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ABSTRACT

Members of the National Center for Research in Mathematical Sciences Education Working Group on the Teaching/Learning of Quantities began research in 1991 to identify the teacher characteristics associated with individuals considered by their peers as leaders or early adopters--whose mathematics teaching already exemplifies the spirit of the (NCTM) National Council of Teachers of Mathematics Standards. This article describes findings from a preliminary study which sought information on middle school mathematics teachers' conceptions of teaching, their pedagogical and content knowledge, and their teaching behaviors. Data were obtained on teacher characteristics by conducting extensive interviews, an assessment of content knowledge, and observations during a series of 3-hour seminars over a 3-month period. A framework containing five categories of conceptions of mathematics teaching was used. The five categories are: what mathematics is, what it means to learn mathematics, what it means to teach mathematics, what the roles of the teacher and the students should be, and what constitutes evidence of student knowledge and criteria for judging correctness, accuracy, or acceptability of mathematical results and conclusions. Findings revealed that these teachers: focused on problem solving, conceptual relationships and understanding, and communication in mathematics; had a comprehensive knowledge of the mathematics they were teaching; and participated in their own professional growth by attending conferences and inservice programs, completing graduate studies, and continuing to seek encouragement and support for their reform efforts. (MKR)

Reflective Practitioners Reform School Mathematics

by Randolph Philipp, Alfinio Flores, and Judith Sowder

For more than a decade, the National Council of Teachers of Mathematics (NCTM) has guided the reform of school mathematics. Early in 1989 it appointed a commission that produced a set of standards for teaching mathematics. The standards, said the commission, rest on two assumptions (NCTM, 1991, p. 2):

- Teachers are key figures in changing the ways in which mathematics is taught and learned in schools.
- Such changes require that teachers have long-term support and adequate resources.

The assumption that teachers are linchpins in the fundamental reform of school mathematics is widespread. Little discussion, however, has focused on the characteristics of teachers who use classroom approaches that embody the vision originally set out by the NCTM *Curriculum and Evaluation Standards for School Mathematics* (1989).

Members of the NCRMSE Working Group on the Teaching/Learning of Quantities began research in 1991 to identify the teacher characteristics associated with individuals considered by their peers as leaders or early adopters—whose mathematics teaching already exemplifies the spirit of the NCTM *Standards*. This article reports on preliminary work with middle school mathematics teachers. During the first year, researchers obtained data on teacher characteristics by conducting extensive interviews, an assessment of content knowledge, and observations during a series of three-hour seminars over a three-month period. They sought information on teachers' conceptions of teaching, their pedagogical and content knowledge, and their teaching behaviors.

While the study set out to identify the characteristics of teachers associated with reform-oriented teaching, early findings also support the assumption that teachers require long-term support and adequate resources. Teacher educators and teacher leaders should find these experiences replete with ideas for designing programs that would support the efforts of teachers ready to change their teaching behavior.

Teachers selected for the study had often been asked to participate in local and state-level projects for mathematics teachers in curriculum and assessment, research, leadership, and professional development areas. They were active in professional development programs and had established reputations as reform-oriented teachers within the local mathematics education community. They described documents such as the *Mathematics Framework for California Public Schools* (California Department of Education, 1985) and the NCTM *Standards* (NCTM, 1989) as having a profound and liberating effect on them and an incalculable influence on their teaching. For these teachers, the publications seemed to validate their teaching practices.

Conceptions of Mathematics Teaching

In designing their study, the researchers sought a framework for their examination of teachers' conceptions of teaching. They used a recently proposed framework consisting of five categories, each with a developmental perspective (Thompson, 1991). The five categories of conceptions include: (a) what mathematics is; (b) what it means to learn mathematics; (c) what it means to teach mathematics; (d) what the roles of the teacher and the students should be; and (e) what constitutes evidence of student knowledge and criteria for judging correctness, accuracy, or acceptability of mathematical results and conclusions. Data on teachers' conceptions of mathematics teaching were analyzed according to the five categories.

What Mathematics Is

While teachers in this study were not asked specifically what mathematics is, researchers used their answers to indirect questions to obtain evidence of their thinking. Among the indirect questions were those on the kind of mathematics teachers believed should be taught in schools. Rules were not the focus of the mathematics taught by these teachers; they were adamant that school mathematics should not focus on the learning of algorithmic skills. One of their "toughest" jobs, according to these teachers, was structuring their classes so their students learned to discover solutions on their own and to develop their own rules. Many students, they said, thought mathematics involved learning what rule to apply.

