, 2, ..., n$ to be near a curve of the form $N = k10^{ct}$ where c and k are constants that we need to estimate.

Taking the logarithm to the base ten of $N = k10^{ct}$ gives $\log N = \log k + ct$. A straight line can be fitted to the n points $\{t_i, \log N_i\}$ either by eye or using a formal statistical procedure. The line's slope is an estimator of c, and $\log k$ is estimated at the intersection point of the line and the $\log N$ axis.

Features of Transformations

Most of the transformations in the previous section can be represented by real valued functions of a real variable - the kind of function studied in beginning mathematics courses. For example, the change from degrees Celsius (C) to degrees Fahrenheit (F) is accomplished by $F = 32 + (9/5)C$, and the speed (s) in miles per hour for a mile that was run in m minutes is $s = 60/m$. A few of the transformations in the examples in the previous section have a slightly different nature. The domain {MALE, FEMALE} of the transformation that takes survey answers to one and zero values is not a subset of the real numbers. The range of the transformation that takes peoples' ages to the groups twenties, thirties, and so forth is a set of classes, not a set of real numbers.

It can be worthwhile to discuss in class some distinguishing features of real valued functions of a real variable as an aid in constructing transformations and in assessing them. Of course, the property of all functions that each element in the observations, that is, the domain, corresponds to exactly one value in the range is necessary in order to have a deterministic conversion without ambiguity.

The property of invertibility or possessing an inverse function insures that no information is lost in the transformation since we can find the original measurement using the inverse function. For the example $F = 32 + (9/5)C$, the temperature in degrees Celsius can be determined from F with the inverse function $C = (5/9)(F - 32)$. However, the age-groups example does not have this property.

Another aspect of functions is that some are increasing and some are decreasing. The example $F = 32 + (9/5)C$ is increasing since increasing C increases F. The example $s = 60/m$ is decreasing since increasing the time (m) to run the mile decreases the average speed (s).

Discussing these issues in class can be especially important in a statistics class composed of students who have minimal mathematical backgrounds.

A Word of Caution

A wide variety of functional forms can be useful. However, many can cause problems when the observations or the transformed observations are manipulated. There can be complications even for functions which might seem to be the easiest to handle - those that are invertible, continuous, and increasing. Let's examine two examples in which the data are manipulated to determine the arithmetic mean.

The first example is well behaved. The arithmetic mean and a linear transformation commute, that is, the arithmetic mean follows the same transformation as the data. Unfortunately, this is not typical. The second example illustrates the usual state of affairs.

In the example $F = 32 + (9/5)C$ with $C = (5/9)(F - 32)$, the arithmetic mean of the temperatures in degrees Celsius ($\overline{C}$) and the arithmetic mean of the transformed temperatures in degrees Fahrenheit ($\overline{F}$) correspond in the transformation and inverse transformation pair. The mean can be computed in either scale and transformed back and forth at will since

$$\overline{F} = \sum F_i/n = \sum(32 + (9/5)C_i)/n = \sum 32/n + (9/5)\sum C_i/n = 32 + (9/5)\overline{C}$$

where the sums are over $i = 1$ to $i = n$ for n data values. Further study shows that linear transformations allow the interchange of arithmetic mean and transformation, but no other functional form does.

Next, reconsider the photo-paper example. The lengths of the sides (S) were measured and the area computed using the nonlinear transformation $A = S^2$. The problem is that the mean of the lengths of the sides ($\overline{S}$) does not correspond to the mean of the areas ($\overline{A}$). Considering the synthetic observations 1, 1.5, 2, 3, 4, 4.5, and 5 representing the lengths in inches of sides of seven squares of photo paper, $\overline{S} = (1 + 1.5 + 2 + 3 + 4 + 4.5 + 5)/7 = 3$ inches and $(\overline{S})^2 = 3^2 = 9$ square inches. However, $\overline{A} = (1 + 2.25 + 4 + 9 + 16 + 20.25 + 25)/7 \approx 11.07$ square inches, not 9 square inches. Attempting to find the mean area by squaring the mean length of sides fails, as does trying to find the mean length of sides by taking the square root of the mean area.

When mixing transformations and calculations caution should be exercised. Sometimes, it is not advisable to transform or take the inverse transformation of a computed number.

Reference

Hoffman, M. S. (1992). *The World Almanac and Book of Facts, 1993*. New York, NY: World Almanac.

Students' Perceptions of Myths About Mathematics

by

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One aim of teaching mathematics is to help students understand mathematical concepts and processes. Recent research in mathematics education has shown that knowledge of appropriate facts and procedures alone cannot guarantee a student success with mathematics. Other factors such as problem solving strategies, anxiety, frustration, enjoyment, and *beliefs* about mathematics influence mathematical performance (Garofalo, 1989; McLeod, 1988; Garofalo & Lester, 1985; Schoenfeld, 1985).

Many commonly held unhealthy beliefs about mathematics are based on myths about the subject. These beliefs have to do with (1) the nature of mathematics and mathematical tasks and (2) preconceived notions about oneself and others as doers of mathematics (Schoenfeld, 1987; Garofalo, 1987). Such unhealthy beliefs constitute "math myths." Math myths are believed to influence how and when students study mathematics (Garofalo, 1989). Frank (1990) suggests that the myths can result in false impressions about how mathematics is done. Kogelman and Warren (1978), who have worked with math-anxious and math-avoidant students, claim that belief in math myths can contribute to math anxiety and math avoidance. Such beliefs can impede the reform of mathematics education that is envisioned in the *Curriculum and Evaluation Standards for School Mathematics* (NCTM, 1989).

This article reports the results of a study of the math myths undergraduate students in a two-year college program believe. The goals of the study are: (1) to provide data that confirm the perceptions of math myths and (2) to suggest strategies to overcome these myth

How do undergraduate students perceive mathematics? To address this question, two hundred ninety five (295) undergraduate students in a two year college were surveyed. The college is one of the nine schools/colleges in a suburban private comprehensive university. The sample included freshmen and sophomores, enrolled in core mathematics courses in college algebra (alg. $n = 44$), and precalculus (prec. $n = 176$) to fulfill the math requirement for the Associate Degree in Arts, and elective courses in calculus (calc.I $n = 20$ and calc.II $n = 53$). Over ninety percent of the students who complete their first two years in the college go on to earn their Bachelor's Degree at the university or at other colleges and universities. Participants consisted of 152 males and 142 females. One participant did not indicate his or her sex, and two did not indicate their math courses.

A survey based on the twelve math myths identified by Kogelman and Warren (1978) was administered during the Spring of 1990. Respondents were to either agree or disagree with each statement. In addition, they were directed to write a paragraph describing how one of the myths contributed to their own math anxiety or math avoidance or to an acquaintance's math anxiety or avoidance. Chi-Square tests were used to test for significance of sex and course differences in the proportions of students agreeing/disagreeing with the myths. In addition, a Spearman's correlation was calculated between the myths and course, with course ranked in terms of level. A negative value of r indicates a decreased agreement with a particular myth statement. Table 1 contains a list of twelve math myths identified by Kogelman and Warren (1978).

