DOCUMENT RESUME

ED 366 585		SP 035 004
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TITLE	The Relationship between Teachers' Knowledge and Beliefs and the Teaching of Elementary Mathematics.	
PUB DATE	Feb 94	
CONTRACT	F-91-29; TPE-9253078	
NOTE	21p.; Paper presented at the Annual Meeting of the American Association of Colleges for Teacher Education (Chicago, IL, February 16-19, 1994).	
PUB TYPE	Speeches/Conference Papers (150) -- Reports - Research/Technical (143)	
EDRS PRICE	MF01/PC01 Plus Postage.	
DESCRIPTORS	Comparative Analysis; *Educational Practices; Educational Research; Elementary Education; *Elementary School Mathematics; *Elementary School Teachers; Higher Education; *Knowledge Level; *Teacher Attitudes; *Teacher Education; Teacher Educators; Teacher Effectiveness; Teaching Experience; Teaching Models	
IDENTIFIERS	Experienced Teachers; NCTM Curriculum and Evaluation Standards; *Preservice Teachers	

ABSTRACT

This study was conducted to explore the relationship between elementary school teachers' professed teaching practice and their beliefs about and understanding of elementary mathematics. The Curriculum and Evaluation Standards for School Mathematics, published by the National Council of Teachers of Mathematics (1989) was used as the criterion for teaching practice. The Teaching Policy Assessment, consisting of 10 vignettes illustrating contrasting teaching models, was administered to 140 practicing and preservice teachers to determine how likely they were to teach in ways consistent with the standards. Pedagogical beliefs were measured by Peterson's Belief Scales (1989), and mathematics understanding was measured by Riedesel and Callahan's (1977) elementary mathematics tests for teachers. Multiple regression analysis revealed that beliefs made a significant contribution to the model, that mathematics understanding made no direct contribution; and that the beliefs-by-mathematics-understanding interaction contributed significantly. Teachers who report teaching in ways that are consistent with the standards believe that children construct knowledge, that problem-solving is a context for learning computation skills rather than a culminating experience, and that children's natural development should determine the sequence of topics in elementary mathematics instruction. The beliefs-by-mathematics-understanding interaction indicates that greater mathematical understanding may enhance the influence of this type of belief. (LL)

The Relationship between Teachers' Knowledge and Beliefs and the Teaching of Elementary Mathematics

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A paper presented at the 1994 Annual Meeting of the American Association of Colleges of Teacher Education

Chicago, Illinois

February 18, 1994

This work is supported by Grant No. TPE-9253078 from the National Science Foundation and Grant No. F-91-29 from the Mark Diamond Research Fund. Opinions, findings, and conclusions expressed are those of the author and do not necessarily reflect the views of the National Science Foundation or the Mark Diamond Research Fund.

Abstract

Elementary teachers were studied to determine the relationship between their professed teaching practice and their beliefs about and understanding of elementary mathematics. The teachers studied were 140 practicing and pre-service teachers in western New York State. The *Curriculum and Evaluation Standards for School Mathematics*, published by the National Council of Teachers of Mathematics (NCTM, 1989) was used as the criteria for teaching practice. The Teaching Policy Assessment, consisting of ten vignettes illustrating contrasting teaching models, was used to measure teachers' likelihood of teaching in ways that are consistent with the *Standards*. Pedagogical beliefs were measured by Peterson's (1989) Belief Scales. Mathematics understanding was measured by Riedesel and Callahan's (1977) elementary mathematics tests for teachers. Multiple regression analysis was used to determine the contributions of each of the variables to *Standards*-consistent teaching. Beliefs were found to make a significant contribution ($p < .001$) to the model. Mathematics understanding made no direct contribution. The beliefs-by-mathematics-understanding interaction also made a significant ($p < .10$) contribution. Teachers who report teaching in ways that are consistent with the *Standards* believe that children construct knowledge, that problem-solving is a context for learning computation skills rather than a culminating experience, and that children's natural development should determine the sequence of topics in elementary mathematics instruction. The beliefs-by-mathematics-understanding interaction indicates that greater mathematics understanding may enhance the influence of this type of beliefs. Reform leaders should give much attention to teachers' beliefs about mathematical pedagogy.

The Research Problem

Purpose of the Study

The purpose of this study was to explore teachers' pedagogical beliefs and mathematics understanding together, with particular attention to their relation to teachers' professed teaching practice. If teacher educators can gain a better understanding of the teachers who are today accepting the challenge to implement the NCTM Standards (National Council of Teachers of Mathematics, 1989), they can perhaps have more success in helping new teachers to do the same. They may also be able to design in-service programs to enable practicing teachers who are not implementing the Standards to begin to do so. These will be necessary elements of sustaining the current momentum and actually changing the way mathematics is taught in this country.

Significance

It is the responsibility of teacher educators to guide the preparation of new teachers in becoming highly competent professionals. In mathematics education, this involves motivating and enabling these new teachers to make use of the latest research knowledge and understanding about teaching instead of following less effective traditions. To do this, teacher educators must have a good understanding of the factors involved in distinguishing between those who do and those who do not teach in ways described in the Standards. This study is designed to contribute to that understanding.

