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ABSTRACT

This training packet on mathematics as strategic problem solving is 1 of 10 developed by the Study of Adult Basic Education (ABE)/English as a Second Language (ESL) Training Approaches Project to assist ABE instructors, both professionals and volunteers. The packet is intended to stand alone and encompasses a two-session workshop series with activities scheduled for participants to accomplish between sessions. Ideally, the sessions should take place about 1 month apart. Introductory materials include information about the series and the training packet, a workshop overview (objectives, time, materials checklist, preparations checklist), and workshop outline for each session. Trainer notes for each session include a checklist of tasks to be completed before the session and an outline of activities with necessary materials and times. Topics covered in the sessions include the following: attitudes toward math education; goals of ABE/General Educational Development math education; problem solving in the classroom; feelings about math; and developing a math lesson. Time is allowed for preparation for the home task and feedback on the home task. Trainer's supplements follow each session. Other contents include sample flyers, participant questionnaire, and masters for all handouts and transparencies needed in the sessions. Background readings include a summary of the 13 standards, Math-Moron Myths, and How Can We Best Help ESL Students with Math? (YLB)

STUDY OF ABE/ESL INSTRUCTOR TRAINING APPROACHES

MATHEMATICS: STRATEGIC PROBLEM SOLVING

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Training Packet for
a Two-Session Workshop on

MATHEMATICS: STRATEGIC PROBLEM SOLVING

Study of ABE/ESL Instructor Training Approaches

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Introduction to
THE ABE/ESL INSTRUCTOR TRAINING SERIES

Scope and Content

The Study of ABE/ESL Training Approaches Project has developed eight training packets to assist ABE and ESL instructors, both professionals and volunteers. Packet topics were selected based on a national review of training content and practices and on recommendations from selected experts representing ABE, ESL, and volunteer programs across the United States.

Packet topics include:

1. The Adult Learner
2. Planning for Instruction
3. Group/Team Learning
4. Monitoring Student Progress
5. Volunteers and Teachers in the Classroom
6. Communicative ESL Teaching
7. Mathematics: Strategic Problem Solving
8. Whole Language Approach

There is no suggested sequence implied in the above listing. Each packet is intended to stand alone. Each encompasses a two-session workshop series with activities scheduled for participants to accomplish between sessions. Ideally, the two sessions should take place about one month apart. Packets include detailed instructions for workshop leaders and masters for all handouts and transparencies needed in the workshops.

Key Assumptions about Adult Learning

All packets have been designed to guide workshop leaders to model the adult learning principles upon which the packets are based. These principles apply to the training of instructors as well as to educating adult students. Based on the literature about adult learners and the experience of skilled adult educators, *it is assumed that adults learn best when:*

- they feel *comfortable* with the learning environment and they attempt tasks that allow them to *succeed* within the contexts of their limited time and demanding lives.
- they provide *input* into the planning of their own learning goals and processes.
- they have opportunities to engage in *social learning*, i.e., they learn from peers as well as from an instructor.

- they have a *variety* of options appropriate to their learning styles (including sensory modalities, ways of thinking, and both individual and group learning) and have opportunities to analyze and expand their modes of learning.

- they are able to associate new learning with previous *experiences* and to use those experiences while learning.

- they have an opportunity to apply *theory/information* to practical situations in their own lives.

In accord with these assumptions, each packet employs research-based components of effective training and staff development: *theory, demonstrations, practice, structured feedback,* and *application with follow-up.* Key research findings on these components are:

1. The *theory* that underlies any new practice is a necessary but insufficient component of training.

2. *Demonstrations* that illustrate new practices and reinforce their use are essential to full comprehension and implementation.

3. Instructors need to *practice* new approaches in a safe environment and to receive *structured feedback* on their attempts.

4. New approaches need to be *applied* over time in a real situation — preferably ones where continuing feedback and analysis are possible (e.g., peer coaching or mentoring).

Research indicates that long-term change is likely to occur only when all of the above conditions are met.

We hope you will find that these training packets produce effective, long-term results.

About the MATHEMATICS: STRATEGIC PROBLEM SOLVING TRAINING PACKET

This training packet employs selected research-based components of effective training and staff development in the following manner:

THEORY: An inductive format is used in presenting theory. This approach requires participants to extract theory from the experiential activities rather than memorize theory from a lecture (a deductive format). Through analyzing their own experiences, participants learn about and internalize theory by discussing it with others.

DEMONSTRATION: A video of an adult ABE math class combined with a focus handout enables participants to identify and analyze the components of problem-solving math lessons.

PRACTICE: During Session One, participants establish goals for ABE/GED math instruction and identify and apply problem-solving strategies.

Between Sessions One and Two, through an interim task assignment, participants have the opportunity to observe a student applying problem-solving strategies and to reflect on the process and the outcome.

STRUCTURED FEEDBACK: During Session Two, participants have the opportunity to share and compare their teaching experiences in the interim task assignment.

APPLICATION: Participants use the knowledge they have gained through the workshop and experience to develop their own problem-solving math lesson.

REFLECTION ON WORKSHOP PROCESSES: During the workshop, participants analyze the type of thinking and learning that the workshop activities stimulate. As a result, participants become conscious of the theories and assumptions that underlie and guide the monitoring of student progress.

During these training sessions, "learning by doing" will be the focus. Participants will experience the problem-solving approach to math education and then reflect upon, analyze, and generalize from their experience.

About the Participants...

This training packet is designed for persons involved in some aspect of adult math instruction — for example: teaching, tutoring, or supervising. It is important for participants to attend <u>both</u> sessions.

WORKSHOP OVERVIEW

Objectives: By the end of Sessions One and Two, participants will be able to:

1) develop goals of ABE/GED math education;

2) identify and apply six problem-solving strategies;

3) recognize and respond to emotional/attitudinal obstacles students may face when trying to apply problem-solving strategies.

Time: Total time required for the workshop: approximately 8-10 hours:

- Session One: 3 hours
- Interim Activities at Home Sites: approximately 2-4 hours over a one-month period.
- Session Two: 3 hours

Materials Checklist: Hardware:

- VHS Player (1/2 inch) and Monitor
- Overhead Projector

Software:

- Video
- Packet Handouts
- Packet Transparencies
- Blank Transparencies and Transparency Pens
- Flip chart paper (or large sheets of newsprint), tape, and markers
- Paper Straws (two per participant during Session One)
- Enough scissors for participants to share (during Session One)
- Sugar Cubes (one or two boxes for each small group Session Two)
- Two very small empty boxes of different sizes for each small group (or use the pattern sheets on pages 27 and 28 to make the boxes) (Session Two)

BEFORE SESSION ONE

The following tasks should be completed *before* Session One of the workshop:

- ☐ **Send out flyers** announcing the workshop series. (See pages 29+ for a sample.)
- ☐ **Send the Participant Questionnaire** (see pages 29+) to all persons responding to the flyer. The suggested maximum number of participants for each workshop series is 30 persons.
- ☐ **Tally the results of the Participant Questionnaire.** (This can be done easily on a blank copy of the Participant Questionnaire.) You may want to make a transparency of those results to share with participants.
- ☐ **Arrange for a place** to hold Session One and make sure it has sufficient space and movable chairs for small groups. Ideally, the room should be set up with tables seating four to six participants each. Arrange for any refreshments that will be available.
- ☐ **Order A/V equipment** (VHS player and monitor; overhead projector.) Before the session begins, check to see that all A/V equipment is working.
- ☐ **Duplicate all handouts for Session One** (H-1 through H-13) and arrange them into packets. Staple those handouts with more than one page (e.g., staple H-10-a and H-10-b together). By providing one packet of materials to each participant at the start of the workshop, constant handling of materials during the session can be avoided.
- ☐ **Make transparencies** from the transparency masters for Session One (T-1 through T-8).
- ☐ **Read the Trainer Notes for Session One** (pages 7-12). Review handouts H-1 through H-13 and transparencies T-1 through T-8.
- ☐ **Each participant needs 2 paper straws.** There should also be enough scissors for participants to share.

WORKSHOP OUTLINE
SESSION ONE (THREE HOURS)

MATERIALS	ACTIVITIES	TIME
H-1*	I. Introductions/Workshop Overview • Agenda, Objectives	5 min
H-2	II. Attitudes Toward Math Education: Survey • Small Group Discussion/Large Group Sharing	20 min
H-3, T-1* H-4, T-2 T-3	III. Goals of ABE/GED Math Education A. Presentation: • NCTM Standards: Emphasis on Standard I • Goals of ABE/GED Math Education B. Whole Group Discussion: • Additional Goals for ABE/GED Math Education	25 min
H-5 Video H-6, T-4 T-5 H-7, T-6	IV. Problem-Solving in the Classroom A. Demonstration • Introduction to the Video • Video • Whole Group Discussion B. Pencil and Paper Computation and Mental Math • Decisions about Calculation Procedures • Discussion of Mental Math • Mental Math Application Activity	 20 min 15 min
	BREAK	15 min
H-8, T-7 Paper Straws H-9, T-8 H-10, H-11	C. Presentation/Practice: • Defining Problem-Solving Strategies D. Applying Problem-Solving Strategies • Individual Application • Whole Group Sharing E. Problems, Strategies, Solutions • Activity: Problem Exchange	 25 min 15 min 25 min
H-12	V. Interim Task Assignment • Distribution of Interim Task Assignment and Explanation of Task	10 min
H-13	VI. Session One Evaluation	5 min

* "H" = "Handout," "T" = "Transparency"

TRAINER NOTES: SESSION ONE

REGARDING THE SUGGESTED TIMES: All suggested times are the result of field testing within a three- to four-hour timeframe. Feel free to adjust the suggested times to meet the needs and experience levels of the participants. In addition, it is important to become familiar with the materials prior to the workshop in order to select specific activities if sufficient time is not provided or some activities take longer than anticipated. Familiarity with the materials also will enable you to personalize the materials by adding anecdotes where appropriate. If more than three hours are available for the training, the suggested times can be expanded to allow for additional sharing and discussion.

REGARDING THE ROOM SET-UP: Since the workshop includes both large and small group work, arrange the room so that participants can move about fairly easily. Try to make certain that the flip charts, overheads, or videos can be seen by all participants. In less than ideal settings, you may have to consider eliminating the use of overheads or flip charts.

REGARDING TRAINING PREPARATION: Before reading through these notes, you should carefully read the articles included as background information (pages 71+). If you feel participants would benefit from reading any of these articles, duplicate them and include them in the Interim Task Assignment Packet (Handout 11).

Refer to the Workshop Outline on the previous page as you go through these notes.

MATERIALS

I. Introductions/Workshop Overview (5 minutes)

If the participants do not know one another, have them introduce themselves one by one to the large group by stating their name, program, and subject/level they are currently teaching. Be sure to move the group along, having each participant speak only a few moments. (If the group is large, ask for a show of hands as to subject/level and program; then have participants personally introduce themselves to people near them.) The purpose of the introductions is to make the participants feel comfortable and to give them a sense of who the other participants are.

H-1* Direct participants' attention to H-1. Go over the agenda and the session objectives. Answer any questions.