Table 1: Percents of Students Agreeing With and Writing Paragraphs on Each Myth

		Agree #	Paragraph*
M1	Some people have a "math mind" and some don't	92	48.0
M2	There is a best way to do a math problem	51	6.6
M3	Math is not creative	44	2.5
M4	You must always know how you got the answer	81	4.0
M5	It is always important to get the answer exactly right	56	9.1
M6	It is bad to count on your fingers	23	2.0
M7	Men are better in math than women	11	0.5
M8	Mathematicians do problems quickly in their heads	49	1.0
M9	There is a magic way to doing math	22	2.5
M10	Math requires a good memory	66	11.1
M11	Math is done by working intensely until the problem is solved	77	7.6
M12	Math requires logic, not intuition	71	5.1

n = 295
* 97 missing (no comment)

The percentage of respondents agreeing with each myth is shown, as well as the proportion of students who chose the myth as the subject of their paragraphs. At least one-half of the respondents agreed with the statements of M1, "Some people have a 'math mind' and some don't": M2, "There is a best way to do a math problem"; M4, "You must always know how you got the answer"; M5, "It is always important to get the answer exactly right"; M10, "Math requires a good memory"; M11, "Math is done by working intensely until the problem is solved"; and M12, "Math requires logic, not intuition." At least one-third of the respondents agreed with the statements of M3, "Math is not creative" and M8, "Mathematicians do problems quickly in their heads." Less than one-quarter of the students agreed with M6, "It is bad to count on your fingers"; M7, "Men are better in math than women"; and M9, "There is a magic way to doing mathematics." "Some people have a 'math mind' and some don't" was the most agreed-with myth while "Men are better in math than women" was the least agreed-with myth.

Table 2 presents how the sexes differ in proportions of agreement with selected myth statements.

Table 2: Percent Agreeing, and Significance of Sex Differences for Selected Myths

Myths		Females (n = 142)	Males (n = 152)	p [1]
M1	Some people have a "math mind" and some don't	95.1	88.8	.05*
M3	Math is not creative	48.2	39.7	.14
M4	You must always know how you got the answer	86.6	76.3	.02*
M11	Math is done by working intensely until the problem is solved	80.1	73.5	.13
M12	Math requires logic, not intuition	75.9	65.8	.06

1 χ^2 p value with 1 d.f.

* $p \leq .05$

Sample sizes: M1(135 = f, 135 = m), M3 (68 = f, 60 = m), M4 (123 = f, 116 = m), M11 (115 = f, 111 = m), and M12 (107 = f, 100 = m).

A statistically significant difference is shown for M1, "Some people have a 'math mind' and some don't," and M4, "You must always know how you got the answer," with larger proportions of females agreeing with the statements.

Table 3 presents how the respondents in different levels of math courses differ in proportions of agreement with selected myth statements.

Table 3: Percent Agreeing, and Significance of Course Differences for Selected Myths

Myths		Alg n = (44)	Prec (176)	CalcI (20)	CalcII (53)	p^2	r^3
M3	Math is not creative	56[1]	46[1]	20	36	.03*	−.15*
M5	It is always important to get the answer exactly right	75	59	45	30	.00**	−.27*
M8	Mathematicians do problems quickly in their heads	64	48	50	38	.09	−.14*
M9	There is a magic way to doing mathematics	34	19	25	17	14	−.10
M12	Math requires logic, not intuition	77	70[1]	50	75	13	−.04

Note
1 one student in the sample did not respond to this statement
2 χ^2 p value with 3 d.f
3 Spearman's correlation coefficient
* $p \leq$ 05
** $p \leq$ 01
Total (n = 293)

A statistically significant difference is shown for M3, "Math is not creative" ; M5, "It is always important to get the answer exactly right"; and M8, "Mathematicians do problems quickly in their heads."

The students' paragraphs provide insights into students' beliefs about the myths. Most of the respondents wrote their paragraphs on M1: "Some people have a 'math mind' and some don't." This could have been due to the fact that (1) most respondents agreed with this myth or (2) this myth is the first one listed.

Some examples of the paragraph descriptions follow:

M1: *Some people have a "math mind" and some don't.*

"Personally, math is not my best subject. I get frustrated when I am doing a long problem. I find it hard to keep up my concentration. If I do not get the answer correct, I panic and go crazy! I'm just not a 'math minded' kind of person. To me math isn't creative and I honestly do not enjoy it. Sorry."

M2: *There is a best way to do a math problem.*

"In my grade school, the teacher showed only one way to do problems. She always insisted that we did our problems in the same way. My brother was good in math and taught me other ways of doing the same problems. The teacher was never pleased with my work, whenever I used my brother's method. This gave me some anxiety."

M5: *It is always important to get the answer exactly right.*

"In the grade school, my math teachers always insisted on the right answer. I then had the feeling that every problem has an exact answer and that it is most important to find it. This idea intimidated me."

M7: *Men are better in math than women.*

"My mom had always told me that math is meant for men and not women. She believed that the humanities are for women and she encouraged me to study them very hard. At home, this idea provided me with an excuse for not doing well in math. But in school, I experienced some fears and anxiety whenever the algebra teacher called on me to do some math for the class."

M8: *Mathematicians do problems quickly in their heads.*

"I am very slow when it comes to mathematics. I write and cross out a lot of numbers when I am solving problems. My teachers only considered problems for a few seconds and write down the right statements. When I try to do the same and don't know the answer, I feel stupid and frustrated. "

M10: *Math requires a good memory.*

"A teacher once told us in a class that you must have a good memory to succeed with math. I don't think that I have a good memory because I can't always remember the formulas or procedures involved in the problem. Sometimes I mix formulas around for different problems which is really bad. This made me to panic any time I was to do a problem on the board. What I need to do is to really memorize all the formulas and procedures to do well in math."

Discussion and Classroom Implications

The second goal of this study is to suggest strategies to dispel these myths. The following suggested strategies are based in part on the results of this study and on my experiences in using some of them. These strategies involve: (1) changes in the teaching and learning environment, (2) applications of technology in the classroom, (3) restructuring of teacher preparation programs, and (4) educating the public about mathematics.

Changes in the teaching and learning environment

Results of this study suggest that agreement with some math myths can be reduced with more mathematical preparation of students (see Table 3). We (in the mathematics community) need to encourage and motivate students to take more math courses beyond the required minimum. Course contents should reflect the present and future uses of mathematics in the society as well as provide for further scholarship in the discipline.

Teachers can discover and discuss some math myths at the beginning of a semester. This provides the teacher with information about students' math misconceptions as well as enabling students to see that they are not alone in their math misconceptions. Discussion of math myths provides a teacher with an opportunity to explain to the students that the myths are unhealthy beliefs about mathematics. Here is a student's comment following one of such discussions:

> "I have a long standing insecure feelings toward mathematics which may have started from the grade school. I had always believed that I am the worst student. Math is not made to be fun. [There are] No rewards and this can frustrate me. But following our discussion of the math myths, I am determined to stick it out with you."

