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ABSTRACT

This report describes a collaborative effort between a research assistant and a fourth-grade teacher to develop a classroom community for teaching and learning mathematics which fosters problem solving, reasoning, intellectual risk taking, appreciation of diversity, trust, and shared ownership. The characteristics are consistent with suggested reforms in mathematics education. The authors draw upon observations, student and teacher interviews, and student, teacher, and researcher journals to describe the changes taking place in the classroom and the relationship of these changes to students' learning about fractions. Classroom community norms developed as the students and teacher participated in mathematical activities, explored new roles, and began to appreciate the diverse contributions of individuals and the group. The classroom community contributed to students' understanding of fractions, as well as their understandings of new ways to participate as learners of mathematics. Contains 24 references. (Author)

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Julie J. Ricks and
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Center for the Learning and Teaching of Elementary Subjects

Institute for
Research on Teaching

College of Education

Michigan State University

MSU is an affirmative action/equal opportunity institution

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The work is designed to unfold in three phases, beginning with literature review and interview studies designed to elicit and synthesize the points of view of various stakeholders (representatives of the underlying academic disciplines, intellectual leaders and organizations concerned with curriculum and instruction in school subjects, classroom teachers, state- and district-level policymakers) concerning ideal curriculum, instruction, and evaluation practices in these five content areas at the elementary level. Phase II involves interview and observation methods designed to describe current practice, and in particular, best practice as observed in the classrooms of teachers believed to be outstanding. Phase II also involves analysis of curricula (both widely used curriculum series and distinctive curricula developed with special emphasis on conceptual understanding and higher order applications), as another approach to gathering information about current practices. In Phase III, models of ideal practice will be developed, based on what has been learned and synthesized from the first two phases, and will be tested through classroom intervention studies.

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Abstract

This report describes a collaborative effort between a research assistant and a fourth-grade teacher to develop a classroom community for teaching and learning mathematics which fostered problem solving, reasoning, intellectual risk taking, appreciation of diversity, trust, and shared ownership. These characteristics are consistent with suggested reforms in mathematics education. The authors draw upon observations, student and teacher interviews, and student, teacher, and researcher journals to describe the changes in taking place in the classroom and the relationship of these changes to students' learning about fractions. Classroom community norms developed as the students and teacher participated in mathematical activities, explored new roles, and began to to appreciate the diverse contributions of individuals and the group. The classroom community contributed to students' understandings of fractions, as well as their understandings of new ways to participate as learners of mathematics.

"ONE OF US IS NOT AS POWERFUL AS ALL OF US": BUILDING A COMMUNITY FOR TEACHING AND LEARNING MATHEMATICS[1]

Julie J. Ricks and Cassandra L.Collar[2]

The quote above has special meaning for Cassandra Collar and Julie Ricks, and for the fourth-grade students in Cassandra's classroom.[3] This motto, which Cassandra has been hanging in her classroom for several years, seems to capture much of what Cassandra, Julie, and the students learned about building a community of learners who were willing to solve problems, trust one another, take intellectual risks, appreciate diversity, and take shared ownership and responsibility for their learning and learning environment. To strive to develop these characteristics in mathematics learners is consistent with recent calls for reforming mathematics curriculum and teaching (National Council of Teachers of Mathematics [NCTM], 1989, 1991; National Research Council [NRC],1989).

Cassandra's fourth-grade students are a diverse group of learners who brought their own special strengths, interests, and questions to their classroom at Emerson school. In this report, we[4] describe our efforts to put into practice recommended changes in mathematics teaching and learning, and what happened when Cassandra taught mathematics in new ways during the 1991-92 school year.

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[1] The original version of the phrase that Cassandra adopted was "One of us is not as smart as all of us," which Cassandra modified to reflect her concern for students' self concepts.

[2] Julie J. Ricks, doctoral candidate in counseling, educational psychology and special education, is a research assistant with the Center for the Learning and Teaching of Elementary Subjects. Cassandra L. Collar is a fourth-grade teacher in an MSU Professonal Development School (Emerson is a pseudonym). Their work has been supported by the Center and Michigan Partnership for New Education. The authors would like to acknowledge Pamela Schram and Lauren Pfeiffer for their helpful comments on earlier drafts and the Math Study Group participants for their support and encouragement of our work.

[3] We are reluctant to refer to the classroom in this study as "Cassandra's classroom," since we tried to promote a sense of *joint* ownership and mutual construction of classroom activities and norms. However, each phrase we substituted seemed awkward and cumbersome. It is difficult to change a long tradition of teacher control and authority in the classroom, but we attempted to make this goal central to our work and discussions. Adopting new language sometimes helps us to create new paradigms. We welcome any suggestions.

[4] In this report, we use a combination of first and third person. We feel most comfortable telling our story in first person, but for clarity, we also refer to ourselves in third person.

Our Context for Inquiry

Cassandra and Julie have been participants in a Mathematics Study Group at Emerson-MSU Professional Development School (PDS) since the fall of 1989. Over the years, teacher candidates from MSU have done their student teaching at Emerson Elementary School, located not far from the MSU campus, and several teachers have been involved in research studies and teacher education programs with faculty from the College of Education. In the Spring of 1989, Emerson and MSU entered into a new relationship as a PDS.[5] Tomorrow's Schools (Holmes Group, 1990) describes a PDS as a "school for the development of novice professionals, for continuing development of experienced professionals, and *for the research and development of the teaching profession*" (p. 1). PDSs are settings where the nature of university/school relationships are redefined in the context of a restructured school environment and where teachers and other practitioners collaborate with university faculty to improve teaching and learning for K-12 students, improve the education of new teachers and other educators, and make supporting changes in both the schools and universities as organizations. In PDSs, teachers and other professionals are provided with time and opportunities to engage in many reform activities. Assumptions--about what curriculum is most worthwhile for students to learn, about how students learn and what they are capable of learning, and about what forms of pedagogy best promote more meaningful and empowering learning--are continually challenged in PDSs.

The first year that Emerson was a PDS, the Math Study Group was one of three projects organized by school and university faculty based on mutual interests. Cassandra and Julie, along with four other teachers, two university faculty, and two interns, were the initial members.[6] During the first meeting of the group, most of the teachers expressed feeling an ambivalence towards mathematics. They were not there because mathematics was their "favorite" subject or

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[5] For an elaboration of our PDS beginnings, see Rosaen and Hoekwater (1990) and Schram and Berkey (1992).

[6] The transition teacher, one third-grade teacher, and three fourth-grade teachers (including Cassandra) were members of the Math Study Group. Pamela Schram is a mathematics educator at MSU, and Richard Prawat is an educational psychologist. Ricks worked as the research assistant for the project. The interns were two recent MSU graduates, hired to teach mathematics in two third- and fifth-grade classrooms. One of these interns, Karen Sands, subsequently took on additional roles and responsibilities (e.g., classroom observations and student and teacher interviews), and has remained a Math Study Group participant.

because they felt it was a strong area in their curriculum. On the contrary, most of the teachers expressed a desire for assistance in improving their mathematics teaching. They thought the Math Study Group could provide a setting where they could begin to make sense of mathematics teaching and learning in discussions with other teachers and university faculty. Also, Julie was not there because mathematics was her expertise. In fact, she admitted to the group that she participated in a "math anxiety" discussion group at the university which focused on the sources of math anxiety and conceptions of mathematics that perpetuated this kind of reaction in students. The last thing she expected to do as a research assistant was to work on a *mathematics* project, but she saw it as an opportunity to learn about the discipline of mathematics and the teaching and learning of mathematics in elementary school classrooms.

During the first two years of the Math Study Group (1989-90 and 1990-91) participants met weekly or every other week to explore more conceptually based approaches to mathematics instruction, curriculum development, and student assessment. We read and discussed sections from the Curriculum and Evaluation Standards for School Mathematics (NCTM, 1989) and other literature and research about conceptual approaches to mathematics teaching and learning. We designed a measurement and a place value unit, analyzed student interviews, and viewed videotapes of each other's and other teachers' classrooms. We also worked on mathematics problems in an effort to make sense of the mathematics ourselves. Through these collaborative problem-solving activities, the Math Study Group members began to draw parallels between their own experiences and the experiences that students have in the classroom--both the pleasure in figuring something out together and hearing other members' ideas, and the frustration of not being able to articulate one's ideas and of occasionally being stumped by the problems. During each of the first two years, the large group meetings were the primary way that the Math Study Group participants interacted.[7] The university participants (Julie, Pam, Dick, and later Karen) interviewed teachers and students, and observed, audiotaped, and videotaped in classrooms but did not form sustained collaborative relationships with individual teachers.

[7] For an elaboration about Math Study Group, see Schram, Prawat, Ricks, Sands, Collar & Seales (1991).

During the third year of the project, which is the focus of this report, Math Study Group participants explored ways in which university participants and teachers could work in pairs and smaller groups. Near the end of the fall semester, Cassandra and Julie began to have conversations about how they might work together on a more regular basis. Our participation in the Math Study Group provided us with opportunities to think differently about mathematics teaching and learning and about how to study the kinds of changes taking place in classrooms. Cassandra felt that she had gained much from discussions with other teachers and from having a variety of people visit her classroom and share their perspectives about what they observed. Julie felt that observing in a variety of classrooms had contributed to her knowledge of elementary mathematics practices and her understanding of teachers' planning and thinking.

We welcomed the opportunity to develop a sustained, collaborative relationship for a variety of reasons. Cassandra made several changes in her mathematics teaching during the 1989-90 and 1990-91 school years, and studying her practice became an important goal for the 1991-92 school year. She wanted to have someone consistently observe in her classroom, to talk with about ideas to try, and to help her document her own and her students learning. Julie wanted to observe teaching and learning over an extended period of time; learn more about how a teacher plans, implements, and reflects on lessons and curriculum; and learn how to do research in collaboration with a teacher. Working together provided a means for us to pursue these goals. We agreed to work together on "some kind of project" around the changes in Cassandra's classroom, beginning after winter break.

Beginnings of a Collaborative Effort

Before school resumed in January, we met at Cassandra's house to discuss and plan our work together. An important aspect of this and ongoing conversations was simply getting to know each other better. We recognized that changes in our thinking and practice had been developing over a long period of time, beginning before either of us had joined the Math Study Group. Richardson (1990) has noted that a combination of factors must be considered in determining the impetus for, nature of, and consequences of changes in teacher practice. Discussing our personal and professional histories provided us with a way to understand how we'd gotten to where we were now (Ricks & Collar, in preparation). We'd been in the Math Study Group together for two and a half years, and had a sense of the strengths and limitations we brought to our collaboration. We were deeply interested in children's learning and in the ways students felt about themselves as learners. We were willing to take risks and to venture into new territory, both in terms of new roles for students, teachers, and researchers and in terms of mathematical content. We felt that our knowledge and understanding of mathematics was limited, but we were committed to exploring this domain. We were clear at the outset that neither of us had any "answers"; that we would be learning as we went along. We also knew that we weren't working in isolation. We could call upon the other Math Study Group members to help us think things through, and we knew that we could always ask Pam, our "math expert,"[8] for help or for resources.

