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ABSTRACT

This guide is support material for geometry teachers in middle schools or high schools in South Carolina. The guide describes the content of each program in the television series and suggests further learning activities for the students. The geometry that underlies the world around us is presented through applications. Contents of the series include points, lines, planes, flatness, sighting a two-dimensional plane through the three-dimensional world, symmetry, angle measurement, intersecting lines, parallel lines, corresponding angles, right angles, perpendicular lines, Pythagorean Theorem, slope, and transformations, among other things. Many experiments and hands-on activities are suggested to help in concept understanding. The tone of both the programs and activities is casual. It is assumed that teachers will "formalize" the plane geometry according to their own methods of teaching. (MPN)

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LANDSCAPE OF GEOMETRY

Teacher's Guide/Student Workbook

South Carolina ETV

Office of Instructional Technology Development • S.C. Department of Education
South Carolina ETV

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CONTENTS

An instructional television series presented on South Carolina ETV
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INTRODUCTION

For some the fascination of mathematics is in its abstract purity. Others are more excited by what they can "do" with mathematics, and how it relates to their lives.

Traditional support material for geometry teachers emphasizes almost exclusively the abstract side of plane geometry. This series supports the other side — *applied* geometry. It presents a way of teaching the geometry curriculum by investigating the plane geometry underlying everyday objects and human activities. These explorations can provide fascinating and enriching support for pure geometry concepts.

This guide describes the content of each program, further explores the concepts introduced in the series, and suggests learning activities for the students.

The series plays "fast and loose" with the concept of transformations, in order to stress the analogy with real-life movements. Although single concepts in geometry are explored in detail — both in the programs and in the activities in this guide — the tone of both the programs and activities is casual. It is assumed that teachers will "formalize" the plane geometry according to their methods of teaching.

Modelling

Applied geometry is essentially the use of plane geometry to model the world around us. The student can momentarily escape the concepts of "right" and "wrong" in pure mathematical logic and wallow in the pleasant greys of "useful" and "not so useful."

The Squint Factor

Two-dimensional modelling of our three-dimensional world has enormous value, but it's important that the students be constantly aware that they *are* modelling. Throughout the programs and the activities this process is casually described as the "squint factor" — the process of selecting a two-dimensional plane (either a slice or a projection plane) with which to explore solid objects and movements.

To further stress this modelling process, some activities explore other geometries, with other "rules," which we — the "masters" — can employ as "servants," just as we employ plane geometry as a tool to further our understanding.

The Advantage of Errors

Applied geometry necessarily involves measuring and arithmetic skills. Many of the instruments that students need to use in the activities are crude, and the question of accuracy is only occasionally approached. If and when questions of accuracy arise (indeed, the teacher may wish to encourage them), these questions can lead to further useful activities (e.g., the testing of accuracy, angle measurement).

PROGRAM 1
THE SHAPE OF THINGS

THE PROGRAM This program introduces the geometry underlying the world around us. It attempts to dispel the fear that geometry is complicated and confusing; rather geometry is a quest to explore simple shapes beneath objects and even movements in real life.

The program and supporting activities cover:

- A description of points, lines, and planes and how they relate to each other through transformations
- The nature of flatness, and how "non flatness" can change the rules of geometry
- The application of geometry to the exploration of the world around us, including:
  - breaking down complex figures into simple ones
  - changing and distorting shapes to help us model, scale, and manipulate the world
  - creating copies of objects and movements by using the principle of congruency
- An introduction to the "Squint Factor" the process of sighting a two-dimensional plane through the three-dimensional world
- A hint of the "internal geometry" that pure product of the mind that allows us to automatically "line up" the world around us
- A means of using geometry to plot the movement of objects
- An extension of the idea of shape to interpret patterns such as **symmetry**.

ACTIVITIES The activities for this program require no prior study of geometry. They make basic inquiries into some of the fundamental concepts upon which geometry is built and into ways we can use geometry as a tool to investigate and change the world around us.

Join the Flat Earth Club

1. Is the flat earth club made up of a bunch of kooks? Not at all. Members include practical geometers. Examine the map in Figure 1.1. Why is it out of date? Because it comes from a time when the earth was thought to be flat? Compare it to a modern map of the same area. Have we improved the accuracy? Now that we know the world is round, have we improved the map by making it "rounder?"

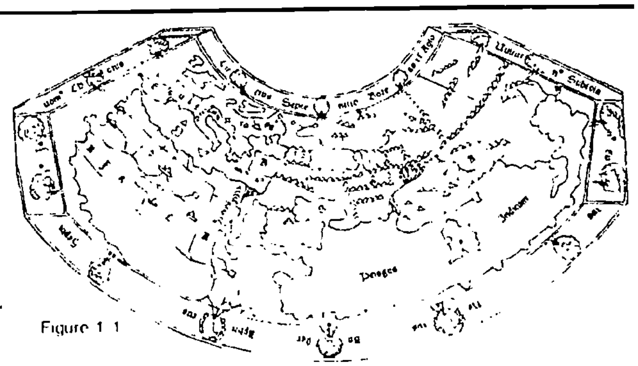


Figure 1.1

It's a Dirty Double-Crossing Earth

2. Plane geometry deals with shapes on a flat, impossibly thin surface. It's a *practical* way of thinking of the earth.

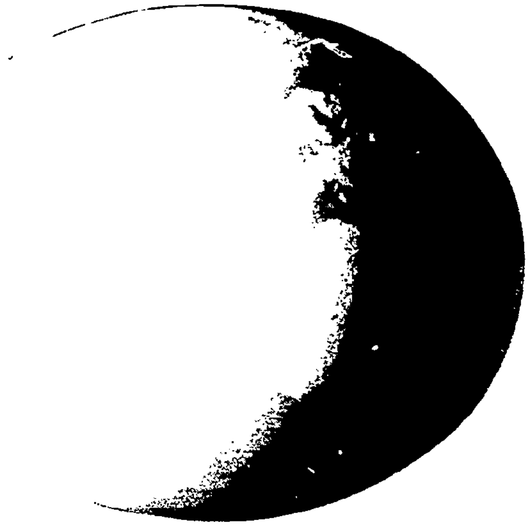
We could use other geometry models, such as those in Figure 1.2. But in each case we would have to use a very different geometry, with very different rules. To see how different, we must consider how straight lines cross — first on a plane, and then on a different surface.

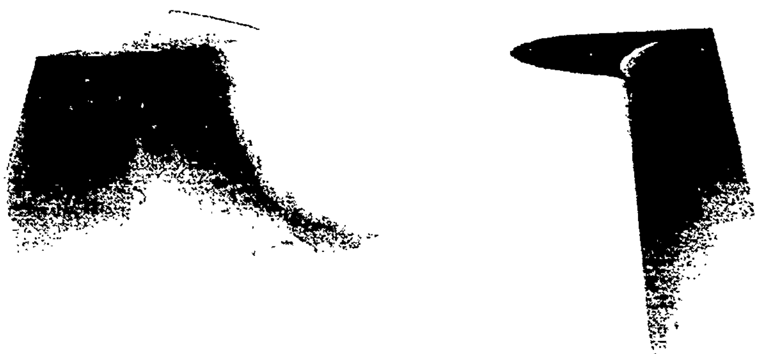
Take a square of tracing paper. Draw two lines corner to corner that cross in the middle of the page.

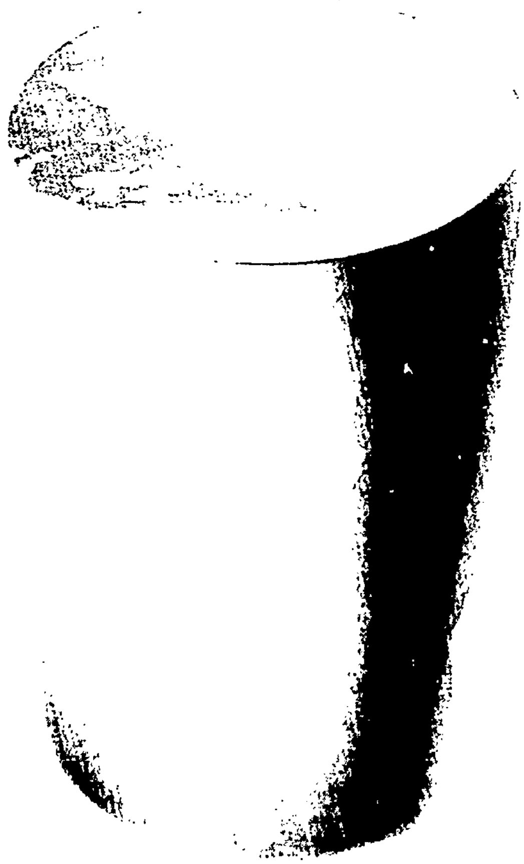
Imagine them extended endlessly. Do you think they will ever meet and cross again?

Now destroy your surface by rolling it into a cylinder so that the ends overlap a few centimeters. How many times do they meet now? Imagine the cylinder extended. Does your answer change?

Using the geometry of a sphere, it's a dirty double-crossing earth. With a cylinder it's even worse! Plane geometry, with its simple crossings, is just one very simple way of modelling the world around us.







Pointy-Headed People in the News

3. [illegible]

Examine a black and white newspaper illustration with a magnifying glass. What do you see? Do the same with a variety of printed illustrations. Is there a relationship between the number of dots and the quality of the picture? What happens when dots are closer together? Farther apart?

[illegible]

Television Never Hands You a Line

... It gives you a lot of them

4. If we think of a line as single point transformed or moved in one direction (e.g., the border of a newspaper illustration), we have another way of looking at a surface built up from a line (of points), transformed sideways to build up the surface.

Take a very close look at a black-and-white TV screen. What do you see? If your classroom or school has a monitor/video system, the effect is dramatic if you can get a solid "black" signal on it, and then turn up the brightness control.

Find out how many scan lines the North American TV system uses. Do the same for European systems. (Refer to Activity 3.) Would you expect the North American system, or one of the European systems, to give a better quality picture?

Splitting Hairs

... Or ... swordsmanship and geometry

5. Models are essential in science, engineering, building — in a whole range of human activities. But *three-dimensional* models can often be a nuisance. Do you know why?

The cheapest and most convenient "modelling clay" is **plane geometry**. But plane geometry has only two of three possible dimensions ... it's a "shadow" of the real world.

Plane geometry can be a kind of swordsmanship — taking the right "slice" of life.

Be an engineering swordsman — a draftsperson. Your tools are basically a pencil and a straight line. Select an object and choose from it several "slices" (or cross-sections). How many slices do you need to capture all the details of the object?

If You Can't Cut It ... Squash It

6. Hold a glass jar up to the sunlight with the neck pointed toward the sun. How much of the jar can you see in the shadow? Could you draw the jar after viewing this shadow alone? Hold it now so that one side faces the sun. Do these two views give you enough information to draw the jar?

A geometry method that is as powerful as "slicing" an object with a two-dimensional plane is "squashing" or **projecting** it onto a plane, as if it were transparent.

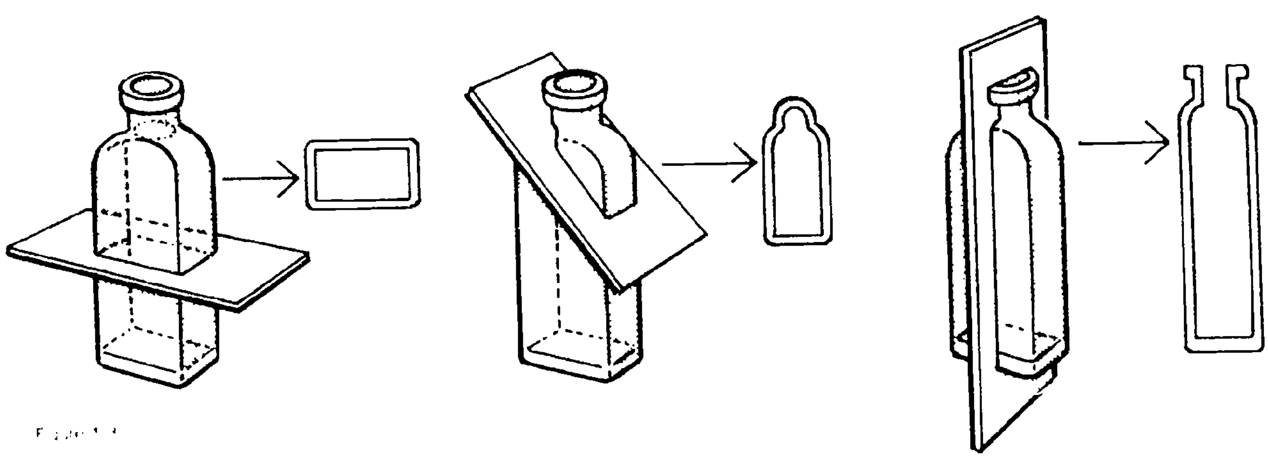
Continue your career as a draftsperson. Choose an object close by and examine it carefully. Sketch the object as if it were squashed onto three different planes. Are three views too many? Not enough?

Now imagine that the object is transparent. What extra lines would you see?

KISS ... Keep It Simple Stupid

KISS is a word computer programmers use to remind themselves ... keep it simple stupid. It represents the kind of thinking that applies to geometers too.

7. Browse through a collection of magazines. You'll notice that buildings come in a great variety of shapes. At first glance their underlying geometry often looks complicated. But we can use geometry to "take apart" a building into a few basic shapes. Often a single shape will repeat itself over and over again.



Each student selects a different building. See if you can take it apart into simple underlying shapes, with the fewest possible sides.

Mind Your Points and Lines

8. How good is your "inner geometry" -- your ability to recognize patterns? See if you can find an old, broken camera — the cheap plastic kind -- with the shutter still working. Remove the back. This is a way of "exposing" the eye to an object for somewhere between 1/25 and 1/50 of a second (depending on the model of the camera). Have a friend hold an illustration you haven't seen before in front of the camera. "Click."

Which objects do you instantly recognize? Does it matter whether they were held the "right way up?"

Play it Again, Sam

Geometry explores shapes made between lines. A vital part of this exploration is identifying shapes that are exactly the same as others. These identical shapes are called **congruent** shapes.

9. Go orienteering. (Orienteering is, roughly speaking, geometry on a grand scale.) You   a point   must transform yourself from place to place to create lines that build to match an original shape. Add running shoes and a watch and you have a popular sport   getting to a specific place faster than anyone else.

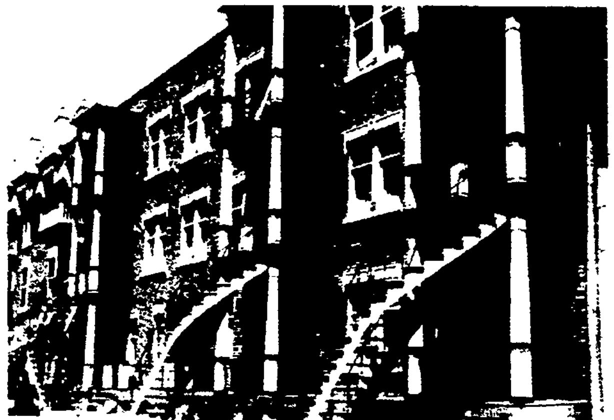
Divide into pairs. Each student marks out an orienteering course around the schoolyard, and translates it into instructions for a second student. Use paces as a unit of measurement, and left, right, forward, backward, quarter turn, half turn, etc., to indicate directions.

The second student then attempts to create a congruent figure on the ground. After the exercise (or instead of), each pair of students should sketch the schoolyard and "pace out" their route and their partner's on the sketch. Compare your results. Discuss as a class how the orienteering "geometry" compares to plane geometry in recreating congruent figures.

Over and Over Again, My Friend

10. Many patterns are made up of a single congruent shape that is repeated in one way or another to build up a larger shape. These patterns are called **symmetry**.

Look about you for patterns of symmetry. Are these patterns useful? (Hint: If windows are congruent, they are cheaper to make, install, etc.) What other values does symmetry have?



PROGRAM 2
IT'S RUDE TO POINT

THE PROGRAM This program examines straight lines and rays, and explores the concept of **direction** by first considering two or more points forming a pointer, and then the angle between one direction and another.

In real life we apply these concepts in navigation, the physics of light, mapping, animal perception, technical drawing, and history.

ACTIVITIES The activities expand the concepts of the programs to:

- Review point, line, and plane concepts (from Program 1)
- Explore straight lines as:
  - the shortest distance between two points
  - a point transformed in a direction
- Relate corners and the vertex of angles, and consider the difficulty of working with curves — "cornerless" lines
- Compare light rays and rays to build a concept of a rotating line through a complete turn to sweep all possible angles
- Explore several methods of angle measurement
- Use practical examples to practise angle measurement, construction, and the addition and subtraction of angles.

No Rules For Schools

1. Suppose you had no straight edge? How would you draw a straight line? Think about the building you are in right now. How were the edges of the walls made straight? The ceiling? The roof? Have you ever seen people carrying giant rulers around a building site?

2. Draw a curved line (any kind of curve) and a straight line. Examine them together. You can see they are different but can you actually describe the difference? Write a description so that someone who has never seen a straight line would be able to draw one.

Straight lines are so simple and obvious, even geometers have difficulty describing them in more simple ways.

Throwing a Straight Curve

Geometers describe a straight line as the shortest distance between two points.

3. Insert a thumbtack in either side of a blackboard. Draw a string tightly between the tacks, then pull it away from the blackboard and rub chalk on it. Now pull the string back and snap it against the blackboard. What happens? This is a technique used by carpenters and masons to draw a straight line. Loosen the string and repeat. Experiment with different lengths. Can you describe a straight line by referring to the two anchor points and the length of the string?

4. Use a piece of string to find the shortest distance between two distant points — say

Halifax and Vancouver — on a map of North America. Note towns and cities close to or on this straight line.

Now use a globe. Repeat the same procedure. What happens? Which is the "real" straight line? If you want to fly the shortest route from Halifax to Vancouver, what line should you follow?

A straight line is the shortest distance between two points on a **plane**. This activity shows that we can't always expect plane geometry to do a good job of modelling the real world.

Send a Point to Do a Line's Job

5. Geometers, engineers, builders, and surveyors describe a straight line in another way. Examine the sign in Figure 2.1. It is a shorthand way of describing a line.

On a map draw a line from your location to the nearest town or city centre. Is the line straight?

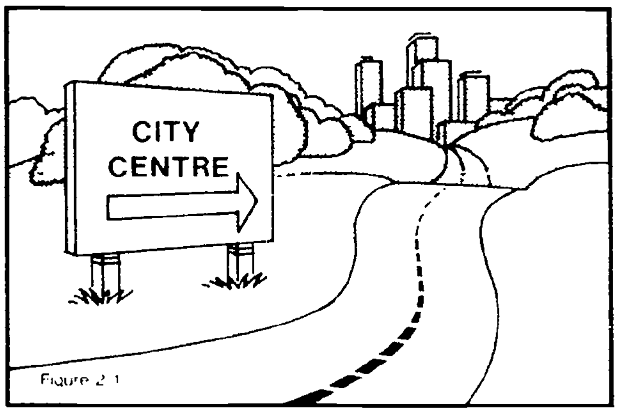


Figure 2.1

Using signs such as the one in Figure 2.1, we all agree on a *convention*. It's understood we will not go straight, but follow the winding line of the road.

Straight as the Crow Flies

6. Scientists and mathematicians draw signs, using the "as-the-crow-flies" convention. Tape a piece of paper to your desktop. Draw *as simply as you possibly can* an accurate sign indicating where the door in your classroom is.

One "standard" sign is shown in Figure 2.2. Think of it as a set of instructions for sending, or **transforming**, a point (A) in *one direction*, without change, until it passes through the door. This kind of line is often called a **ray**. Examine it. Can you tell from the instructions

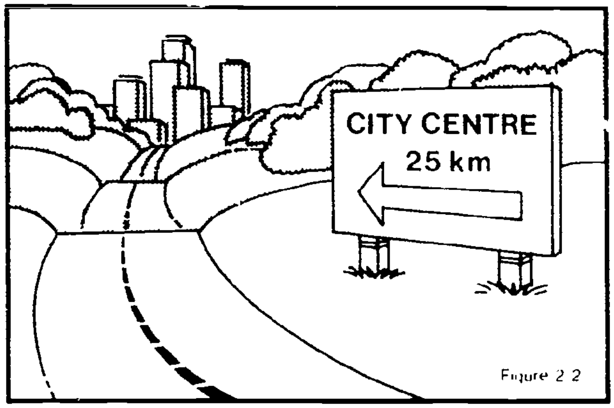


Figure 2.2

where a ray stops?

If we write an additional instruction on our sign, we can describe how to transform a point to create a straight line of fixed length — often called a **line segment**.

What information is needed to change your ray to the classroom door into a line segment?

Geometry? It's "O-Fence-Ive"

We can "build" a ray by transforming, or **sliding**, a point indefinitely in a certain direction. But we can slide more than a point. The archer in the program **transformed** an entire short line – the arrow — sliding it along its path.