Finally, my experiences suggest that employing a variety of teaching strategies in classes could be helpful in dealing with math myths. Some successful strategies that I have found to foster active student involvement include:

1. Making problem solving a focus of mathematics teaching and learning. Students' responses to open ended questions usually bring to light their misconceptions about mathematics.
2. Incorporating activities where students can work cooperatively. This instructional format provides for the challenge and modifications of students' beliefs.
3. Using writing assignments in math classes. Students' explanations in writing can bring their beliefs to a conscious level.
4. Incorporating projects that are applications of skills and concepts into math classes.

5. Integrating history of mathematics into math classes to provide an opportunity for students to learn of the development of and creativity in mathematics.

6. Employing a variety of strategies to assess mathematics learning. The National Research Council (NRC, 1989, p. 70) provides the following suggestions:

   To assess development of a student's mathematical power a teacher needs to use a mixture of means: essays, homework, projects, short answers, quizzes, blackboard work, journal, oral interviews, and group projects. Only broad-based assessment can reflect fairly the important higher-order objectives of mathematics curricula...

Applications of Technology in the Classroom

My experiences indicate that students who believe in math myths are weak in algebraic skills, lack perseverance and understanding of what mathematics is about. Use of calculators and computers can enhance the learning of mathematics for students. The technology helps students realize that math does not involve only the memorization of rules and formulas. Any teacher can practically use these tools to free students from complex computations, and by doing so create an opportunity for them to learn mathematics with understanding, insight, and intuition.

Restructuring Teacher Preparation Programs

Central and critical to the implementation of any recommendations to dispel math myths is the classroom teacher. Students' paragraph descriptions in this study blamed the teacher for being responsible for their misconceptions and anxieties about mathematics. This situation has been brought about (unknowingly) through the teachers' classroom behaviors. Teacher preparation programs should restructure their offerings to take into account the realities about the "type" of students in our educational institutions. Many of these students have numerous misconceptions about mathematics (as evidenced in this study). According to *Curriculum and Evaluation Standards for School Mathematics* (NCTM, 1989, p. 233), "...teachers structure experiences that form the basis of students' beliefs about mathematics. These beliefs exert a powerful influence on students' evaluation of their own ability, on their willingness to engage in mathematical tasks, and on their ultimate mathematical disposition." Teachers are faced with an environment that is different from what they experienced as students in math classes (NCTM, 1991). Teacher preparation programs should prepare teachers to teach courses that are relevant to students' present and future needs. Professional development activities should among other things, make teachers:

1. aware of and understand the existence of math myths,
2. aware of the means (techniques) of identifying students that believe in math myths, as well as the multiple follow-up procedures,
3. explore mathematical applications,
4. develop confidence in their abilities to help their students learn mathematics, and

5. have similar expectations for success and achievement in mathematics for male and female students.

Eradication of Ignorance in the Public

This study indicated that agreement with some math myths was more for females than for males (see Table 2). A female student in this study wrote in the paragraph descriptions "My mom had always told me that math is meant for men and not women...." A great deal of mythology about mathematics pervades this society. Some believe it is "all right" for educated persons to be mathematically illiterate or ignorant. John A. Paulos, author of *Innumeracy: Mathematical Illiteracy and Its Consequences*, believes that poor education and cultural attitudes may be responsible for the nation's mathematical illiteracy (1989). Mathematics is further viewed as a masculine subject (Sherard, 1981). The need exists for the mathematics community to educate the public about the realities of mathematics as distinct from math myths. Programs that are successfully educating women in mathematics should be given more publicity. This can go a long way in dispelling the myth that "men are better in math than women." We should not forget "...that ignorance in parents and teachers begets ignorance in students." (NRC, 1991, p. 12).

In conclusion, math myths exist and we need strategies to dispel them. We may need further studies to explore:

1. the effects of other variables on math myths, and
2. the relationship between belief in math myths and student achievement.

Only by expanding our knowledge of student beliefs about mathematics and how these beliefs affect students' achievement, will we continue to develop and implement appropriate curricular and methodological changes to improve their experiences and achievement in the classroom. These tasks may call for new research questions and methods.

Acknowledgement

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References

Frank, M. (1990). What myths about mathematics are held and conveyed by teachers? *Arithmetic Teacher*, 37, 10-12.

Garofalo, J. (1987). Metacognition and school mathematics. *Arithmetic Teacher*, 9, 22-23.

Garofalo, J. (1989). Beliefs and their influence on mathematical performance. *Mathematics Teacher*, 82, 502-505.

Garofalo, J., & Frank, K. L. (1985). Metacognition, cognitive monitoring, and mathematical performance. *Journal for Research in Mathematics Education*, 16,163-76.

Kogelman, S., & Joseph, W. (1978). *Mind over mathematics.* New York: McGraw-Hill.

McLeod, D. B. (1988). Affective issues in mathematical problem solving: Some theoretical considerations. *Journal for Research in Mathematics Education*, 19,134-41.

National Council of Teachers of Mathematics. (1989). *Curriculum and evaluation standards for school mathematics.* Reston, VA: The Council.

National Council of Teachers of Mathematics. (1991). *Professional standards for teaching mathematics.* Reston, VA: The Council.

National Research Council. (1989). *Everybody counts.* Washington, DC: National Academy Press.

National Research Council. (1991). *Moving beyond myths: Revitalizing undergraduate mathematics.* Washington, DC: National Academy Press.

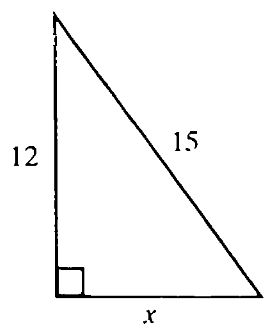
Odafe, V. (1992). Dispelling math myths: Some suggestions. Paper presented at the annual meeting of the Eastern Educational Research Association. Hilton Head, SC.

Schoenfeld, A. H. (1985). *Mathematical problem solving.* Orlando, FL: Academic Press.

Sherard, W. H. (1981). Math anxiety in the classroom. *The Clearing House,* 55, 106 -110.