One of our goals during that first meeting was to generate questions that would guide our inquiry. We knew that our study would be related to Emerson's school-wide focus of "Creating Visions of Literacy," and to the Math Study Group's work plan, which stated the following:

> The development of mathematical literacy for students and Study Group participants is a primary goal of the Math Study Group. To facilitate this goal participants will work to create learning communities that include supportive discourse through
> • collaborative problem-solving
> • intellectual risk-taking
> • the appreciation of group efforts and individual contributions
> • the thoughtful selection, construction, and assessment of meaningful tasks
> • the development of shared language, ways of knowing, ways of communicating, and norms of interaction
> • reflection
> • formative assessment

The questions that guided the Math Study Group's inquiry were as follows:

- What forms might these learning communities take in different classrooms?
- In what ways do these learning communities contribute to the mathematical literacy of participants?
- By what criteria might learning communities be assessed?
- What are some of the alternative means of assessment that can provide insight into students' thinking and understanding of mathematical ideas?

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[8] Math Study Group members often referred to Pam Schram as the "math expert". Over time, Pam helped group members begin to reconsider what it meant to be and "expert,"and to understand that even people with considerable mathematics knowledge were continually learning and exploring in the domain of mathematics. However, we continue to affectionately refer to Pam as our "math expert."

- How can information regarding student learning be communicated to students, parents, community members, and educators?
- How do the roles of classroom teachers, university faculty, teacher candidates and students change to support the development of learning communities which promote mathematical literacy?

These goals and questions reflected the kinds of issues discussed in NCTM's Curriculum and Evaluation Standards (NCTM, 1989) and Professional Standards (NCTM, 1991). The recommendations in these documents represent a view of mathematics teaching and learning that is quite different from traditional elementary and secondary school mathematics (Lampert, 1988). Although these documents helped Math Study Group participants envision *what* their classroom communities might look like, they were less helpful in terms of *how* to go about making the kinds of recommended changes. As Lampert has noted (citing NCTM, 1988),

> Very little research has examined either the intellectually generative sort of mathematical activities espoused by NCTM or the role that the classroom culture plays in the social construction of a view of mathematical knowledge that would support such activities; studies of this sort are needed if we are to understand what it will take to transform standards into practice. (p.7)

We hoped that our through our work together, we would gain a better understanding of what it would take to transform standards into practice. The goals and questions of the Math Study Group provided the overarching framework for our collaborative study. Within this framework, we focused on the teaching and learning of fractions, the unit Cassandra had begun just before winter break and which continued when school resumed. We identified some some resources that would help us think about fractions (e.g., the Standards and various books and articles),[9] and discussed some ideas to try in the classroom, such as using student journals as a vehicle for interaction between students.

Part of our conversation that afternoon involved coming to some understanding of how what were were going to do was "research." We knew that what we planned differed in significant ways from what we traditionally thought of as research, because our roles would be intertwined in ways not typical for teachers and researchers. Others have tried to describe these innovative

[9] Other resources that were helpful included Ball (1990a); Bouck, Jones, and Pierce (1991); Corwin, Russel, and Tierney (1991); and Steffe, Olive, Battista, and Clements (1991).

teacher-researcher relationships (see, e.g., Brown, 1991; Campbell, 1988; Lieberman, 1992), and we knew that there was little consensus in the field about where what we would be doing fit in the standard scheme of research methodologies. We thought of our work together as an inquiry into how to develop, study, and evaluate a classroom learning community and its role in supporting mathematical literacy, in this case, understandings and applications of fraction concepts. We knew that we would both be participants and both be observers interested in the goings-on in a fourth-grade classroom.

With the Math Study Group's work plan as a starting point, we discussed the kinds of things we might need to pay attention to as we taught, observed, interviewed children, collected student work, and reviewed videotapes. We generated several questions, which became the first page of our dialogue journal:

C & J Planning Session 1/5/92
Establishing a focus: Discourse Community

<u>Questions:</u>

•What is the nature of discourse during math?

•Who's talking? When? How often? How long?
% time teacher talk? % time student talk?

•Who are they talking with/to?

•Large Group vs Small Group Discourse

•How is teacher decision-making different when creating supportive discourse community is the focus?

•In what ways can we assess what students are thinking?
Tape record small group discussions
Collect student journals
Interviews

•How do students define and understand the norms of discourse?

•How are students talking fractions?
procedural/conceptual distinction?

•What are they talking about?

•Verbal vs. Written Discourse

•What factors seem to stimulate/inhibit discourse?
activities? grouping?
content issues -- familiar/novel?

•What is the relationship between collaborative dialoguer between adults and collaborative dialogue between students?

•How does T. define and understand the norms of discourse?
Need to be quiet to think
Need to formulate ideas before sharing them
We formulate ideas by listening to others
What we say should be connected to what others

• Review fall student interviews

In addition to generating study questions that afternoon in January, we planned *how* we would work together each week. As part of the PDS restructuring efforts, the Math Study Group teachers had reassigned time each week to pursue project activities, including working with university participants and each other, designing curriculum and assessment, and analyzing and writing about their learning. We decided to meet each Monday afternoon, during Cassandra's reassigned time, to discuss what had been going on in the classroom, review videotapes, plan classroom activities, and perhaps do some writing. Julie would observe in the classroom each Tuesday (other days during the week varied) to take fieldnotes, video- or audiotape, collect student work, and interview students. We would use this data during our Monday afternoon meetings.

At the end of our first "real" working session, we felt excited about pursuing our work together. We felt we had a good start on thinking about the aspects of the learning community that we were interested in studying, and a plan for how we would do that. The next day, school resumed, and we began our journey with enthusiasm and a sense of direction.

Developing Visions of Community and Content

As we mentioned previously, Cassandra made several changes in her thinking about and teaching of mathematics during the 1989-90 and 1990-91 school years. Before we share what happened from January through May of 1992, we provide a brief glimpse of those changes to provide a context for the rest of our story.

Visions of Community

Cassandra's visions had developed over time as she reflected on her past experiences, participated in the Math Study Group, and read and discussed reform documents such as the NCTM Curriculum and Evaluation Standards (NCTM, 1989) and Professional Standards (NCTM, 1991). Often, the Math Study Group focused on identifying and understanding the "big ideas" in a mathematical area (e.g., measurement and place value), and Cassandra sought out problems, activities, and representations that embodied these big ideas, rather than focusing on isolated facts and algorithms. Students often worked in small groups on these problems, then returned to the large group to discuss their ideas, rather than doing individual seatwork. Cassandra began to emphasize having students explain their thinking and the processes by which they got solutions, rather than just giving answers. "How did you figure that out?" "How do you that?" and "Can you prove that?" became common questions as students shared their thinking. She incorporated new words in the discourse around mathematics: "conjectures," "revisions," "agreeing," and "disagreeing" became both ways of talking about and ways of doing mathematics. Building on each other's ideas, making connections with what the last person said, and listening for the purpose of making connections were features of participation that she stressed with students during mathematical discussions. Curriculum planning became more dynamic as Cassandra drew on students' learning and interests to guide her decisions, and she devoted more time to working on math problems, both during the course of the day and by working on a problem or set of problems over the course of several days. By the Fall of 1991, these were salient features of Cassandra's mathematics teaching.

During an interview in October 1991, Cassandra talked about her "vision of a learning community that promotes mathematical literacy":

> My vision for [the students] is to have a vision of themselves as a member of a community of learners and thinkers....They'll see themselves as language writers and communicators ... as thinkers and problem solvers and that they will be able to express this, the sharing through writing and through words and through art.... The second part of my vision is that the students will see themselves as players, you know, as being able to see math and their own role as people who play with numbers.... I want them to see themselves as mathematicians and inventors. That mathematicians are inventors and that they too are inventors.... Part of their language use and writing is inventing things--using this number play to build problems and inventing ways to use their language and to use their written expression and artistic medium to invent new ways to look at the worl around them using number. Another vision is that math is possibility thinking--that math is possible for them--that math has to do with worlds out there and connections that they maybe have never thought of before.

Cassandra's own experiences with mathematics had shaped her visions for students and for her own continued learning:

> As I've grown in the Math Study Group ... in my math classes that I've been taking, and as meditate on the different experiences that I've been going through the last three years.... I've learned to play with number....With my own science interest in inventing, as I've looked at probability and geometry and problem solving as being ways of inventing ... and using my own language and seeing the connections within my own life.... I began to look at what the curriculum had been, what the textbook lays out for the kids and ... there was too much and no depth.... I wanted to get at using talents and strengths that the kids already had at their developmental level--maybe they can write in their own language, they love to draw, and they can talk. Every kid in my class can talk. Understanding themselves as possibilities, they in their own life are constantly everyday creating and inventing and relating to others around them to get what they need, that math is, in that respect, an intricate part of their existence and that it is a possibility for them. I began to see that for myself.... I began to see number as something I can use, I could already talk about it, I could write about it in my journal, I began to build

language for myself about it.... I began to feel more confident and I began to see it as a possible thing, as a friendly thing. Then I began to see if I would approach the students this way ... then all the vision I have for social studies and science and reading is the same vision I have for math.

Cassandra thought that being mathematically literate included being able to do basic operations with numbers--addition, subtraction, multiplication and division--but a more important aspect of literacy was being able to use and communicate about mathematics:

> A person is mathematically literate if they can ... think through problems they have in their daily lives and they can come to solutions.... Part of mathematical literacy is being able to write about it and talk about it with someone else to be helpful because you can do all the subtraction and adding and division and multiplication but if you can't use it to better your environment and to help other then I don't consider that literacy.... So if math is not being communicated ... then I think you're really handicapped. So for me literacy would go more into can they share or share with someone in building solutions.

Cassandra also described the way the learning community contributes to mathematical literacy:

> If the learning community is one in which they listen to each other's ideas and they use appropriate language that's positive and nurturing to each other, then within the community the members are free to be inventors and possibility seekers and see themselves as able to play with numbers, able to consider what number is to them and they have a vision for themselves as well as contributing to a community of supporting one another. But if you don't have this kind of nurturing and sharing amongst each other ... I think people go into themselves and become very selfish and you aren't as strong. I mean, my motto in the classroom is that "one of us is not as powerful as all of us." And I still believe that.... So community is very important all your life, when you're in relationships ... you're problem solving and that's a prime focus of mathematics.