* "H" = "Handout," "T" = "Transparency"

II. Attitudes Toward Math Education (20 minutes)

H-2

Direct participants' attention to H-2. Explain that these are statements about which many math teachers and students have strong opinions. The responses to these statements influence attitudes and approaches toward math education.

Have participants work in groups of 4-6 people. Assign each group one of the three statements on H-2 and a position of either agreement or disagreement with the statement. Have each group try to build a case for their assigned position and discuss how this opinion would influence their teaching methods. Allow ten minutes for small group discussion.

> The purpose of the survey is to make participants aware that differing opinions exist and that these opinions can affect math instruction. The purpose is not necessarily to reach group consensus on the issues raised.

Have a volunteer from each group present their group's position to the large group. Allow ten minutes for large group discussion. (See the possible responses on pages 14-15.)

III. Goals of ABE/GED Math Education (25 minutes total)

A. NCTM Standards: Emphasis on Standard One (15 minutes)

H-3, T-1

Direct participants attention to H-3. Project T-1 on the overhead projector. Explain that the National Council of Teachers of Mathematics has spent the last several years developing standards for math instruction. It has completed standards for grades K-4, 5-8, and 9-12 and is presently extending these to adult math education. The first four standards are considered the basis for establishing a strong foundation in mathematics. This workshop focuses on Standard I: Mathematics as Problem Solving. Have participants read H-3 and ask for comments and reactions from the group. *(Note: All the standards for ABE Skills (5-8) are included in the background readings on pages 71+.)*

H-4, T-2

Keeping in mind Standard I, Mathematics as Problem Solving, participants will next consider classroom goals. Direct the participants' attention to H-4. Project T-2 on the overhead projector. Go over the ABE/GED math instruction goals and examples, perhaps by underlining key words with a transparency pen. Ask for comments and reactions from the participants.

B. Whole Group Discussion: Additional Goals for ABE/GED Math Education (10 minutes)

T-3 — Project T-3 on the overhead projector. Elicit additional goals relevant to adult math education from the participants. Write their ideas on T-3. *(Note: Keep T-3 with the participants' comments as it will be used for review in Session Two.)*

IV. Problem-Solving in the Classroom (110 minutes total)

A. Demonstration: Video Presentation and Discussion (20 minutes)

The suggested video is: *Changing the Rules: Teaching Math to Adult Learners,* New Readers Press, Syracuse, New York, 1990.

This video shows teachers how to make math more achievable and useable by addressing math anxieties, relating abstract concepts to concrete application, and by "spiraling" math teaching — integrating more advanced concepts to earlier stages of the curriculum.

The video section to be shown in Session One begins about four minutes into the video (right after the introduction of Rule 2, "Students should memorize formulas before they start to solve problems.") This section of the video lasts approximately ten minutes. Stop the tape after the comment: "Lessons that build from the concrete to the representational and finally to the abstract help learners to understand and apply different types of mathematical thinking."

Introduce the video presentation by telling the participants that they will watch an adult class that is using a problem-solving approach to begin the study of area. While the participants watch the demonstration, ask them to keep in mind the first NCTM Standard, Mathematics As Problem Solving, as well as the goals of ABE/GED math education discussed earlier.

H-5 — Direct participants' attention to H-5, the Video Response Sheet. This sheet contains focus questions for participants to consider while watching the video. Participants may take notes on H-5 while viewing, if desired.

Video — Show the video. After the video, use the questions on H-5 to guide a large group discussion.

B. Pencil and Paper Computation and Mental Math (15 minutes)

Use the response to the last question on H-5 ("When did the students use mental math? Pencil and paper calculation? Calculators?") to point out that mental math is an essential skill in mathematics problem solving.

H-6, T-4

Direct participants' attention to H-6. Project T-4 on the overhead projector. Using the flow chart on H-6, discuss the decisions the students in the video made about the calculation procedures.

T-5

Project T-5 on the overhead projector. Explain that most mental math procedures involve simplification and estimation. Elicit from the large group situations in everyday life where they need to be able to do mental math. Write their responses on T-5.

H-7, T-6

Direct participants' attention to H-7, Problem Sheet A. Project T-6 on the overhead projector. Very briefly discuss the following questions with the large group. (See page 16 for possible solutions.)

- What are possible math questions in this situation?
- How would students use estimation and simplification to solve these questions?

Inform participants that "Simplify and Estimate" is the first of six problem-solving strategies that will be explored during the rest of workshop. Invite the participants to take a 15 minute break, telling them they will have the opportunity to examine other strategies when they return.

B R E A K (15 minutes)

C. Presentation/Practice: Defining Problem-Solving Strategies (25 minutes)

H-8, T-7

Direct participants attention to H-8. Project T-7 on the overhead projector. Explain that students often attempt to solve problems through rote memorization of rules. If the rule is forgotten, many students feel frustrated and defeated. The purpose of problem-solving strategies is to enable students to reach a solution by using a variety of modalities suited to their own reasoning processes and to the specific problem presented.

The six strategies presented on H-8 are useful methods of problem solving. Discuss each strategy with the participants by:

- Briefly describing the strategy.

- Inviting the participants to try to solve the example problem either alone or with someone else. (There will often be a variety of solutions.)

- Asking the participants to share their results and their cognitive processes as soon as the majority of participants have completed each example problem.

> Spend only 4-5 minutes discussing each strategy and example problem. When discussing strategy 6, pass out scissors and two straws to each participant. If there is universal trouble in finding solutions to any of the example problems, give participants a hint from the possible solutions on pages 17-18.

Point out that often these strategies can be combined. Ask participants where, if at all, they used more than one strategy in solving the above problems. (Strategy 1, Simplification and Estimation, and Strategy 2, Trial and Error, are most commonly combined with other strategies.)

Emphasize that strategies are only applied when they are <u>needed</u> both in terms of student learning style and problem difficulty.

D. Applying Problem-Solving Strategies (15 minutes)

H-9 Direct participants' attention to H-9, Problem Sheet B. Individually or with a partner have participants decide which of the six problem-solving strategies on H-8 would be helpful in solving the three problems on H-9. Allow ten minutes for discussion.

> The emphasis of this activity should be on identifying <u>strategies</u> — not on finding solutions. Reassure participants that you will hand out the solutions (H-11) later in the workshop.

T-8 Project T-8 on the overhead projector. Ask the large group what strategy(s) would be most useful for each problem and how they would use that strategy(s). Write their responses on T-8. Allow five minutes for large group discussion.

E. Problems, Strategies, Solutions (25 minutes)

H-10 Direct participants attention to H-10, the Problem Exchange Activity. Have the participants work in pairs for this activity. Go over the three steps explained on H-10-a and make sure that all participants understand the activity.

Watch the time carefully. Encourage the pairs to get together and share their ideas even if they have not totally finished solving their problem.

Ask a volunteer from the combined pairs to briefly explain their solution and how they reached it to the large group.

When the Problem Exchange Activity is completed, pass out H-11, Possible Solutions for Problem Sheets B and C. This handout is for participants to take home and study, if desired. Do not take time during the workshop to discuss H-11.

H-11

V. Interim Task Assignment (10 minutes)

H-12

Direct participants' attention to H-12, the Interim Task Assignment packet. Go over the instructions on H-12-a and make sure all participants know what is expected of them. Answer any questions. Remind participants of the date and time of Session Two.

VI. Session One Evaluation (5 minutes)

H-13

Direct participants' attention to H-13, the evaluation of Session One. Ask participants to complete the evaluation.

TRAINERS' SUPPLEMENTS

ATTITUDES TOWARD MATH EDUCATION: SURVEY

It is important that participants come up with their own opinions in this activity. There should be no difficulty in getting a response from the group; however, if there is hesitation, you may choose to use some of the reasoning below to stimulate discussion.

1. **Some people are just not good at math and will never do well in math class.**

   Agree: People have varying abilities. It is cruel to lead people to expectations that they never will be able to fulfill.

   Studies have shown that men are better at math than women.

   Some people are predominantly right-brained (global), while others are predominantly left-brained (analytical). This orientation determines in part how well people will perform in analytical subjects like math.

   Disagree: Math skills may come more quickly for some than others, but it is self-defeating to assume that someone can't learn. Teaching at the level of each learner is the responsibility of a good teacher.

   Math doesn't consist of just one skill. Just because one aspect of math is difficult doesn't mean all aspects will be. Keep students motivated to try.

   This is what strategic problem-solving should be about: with the right approach, anyone can learn math.

2. **Students must understand the reasoning behind mathematical procedures.**

   Agree: Doing a mathematical operation without understanding the reasoning behind it is like memorizing nonsense.

   If you understand what you are doing, you can apply that understanding to new types of problems.

   Mathematics is basically the study of logic. Understanding the underlying reasoning is the whole basis of learning mathematics.

Continued

ATTITUDES TOWARDS MATH EDUCATION, Continued

Disagree: Not everyone is capable of understanding all mathematical laws. The important thing is that students be able to apply mathematics. (For example, you don't need to understand how a car works in order to drive it.)

Teachers waste time and confuse students trying to teach them number systems, etc. Time would be better spent in learning and practicing computation.

Students who could do well in math are turned off by time spent in arguments that should be left to mathematicians.

3. **There are many ways to reach a solution in almost every math problem.**

Agree: The important thing is for students to reach a useable answer. It is not so important how they get there.

Being aware of several methods of reaching a solution gives students flexibility to think about math and to try out their own ideas — rather than simply memorizing.

Real-life math demands flexibility in reaching solutions.

Disagree: It is confusing enough to learn one way to do something — why make it harder on students?

Mathematicians have spent centuries developing efficient ways to solve problems. It is a teacher's responsibility to teach students these efficient ways.

It is not necessary to know more than one way to solve a problem. It is more important to know <u>one</u> way <u>well</u>.

POSSIBLE SOLUTION FOR PROBLEM SHEET A

This is only <u>one</u> problem from the situation and <u>one</u> method to reach the solution. The participants may come up with equally valid problems and solutions.

SITUATION: You can buy yogurt three ways:

- a six-pack of four-ounce containers for $2.53
- an eight-ounce container for $0.74
- a sixteen-ounce container for $1.25

PROBLEM: Which is the best buy?

SOLUTION: <u>Estimate</u>:

6 x 4 = 24 ounces in the six-pack
$2.53 can be rounded to $2.50
10 cents x 24 ounces = $2.40
So each ounce costs about $0.10

The eight-ounce container costs $0.74
9 cents x 8 ounces = $0.72
So each ounce costs about $0.09

The sixteen- ounce container costs $1.24
8 cents x 16 ounces = $1.28
So each ounces costs about $0.08

Then imagine or sketch a table:
8 < 9 < 10
Therefore the sixteen ounce container should be the best buy.

POSSIBLE SOLUTIONS TO THE SAMPLE PROBLEMS ON H-8

Review Strategy #1: Simplify and estimate.

Example problem: Determine an approximate answer: 40×375.

Possible solutions: $40 \times 400 = 16{,}000$; since the number is slightly less, the answer may be about 15,000.

$10 \times 375 = 3{,}750$ which is about 3,800 and
$3{,}800 \times 4 = 15{,}200$.