Need

Since Shulman identified teacher knowledge as the "missing paradigm" (Shulman, 1986), there has been increasing attention given to the fact that teaching does not occur in an intellectual vacuum. It has become more and more clear that a teacher's special knowledge of subject matter is an important component of good teaching. Within the subject of mathematics, a growing number of studies (Ball, 1988b; Duckworth, 1987; Guyton & Farokhi, 1987; Larson & Choroszy, 1985; Movshovitz-Hadar & Hadass, 1990; Schram, 1988; Tirosh & Graeber, 1990) has shown that teachers may lack a deep understanding of mathematics. These studies have tended to focus on a narrow component of mathematics understanding, such as understanding of division. Furthermore, they imply, but cannot provide evidence that a shallow understanding of mathematics is associated with poor mathematics teaching.

The current study extends previous work in two important ways. First, a broader range of mathematics understanding at the elementary level is provided. The emphasis is on flexible and creative understanding of mathematical concepts and representations. Second, both pedagogical beliefs and mathematics understanding are considered together with the focus on their relation to self-reported classroom practice.

Background

Knowledge

Mathematics Understanding Studies

Ball

Evidence of teachers' lack of depth of mathematical understanding was presented by Ball (1988b). In this study, 19 prospective elementary and secondary teachers were interviewed to determine their understanding of division by fractions, division by zero, and division in an algebraic expression. These subjects were asked to explain and generate representations for these mathematics concepts. They were also asked how they would respond to students asking why certain mathematics procedures worked. It was found that most of the subjects could calculate correctly, and many could state correct rules. However, most of these prospective teachers could not adequately explain or represent the mathematics involved.

Davis: The "Disaster Studies"

This ability to compute without understanding is not surprising to mathematics educators. A group of studies, termed by Davis (1984) as the "disaster studies," has shown that large numbers of students have exactly this problem. Further, these studies have shown that incorrect basic ideas at the most fundamental levels are often hidden by these students' abilities to produce correct answers on traditional formal tests. Since the National Assessment of Educational Progress began testing students in mathematics in 1974, the results have consistently indicated the same problem. Children are able to compute fairly accurately, but their difficulties solving problems indicate a poor basic understanding of mathematics. If these conditions persist in children, it is logical to suppose that they may also exist in teachers.

Duckworth

A special depth of understanding of elementary arithmetic ideas is possible, as demonstrated by Duckworth (1987). In her study, teachers were confronted with many experiences of cognitive conflict as they attempted to construct meanings for such seemingly simple concepts as division of whole numbers. These teachers were surprised to discover that, although they knew the procedures for long division, they had quite undeveloped ideas about why these procedures worked or what the procedures meant on a conceptual level. They often found themselves frustrated and confused as they attempted to make sense out of procedures that they had been taught as children.

The fact that teachers such as those in this study could substantially deepen their conceptual understanding of elementary arithmetic has at least two implications. First, an understanding of elementary arithmetic is an area in which teachers may vary significantly. Second, a deepening of understanding, and the process by which they were able to deepen their understanding, might enable teachers to change the way they teach elementary mathematics.

Movshovitz-Hadar and Hadass

In a study somewhat akin to Duckworth's, Movshovitz-Hadar and Hadass (1990) used the technique of introducing cognitive conflict to test teachers' knowledge fragility/stability. Pre-service secondary mathematics education students were presented with a mathematical paradox. The investigators observed and analyzed the behaviors of the students. The results indicated, among other things, that, for these education students, school mathematics is not a well-sorted-out topic without room for discussion or controversy. Only a small minority of the subjects was able to demonstrate the mathematical sophistication necessary to resolve the mathematical paradox.

Tirosh and Graeber

Tirosh and Graeber (1990) recognized and attempted to remedy the problem of measuring conceptual mathematics understanding. They selected for their study teachers who were able to correctly compute 4 divided by 0.5, but who indicated a belief that quotients must always be greater than dividends. They used interview techniques to present these teachers with the cognitive conflict between what they said they believed and what their computation results showed. Of 21 subjects that were involved in this interview, only 6 were able to correct themselves when confronted with the conflict. Fifteen teachers continued to justify incorrect and inconsistent thought. One subject never even recognized the conflict. This person expressed a conviction that "math is a series of meaningless rules with nothing to do with reality."

Wheeler

In a study that combined qualitative techniques with quantitative techniques, Wheeler (1983) tested and interviewed 62 pre-service teachers about their understanding of zero. On problems where neither the divisor nor the dividend was zero, 90.4% of the pre-service teachers made no errors. On problems where the dividend was zero, 75% of the pre-service teachers made no errors, but 3.8% of the pre-service teachers gave incorrect answers on all of these items. On problems where the pre-service teachers were asked to divide by zero, only 23.1% made no errors. 63.5% of the pre-service teachers gave incorrect answers on all of the division by zero items.

In the interview, many of the pre-service teachers could not adequately elaborate on the question, "What is zero?". Furthermore, only 23.1% could accurately explain why 0 divided by 0 is not possible. The researchers concluded that these pre-service teachers did not adequately understand zero. "This study suggests that teacher candidates are not, in many respects, adequately knowledgeable about zero. Consequently, these future teachers are not sufficiently prepared to teach concepts of zero. Concerted efforts are needed to better equip teachers for dealing with zero." (p 155)

Mathematics Understanding Related to Teaching

Battista

An examination of the relation between knowledge and teaching was conducted by Battista (1986). This researcher gave numerous tests to his undergraduate methods students and examined the intercorrelations between the various cognitive and mathematics anxiety measures as well as measures of teaching performance. The results were counterintuitive in that mathematics knowledge was shown to be unrelated to teaching performance. His conjecture was that the measurement instruments may not have been powerful enough to detect a relationship. This conjecture is plausible, because the measure of teaching performance was a subjective evaluation on a 5 point scale completed by co-operating teachers during practicum experiences, and the mathematics scores were course grades and their related mathematics tests.