7. Be a landscape architect. Draw a plan view (from above) of a picket fence. It should be five pickets long; the pickets 10 cm wide and 2 cm thick. Each picket should be 1 cm apart.

No problem? Fine, but who ever heard of a five-picket fence? Repeat the above for a 1000-picket fence. You refuse? Who can blame you? Instead, reduce the problem to geometry and design a transformation instruction to **slide** a short line (don't worry about the thickness of the picket) along a ray to produce the fence.

(Remember: Transformation instructions are *conventions*. They are neither correct nor incorrect; they are simply clear or confusing, "standard" or unfamiliar, depending on their design.)

Throw a Little Light in the Corners

8. Light rays and geometry rays often behave in very similar ways.

Figure 2.3

Suppose the beam of a lighthouse rotates so that after 20 s (seconds) it points back to the start. On paper choose a point to represent the light itself, and then transform, or rotate, a geometry ray to represent this light beam.

If it must rotate once every 20 s, draw the direction of the ray after 5, 10, and 15 s. Indicate these directions on your diagram.

An actual lighthouse lamp rotates around some form of pivot. When we rotate a geometry ray, we call the "pivot" point a **vertex**. Label the vertex of your diagram.

It Takes Two to Angle

All angles are *relative* measurements. They measure a *change* in direction, or a *difference* in direction *from some other direction*.

9. Angles only make sense if they are between two straight lines or rays pointing in different directions. The change in direction is called an **angle**.

On your lighthouse diagram (Activity 8), erase all the lines except the five-second ray. Does this alone tell you anything about a change in direction? Draw in the start line again. Can you see a change now? A starting direction is often emphasized by calling it a **prime ray** or a **base line**. Choose a prime ray and label it on your diagram; then label 5, 10, and 15 s lines.

Consider the prime ray and the 10 s line. Is there really any change in direction here? Although it doesn't look like an angle, it is one of the most important angles of all — the **straight angle**.

Boxing the Compass

10. For centuries boxing the compass has been a serious learning activity for young sailors.

Examine the compass rose in Figure 2.4. This is an ancient method of dividing up angles. From your general knowledge, which ray, if any, is considered the prime ray? Why?

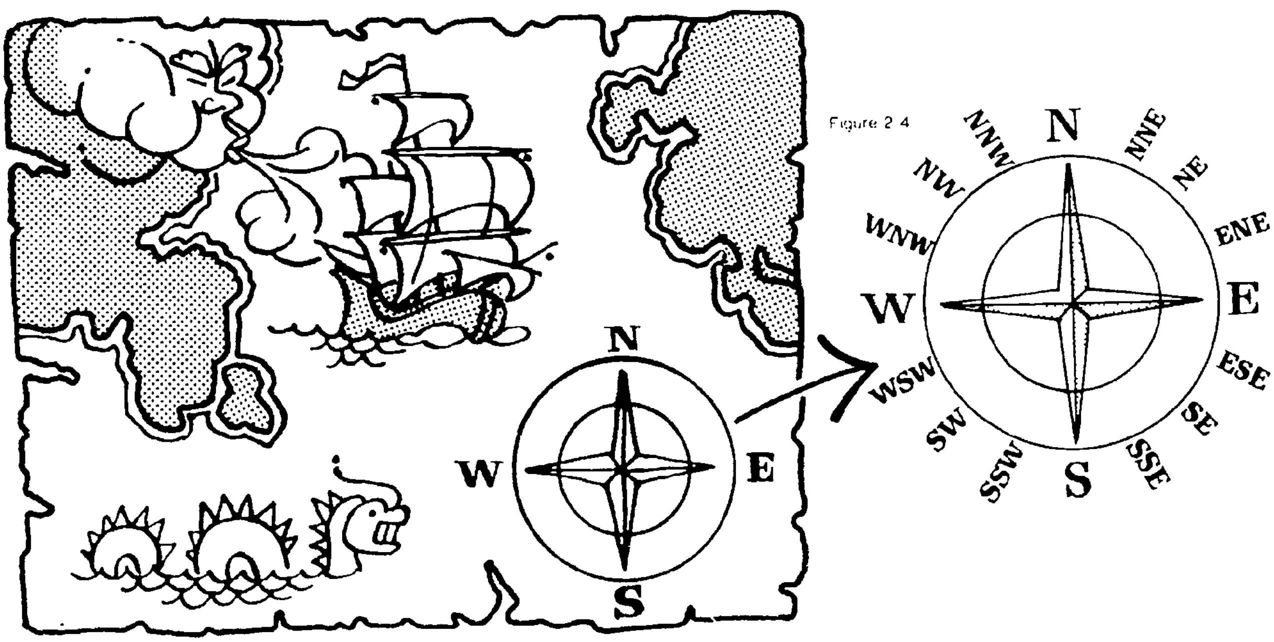


Figure 2.4

In this measurement system one full rotation of a line back to its starting point is divided into 32 angles or **compass points**. The initials of the 16 most important points are around the edge of the compass rose in Figure 2.4. Study the rose and then see if you can list their names in order, beginning with North and working clockwise.

These points are further divided into quarter points. Each point has a name: e.g., Northwest by North, 3/4 North. The drill of reciting these points by heart is known as "boxing the compass."

Rock Around the Clock

Or . . . Bandits at ten o'clock! Dive! Dive!

11. A casual way of dividing angles is into 12 clockface directions. This division is particularly popular with pilots for describing directions in a vertical (up and down) plane, as opposed to a horizontal (e.g., the sea) plane.

Draw a nose-on view of an aircraft. Place a second aircraft at the five o'clock position from the first — *from the pilot's point of view*, not yours.

Learning Geometry by Degrees

The most common way of dividing angles for geometers, scientists, engineers, and many others is into **degrees**.

12. Examine a protractor. Find its vertex. Find the line joining the vertex and 0°. Rotate it (in your head) to the opposite side of the protractor. How many degrees does it rotate through in *half* a complete circle? Continue this rotation to complete the circle. How many degrees does the line rotate through?

New-Fangled Angles

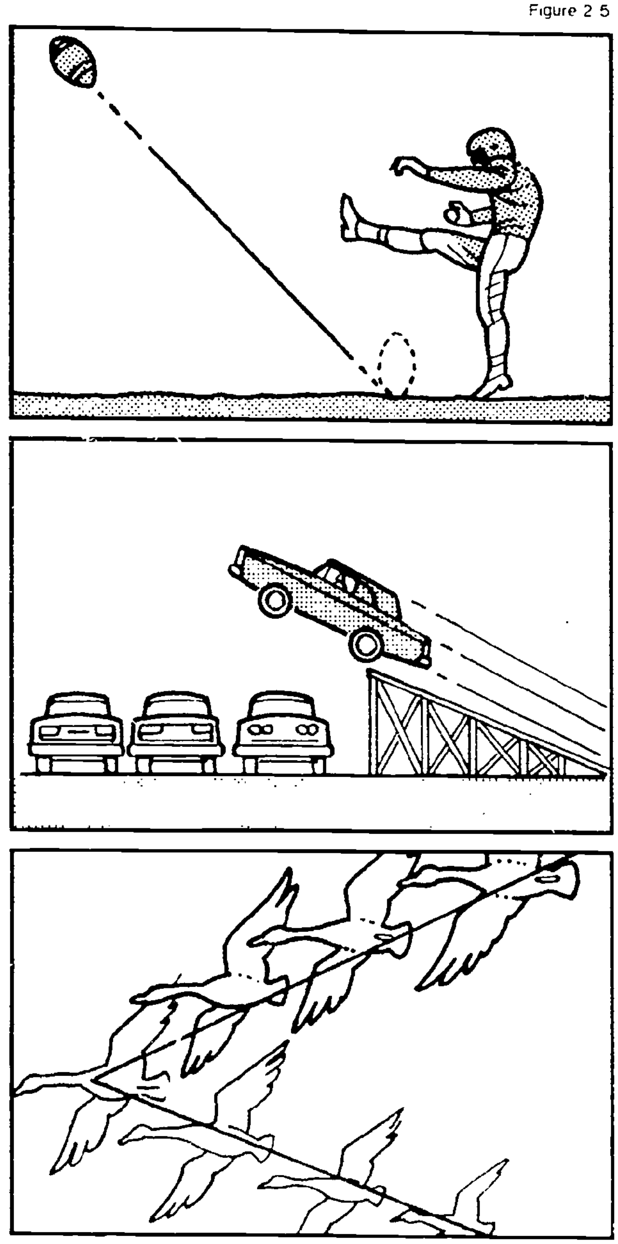
13. Now that we use the metric system, our method of dividing a circle into 360 might seem outdated. Although 360 is really quite a useful number (why?), the system could be improved. Why not invent your own angle measurement system, using the metric system.

Calculate the smallest total number of divisions for a full circle that could be divided by *every* number from 10 down. Try to think of a more entertaining term than "degree" to name the smallest division. (You might follow the example of many modern scientists — name the increment after yourself, or honor a classmate.)

Galloping Off in All Directions

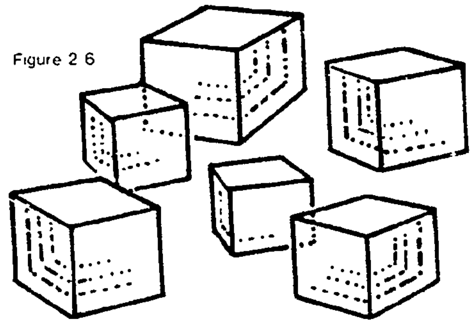
14. Angles are an intriguing way of looking at the world around us. Build up a chart of "critical" angles in real life. Measure the angles and transform their geometry to the chart.

Figure 2 5



There's Nothing Bigger than a Small Angle

Figure 2 6



15. Many crystals in nature take on a variety of angles that make them unusual and unique. Examine the crystals of salt in Figure 2.6.

Draw larger versions of these crystals. Which angles are larger — the ones of the crystals or the ones in your diagram?

Unlike lengths, which often must be reduced or increased in order to transform them (for example, onto a diagram or a blueprint), angles can't be scaled up or down. Even though the angle measurement doesn't change, the distance between the rays can grow immeasurably large.

Slicing Up the Pie

16. We can add and subtract angles as easily as we add and subtract lengths. As a result angles are a useful way of showing how a number of parts add up to make a whole — in a diagram called a **pie chart**.

Draw a pie chart to compare:

a) The number of students in your class who ride a bus (or subway) to school, compared to those who don't

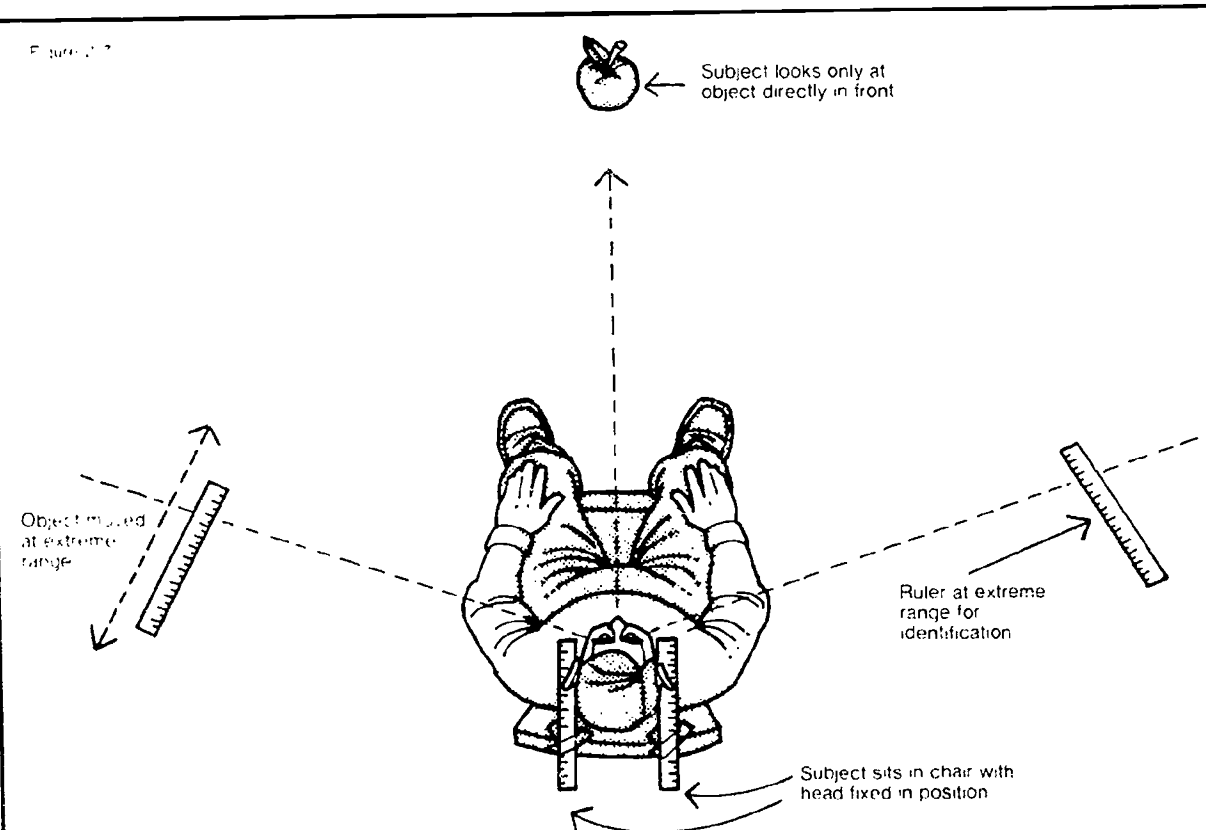
b) The number of boys compared to the number of girls

c) The number of blondes, redheads, and brunettes.

In each case represent each group as an angle and as a percentage.

Seeing Angles

17. For both animals and humans, survival can depend on how well they see. For example, test pilots need to have a *wide* angle of vision.



With a classmate, design a test to measure your angle of vision. At what angle do you detect movement? At what angle do you recognize the object? Compare the angle for one eye and two eyes.

Another advantage for survival is overlapping vision. Can you measure the vision angle overlap for your eyes? Why is this useful?

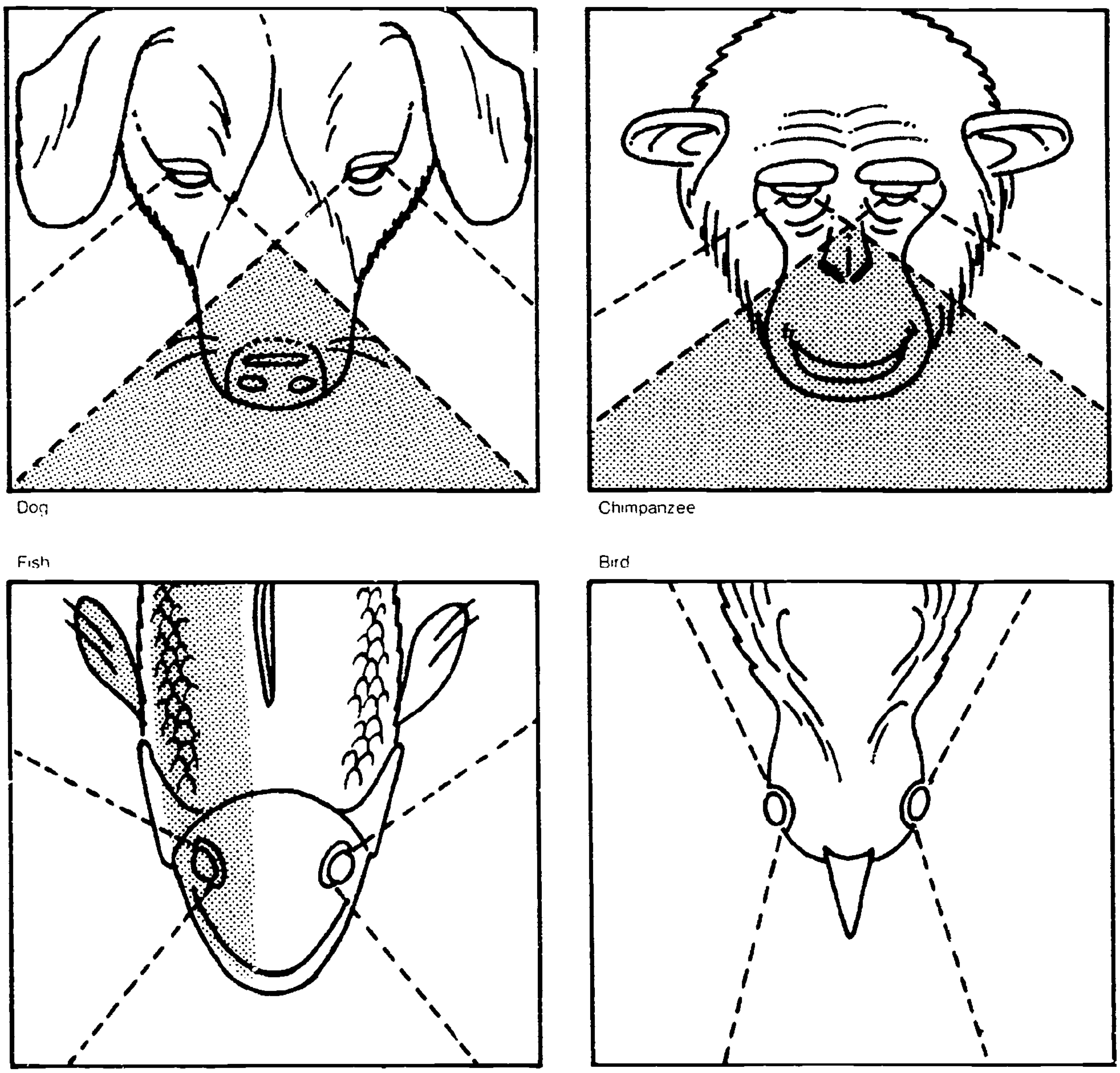
Close one eye, choose a point within reach in front of you, and try to put your finger on it. Try several times, choosing different points. Is your accuracy worse with just one eye?

Since you don't fail completely, you must be using some means of judging distance even with one eye. What might this be? Try "mixing up" your world a little by tilting your head against a wall or even standing on your head. Repeat the "one-eye" test. Does your accuracy improve or get worse?

18. Survival often means *fight* or *flight*. Within the animal kingdom there is a varied mix of wide-angle and overlapping vision.

Examine Figure 2.8, then make a chart comparing these animals' vision. Is there any pattern that seems to link wide-angle vision with fight or flight? What about overlapping vision?

Figure 2.8



PROGRAM 3
LINES THAT CROSS

THE PROGRAM

This program explores the intersection of straight lines and the important geometric properties of that intersection. The concept of supplementary angles is introduced and the properties of opposite and supplementary angles are developed.

The properties of intercepting straight lines have many applications. They are indispensable when surveying, using levers, drawing signs and symbols, and studying the occult science of geomancy.

The program provides:

- An opportunity to explore intersecting straight lines thoroughly, giving every student a chance to absorb and feel comfortable with this first geometric "structure" and the logic of its properties
- A review of straight lines, angles, and rays
- A chance to preview parallel lines, triangles, and more complex geometric figures.

ACTIVITIES

The activities expand the concepts of the program by:

- Reviewing straight lines, and angle measurement and construction (see Program 2)
- Exploring the difference between lines that meet and lines that cross
- Considering an intersection as a kind of transformation — the rotation of a line about a point
- Studying and verifying the logic of equal and opposite angles
- Constructing a variety of intersections, noting the properties of particular angles of intersection, and examining a wider variety of applications in the real world
- Introducing angle bearings and position fixes in order to lay the foundations for the exploration of triangles (see Program 6)

Criss-Cross

When two straight lines meet and carry on geometry calls it an intersection. In everyday terms, we use the word "cross" to express the notion of lines, objects, and ideas intersecting.

1. Make a list of words and expressions that use the term "cross." How many suggest intersecting straight lines? (e.g., railway crossing, cross-examine)

Since ancient times, the words "cross" and "crossing" have often been linked with force, power, or magic.

How many of the "cross" words and phrases in your list seem linked to forces or power — real or supernatural?

The Clash of Symbols

Symbols are one of our most powerful tools — and some of our most powerful symbols use intersecting straight lines.

2. Research and gather as many symbols as you can that involve crosses (e.g., the classic medical symbol). How many of these symbols seem linked to power or force? You might even consider the supposed meaning of lines on our skin in such studies as palmistry and physiogonomy.

Greek? It's All Geometry to Me

The most powerful symbols of all are letters. Geometry can help us describe and "standardize" our alphabet.

3. Can you continue the following progression?

A   EF

BCD   G

Figure 3.1

If so you've passed a common test used to determine genius! (Hint: A little lateral thinking — distinguish between curved and straight lines)

4. How would you describe a letter if you couldn't draw it? Assign letters of the alphabet among the class. With a ruler and protractor draw, then write, the **geometric** instructions that would allow someone else to draw a letter without seeing it.

Describe sides in relative length (fractions or decimals) of the longest side and use angles to describe meetings and intersections. Do not measure and indicate *every* angle; only the minimum number necessary to draw the letter.