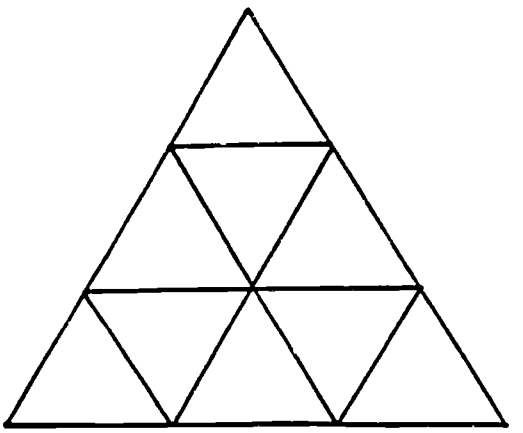
Introduce Strategy #2: Experiment with various solutions (trial and error).

Example problem: Discover a series of operations on the numbers 3 and 8 (used as often as needed) which will yield the number 7.

A possible solution: $[(8 \times 8) - (3 \div 3)] \div [(3 \times 3)]$

Introduce Strategy #3: Visualize the problem.

Encourage the participants to sketch the problem while searching for a solution.



Continued

POSSIBLE SOLUTIONS TO THE SAMPLE PROBLEMS ON H-8, Continued

Introduce Strategy #4: Make a diagram or chart.

Again, encourage the participants to sketch some form of diagram or chart.

Example problem: What possible combinations of four friends can work in a weight room when only two are permitted in at the same time.

Possible solutions: Assigning letters to each of the friends: A, B, C, D.

	A	B	C	D
A		AB	AC	AD
B			BC	BD
C				CD
D				

Introduce Strategy #5: Find patterns.

Example problem: What pattern produces the numbers: 144, 36, 12, 6? What is the next number? Give hints if needed.

Solution: 144/4 = 36, 36/3 = 12, 12/2 = 6, 6/1 = 6
(therefore 6 is the next number)

Introduce Strategy #6: Use realia.

Example problem: Which is larger: 1/2 or 3/4?

Suggest the participants illustrate the answer by marking and/or cutting the two paper straws in their packets.

BETWEEN SESSIONS

The following tasks should be completed *before* Session Two of the workshop:

- ☐ **Send out reminder flyers of Session Two** to Session One participants only (See pages 29+ for a sample.) This notice should remind participants to bring their handout packets from Session One with them to Session Two, especially their interim task assignment packet (H-12).

- ☐ **Arrange for a place** to hold Session Two and make sure it has sufficient space and movable chairs for small groups. Ideally, the room should be set up with tables seating four to six participants each. Arrange for any refreshments that will be available.

- ☐ **Order A/V equipment** (VHS player and monitor; overhead projector). Before the session begins, check to see that all A/V equipment is working.

- ☐ **Duplicate all handouts for Session Two** (H-14 through H-17) and arrange them into packets. Duplicate a few extra sets of the interim task assignment packet (H-12) from Session One for those participants who forget to bring theirs to Session Two.

- ☐ **Prepare transparencies for Session Two** (T-1, T-2, T-3, and T-7 from Session One will be used for review). Make enough copies of T-10 for one per group of 4-6 participants. Have at least one transparency pen available for each group. Have at least one blank transparency for each group as well.

- ☐ **Read the Trainer's Notes for Session Two** (pages 21-25). Review handouts H-14 through H-17 and transparencies T-9 and T-10.

- ☐ **Supply a container of sugar cubes** for each small group. Collect a variety of small boxes (such as candy or jewelry boxes) so that each group has two boxes — or use the patterns included on pages 27 and 28 to make two small boxes for each group.

WORKSHOP OUTLINE
SESSION TWO (THREE HOURS)

MATERIALS	ACTIVITIES	TIME
H-14*	I. Introductions/Workshop Overview • Agenda, Objectives	5 min
Video	II. Feeling About Math • Video Presentation • Large Group Discussion	10 min
T-1 T-2, T-3 T-7	III. Review of Session One: • NCTM Standard I • Goals of ABE/GED Math Education • Six Problem-Solving Strategies	10 min
H-12, H-15 Blank Transparencies Pens	IV. Sharing of Interim Task Assignment A. Small Group Discussion B. Small Group Presentations to the Large Group	 20 min 25 min
Video Boxes, Sugar Cubes T-9	V. Developing a Math Lesson A. Video Lesson and Real-life Application B. Experimentation C. Developing a Formula for Volume	 15 min 5 min 10 min
	BREAK	15 min
H-16, T-10 T-10 (group-made transparencies), T-7	D. Strategic Problem Solving — Volume • Small Group Activity: Development of a Lesson E. Small Group Presentation to the Large Group	15 min 25 min
Newsprint, Markers, Tape	VI. Reflection	20 min
H-17	VII. Session Two Evaluation	5 min

* "H" = "Handout," "T" = "Transparency"

TRAINER NOTES: SESSION TWO

REGARDING THE SUGGESTED TIMES: All suggested times are the result of field testing within a three- to four-hour timeframe. Feel free to adjust the suggested times to meet the needs and experience levels of the participants. In addition, it is important to become familiar with the materials prior to the workshop in order to select specific activities if sufficient time is not provided or some activities take longer than anticipated. Familiarity with the materials also will enable you to personalize the materials by adding anecdotes where appropriate. If more than three hours are available for the training, the suggested times can be expanded to allow for additional sharing and discussion.

REGARDING THE ROOM SET-UP: Since the workshop includes both large and small group work, arrange the room so that participants can move about fairly easily. Try to make certain that the flip charts, overheads, or videos can be seen by all participants. In less than ideal settings, you may have to consider eliminating the use of overheads or flip charts.

REGARDING TRAINING PREPARATION: Before reading through these notes, you should carefully read the articles included as background information (pages 71+). If you feel participants would benefit from reading any of these articles, duplicate them and include them in the Interim Task Assignment Packet (Handout 11).

Refer to the Workshop Outline on the previous page as you go through these notes.

MATERIALS

I. Introductions/Workshop Overview (5 minutes)

Ideally, all participants in Session Two will have taken part in Session One. However, if there are any newcomers to the group, have them introduce themselves one by one to the large group by stating their name, program, and subject/level they are currently teaching. Be sure to move the group along, having each participant speak only a few moments. Allow the group as a whole a few minutes to get reacquainted by introducing themselves to people near them.

H-14 Direct participants' attention to Handout 14 (H-14). Go over the agenda and the session objectives. Answer any questions.

II. Feelings About Math (10 minutes)

Video

Show the one-minute portion at the beginning of the video where three students talk about their experiences and feelings toward learning and using math. Stop the video at the frame, "The Rules."

After the video presentation, facilitate a brief large group discussion on the topic of feelings about math. Ask for volunteers to share with the large group how they felt about math in high school and college. Encourage participants to share both positive and negative feelings. Ask participants how they think their feelings about math influenced their successes (or frustrations) in studying math.

Inform participants that during the sharing of their interim task assignments they will have the opportunity to further explore the role of feelings in math education.

III. Review of Session One: NCTM Standard I, Goals of ABE/GED Math Education Problem Solving Strategies (10 minutes)

T-1

T-2, T-3

T-7

This portion of the workshop will be a brief review of Session One material. To review NCTM Standard I, project T-1 on the overhead projector. Stress this standard's emphasis on inductive learning. Then review the Goals of ABE/GED Math Education by briefly reviewing the goals discussed in Session One (use T-2 and T-3 to facilitate this review). Finally, summarize the six problem-solving strategies by projecting T-7 on the overhead projector and briefly reviewing each strategy.

IV. Sharing of Interim Task Assignments (45 minutes total)

A. Small Group Discussion of Interim Task Assignment (20 minutes)

H-12

Participants will now work in small groups (4-6 people) to share and discuss the results of their interim task assignment (H-12). *For this activity, have participants group themselves according to the level of math taught.*

H-15
Blank Transparencies, Pens

After participants have grouped themselves according to level taught, direct their attention to H-15. Explain each step of the small group task detailed on H-15 and answer any questions. Distribute one blank transparency and one transparency pen to each group (to be used if they desire).

Inform the groups that you will be available to provide assistance while they are working. Circulate among the groups, offering assistance as necessary. It is a good practice to monitor the progress of the groups, moving them along and encouraging them to prepare their presentations for the large group.

When ten minutes remain in the time allotted for completion of this activity, make an announcement to the large group. This will serve to keep the participants on task.

B. Small Group Presentations to the Large Group (25 minutes)

Have one or two volunteers from each small group come before the large group to present either their small group's summary or their chosen experience. They may use notes and/or their transparency.

Adjust the time allotted for each presentation to the total number of small groups (e.g., if there are five small groups, allow five minutes per presentation).

V. Developing a Math Lesson (70 minutes total)

A. Video Lesson and Real-life Application (15 minutes)

Video

Reshow the section of the video where teacher and students develop a lesson on area. (See page 9 for exact specifications).

After the video, facilitate a brief large group discussion. Ask participants for real-life situations in which students might need to know how to figure the volume of a rectangular solid.

B. Experimentation (5 minutes)

2 Boxes Per Group; Sugar Cubes

Pass out two small boxes to each group. (Boxes for a necklace or candy sampler boxes would be a good size. If no boxes are available, make boxes from the patterns on pages 27 and 28 before the session). Pass out enough sugar cubes to each group to fill their boxes. Have participants of each group fill their boxes with their sugar cubes.

C. Developing a Formula for Volume (10 minutes)

T-9

Project T-9 on the overhead projector. Ask each group how many sugar cubes fit into their boxes. Counting the sugar cubes gives the volume of the box. How can they count the cubes? On T-9 write the different ways to develop a formula for volume. Compare "cubic

inches" to sugar cubes. Together with participants, translate their formulas into cubic inches.

Ask participants how the problem-solving strategies of trial-and-error, using realia, and finding patterns helped them come up with the formula for volume.

B R E A K (15 minutes)

D. Strategic Problem Solving — Volume (15 minutes)

H-16

Direct participants attention to H-16. Explain that each small group will develop a lesson on volume for a class of ABE/GED students. Have them use H-16 to describe a problem situation, develop a problem, and describe how problem-solving strategies would be presented and applied in class.

T-10 (one per group)

Pass out one copy of T-10 and one transparency pen to each group. Have a volunteer from each group copy the lesson onto T-10 for presentation to the large group. Inform the groups that you will be available to provide assistance while they are working. Circulate among the groups, offering assistance as necessary. It is a good practice to monitor the progress of the groups, moving them along and encouraging them to fill out T-10 for their presentations to the large group.

> When five minutes remain in the time allotted for completion of this activity, make an announcement to the large group. This will serve to keep the participants on task.

E. Small Group Presentations to the Large Group (25 minutes)

T-10 (Group-Made Trans-parencies)

Have one or two volunteers from each group present their lesson to the large group. Adjust the time allotted for each presentation to the total number of small groups.

T-7

After the presentations, project T-7 on the overhead projector. Ask participants how these six problem-solving strategies were reflected in *their* lessons on volume. (Did they simplify and estimate? Did they visualize the problem before they did it? At any time did they need to make a diagram or a chart? Did they use patterns? How did the use of realia make the concept of volume clearer?)