Lampert

Not all studies show a deficit of necessary teacher understanding of mathematics. Lampert (1986) describes her own series of lessons in fourth-grade mathematics in which she demonstrates a deep knowledge of mathematics as well as sophisticated pedagogical skill. Her lesson sequence involved three types of activities: coin problems, multiplication stories, and meaningful use of numbers. Through these activities Lampert engaged her students in a collaborative model of teaching in which teacher and students constructed knowledge structures and representational systems together. The results of this work can be seen in the descriptions of the students' thinking and discussion. Clearly, the children in this study developed the principled conceptual understanding of multiplication that is desired by educational reformers. It must be noted and stressed that Lampert is not a typical case. She is a mathematics teacher-educator who continues to teach in an elementary school as part of her professional involvement with the field. Because the pedagogy and the understanding of mathematics demonstrated by Lampert are both exemplary, this case can be taken as evidence that there may be an association between this deep understanding of mathematics and improved pedagogy.

Carpenter et al (descriptive study)

Teachers' specific pedagogical content knowledge related to children's solutions of addition and subtraction word problems has been studied (Carpenter, Fennema, Peterson, & Carey, 1988). These researchers indicate that one flaw of studies of teacher knowledge of mathematics has been the use of global measures of teacher knowledge not directly related to instruction in classroom (Romberg & Carpenter, 1986). Carpenter et al. (1988) were able to avoid this flaw, primarily because their ongoing program of research has developed a classification system of problem types that describes how first grade children solve addition and subtraction word problems. This body of knowledge was used as a foundation for their 1988 study.

Pedagogical content knowledge includes knowledge of concepts and procedures that learners bring to a task, misconceptions, stages of understanding, techniques for assessing understanding and diagnosing misconceptions, instructional strategies to connect new to old knowledge, and instructional strategies to eliminate misconceptions. Carpenter et al.'s (1988) study focused on teachers' understanding of how children think about mathematics and on knowledge of their own students' thinking.

The following research questions were explored by Carpenter et al. (1988):

1. What do teachers know about the distinctions between different addition and subtraction problem types?
2. What do teachers know about the strategies children use to solve different problems?
3. How successful are teachers in predicting their own students' success in solving different types of problems and identifying the strategies used by children to solve problems of the different types?
4. What is the relation between different measures of teachers' pedagogical content knowledge and their students' achievement?

It was found that teachers could distinguish between the major types of problems, and that they could accurately write problems for the different structures. They did not have a coherent framework for classifying problems.

None of the measures of teachers' knowledge of problems, problem difficulty, or strategies was significantly correlated with student achievement or even with teachers' ability to predict either their own students' success or the strategies the children would use. It seems likely that this lack of correlation could be explained by the very low variability in the scores of the teachers' general knowledge of problems and strategies.

Carpenter et al (intervention study)

Following the descriptive study of 1988, Carpenter led a team of researchers in an investigation of the effects of providing teachers with research-based knowledge about problem types and solution strategies (Carpenter, Peterson, Chiang, & Loef, 1989). The purpose of their study was to investigate whether providing teachers with this knowledge would influence the teachers' instruction and the students' achievement. Specifically, the experimental group of teachers was provided with knowledge about different problem types, knowledge of different solution strategies used by first grade children in solving the different types of problems, and knowledge of how children's knowledge and skills evolve. The study was designed to explore the effect of this knowledge on how teachers teach, what teachers teach, teachers' ability to assess students, and students' meaningful learning and problem-solving.

After the intervention, teachers in the treatment group were found to be more "cognitively-based" in their beliefs and teaching than were teachers in the control group. (Cognitively-based is the researchers' term for a constructivist orientation to the teaching and learning process.) There were negligible achievement differences between children in the treatment teachers' classes and children in the control teachers' classes.

However, the children in the treatment teachers' classes showed more developmentally advanced methods of solving problems. They were equal to the control children in knowledge of number facts in spite of the fact that control teachers spent much more time and energy on memorization of facts believing this to be a prerequisite to problem-solving.

The most significant result of their study was the apparent change in the treatment teachers' beliefs and teaching practices. Although the teachers were not assessed prior to the intervention, and it is possible that the treatment group was more constructivist than the control group to begin with, the conclusion of the researchers is that the intervention modified the teachers' beliefs and practices. If this is the case, then apparently specific kinds of knowledge have the potential to change the way teachers teach mathematics.

A Review (Fennema and Loef)

The "state-of-the-art" in research on teacher knowledge is presented in a newly published review of research (Fennema & Loef, 1992). Fennema and Loef (1992) elucidate the problems that have been present in past studies attempting to link teacher knowledge with student achievement. The lack of adequate measurement techniques for examining teachers' mathematical understanding, the use of unsophisticated data analysis, and the lack of attention to the intervening variable of what teachers do in the classroom are all acknowledged as flaws of early research on teacher knowledge.

More recent studies carried out in the interpretive tradition are shown to lead to different conclusions. In three such studies Fennema and Loef (1992) illustrate that constructivist teaching occurred in classrooms where the teachers had an unusually deep understanding of mathematics. While these studies are used to suggest that teachers' knowledge plays a vital role in constructivist teaching, the reviewers admit that such studies are inconclusive regarding the impact of constructivist teaching on student achievement.