For Cassandra, "transforming standards into practice" meant constructing new classroom norms and participation structures. Much of the recent literature addresses what is involved in this process: Redefining roles for teachers and students; changing expectations for student learning and what is considered evidence of student learning; reexamining and broadening the sources of authority for knowledge; revising mathematical tasks to promote discourse, problem solving, flexible thinking, and reasoning (see, e.g., Ball, 1990b; Lampert, 1988; Putnam, 1992; Shoenfeld, 1992).

Cassandra's vision included some fairly specific ideas about how she hoped she and her students could participate in the classroom. She believed that students needed some quiet time to think and formulate their ideas. Often after introducing a problem, she asked the students to "just think quietly to yourself for a minute about what this problem is about and how you might solve it before you get started working in your groups" (Or as Cindy, one of the students, put it, "She'll give us a couple of minutes to work on it by ourselves and get our own ideas going"). During small-group activities, she expected all of the group members to participate in the mathematical tasks, to work as a team and to either come to consensus or be able to explain why they hadn't been able to. During large- and small-group discussions, she wanted students to feel free to offer their ideas, conjectures and solutions, and to be able to explain their thinking. These explanations, rather than just the answers students gave, were the evidence Cassandra relied on to assess understanding.

She expected students to listen to each other, not only because it is polite, but because this was a way to formulate new ideas. Cassandra also stressed listening as a means for students to "make connections" with what others were saying in the interactive flow of the discussion, in contrast to discourse in traditional classrooms where individual students may listen primarily for the purpose of responding to teacher questions directed at them (Mehan, 1979). Her emphasis on students listening to *each other* also embodied her hope that, over time, she would be able to move away from center stage in the classroom as discussion leader and authority. Cassandra wanted students to be able both to develop and appreciate a variety of approaches to solving mathematical problems and a variety of ways of thinking. She referred often to the motto she had hanging in her room, "One of us is not as powerful as all of us," as a way to promote the idea that she and the students could learn from one another.

Visions of the Content

Cassandra had been thinking about fractions ever since the Math Study Group designed the measurement unit during the Winter and Spring of 1990. In fact, in February of that year, she began to create her own concept map of fraction ideas and topics she wanted to pursue with her students. (See Figure 1.)

As the Math Study Group explored, discussed, and taught about place value the following year, Cassandra continued to think about how she could go beyond the typical treatment of fractions in the fourth grade (a two-week lesson near the end of the year). She decided that the study of fractions could provide a rich context from which she and the students could explore many related mathematical concepts--multiplication and division, place value, measurement, graphing, and sets. Cassandra kept modifying her concept map of fractions, drawing on readings and discussions with other teachers, and on our discussion about fractions during our initial planning meeting in January 1992. At the beginning of January, she drew these concept maps in her journal. (See Figure 2.)

Comparing the February 1990 and the January 1992 concept maps illustrates some of the changes in Cassandra's thinking about fractions. One difference was that "fractions as parts of the whole" moved from being the central idea in her earlier map to being one of several ways to think about fractions in her later maps. Hiebert (1992) and Ball (1990a) have noted that the part/whole interpretation of fractions is the most common interpretation, and the one most often introduced to students in elementary school. Cassandra's broadening conception of the terrain of fractions helped her to consider a wider range of interpretations, representations and mathematical tasks for her students. By the time we started working together, Cassandra had already thought deeply about the connections between fractions and other mathematical concepts.

Throughout our collaboration, we shared the experience of learning the subject matter ourselves. Much of our work together involved sorting out our own understandings of fractions as we explored this domain with students. Sometimes, we felt uncertain about our mathematical knowledge and our interpretations and understandings of fraction concepts. Our concerns about subject matter took different forms because we had different roles in this collaborative effort. For Cassandra, the questions were centered around her role as a teacher: What do I need to know in order to teach students? I want classroom discussions to be shaped by the students' questions and understandings, but what if they venture into territory I'm not ready or able to venture into? For Julie, the questions about subject matter were centered around her role as a research assistant:

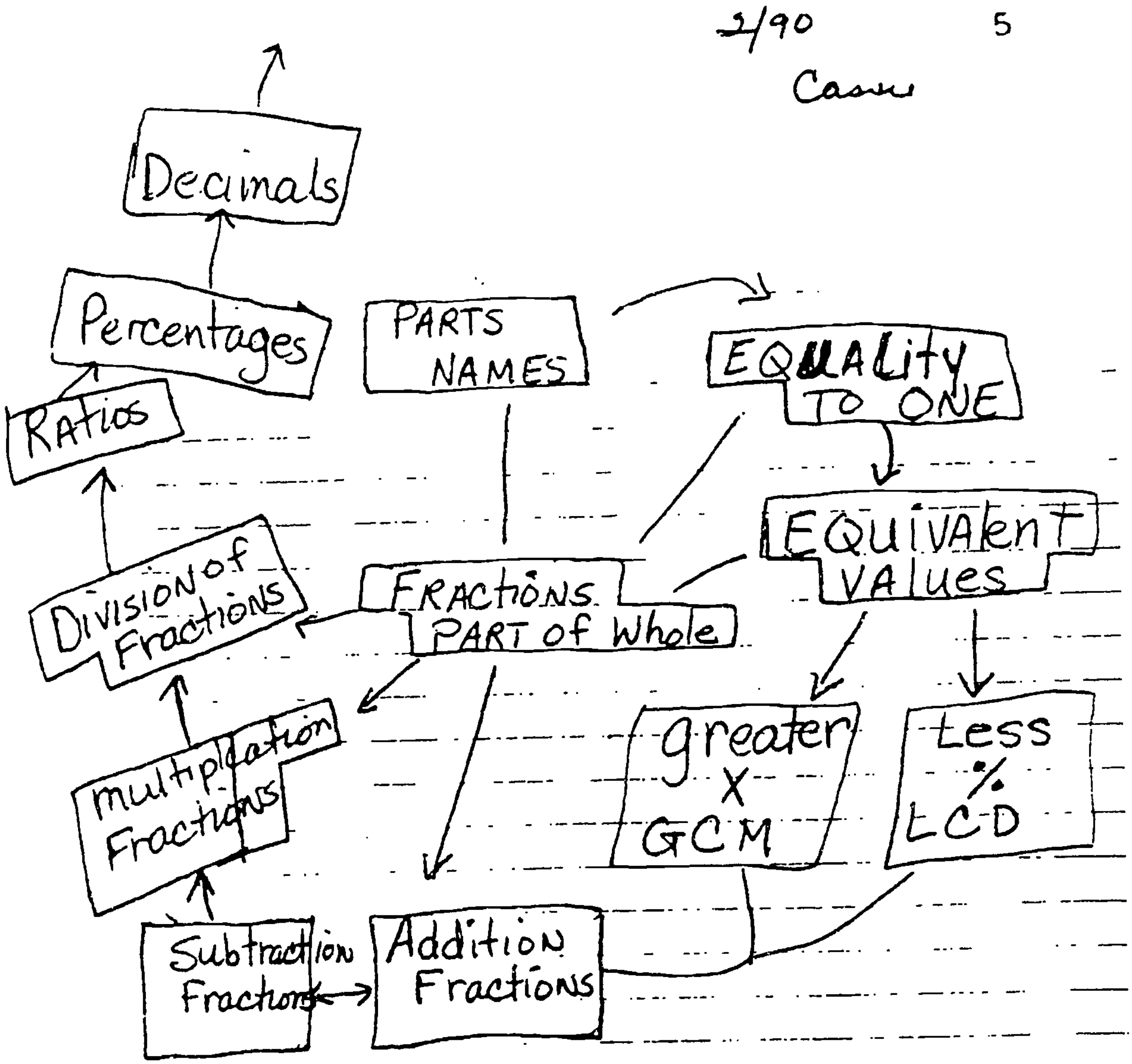

Figure 1. Cassandra's February 1990 concept map.

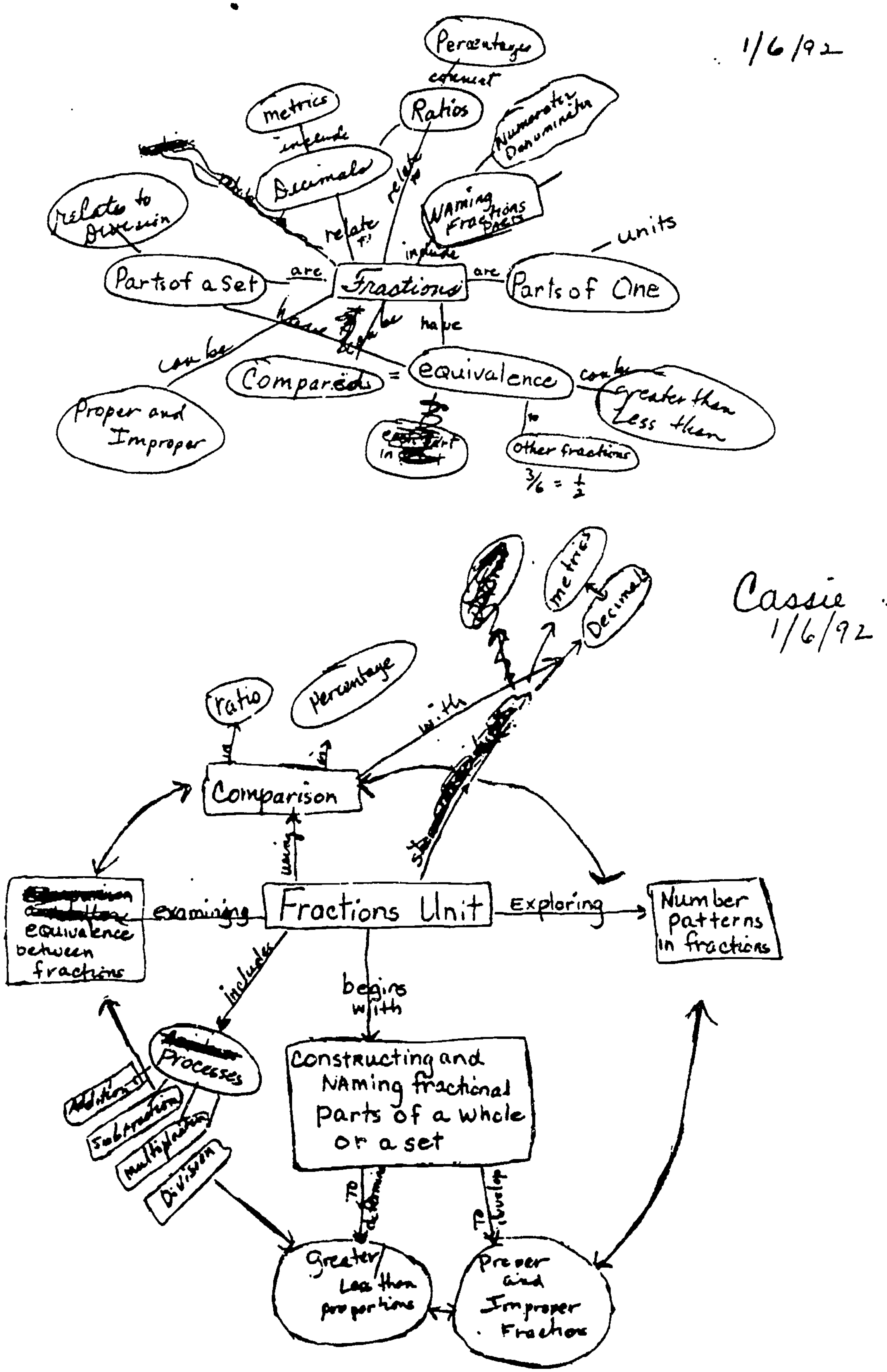

Figure 2. Cassandra's January 1992 concept maps.