VI. Reflection (20 minutes)

The purpose of this activity is to allow all participants to express their feelings toward the problem-solving strategies and to provide closure to the workshop.

Newsprint,
Tape,
Markers

Hang several sheets of newsprint (at least one sheet for every five participants) on the walls of the training room. Have at least one large marker available for each sheet of newsprint. On one of the sheets, write in large letters, HOW DO YOU PLAN TO IMPLEMENT PROBLEM-SOLVING STRATEGIES IN YOUR MATH CLASS?

Allow about 10 minutes for participants to walk around and write their answers on the sheets of newsprint. Encourage participants to read what others have written before them and to respond to those ideas as well as to express their own opinions.

After about 10 minutes, ask participants to return to their seats. Summarize the comments written on the sheets to bring closure to the workshop. This can be done by walking from sheet to sheet and reading or summarizing the comments and then reflecting on them. The participants should be invited to respond as time permits.

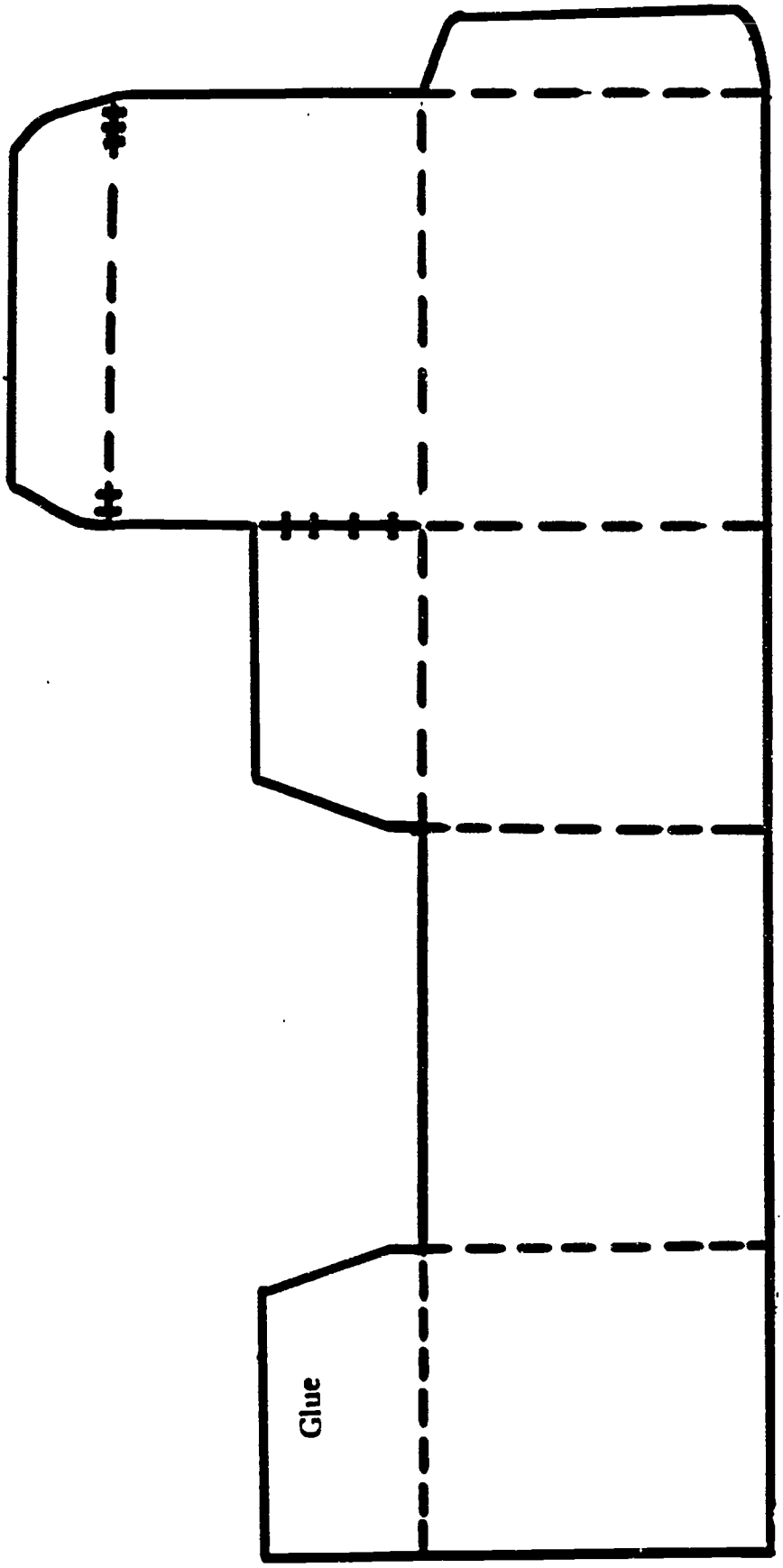
> If time is running short, the reflection activity can be done orally — without writing on the newsprint. Conduct a large group discussion of the question (HOW DO YOU PLAN TO IMPLEMENT PROBLEM-SOLVING STRATEGIES IN YOUR MATH CLASS?) and follow this discussion with summary statements which bring closure to the workshop.

VII. Session Two Evaluation (5 minutes)

H-17

Direct participants' attention to H-17, the evaluation of Session Two. Ask participants to complete the evaluation.

TRAINERS' SUPPLEMENTS



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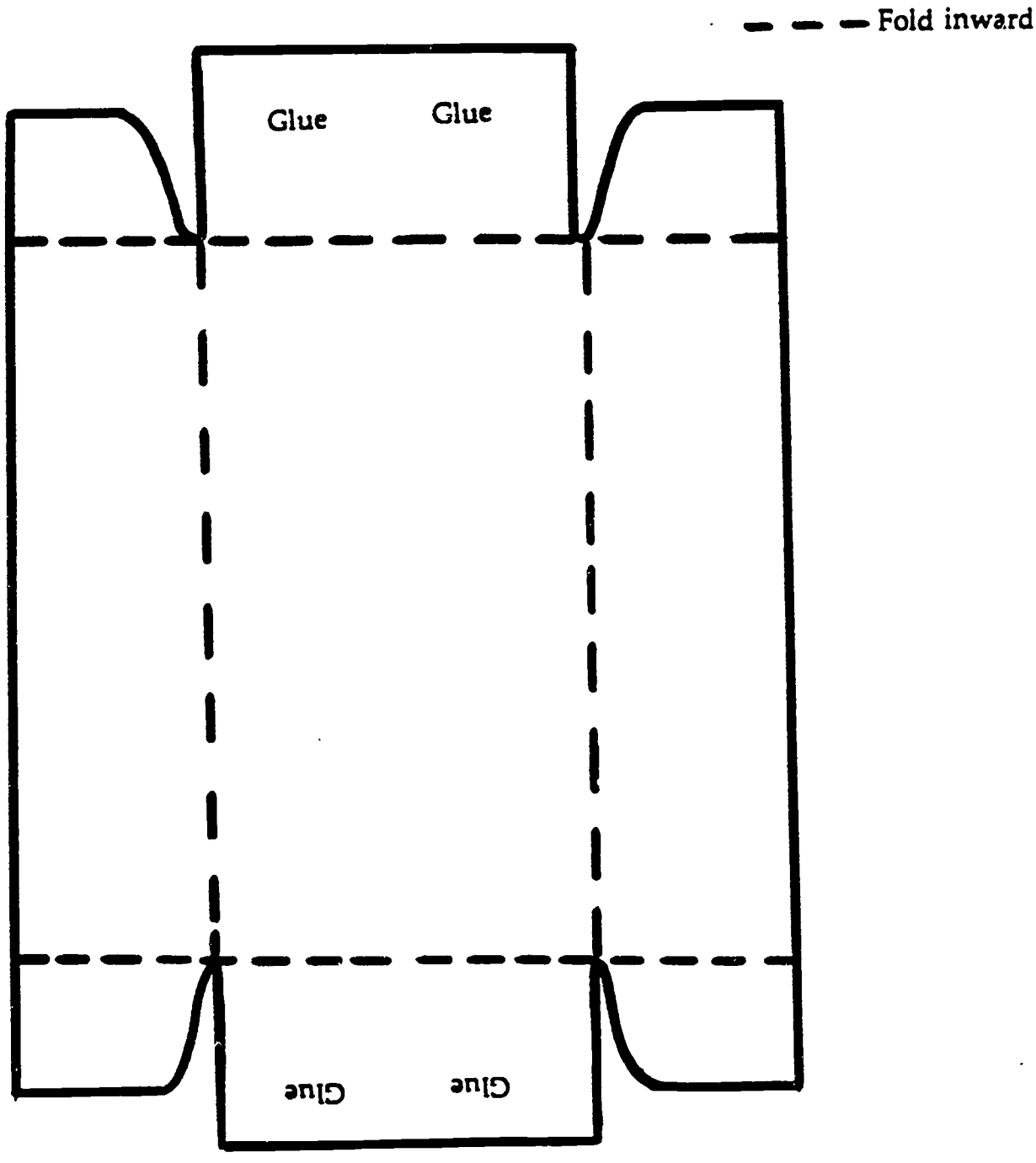
Box Pattern I



Box Pattern II

SAMPLE FLYERS

AND

PARTICIPANT QUESTIONNAIRE

You Are Invited
to Participate in
A Two-Session Workshop on

MATHEMATICS: STRATEGIC PROBLEM SOLVING

Participants will learn to:

1) Develop goals of ABE/GED math education.

2) Identify and apply six problem-solving strategies.

3) Recognize and respond to emotional/attitudinal obstacles students may face when trying to apply problem-solving strategies.

4) Create math lessons using problem-solving strategies.

Date of Session 1: _____ Time: _____

Date of Session 2: _____ Time: _____

Location: _____

Trainers: _____

Sponsors: _____

Please complete and return this portion to: _____

_____

☐ Yes, I would like to attend the two-session workshop on Mathematics: Strategic Problem Solving. I agree to attend both sessions. If I am accepted, please send me a participant questionnaire. Send to:

Name: _____ Telephone: (____) _____

Job Title: _____

Address: _____

_____
(City) (State) (Zip)

School/Program: _____

MATHEMATICS: STRATEGIC PROBLEM SOLVING WORKSHOP PARTICIPANT QUESTIONNAIRE

If you plan to attend the workshop on Mathematics: Strategic Problem Solving, please complete this form and send it to the address at the right by _____.
(date)

Thank you! We look forward to seeing you at the workshop.

Name: _____ Phone: _____

Address: _____

School/Program: _____

1. What is your educational background? _____ Field _____

2. Are you teaching now? ☐ Yes ☐ No
   If yes, what is your position? Check all that apply:

   ☐ Adult Basic Education Math Teacher

   ☐ Adult Basic Education Reading Teacher

   ☐ ESL Teacher

   ☐ Administrator

   ☐ Volunteer

   ☐ Other: _____

3. Please indicate the number of years you have taught each of the groups listed below. (If you have taught for less than one year, write "1.")