An important contribution of Fennema and Loef's (1992) review is their analysis of teacher knowledge within the framework of situated knowledge. Situated knowledge is described as knowledge that arises from interaction with problems encountered outside of a formal learning situation. This is to be contrasted with knowledge that is presented formally with the expectation that such a presentation will enable the learner to apply that knowledge later in various situations. Situated knowledge is said to be personal and transferable, while formal knowledge is said to be rigid and specific. School knowledge is said to be "...fragmented, isolated from reality, too explicit to be transferred, and ... quickly forgotten." (Fennema & Loef, 1992)

Teachers' knowledge of mathematics, at least in the early part of their careers, has typically come from their own school learning. It is this mathematics that is available to the teachers to teach to their students. Fennema and Loef (1992) suggest that as teachers gain more experience in the classroom, it is possible for them to construct new, situated knowledge of mathematics that is more pedagogical in nature. They conclude that the idea of situated knowledge is relatively new and offers exciting possibilities for new research directions.

Beliefs

Cohen (A "Change" in Beliefs)

In a case study that highlights the importance of examining beliefs and actions, Cohen (1991) describes one teacher's change in pedagogical beliefs. According to this teacher's own testimony, her view of teaching mathematics had undergone a radical change. In place of her previous adherence to a textbook-bound, rote drill and practice approach, she had changed her focus to mathematical understanding. She now made use of manipulatives and activities designed to help students make sense of mathematics.

Although this teacher had experienced a change in her beliefs about how mathematics was to be taught, and in spite of the fact that her teaching behavior had changed, she continued to hold a view of mathematics that Cohen described as traditional. Her view of mathematics was consistent with a Naturalist worldview. This, in combination with an evidently shallow understanding of mathematics, led to many inconsistencies in her teaching. She was attempting to patch new frameworks for teaching onto old conceptions of subject matter. The result was a less than radical change in the mathematics education that the children were receiving (Cohen, 1991). This case study illustrates the different levels of beliefs and their pervasive influence over practice.

Wood et al (Second Grade Study)

A more thorough change in beliefs about mathematics teaching is described by Wood, Cobb, and Yackel (Wood et al., 1991). In another case study, the researchers tell the story of a second grade teacher and how she came to view teaching of mathematics in a constructivist way. This teacher was one of the participants in a more lengthy study of second grade mathematics teaching. A primary cause in her coming to change her beliefs occurred when she began asking children to explain their thinking about a mathematics problem. She was surprised to learn that the children could actually think about the problem on their own. Previously, in her 15 years of teaching experience, she had not given children this opportunity to express their own thinking about mathematics.

The researchers describe the process by which this teacher changed her beliefs:

> As the project teacher used the instructional activities we had developed, interacted with her students, and engaged in communicative discourse, she encountered situations that were in sharp contrast to her previous experiences in teaching mathematics. These contradictions created conflicts, dilemmas, and surprises that in turn proved to be learning opportunities for her as well as for the students. In the process of resolving these contradictions, she developed a form of practice compatible with constructivism in which her beliefs about her role, the students' role, and the nature of mathematics changed dramatically.
>
> (p. 588)

An important implication of the report of this teacher's change is that a fundamental belief change that includes beliefs about subject matter, seems to be a necessary first step in becoming a constructivist teacher. Even after this teacher became re-oriented toward a constructivist approach, she still needed to learn a new way of teaching. This is in contrast to Cohen's (1991) case where a new way of teaching was attempted with only a partial belief change.

Watts (Orientations to "Problem Solving")

In a study that lends support to Cohen's (1991) interpretation of beliefs residing at different levels, Watts (1991) studied 36 practicing teachers' and principals' beliefs about reform of elementary mathematics education. About half of the respondents indicated that they agreed with reformers that mathematics education was in need of change. There was reportedly agreement among the practitioners that problem-solving should be used to provide a basis for developing students' higher-level thinking and reasoning skills. However, in an analysis of her data concerning problem-solving, Watts discovered that these practitioners held three categorically different orientations toward problem-solving. Of these three orientations, only one, the "holistic orientation", adequately represents the orientation expressed in the Standards: the notion of using problems as the context for developing mathematical concepts and skills. Only 4 of the 36 respondents were found to hold this orientation.

The respondents' agreement that mathematics education should focus on problem-solving evidently reflected their explicit belief. However, their underlying meanings for problem-solving indicated their implicit beliefs. The difference between explicit and implicit beliefs resulted in apparent agreement with reformers about the need for problem-solving, but in actual disagreement with reformers about what that meant. Although Watts did not examine teaching practice, one could surmise that none of the 32 respondents with problem-solving orientations different from the Standards would have been teaching problem-solving in ways that conform to the Standards.

Knowledge and Beliefs Together

Peterson et al (1989)

Although the interaction between knowledge and beliefs is logically and theoretically of crucial importance, no studies have been found that examine this interaction. One study (Peterson et al., 1989) examined teachers' beliefs and teachers' pedagogical knowledge as separate variables. Although one of the stated research questions referred to the relationship between pedagogical content beliefs and pedagogical content knowledge, the researchers focused their attention on other questions and concluded that little is known about this crucial relationship.

An important contribution of this research (Peterson et al., 1989) was the development of four scales used to assess teachers' beliefs about elementary mathematics and elementary mathematics instruction. These scales measure beliefs about the relationship between skills and problem-solving in teaching elementary mathematics, the basis for sequencing of mathematics instruction, the learner's role, and the teacher's role. The first three of these scales were used in the present study.

Carpenter et al (1989)

Another study that made use of the Peterson (1989) scales was conducted by Carpenter, Peterson, Chiang, and Loef (1989). Again, knowledge and beliefs were both studied, but the interaction between these two variables was not studied. These researchers provided a summer training program for selected teachers in which both knowledge and beliefs were treated. Results showed that teachers who had received this training changed in terms of their knowledge, their beliefs, and their teaching practices. However, no attempt was made by the researchers to assess the relative importance of each of these variables or the interactions between them.