How can I help Cassandra think about teaching fractions when I don't have the background? What sense will I make of students ideas and understandings if I don't know the terrain of fractions myself? Will I overlook important aspects of mathematical content in the classroom interactions? Or might I see things that a "math person" would take for granted?

We knew that the questions we had about subject matter knowledge were not uncommon; in fact, it was a recurring theme in much of the reading we'd done about teaching and learning for understanding. Again and again, the issue of teachers' subject matter knowledge was raised as a concern if teachers were to take on new roles as guides in the mathematical territory (see, e.g., Ball, 1990b; Lampert, 1988; Prawat, 1989). Rather than interpret our limited knowledge as a roadblock, however, we welcomed the opportunity to become mathematical learners ourselves. We tried to pay attention to our experiences as learners and use these as a lens through which to understand students' learning experiences. We sometimes wanted someone to give us the answers. The students have also at times wanted us to give them the answers. We often came up with multiple interpretations of mathematical problems and were surprised that there was not "one right answer," or that there were many approaches to getting an answer. In turn, we emphasized this aspect of mathematical learning with our students. We knew that it sometimes took us a long time to solve complex mathematical problems and that our understandings of mathematical concepts changed over time. We assumed that the students would also need to have extensive time to work through problems and chances to revisit ideas several times. We also at times felt frustrated, helpless, and wanted to give up. We anticipated that some of our students might feel this way too. We wanted students to experience the excitement we felt about becoming mathematical learners and knew that they would need a supportive and invigorating learning environment in which to do so. It was these visions of the community and of the content that we brought to our collaborative inquiry.

Building a Community Around the Exploration of Fractions

Earlier, we described features of the learning community that the Math Study Group participants were working towards and the questions that guided our inquiry. In this section, we share what we have learned about these goals and questions, through a presentation and analysis of classroom vignettes, excerpts from student interviews and journals, and selections from our individual and shared journals. In the final section, we consider issues and questions that will continue to guide our work.

Constructing Classroom Norms

In mid-December, Cassandra gave each student a "fraction packet," consisting of a set of worksheets which used a circle representation to show 1 whole, 1/2, 1/3, 1/4, 1/5, 1/6, 1/8, 1/9, 1/10, and 1/12. (Seymour, 1988). During the weeks before winter break, the students used the fractional pieces they had cut and colored to investigate how the unit circle could be divided into many different "fraction families", as they were referred to in the worksheets.

"When I say 'fractions,' what comes to mind?" was the question that Cassandra asked students to write about in their journals on the first day back after winter break. Many of the students began by listing some fractions--1/2, 1/3, 1/4, 1/5, and so on. Most wrote words related to the part/whole aspect of fractions (whole, unit, pieces, parts, slices) and to the action involved in getting fractional parts of a whole (cutting, splitting, dividing, broken up). Some students also included words such as "equal," "congruent," and "same shape," and many indicated that fractions are a "part of math," involve "numbers," and can be used in "math problems". Several listed operations (multiplying, adding, subtracting, and dividing), and a few children listed fraction terms such as "numerator," "denominator," "mixed numbers," "improper," and "decimal". Some of the children noted the word "shapes" and/or listed particular shapes, especially "circle," which is the representation they'd been working with prior to Christmas vacation.

Cassandra asked the students to make a concept map in their journals showing how the ideas they had listed were related, then the class made a group concept map. The following week, Julie interviewed students in pairs about the concept maps they'd drawn in their journals. During the interview with Jenny and Tammy,[10] Jenny illustrated the relationship between the unit and the fractional parts by drawing a circle and then dividing it into five wedges. Julie asked her if there

[10] All student names are pseudonyms and, to the extent possible, reflect students' ethnic backgrounds.

could be any number of parts, and she said, "Just as long as it's not seven, or ...what's that other number?" but couldn't explain why she thought that. When asked if she had any questions for Jenny, Tammy clarified Jenny's earlier statement, "No, but I think that when she [Jenny] said that, um, you can't do seven--well, when we were doing our fraction booklets, there wasn't [sic] no seven or elevens it it."[11] Tammy's explanation brought to the surface our own questions about the possibilities and limitations of various representations for highlighting various fraction concepts. We were concerned that Jenny's understandings may have been limited by the particular representation they'd been working with, which *provided* students with the fractional parts of a given circle. The following week, Cassandra decided to have students construct sevenths and elevenths pieces for the unit circle they'd been using in their fraction packets.[12]

In small groups, the students generated strategies for creating sevenths or elevenths pieces for the given unit circle. The following day, the small groups shared their ideas. Cassandra asked the children to form a "gallery"[13] by arranging their desks in a small semicircle to create space in the center where the small groups would sit as they shared their ideas.

> OK, why I'm arranging things this way is so that all of you can have a chance to listen to this group. We haven't tried this before. Every one of the groups had an opportunity to think about their own strategies for how to create--on our circles--a 1/7 family and a 1/11 family. Every group is going to have a chance to get their ideas out. We want to hear everybody's ideas. Maybe there's more than one solution. We need to know that.

She explained that the people in the gallery should take notes in their journals as they listened to each group, so that they could ask questions of the small group members. Cassandra sat on the floor, in the open end of the U, facing the group, and asked the first small group (Rick, James, Aaron, Matt, Leslie, and Mary) to come to the center. Rick was designated by the group members as the spokesperson, but Cassandra reminded them to "make sure the rest of the group talks also." Rick explained the group's strategy:

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[11] In the fraction packets, representations for 1/7 and 1/11 were omitted.

[12] Cassandra's decision to have students construct fractional pieces was based, in part, on our discussions about Ball's (1990a) notion of "representational contexts", which addresses the advantages and disadvantages of having students construct their own mathematical representations.

[13] Normally, desks are arranged in a double row, square-shaped U, so that students can see each other as they are talking. However, Cassandra felt that students in the back row of the U still felt far away, and she wanted to experiment with a more intimate arrangement.

Rick: So far we've got three sevenths. And we made it by dividing 1/7--wait ... (Rick opens his journal, then Aaron hands him a piece of paper, which Rick reads.) One divided by 7....We found out that it was 0.1428571. We needed to convert 0.1428571 into a percentage. I ended up getting 53 degrees.

For the next several minutes, Cassandra questioned the group about this strategy. Rick did most of the explaining, even when Cassandra urged the other group members to participate:

Cassandra: What does he mean group? What were you doing? If this was his idea and you were going along with it, what did you put in your notes? Anybody in the group may speak out. Just these people aren't going to talk right now [referring to the students in the "gallery"]. Anybody in the group have any ideas? What did you put? (long pause)....You don't know what he was doing? What were you doing, James?

James: We need to convert 1/7 into decimals.

Cassandra: Why?

James: Because we wanted to come up with the size of the pieces that would be 1/7.

With much effort, and many questions from Cassandra, Rick was eventually able to explain his procedure: Divide 1 by 7, get a decimal, convert the decimal to a percentage, use this percentage to figure out how many degrees of the circle the 1/7 piece would be, and then use the protractor to measure the 1/7 pieces on the circle. At several points throughout the exchange which was primarily between Cassandra and Rick, Cassandra tried to pull in other group members, "Did anybody in the group write down what he did on the calculator? to help him out? He needs some help-- you're letting him run the whole group". When no one in the group responded, Cassandra turned to the gallery, "Does anybody in the gallery have questions for this group so far?".

David: Why did you divide 1 divided by 7, because on the protractor there's 360 degrees--divided by 7.

James: We did that first but I think Rick didn't want to do it.

Rick: No, I was doing something wrong--I was pushin' percent [on the calculator] instead of divide. So I needed to do that over again.

Cassandra continued questioning Rick about the strategy that he'd initiated in the small group, often repeating and rephrasing his explanations. She also took the opportunity to open up a discussion about the process of working together in groups.

Cassandra: OK, do you have any questions to ask each other? What questions might you put in your journals? Talk to each other. I'd like to hear what questions you're gonna ask each other before you start again.

(Leslie points to Mary's journal and motions to Mary, as if urging her to speak)

Cassandra: Mary, you had an idea you want to share with us?

Mary: I had an idea yesterday that if I traced my 1/6 piece and then I put my 1/8 and I traced my 1/8 piece over my 1/6 it has to be smaller than the 1/8.... It's hard to explain. If I put a line between both of them ... where 1/7 would be.

Cassandra: Did that work?

Mary: I'm not really sure yet.

Rick: I tried that before, but six of the seven pieces looked like 1/7, but the last piece was 1/8--was equal to 1/8.

Cassandra: What questions would you like to ask each other about that? Or what suggestions might you pose to each other? (long pause, no response) Can I ask you another question? How hard was it to work as a group on this problem?

(In unison): Hard!

Cassandra: Can you offer--would you please share in your group so the rest of the students can hear--what your problems were? What do you think made it hard?

Rick: No one was really helping me

Leslie: When Rick was writin' down all this stuff, some of it didn't make sense to me, and he kept saying it and saying it, and every time he said it, it got me more confused. And so you ask what he was doing and he tells us, you just get lost.

Cassandra: Talk to each other. Explain to each other what you'd like to have different when you come back to work together. James, what do you want to have different?