   ☐ Adults

   ☐ High School/Junior High Students

   ☐ Elementary/Preschool Students

4. In which of the following settings do you currently teach? Check all that apply:

- ☐ Classroom
- ☐ One-on-One Instruction/Tutoring
- ☐ Learning Laboratory/Language Laboratory
- ☐ Computer Laboratory
- ☐ Other: _____

5. What levels of math students do you work with?

- ☐ Literacy Level
- ☐ Basic Math
- ☐ GED Math
- ☐ College Level
- ☐ Some of my groups are multilevel
- ☐ All of my groups are multilevel

6. Have you received prior training in math education? Check all that apply:

- ☐ College courses in adult math education
- ☐ College courses in secondary math education
- ☐ College courses in elementary math education
- ☐ Workshops/Conferences on adult math education
- ☐ Other: _____

REMINDER!

Session Two of
the Workshop on

MATHEMATICS: STRATEGIC PROBLEM SOLVING

Date: _____ Time: _____

Location: _____

Please remember to bring the following:

1. Completed Interim Task Assignment
2. Materials from Session One

Please complete and return this portion to:

_____

_____

☐ Yes, I will attend Session Two of the workshop on Mathematics: Strategic Problem Solving.

☐ No, I am unable to attend Session Two.

Name: _____ Telephone: (____) _____

Job Title: _____

Address: _____

_____

(City) (State) (Zip)

School/Program: _____

_____

HANDOUT MASTERS

MATHEMATICS: STRATEGIC PROBLEM SOLVING: SESSION ONE

AGENDA

1. Introductions/Workshop Overview

2. Attitudes Toward Math Education

3. Goals of ABE/GED Math Education

   - NCTM Standards: Emphasis on Standard I
   - Additional Goals

4. Problem-Solving in the Classroom

   - Video Demonstration and Discussion
   - Pencil and Paper Computation and Mental Math
   - Six Problem-Solving Strategies

5. Interim Task Assignment

OBJECTIVES OF SESSIONS ONE AND TWO

Participants will be able to:

1. Develop goals of ABE/GED math education.

2. Identify and apply six problem-solving strategies.

3. Recognize and respond to emotional/attitudinal obstacles students may face when trying to apply problem-solving strategies.

4. Create a math lesson using problem-solving strategies.

ATTITUDES TOWARDS MATH EDUCATION:

SURVEY

Agree or Disagree

Directions: Your group will be assigned one of the following statements and a position of either agreement or disagreement. Through discussion, build your case for your assigned position and discuss ways in which this position would influence your teaching methods. Be prepared to share your position with the large group.

1. **Some people are just not good at math and will never do well in math classes.**

2. **Students must understand the reasoning behind mathematical procedures.**

3. **There are many ways to reach a solution in almost every math problem.**

NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS: STANDARDS FOR MATH INSTRUCTION*

STANDARD 1: MATHEMATICS AS PROBLEM SOLVING

- use problem-solving approaches to investigate and understand mathematical content;
- formulate problems from situations within and outside mathematics;
- develop and apply a variety of strategies to solve problems, with emphasis on multistep and nonroutine problems;
- verify and interpret results with respect to the original problem situation;
- generalize solutions and strategies to new problem situations;
- acquire confidence in using mathematics meaningfully.

* From the National Council of Teachers of Mathematics, Inc., *Curriculum and Evaluation Standards for School Mathematics.* Reston, Virginia. 1990.

GOALS OF ABE/GED MATH CLASSES

In addition to passing the math section of the GED test, there are other goals in an ABE/GED math class. Here are some of them.

Acquire skills that enable students to recognize and carry out the mathematical functions of their everyday lives. Among these are:

- balancing a check book
- making and using a budget
- determining the price/value of goods
- understanding sales and finance terms
- making schedules
- determining distance and mileage
- comprehending and computing pay rates

Develop problem-solving abilities applicable in mathematical and non-mathematical contexts. Among these are:

- noticing patterns
- searching for a variety of solutions
- formulating reasonable estimates
- reasoning from the concrete to the abstract
- collecting data to use in table/or graphs
- deriving formulas

VIDEO RESPONSE SHEET

As you watch the video of a mathematics class in action, think about the following questions:

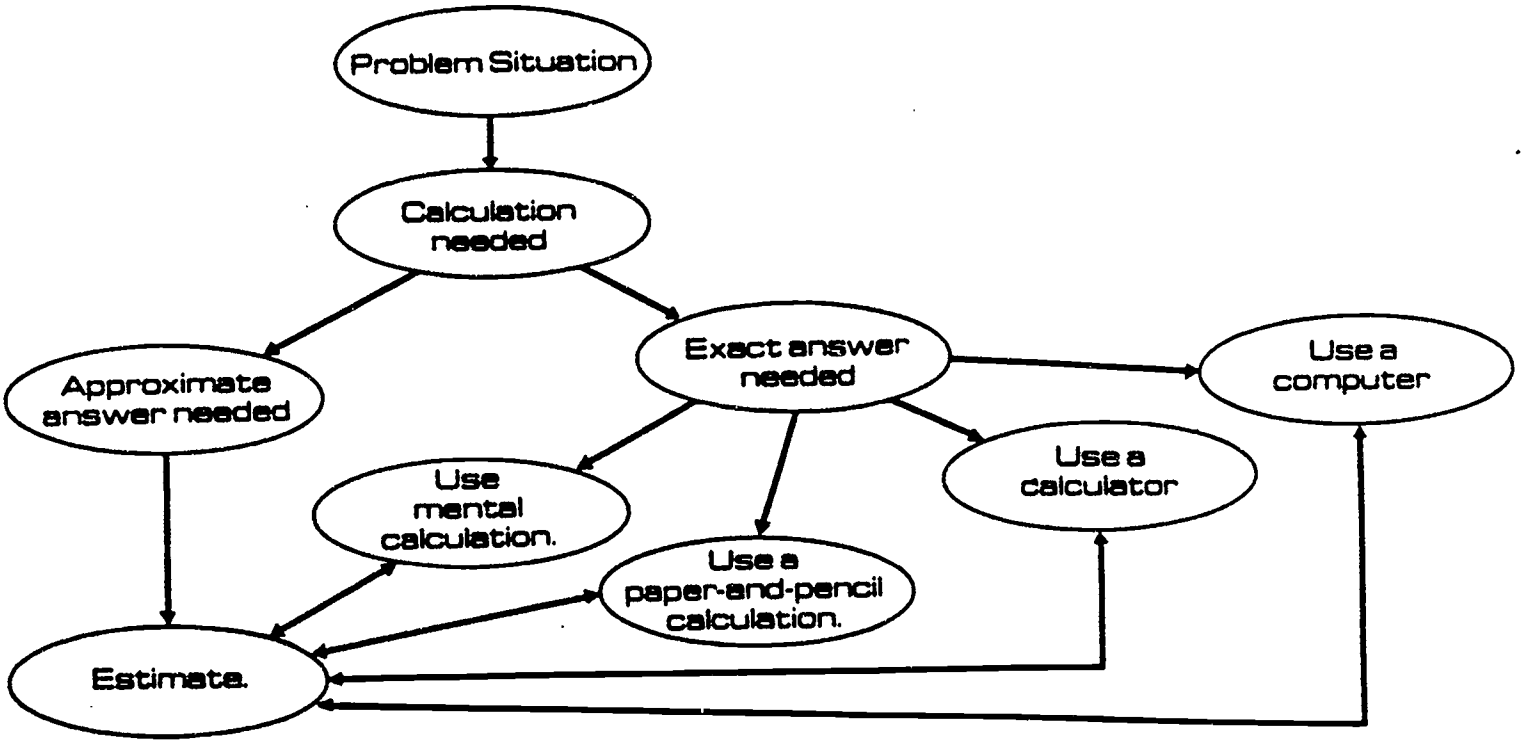
1. What problems are the students trying to solve?

2. What do they do to try to solve them?

3. How is NCTM Standard I, Mathematics As Problem Solving, implemented by the teacher and the students?

4. When do the students use mental math? Pencil and paper calculation? Calculators?

DECISIONS ABOUT CALCULATION PROCEDURES*



* From the National Council of Teachers of Mathematics, Inc., *Curriculum and Evaluation Standards for School Mathematics.* Reston, Virginia. 1990.

PROBLEM SHEET A

You can buy yogurt three ways:

1) a six-pack of four-ounce containers for $2.53

2) an eight-ounce container for $0.74

3) a sixteen-ounce container for $1.25

- What are possible math questions in this situation?
- How would students use estimation and simplication to solve these problems?

SOME PROBLEM-SOLVING STRATEGIES

STRATEGY	EXAMPLE PROBLEM
1. Simplify and estimate.	1. Quickly determine a closely approximate answer to 40 × 375.
2. Experiment with possible solutions (trial and error).	2. Discover a series of three operations with repeated numbers 3 and 8, which yield the number 7.
3. Visualize the problem.	3. You need to cut nine equilateral, congruent triangles out of a piece of rigid material. How can the triangles be arranged for the most efficient cutting — so that the sides of more than one triangle may be cut at one time (i.e., the smallest total perimeter)?
4. Make a diagram or a chart.	4. Carlos, Lamont, Gus, and George like to work out in the weight room. Only two people can use the weights at the same time. How many different combinations of friends can work out together?
5. Find patterns.	5. What is the next number in the series: 144, 36, 12, 6?
6. Use realia.	6. Compare fractions 1/2 and 3/4 by marking straws in halves and fourths and then cutting and comparing them.

APPLYING PROBLEM SOLVING STRATEGIES:

PROBLEM SHEET B

PROBLEM I

The library is open from 9:00 a.m. to 5:00 p.m. on Monday, Wednesday, and Friday. It is open from 9:00 a.m to 8:00 p.m. on Tuesday and Thursday. On Saturday it is open from 10:00 a.m. to 2:00 p.m., and on Sunday it is closed. How many hours a week is the library open?

Possible Strategies:

PROBLEM II

You want to paint a window frame whose top is 8 feet from ground level. The only solid place to put the bottom of the ladder is 4 feet from the house. What is the shortest ladder you can use?

Possible Strategies:

PROBLEM III

There is a special at the local fast food restaurant. You can buy 3 hamburgers for $5.00. You have $307.00 to feed the 225 employees at your company picnic. How many hamburgers will you be short or how many extra hamburgers will you have?

Possible Strategies:

PROBLEM EXCHANGE ACTIVITY

STEP I:

Each pair of problem-solvers will:

- Choose one situation from the next page.
- Develop a problem.
- Determine the steps necessary to reach a solution.
- Identify the strategy(s) needed to solve each step of the problem.
- Solve the problem.

STEP II:

- Exchange problems (but not strategies or solutions) with another pair of problem-solvers.
- Determine the steps necessary to reach a solution in the problem you receive.
- Identify the strategy(s) needed to solve each step of the problem you receive.
- Solve the problem.

STEP III

Both pairs of problem-solvers will then:

- Get together to discuss their problems.
- Discuss reasons for the strategies used and the solutions obtained.

NOTE: THE STRATEGIES AND SOLUTIONS THAT EACH PAIR DEVELOPS MAY BE DIFFERENT.