Research Design

The Model and Research Hypotheses

Teaching practice, as reported by the teachers, was taken as the dependent variable in this study. Mathematics understanding and pedagogical beliefs were the independent variables. (See Figure 1.).

******************* Insert Figure 1 about here. ********************

It was hypothesized that both beliefs and mathematics understanding would be uniquely related to teaching practice. It was further hypothesized that there would be an interaction effect between beliefs and mathematics understanding and teaching practice.

Sample

One hundred-forty pre-service and in-service teachers in upstate New York volunteered to participate in the study. Table 1 illustrates the demographic distribution of the participants. Table 2 illustrates the experience level distribution of the participants.

******************* Insert Table 1 about here. *******************

******************* Insert Table 2 about here. *******************

Measurement

Self-Reported Teaching

An indication of teachers' typical classroom practice was obtained by asking teachers to identify vignettes that would be most likely to typify their own classroom teaching. Half of these vignettes were drawn from the vignettes published by NCTM in the Professional Standards (National Council of Teachers of Mathematics, 1991), and they were used to indicate "Standards-consistent" teaching. Since the NCTM vignettes were designed to illustrate desirable components of Standards-consistent mathematics teaching, it was necessary to provide vignettes for a contrasting model of "non-Standards-consistent" mathematics teaching. The purpose of this was to prevent subjects from identifying the type of response the researcher was "looking for" and giving only those responses. Therefore, five vignettes characterizing recommendations for effective behavioristic / didactic (intended to characterize "non-Standards-consistent") teaching were developed.

The vignettes representing non-Standards-consistent teaching were written to capture some essential elements of the effective teaching model (Hunter, 1982; Hunter, 1987). This model reflects a behavioristic / didactic approach seen by many to be a good teaching model. While it is not the purpose of this research to evaluate the effectiveness of that model of teaching, certain aspects of it do contrast with the vision put forward in the NCTM Standards. This contrast was used to provide the necessary discrimination power for distinguishing between Standards-consistent and non-Standards-consistent mathematics teachers.

Beliefs

In their work investigating Cognitively Guided Instruction (Carpenter et al., 1989; Peterson et al., 1989), researchers at the Wisconsin Center for Education Research and Michigan State University, have developed four scales that have been used to measure teachers' beliefs. These scales measure beliefs about how children learn mathematics, the relationship between skills and problem-solving in teaching mathematics, the basis for sequencing topics in mathematics instruction, and the role of the teacher in mathematics instruction. The first three of these scales were used by permission in this study. (The fourth scale was felt to be overly redundant with the Teaching Policy Assessment developed specifically for this study, and it was not used.)

Each of the scales included 12 items that represented a continuum. Six of the items in each scale were worded in such a way that agreement constituted placement near one end of the continuum. The remaining six items for each scale were worded in the opposite direction.

For the first scale, the continuum goes from the belief that children receive knowledge to the belief that children construct their own knowledge. The developers of the instrument report a reliability coefficient (Cronbach's alpha) of 0.81 from their administration of this scale.

For the second scale, the continuum goes from the belief that mathematics skills should be taught in isolation to the belief that skills should be taught in relationship to understanding and problem-solving. The developers of the instrument report a reliability coefficient (Cronbach's alpha) of 0.79 for this scale.

For the third scale, the continuum goes from the belief that formal mathematics should provide the basis for sequencing topics for instruction to the belief that children's natural development of mathematical ideas should provide the basis for sequencing topics for instruction. The developers of the instrument report a reliability coefficient (Cronbach's alpha) of 0.79 for this scale.

Mathematics Understanding

Teachers' ability to flexibly and creatively interpret mathematical tasks was assessed with a 30-item conceptual mathematics test. The mathematics test measured teachers' understanding of the fundamentals of elementary mathematics. Rather than focusing on procedures, the instrument assessed conceptual understanding. In addition to this, for some of the items, subjects were asked to indicate how they determined a correct answer. Scores are provided to indicate overall number of correct answers and types of thinking indicated.

Data Analysis

Descriptive

Mathematics Understanding

The maximum possible mathematics score would have been 57 points. In addition to simply counting each correct response, three of the items were coded to indicate the solution strategy used. These coded responses were scored according to the degree of flexibility and sophistication displayed. The score from these coded responses was added to the raw score of items correct.

Items on the mathematics test dealing with part/whole operations were the most difficult for these teachers. A second category of items that many teachers found difficult were the ones dealing with the fundamentals of the number system. Teachers did not, in general, display great facility with concepts of base and place value. The easiest categories of items were the geometry/measurement items and the whole number operations items.

The coded responses that indicated the teachers' solution strategies showed that the most common strategy, in general, was a trial-and-error strategy. The second most common strategy was the use of a traditional algorithm. It is notable that a traditional algorithm was not usually the most efficient method of solving these problems. In many cases the problems were designed to be more easily solved by estimation or by reasoning than by the traditional algorithm.

Table 3 provides the mean and standard deviation for the mathematics test.

Beliefs

Overall, a score of 72 on the three belief scales would have been a neutral response. The maximum possible score, which would indicate a maximum possible cognitively-guided orientation, would have been 144. The mean and standard deviation for pedagogical beliefs are shown in table 3. The teachers' generally cognitively-guided beliefs are examined in more detail in the following paragraphs.