Problem solving was considered an extremely important aspect of mathematics by all of the teachers. At times they seemed to view problem solving as one of many topics in the mathematics curriculum, and at other times they considered it an approach they used to teach mathematics. One teacher commented: "I think problem solving kind of drives what's going to happen. I try to keep my teaching involved around some kind of problem situation."

Another teacher spoke of mathematics as a language that was connected to the real world:

> To me, to learn mathematics means to get to a point when you understand that there is no such thing as learning anything in isolation. I tell my kids that mathematics is like a foreign language, and that it's just sort of a language that explains the way things work in the world. . .to learn mathematics to me is to understand the applications, the usefulness of that language in describing the world around us.

What it Means to Learn Mathematics

Teachers believed that students learn mathematics by being immersed in doing mathematics. They said students learn a great deal of mathematics outside of school that is not recognized in classrooms, "I think the kids have a really pretty good understanding sometimes of quantities, but that doesn't get explored; it doesn't get emphasized at all; it gets almost quashed in the race to learn how to add four-digit numbers. . . ."

Teachers encouraged students to explore and make conjectures and they had them discuss and evaluate the solutions put forward by students during discussions. When teachers prepared activities for their classes they used problems suggested by newspapers or happenings in their schools or they approached traditional topics in unusual ways. One teacher was amazed that the text devoted only three pages to perimeter and area; this, he thought, indicated students were expected to learn these difficult concepts by looking at examples of regular figures or learning formulas. Yet in his experience:

> When students are learning mathematics. . .what I see is the students who are willing to explore and make mistakes without getting all flustered, students who are willing to not depend on formulas or numbers or things like that in order to solve a problem, students that are willing to talk to each other and with each other about what might work and what might not work.

What it Means to Teach Mathematics

The teachers in this study felt that the development of an understanding of conceptual relationships was an essential focus of their teaching. They were dissatisfied when they, as learners, did not understand the deeper conceptual relationships. Their comments are represented by the following:

> My primary focus is understanding. I want those kids to know not just how to do it, but why it works. And sometimes they get tired of that and they want the shortcut, but I really want them to know why it works. I think if you understand why, then you're not going to forget. And you're going to have a background to pursue further.

Teachers felt constrained by curriculum guidelines and textbooks. While their conceptions of appropriate content were tied to their district's curriculum guidelines, some interpreted the guidelines liberally. Either they rarely used textbooks or they used textbooks to cover certain topics at certain points in the year. Most selected other materials and ideas to enrich texts. The goal, according to one teacher, was a "concept-oriented program where I'm putting together units of concepts rather than just chapters in a book and still covering concepts that I'm supposed to be covering, but hopefully in a more meaningful way for the kids."

Testing programs also affected, to some extent, what teachers taught. Only one of the teachers did not let the standardized tests requirements influence her curriculum choices, "I'm doing some other potentially more meaningful things with these children, and maybe we just don't know yet how to measure these things. . . . And maybe the impact of what I do won't even be known for ten years." Another teacher expressed the conflict he felt in the effort to "feed into what we are trying to do, the direction we're going [in the reform movement]," yet knowing students "are going to be assessed in some kind of standardized, multiple-choice thing at the end of the year."

Teachers who are reflective practitioners, according to this study, may be more likely to move away from the traditional model of mathematics teaching.

Dealing with diversity and individual differences were concerns of all teachers. They did not accelerate their brighter students. The schools where some taught had made conscious attempts to avoid tracking strategies and to mainstream all students. One teacher believed that accommodating differences—cultural, learning, and behavioral—was his biggest challenge, "I kind of keep my expectations at a level where kids are going to be successful if they just do what I ask them to do. I challenge them, but I don't make them frustrated." Another focused on providing students with multiple entry levels:

> And, so what I try to really concentrate a lot on is: Am I presenting this information in ways that allow multiple entry levels and allow

accessibility by kids of all abilities? And am I trying to teach in a variety of ways? And am I aware? How do I know? What do I do, and how do I know if the kids are not getting it?. . . .I try to be very, very conscious of the reality that all of these people in the room have an incredible variety of prior knowledge and experience, and they're all coming into this learning that I think I want to have happen at exactly the same time with this diversity.