PROBLEM SHEET C

SITUATION I

You have been invited to Aunt Agatha's for dinner. If you go on the expressway, you can drive 50 to 65 mph. If you go through town, you will need to drive at only 25 to 35 mph. You have two choices. One way is 13 miles long, with 8 miles of expressway driving and 5 miles through town. The other way is all expressway, but the distance is 21 miles.

SITUATION II

The president of the poor country Wanegel is giving a dinner party. He has invited 6 additional officials from Wanegel, 8 officials from his equally poor neighbor, Carocal, and 6 officials from the very rich country of Bruntary. The banquet table has 10 chairs on each side and 1 chair each at the head and the foot.

SITUATION III

George earns $7.00 an hour for his regular 40 hour week and time-and-a-half for overtime. He generally works 7 to 10 hours of overtime each week. He gets paid once a month.

POSSIBLE SOLUTIONS FOR PROBLEM SHEET B

Problem I. (chart)

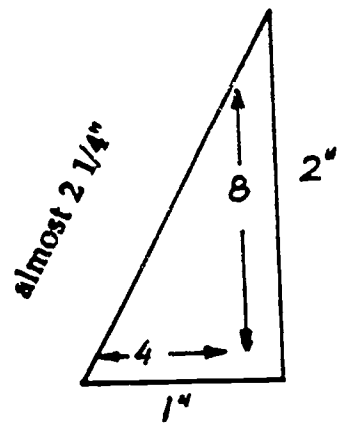
_____	Mon.__	Tues.__	Wed.__	Thurs.__	Fri.__	Sat.__	Sun.__
9 - 10	X	X	X	X	X		
10 - 11	X	X	X	X	X	X	
11 - 12	X	X	X	X	X	X	
12 - 1	X	X	X	X	X	X	
1 - 2	X	X	X	X	X	X	
2 - 3	X	X	X	X	X		
3 - 4	X	X	X	X	X		
4 - 5	X	X	X	X	X		
5 - 6		X		X			
6 - 7		X		X			
7 - 8		X		X			

8 + 11 + 8 + 11 + 4

3 × 8 = 24 2 × 11 = 22 22 + 24 + 4 = 48

48 hours open

Problem II. (Diagram to scale)



1 foot = 1/4 inch

Shortest ladder = about 9 feet

PROBLEM SHEET B (cont'd)

Problem III. (Simplify and estimate)

$307.00/$5.00 = approximately 60 (number of $5.00 bills that you have)

3 × 60 = 180 (number of hamburgers you can buy)
hamburgers # of $5.00

225 - 180 = 45
total people total hamburgers approximately hamburgers short

POSSIBLE SOLUTIONS FOR PROBLEM SHEET C

Problem I. (diagram, simplify and estimate)

8 miles 5 miles

1st way

about 57 mph about 30 mph

about 1 mile/min. about 1 mile/2 min.

21 miles

2nd way

57 mph

1st way: 8 + 10 = 18 minutes PROBABLY THE SHORTEST

2nd way: 21 minutes

Problem II. (patterns, diagram, experiment with possible solutions, students may use realia: moving actual markers around a cardboard rectangle)

As one math teacher sees this situation:

Everyone wants to sit by the wealthy people of Bruntary.
If the officials from Wanegel come in first, they will sit as spread out as possible so as to increase their chances of sitting by the people of Bruntary.
Then when the officials from Carocal come in, they will try to block the people of Wanegel from sitting by the wealthy people of Bruntary.
After a lot of trial and error, I came up with this possible seating.
What is yours? Why?

B W C B W C B C W C

C B W C B B W C B C

PROBLEM SHEET C (cont.)

Problem III. (Simplify and estimate)

4 × 40 = 160 hours in a month

about 4 weeks in a month 40 hours in a week

George must work about 8 or 9 hours a week on the average, maybe:

4 × 8 (plus a little) = 35 hours overtime a month

$7.00 straight time. Half of $7.00 is about $3.00. So time and a half = approx. $10.00

$7.00 × 160 hours = $1,120 and $10.00 × 35 hours = $350.00

$1,120	pay for straight time
+ $350	pay for time and a half
$1,475	pay a month (approximately)

INTERIM TASK ASSIGNMENT

To be completed for Session Two.

Six problem-solving strategies are:

1. simplify and estimate
2. experiment with possible solutions (trial and error)
3. visualize the problem
4. make a diagram or chart
5. find patterns
6. use realia

During the hiatus between Session One and Two, working with your own students, you will:

1. Use what you have learned in Session One to help an adult math student (or your class) develop a math problem and solve it using one or more problem-solving strategies. Choose from any of the situations on Problem Sheets A, B, or C - or develop your own situation. You will:

   - Develop the problem together with the student(s) by examining options.
   - Allow the student(s) to attempt to solve it.
   - Ask the student(s) what strategies he/she is using.
   - Review, with the student(s), the six strategies discussed in the workshop.
   - Have the student(s) choose from the six strategies to try the problem again (even if the correct solution was reached on the first try).
   - Discuss with the student(s) the usefulness of the strategies and how they might be extended to other problems.

2. Fill out the result sheet on the next page. Be prepared to report back to the group at Session Two.

Instructions for Volunteers/Tutor/Aides

If you work one-on-one with a student (or students), complete the above steps with one student.

If you work in a classroom setting, discuss the task assignment with the classroom teacher. Complete the above steps with one or more students, as negotiated with the teacher.

INTERIM TASK RESULT SHEET

1. What was the student's (students') attitude about math? How did he/she express this attitude?

2. How did the student(s) first approach the problem?

3. How did the student's (students') attitude toward math and problem-solving influence his/her choice of strategies?

4. What new strategy(s) did the student(s) choose to use after the discussion of possible problem-solving strategies?

5. Were the strategies successful or unsuccessful? Describe.

6. Was there any change in the student's (students') attitude toward math and problem-solving?

7. How do you feel the process of using problem-solving strategies (NCTM Standard I) reflects a goal or goals of ABE/GED math education? Please explain on the back side.

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SESSION ONE EVALUATION

MATHEMATICS: STRATEGIC PROBLEM SOLVING

Date: _____ Workshop Location: _____

1. What is your educational background? _____ Field: _____

2. What subject(s) do you teach?

   ___ Adult Basic Education/Math

   ___ Adult Basic Education/Reading

   ___ English as a Second Language

   ___ Other: _____

   ___ I am not teaching right now.

3. In which setting(s) do you teach?

   ___ Classroom

   ___ Individual Instruction

   ___ Other: _____

4. Please check the ONE statement that best describes how useful you found the workshop.

   ___ Very valuable; I plan on incorporating things I learned into my work with students.

   ___ Valuable; the workshop was a good review of things I already knew.

   ___ Somewhat valuable; I learned some things but I am not sure how I will be able to apply them.

   ___ Barely valuable; the information presented was not helpful to me.

   ___ A waste of time.

   ___ Other: _____

5. Below is a list of potential benefits of the workshop. Please check all that apply to you:

___ I have a clearer idea of my goals in teaching mathematics to adults.

___ I understand and will try to apply NCTM Standard I, Mathematics as Problem Solving.

___ I have used and am comfortable with the six problem-solving strategies discussed in this workshop.

___ I will share what I have learned with others.

___ I will read more about the topics we covered.

___ I will get together again with people I met here.

___ I will seek other opportunities for training.

6. Please rate the extent that you agree with each of the following statements. Circle ONE number for each statement.

	Strongly Agree	Agree	Disagree	Strongly Disagree
The training facility was comfortable.	1	2	3	4
The trainers were well-prepared.	1	2	3	4
The trainers gave clear instructions.	1	2	3	4
The trainers were responsive to participants' needs.	1	2	3	4

7. What was most valuable to you about the workshop? _____

_____

8. What suggestions do you have for how the workshop might be improved?

_____

9. Please add any other comments. _____

MATHEMATICS: STRATEGIC PROBLEM SOLVING: SESSION TWO

AGENDA

1. Introductions/Workshop Overview

2. Feelings About Math (Video Presentation)

3. Review of Session One

4. Sharing of Interim Task Assignments

   - Small Group Discussion
   - Small Group Presentations to the Large Group

5. Developing a Math Lesson

   - Video Lesson and Real-life Application
   - Experimentation/Developing a Formula
   - Strategic Problem Solving: Volume
   - Presentation of Lessons to the Large Group

6. Reflection

7. Session Two Evaluation

OBJECTIVES OF SESSIONS ONE AND TWO

Participants will be able to:

1. Develop goals of ABE/GED math education.

2. Identify and apply six problem-solving strategies.

3. Recognize and respond to emotional/attitudinal obstacles students may face when trying to apply problem-solving strategies.

4. Create a math lesson using problem-solving strategies.

MATHEMATICS: STRATEGIC PROBLEM SOLVING

SMALL GROUP SHARING

I. Using the Interim Task Assignment Packet (H-12), each member of your group will:

A. Describe which problem your student(s) attempted to solve.

B. Discuss how your student's (students') emotional attitudes toward math affected his/her problem-solving attempts.

C. Describe strategies chosen before and after the presentation of problem-solving strategies and the success the student(s) had in solving the problem.

D. Discuss how the experience reflected ABE/GED goals of math education.

II. After all members of your group have finished Part I, your group will decide to do either A or B:

A. Summarize the experiences of your group. Either prepare a brief oral report or use a blank transparency to summarize your group's experiences. One or two volunteers from your group will then present this summary to the large group.

B. Choose one experience from the group to share with the small group. Either prepare a brief oral report or use a blank transparency to summarize this experience. One or two volunteers from your group will then present this experience to the large group.

DEVELOPING A MATHEMATICS LESSON: STRATEGIC PROBLEM-SOLVING — VOLUME

Directions: With your group, develop a lesson on volume for a class of adult students. A volunteer from your group will copy this lesson on Transparency 10 for presentation to the large group.

PROBLEM SITUATION:

PROBLEM:

PROBLEM-SOLVING STRATEGIES

Presentation to Students:

Application by Students:

SESSION TWO EVALUATION

MATHEMATICS: STRATEGIC PROBLEM SOLVING

Date: _____ Workshop Location: _____

1. What is your educational background? _____ Field: _____

2. What subject(s) do you teach?

   ___ Adult Basic Education/Math

   ___ Adult Basic Education/Reading

   ___ English as a Second Language

   ___ Other: _____

   ___ I am not teaching right now.

3. In which setting(s) do you teach?

   ___ Classroom

   ___ Individual Instruction

   ___ Other: _____

4. Please check the ONE statement that best describes how useful you found the workshop.

   ___ Very valuable; I plan in incorporating things I learned into my work with students.

   ___ Valuable; the workshop was a good review of things I already knew.

   ___ Somewhat valuable; I learned some things but I am not sure how I will be able to apply them.

   ___ Barely valuable; the information presented was not helpful to me.

   ___ A waste of time.

   ___ Other: _____

5. Below is a list of potential benefits of the workshop. Please check all that apply to you:

___ I have a clearer idea of my goals in teaching mathematics to adults.

___ I understand and will try to apply NCTM Standard I, Mathematics as Problem Solving.

___ I have used and am comfortable with the six problem-solving strategies discussed in this workshop.

___ I will share what I have learned with others.

___ I will read more about the topics we covered.

___ I will get together again with people I met here.

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The trainers were responsive to participants' needs.	1	2	3	4

7. What was most valuable to you about the workshop? _____

_____

8. What suggestions do you have for how the workshop might be improved?

_____

9. Please add any other comments. _____

TRANSPARENCY MASTERS

NCTM STANDARD 1:

MATHEMATICS AS PROBLEM SOLVING

- use problem-solving approaches to investigate and understand mathematical content;
- formulate problems from situations within and outside mathematics;
- develop and apply a variety of strategies to solve problems, with emphasis on multistep and nonroutine problems;
- verify and interpret results with respect to the original problem situation;
- generalize solutions and strategies to new problem situations;
- acquire confidence in using mathematics meaningfully.