Let Them Eat Cardboard

When many lines meet at a point but do not **intersect**, we need a compass to measure the different angles they form. But when straight lines intersect we can use our brains, instead of a compass, to do most of the measuring.

In the program our brave knight slashed up a cake with his sword to explore the geometry of intersecting lines. Since cake is messy, we recommend you use cardboard.

Figure 3.2

5. Duplicate the geometry of the cake in Figure 3.3 on paper or cardboard. "Slice" the cake along the straight lines.

Figure 3.3

Select any one slice of cake:

a) Measure its angle. One and only one angle size can be added to this to make a straight angle of 180°.

b) Use simple arithmetic to predict the measurement of this additional angle. Then combine your selected piece of cake with another piece to produce a straight angle.

c) Measure the angle of this additional piece of cake. Does it match your prediction?

d) An angle that is added to another to make a straight angle is called a **supplementary angle**. Does it matter in what order two angles are placed to make up a straight angle?

6. Study the cake in Figure 3.3:

a) The "Happy" angle is a part of how many straight angles? In each straight angle which angle is **supplementary** to the "Happy" angle? Since an angle supplementary to the "Happy" angle can have only one measurement, what conclusion can you draw about angle "Birth" and angle "Day," which are opposite angles?

b) Use the same reasoning with each of the other three slices to explore the geometry of the entire cake. Confirm your predictions by comparing the cardboard slices.

c) Review this reasoning process. Notice you have not had to refer to any *specific* angle except the straight angle. As a result your "proof" applies to any pair of intersecting straight lines, no matter what angles lie between them.

Intersection Inspection

Squint and you'll find many intersections lurking around you.

7. Collect several examples of intersecting straight lines from the world around you. Sketch them. "Apply" geometry: measure one angle and predict the others. Confirm your results by measurement. Which angle(s) occurs most often?

Geometry for Your Prying Eyes

Levers are some of our most important machines. For example, they help us pry the lid off pop bottles. Geometry can help us pry the lid off levers.

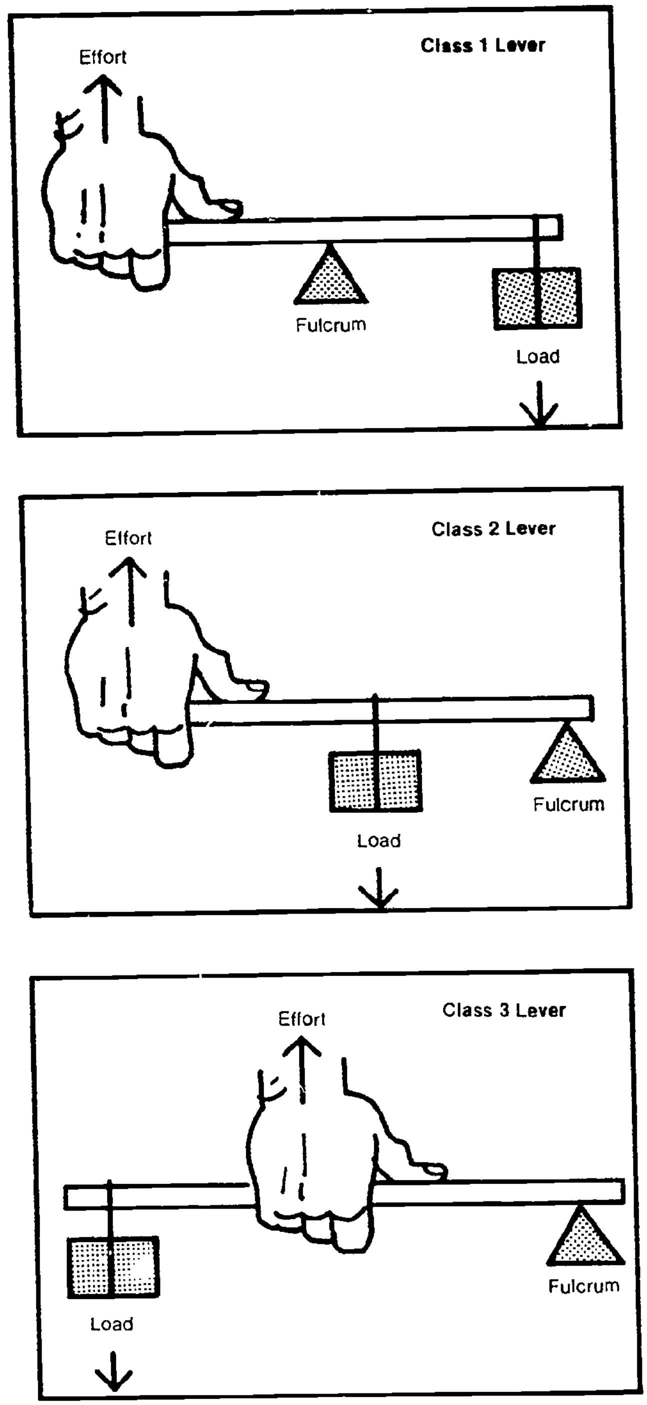


Figure 3 4

8. Physics describes a lever as an arm fixed to a fulcrum. A force acts on the arm to move a load.

Collect or list as many levers as you can. Categorize them according to the three classes listed in Figure 3.4, then "squint" at them. Can you see the geometry of intersecting straight lines in all your lever examples?

I'm Not Fooling Around ... Honest

9. Construct a classroom catapult using a ruler, an eraser, and a suitable object to throw. Just as we can tinker with shapes to build geometry, our geometry models can also tinker with *time*. Draw "before-and-after" positions of the lever, both on the same fulcrum point. Can you see the intersecting lines now?

Explore how the catapult works best, and draw its geometry as a **transformation** instruction for a single line, to be rotated about a point.

What Advantage Does a Lever Give Us, Anyway?

As a lever moves the geometry of intersecting straight lines helps us see that the angle created by the movement of one arm of the lever *must* equal the **opposite** angle movement of the other arm. If the angles stay equal but the arms are different lengths, something has to "give." The result? A force at one end produces a different force at the other end. Speed of movement at each end is also different.

10. In Figure 3.5 arm A and arm B determine the advantage of the lever. If A is two times as long as B, then moving the load takes one-half the effort. If A is four times B, then the force need only be one-quarter of B.

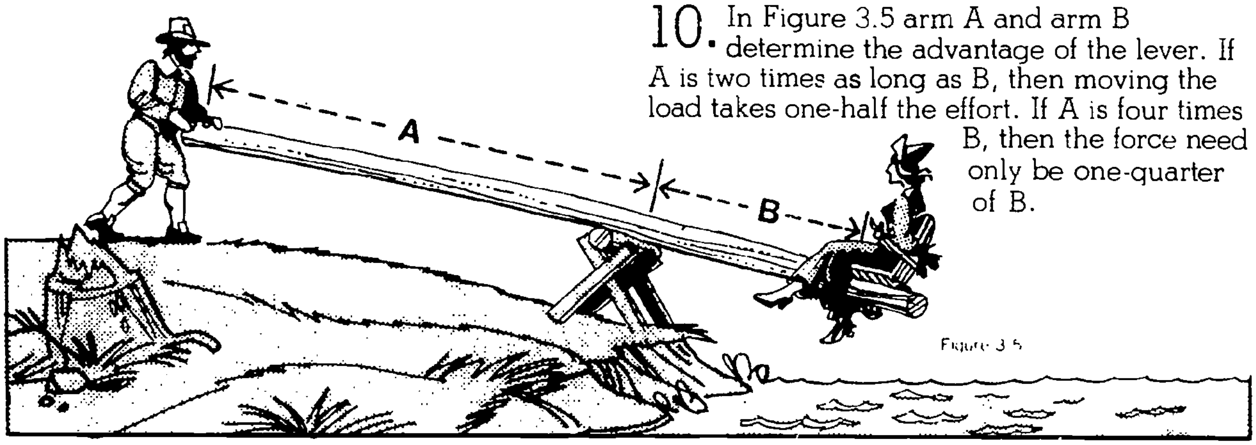


Figure 3.5

Be Lever Mad

11. Why should Rube Goldberg have all the fun? Draw your own zany contraption such as the one in Figure 3.6 to perform a vital job.

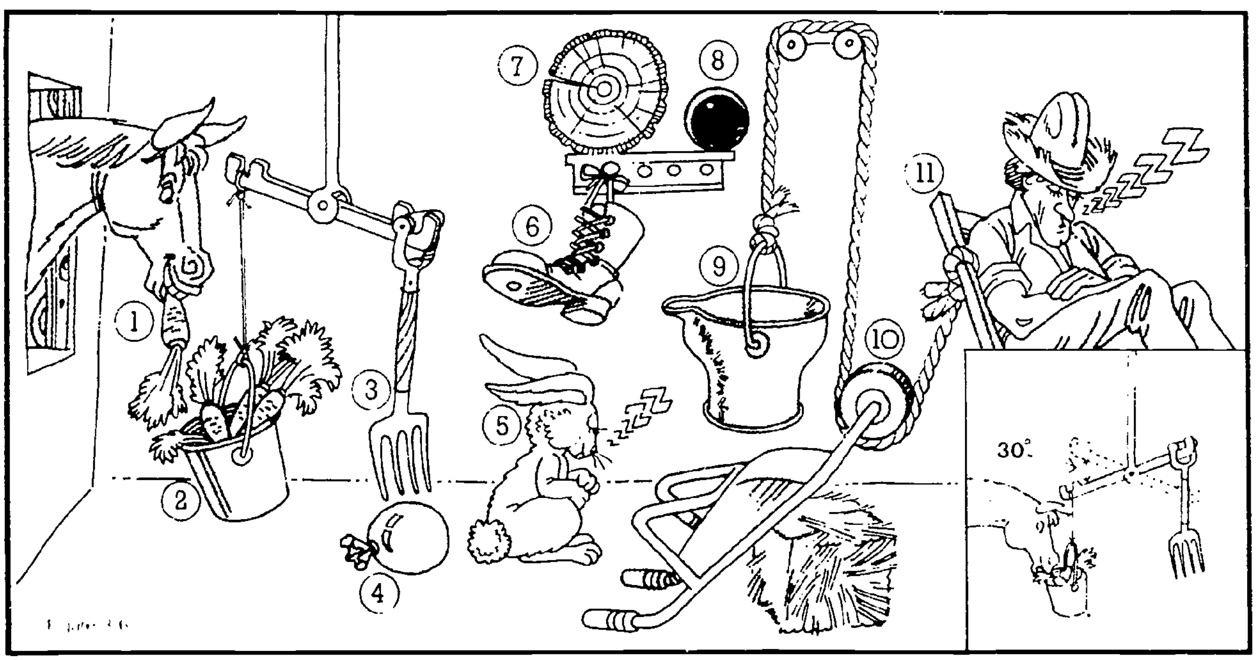


Figure 3.6

Rube Goldberg used plenty of levers in his designs. So should you, but you can go one better. Estimate the lever "throw" from start to finish and draw the geometry of the lever action in each case. This will give your contraption an air of scientific respectability.

"X" Marks the ... Er ...

Any fool pirate can bury a treasure. It takes a smart one to map it so that it can be found again. An "X" is just the way to mark the spot.

12. Here's a team game to test your pirate skills.

Divide the class into pirate crews of six with appropriate names. Each crew further divides in half — one group of "squirrellers" and one group of golden retrievers.

Each crew of six requires a nail (treasure), magnet (close-range retrieval equipment), a pole or rod as a temporary location device, and a sighting device — a handbearing compass (See Figure 3.7).

The squirrellers move to a field out of view of their teammates. (There is nothing to hide from the other pirate crews, however, and a common field may be used for the contest.)

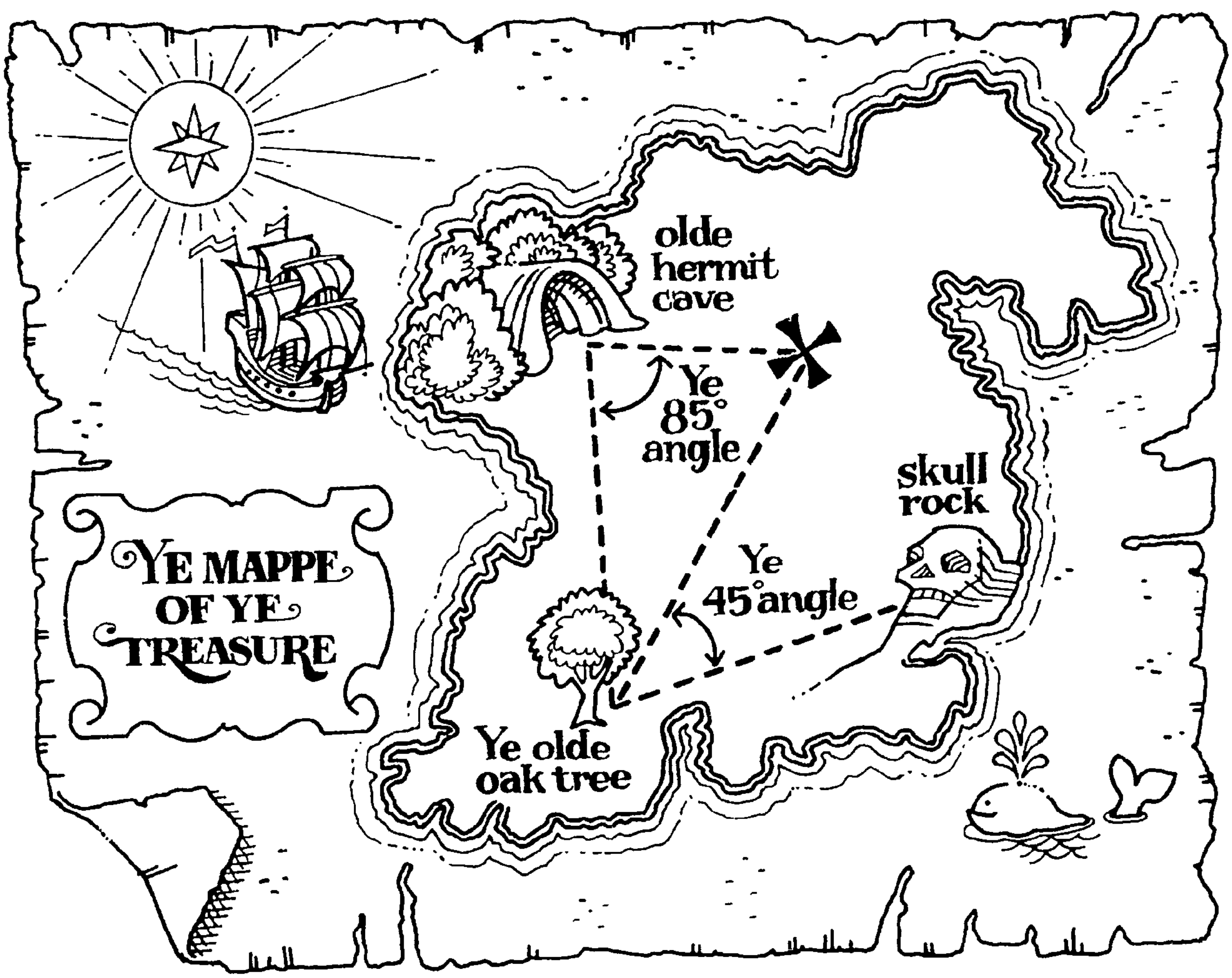
The nail is then hidden beneath grass (or snow) and the rod is temporarily implanted at this spot because the nail can't be seen at a distance. The squirrellers select two visible landmarks. Determine the minimum number of sightings necessary to locate the treasure.

Measure the angles formed by lines connecting the landmarks and the treasure.

Transfer these angles to a treasure map.

The golden retrievers from each crew then go to the field and, on a signal, are handed treasure maps and equipment.

Figure 3.7



If the squirrellers and the retrievers in turn measure the angles carefully, the retrievers will quickly find the treasure. "Yo-ho" and may the best pirates win!

Dam Busters

The Dam Busters had a clever way of using two light rays to fly a constant height above the earth.

13. Tape two flashlights to each end of a rod or stick so that they face toward each other. Each should make an angle with the rod of about 45°. Now hold the rod above a shaded floor so that the beams point downward. What happens as you move the rod toward and away from the floor? Change the angle of both lights to 90°, then 10°.

What advantages and disadvantages do different angles have for your rangefinder?

14. How did the pilots know when to release the bomb?

You can reverse the rangefinder above to produce a bombsight. On the dam were two towers X meters apart. The bomb needed to be released at distance Y.

Draw a plan view of the event on a piece of paper. Can you design a bombsight, using a piece of wood and nails, that would allow you to release the bomb at the right time? Draw the bombsight beside the plan and indicate the **congruent** angles.

X-Ray Vision

15. Cameras can be pretty expensive. Would you believe that you can design and build your own using the principles of intersecting straight lines?

First, let's explore the characteristics of light rays. Look outside a window at a tree. Stare at its tip momentarily. What you see is a light ray striking your eye. Now go to another window and you will still see the tree tip — only this time a different ray has been intercepted by your eye. The trick is to isolate "one" ray coming from the tree tip by filtering out all the others.

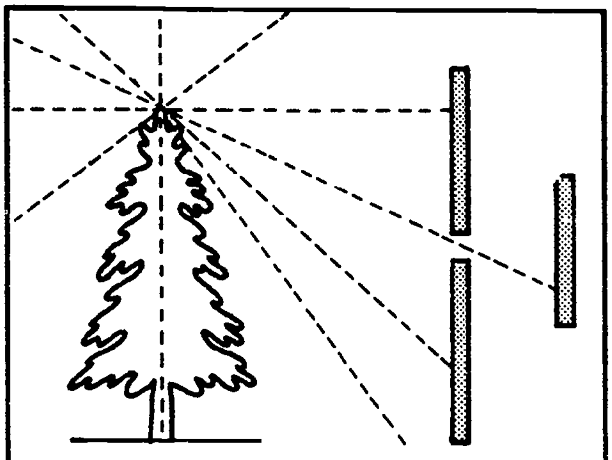


Figure 3.8(a)

Continue Figure 3.8 (a) and imagine other parts of the tree giving off light rays. Selecting the base of the trunk, we can once again isolate a "trunk" ray.

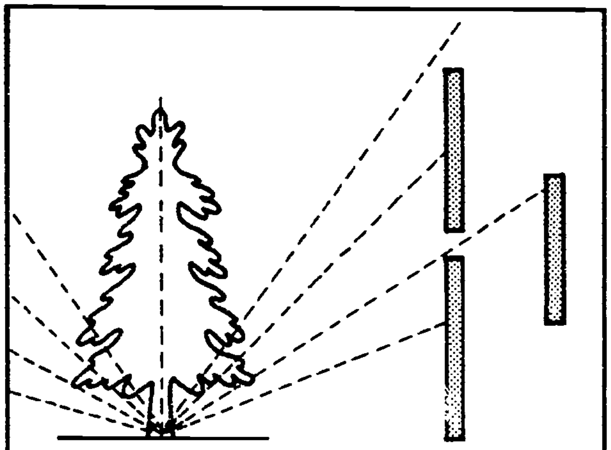


Figure 3.8(b)

Draw rays from other parts of the tree through the pinhole or point. It is now necessary to intercept these rays on a plane a short distance beyond the pinhole filter so that we can pick up an image — much the way a screen captures the image from a slide projector.

Draw the complete image of the tree on the plane. What is peculiar about the image? How will it differ from the original object?

The final step is yours. Design a simple camera that you could mass produce in your basement — and you are on your way to becoming a millionaire.

PROGRAM 4
LINES THAT DON'T CROSS

THE PROGRAM

This program explores the nature of parallel lines by:

- Discussing the concept of "same distance apart"
- Considering a sideways slide transformation
- Chasing the outer limits of intersecting straight lines.

The important property of corresponding angles is developed and several activities revolve around practical uses for the relationship between corresponding angles and parallel lines.

We apply these concepts in real life in military drill, celestial navigation, weaving and dressmaking, coastal piloting, and science fiction.

ACTIVITIES

The activities develop the concepts of the program and explore:

- Our ability to recognize and even "predict" parallel lines when we can't see them
- The difficulty of actually describing how to construct them
- The dilemma of "proving" that parallel lines never meet
- The measurement of corresponding angles by approaching the limits of intersecting straight lines
- The particular characteristics of two intersecting sets of parallel straight lines that form a parallelogram
- The extension of "same distance" to include "in the same direction" as another method of constructing parallel lines
- Practical exercises in constructing parallel lines and measuring corresponding angles.

Parallel Lines — The Eyes Have It
... Even if the mind doesn't

1. Look around you. Count straight lines that appear to stay the same distance apart. Popular, aren't they? Discuss why.

2. Your brain is pretty good at recognizing parallel lines — but it can be fooled.

In Figure 4.1 which pair of lines is parallel and which is not? Try to confirm your guess in each case by extending the pair of lines to see if they meet.