On the scale examining whether teachers believed learning to be transmitted or constructed, teachers displayed a slightly constructivist orientation. However, there was a great deal of variability on this scale. Scores within one standard deviation of the mean extended well into the neutral range of the scale. Teachers appeared to be rather unsure of whether children receive knowledge or construct it themselves.

On the scale examining teachers' beliefs about problem solving, teachers leaned strongly toward the belief that mathematics skills should be taught in the context of problem solving. However, there was a great deal of disagreement about this. In spite of the high amount of disagreement, the majority of teachers expressed a belief in problem solving as a context for learning. The disagreement was a matter of degree.

On the scale examining teachers' beliefs about the sequencing of topics for mathematics instruction, teachers leaned toward a cognitively-guided orientation. They felt that children's cognitive development should be considered in choosing an instructional sequence of topics. Unlike on the previous two surveys, the teachers were relatively agreed about this.

Self-Reported Teaching

In order to be considered a Standards-consistent teacher, a respondent needed high scores on three components of the teaching assessment scale. The teacher needed to embrace the NCTM vignettes, reject the didactic vignettes, and demonstrate consistency. Each of these components was scored on a 20 point scale, and the three scores were summed. This made a total possible score of 60. The mean and standard deviation are shown on table 3.

Teachers were much more willing to embrace the NCTM vignettes than they were to reject the didactic vignettes. The didactic vignette that appealed to the teachers the most was one in which a teacher was showing children how to use key words to solve word problems. Another appealing didactic vignette showed a teacher providing a carefully-worded list of steps for finding common denominators.

The NCTM vignettes that were used were from the Professional Standards (National Council of Teachers of Mathematics, 1991).[1] The teachers responded with great enthusiasm to most of these vignettes. The vignette that they were most enthused about was the vignette illustrating children and a teacher discussing various possibilities for a basketball score.

**************** Insert Table 3 about here. ************

Correlations

The zero-order correlations were examined in order to provide a preliminary picture of the relationships among the variables. It had been hypothesized that both beliefs and mathematics understanding would be related to teaching practice. The zero-order correlations support the relationship between beliefs and teaching but not between mathematics understanding and teaching. Table 4 displays these correlation coefficients.

[1] Page 30: Exploration of multiples of 3 and multiples of 8; Pages 45-47: Basketball score; Pages 58-59: Picnic problem; Pages 90-91: Fraction addition; Pages 96-98: Names in a grid.

**************** Insert Table 4 about here. ************

Multiple Regression

A better view of the relationships among the variables was obtained through multiple regression analysis. This analysis permitted an examination of the unique contribution of each variable. It also allowed an examination of any interaction effect between mathematics understanding and beliefs on teaching. Table 5 displays the results of the hierarchical analysis in which the beliefs variable was entered, followed by the mathematics understanding variable, and finally by the math by beliefs interaction. This table illustrates the significance of the changes in the model as each variable was entered. The full regression model, with all variables entered, is displayed in table 6.

Once again, a significant ($p < .001$) relationship emerges between pedagogical beliefs and teaching. Also, the relationship between mathematics understanding on teaching appears to be non-existent. 'low however, we see a significant ($p < .10$) interaction effect. The effect of peaagogical beliefs on teaching appears to be mediated by mathematics understanding.

**************** Insert Tables 5 and 6 about here. ************

Interaction

As with any interaction, there is more than one way to view this effect. Figure 2 illustrates the interaction effect with beliefs mediating the relationship between mathematics understanding and teaching. This figure was obtained by solving the regression equation using values one standard deviation above and below the mean for each of the variables. What this indicates is that for teachers who have cognitively-guided beliefs, the relationship between mathematics understanding and teaching is positive. (High math scores are associated with more Standards-consistent teaching.) At the same time, for teachers who lack cognitively-guided beliefs, the relationship between mathematics understanding and teaching is negative. (High math scores are associated with less Standards-consistent teaching.)

**************** Insert Figure 2 about here. ************

Implications

Teacher-educators should not assume that simply strengthening teachers' mathematical content knowledge will help them teach according to the NCTM Standards. While this study cannot provide insight into any causal nature of the relationships examined, it is reasonable to assume some degree of causality. To whatever degree causality exists, an improvement in mathematics understanding will be beneficial only to those who have cognitively-guided beliefs. Any strengthening of mathematical content knowledge for teachers should be undertaken in ways that are philosophically in agreement with the epistemology embodied in the NCTM Standards.

In light of these findings, educational leaders seeking to implement the NCTM Standards will need to be primarily concerned with the beliefs of teachers and pre-service teachers. They should give attention to the different philosophical and psychological underpinnings of constructivism and behaviorism. They should explore whether these two divergent views of human learning are compatible or competitive. Research findings that indicate that children construct their own mathematical understandings, and that they do so in a sequence that is not always predictable on the basis of the structure of mathematics, should be widely circulated and discussed.

Limitations

The Sample

Although the present sample had the demographic characteristics appropriate to the investigation, it suffered from two serious drawbacks. The all-volunteer nature of the sample necessarily resulted in a restriction of range. Only those teachers interested in mathematics education and confident about their mathematics teaching abilities would be expected to volunteer to participate. A second drawback was the size of the sample. Although the overall R^2 was large enough to detect with a sample of 140 teachers, the increments in R^2 associated with each of the independent variables were much smaller. The sample size of 140 may have lacked adequate power to detect such small increments.

A better study will attempt to recruit participants at the school-district level. If entire school districts can be persuaded to participate, then a decision of the administration rather than individual teachers' decisions will determine the nature of the sample. While this procedure may still result in some selectivity, its effect would be negligible compared to the selectivity in the present study.