James: We need more protractors--like five--so that all of us would have protractors and not just him.

Rick: There was only one protractor.

Cassandra: So you need more protractors. OK, Matt, what would you like to see different, that would be more helpful next time you work together?

Matt: All of us should be helping, not just one person.

Cassandra: Leslie?

Leslie: That one person won't be doing all the talking and so if he was going slow enough so that we could understand.

Cassandra: Mary?

Mary: If somebody's doing something and you don't understand, you can always ask them like a little more questions to get more information out.

Aaron: I'd like Rick to tell us what he's doing.

Cassandra: How do you get that to happen? How do you get someone in your group to tell you more?

Aaron: Ask him.

Cassandra: What might you ask Rick--from what you know now, what you've been doing--James, what might you ask Rick to help you understand more?

James: Rick, can you please explain what you're doing?

Cassandra: What if he does? Then what could you say?

James: Say thank and move on

Cassandra: But what if you *don't* understand? Aaron?

Aaron: You can ask him to stop, and ask him how he's doing this stuff.

Leslie: You could ask him to please say it in words that we can understand.

Cassandra: What if he has explained it in words that he thinks you understand?

Leslie: Tell him you don't understand the words he's saying.

Cassandra: Could you ask, "Is there another way that you could share this? Could you draw me a picture?" Would that help, those of you in the group?

(Everyone nods)

Cassandra: Should Rick have to be the only one doing all the thinking in the group?

(Everyone shakes heads no)

Cassandra: What makes a group?

James: Helping each other.

Cassandra: Does asking questions of each other help you?

(Everyone nods)

Cassandra: Ok, we're going to stop with this group, because I see they have more work to do, and they've got some ideas to get started.

Without having closely watched this small group work together the previous day, we nevertheless got a glimpse of what happened when they approached the task of dividing the unit circle into sevenths. For Cassandra, this glimpse provided an opportunity to address some issues about participation that she felt were important. It was clear that Rick came up with an idea that the rest of the group didn't really understand, that he did calculations that didn't make sense to them, and that he was the only one who could attempt to explain the strategy. In urging the small-group members to think through the problems they'd had in working together and the changes they could make, Cassandra was making explicit her expectation that the students work collaboratively. Issues of ownership, control and authority also surfaced. We suggest that a clear message in this exchange was, "If you don't understand something, you have not only a right, but a responsibility to ask for clarification."

The other small groups each had an opportunity to share their strategies for finding the 1/7 and 1/11 pieces, and in each, students had used some variation of an estimation strategy to find the fractional pieces:

Beth:	What we were doing was, I asked Cindy what she was doing cause she was gonna make 1/11, and she took her 1/10 and 1/12, and she put 'em over top of each other, and then she put 'em down on her paper, and then she traced her 1/10 and then she took it off and then she traced her 1/12 and she said that she found out 1/11.
Cindy:	I just made a line in between the 12th and the 10th, right in between there because it has to be somewhere between the 10th and the 12th.
Jacob:	I was doing 1/7 and I took my ruler and took my 1/10 and I made it just a little smaller than the 1/10--I mean I made it just a little thinner than the 1/6 and I used my ruler and made it so it was all sevenths right here (shows his journal page) ... take the 1/6 part, fold it just a little, and then trace around that.
Carl:	Yesterday we did basically what Cindy's group did, we measured between the things, but we used a compass. And we measured with a ruler how it would be exactly in the middle.
David:	We used the compass and we marked spots till we got all the way around
Cassandra:	What did the spots represent?
David:	Where the lines were going to be. We drew lines to the middle, then we measured 'em again so the lines would touch.
Cassandra:	How did you know where to put the spots? How many lines to make?
David:	First we drew a circle, and we came up with six pieces, so then we made it a little smaller,but it wasn't small enough, so then we did it a second time.

Cassandra: My question is, were you estimating--making a guess--where those lines should be? You said you had some measurement tools. Did you decide that every five degrees you would make a mark? How did you decide that?

Carl: Well, we measured right between the 1/6 and the 1/8 and put the compass right at that angle, and we kept on moving it in until we got it at the right place.

David: We used the compass to check.

Antonio: If you make a big circle, and if you cut--make two pieces real big and the other two as small as you want the pieces, and then cut the big one till the pieces are the same size as the other two.... I tried it but I get all the sides the same size, but I gotta get two of the sides bigger than the other two.

Cassandra: What's the bigger have to do with it?

Antonio: The bigger half so we can cut the bigger half.

Cassandra: You make the circle bigger than normal?

Antonio: The piece of the circle--you would have to cut that one into a half until you get the size you wanted.

Cassandra: So you think that cutting the circle into a large piece, and then cutting that piece into halves, you'll eventually get to 1/7.

(Antonio nods)

Sarah: We also had the idea with a square--we thought that if you drew a line in the middle for half, and then drew a line for 1/4 and keep doing that you can get 1/7.

Throughout the discussion on this day, Cassandra focused on what students might learn from each other (e.g., "Will you be able to use any of the ideas you heard when you go back to work in your groups?"), rather than on determining what strategies were best. All of the ideas were accepted as viable alternatives to solving the problem that was posed, at least until the student had a chance to work further in their groups. Students had taken intellectual risks by formulating strategies which involved the use of new tools (compasses, protractors, and rulers), and which involved putting together their as-yet fragile ideas about fractions (e.g., continually dividing the whole into halves to get to the size you need). Sharing their ideas with the large group helped the students reflect on their strategies and plan their next steps. This shared reflection is a kind of formative assessment that helped both the students and us to examine their learning, especially since they returned to this problem several times during the following weeks.

We can't claim that the discussion this day caused changes right away in the ways that students worked together in groups. Changing classroom norms was an ongoing process; it took many more experiences and many more conversations before students approached group work *as* a group. Over time, however, students adopted strategies for hearing everyone's ideas during small-group discussions ("Let's go around and say our ideas, OK?") and for documenting what they were doing ("I'll write down the ideas for when we share"). We also noticed changes in the ways they presented their findings to the class. More often, the *group* was the referent for ideas ("We thought..." or, "In our group, we..."), students referred to one another's ideas ("Katy thought if we kept folding it we'd get the 1/8 piece"), and they were comfortable describing their disagreements ("We never really decided, because Lisa, Kelly and Scott thought that congruent was the same as equivalent and me and Jenny thought they were different things"). In time, students began to formulate and articulate their meanings of the changing classroom norms. Their interactions with one another and with Cassandra changed; One of the most salient features was the students' view of the power of the group in solving mathematical tasks.

One of Us Is Not as Powerful as All of Us

In a discussion about creating and using community in her teaching, Ball (1990b) speaks about developing "each individual child's mathematical power *through the use* of the group" (p. 19). We're not sure that Ball would agree with the motto which hung in Cassandra's classroom, "One of us is not as powerful as all of us" but for the students, this motto captured what many of them had come to value about being in this classroom. Children frequently offered a "more heads are better than one" explanation for why they shared ideas in large and small group discussions, why they worked in small groups, and why they needed to listen.

During an interview in May, we asked students to describe "What it's like in your classroom during math"

James: The teacher tells us to go get our math journals and a pen and a clean piece of paper. Then she starts talking about fractions and we get into a big discussion.... Leslie starts out with like a agreement like someone says something and she says, "I disagree with that" and we all start going, "Yeah, I disagree too," and then we get into this big discussion like "I disagree" and "I agree" and we just talk about fractions.

Tammy: When people have different ideas we just stop and talk it over.

David: We just try to give each other ideas ... we figure out problems and put out ideas.

Cindy: Sometimes we write a paragraph about what would help other kids....We have posters, like "Make connections."... A lot of teachers just tell the answers.

Carl: Sometimes--very rarely--we get out the math books, like as a reference. Some people agree with one person, some people agree with another. We raise our hands and tell why we think that ... then another person would make people stronger on their ideas.

Debbie: Mrs. Collar asks the class what they think ... sometimes we find out like that we were really trying to say the same thing.

Their descriptions emphasized discussion, sharing ideas, and solving problems together. These same themes were reflected in the students' journals,[14] when Cassandra asked them late in January to write about what they envisioned as a "perfect math class".

> The class seems like it is just one big brain and it makes it easier to work with everybody then alone. Sometimes it is nice to work alone but it is not as fun.... Challenging problems are fun because they make you think more and so other people get different problems so you get to work it out and get a problem together. (David)
>
> They're writing thier ideas, thier papers are filled with ideas.... And everyone is working very hard.... And they have big groups and their sharing ideas, and combinging them together to get a right answer. And thier all going to get back to thier desks and going to put their ideas on the chalkboard. Thier trying to make a 1/20, and thier having fun doing it. I think that they must like math a lot. (Sarah)
>
> The room is filled with numbers. They are all working on one graph. A picter graph. I would feel comtble being with all those people sareing there iders, with others. Every one is in a group shareing there iders with ech other. (Leslie)

Students also viewed their group work as a means for learning from each other. During an interview in May, we asked, "Why do you think that sometimes the whole class works together, sometimes you work in small groups, and sometimes you work alone?"

James: When we're in groups it might be a little harder for you to understand so that's why you have partners and stuff.

Debbie: Because she has a sign that says, "One of us is not as powerful as all of us."

Cindy: Certain people learn better by working individually. I like to have conversations.

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[14] These excerpts are written as they appeared in students journals; the misspelled words are not typographical errors.

Tammy: Because she could put a small group with something they don't know about. More people could help.

Carl: If it's something that's hard for a lot of people, we'll do a large group. If it's hard for just a few people, small groups.

Discussions about listening surfaced in Cassandra's classroom, as in most fourth-grade classrooms, at times when too many people were talking at once. During one class discussion, Beth was trying to explain a pattern she noticed in the denominators of the fractions that were in their fraction packets. Several students in the back row on one side of the room were talking. Cassandra asked the students in the back row to "join our thinking," and asked, "Why is it important to be listening to the people who are talking?" The students' comments, we think, reflected a purpose for listening that is different from "being ready to show the teacher what you know," a purpose often perpetuated in the question-answer format of most traditional classrooms. What we heard was that students are listening to learn, to understand, to explain, to make connections, to challenge, to concur.

Molly: If you listen to them--the person who's talking--then you can understand more.

Amy: So you can make connections.

Cindy: So you can learn from what other people say

Antonio: If someone new comes in and wants you to explain it to them, you'll know what to tell them.

Sarah: If you're like playing around and you're not listening, and then you start listening and then they[15] call on you and you were listening before, but not way back before, you don't know what to say.

Beth: If you didn't listen, we might start to bring up an important part. You might miss an important part and you might miss the whole thing.