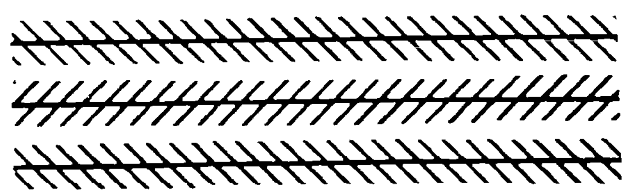


Figure 4.1

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Organization for the Nation

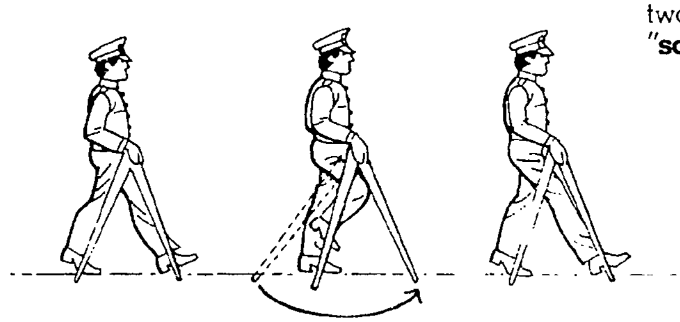
3. Squint hard enough and you can find geometry under an idea. Organization? Orderliness? Hiding under these ideas is the parallel straight line.

Are your classroom desks in parallel straight lines? The parking lines in your school lot? How many examples can you find where the concept of organization and order really means parallel straight lines? Sketch them, and their underlying geometry.

The Thin Red Parallel Lines
Or ... trooping the geometry

4. Ceremonial drills (such as the British tradition of trooping the color) sometimes require marching in long parallel lines. Form up in ranks of three, and try marching abreast in line while maintaining your geometry.

Discuss these military drills. How do the troops maintain their rank? Why do you think these geometries are, or were, useful? Do they help you to *describe* how parallel lines in general can be constructed?



Front Rank, One Pace Forward ... March

5. The military uses a kind of **transformation** instruction to widen the space between two rows of soldiers. Think of a rank of soldiers transformed, or slid a pace forward, to form a new rank.

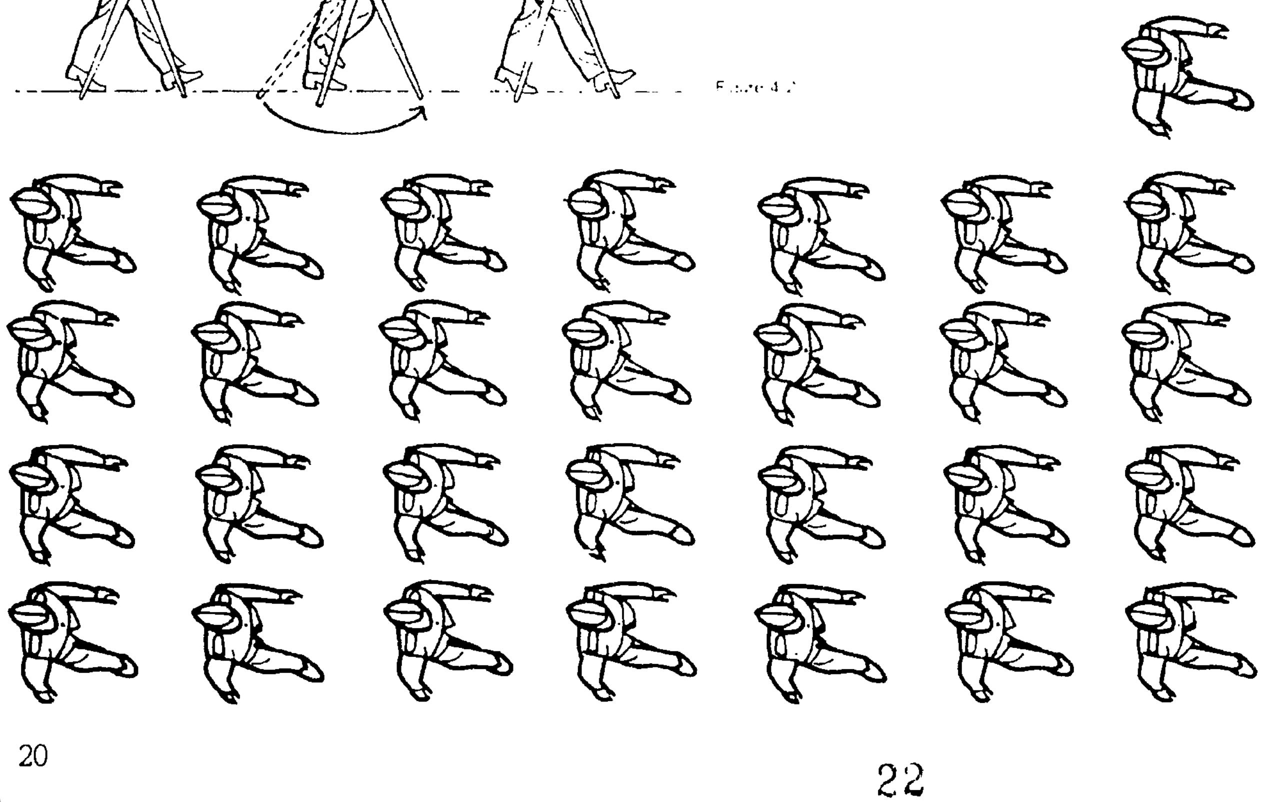
Geometers use the same technique to describe a second parallel line as a **slide** transformation. Draw a straight line on a piece of paper. Write a transformation instruction to create a parallel line 5 cm away.

It's Really Just the Same Difference

6. Some military services use a giant geometry tool — called a pace stick — to help create parallel lines. What tool does the pace stick resemble?

Drill sergeants may *know* parallel lines when they see them — even order them up. But can they actually describe them? Try it yourself.

Divide the class into pairs. Each of you writes an exact description of a pair of parallel lines. Exchange descriptions. Following *only* the instructions in the description, try to construct two parallel lines. What exactly does the "**same distance apart**" mean?



Space Invaders

7. You're quite the space traveller — each year the earth makes one complete circuit around the sun. If you squint at the earth in space, you can draw a **base line** between the two far points of travel.

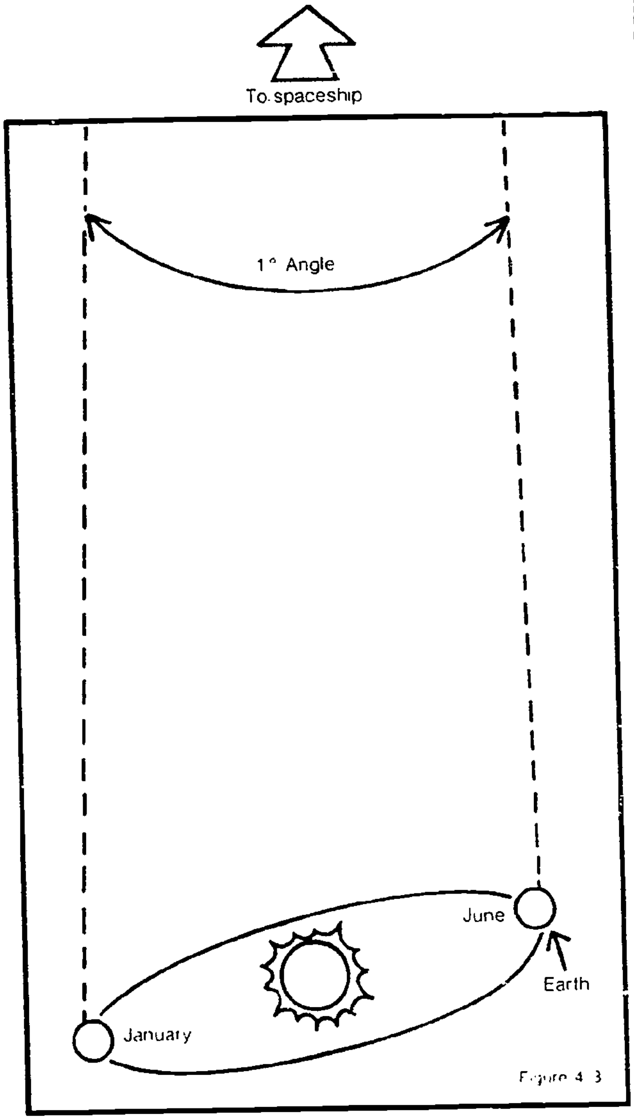


Figure 4.3

For six months, a UFO has been stationed in deep outer space. Before we launch a space probe, we need to determine how far away the UFO is so that we know how much fuel to take on board. We know this much: when we used the extremities of the earth's orbit as a base line, our sightings intersected the spaceship at an angle of 1°.

Using a scale drawing (be prepared with a long piece of paper), calculate how far away the spaceship is.

If this were the smallest angle an astronomer could measure accurately, which heavenly bodies could we fix using this technique?

Shadow Play

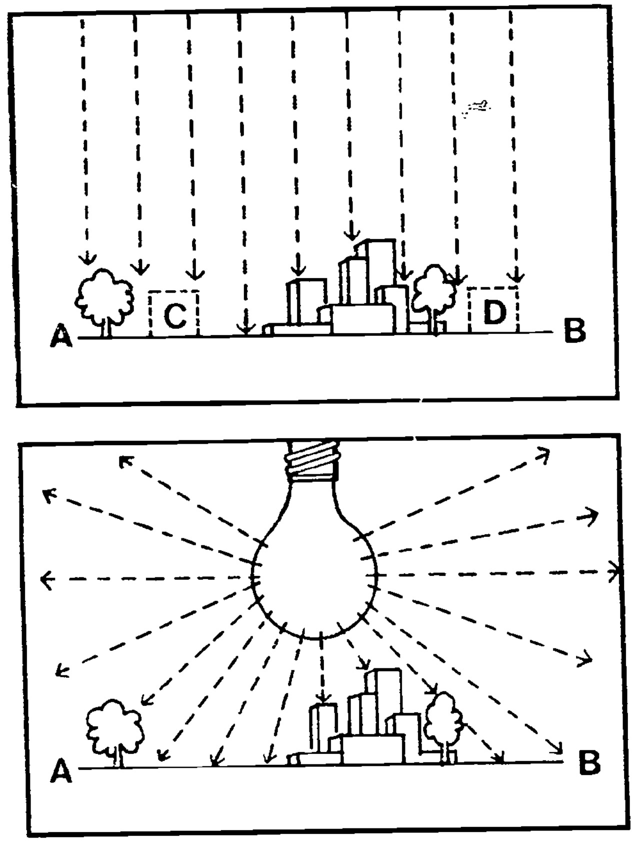


Figure 4.4

8. Hold your hand 30 cm from a piece of paper (a) in sunlight; and (b) near a lightbulb. Examine the sharpness of the shadows. Can you explain the difference?

Draw light rays from the sun and from the lightbulb as in Figure 4.4. Does your drawing help explain why one set of shadows is sharper?

Examine Figure 4.4. The earth line (AB) intersects many pairs of light rays. When the light source is close (lightbulb) each ray intersects line AB at a different angle. But when the light source is very far away (sun and stars) we can't measure any difference between two angles, such as C and D. Measure them yourself.

For our purposes, treat the light ray as **parallel**. An important geometric property of parallel lines is: when a straight line (AB) intersects a pair of parallel lines, the **corresponding** angles (C and D) are equal.

Parallel Rules the Waves

To a sailor land is a blessing and a curse. It may cause ships to sink, but it also acts as a signpost on water.

9. Refer to the chart (Figure 4.5). It is night. You know you are somewhere in Boundary Pass, but where? You see a light flashing every four seconds and identify it as the light at Gowlland Point on South Pender Island.

Using a compass, and allowing for magnetic deviation (a skill worth investigating), you determine that the light bears 320° from true North.

How do you draw this line on the chart? Note there is a compass rose. Draw a ray bearing 320° true North from the compass rose. Does it run through the light?

To find a 320° angle that does run through the light is simple, using the principle of parallel lines and **corresponding** angles.

Extend the ray that runs North-South through the centre of the compass rose from one end of the chart to the other. We will call this your "runway."

Now draw a circle on tracing paper having a diameter twice the compass rose. Trace a **prime ray** (0-180°) on it, and at the centre

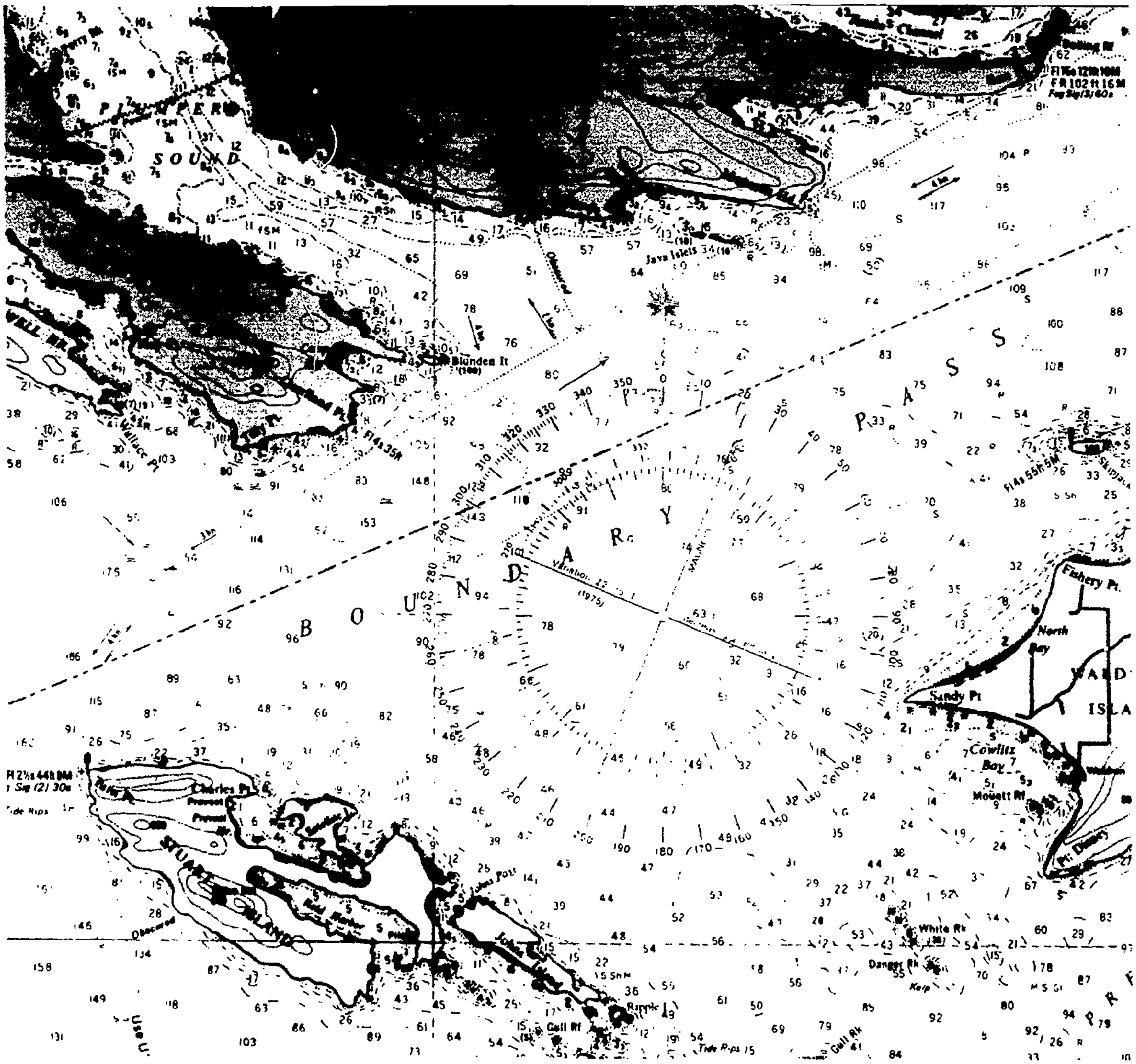


Figure 4 5

mark off an angle of 320° from the prime ray.

Now place the prime ray of your circle on top of the prime ray ("runway") of the compass rose. Slide your circle up and down the runway until the 320° angle on your tracing paper intersects the light.

Using this method, you have established what is known in navigation as a *line of position*. You know you are somewhere on that line (much better than knowing nothing at all)!

10. That's the hard way of doing it. Coastal navigators use an easier way. Inspect your line of position and compare it to the line you drew at 320° through the compass rose. Notice there are **corresponding** angles on the intersecting North-South line. Because the **corresponding** angles are equal, the two bearing lines must be **parallel**; so navigators use a parallel rule. Can you design a parallel rule out of cardboard?

11. You have taken another bearing on the light at the east point of Saturna Island. It bears 50°. Use your parallel rule to determine a second line through your boat. Place the rule at the correct angle on the compass rose, then "step" the rule until one edge runs through the light.

Do two "lines of position" determine precisely where you are?

The Same Difference ... Revisited

If you have tried to describe *exactly* what you mean by "the same distance apart," you'll know it's not as easy as it seems.

12. Examine your parallel rule closely. It is formed by *two* pairs of parallel lines. Remove one long side. Think of the remaining three pieces as a "set of rules" for creating the fourth. What conditions must be met *exactly* by one long line and two shorter lines before a second long line can be drawn parallel? Describe this condition and you will describe *exactly* what "the same distance apart" means.

After all this boating, we could do with ...

Drip-Dry Parallels

Underlying the parallel rule we can find a figure called a **parallelogram**. It is a four-sided figure, the opposite sides parallel to each other.

13. Flex the parallelogram. Would you call it a rigid or flexible figure? Draw a parallelogram on a piece of paper. How far can you "collapse" it? Pull on your parallel rules in various directions. Which way does the figure stretch? Stay the same?

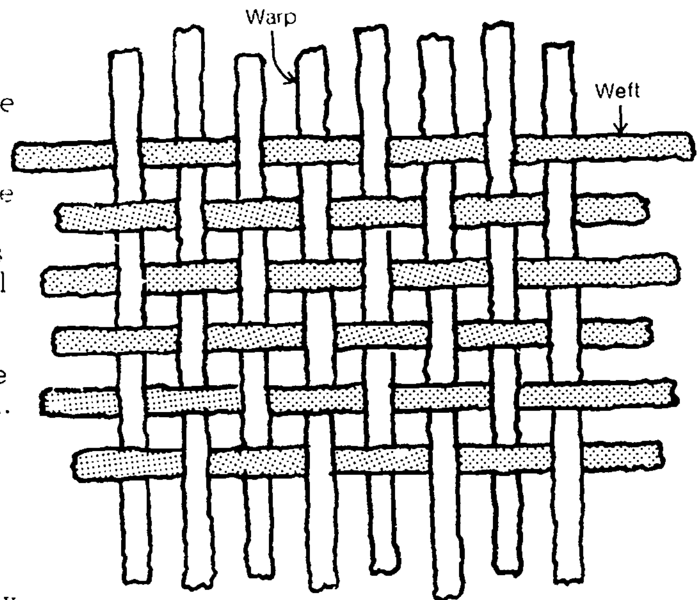


Figure 4.6

14. If you squint you can find a repeating pattern of these parallelograms in any piece of woven cloth. Do these tiny parallelograms "flex?" From your experiments with the parallel rule, which way would you expect the cloth to flex?

Most of you are probably wearing at least one piece of woven cloth. Examine it closely. Determine the direction of the *warp* and *weft* and *bias*. Test your predictions by stretching the cloth along the warp, weft, and bias. Do the results confirm your predictions?

Fairly Parallel

15. It may seem like a silly question, but are parallel lines a "fair" kind of geometry? How many sports can you list in which parallel lines play a part? Sketch some of these sports, altering the lines so they are no longer parallel. How does this change the sport?

A Slice of Life

16. In plane geometry only lines can be parallel. In three-dimensional geometry, lines *and* planes can be parallel. Can you sketch some objects built from parallel planes?