Lucky Larry #12

The problem said to find the missing side of the triangle. Lucky Larry found a short cut to the Pythagorean Theorem which worked — once.



$$x = (15 - 12)^2$$
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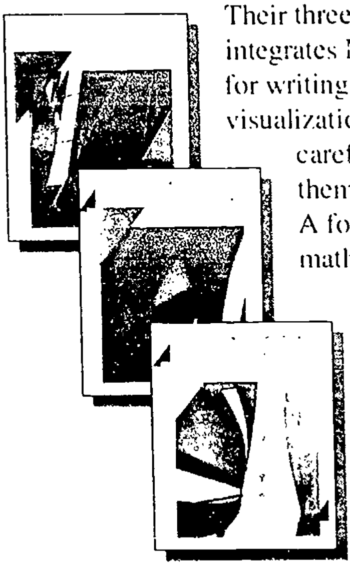

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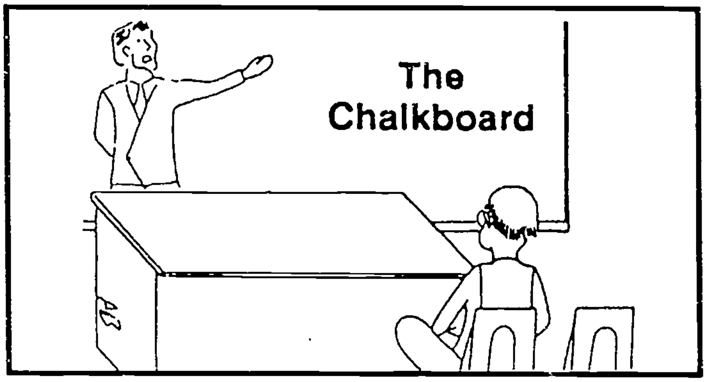

Edited by

Judy Cain and Joseph Browne
Tompkins Cortland Comm. College, Dryden NY 13053
Onondaga Comm. College, Syracuse NY 13215

This column is intended as an idea exchange. We hope to facilitate an open exchange of ideas on classroom management, teaching techniques, tips for helping students get past the usual stumbling blocks, techniques for improving student participation, etc. We know there are lots of good ideas out there, and this is your chance to share them. Please send your contributions to Judy Cain.

Horner's Method in Algebra

Polynomials and their evaluation are discussed in algebra, precalculus, and beginning calculus; usually the method employed is substitution. Horner's method, which arises often in computer science classes, can be illustrated easily in a handout. (NOTE: Horner's method involves rewriting $ax^3 + bx^2 + cx + d$ as $x(x(ax + b) + c) + d$ and then evaluating from the inside out.) Then the students can do the following project(s).

Students can be given several polynomials of various degrees to evaluate by both substitution and Horner's method. Then the students can count the number of multiplications required by each method and try to predict the reduction in multiplications when Horner's method is used on polynomials of other degree.

On his own, one of my algebra students computed the difference in multiplications for polynomials of degree 0 through 15 and then graphed (via a spreadsheet) the difference vs. the degree. The resulting graph is quite interesting.

Submitted by Rosemary Hirschfelder, The University of Puget Sound, Tacoma WA 98416

Involving Every Student

Many times teachers come across students in mathematics classes who either are not participating or are overzealous. The non-participating students may be left out while the overzealous dominate the class activities; either situation demands the attention of the mathematics teacher. I have succeeded in involving non-participating students through the following techniques:

1. I teach some units using small-group instruction. Students are given instructions and practice on how to work cooperatively. The non-participating students seem to open up more easily in a small group than in a large class. Small group instruction has the further advantage of empowering students to take charge of their learning.
2. I require my students to solve problems using interactive technology. The non-participating students are seen eagerly working with calculators or computers. Shyness and feelings of intimidation which sometimes prevent them from participating in class activities disappear when they work with a piece of technology. Furthermore, I have found them to persist more with a problem when they are solving it technologically than when they are using only pencil and paper.
3. I call on non-participating students to answer questions, or to agree (or disagree) with other students' solutions. However, it is important to be tactful and avoid an embarrassing situation by choosing questions that the non-participator can answer correctly. The goal here is to break the ice in a positive manner.
4. I hold a conference with each non-participating student, to talk about anything and just to provide a way to get to know one another informally. At the end, I give him or her a question to which he or she is to prepare the answer for the next class..We agree that I will call on him or her to provide the solution to the class.

On the other hand, I channel the energies of the overzealous into helping other students, especially in a small-group setting. The class benefits from them and they in turn learn more from helping others.

Submitted by Victor U. Odafe, Bowling Green University, Bowling Green OH

Thinking in Large Mathematics Lectures

Most students do not expect to be forced to think in large mathematics lectures. They are used to taking notes as best they can and sorting the ideas and examples out later. Often, due to large workloads and other commitments, this is not done before the next lecture; and in subsequent lectures, students make little or no sense of the material under consideration.

I find that students want to think about the mathematics that they are learning; and they feel more positive about the discipline if they feel that they have been involved in the class. Working in a lecture of 150 or more students, I involve students by using several key words followed by specific instructions to the class. For example, I might present a problem or an idea and say "Put your pen down and think about how you will start." Then I say, "Now every student in every second row, stand up and turn around; discuss your ideas with the person sitting behind you." The first time I do this, there is some confusion; but they soon sort out which row is correct, and from then on, this process takes no time at all. I walk up into the lecture theatre and after a few minutes, I say, "Now please take your seat and write down how you will start the problem." This is an effective way to stop discussion and focus student attention on their own ideas.

I have used this procedure in lectures of all sizes from 50 to 250 students. Students report that they enjoy this technique and that their achievement is enhanced. The fact that they feel confident when they approach difficult problems is a response to the common complaint "It's easy when I watch you do problems in class, but on my own I don't even know where to start."

Submitted by Doug Pitney, University of Western Australia, Nedlands 6009, Western Australia

Lucky Larry #13

Lucky Larry used an amazing variation of canceling to combine fractions as follows:

$$\frac{x}{2(x+2)} - \frac{2}{x(x+2)}$$

$$\frac{x}{2(\cancel{x+2})} - \frac{2}{x(\cancel{x+2})} = \frac{x-2}{2x}$$

This "method" produces right answers whenever all of the constants are the same, e.g.

$$\frac{x}{a(\cancel{x+a})} - \frac{a}{x(\cancel{x+a})} = \frac{x-a}{ax}$$

Submitted by Dona Boccio
Queensborough Community College
Bayside NY 11364

Students need to know you care about them. I give students my home phone number. How many kids do you think call me? Hardly any. But they know they could. Maybe you don't want them calling you. Great! Give them the wrong number. They're not going to call anyway. Even the ones who aren't dyslectic think they are, so they'll just think they copied it down wrong.

Herb Gross

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Software Reviews

Edited by Shao Mah

Title:	MicroCalc 6.1 for IBM-PCs/compatibles
Author:	Prof. Harley Flanders
Distributor:	MathCalcEduc 1449 Covington Drive Ann Arbor MI 48103-5630 Phone: (313) 761-4666
Price:	First 50 site-licensed disks of MicroCalc v. 6.1 cost $750; for v. 5.5 it's $550 for either coprocessor or non-coprocessor version ($650 for both together). There are high-school versions that omit the multivariable calculus material.

MicroCalc is an easy-to-use, interactive symbolic mathematics program for simplifying calculations, graphing, and much more. Faculty and students can use the program for demonstrations, computations,what-if experimentation, a variety of plots, and most of the manipulations found in the first two years of college mathematics (calculus, linear algebra, and more). The fact that it has all these different roles reflects the product's strength and excellence.

MC requires at least a 286 with an EGA or VGA monitor, preferably color. For others, version 5.5 is available as well. Each is available on a site-licensed basis of at least 50 disks.