Cassandra: You need to consider their ideas and see if you agree or disagree with these ideas. You might agree and you'll come together with that other person and be stronger. Or you might disagree and cause that other person to think about their idea more and you both will grow.

The students' understandings of classroom norms for working together, for sharing ideas, and for listening were constructed as they participated in mathematical tasks and activities.

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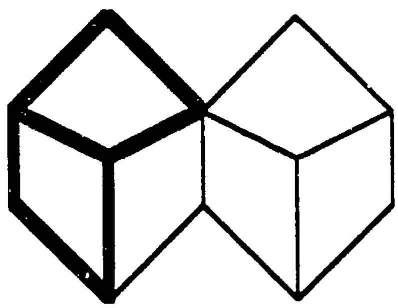
[15] During most discussions, the students called on each other, rather than Cassandra calling on students.

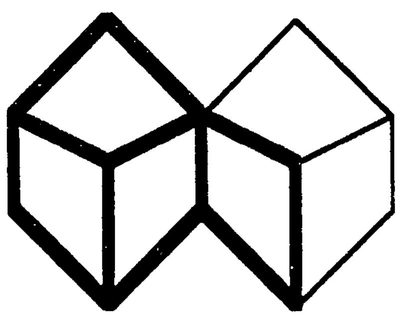
Expectations--both Cassandra's and the students--were woven into the fabric of mathematics lessons; they were contextualized in terms of the mathematical experiences students had. Often, classroom discussions zig-zagged between talking about *fractions* and talking about *classroom norms*. During a lesson in mid-May, for example, the students worked with pattern blocks to fill in a whole region which was the shape of two hexagons side by side, as a way to review the idea that what constitutes fractional parts depends on the whole you start with (i.e., for different-sized wholes, various pattern blocks represented different fractional parts). After working through the problems on a worksheet, Leslie commented that it had really helped her to draw in the lines on the figures to represent the fraction pieces, that this really helped her understand. Cassandra said she'd noticed several students doing that, and was glad they were comfortable using the strategy of "drawing pictures to help us with our thinking." Cassandra asked if there were any other strategies that helped them. Mary suggested that when she worked with other people she understood better; this initiated a discussion about the about working together. Leslie agreed with Mary, saying, "It's kind of like that slogan you have up there--'One of us is not as powerful as all of us.'"

Later in the discussion, Leslie commented that different people were good at different things, "Like Debbie might be better at math than me, but Mary might be better at reading than Debbie." Beth said, "I have something to add to Leslie's" and suggested that the strengths in different areas were like a scale that could be balanced out "if you lived close together and you could study each night." Rick explained that in other classes he'd been in, it was more important to do your own work and not worry about if other people understood, saying, "Personally I don't really care about the people who don't understand." Cassandra commented, "That happens a lot. Some people here voiced the opinion in our discussion earlier that they appreciate it, and it makes them feel that it's been a learning experience all the way around for the group." Cindy agreed with Beth, "If you have a group, and someone's good at math, someone's good at spelling ... and you put 'em together, and they all help each other, soon everybody will be good at everything." Leslie countered, "Rick, I kind of disagree with your opinion. I mean, I wouldn't want to not help other people, because like I said in the beginning, if you help someone, maybe you discover something new. You can learn from them and also help them. And you can discover how they think". Rick, however, maintained his position: "I don't care about how they think."

Cassandra ended this part of the discussion by suggesting that "because we don't live alone, we need to be able to communicate," then announced, "We're going to go on now with our fractions." Using the double hexagon on their worksheet as the referent, she asked students to determine for different pattern blocks, "How many are in the 'family'?" (e.g., How many diamond shape pattern blocks fill up the double hexagon?) and "If only two of the family are home, how would we write that fraction?" (e.g., 2/6). When she asked "What if four of the family are home?" Rick went to the board and wrote 2/3. Cassandra asked him to explain what he meant, and he drew a representation of the double hexagon on the board, showing where the six diamonds would fit. Then he shaded two of the diamonds and said, "1/3rd would be that one ... or 2/6":



He then shaded two more diamonds and said, "And 2/3 would be that one ... or 4/6ths":



Leslie raised her hand, and Rick said, "Why you don't understand, Leslie?"

Leslie: 'Cause, Rick, you said *one* third, then you drew these *two* (comes to board and points to the left two diamonds) You said 1/3 is equal to this one and then you colored in these (motions to the two diamonds) and then you colored it over.

Rick: Yeah, both of 'em is 2/6

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Leslie: You only said *one* third, though!

Rick: 1/3rd equals both of these (points the the left two diamonds).

Becky: (comes to board) Well, he's saying that 1/3 is equal to 2/6 because there's 1, 2, 3, 4, 5, 6 pieces (counts and points to each diamond) ... and then there's--he colored in two of them--two sixes, and he said it's equal to *one* third.

(Leslie shakes her head)

Rick: You understand now?

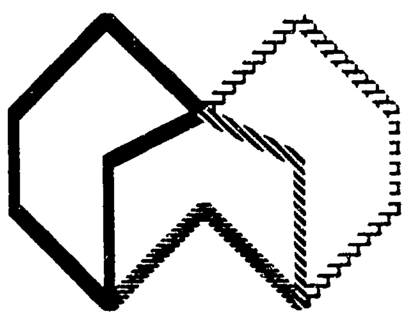
Leslie: No

Cassandra: What does 1/3 mean, Leslie?

Leslie: 1/3 is one out of three things.

Cassandra: Ok, Rick, can you show her one out of three things up there?

Rick demonstrated by erasing the lines that showed where the six diamonds were on the figure, and reshading the three areas using different colored chalk, so that the figure now looked like this:



As Rick worked on the drawing, Cassandra commented, "Rick's doing a very patient job; he's already said to us quite openly that he doesn't enjoy doing this, and yet I think he's doing a very patient job of helping us to understand" For several more minutes, other students explained Rick's idea in their own words, often coming to the board to refer to the drawing, until most of the students agreed that they understood how 1/3 and 2/6 covered the same area on the hexagon.

This lesson had zig-zagged from work on fractions, to talk about working together, and back to talk about fractions. For many of the students, their beliefs about and understandings of classroom norms were influenced by the fact that they were experiencing new roles and constructing new niches for themselves in the classroom. They had begun to appreciate the diverse contributions that they could individually and collectively make to the classroom community.

Creating Niches: A Place for Everyone?

One of the dilemmas of being a teacher is to design a classroom community in which all students can participate. Cassandra, like most teachers, had a wide range of students who brought different prior experiences, attitudes, beliefs, assumptions, and abilities to bear on the ways that they participated in the community. We tried to pay attention to the ways that students modified their roles as we explored new ways of interacting together.

When we interviewed students in November, many of them named two of their peers--Rick and Davi--as "someone who is good at math." Rick and David were frequently treated as "experts"; other students deferred to their ability to solve problems and get answers quickly. However, Rick and David carved out quite different niches as the classroom experts. Rick claimed, "I don't really care about the people who don't understand" and he couldn't really see how working with others was helpful. In contrast, David wrote in his journal that the perfect math class

> seems like it is just one big brain and it makes it easier to work with everybody then alone. Sometimes it is nice to work alone but it is not as fun.... Challenging problems are fun because they make you think more and so other people get different problems so you get to work it out and get a problem together.

More often, David moved the spotlight from himself to others in the group--urging Debbie to show her drawing, handing Carl the notes the group had taken.

As we watched and listened to children, we noticed them exploring and creating many niches, as they participated in mathematical activities. In February, Julie noted the following in her journal about the interaction between Jacob and a new student, Scott.

> As far as I can tell, Jacob started out helping Scott first, then Aaron joined in. At some point, Jacob or Aaron (or both) called Rick over to help them explain things to Scott. I think Jacob said something like, "Rick'll have to tell you". Jacob seemed to be deferring to Rick's "expertise" for some reason. Yet, Jacob was the one who stuck it out with Scott for the whole time. (Rick and Aaron wandered away--did they get frustrated?) Jacob was the one who tried to help Scott understand the different fractions by connecting them to a real-life example. And Jacob was the one who kept trying different avenues for making fractions meaningful to Scott. Rick may have the expertise, from Jacob's perspective, but

Jacob was patient, persistent, thoughtful and resourceful in this interaction with a new student.... Somehow it seems important to acknowledge Jacob's kind of contribution to the community and to the knowledge being constructed. Jacob's contributions are as important as the kinds of contributions that students like Rick and David make when they offer ideas that push and challenge the class's understandings. Both kinds of contributions are important.

Over time, "being good at math" was a niche that began to take on new dimensions. We noticed that, as the norms for discourse changed, many students seemed to blossom as they tried out being questioners, challengers, discussers, helpers. For example, in January Leslie seemed frustrated by the way that Rick explained his procedure for finding sevenths,

When Rick was writin' down all this stuff, some of it didn't make sense to me, and he kept saying it and saying it, and every time he said it, it got me more confused. And so you ask what he was doing and he tells us, you just get lost

But she never challenged the veracity of his approach. By May, however, Leslie challenged his assertion that 2/6th and 1/3rd were the same, "Cause, Rick, you said *one* third, then you drew these *two* ... you said 1/3 is equal to this one and then you colored in these ... you colored over it."

When we interviewed students in May, we saw evidence that they were beginning to value different ways of participating in mathematical discussions and activities. Many students still named Rick and David as good at math, but more of them included other students as well.

Carl: Sarah asks a lot of questions and that's how she gets good at math.... And Cindy--sometimes she proves that other people are wrong, and she's right ... and Leslie--these are the people who bring up discussions.

Cindy: Sarah--she asks a lot of good questions and knows a lot about fractions. She asks questions and makes good points. When we get in math discussions, we both talk a lot.

David: Lots of people are pretty good at math. Rick, Carl, Stephanie--They know how to explain if someone needs help and they know how to work on problems.

Students had begun to make sense of, accept, and help shape new ways of interacting in the classroom. In many ways, the community that we have described in the sections above are consistent with Cassandra's vision of the learning community and the features outlined by the Math Study Group. Students, for the most part, valued working together on problems, recognized the importance of communicating their ideas, and had opportunities to invent and explain their own ways of learning about fractions within the context of challenging tasks. Their experiences, we think, also contributed to the students feeling confident in their own abilities and to their mathematical literacy in the domain of fractions. We've provided only glimpses of the fraction unit which continued from mid-December through May. During these months, Cassandra and the students explored a range of concepts, using a variety of representations. For the most part, the focus was on the interpretation of fraction as a part/whole relationship using both regions and sets as the whole. Their fraction packets (a circle), geoboards, fraction bars, pattern blocks, and a variety of sets generated "representational contexts" (Ball, 1990a) from which they discussed part-whole relationships, patterns, equivalence, congruence, terminology (e.g., numerator and denominator), and the connections between concrete materials, pictures, and symbols.