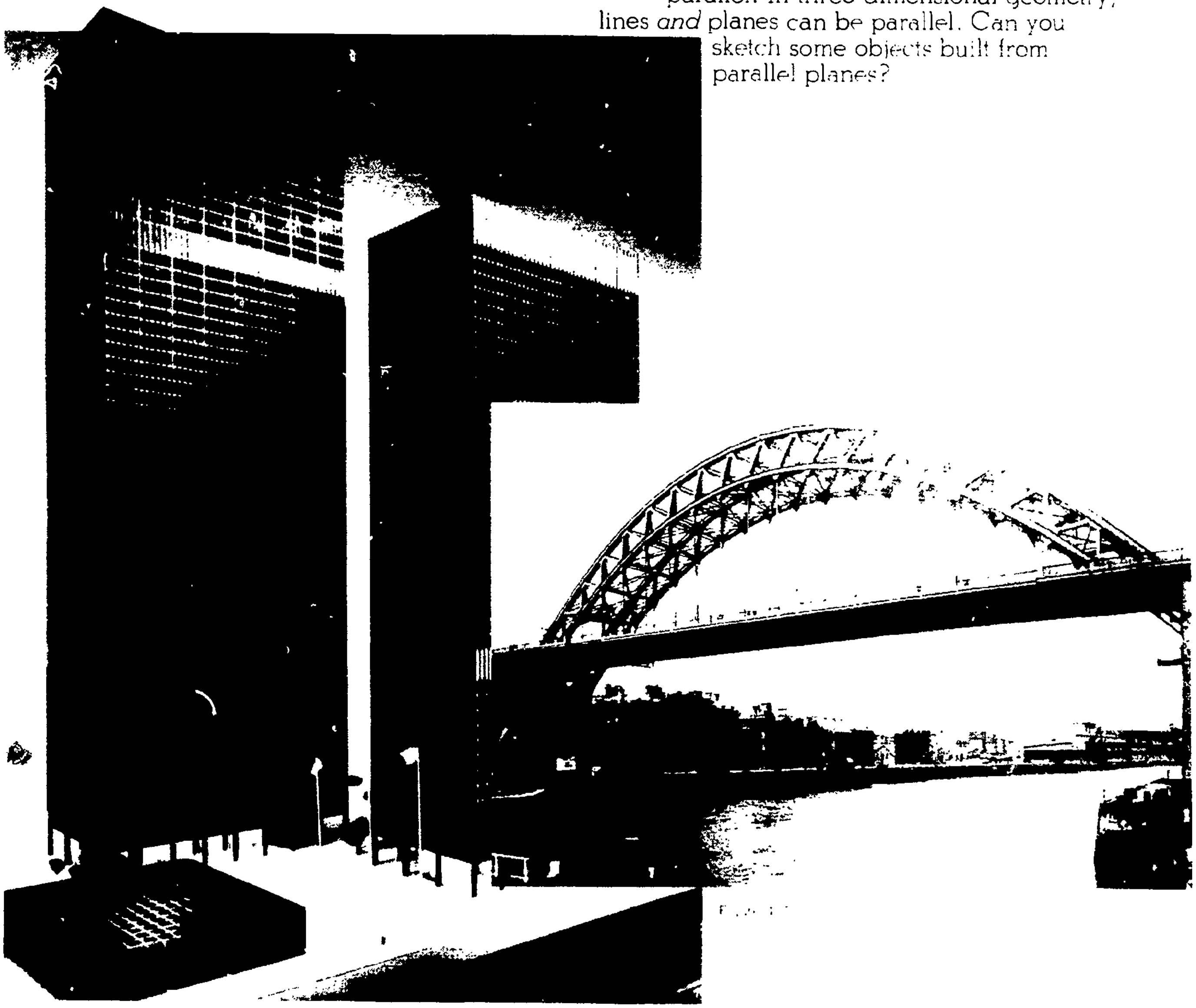
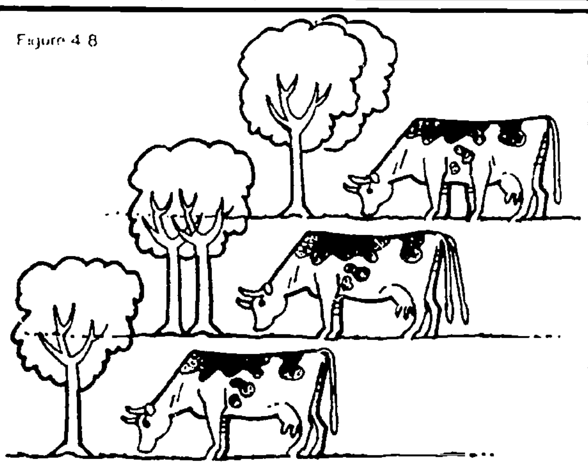


Figure 4 8



Parallel Universes

17. You can easily imagine parallel lines on a plane, and parallel planes in three-dimensional worlds. Can you "step-up" another dimension and imagine whole three-dimensional worlds, side by side?

Some scientists claim that parallel universes are not only possible, but that an infinite number may exist. This is a favorite theme for science fiction writers. Browse in your library and see if you can find a novel to read that is based on parallel universes (e.g., Isaac Asimov, *The Gods Themselves*). Write a review.

PROGRAM 5
UP, DOWN, AND SIDEWAYS

THE PROGRAM This program examines some properties of the right angle. Its importance in geometry equals its importance in the physical world, underlying the relationship of **horizontal** and **vertical**. The geometric relationship of **perpendicular** is introduced and the method of constructing perpendiculars is compared to real-life examples. Our first encounter with triangles in the series is the right-angled triangle — historically an important building technique.

In everyday life we apply the properties of the right angle in the construction trade, the physics of gravity, and techniques of wire and rope rigging.

ACTIVITIES The activities support and supplement the concepts of the program by exploring:

- Our sense of up and down
- The geometry of falling, and "not falling" introducing concepts of the right angle
- The use of the geometry of right angles to "support" movement, shape, and forces, through the concept of the "centre of gravity"
- The importance of right angles, which results in a convenient concept of **complementary angles**
- Right angles on the surface of the earth — a review of the concept of squinting and angle construction
- An introduction to the right-angled triangle and Pythagorean theorem (in preparation for Program 6)
- A preview of analytical geometry techniques based on the right angle, which are the foundation of transformation instructions (in preparation for Program 8).

Crooked Up and Crooked Down

Although there is a *tendency* for objects to fall straight toward the centre of the earth, different things "foul up" a nice straight line. Can you name some?

1. We may see a skydiver falling straight down, but can we *really* describe his or her path? What changes the course of his fall? What effect will the rotation of the earth – 1600 km/h at the surface – have? In which other directions does the skydiver move?

In the real world falling things may only roughly match an ideal **vertical** straight line that passes through the centre of the earth. Still, it's a *useful* way of exploring the geometry of falling.

Because gravity can kill, we fight the geometry of falling with ... the geometry of *not falling*.

The Geometry of Not Falling

The geometry of not falling depends on an important event.

2. When a vertical line intersects the earth's surface, what angle does it make with the earth?

With a piece of sticky tape, fasten string to a protractor so that it lies along the line between 0° and the vertex. Fasten a weight to one end of the string just below the protractor. Hang the other end over a nail or other convenient support. Fill a bucket or a large beaker with water and add a little chalk dust to it. Place the bucket beneath the protractor and lower the string until the vertex of the protractor is at the surface of the water. Allow the weight and water surface to come to rest. What angle does the "line of fall" of the weight make with the surface of the water?

Right Angles and Wrong Angles

3. Stand a ruler upright so that it makes an angle of 90° with a tabletop. Balance an eraser on the tip of the ruler.

Now tip the ruler to 80°. Does the eraser balance? Repeat this procedure several times, reducing the angle by 10° each time. Which angles support the eraser?

The 90° degree or **right angle** is an important prop to prevent falling. An object falls at 90° to the ground. By "building" at 90°, we "get in the way" of a falling object and prevent its fall.

4. Look around you. What examples can you find that seem to use right-angle props at the ground to keep them upright?

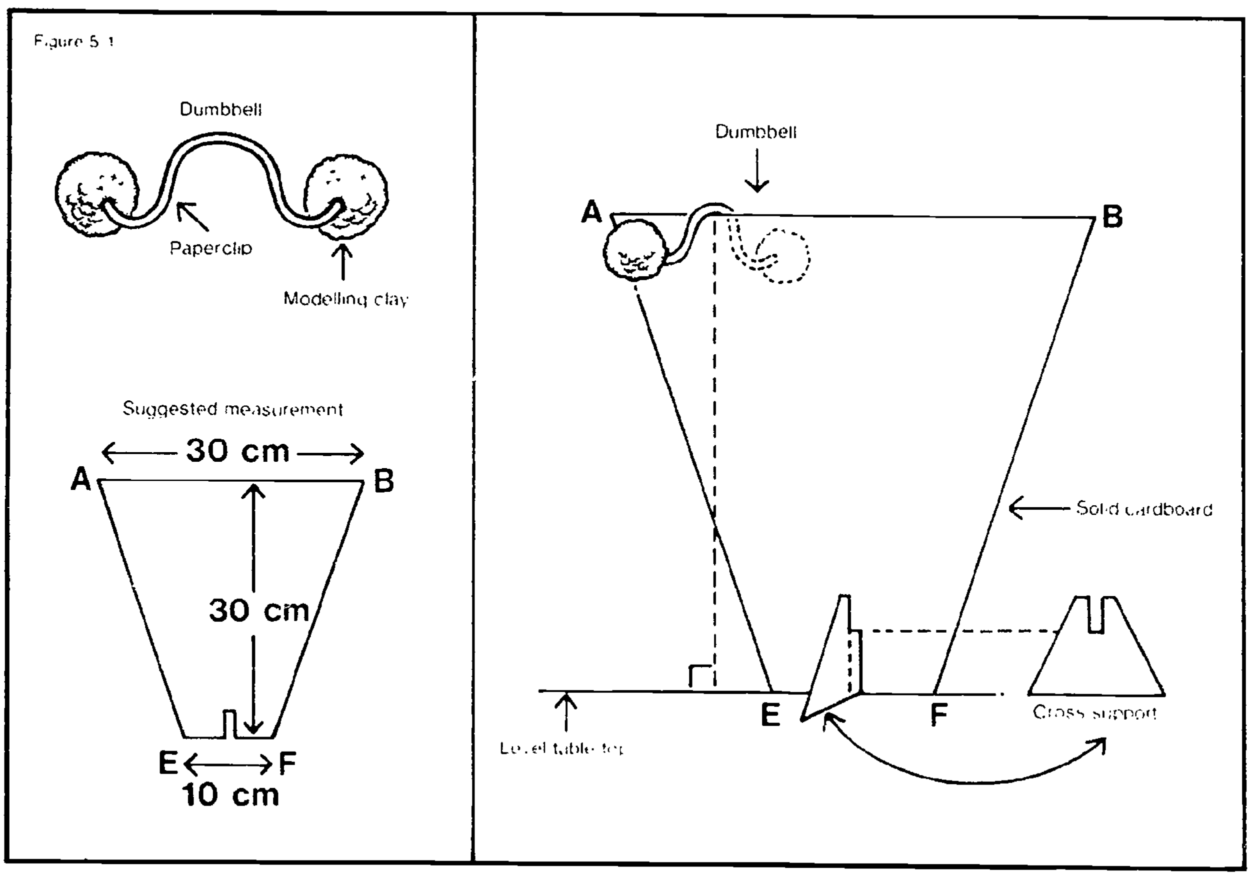
How many things can you see that stay upright perfectly well, *without* appearing to use any right angles at the ground?

The Course of the Force

5. We can use geometry to describe movement and shapes. In the previous activity you were probably describing shapes. But falling and not falling have to do with **force** first, and shape only second. We can use geometry to describe these forces.

Make the apparatus in Figure 5.1 from

Figure 5.1



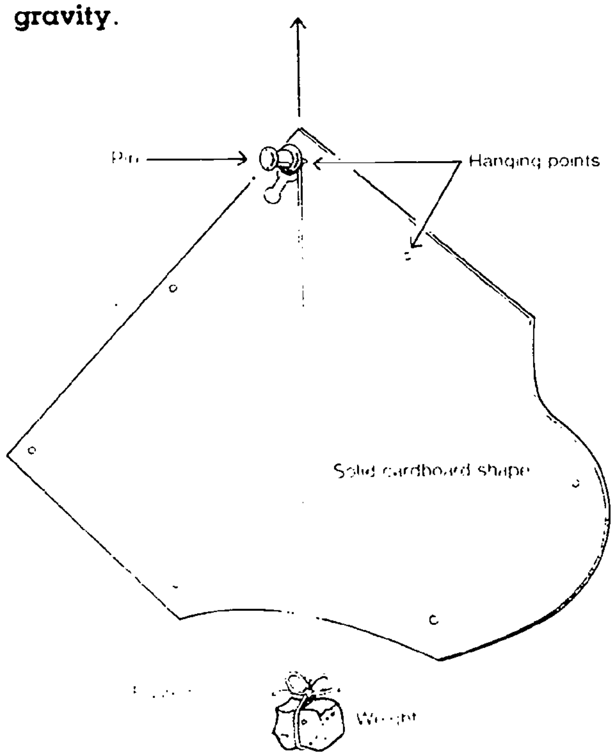
cardboard. The modelling clay weight should be as heavy as possible without buckling the cardboard. Experiment by placing the weight at several places between A and B. The force of gravity pulls the weight so that it "wants" to fall straight down.

In each case draw a 90° angle from your desk top to the weight. Note what happens when the vertex of the right angle falls *between* points E and F, and outside of them. Using these observations can you devise a rule to describe **balance**?

Remove the clay from the paperclip, and repeat the activity. Why do you think the results have changed?

Shape Up or Fall Down

Shape, weight, and **weight distribution** affect how well buildings or other objects "stand up." We can think of the weight as "acting around" a single point called the **centre of gravity**.



6. Cut an irregular shape from a piece of cardboard. Pierce holes around the edge with a pin, and hang the cardboard as in Figure 5.2.

Trace the line of thread across the cardboard in each case. The point where the lines intersect is the **centre of gravity** the cardboard will balance on this point.

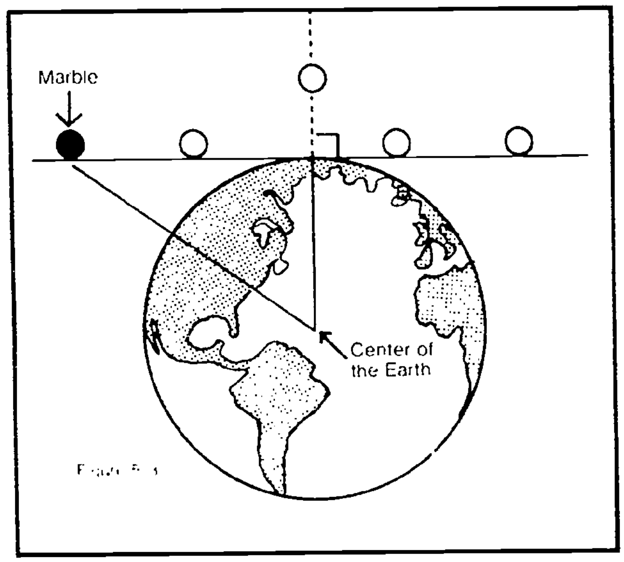
Repeat the experiment with a large blob of modelling clay pressed to one edge. How does it change your results?

Can you rewrite your rule for balance in Activity 5, referring to the centre of gravity?

Falling Sideways

... Possible or impossible?

7. Does a straight line exist in your classroom, such that a marble will roll down the line for half its length, and up the line for the other half, *without the line being moved or changing direction*?



Draw a circle to represent the earth. Draw the vertical line of fall of a marble, from above the earth down to the centre. At the point where it touches the earth, draw a horizontal straight line -- a giant table — making an angle of 90° to the vertical line.

Now place a marble at one end of the table. Measure the distance from the marble to the centre of the earth. Place your marble on the horizontal line at the point where it touches the earth. Measure again to the centre of the earth. Has the marble moved closer to the centre of the earth? Take intermittent measurements as the marble travels to the opposite end of the table.

Look at the surface of your desk. If it is perfectly horizontal, then *any* line on it is that magical line which goes up and down at the same time. But why doesn't a marble placed at the edge of a horizontal desk roll to the middle?

Ride on a "Geo-Yo-Yo"

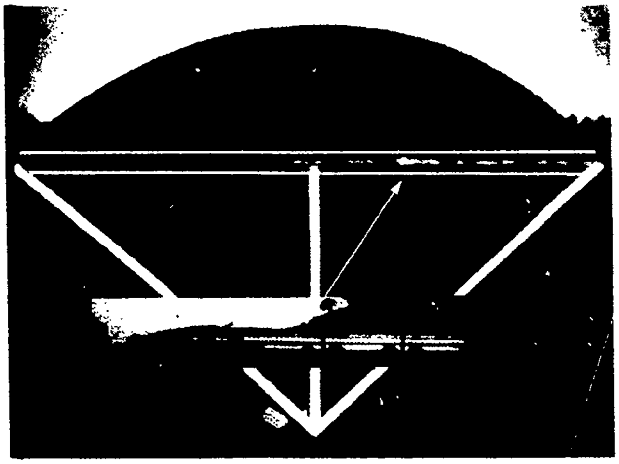


Figure 5.4 Train travelling through tunnel

8. Imagine a tunnel cut straight through the earth from Vancouver to St. John's, Newfoundland.

Draw a diagram that shows such a tunnel as a straight line cutting across a circle. Measure the distance from the tunnel to the centre of the earth at various points along the tunnel. Now describe what would happen to a train travelling through the tunnel.

Around and Around and Around It Goes

A right angle can also help us explore how a lasso is thrown, as well as a discus and a hammer in track and field.

9. Swing a purse or bag around your head. Where does the pull seem to come from? Can you predict in which direction the weight would fly if you let it go?

10. Find a wind-up or battery-operated car, truck, train, etc. Fix the front wheels straight ahead. Fasten one end of a piece of string to the vehicle's side and attach the other end to a nail or pin that keeps the vehicle travelling in a circle. Tape a felt-tipped pen to the vehicle so it traces the circular path on a large piece of paper taped to the floor.

Now quickly release the string from the nail, carefully noting the exact point of release. Repeat several times, each time measuring and drawing the angle between the nail, the point of release, and any point on the new line of travel.

Sidling Up to Down

11. Carpenters, stonemasons, and other builders use the right angle as one of their basic tools. Although they may use plumb bobs, they often use another method of determining vertical lines -- by finding the vertical from the **horizontal**.

What is a level? How does it work? Find three small test tubes with corks. In the first put water and a little soap; in the second, cooking oil; and in the third, liquid honey. Leave a little air trapped under the cork. What happens to the bubble in each test tube as you tilt it. What liquid makes the best level? Why?

Draw a carpenter's level containing such a bubble tube. How long should it be? What advantage does a long level have? A short one?

Now draw a level that *adds* 90° to the bubble tube. This allows you to find a line **perpendicular** to the horizontal — in other words, a **vertical** line.

Free with Every Right Angle

... Complementary angles!

12. Two angles that add up to 90° are known as **complementary angles**.

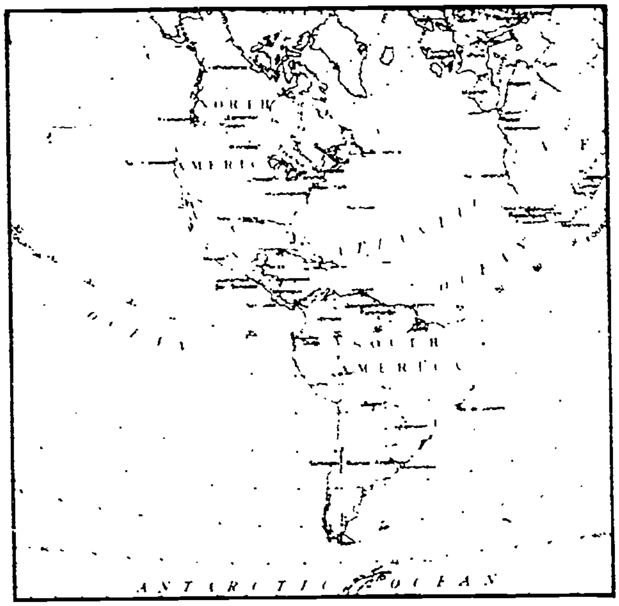
Squint at right angles in the world around you, and see if you notice any complementary angles within them. Measure them. Which pair of angles are the most common complementary angles?

Fiddle with a Grid'll Get You Someplace

13. Navigators and cartographers use a grid system on the surface of the earth to keep from getting lost.

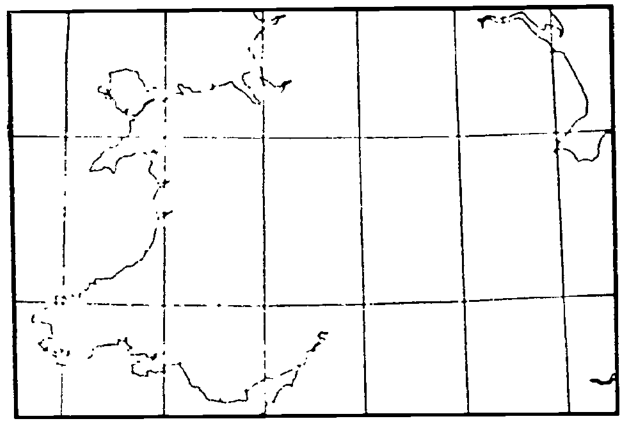
Look at several large-scale maps of the continents. Notice the grid of lines on the maps. What are these lines called? See if you can find maps that have grid lines that meet at a variety of angles. Measure some of the angles at which they meet.

Now look at a globe. The same grid system of **latitude** and **longitude** can be found on the globe. Use the corner of a piece of paper to test different angles where grid lines meet on



[illegible] can you find any which *do not* [illegible]

You have been exploring another reason why flat maps often poorly represent the curved surface of a globe.



Stars and Stripes

14. You can learn to navigate by the stars, using lines of latitude and your knowledge of parallel lines and right angles.

Make a simple sextant for measuring altitude (see Figure 5.6). Besides the sextant, you may need a flashlight to help illuminate the sighting edge of the protractor, and an assistant, to observe the angle while you sight.