MC deserves an award for ease of use with its menu-driven format. My review copy consisted of a single 1.44Mb diskette that included both coprocessor and non-coprocessor versions, a demo program, and the manual. Though the lack of a printed manual (the one included is on disk only) had me concerned initially, I'm not now. *Au contraire.* I admire the fact that none is even needed.

The opening screen – in bright red mostly – offers the three choices of beginning, intermediate, and advanced calculus topics, plus the fourth: quit. Navigate either by mouse or arrow keys to select your choice. The first three each have their own menus when you select them. There are also function-key combinations listed on screen to serve as optional shortcuts to typing in square roots, inverse circular/trig functions, and the like.

The topics run the gamut of calculus, though not of algebra. You can create tables of values, graph functions, calculate derivatives explicitly and implicitly, apply such methods as Newton's and bisection, and calculate Riemann sums, integrals, and extrema. Similarly, you can work with polar and parametric (2-D and 3-D) plots, partial derivatives, multiple integrals, graph surfaces, explore series, Taylor polynomials, solids of revolution, conic sections, vector algebra, linear algebra (some of which I confess I have not yet tried), graphical solutions of systems of ordinary differential equations, and more. In fairness, one could make his or her list shorter detailing what MicroCalc won't do rather than what it will. There are probably 60 to 75 modules available; I've named a small fraction. I should add, however, that one can print out graphs and other results.

MicroCalc impressed me by its attention to detail, something I insist on even in my teaching. For instance, I insist that my students label axes. This is undermined by most programs, but MicroCalc even gets this little thing right.

If such details are not important to you, surely MicroCalc's user-friendliness will be. Besides the fact that no manual is really needed for the expected computations and calculations, one thing I particularly noted was how certain defaults are highlighted based on your most likely next choice. Case in point: If you've just taken the first derivative, the highlight is on the second derivative. After taking the second, the highlighted is the third derivative. Just press <ENTER> and that next choice is activated. (Of course, if you prefer something else, just navigate around or press the ESCape key.)

Not all is perfect with MicroCalc. In two of the modules in version 6.0, the prompt to press a certain key caused the top line with the y-axis label to erase. This may have been corrected in v. 6.1. Two other things I've noticed so far – and they are minor – are:

- On my 386, MicroCalc freezes on computing the fifth derivative of the arcsine of the square root of x. The only way out is to re-boot the computer. (Prof. Flanders wrote to tell me that occasionally a large symbolic calculation can overwrite the operating system. He may have fixed this by the time you read this review.)
- The implicit differentiation does not really give answers as functions of x and y, as one expects. Rather, it uses an intermediate calculus technique to describe the answer.

To see whether I was being unreasonable on the first point, I tested Derive's ability to calculate the same fifth derivative on the same 386, and it reported back the result in a few seconds (3.2 seconds according to Derive).

Nevertheless, it is a marvelous testament to a product that this is all I could come up with in the way of criticism so far. And, though I still remain a big booster of Derive, I must say that MicroCalc's sheer ease of using all the features commends it to your attention for departmental consideration. To that add MC's terrific ease and pleasure in graphing; it's at least as good as the graphing in Derive, and on several counts is better.

My reviewing MicroCalc has persuaded me that, at the very least, it is a serious yet fun tool for illustration and for relieving some of the tedium found in our chug-and-plug calculus courses. It may not guarantee that our students excel or that we teach better. That much is probably more up to all of us. But I do commend MicroCalc to everybody's attention as a worthy product with a lot of work and thought behind it. It might just be what you are looking for.

Reviewed by: Michael W. Ecker, Pennsylvania State University, Wilkes-Barre Campus, PA.

Title: Personal Tutor, Algebra I
Author: Phyllis Kretzmann Townswick
Distributor: Stacks 1 and 2 are available from:
Intellimation
P. O. Box 1992
Santa Barbara CA 93116-1922
(800) 443-6633
All five stacks available from:
Phyllis Townswick
541 Shady Wood Way
Madison WI 53714
(608) 221-2719
Computer: Apple Macintosh, with hard drive recommended
Minimum RAM >1 Mb
Minimum System Version 7.0
Hypercard 2.1
Price: $45 for the first stack and $30 for each additional stack, or in lab packs: $180 for the first stack and $120 for each additional stack

Phyllis Townswick's Personal Tutor is a cut above the usual drill and practice tutorial for elementary algebra. By utilizing the capabilities of Hypercard, Professor Townswick has produced a program that provides students with more guidance and more opportunities for interaction. Each stack comes with a workbook in addition to the computer-generated lessons, and students need only operate the mouse in order to navigate through the tutorial.

An innovative screen format features a "chalkboard" or notebook on the right side of the screen and a space for hints, explanations and a directory of options on the left. The student selects a topic and a type of problem from the menus, and the program generates examples of that type. After attempting the problem with pencil and paper, the student can ask for general or specific hints, the complete solution, or a step-by-step explanation that calls for student input at each stage of the solution. As steps of the solution appear on the notebook half of the screen, commentary and instructions appear on the left. The solutions include number lines, graphs, and animated icons offering reminders of correct procedures.

The first stack, "Integers and Rationals," offers operations on signed numbers, including decimals and fractions, and order of operations. Addition and subtraction of integers are illustrated on the number line, and the order of operations problems feature a "funnel" icon that keeps track of the next appropriate operation to perform.

"First Degree Equations" is the best of the five stacks currently available. The student can click on each side of an equation to see the next step in its solution performed; an icon of a scale appears in unbalanced position until the current operation is performed on both sides of the equation.

The "Word Problems" stack provides an excellent tutorial in approaching and modeling word problems. Unlike most programs, whose first "hint" is the equation itself, Personal Tutor gives hints (and answers) on choosing the variable, setting up a chart to organize the information (and fills it in block-by-block if desired), writing an equation in words and then in symbols, and finally solving the equation. This is the best treatment of standard word problems I have seen in tutorial programs.

There are also good stacks on "Graphing Linear Equations" and "Systems of Linear Equations." I especially liked the lessons on slope, which show graphs comparing lines of different slopes for each computational problem, and on graphing lines by the slope-intercept method. The lessons on systems show a "Minigraph" with the solution of each problem. In all of these lessons, the graphs are always present to remind the student of the meaning behind the computations.

There are a few problems with formatting and organization in these early versions; one hopes that infrequent bugs that keep the user interface from being perfectly smooth can be eliminated in the near future. Also, the commentary half of the screen could be improved by deleting some of the repetitive menu instructions directing users to another example, the review lessons, and so forth. Perhaps some sort of distinction between mathematical comment and menu directions (different fonts?) would help. There are some places (but not many) where I would alter the content of a lesson. For example, subtraction of integers is illustrated on the number line, but the computational skill of "adding the opposite" is not shown in the solutions. I would like to see both.