Teachers working in private schools with students from upper socio- economic levels also accommodated student differences. To do so, they avoided closed and narrow mathematics lessons.

The Roles of Mathematics Teachers and Students

The roles of these teachers were affected by their beliefs. To them, the goal of mathematics teaching ought to be the development of children's conceptual understanding of mathematics. They believe that children develop mathematical understanding when teachers provide them with experiences that allow them to do mathematics and ask them probing questions about their work.

Most of these teachers see themselves as facilitators who lead children as they "figure things out on their own." As one put it, a teacher "can either be—and this is a very old cliche—a sage on the stage or a guide on the side, and when the teacher is a sage on the stage, that's a teacher who's going to have students who depend on the teacher for answers." Students of the sage, he said, grow to depend on the teacher for answers and hints when stuck and students of the guide grow to see that mathematics problems exist all around them and that there are other sources of mathematical authority than the teacher. He offered the advice, "What the teacher needs to do—and this is really important—is to back off a little bit."

Students' roles can be inferred from these teachers' views of their roles. One put it more explicitly, "I don't have to then tell them that a centimeter is . . . , I mean 2.54 centimeters makes an inch, because they'll just sort of pick that up. I guess I trust more kids' abilities to be inductive about things than what I was ever taught how to do."

What Constitutes Evidence of Student Knowledge

Student responses to verbal questions were the primary means used by these teachers to obtain evidence of student knowledge. An atmosphere of acceptance, they said, is a necessary component of their questioning strategy. This atmosphere also permits students to be honest when they "don't get it." Said one teacher, "I try to have an atmosphere in my classroom of it being wonderful if you're able to articulate what you don't know. Students know that 'why' is going to be the next thing I ask because I really want them to understand" said one: "Can you explain why this works? Why did we do this? How does this happen? How would you do this and why?"

Observations of students provided these teachers with crucial information about students. In their words, they watched their students' work habits when given problems, their use of manipulatives to model mathematical processes, their selection and use of calculators, and their approaches to problems.

Journal writing was also used by teachers as a means of assessment. Students were asked to keep mathematics journals. The quality of the explanations in their journals provided teachers with additional information.

Although the teachers in this study seemed to attend to the thinking processes of their students, descriptions of their use of assessment information seldom mentioned individual students. Mathematics educators stress using knowledge of students' understanding to make curricular decisions (Carpenter, Fennema, Peterson, & Carey, 1988). Group members appeared to use the information gained through assessment activities to guide their classes as a whole; as one teacher put it, "There's a lot of constant monitoring of where they're at and adjusting what I have to do—kind of based on the way they react and based on the kinds of work I see them do."

Knowledge of Middle School Mathematics

Middle school teachers in this study had a comprehensive knowledge of the mathematics they were teaching and a good understanding of the relevant concepts and procedures. The study had acquired evidence of the mathematical knowledge of these teachers using a written test with a subsequent discussion of responses, interviews, and discussions of problems during research seminars. The written test covered concepts related to rational numbers, fractions, percents, ratios, and rates. Teachers demonstrated a thorough knowledge of the school mathematics appropriate for the grades they were teaching. They had constructed their knowledge in ways that enhanced making connections and using alternate representations and meanings.

Mathematical knowledge, for these teachers, was interwoven with pedagogical knowledge. They knew of numerous ways to teach concepts but made a decision to

teach a concept or topic in a certain way because they thought it the best approach for their students. Those concepts and tools that were accessible to their students were of greater interest to them than concepts that required other more advanced mathematical tools. As an example, when presented with the problem,

> A 6th-grade student says that ¾ + ½ is ⅝ and justifies her reasoning as follows: "If I made 3 out of 8 free throws in the morning, and 5 out of 12 free throws in the afternoon, then altogether I made 8 out of 20 free throws." How would you respond to that student?,

the teachers had mathematical knowledge that would enable them to explain the problem using algebraic notation and variables, and the concept of weighted averages. They thought such an approach was pedagogically inappropriate for 6th -grade students and sought to develop approaches that would be suitable for students at this level.