GOALS OF ABE/GED MATH EDUCATION

In addition to passing the math section of the GED test, there are other goals in an ABE/GED math class. Here are some; what are others?

Acquire skills that enable students to recognize and carry out the mathematical functions of their everyday lives. Among these are:

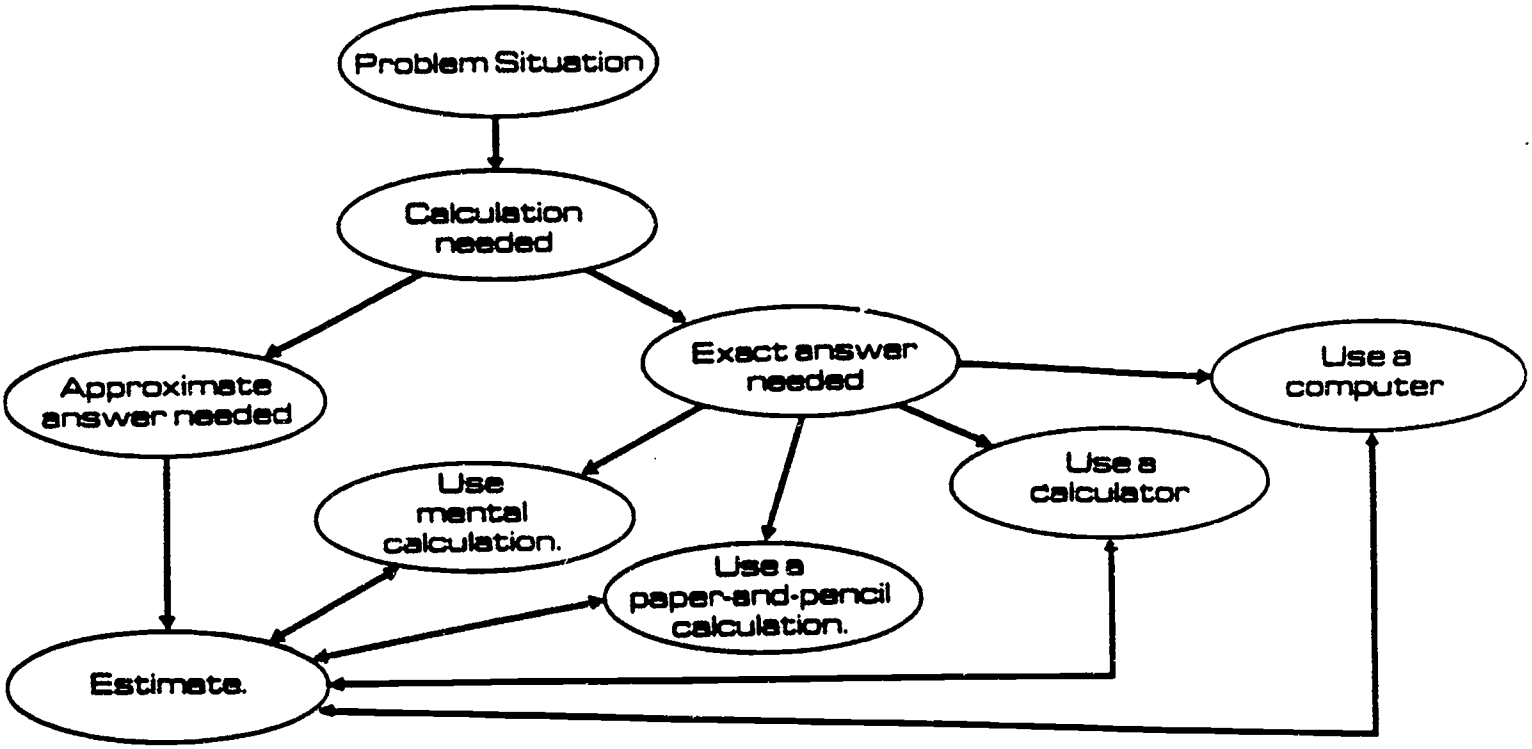
- balancing a check book
- making and using a budget
- determining the price/value of goods
- understanding sales and finance terms
- making schedules
- determining distance and mileage
- comprehending and computing pay rates

Develop problem-solving abilities applicable in mathematical and non-mathematical contexts. Among these are:

- noticing patterns
- searching for a variety of solutions
- formulating reasonable estimates
- reasoning from the concrete to the abstract
- collecting data to use in tables, graphs
- deriving formulas

ADDITIONAL GOALS
FOR
ABE/GED MATH EDUCATION

DECISIONS ABOUT CALCULATION PROCEDURES*



* From the National Council of Teachers of Mathematics, Inc., *Curriculum and Evaluation Standards for School Mathematics.* Reston, Virginia. 1990.

EVERYDAY SITUATIONS NEEDING MENTAL MATH

PROBLEM SHEET A

You can buy yogurt three ways:

1) a six-pack of four-ounce containers for $2.53

2) an eight-ounce container for $0.74

3) a sixteen-ounce container for $1.25

- What are possible math questions in this situation?
- How would students use estimation and simplification to solve these problems?

SOME PROBLEM-SOLVING STRATEGIES

STRATEGY	EXAMPLE PROBLEM
1. Simplify and estimate.	1. Quickly determine a closely approximate answer to 40 × 375.
2. Experiment with possible solutions (trial and error).	2. Discover a series of three operations with repeated numbers 3 and 8, which yield the number 7.
3. Visualize the problem.	3. You need to cut nine equilateral, congruent triangles out of a piece of rigid material. How can the triangles be arranged for the most efficient cutting — so that the sides of more than one triangle may be cut at one time (i.e., the smallest total perimeter)?
4. Make a diagram or a chart.	4. Carlos, Lamont, Gus, and George like to work out in the weight room. Only two people can use the weights at the same time. How many different combinations of friends can work out together?
5. Find patterns.	5. What is the next number in the series: 144, 36, 12, 6?
6. Use realia.	6. Compare fractions 1/2 and 3/4 by marking straws in halves and fourths and then cutting and comparing them.

APPLYING PROBLEM SOLVING STRATEGIES:

PROBLEM SHEET B

PROBLEM I

The library is open from 9:00 a.m. to 5:00 p.m. on Monday, Wednesday, and Friday. It is open from 9:00 a.m to 8:00 p.m. on Tuesday and Thursday. On Saturday it is open from 10:00 a.m. to 2:00 p.m., and on Sunday it is closed. How many hours a week is the library open?

Possible Strategies:

PROBLEM II

You want to paint a window frame whose top is 8 feet from ground level. The only solid place to put the bottom of the ladder is 4 feet from the house. What is the shortest ladder you can use?

PROBLEM III

There is a special at the local fast food restaurant. You can buy 3 hamburgers for $5.00. You have $307.00 to feed the 225 employees at your company picnic. How many hamburgers will you be short or how many extra hamburgers will you have?

Possible Strategies:

DEVELOPING A FORMULA FOR VOLUME

DEVELOPING A MATHEMATICS LESSON: STRATEGIC PROBLEM-SOLVING — VOLUME

PROBLEM SITUATION:

PROBLEM:

PROBLEM-SOLVING STRATEGIES

Presentation to Students:

Application by Students:

BACKGROUND READINGS

This is a summary of the thirteen standards prepared by the National Council of Teachers of Mathematics for grades 5-8. It is relevant for teaching basic mathematics skills at any level. For a more extensive discussion refer to the *Curriculum and Evaluation Standards for Mathematics* listed in the bibliography (H-12-c).

➤ STANDARD 1: MATHEMATICS AS PROBLEM SOLVING

In grades 5-8, the mathematics curriculum should include numerous and varied experiences with problem solving as a method of inquiry and application so that students can—

- ♦ use problem-solving approaches to investigate and understand mathematical content;
- ♦ formulate problems from situations within and outside mathematics;
- ♦ develop and apply a variety of strategies to solve problems, with emphasis on multistep and nonroutine problems;
- ♦ verify and interpret results with respect to the original problem situation;
- ♦ generalize solutions and strategies to new problem situations;
- ♦ acquire confidence in using mathematics meaningfully.

➤ STANDARD 2: MATHEMATICS AS COMMUNICATION

In grades 5-8, the study of mathematics should include opportunities to communicate so that students can—

- ♦ model situations using oral, written, concrete, pictorial, graphical, and algebraic methods;
- ♦ reflect on and clarify their own thinking about mathematical ideas and situations;
- ♦ develop common understandings of mathematical ideas, including the role of definitions;
- ♦ use the skills of reading, listening, and viewing to interpret and evaluate mathematical ideas;
- ♦ discuss mathematical ideas and make conjectures and convincing arguments;
- ♦ appreciate the value of mathematical notation and its role in the development of mathematical ideas.

➤ STANDARD 3: MATHEMATICS AS REASONING

In grades 5-8, reasoning shall permeate the mathematics curriculum so that students can—

- ♦ recognize and apply deductive and inductive reasoning;
- ♦ understand and apply reasoning processes, with special attention to spatial reasoning and reasoning with proportions and graphs;
- ♦ make and evaluate mathematical conjectures and arguments;
- ♦ validate their own thinking;
- ♦ appreciate the pervasive use and power of reasoning as a part of mathematics.

➤ STANDARD 4: MATHEMATICAL CONNECTIONS

In grades 5-8, the mathematics curriculum should include the investigation of mathematical connections so that students can—

- ♦ see mathematics as an integrated whole;
- ♦ explore problems and describe results using graphical, numerical, physical, algebraic, and verbal mathematical models or representations;
- ♦ use a mathematical idea to further their understanding of other mathematical ideas;
- ♦ apply mathematical thinking and modeling to solve problems that arise in other disciplines, such as art, music, psychology, science, and business;
- ♦ value the role of mathematics in our culture and society.

➤ STANDARD 5: NUMBER AND NUMBER RELATIONSHIPS

In grades 5-8, the mathematics curriculum should include the continued development of number and number relationships so that students can—

- ♦ understand, represent, and use numbers in a variety of equivalent forms (integer, fraction, decimal, percent, exponential, and scientific notation) in real-world and mathematical problem situations;
- ♦ develop number sense for whole numbers, fractions, decimals, integers, and rational numbers;

- understand and apply ratios, proportions, and percents in a wide variety of situations;
- investigate relationships among fractions, decimals, and percents;
- represent numerical relationships in one- and two-dimensional graphs.