What distinguishes this tutorial from the field is its instructional technique. Considerations of pedagogy are conspicuously absent from most drill and practice packages, and this may be at least partly a consequence of the limitations of the medium. Professor Townswick has made significant advances in this area, and the lessons in Personal Tutor more closely approximate the teaching style that most of us would use ourselves. If you are considering a tutorial package to supplement your elementary algebra classes, then you would do well to consider Personal Tutor. Each stack comes with readable documentation and instructions as well as a workbook whose examples can be accessed through the software.

The author plans to offer additional stacks, including Polynomials and Factoring, by the Spring of 1994.

Reviewed by: Katherine Yoshiwara, Los Angeles Pierce College, 6201 Winnetka Avenue, Woodland Hills, CA 91371.

Send Reviews to: Shao Mah, Editor, Software Reviews
The AMATYC Review, Red Deer College, Red Deer, AB, Canada T4N 5H5

At Bunker Hill we have this paradox. You can be "behind" in a self-paced course. They're so nervous about kids not graduating on time that to get an "in progress" grade they want the kid to have finished at least 60% of the course. Is that really self-paced? It's just a different pace.

Herb Gross

The Problem Section

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Greetings, and welcome to still another Problem Section!

At this time our shortage of problems is not so acute. The down side of this for some participants is that marginal submissions – worthy of being published, but only as room allows – will continue to be delayed as better problems are sent in and move ahead in the queue. I apologize to anybody who has sent in material and not had it published.

However, some of the problems I have on hand are too abstract, too easy, unsolved, or not quite right. So, I still solicit higher-quality proposals.

The AMATYC Review Problem Section seeks lively and interesting problems and their solutions from all areas of mathematics. Particularly favored are teasers, explorations, and challenges of an elementary or intermediate level that have applicability to the lives of two-year college math faculty and their students. We welcome computer-related submissions, but bear in mind that programs should supplement, not supplant, the mathematics.

When submitting material for this department, please note that we have separate editors for problems and for solutions. Send new **problem proposals**, preferably typed or printed neatly with separate items on separate pages, to the Problem Editor. Include a solution, if you have one, and any relevant comments, history, generalizations, special cases, observations, and/or improvements. Enclose a mailing label or self-addressed envelope if you'd like to be assured a reply. All **solutions** to other's proposals, *except Quickies*, should be sent directly to the Solutions Editor.

Dr. Michael W. Ecker (Pennsylvania State University—Wilkes Barre)
Dr. Robert E. Stong (University of Virginia)

New Policy

A couple years ago I received a cranky but legitimate complaint from veteran problemist Prof. Murray Klamkin. He urged me strongly to stop requiring proposers to send copies of solutions of their own proposals to the Solutions Editor. That is, I should forward proposers' own solutions to Dr. Stong so as not to inconvenience them. Effective this issue I belatedly initiate this suggestion as new policy, thus bringing us in line with other problem sections.

Baker's Dozen

This column has a kind of anniversary. If you glance at the problem numbers, note that we are on set Z. Any suggestions on how to number the next baker's dozen of years of semi-annual columns?

Quickies

Quickies are math teasers that typically take just a few minutes to an hour. Solutions usually follow the next issue. All correspondence to this department should go to the Problem Editor, not the Solutions Editor.

Comments on Old Quickies

Quickie #10. Consider two planets with no air resistance and different constant gravitational accelerations g and G. Suppose a projectile is launched straight up with the same constant speed on both planets. Let $s(t)$ and $S(t)$ respectively denote the projectile positions at time t on the two planets. Prove that the maximum heights achieved on the planets are inversely proportional to the gravitational accelerations: $\frac{s_{max}}{S_{max}} = \frac{G}{g}$.

If v_0 is the initial speed then $s(t) = -.5gt^2 + v_0t$ and $S(t) = -.5Gt^2 + v_0t$. By the usual equation for the axis of symmetry of a parabola, the maxima for s and S are achieved at $t = \frac{v_0}{g}$ and $\frac{v_0}{G}$, respectively. The corresponding maximum values for s and S are $s_{max} = s\left(\frac{v_0}{g}\right) = \frac{.5v_0^2}{g}$ and $S_{max} = S\left(\frac{v_0}{g}\right) = \frac{.5v_0^2}{G}$. (This is pretty well-known, but I prefer not to rely on excessive formulas.) The claim now follows immediately.

Quickie #11. Adjoin the origin to the graph of the relation $\frac{x}{y} + \frac{y}{x} = 4$. Prove that this full graph may be regarded as the union of two functions that intersect at the origin at a 60° angle.

Rather than use implicit differentiation, put $r = \frac{y}{x}$ and multiply by r to obtain a quadratic in r. Use the quadratic formula and replace r again to obtain $y = \left(2 \pm \sqrt{3}\right)x$. Use trigonometry or form two vectors and use the formula for the cosine of the included angle to verify the result now.

New Quickies

Quickie #12: Proposed by J. Sriskandarajah, University of Wisconsin, Richland.

If the sides of a quadrilateral are 3, 13, 12, and 4 units, find its area.

<u>Comment</u>: I believe the intent is that these be the side lengths listed in a cyclic order. In any case I ask: Is the quadrilateral unique (up to congruence)?

Quickie #13: Proposed by the Problem Editor.

Suppose a positive integer n is chosen in such a way that the probability of choosing $n + 1$ is $\frac{9}{10}$ that of choosing n (for all n). What is the probability of choosing 2? What is the mathematically expected (average) choice?

Quickie #14: Proposed by Frank Flanigan, San Jose State University.

Investigate the limit of $x^{\frac{1}{\ln x}}$ as x increases without bound.

Quickie #15: Proposed by Michael H. Andreoli, Miami Dade Community College.

Find a divergent alternating series whose n^{th} term nevertheless approaches 0.

New Problems

Set Z Problems are due for ordinary consideration October 1, 1994. Our Solutions Editor requests that you please not wait until the last minute if you wish to be listed or considered. Of course, regardless of deadline, no problem is ever closed permanently, and new insights to old problems – even Quickies – are always welcome.

Problem Z-1. Proposed by the Problem Editor, Pennsylvania State University, Wilkes-Barre Campus.

Characterize all invertible, odd, fifth-degree polynomials. That is, determine necessary and sufficient conditions on the real numbers a and b to make $p(x) = x^5 + ax^3 + b$ invertible. (Non-monic versions of these polynomials are just scalar multiples of the monic $p(x)$ shown.)

Comment: This is not hard but not quite as easy as X-2 was, solved this issue.

Problem Z-2. Proposed by Leonard Palmer, Southeast Missouri State University, Cape Girardeau, MO.

If you look at a table of primes you will be able to pick out pairs of primes that differ by 10. Let's call these ten-primes.

Find all triples of consecutive ten-primes $(p,\ p+10,\ p+20)$.

Problem Z-3. Passed on by Charles Ashbacher, proposed by Ruiqing Sun, Beijing Normal University, Beijing, People's Republic of China, and translated by Leonardo Lim, Mount Mercy College, Cedar Rapids, IA.