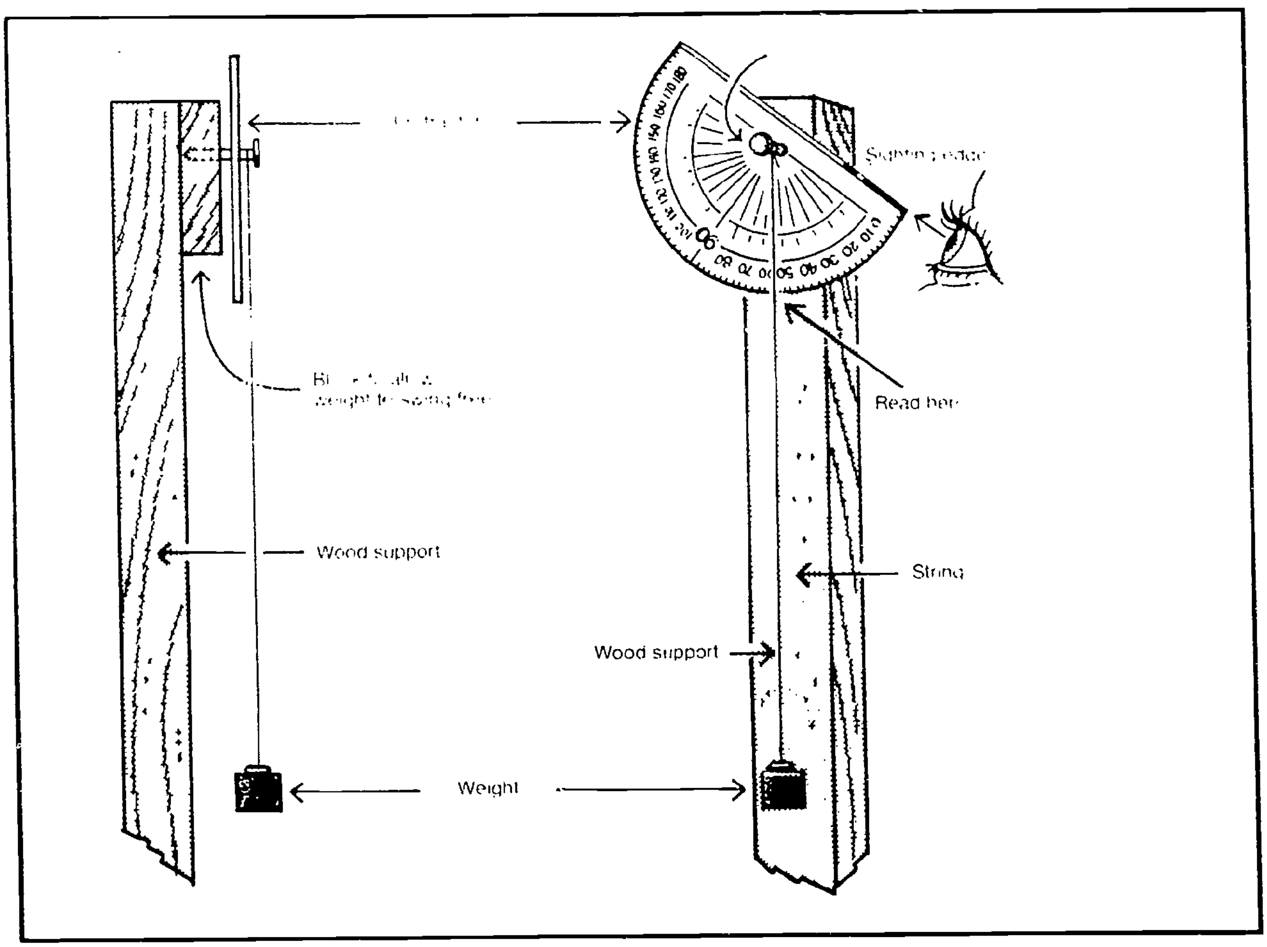


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Figure 5.7

First, rearrange the universe as all navigators do. Make the centre of the earth into the centre of the universe. All the stars revolve about the earth — all except the polestar, which stays fixed almost directly above the North Pole. Refer to your drawing of lines of latitude. What angle does a ray from this star make between the centre of the earth and the equator?

On a clear evening locate the polestar, using its Pointer stars in the Big Dipper (see Figure 5.8).

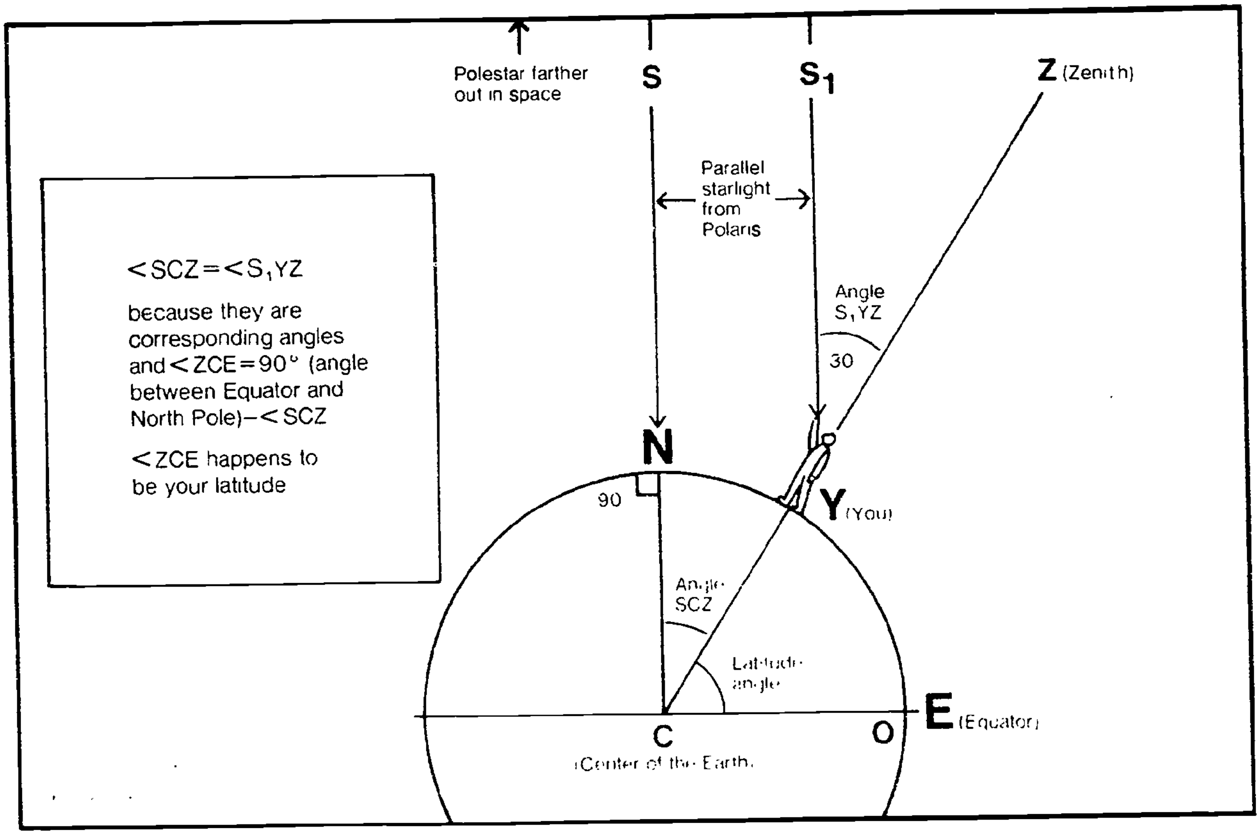
Now, you need to know your own **zenith**. Look straight up. Spin around rapidly. Before you fell down did you notice a point **vertically** over your head that didn't spin? That's your zenith.

It's a little easier, though, to use the plumb bob. It points both to the centre of the earth and to the opposite direction — the zenith.

While you sight the star, have your assistant record its angle to the zenith using the plumb bob. Now you can roughly find your latitude on earth.

You now know that the angle between the equator and the North Pole at the centre of the earth is 90°. The angle at the centre of the earth between the equator and the point where you stand (Figure 5.9 — angle YCE) is your **latitude** measured in degrees, which you don't know.

Figure 5.8



But here is a stroke of good fortune. Refer to Figure 5.9. You have measured angle S YZ at the surface of the earth. Because rays from the stars are parallel when they strike the earth, they intersect your zenith line everywhere with the same **corresponding angle**. Imagine the angle of your sight **sliding** along the zenith to the centre of the earth.

Angle S YZ corresponds to the angle SCZ at the centre of the earth. And this angle is the **complement** of your latitude (i.e., the angle that must be added to your latitude to make 90°). Since only two angles can together make up 90°, subtract your sighting from 90° to find your latitude!

Confirm it on a map, O mighty Navigator.

Right Angles: A "Rigger-Us" Approach

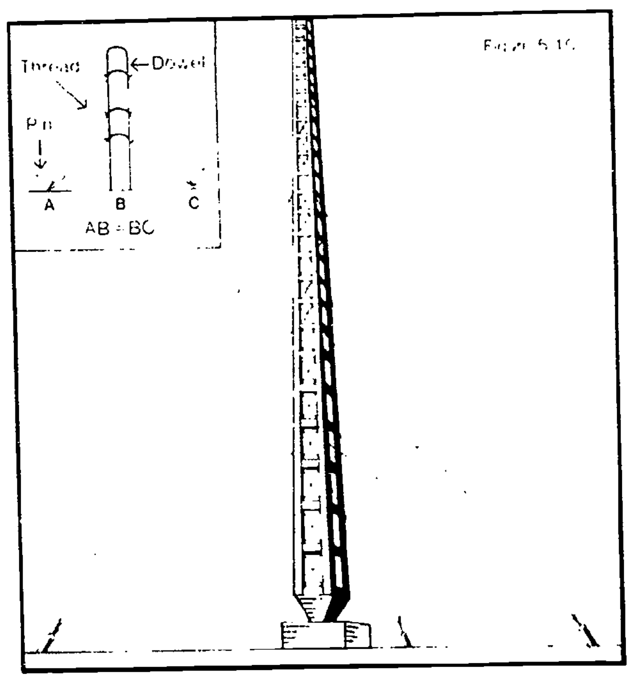
15. Right angles are of enormous importance in our modern world. Often they have nothing to do with "up and down." When we leave the world of "up and down" we leave behind the terms **vertical** and **horizontal**. Instead we say one line is **perpendicular** to another when they meet at a right angle.

Riggers erect masts — radio masts and ships' masts. Discuss why a plumb bob or a level might be inconvenient when swinging several thousand kilograms of metal pole 50 m high.

Riggers depend on right angles to erect masts. And geometers use the same technique to construct perpendiculars.

Erect a radio mast, using a dowel, thread, and

Figure 5.10



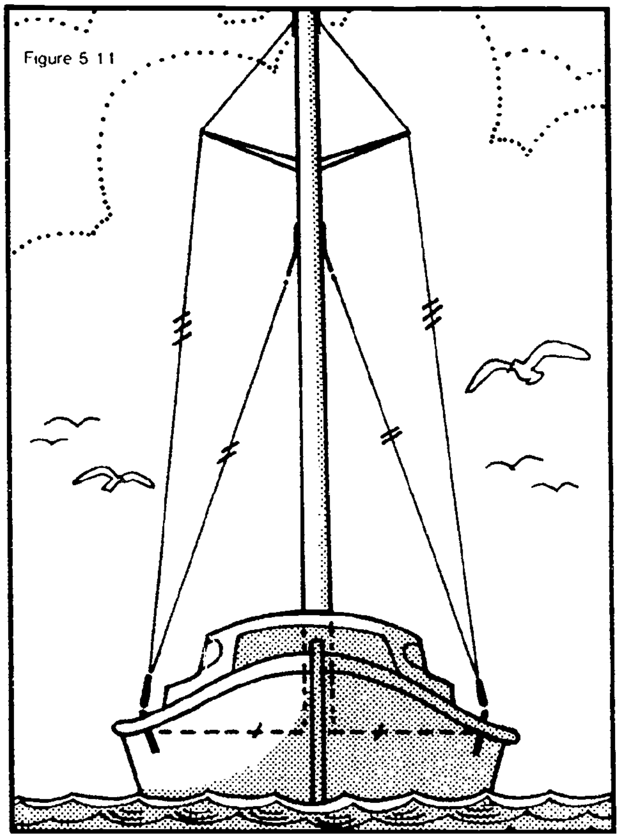


Figure 5 11

pins, as shown in Figure 5.10. (You will have to do this in two planes to keep the mast upright.) Use a protractor to make certain the mast is **perpendicular** to the ground.

Now dismantle your mast, cutting the threads carefully at the pins. Compare the lengths of the threads. If you think about it, you can now devise a way of getting the mast upright using the lengths of the pairs of threads, and without the use of a protractor.

16. Instead of using equal lengths of string, geometers use a compass to "cut" equal lengths on paper. Can you adapt your rigger's technique to use a compass on paper and draw one line perpendicular to another?

Mysteries of the Great Pyramids

17. Researchers have discovered that the great pyramids of Egypt were built with very accurate geometry, including a great number of right angles. How did they do it?

Cut out three narrow strips of cardboard; the first, three units long, the second, four units, and the third, five units. Pin the ends together in any way you wish, to form a three-sided figure — a **triangle**.

Measure the angles between each of the sides of this triangle. The Egyptians knew this secret of geometry and stretched ropes to lay out very large right angles quite accurately.

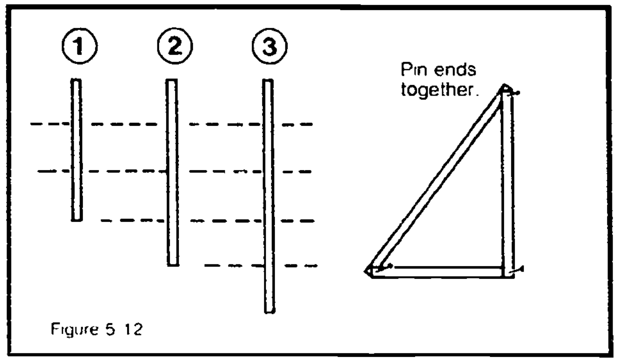


Figure 5 12

It's Easy for You to Say, Pythagoras

18. The Greek mathematician Pythagoras explained why the three-four-five triangle forms a right angle, and gave us a way of finding *any number* of triangles with right angles.

Using a protractor, draw a triangle with a right angle. Label the side *opposite* the right angle "a." This side is called the **hypotenuse** of a right-angled triangle. Label the other two sides "b" and "c."

Erect a perpendicular at either end of side a. (You can do this in two directions — both will give the same results, but one direction will result in messy construction.) Use a compass to make the length of these two sides exactly equal to a. Join the tops of these two sides. You have constructed a four-sided figure with all its angles right angles and all sides equal to each other. What is this figure?

Repeat to create the same figure on sides b and c.

Pythagoras proved that the area of the **square** on the side opposite the right angle is equal to the sum of the areas of the squares on the other two sides.

Test this by cutting out the two smaller squares, then cut them up to fit on the third. (Note: This puzzle may be very time consuming as it is not as simple as it appears.)

19. The Pythagorean theorem can be expressed in algebra as well: $a^2 = b^2 + c^2$. Measure the sides of the figure you've drawn and square them to confirm this. Draw another right-angled triangle and repeat.

PROGRAM 6
TRUSSWORTHY

THE PROGRAM This program explores the nature of the triangle. It describes the triangle in terms of sides (e.g., isosceles) and in terms of angles (e.g., right angled). The useful properties of triangles are explored in the "real world" — in particular, their stability and rigidity. Two methods of constructing congruent triangles are considered in the program, and the third method in the Activities section that follows.

Examples of how we use the properties of triangles are drawn from the disciplines of geomorphology, engineering, surveying, and archeology.

ACTIVITIES The activities expand on the concepts of the program to examine:

- The angle properties of different triangles
- The side properties of different triangles
- Nature's "triangles," and the use of the right-angled triangle to describe any slope
- The particular stability of triangles that relate to some ancient monuments
- The use of the SSA method of constructing congruent triangles to measure slopes
- The use of the SAA and SSS method of constructing congruent triangles in surveying and archeology
- The rigidity of the triangle and its importance in engineering
- The perimeter of a triangle as a function of materials cost.

Let's Go "Trianguleering"

Using straight lines, triangles are the simplest possible closed shape — a figure that encloses a surface.

1. Mountaineers climb mountains; "trianguleers" climb triangles (or, at the very least, hunt for them).

Do the very least. Hunt the world around you for triangles. You'll find many common triangles perched on buildings. Why?

2. In construction, a roof is an important enclosure, and straight lines are useful. But why the triangle for roofs? Leaf through magazines and look for triangles that underlie roof shapes. See if you can find:

a) Triangles with very small angles

b) **Acute**-angled triangles, with all angles less than 90°

c) **Obtuse**-angled triangles in which one angle is greater than 90°

Why is there such a wide range of triangles in roof shapes? What advantage does one shape have over another? Keep these questions in mind as you do the other activities. You may find other reasons for triangle shapes in roofs.

Triangles Fight Decay

3. Some triangles don't fall down. Which ones?

Draw on cardboard a variety of triangle shapes: acute, obtuse and right angled. Cut them out. Stand each one up on one of its sides. Can you find any that will tip over (in the plane of the triangle)? Which types refuse to tip on any base, no matter how small you make the base?

The Slump of the Hump

4. If you live anywhere near a gravel, cement, or similar mining industry you can view man-made triangles in the making. What is the name of the three-dimensional shape of a newly formed gravel pile? What prevents it from being an ideal shape in reality? If you can find other piles of this kind, try to sight their **slope** angles and draw a triangle cross-section of the matching "ideal" pile.

5. Falling materials such as sand, dust, and snow all tend to settle at a particular **angle of repose**.

Obtain several dry materials such as sand, cement powder, flour, aquarium gravel, etc. Experiment by allowing them to trickle gently down, from any particular height, and settle. Measure the angle of repose in each case. Repeat after wetting them.

Whole towns have been wiped off the face of the earth because engineers failed to carry out simple tests such as these.

Greasy Skid Stuff

6. Different materials, under different conditions, have different angles of repose.

Based on your observations in Activity 5, can you suggest theories for the geometry of avalanches, landslides, and mud slides? Investigate these events to see if your theories are correct.

Earth Pimples

7. Some of nature's most spectacular triangle shapes come from volcanoes. Browse in a library and collect pictures of volcanoes. Squint and see them as "ideal" triangles. Measure the angles of the slope. Are any of these shapes **isosceles** triangles? **Equilateral** triangles?

The Dope on the Slope

Nature very seldom shows us ideal triangle shapes. In spite of this, thanks to Pythagoras, we can "create" triangles beneath slopes to help us study them. This is an important tool of geomorphology — the study of landforms. With the geometry of the right-angled triangle we can **transform** the shapes of slopes onto maps and diagrams.

8. On your desk, prop up one end of your ruler with an eraser. You have created a slope. You can describe this slope in several ways. One way is to refer to the angle between the ruler and the desk. Use a protractor to describe your slope.

9. In physics, slopes are considered **machines** which help ease an object upward against the force of gravity.

It's useful to compare how far *up* an object travels, to how far *along* it travels. This information is much more important than knowing the twists and turns of the slope as it climbs.

Prop up a second ruler or another straight edge so that it falls vertically from the top end of your slope. Can you see the right-angled triangle made with this line, the slope, and your desk? Measure the **height** an object travelling up the slope would climb. Measure the **horizontal** distance (i.e., the base line of your triangle) it would travel.

Represent this:

a) As a fraction: $\frac{\text{vertical}}{\text{horizontal}}$

b) As a percentage

c) By changing the scale to calculate how many units *along* the car would travel for one unit of *vertical climb*. Express it as "1 in 15" (for example) or 1:15.

Straight Up the Stairs

... Except, when you think about it, stairs very often don't go straight up. They wind from landing to landing. Why?

10. Stairs are a form of slope, or, as physicists call it, an **inclined plane**. If you squint at some stairs you can draw a straight line through them to turn them into a ramp.

What advantage do stairs have? It depends on how steep they are. Examine the stairs around your school and draw the slope triangle of each set. Which would you define as steep and "gentle" ones? Which feel easier to climb? Why isn't the slope of all stairs made as gentle as possible?

11. Measure the height of a nearby building that is more than two stories high.

Use your own angle measurements of stairs as a guide to choose a comfortable slope for the staircase.

Calculate how far away the stairs must begin, if they do not wind, to reach the roof.

Look up the height of a major landmark, such as the CN Tower in Toronto. Calculate how far away an "unbent" staircase would have to begin in order to reach the top.

Making the Grade

Road and railway slopes are called **grades** and are usually expressed either as a percentage or as "1 in something."

Hills that seem steep while you are driving are, in fact, surprisingly gentle. Railway grades are very slight slopes indeed. You can measure these slopes yourself with a few simple tools.

12. Some ramps that rise above a level floor can be measured directly. See if you can find one (dock loading ramps, ramps in apartment parking lots, etc.).