The Teaching Policy Assessment

The nature of the Teaching Policy Assessment captured the "spirit" of the Standards, but its low reliability (alpha coefficient 0.54) was a limiting factor. The fact that reading and responding to vignettes is time and energy consuming makes it difficult to design a similar instrument with enough items to provide a high reliability. Based on the reliability coefficient of the ten vignettes that were used on this instrument, even doubling the number of vignettes would not improve the reliability to the desired .80 level. If the idea of using vignettes is to be adopted for another study, some development work will need to be done in order to try to improve the average inter-item correlations of the vignettes included in the instrument.

The Mathematical Thinking Component

The requests for teachers to explain their solution processes was a very helpful part of the mathematical assessment. The fact that only three items requested this information was a limitation of the current assessment. A future study should request teachers' thinking processes for many more of the mathematics items. A larger number of these items will give a pencil-and-paper mathematics assessment some of the capabilities of an interview-based assessment.

References

Ball, D. (1988b) Knowledge and reasoning in mathematical pedagogy: Examining what prospective teacher bring to teacher education. Unpublished doctoral dissertation, Michigan State University.

Battista, M. T. (1986). The relationship of mathematics anxiety and mathematical knowledge to the learning of mathematical pedagogy by preservice elementary teachers. School Science and Mathematics, 86(1), 10-19.

Borko, H., Eisenhart, M., Brown, C. A., Underhill, R. G., Jones, D., & Agard, P. C. (1992). Learning to teach hard mathematics: Do novice teachers and their instructors give up too easily? Journal for Research in Mathematics Education, 23(3), 194-222.

Carpenter, T. A., Fennema, E., Peterson, P. L., & Carey, D. A. (1988). Teachers' pedagogical content knowledge of students' problem solving in elementary arithmetic. Journal for Research in Mathematics Education, 19, 385-401.

Carpenter, T. P., Peterson, P. L., Chiang, C., & Loef, M. (1989). Using knowledge of children's mathematics thinking in classroom teaching: An experimental study. American Educational Research Journal, 26, 499-531.

Cohen, D. K. (1991). Revolution in one classroom (or, then again, was it?). American Educator(Fall 1991), 16-23 & 44-48.

Davis, R. B. (1984). Learning mathematics: The cognitive science approach to mathematics education. Norwood, NJ: Ablex.

Duckworth, E. (1987). Some depths and perplexities of elementary arithmetic. Journal of Mathematical Behavior, 6, 43-94.

Fennema, E., & Loef, M. (1992). Teachers' knowledge and its impact. In D. A. Grouws (Eds.), Handbook of research on mathematics teaching and learning New York: Macmillan.

Guyton, E., & Farokhi, E. (1987). Relationships among academic performance, basic skills, subject matter knowledge, and teaching skills of teacher education graduates. Journal of Teacher Education, 38(5), 37-42.

Hunter, M. (1982). Mastery teaching. El Segundo, CA: TIP Publications.

Hunter, M. (1987). Improved instruction. El Segundo, CA: TIP Publications.

Lampert, M. (1986). Knowing, doing, and teaching multiplication. Cognition and Instruction, 3, 305-342.

Larson, C., & Choroszy, M. N. (1985). Elementary education majors performance on a basic mathematics test. (Paper presented at the Annual Meeting of the Research Council for Diagnostic and Prescriptive Mathematics No. ED 287 728). Austin, TX.

Movshovitz-Hadar, N., & Hadass, R. (1990). Pre-service education of math teachers using paradoxes. Educational Studies in Mathematics Education, 21, 265-287.

National Council of Teachers of Mathematics (1989). Curriculum and evaluation standards for school mathematics. Reston, VA: National Council of Teachers of Mathematics.

National Council of Teachers of Mathematics (1991). Professional standards for teaching mathematics. Reston, VA: National Council of Teachers of Mathematics.

Peterson, P. L., Fennema, E., Carpenter, T. P., & Loef, M. (1989). Teachers' pedagogical content beliefs in mathematics. Cognition and Instruction, 6, 1-40.

Romberg, T. A., & Carpenter, T. P. (1986). Research on teaching and learning mathematics: Two disciplines of scientific inquiry. In M. Wittrock (Eds.), Handbook of research on teaching (pp. 850-873). NY: Macmillan.

Schram, P. (1988). Changing mathematical conceptions of pre-service teachers: A content and pedagogical intervention. (Paper presented at the Annual Meeting of the American Educational Research Association No. ED 302 549). East Lansing, MI: National Center for Research on Teacher Education.

Shulman, L. (1986). Those who understand: Knowledge growth in teaching. Educational Researcher, 51(2), 4-14.

Tirosh, D., & Graeber, A. O. (1990). Evoking cognitive conflict to explore pre-service teachers' thinking about division. Journal for Research in Mathematics Education, 21, 98-108.

Watts, L. T. (1991) Teachers' and principals' perceptions, beliefs and attitudes about proposed reform in elementary mathematics teaching, curriculum, evaluation and learning. Unpublished doctoral dissertation, State University of New York at Buffalo.

Wheeler, M. M., & Feghali, I. (1983). Much ado about nothing: Preservice elementary school teachers' concept of zero. Journal for Research in Mathematics Education, 14, 147-155.

Wood, T., Cobb, P., & Yackel, E. (1991). Change in teaching mathematics: A case study. American Educational Research Journal, 28, 587-616.