Given a, b, x, and y (x and y distinct) such that:

1) $ax + by = 3$
2) $ax^2 + by^2 = 7$
3) $ax^3 + by^3 = 16$
4) $ax^4 + by^4 = 42$

Determine the value of $ax^5 + by^5$.

Problem Editor's Comment: What if $x = y$?

Problem Z-4. Proposed by Charles Ashbacher, Geographic Decision Systems, Cedar Rapids, IA.

(The fuller version of the following problem was proposed by Paul Erdos and appeared as problem 294 of the *Canadian Mathematical Bulletin*. It reappears listed as an unsolved problem in Stanley Rabinowitz's *Index to Mathematical Problems 1980-84*. Just the half we know to be solvable is included here.)

Let phi(n) be the Euler phi function. Prove that there exist infinitely many natural numbers n such that phi(n) < phi(n – phi(n)).

Problem Z-5. Proposed by the Solution Editor, University of Virginia.

Suppose that you make a purchase at the store for which the price is some number of dollars and k cents. In your pocket or purse you have p ($p \geq k$) pennies and n other coins (not pennies). Desiring to get rid of as many pennies as possible, you randomly draw coins until you get the needed k pennies. How many coins do you expect to have to draw?

Set X Solutions

To Diverge Or Not To Diverge

Problem X-1. Proposed by Leonard M. Wapner, El Camino Community College, Torrance, CA.

Does the sequence $\sin^1 1, \sin^2 2, \ldots, \sin^n n, \ldots$ converge?

Solutions by Robert Bernstein, Mohawk Valley Community College, Utica, NY; Stephen Plett, Fullerton College, Fullerton, CA; and Larry Thomas, University of Virginia, Charlottesville, VA. Charles Ashbacher, Geographic Decision Systems, Cedar Rapids, IA provided extensive UBASIC calculations.

(Following Stephen Plett) Hurwitz's theorem says that for any irrational number ξ there are infinitely many rational numbers $\frac{n}{d}$ with $\left|\xi - \frac{n}{d}\right| < \frac{1}{\sqrt{5}d^2}$. Thus $|d\pi - n| < \frac{1}{\sqrt{5}d}$ for infinitely many n. For these n we have

$$|\sin n| = |\sin(d\pi - n)| < |d\pi - n|$$
$$< \frac{1}{\sqrt{5}d} = \frac{2}{2\sqrt{5}d}, \text{ by Hurwitz}$$
$$< \frac{2}{n}$$

since $2\sqrt{5}d > \left[\pi - \frac{1}{\sqrt{5}d^2}\right]d > n$. Then $|\sin n|^n < \left(\frac{2}{n}\right)^n$ for infinitely many n gives a subsequence with limit zero.

Now take $\xi = \frac{\pi}{2}$ and there are infinitely many natural numbers n for which $\left|\frac{d\pi}{2} - n\right| < \frac{1}{\sqrt{5}d}$ with d odd. (Stephen notes that the extension to d odd is fairly routine.) Then

$$|\sin n| = \left|\sin\left(\frac{d\pi}{2} - \left[\frac{d\pi}{2} - n\right]\right)\right|$$

$$= \left|\sin\frac{d\pi}{2}\cos\left(\frac{d\pi}{2} - n\right) - \cos\frac{d\pi}{2}\sin\left(\frac{d\pi}{2} - n\right)\right|$$

$$= \left|(\pm 1)\left(1 - \frac{\left[\frac{d\pi}{2} - n\right]^2}{2!} + \frac{\left[\frac{d\pi}{2} - n\right]^4}{4!} - \dots\right)\right|.$$

$$> \left|1 - \frac{1}{10d^2}\right|.$$

Raising to the nth power

$$|\sin n|^n > \left|1 - \frac{1}{10d^2}\right|^n > 1 - \frac{n}{d}\left[\frac{1}{10d}\right]$$

$$> 1 - \frac{5}{3}\left[\frac{1}{5n}\right], \text{ since } \frac{n}{d} \approx \frac{\pi}{2} < \frac{5}{3} \text{ and}$$

$$10d \approx \frac{20}{\pi} n > 5n$$

$$> 1 - \frac{1}{3n}$$

Since this holds for infinitely many n, there is a subsequence converging to 1.

Having subsequences with different limits, the given sequence cannot converge.

Cibuc

Problem X-2. Proposed by the Problem Editor, Penn State University, Lehman, PA.

Consider polynomials of the form $p(x) = ax^3 + bx^2 + cx + d$, where a, b, c, d are arbitrary real constants. In terms of these constants, characterize all such polynomials p that are invertible as functions.

Solutions by Charles Ashbacher, Geographical Decision Systems, Cedar Rapids, IA; Robert Bernstein, Mohawk Valley Community College, Utica, NY; Kenneth G. Boback, Penn State University, Lehman, PA; Joseph E. Chance, University of Texas – Pan American, Edinburg, TX; Mark de Saint-Rat, Miami University, Hamilton, OH; Steve Kahn, Anne Arundel Community College, Arnold, MD; Brandin Meyers (student), Onondaga Community College, Syracuse, NY; Stephen Plett, Fullerton College, Fullerton, CA; Grant Stallard, Manatee Community College, Bradenton, FL; Terry Zeanah, Jefferson State Community College, Birmingham, AL; and the proposer.

To be invertible, p must be monotone and the derivative $p'(x) = 3ax^2 + 2bx + c$ cannot change sign. If a is nonzero, $p'(x)$ is quadratic and can have at most one zero. Thus the discriminant, $4(b^2 - 3ac) \leq 0$ or $3ac \geq b^2$. If $a = 0$, b must be zero to avoid sign changes and c must be nonzero, else $p(x)$ is a constant.

Snuggle Up A Little Closer

Problem X-3. Proposed by Jim Africh, College of DuPage, Glen Ellyn, IL.

If $a_1, a_2, \ldots, a_n$ are n real numbers that sum to 1, show that the sum of their squares is at least $\frac{1}{n}$.

Solutions by Shiv Kumar Aggarwal, Embry-Riddle Aeronautical University, Daytona Beach, FL; Robert Bernstein, Mohawk Valley Community College, Utica, NY; Joseph E. Chance, University of Texas – Pan American, Edinburg, TX; Peter Collinge, Monroe Community College, Rochester, NY; Mike Dellens, Austin Community College, Austin, TX; Mark de Saint-Rat, Miami University, Hamilton, OH; Stephen Plett, Fullerton College, Fullerton, CA; Terry Zeanah, Jefferson State Community College, Birmingham, AL; and the proposer.

The point on the plane $x_1 + \ldots + x_n = 1$ closest to the origin is the foot of the perpendicular drawn from the point to the plane. Starting at the origin one moves in the direction of the normal, (1, 1, ..., 1), until hitting the plane at the point $\left(\frac{1}{n}, \ldots, \frac{1}{n}\right)$ which is the closest point. The square of the distance to the origin is then $n\left(\frac{1}{n}\right)^2 = \frac{1}{n}$, which is the minimum.