A Response to Readers

Readers of the *NCRMSE Research Review* have asked about the rationale behind the choices of the Center's title, The National Center for Research in Mathematical Sciences Education (NCRMSE), and of the Center's logo. According to NCRMSE Director Thomas A. Romberg, they were selected for the following reasons:

The word "national" was used in the title to represent two realities: the research program of the Center focuses on the national reform effort in school mathematics, and its work is carried out by a network that includes approximately 200 of the nation's mathematics educators. The term "mathematical sciences education" reflects the notion shared by these scholars that the school curriculum should include topics such as statistics and discrete mathematics. It also reflects the fact that when NCRMSE was created in 1987, it had a formal relationship with the Mathematical Sciences Education Board of the National Academy of Sciences.

The NCRMSE logo consists of a red torus surrounded by eight linked black orbits. The torus portion of the design represents the reform effort in school mathematics. The black orbits represent the linkages among the Center's seven Working Groups and its management staff. The combination of the torus and its orbits conveys the mission of the Center, which is to provide a research base for the reform movement in school mathematics. (The logo appears on the *NCRMSE Research Review's* first page, in the top left corner.)

Teaching Behaviors

Teaching requires planning and online decision making. These complex cognitive skills are applied in the relatively poorly structured yet dynamic environment of classrooms (Leinhardt & Greeno, 1986). Teachers' descriptions of their planning activities and of a typical day in their classrooms were used to obtain data on teaching behaviors.

Long-term planning, undertaken by all of the teachers, was related to their perceptions of their obligation to follow curriculum guides or texts. A majority began with a top-down approach, considering the "big picture" and breaking it down into units and then into individual lessons. One of the teachers approached planning by identifying the "big ideas" and finding a theme into which she could fit the big mathematical ideas. One teacher mentioned using both the NCTM *Standards* and the *California Framework* as well as her experience as a teacher of 3rd and 4th graders to develop broad units of study for her 5th-grade class. These units were then divided into weeks of study. Only one teacher used a text-oriented approach while trying to develop a concept-oriented rather than a chapter-oriented focus.

Planning involved sequencing that related to students' understanding of previously learned material. According to one of the teachers, "What is important is that the teacher sees how whatever specifically they are teaching, how it fits into what came before it and what comes after it, and how the child is progressing."

Short-term or day-to-day planning, according to these teachers, often occurred shortly before class or "right on the spot." One teacher described daily planning as "thinking stuff—stuff that I think at night and I think in the morning on my way to school." She wrote out lesson plans but she described the process as "more of a brainstorming thing for me: I'll just write a whole bunch of activities that I've seen, or remember, or think I could do." While they create daily lesson plans, these teachers do not feel an obligation to follow them. A basic outline may be erased, "based on what the reaction to a lesson is on one day. Nothing is set in concrete, more like jello."

Resources are essential to the planning processes used by the teachers. They are professionals and have gained broad knowledge of their field from reading, gathering materials, and taking advantage of professional development oppor-

tunities. One described the materials she had accumulated:

> I have been known for a long time as having an incredible resource library. It's too bad I didn't buy stock in Dale Seymour and Creative Publications. And I read all the time, like when the *Arithmetic Teacher* or the *Mathematics Teacher* comes, I read it that day. . . . So if you told me to put together some stuff on volume, I could have a year's worth.

A second teacher described how she used her resources:

> At first I just sat down and off the top of my head thought about, "What do I want them to learn when it comes to geometry? "—and I just wrote down some ideas. Then I start piling all sorts of resources together. I read through the section on geometry in the *Standards;* I read through a lot of curriculum guides on geometry, and through textbooks, to see what they cover. I look at different resource materials, for example, from Marilyn Burns. And I think of ideas for activities. So I have a broad plan throughout the unit of the concepts I want covered, possible activities, and then I sort of plot them in. I throw all my files and all my resource materials on geometry together in a box, and I keep that and look through it for lessons and ideas and take out what I need.

A Typical Day

Teachers in this study described their early teaching as resembling descriptions found in the literature (Romberg & Carpenter, 1986). In the words of one of them:

> The first several years of teaching I really was into "This is the section of the book that we're doing today, and here's the practice problems, and now we'll go over homework, and then I'll teach you how to do it, then you'll practice, and then you'll have some to try before you go home," and that kind of thing.

The teachers teach very differently now, when compared to the model described above. Activities and journals were used by all members of the group. A description of their previous day's class included journal writing toward the end of the class, and it was considered a critical part of their mathematics lesson. One of them commented:

> I have [my students] write for a certain length of time and write everything they can think of. Lots of times, if I'm walking around and see some interesting things in their writing, then we'll discuss those as a way to develop the idea we're working on or reviewing from the previous day.