Figure 1
The Conceptual Model

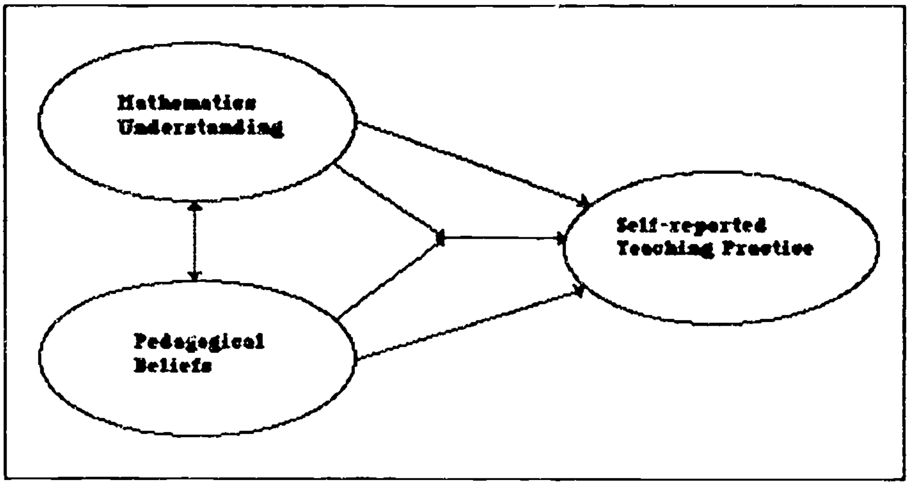
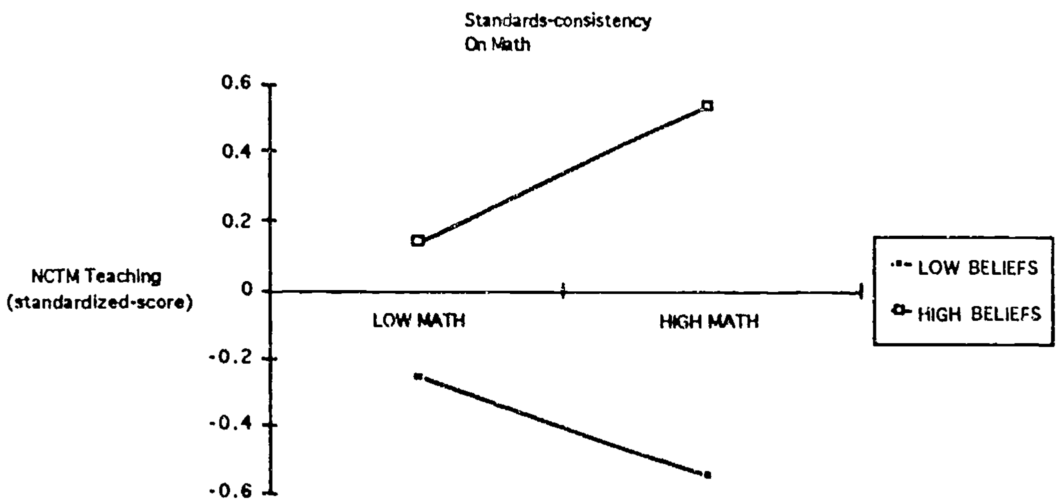


Figure 2
The Beliefs-by-Mathematics-Understanding Interaction (Beliefs Mediating)



Note: Graph represents the solution of the regression equation with "High" and "Low" denoting one standard deviation above and below mean.

Table 1
Distribution of Subjects by Community-type

Community-type	Number of Subjects
Urban	29
Suburban	34
Rural	32

Note: Pre-service teachers could not provide data on school-type.

Table 2
Distribution of Subjects by Experience Level

Years of Experience	Number of Subjects
No Experience	21
Student Teaching	16
1 to 2 Years	14
3 to 5 Years	13
6 to 10 Years	20
11 to 15 Years	15
16 to 20 Years	16
21 to 25 Years	15
26 to 30 Years	4
31 to 35 Years	6
Total reporting experience:	140

Table 3
Descriptive Statistics
Total observations: 140

	Pedagogical Beliefs	Mathematics Score	NCTM Standards-consistency (dependent variable)
N of cases	136	140	140
Mean	93.19	29.11	38.34
S.D.	13.89	9.06	6.31

Table 4
Zero-order Correlations

	Beliefs	MATH	NCTM
BELIEFS	1.00		
MATH SCORE	0.16	1.00	
NCTM	**0.39****	**0.09**	**1.00**

* p-value < 0.05 (2-tailed) ** p-value < 0.01 (2-tailed)

Table 5
Hierarchical Regression Analysis for Standards-consistent Teaching

Predictors	df	R^2 Change	F Change	p-value of R^2 Change
Beliefs	1	.15	24.40	<.001
Math	1	.00	.09	.77
Bel. x Math	1	.02	3.44	.07
Residual	132	Mean Square = .85		

Table 6
Multiple Regression for Full Model

Dependent variable: NCTM Standards-consistent Teaching

Multiple R = .42 Adj. R^2=0.16 F=9.40 Signif F = .00 (df = 3, 132)

Variable	B	SE B	Beta	Correl	Part Cor	Partial	T	Sig T
BELIEFS	.37	.08	.37	.39**	.36	.37	4.54	.00
MATH	.03	.08	.03	.09	.03	.03	0.35	.73
BEL. x MATH	.17	.09	.15	.20*	.15	.16	1.85	.07
(Constant)	-.03	.08					-0.34	.74

** sig. < .001
* sig. < .01

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