Near the end of May, we had an opportunity to better understand how students pulled together the ideas about fractions that they had been exploring for over five months and how they felt about their learning.

Mapping Out the Terrain of Fractions

As we mentioned, in January the students made individual fraction concept maps in their journals, and then created a composite concept map. (See Figure 3.) In May, Cassandra again asked students to make a concept map of fractions in their journals, and again, they constructed a composite map. Their discussion, which spanned two days, revealed ways that students were connecting ideas, as well as ways that they were still wrestling with particular concepts.

After they had constructed the concept map on a piece of chart paper, Cassandra told them, "All right, I want to show you something. This is what we know now" (referring to the map they'd just constructed that was hanging on the wall--see Figure 4.) She continued, "You ready?", and removed the chart paper from the wall, unfolding it to reveal the concept map they'd constructed in Janaury so that it was adjacent to the current concept map. "That's the way your chart looked when we started back months and months ago":

At this, the children responded with "Ooohs," "Wows," and "Hey, that's pretty good!" Cassandra told them, "How you feel about this right now is almost as important to me as what you

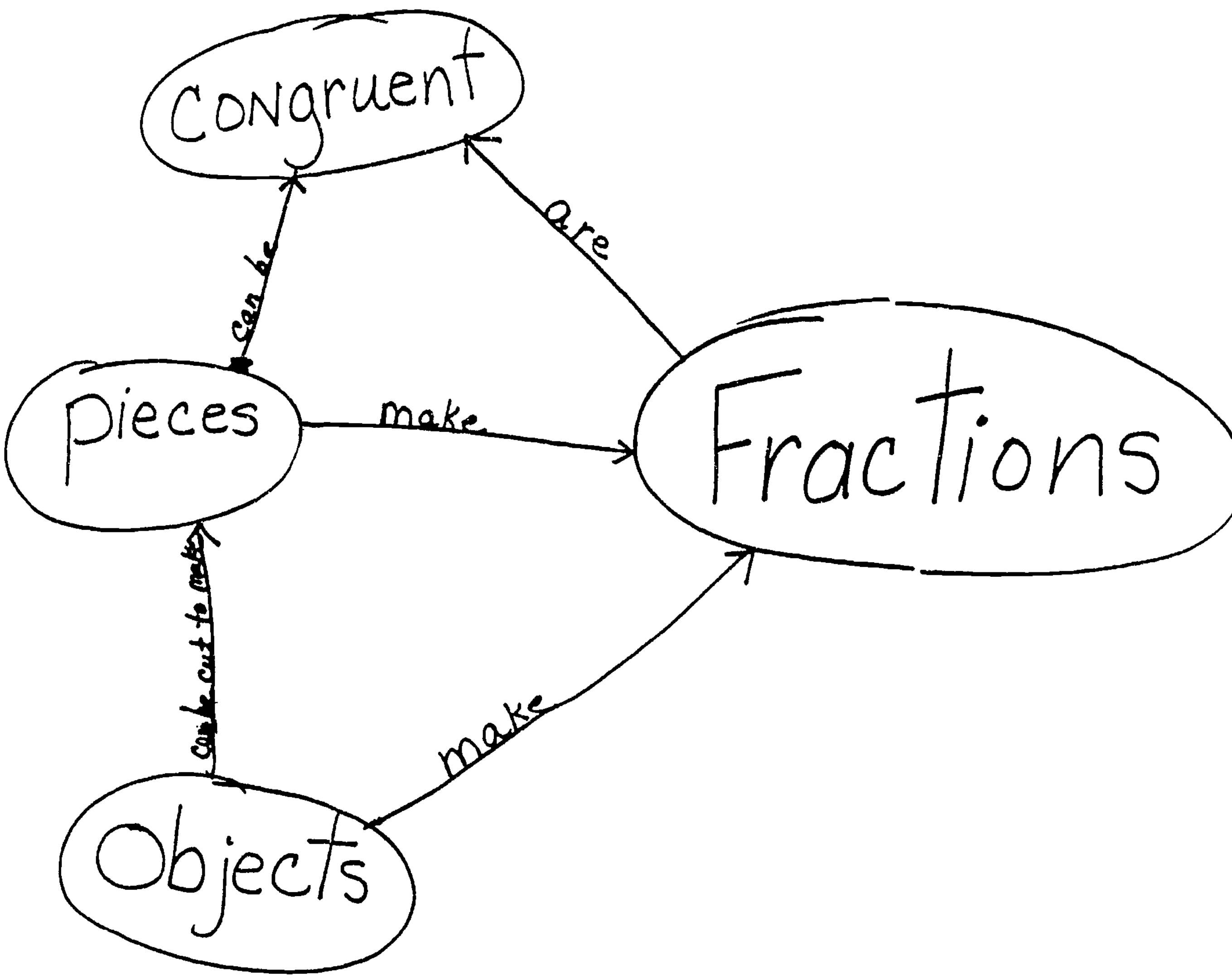


Figure 3. Students' January 1992 composite concept map.

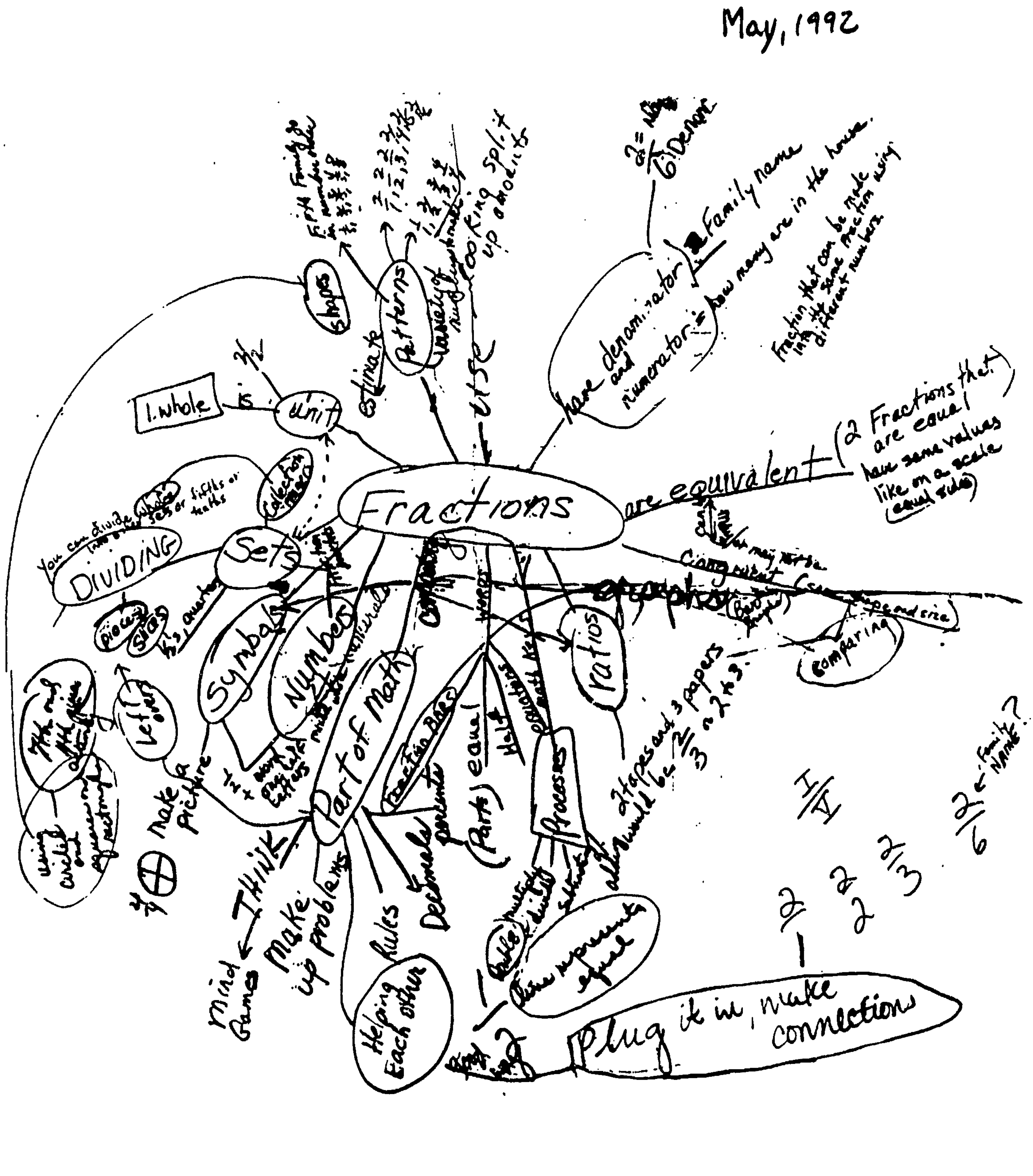


Figure 4. Students' May 1992 composite concept map.

said on the other side. I'd like to hear what your thoughts are, what you've noticed". To the students, this picture was worth a thousand words; it represented what they'd learned about fractions as well as what they'd learned about themselves as learners. Each of the students had an opportunity to comment on their reaction to the concept maps; here we present some of their responses:

Denny :	I just don't think we were aware of what math--the power of math and what math could do and how fractions were related to math and how you could use fractions. Which is basically the basis of math and how math is used.
Antonio :	We were dumb and then we got smart!
Jacob :	I think I'm pretty proud of myself . I mean we just got 1,2,3 ... 14 words up there and now we have about a thousand on the other side.
Cassandra:	Large difference, huh?
Jacob :	Yeah, well I think if you put your mind to it, anything can be possible.
Cindy :	I think we learned a lot since then--I mean we got 3 ideas and about 7 connections up here and the other side and what we have now is about packed full of ideas, so I think we've learned a whole lot from back then to now.
Matthew :	I think we've done a lot in the last few months because I mean fractions is important and I mean if we've got the--I mean if we can do this now, then you can do a lot more. And I think that well, we've improved.
Cassandra :	Did you know you've improved that much?
Matthew :	No, but we've done in the last few months
Amy :	I think we did OK at the beginning of the year--I mean, we didn't have very many kids. Now we have a whole bunch of kids that are helping us make more of connections.
Sarah :	I think we can improve a lot more in here because, I mean, I think a lot of people were kind of shy in the beginning of the year to like tell what other things we could put up there. I think we could have put a lot more up there.
Cassandra :	Maybe at the beginning even?
Sarah :	A lot of people were pretty shy to say something.
Cassandra :	Why do you think that might have been? Why were they so shy to say things at the beginning of the year?
Sarah :	Well, because we hardly knew you.
Cassandra :	But we didn't start this exactly at the beginning.