Measure the height and length of the ramp.

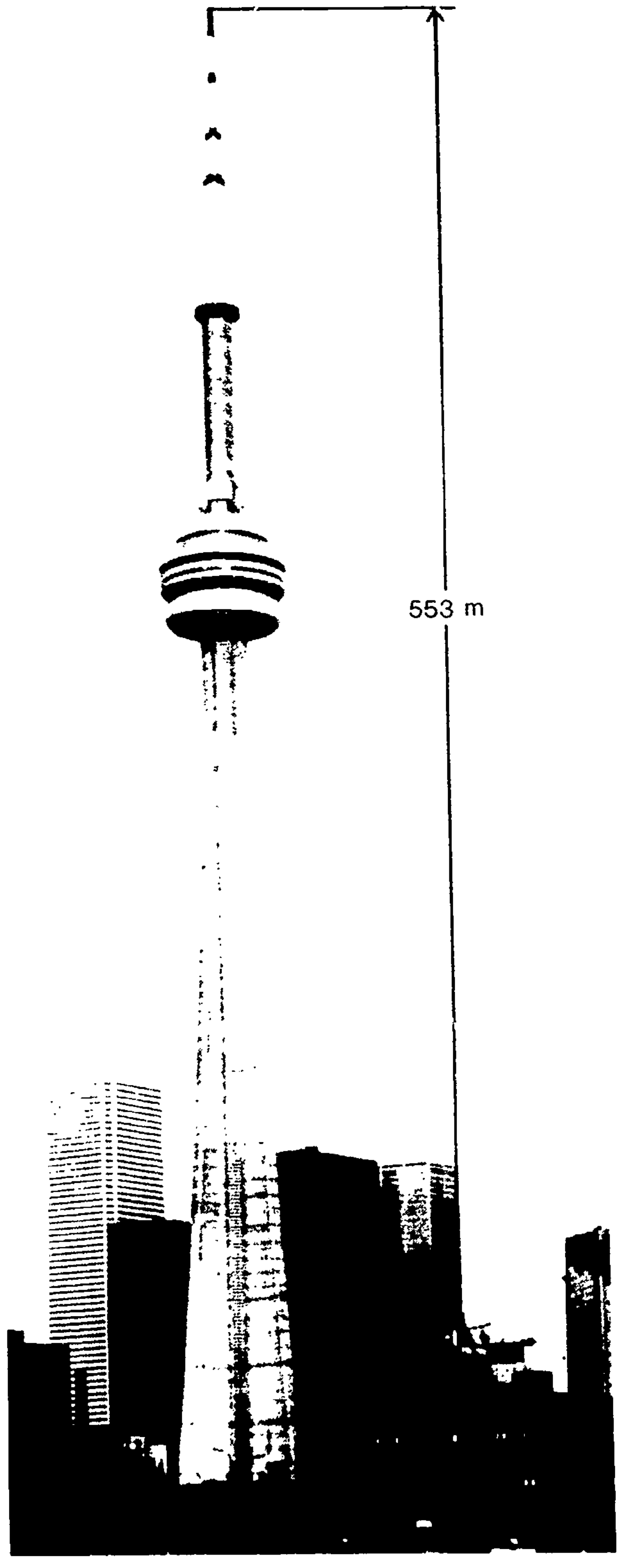


Figure 6.1

Draw it to scale to calculate its slope.

Measure the height and the length of a small section of the ramp and compare it to the slope of the entire ramp.

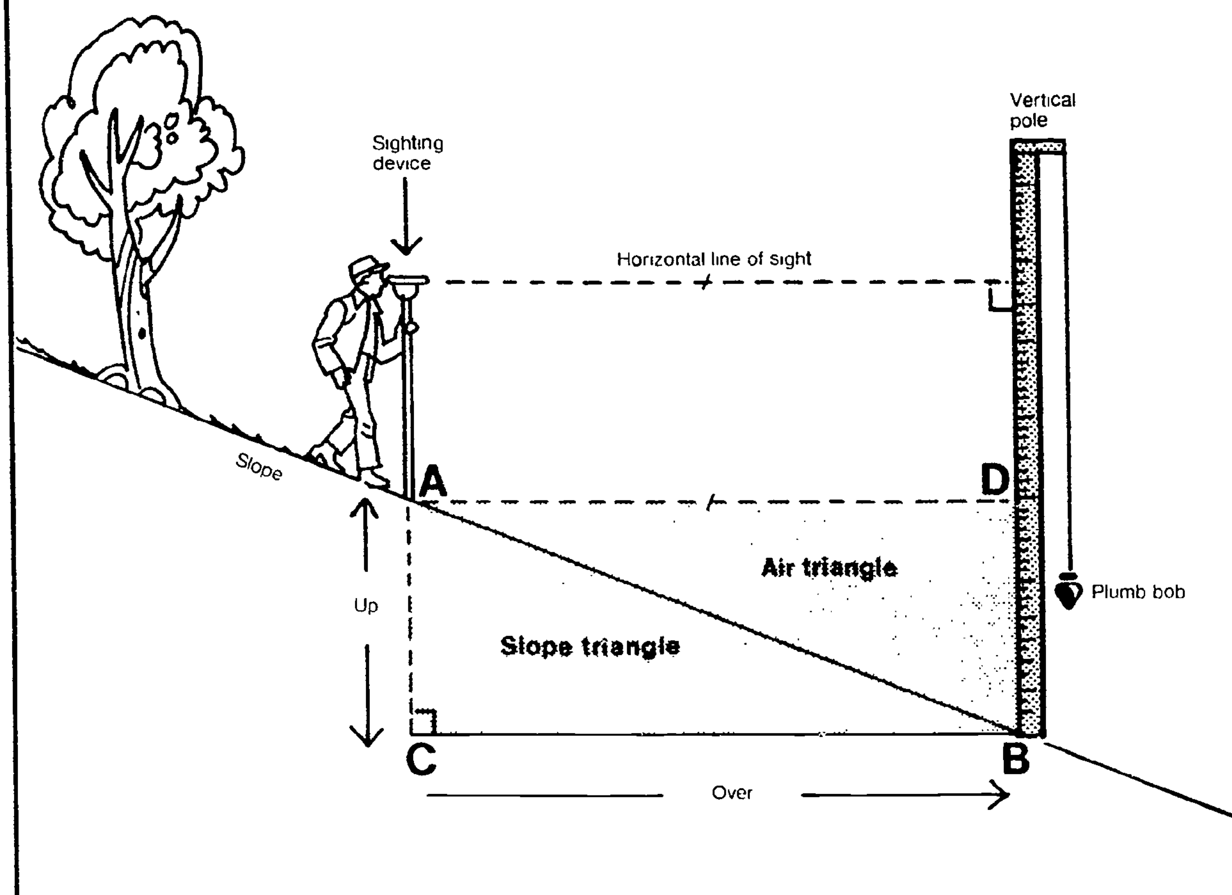


Figure 6 2

Up ... in the Air

13. It's impossible to get "underneath" most road slopes to do our measuring, so we must make use of an important property of triangles to transform our slope triangle in order to create a new imaginary triangle in mid-air — a triangle that is **congruent** with the first.

Examine the method in Figure 6.2 to measure slope. Note the slope triangle, and the triangle in the air — created by **flipping** the slope triangle.

Find a slope and measure it, using this method. You will need a long rod, a protractor sighting device (Program 5, Activity 14). You will also need to devise a method to keep the rod vertical. Remember to allow for the height of your eye above the slope.

(Note: This method can be practised to measure the slope of a set of stairs.)

Pass the Slope, Please

Wedges are portable slopes. Instead of moving a load up a slope to lift it, we can also lift it by shoving the slope under the load, in the form of a wedge. Screws are simply coiled-up wedges.

14. Collect several screws of different *pitch* (distance between the threads). Use a thread wrapped around each screw to measure the length of the slope. Now measure the height of each screw.

Use these two measurements to draw a slope triangle for each screw. Use either a scale drawing or the Pythagorean theorem to work out the slope percentage, and a "1 in something" description of the screws.

Which screw should be the easiest to turn into wood? Test your guess.

Triangle Shorthand

15. A triangle has three sides and three angles. But it can be completely described without mentioning them all.

Pair up with a classmate. Both classmates draw a triangle. Pass "clues" about the triangle (angles and sides) to each other so that you can redraw it. What is the smallest number of clues that will allow you to construct a **congruent** triangle? There are three different combinations of three clues (angles and sides) that will allow you to draw a congruent triangle. Can you find them?

When You Can't Get There from Here

... You can always send a ray instead

Private ownership of land is a cornerstone of the free world ... and that makes **triangles** a vital tool of democratic countries. Ownership of land depends on knowing exactly where it is. In Program 3 we explored how intersecting straight lines can "fix" points.

16. From your schoolyard or another convenient site find a distant point, such as a tower or high building, some distance away. How far away is it?

Choose two fixed points as far apart as possible. Pace the distance between them.

Use a compass at either end of the base line to measure the angle between the tower and the other end of the base line.

Draw the base line to scale on the map and construct the angles you measured at the ends of this line.

Now you have a triangle. Using your scale you can find the distance to the tower from either end of the line.

If possible, use some other means to locate the distant point on your map (e.g., the street intersection of a corner building). How accurate are your measurements?

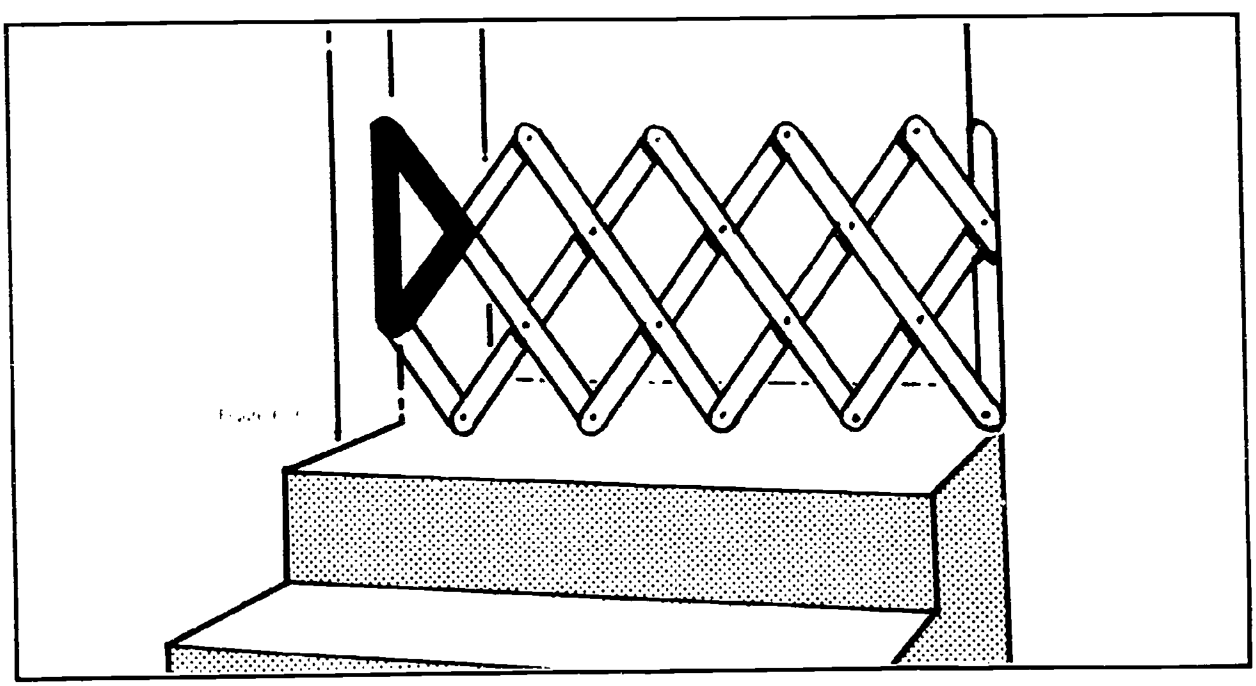
Keep a Stiff Upper Triangle

17. Triangles are simple rigid shapes. We use them in any place where we don't want things to collapse.

Research and collect photographs of as many different kinds of structures as you can that have triangles within their structure. Draw sections of their structure.

Strutting About

18. Triangles are also useful for making collapsible things rigid. Can you think of any objects that "add" an extra side or "strut" to create a triangle, in order to fix the object in a rigid position. Draw them, indicating the added triangle.



"Trussworthy" Shapes

19. What does a bridge have to hold up *in addition* to the load on top of it? The truss weighs little compared to a beam, and requires less material. But which triangles make the best trusses?

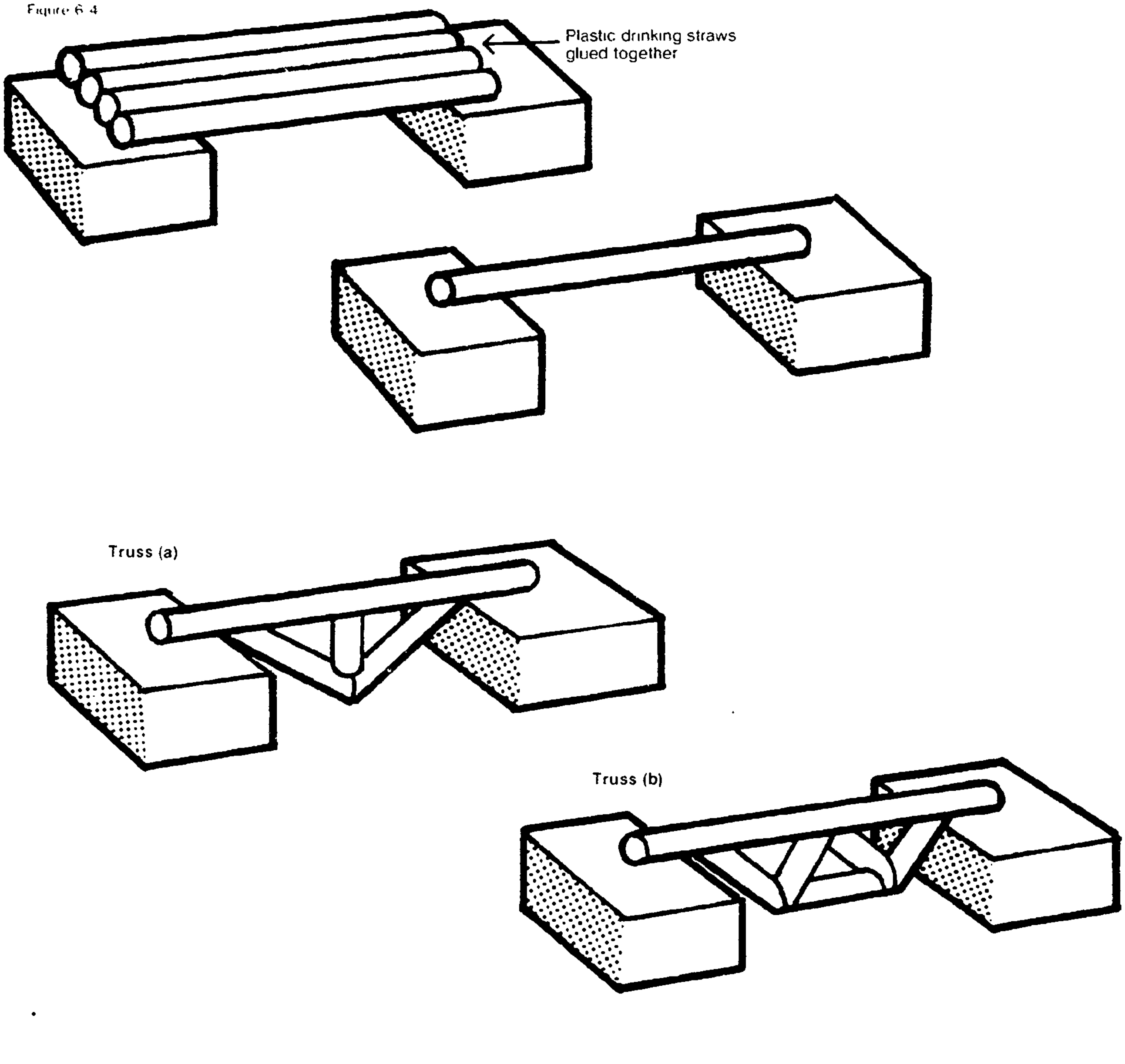
Make a beam using four drinking straws glued together on their edges.

Make several trusses using different patterns of triangles, as in Figure 6.4.

Hang weights from the centre of a simple span and compare the force needed to produce the same "deformation" in a trussed span of the same length.

Which spans are the strongest? The lightest? Have the least material (in centimetres of straw — an important cost factor for engineers)?

Figure 6.4



PROGRAM 7
CRACKING UP

THE PROGRAM This program examines tiles and tiling. The nature and purpose of real tiles are contrasted with geometric tiles. Polygons are introduced, and regular polygons (a special case) are examined for their tiling properties. Tiling is explored as a transformation of a single tile through slides, turns, and flips.

The program considers patterns formed by more than one tile, as well as the process of developing the pattern itself. Tiling games and art are also discussed.

Because there are a variety of formal and semi-formal definitions of tiling and tesselation on the one hand, and such a wide range of geometric patterns in the world around us, on the other, the program makes little of tiling *rules*, leaving it to individual teachers to formalize activities as they prefer.

Examples are drawn from ancient handicrafts, nature, netmaking, and art.

ACTIVITIES The activities approach the geometric tile through a consideration of "real" tiles and tile shapes, including:

- Questions about cracks — natural and man-made — and their advantages and disadvantages
- The purpose of armor — to protect things, living and otherwise
- The beauty and inconvenience of irregular polygons, such as mosaic chips
- An introduction to regular polygons
- Tiling methods through flips, slides, and turns
- Tiling with combinations of shapes
- Games such as tangrams and polyminos
- The tile as an art form, including Escher patterns.

Party Crackers

1. Blow up a balloon. Now crack it. You can't? Of course you can. It's difficult to believe rubber is brittle, but it is when under tension.

Touch a pin to it. A balloon cracks explosively. So does a Pyrex dish when left on a hot stove.

What causes things to crack? Investigate. Consider: cracks in ice; a broken glass in a dishwasher; a broken window; cracks in building bricks and in concrete sidewalks.

Some things will crack no matter what we do to stop it. One way of controlling this is to put the cracks there ourselves to control what happens. Can you think of examples?

"Romin" the Tiles

2. Collect illustrations of stained glass windows and Roman mosaics. Why do you think small pieces of glass and ceramics were used? What is the largest piece of glass you can find around your community? The largest piece of ceramic?

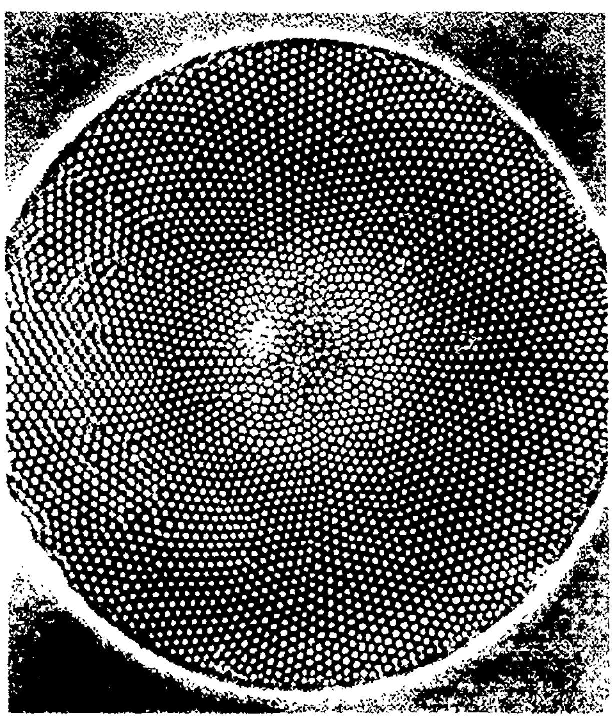
Take a section of each illustration and reproduce a few geometric shapes. Can you find any regular or repeating shapes?

Blocking All the Old Chips

3. Mosaic patterns in stone, glass, and ceramic can be beautiful, but there are certain problems involved in making them. On paper, design a pattern to be made from colored mosaic tiles. Choose a medium (cookie tiles, segments of candied fruit on a cake, chips of colored paper, etc.) and execute your pattern. What problems do you have? Do you think mosaic tilework is a task for skilled or unskilled workers?

Dead Parrots

The easiest way to tile a surface is to use a very regular shape, over and over again. We can employ geometry to describe straight-sided shapes.