➤ STANDARD 6: NUMBER SYSTEMS AND NUMBER THEORY

In grades 5-8, the mathematics curriculum should include the study of number systems and number theory so that students can—

- understand and appreciate the need for numbers beyond the whole numbers;
- develop and use order relations for whole numbers, fractions, decimals, integers, and rational numbers;
- extend their understanding of whole number operations to fractions, decimals, integers, and rational numbers;
- understand how the basic arithmetic operations are related to one another;
- develop and apply number theory concepts (e.g., primes, factors, and multiples) in real-world and mathematical problem situations.

➤ STANDARD 7: COMPUTATION AND ESTIMATION

In grades 5-8, the mathematics curriculum should develop the concepts underlying computation and estimation in various contexts so that students can—

- compute with whole numbers, fractions, decimals, integers, and rational numbers;
- develop, analyze, and explain procedures for computation and techniques for estimation;
- develop, analyze, and explain methods for solving proportions;
- select and use an appropriate method for computing from among mental arithmetic, paper-and-pencil, calculator, and computer methods;
- use computation, estimation, and proportions to solve problems;
- use estimation to check the reasonableness of results.

➤ STANDARD 8: PATTERNS AND FUNCTIONS

In grades 5-8, the mathematics curriculum should include explorations of patterns and functions so that students can—

- ♦ describe, extend, analyze, and create a wide variety of patterns;
- ♦ describe and represent relationships with tables, graphs, and rules;
- ♦ analyze functional relationships to explain how a change in one quantity results in a change in another;
- ♦ use patterns and functions to represent and solve problems.

➤ STANDARD 9: ALGEBRA

In grades 5-8, the mathematics curriculum should include explorations of algebraic concepts and processes so that students can—

- ♦ understand the concepts of variable, expression, and equation;
- ♦ represent situations and number patterns with tables, graphs, verbal rules, and equations and explore the interrelationships of these representations;
- ♦ analyze tables and graphs to identify properties and relationships;
- ♦ develop confidence in solving linear equations using concrete, informal, and formal methods;
- ♦ investigate inequalities and nonlinear equations informally;
- ♦ apply algebraic methods to solve a variety of real-world and mathematical problems.

➤ STANDARD 10: STATISTICS

In grades 5-8, the mathematics curriculum should include exploration of statistics in real-world situations so that students can—

- ♦ systematically collect, organize, and describe data;
- ♦ construct, read, and interpret tables, charts, and graphs;
- ♦ make inferences and convincing arguments that are based on data analysis;

- evaluate arguments that are based on data analysis;
- develop an appreciation for statistical methods as powerful means for decision making.

➤ STANDARD 11: PROBABILITY

In grades 5-8, the mathematics curriculum should include explorations of probability in real-world situations so that students can—

- model situations by devising and carrying out experiments or simulations to determine probabilities;
- model situations by constructing a sample space to determine probabilities;
- appreciate the power of using a probability model by comparing experimental results with mathematical expectations;
- make predictions that are based on experimental or theoretical probabilities;
- develop an appreciation for the pervasive use of probability in the real world.

➤ STANDARD 12: GEOMETRY

In grades 5-8, the mathematics curriculum should include the study of geometry of one, two, and three dimensions in a variety of situations so that students can—

- identify, describe, compare, and classify geometric figures;
- visualize and represent geometric figures with special attention to developing spatial sense;
- explore transformations of geometric figures;
- represent and solve problems using geometric models;
- understand and apply geometric properties and relationships;
- develop an appreciation of geometry as a means of describing the physical world.

➤ STANDARD 13: MEASUREMENT

In grades 5-8, the mathematics curriculum should include extensive concrete experiences using measurement so that students can—

- ♦ extend their understanding of the process of measurement;
- ♦ estimate, make, and use measurements to describe and compare phenomena;
- ♦ select appropriate units and tools to measure to the degree of accuracy required in a particular situation;
- ♦ understand the structure and use of systems of measurement;
- ♦ extend their understanding of the concepts of perimeter, area, volume, angle measure, capacity, and weight and mass;
- ♦ develop the concepts of rates and other derived and indirect measurements;
- ♦ develop formulas and procedures for determining measures to solve problems.

John Allen Paulos

At the request of the Editorial Panel, John Paulos adapted his editorial, "Math Moron Myths" (*New York Times*, 24 April 1991), for readers of the *Mathematics Teacher*.—Ed.

Math-Moron Myths

The field of mathematics education has experienced a great deal of ferment recently. Proposals to vivify the curriculum, to employ technological tools, and to increase teachers' salaries have all been offered. These ideas are all to be applauded, but I sometimes think that exploding five crippling and widely held misconceptions about mathematics would be almost as therapeutic.

Mathematics is computation. Mathematics has as much to do with computation as writing has to do with typing. Imagine that throughout the course of one's education all one ever did in English class was diagram sentences. It wouldn't be surprising if one didn't acquire a very keen appreciation of literature.

Most students—and most adults—can't interpret graphs; don't understand statistical notions; are unable to model situations mathematically; seldom estimate or compare magnitudes; are immune to mathematical beauty; and, most distressing of all in a democracy, hardly ever develop a critical, skeptical attitude toward numerical, spatial, and quantitative data or conclusions.

Mathematics is a rigidly hierarchical subject. A common belief is that first comes arithmetic, then algebra, then calculus, differential equations, and so on. Parts of mathematics display a cumulative aspect, to be sure, but it is less important than many realize.

Often, when I explain sophisticated ideas in probability or calculus to people who might have difficulty adding two fractions, such self-styled enumerates seem amazed at their new insights, much like the Molière character who was surprised to discover that he'd been speaking prose all his life.

Mathematics and narrative are disparate activities. Storytelling is as effective an educational tool in mathematics as it is in other domains. It puts the subject into context and illustrates its limitations. For example, in teaching correlation, a traditional topic in statistics courses, I've given students data demonstrating conclusively that children with bigger feet spell better. (Readers may have noticed this phenomenon themselves.) Should we therefore use foot stretchers to increase spelling scores? No, because the correlation is not causal: Children with bigger feet spell better because they're older.

Note that this little story doesn't contain any number crunching. And innumerable other vignettes can be cited from areas like sports, lotteries, medical fraud, and sex-discrimination cases that communicate mathematical ideas in a nontechnical and topical manner. If the connection between mathematics and ordinary language and thought is established early, then the tables, formulas, and algorithms that come later are justified: they're a shorthand means to get to solutions.

Mathematics is only for the few. "I'm a people person, not a numbers person." "Math was always my worst subject." "I can't even balance my checkbook." These are just a few of the tiresome—to me—comments that people use to explain disabilities in mathematics. It is true that some people have considerably more mathematical talent than others, but this disparity also holds for literary talent, and people rarely apply such arguments to reading.

Almost everybody can develop an understanding of numbers and probabilities, of relationships and logic, of graphs and rates of change, and of the role that these notions play in everyday life. All students can be taught standard heuristic problem-solving methods: assume a solution and work backward, look at an easier or related situation, draw a picture, and so on.

Mathematics is numbing. Finally a romantic belief prevails that a concern with numbers numbs one to the big questions, to the grandeur of waterfalls and sunsets. Too many people cling to the usually unarticulated belief that one must choose between life and love on the one hand and numbers and details on the other. Such sentiments are as prevalent as they are unfounded—and help bring about the dismal test scores we hear so much about.

John Allen Paulos

Numbers are not at odds with love and beauty

John Paulos teaches at Temple University, Philadelphia, PA 19122. He is an author of books on numeracy and innumeracy.

The views expressed in "Soundoff" do not necessarily reflect the views of the Editorial Panel of the *Mathematics Teacher* or the National Council of Teachers of Mathematics. Readers are encouraged to respond to this editorial by sending double-spaced letters to the *Mathematics Teacher* for possible publication in "Reader Reflections." Editorials from readers are welcomed.

How Can We Best Help ESL Students With Math?

by Mary Holt Kimball

Because the assumption persists that mathematics depends only marginally on expertise in English, mathematics typically serves as the first academic course beyond English as a Second Language (ESL) into which counselors place students from foreign countries. However, counselors and bewildered students soon discover that limited English proficiency is a discouraging obstacle to learning mathematics.

First, t shift from the metric to the customary system of weights and measure. nstitutes an immense barrier. Second, foreign-born students who are reasonably fluent in conversational English may still have little command of the language of mathematics. Third and most critical, mathematics problems described in sentences (word problems) create a major obstacle for ESL students.

To serve ESL students well, we need to place them in appropriate classes and change our curriculum. Counselors should use placement tests that distinguish between computational skills and language comprehension. A special curriculum should be established to bridge the gap between ESL classes and mainstream classes.

A curriculum to prepare students for mathematics must be vocabulary-intensive. Students must learn math terminology—*addition, quotient, reciprocal, coordinates*—as well as operational language: "One-fourth *of* twenty," "What is the *difference* between...?" They should learn quantitative prefixes like *mono–, bi–, tri–* and *multi–* and should compile individual glossaries of terms they learn.

Preparation for mathematics should involve hands–on practice with customary weights and measures for volume, length, time, temperature, and money. Math teachers with ESL students can increase their chance of success by employing these techniques:

- Never assume learners already understand mathematical terms. Allow student partners to work cooperatively with flash cards showing the term on one side and the symbol on the other, for example:
  Side A, "is greater than" Side B, ">"
- Check constantly for comprehension of both mathematical concepts and English meaning. Use charts and diagrams to convey concepts, and then identify and teach the related linguistic terms.
- Occasionally elicit "yes-no" or multiple-choice responses rather than time-consuming English-intensive answers. For example, ask, "Is one-fifth *greater than* or *less than* twenty-five percent?" rather than "What is the relationship between one-fifth and twenty-five percent?"
- Encourage accuracy, not speed. In oral question-answer sessions, for example, insist on a silent period to allow undisturbed processing time for decoding English meaning and computing answers.
- Stress *process*, not answers, and limit word problems to an essential few. To emphasize process and facilitate reading of word problems, use group activities and such instructions as, "Underline the mathematics-operation words," "Circle the data needed," and "List or diagram the steps called for."
- In mixed-English classes, have students with different levels of English proficiency participate in small reading groups. Have one student read the problem aloud, another paraphrase it and explain its meaning, still another select the essential data, and one or two students describe the process needed to solve the problem.

Use a buddy system in mixed-English-proficiency classes. Ideally, the buddy speaks the same native language as the newcomer and can help interpret directions or number concepts. While the less proficient student devotes attention to listening, the buddy takes notes to share.

These are just a few suggestions, that, combined with common sense and a sense of humor, can help educators not only survive but actually flourish when the next immigrants appear in class.

[Mary Holt Kimball teaches English as a Second Language at Klein Forest High School in Houston, TX. With Klein Independent School District mathematics teachers, she is engaged in a curriculum-writing project to help foreign-born students make the transition to English-language mathematics.]

Texas Instruments Calculator Loan Program

To support the training of teachers in using calculators for math education, Texas Instruments lends calculators (in limited quantities) for work-shop presentation purposes. The loans are on a short-term basis, and availability is determined by first-come, first-served criteria. For additional information or to reserve your loaner kit, contact Alva Farmer at (214)917-1550.