Activities making up lessons are typified by an extensive use of manipulatives. The teachers choose manipulatives carefully, depending on how they would incorporate them into the learning experience. They were familiar with all commonly used manipulatives and indicate they had good classroom supplies, but they sometimes had to order specific types of manipulatives for their schools.

While whole-class instruction is used by all of the teachers, only half of them used cooperative groups as a regular part of the lessons they described. The other half did not think whole-class instruction was optimal and spoke of using more group work in the future. In the words of one, "I would say that most of my teaching is to the class as a whole, which is kind of dangerous. I have been assessing whether that's the best way to do things."

Typical lessons also include problem-solving discussions that often are linked to activities or reports. Going over homework assignments is not perceived by these teachers as an important component of a typical class. They tended to begin a class with some problem or situation related to a previous lesson in order to assess student understanding and then decide whether to give a previous topic more attention before proceeding with the lesson they had planned for the day.

Asked about the major barriers they face when teaching mathematics the way they believe it should be taught, these teachers listed a lack of time and materials, pressure from parents and administrators to make their mathematics instruction conform to traditional methodologies, and the weak mathematical backgrounds of their students. Said one who took a broader perspective, "School is probably the biggest barrier to teaching mathematics. . . . School is the most bizarre place. There is nothing else like it in the whole wide world." Her comments followed a discussion on the ways in which schools partition time, fragmenting the school day.

Descriptions of long-term and day-to-day planning and of typical classes provided by these teachers are reminiscent of the descriptions of preparation and improvisation provided by Yinger (1990, p. 88), "Preparation expects diversity, surprise, the random, and the wild. To prepare is to get ready, to become equipped, and to become receptive." By collecting many ideas and activities they can draw upon, these teachers seem to feel that they are ready to face the diversity and surprises of the day-to-day lessons that are then, to some degree, improvised. Improvisation, according to Yinger, is a highly responsive act that calls for skill and sensitivity to the

moment and place. It is structured by actions and dependent on knowledge, beliefs, and goals. To this group of teachers, being well-prepared seems to mean being ready for whatever the day holds, being able to be both proactive and reactive, to be actors without lines who are responsive to their audiences' needs.

The teachers in this study viewed their profession as involving constant growth and change. This view is revealed by their participation in professional conferences and inservice programs, their completion of graduate studies, and their approaches to instructional planning. One teacher said that she and her colleagues had considered putting together a three-year program so that after three years teachers could cycle back through a curriculum. But she and her colleagues decided, "No, we never want to do that. Every year should be different from every other year." Change for these teachers was gradual and ongoing. While they faced barriers in their attempts to reform their teaching, they belonged to strong advocacy groups and continued to seek encouragement and support for their efforts.

The quality of reflectiveness appears to undergird the changes made by these teachers as they acquired classroom experience. A reflective practitioner of mathematics teaching, according to Houston and Clift (1990), uses knowledge that is pedagogically based, knowledge of students both as groups and as individual learners, an understanding of the milieu of the school and of the community, and an understanding of how all of these interrelate. Reflectiveness was illustrated in the thinking and practices of these teachers. They frequently had ready answers to questions during interviews, giving the impression that they had thought about the issues. They listened carefully and were able to help others articulate their thoughts during seminars. One spoke about lying awake and thinking about the discussions that occurred during seminars. Others indicated they thought about the discussions long after the seminars.

Because teachers can be viewed as key figures in the reform of school mathematics, this study set out to identify the characteristics associated with teachers whose mathematics teaching already exemplified the spirit of the NCTM *Standards*. A preliminary study designed as a precursor to a larger study, it found that these teachers' beliefs were congruent with those expressed in the NCTM *Standards*. It also found that the teachers were still struggling to revise their teaching approaches so that they provided problem-solving opportunities for students and incorporated effective ways of responding to their individual differences. Teachers who are reflective practitioners, according to this study, may be more likely to move away from the traditional model of mathematics teaching. In some circles, these teachers would be called heroic in that they sought out their own resources and their sources of support to help them reform their teaching approaches. This finding reinforces the second of the two NCTM assumptions (NCTM, 1991, p. 2), that support and resources will need to be provided if larger numbers of teachers are to change the ways mathematics is taught in the nation's schools.

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