Sarah : This stuff was real new to us.

Cassandra : It was real new to you.

Sarah : And now we know more about it, so we can put a lot more things up there too.

Rick : At the beginning of the year when Mrs. Collar said we were going to change a lot, I didn't believe her because I thought we were going to be the same when we got done. Then we kept on going through the year until it's almost the end now, and I've noticed that at Christmas we've improved (?) a lot, cause I was sorting through my stuff over there, and I looked on my folder and it has my name in cursive, and so then read a sheet that it's pretty recent about like a week or so ago, and I noticed that it's improved a lot, and so has everyone else.

Cassandra : So a lot of improving's going on in a lot of areas, huh?

Rick : Mmm hmmm

Cassandra : How do you feel about this today, as far as listening to the class being able to make those connections?

Rick : I felt we could go a lot more than 8.

Cassandra : Than 8 in the begining? Why do you think that happened?

Rick : Well, we didn't know a lot about these fractions 'cause you were just starting to teach us about it, and we didn't know like maybe that 1/3 could be take up the same area as 2/6. We probably only knew--probably about like some of the most popular fractions like 1/2, 1/4, and 1/8.

James : I was going to say about what Amanda said--I think it was Amanda--when she said we--in the beginning of the year we only had like 23 people or something like that and since we started out now we got almost 30 kids and we've improved a lot.

Cassandra : Because of the numbers of kids?

James : Yeah.

Cassandra : How did havin' a whole bunch of kids--did that help?

James : You get more ideas and one might be smarter than the other--the ones that just came in, and other people like that--one can be smarter than the other

Teresa : It took me a long time to understand about fractions and now since I look at that it looks like we learned a lot ... at the beginning of the year I didn't know nothin about fractions, now I understand what fractions are

Cassandra : At least you're getting it so you feel more confident with it?

Teresa : Yeah.

Cassandra : Ok, Lisa, you're really new to our room. Have you learned anything just in the short time you've been here? Or noticed anything about our classroom in math?

Lisa : I learned what congruent means and a little bit more about--I learned more about fractions and um, I always thought that dividing was a totally different thing from anything else

Cassandra : And what do you think now?

Lisa : I think now that fractions and dividing are practically the same thing.

Cassandra : Why?

Lisa : Because when you're dividing, my teacher would take dividing and she'd take a pie--she'd draw a pizza pie and she'd cut into six pieces and that was her example for dividing. But here you guys take a pie and cut it up into six pieces and that's your example for fractions.

Cassandra : There's gotta be a connection.

Conclusion

We thought that writing about our experiences would bring some sense of closure to our work together. Instead, the process of discussing, analyzing, and writing about our experience together has engendered new ideas, many questions, and a sense of the continuous, iterative nature of inquiry into teaching and learning. In closing the door on this paper, we open the door to further inquiry, by exiting with some questions and issues that will guide our future work, together and apart. We hope that in sharing a part of our story, we have communicated both what is unique from and what is commonplace to other teachers and researchers.

Does the Subject Matter?

Primarily, our story was about the development of a classroom community based on features described in the Standards and other documents suggesting reform in mathematics education. Secondarily, our story was about students learning of fractions in a fourth-grade classroom. The main reason that subject matter took a back seat to issues of developing community was that both of us felt tenuous about our own understandings of mathematics. Learning about fractions in the midst of teaching and observing was often frustrating. We speculate that additional subject matter knowledge will extend and enrich our future inquiries. A central feature of our collaboration was reflecting on and posing questions about what we were observing; this provided a context for learning about the kinds of change that can take place in a classroom community, and about the range of fraction concepts that could arise around stimulating mathematical tasks. However, if teachers are to make significant change in their mathematics curriculum and instruction, we wonder if it is enough to pose thoughtful questions in the absence of additional resources for learning mathematics. Do teachers need opportunities to interact with persons more knowledgeable about mathematics? We're not sure. Additional subject matter knowledge, in and of itself, can't replace trust and commitment. We suggest that a combination of a personal and intellectual features are important to consider as teachers and researchers continue to work together in new ways.

Whose Classroom?

Although the classroom environment we described in this report had evolved over several years, in many ways it represents a starting point for thinking about how to move further in the direction of fostering in students a sense of ownership and responsibility for their classroom environment. Even when students and teachers are enthusiastic about making changes, the construction of different classroom norms is a continual process. The responsibility for constructing these norms does not rest solely on the shoulders of the teacher; students, too, must be involved in building a classroom which reflects their interests and needs. In what ways can students be involved in collaborating to build curriculum and design assessment? Some of our experiences this past year are ones we feel are worth building on.

Cassandra shared ways that she "worked on math" outside of the classroom--reading about math, writing in her journal, solving math problems, listening to the students' ideas on the tapes, reading their journals, writing to them in their journals, talking to other teachers. Glimpses of the classroom through the eyes of the students helped us to understand and evaluate how the classroom community was developing, and we shared this aspect of our work with the students. We explained that we were doing research together about "better ways to teach math" and that their ideas were important to us. Cassandra often asked the students for their opinions about what they were doing in the classroom--"What did you think of that activity?" "Would you recommend it for next year?" "What did you learn?" "How did that help you?" Frequently, Cassandra linked these discussions to the research we were doing, noting that the students' ideas were something she could share with other teachers, and that would help other fourth graders. Increasingly, the students were interested in the idea that what they were saying and doing might affect teachers and students in other schools.

Near the end of the year, students began to take some responsibility for the recording equipment, such as turning on tape recorders during small group discussions, or making sure that the videocamera was on and focused. Having students participate in these ways, we think, provided them with a sense of control and ownership of classroom events and was another way for them to participate as a member of the classroom learning community. As Antonio said, "If we're all doing research *together*, it's important to be a part of the work."

Should We Be Committed?

It would be impossible *not* to be committed to one's work if change is to take place in the classroom. We experienced a nearly constant engagement with thinking about our work together--both in and outside of the classroom. We aim neither to pat ourselves on the back nor to complain about this aspect of our work. We mention it only to raise one final issue about mathematics reform. We gained much from our collaborative inquiry, but there was also enough struggle involved that we had to recommit ourselves to our goals from time to time. We suspect that not every teacher would be willing to make the kind of commitment necessary to change his or her practice in the ways that are entailed if the reform documents are to truly be translated into practice. If we are right, we fear that exemplary practice may remain trapped on the pages of reform literature, rather than become the common practice of the classroom.

References

Ball, D. (1990a). Halves, pieces, and twoths: Constructing representational contexts in teaching fractions (Craft Paper 90-2). East Lansing: Michigan State University, National Center for Research on Teacher Education.

Ball, D. (1990b). With an eye on the mathematical horizon: Dilemmas of teaching elementary school mathematics (Craft Paper 90-3). East Lansing: Michigan State University, National Center for Research on Teacher Education. .

Bouck, M.K., Jones, E.M., & Pierce, L. (1991). Fraction sense: Guidelines for quality mathematics teaching. Lansing: Michigan Council of Teachers of Mathematics.

Brown, A. (1991). Design experiments: Theoretical and methodological challenges in evaluating complex interventions in classroom settings. Journal of Learning Sciences, 2, 141-178.

Campbell, D.R. (1988). Collaboration and contradiction in a research and staff-development project. Teachers College Record, 90, 99-121.

Corwin,R., Russel, S., & Tierney, C. (1991). Seeing fractions: A unit for the upper elementary grades. Sacramento: California Department of Education.

Hiebert, J. (1992). Mathematical, cognitive, and instructional analyses of decimal fractions. In G. Leinhardt, R.Putnam, & R.A. Hattrup (Eds.). Analysis of arithmetic for mathematics teaching (pp. 283-322). Hillsdale, NJ: Erlbaum.

Holmes Group (1990). Tomorrow's schools: Principles for the design of Professional Development Schools. East Lansing: Author

Lampert, M. (1988). The teacher's role in reinventing the meaning of mathematical knowing in the classroom (Research Series No. 186). East Lansing: Michigan State University, Institute for Research on Teaching .

Lieberman, A. (1992). The meaning of scholarly activity and the building of community. Educational Researcher, 21 (6), 5-12.

Mehan, H. (1979). Learning lessons. Cambridge: Harvard University Press.

National Council of Teachers of Mathematics. (1989). Curriculum and evaluation standards for school mathematics. Reston, VA: Author.

National Council of Teachers of Mathematics. (1991). Professional standards for teaching mathematics. Reston, VA: Author.

National Research Council (1989). Everybody counts: A report to the nation on the future of mathematics education. Washington, DC: National Academy Press.

Prawat, R. (1989). Teaching for understanding: Three key attributes. Teaching and Teacher Education, 5, 315-328.

Putnam, R. (1992). Thinking and authority in elementary-school mathematics tasks. In J. Brophy (Ed.), Advances in research on teaching: Vol. 3. Planning and managing learning tasks and activities (pp. 161-189). Greenwich, CT: JAI Press.

Richardson, V. (1990). Significant and worthwhile change in teaching practice. Educational Researcher, 19 (7), 10-18.

Ricks, J. & Collar, C. (in preparation). Juxtaposing teacher-researcher collaboration with collaboration in the classroom.

Rosaen, C., & Hoekwater, E. (1990). Collaboration: Empowering educators to take charge. Contemporary Education, 61,144-151.

Schram, P. & Berkey, R. (1992, February). Creating an environment for study and improvement of practice in an elementary school. Paper presented at a meeting of American Colleges of Teacher Education, San Antonio, Texas.

Schram, P., Prawat, R., Ricks, J., Sands, K., Collar, C., & Seales, P. (1991). Teacher empowerment in mathematics: Negotiating multiple agendas [Summary]. Proceedings of the 13th annual meeting of the International Group for the Psychology of Mathematics Education, North American Chapter, 2, 50-56.

Seymour, D. (1988). Fraction circle activities. Palo Alto, CA: Dale Seymour Publications.

Shoenfeld, R. (1992). Learning to think mathematically: Problem solving, metacognition and sense-making in mathematics. In D. Grouws (Ed.), Handbook for research on mathematics teaching and learning (pp. 334-370). New York: Macmillan.

Steffe,L. P., Olive, J., Battista, M.T., & Clements, D.H. (1991). The problem of fractions in elementary school. Arithmetic Teacher, 38 (9), 22- 24.