Problem X-4. Proposed by J. Sriskandarajah, University of Wisconsin, Richland Center, WI, and modified slightly by the Problem Editor.

Find the sum of the finite Arithmetico-Geometric Series

$$ag + (a + d)gr + (a+2d)gr^2 + \ldots + (a + (n-1)\,d)gr^{n-1}.$$

For which values of r is the corresponding infinite series convergent? For such r, derive the sum of the infinite series. Hence or otherwise evaluate

$$-4 - 2 + 3 - \frac{5}{2} + \frac{7}{4} - \frac{9}{8} + \frac{11}{16} - \ldots \text{ (arithmetico-geometric series).}$$

Solutions by Shiv Kumar Aggarwal, Embry-Riddle Aeronautical University, Daytona Beach, FL; Charles Ashbacher, Geographical Decision Systems, Cedar Rapids, IA; Robert Bernstein, Mohawk Valley Community College, Utica, NY; Joseph E. Chance, University of Texas – Pan American, Edinburg, TX; Mark de Saint-Rat, Miami University, Hamilton, OH; Stephen Plett, Fullerton College, Fullerton, CA; Grant Stallard, Manatee Community College, Bracenton, FL; and the proposer.

The sum of the geometric series $1 + r + \ldots + r^{n-1}$ is well known to be $\dfrac{1 - r^n}{1 - r}$

for r not equal to 1. Applying the operator $r\left(\dfrac{d}{dr}\right)$ shows that

$$r + 2r^2 + \ldots + (n-1)r^{n-1}$$

has sum $\dfrac{r - nr^n + (n-1)r^{n+1}}{(1-r)^2}$. Thus the given series has sum

$$\frac{ag(1 - r^n)}{1 - r} + \frac{dgr\,(1 - nr^{n-1} + (n-1)\,r^n)}{(1-r)^2}$$

for r not equal to 1. The infinite series converges for $|r| < 1$, with sum

$$\frac{ag}{1-r} + \frac{dgr}{(1-r)^2},$$

and taking $g = 4$, $a = -1$, d = 2, $r = -\dfrac{1}{2}$, the example has sum $-\dfrac{40}{9}$.

Factor Factory

Problem X-5. Proposed by the Solution Editor, University of Virginia.

Find the factorizations of the polynomial $x^4 - 6x^2 + 1$ into irreducible factors in the ring $Z\left[2\sqrt{2}\right]\left[x\right]$. Hint: There are three such factorizations.

Solved by Robert Bernstein, Mohawk Valley Community College, Utica, NY; Stephen Plett, Fullerton College, Fullerton, CA; and the proposer.

The given polynomial is

$$(x^2 - 3)^2 - 8 = \left(x^2 - 3 + 2\sqrt{2}\right)\left(x^2 - 3 - 2\sqrt{2}\right),$$

which has zeros the square roots of $3 + 2\sqrt{2}$ and $3 - 2\sqrt{2}$. These zeros do not lie in the given ring and hence the irreducible factors are all quadratic and arise by pairings of the linear factors, which can be done in three ways. The other two factorizations are

$$(x^2 + 1)^2 - 8x = \left(x^2 + 2\sqrt{2}x + 1\right)\left(x^2 - 2\sqrt{2}x + 1\right)$$

and

$$(x^2 - 1)^2 - 4x = (x^2 + 2x - 1)\,(x^2 - 2x - 1).$$

Binomial Identity

Problem X-6. Proposed by Stanley Rabinowitz, MathPro Press, Westford, MA.

Let n and k be fixed integers with $0 \le k \le n$ and let $a_i = C(n+i, k)$. (This is a binomial coefficient or number of ways of choosing k things from $n + i$.) Find a formula for a_4 in terms of a_1, a_2, and a_3.

Solved by Robert Bernstein, Mohawk Valley Community College, Utica, NY; Stephen Plett, Fullerton College, Fullerton, CA; and the proposer.

One has $C(n+4,k) = (n+4)\dfrac{C(n+3,k)}{n+4-k}$. Letting $a = \dfrac{n+3}{n+3-k} = \dfrac{C(n+3,k)}{C(n+2,k)}$

and $b = \dfrac{n+2}{n+2-k} = \dfrac{C(n+2,k)}{C(n+1,k)}$, one has $k = \dfrac{(n+3)(a-1)}{a} = \dfrac{(n+2)(b-1)}{b}$

and the last equation gives

$$n\left[\frac{a-1}{a} - \frac{b-1}{b}\right] = \frac{2(b-1)}{b} - \frac{3(a-1)}{a}.$$

Then

$$n = \frac{3b - 2a - ab}{a-b} \text{ and } k = \frac{(a-1)(b-1)}{(b-a)}$$

and substituting in the formula for $C(n+4, k)$ gives the desired form.

> There's a man ten thousand feet up on a mountain. He slips, and, just as he's about to fall to his death, he grabs on to a slender shrub and hangs there, precariously. In desperation, he looks up into the heavens and shouts, "Is there anybody up there?" A voice booms down, "I'm up here!" The man says, "Who are you?" and the voice says, "I'm God." He says, "Can you help me?" and the voice says: "Of course I can help you. That's why they call me God. But first, you have to show me a token of your faith. Let go of that shrub." The man said, "Is there anybody else up there?" We [the community colleges] have been that slender shrub for millions, their only access to higher education.
>
> Herb Gross

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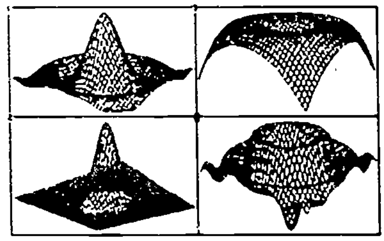

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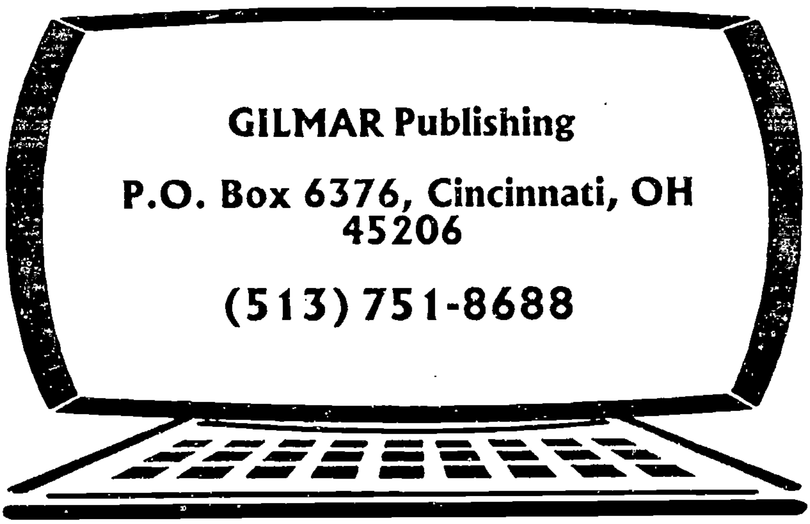

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