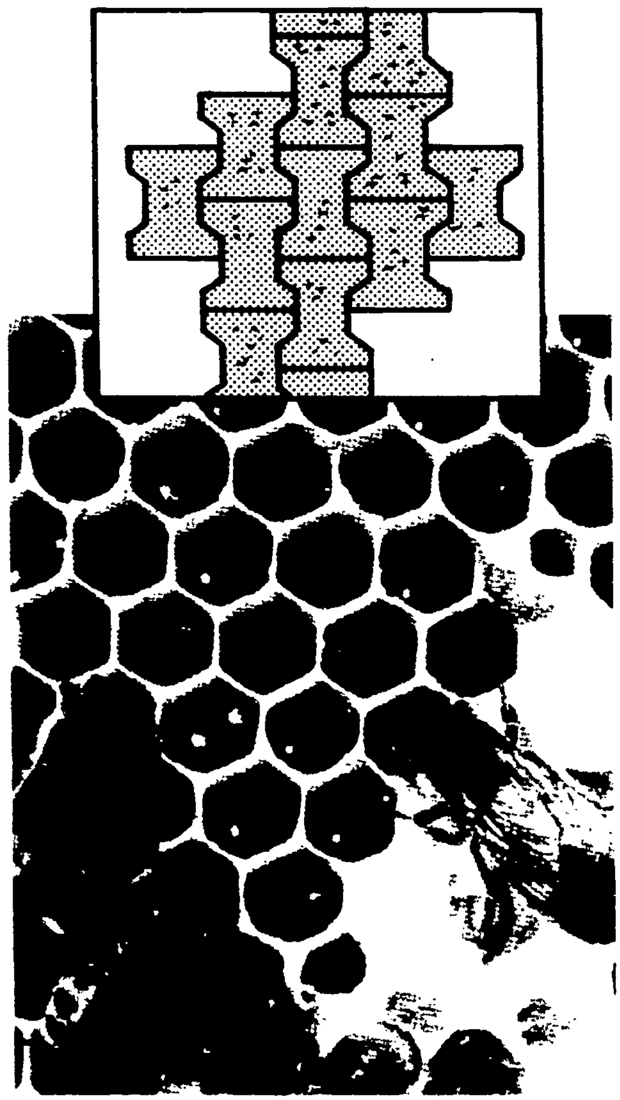
4. Straight-sided shapes with three or more sides are called **polygons**. Regular polygons have all sides and angles equal.

Draw regular polygons with three, four, six, and eight sides (for a challenge, try five sides). What inside angle is repeated in each case? Add up the inside angles. Does the sum of these angles remain the same, reduce, or increase as the number of sides increases?

Slide! Slide! Slide!

We can think of a tiling pattern as if it were created by a single tile **transformed** from place to place.

In geometry we consider tiling the plane, or **tesselation**, as a regular repeating pattern of polygons that fit exactly together, without overlapping.



5. How many of the regular polygons will *tile the plane* by themselves using the simplest transformation — a slide in any direction? Cut out a cardboard pattern of each regular polygon to help you transform them from place to place.

Rotate!

6. How many other regular polygons will tile the plane if you rotate, or slide *and* rotate them?

Flipped In

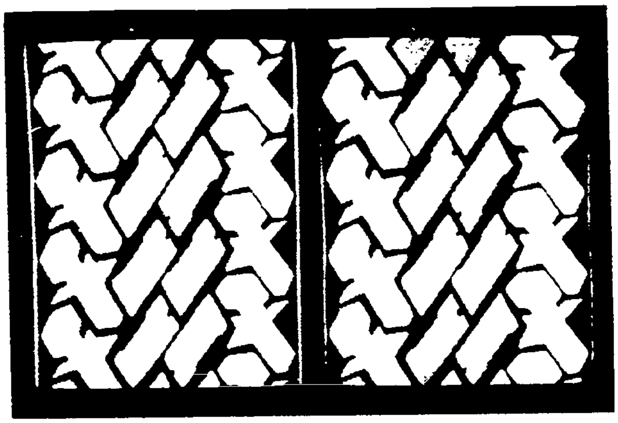


Figure 7.2

7. Cut out an L shape to match Figure 7.2. This is not a regular polygon, but it's one of many, many shapes that will tile the plane in numerous ways. Use your pattern as a guide and on graph paper draw several tiling patterns using slides and rotations.

Now **flip** your shape over. Can you create new tiling patterns with this transformation? Does this transformation help you tile the plane with any of the regular polygons?

Slopping Between the Cracks

More cracks mean more filler, and more work laying pieces.

8. Consider tiling your bathtub with regular polygons. Imagine you have an equilateral triangle, a square, and a hexagon — each with an area of one unit. Which tile will cover the wall with the biggest (lengthwise) crack? The smallest?

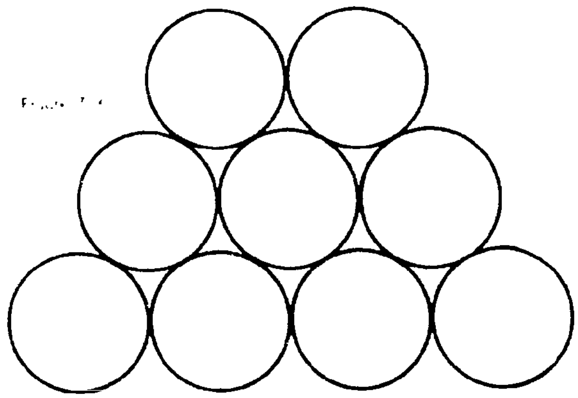
Find the area of an equilateral triangle. Choose one side to be the base, then drop a perpendicular from the opposite vertex to the base. This is the height. The area of a triangle is one-half the base times the height. One way to find the area of other polygons is to divide them into triangles.

Now draw an equilateral triangle, a square, and a hexagon, and compare their areas to their perimeters (the sum of their sides). Which has the longest perimeter (crack) per unit area? The smallest?

9. Based on this study, which regular polygon pattern would you expect to be the most common in bathrooms? Least common? Do a class survey. Does it confirm your guess? Discuss why or why not.

Being Positive About the Negative

Figure 7.3



10. Is Figure 7.3 a bottle rack or a stack of tins? Are the holes circles, or spaces in between?

11. A geometry plane is so thin, the idea of a hole has no meaning. If we use geometry to model the real world, we can think of two shapes together, but not one shape with holes around it.

As a result, we can squint and find examples from the world around of two shapes tiling the plane: a shape and an "anti-shape" — the shape of the "hole" around it.

Find examples in your neighborhood and draw the underlying tiling pattern.

Irregular Regulars

12. If you tried to tile the plane with pentagons or octagons you know it doesn't work. Not with a single shape.

Cut several pentagons out of paper and, using one as a pattern, find out what "anti-shape" must be combined with the pentagon to tile the plane. Repeat with the octagon.

Let's See ... That's 45 and 52 and 65 ...

13. Examine a tiling pattern. At each intersection add up the angles. What is the rule for determining whether a group of corners will fit together in a tiling pattern?

A Tile Sandwich

Doesn't sound too tasty? Here's one that's light and strong and used in the aircraft and mobile home industries.

14. Work in groups of three or four to make a tile sandwich. Each student needs four sheets of standard lined paper and some glue.

Step 1: Place one piece of paper on a desk and run glue across the page in the space between the first and second lines. The next three spaces are left blank, then glue is spread between the fifth and sixth lines. The next three are blank, and so on, to the bottom.

Step 2: Press the next sheet on top. On the new top sheet, spread glue in the space between the third and fourth lines, leave three spaces, place glue between the seventh and eighth lines, leave three more, and so on down the page. Press on a new top sheet and repeat steps one and two again.

Use the same repeating pattern to glue together several groups of sheets.

Step 3: When the sheets dry, slice the "sandwich" at right angles to the lines on the page. Each slice should be about 2.5 cm wide. Gently pull the strips apart until a honeycomb pattern is formed. This is the filling for the sandwich.

Step 4: Finally, smear glue on one side of two sheets of cardboard. This is the "bread." Place the filling between the bread and weigh it down with a few books while the glue dries.

Compare the properties of this sandwich with the paper and cardboard from which it is made.

"Waist" Not, Want Not

... Leg not, sleeve not, etc.

15. If there are no holes in geometry planes, there certainly are in real life. And scraps too. In the garment industry, scraps cost money, so special skill is needed in cutting patterns. A similar skill is used in sawmills to cut the greatest number of boards from a tree, and in the sheet metal industry, where patterns are punched from sheets.

The Challenge: Who can fit the shapes in Figure 7.4 together on a sheet of paper with the least waste? You should try to keep an equal number of each shape on the paper.

Judge the results, either by measuring the area of the scraps, or the area of the shapes.

Sphere Packing

16. New nuclear fuels can be "mined" from the wastes of old fuels. The problem is that these wastes are radioactive. In order to "burn" properly, the recycled fuel must be evenly and densely packed, and its density known exactly.

Atomic Energy of Canada Limited has experimented with a kind of three-dimensional tiling technique which it calls "sphere packing." The fuel is formed into round pellets and poured into cylinders by remote control.

Using plane geometry you can get an idea of this technique. Draw a rectangle 10 cm wide and 25 cm long to represent a cross-section of the cylinder. What is its area? Now choose a sphere size and "tile" the rectangle.

Calculate the area of the circles [$A = \pi r^2$]. Compare the area of the spheres to the area of the rectangle by representing it as a decimal fraction.

The trick is to use spheres of different sizes, poured and shaken between the others, in order to bring the density up to an ideal level say 0.90. Using different sphere sizes, experiment with filling in the spaces.

Assume each sphere costs about the same to make, regardless of size. What is the cheapest tiling pattern you can design that would give a density of 0.90?

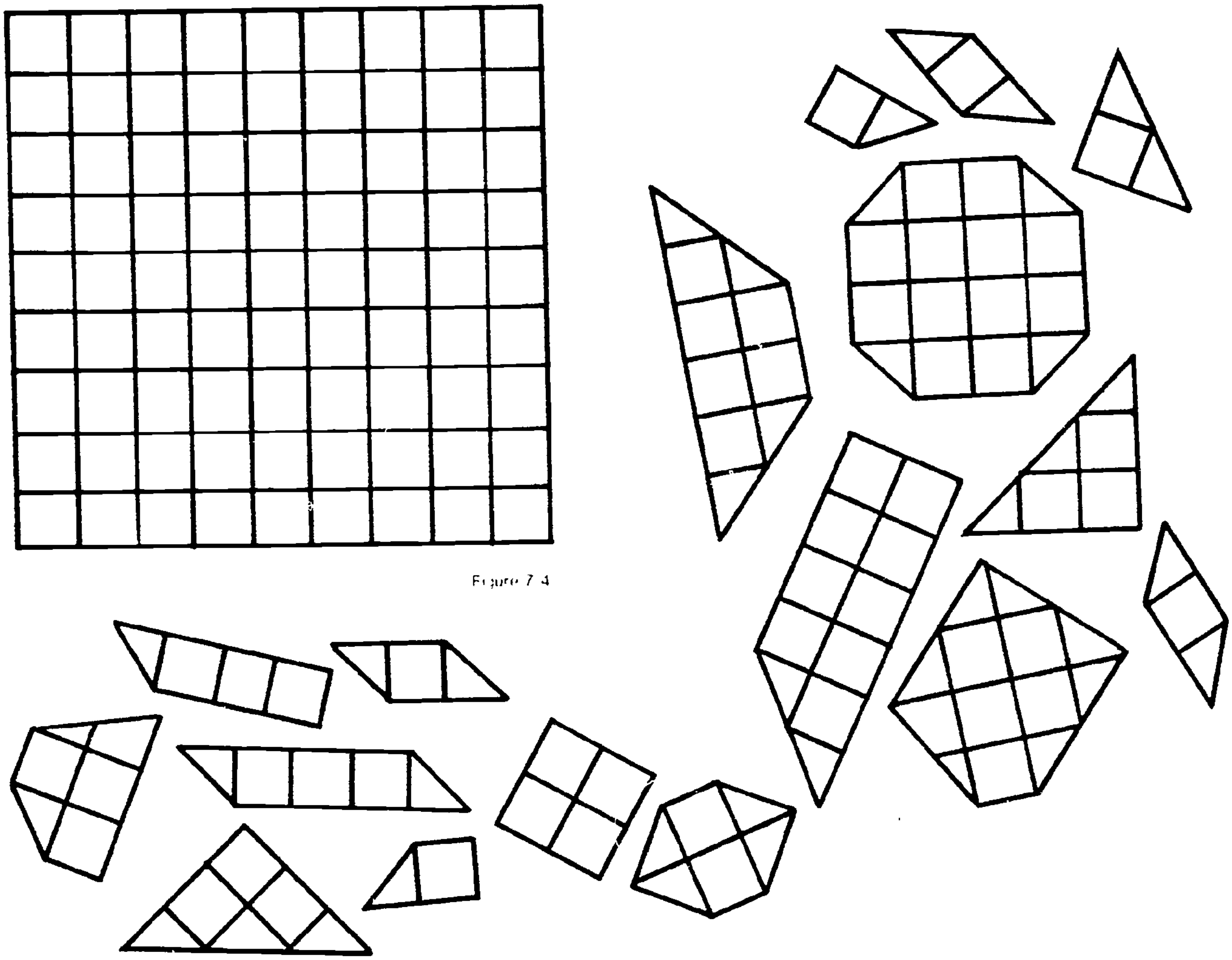


Figure 7.4

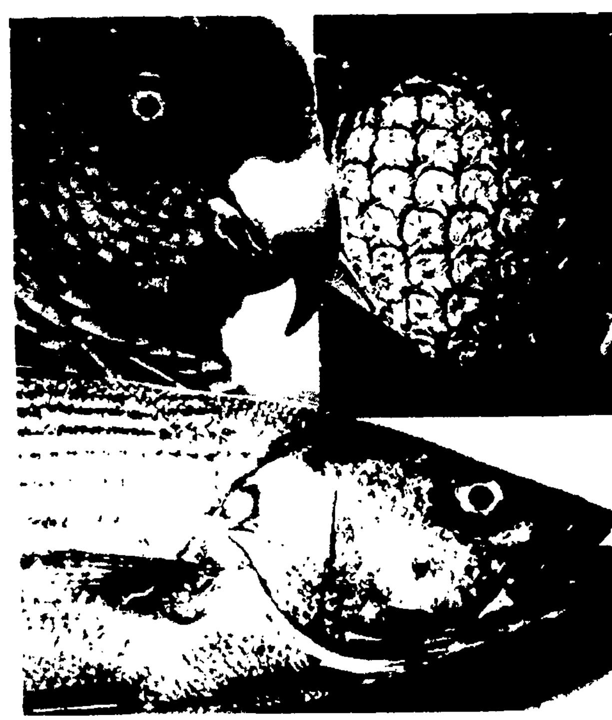
Figure 7.5

Styles of Tiles

Tiles are thought to have been developed in the Middle East where they had an important function as *armor*, protecting soft brick from sun and blowing sand.

17. Are tiles out of style? Not a bit. As a research project investigate the space shuttle. Why does it use tiles? What kind of material are the tiles made of? How do they differ from ceramic tiles used thousands of years ago in mosques in the Middle East?

18. The armor has existed much longer than ceramic tiles. It was developed by nature. What tiling patterns can you find in nature? How many are a kind of armor? Draw the tiling pattern and the shape of an actual [illegible]



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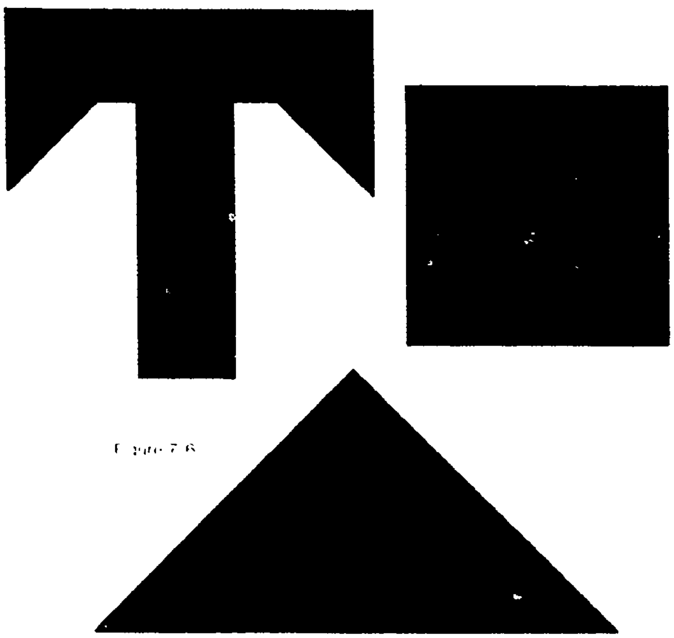
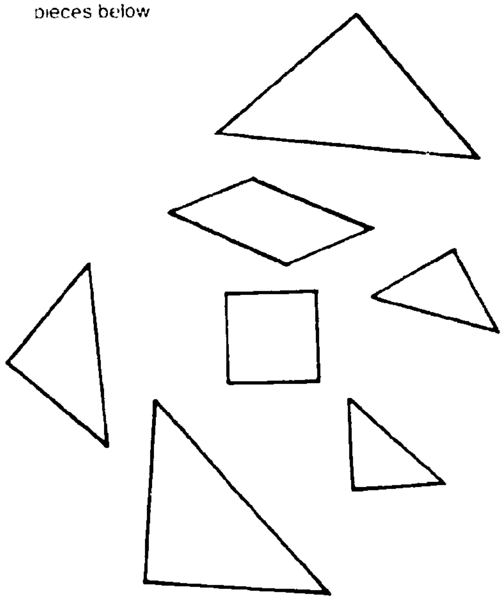
Some real-life tiling patterns also depend on an overlap. Examine asphalt roof tiles and chain mail armor (or other tile-form armor). Draw the tiling pattern and an individual tile.

Tangrams

19. A tangram is an ancient Chinese puzzle, still popular today. Cut out the shapes in Figure 7.6. Using these geometric shapes, construct the tangram "pictures" in Figure 7.6.

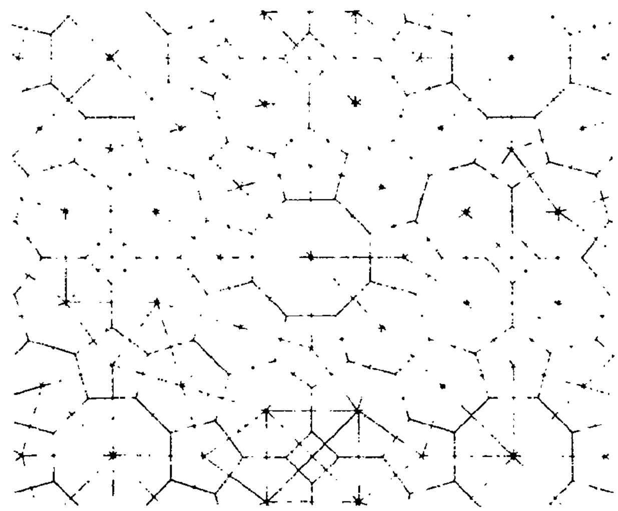


Form the shapes from all the pieces below



Polyminos

20. A domino is a rectangle made up of two squares. You can have fun playing with other "mino" shapes. How many *tri-minos* can you make (distinct shapes made up of three squares)? There are five *tetrominos* of four squares, and 12 *pentominos* of five squares. Can you draw them?



21. Artists use tiling techniques to make many pretty patterns. Make an intricate pattern using four different colors from Figure 7.7. You'll be amazed at the variety in the class when you compare all the patterns.

Now You See It, Now You Don't

22. M.C. Escher, an artist and an engraver, is famous for exploring tile patterns in his art. Patterns such as those in Figure 7.8 are known as Escher patterns. Devise one of your own.

PROGRAM 8
THE RANGE OF CHANGE

THE PROGRAM

The program explores the concepts of **transformations.** Preceding programs in the series considered transformations in a "freewheeling" way, particularly through the analogy of movement. This program begins by considering the concept of change in the world around us and discussing the importance of shape and place as a way of describing change.

Geometry is introduced as a means of describing "before and after" through the idea of a transformation. The program then turns to the Cartesian "playing field" — a grid of points described by the intersection of straight lines at right angles. Translations, rotations, and reflections are then explored on this grid.

ACTIVITIES

Until the recent explosion in computer graphics displays, real-life parallels to the process of transformation have been quite limited. There are, of course, techniques of analytical geometry, but most are inaccessible to beginner students in plane geometry.

The spread of computer graphics displays at least brings students in contact with a close approximation of geometric transformations.

The following activities explore the nature of transformations by:

- Exploring the nature of change in the world around us, and the relationship of life to change
- Considering ways of describing change via a "before-and-after" process
- Developing a point map of the plane, and contrasting this with Euclidian shape descriptions
- Examining the kinds and purposes of grid systems
- Exploring a simple computer graphics problem as a parallel to pencil and paper work in transformations.

The Range of Change

1. The world, indeed the whole universe itself, is in a constant state of change. Can you think of anything that doesn't change? How many different kinds of change can you think of [illegible]

Look around you. How many objects seem to involve a change of shape or position? (e.g., consider the patterns on wallpaper)

A Step at a Time

2. Something we take for granted is actually quite complicated. Briefly describe the act of walking. Were the words you used exact and concise or vague and fuzzy?

Now try to use the "language" of geometry which may be, for your purposes, far more precise. Draw a simple stick person to record the transformation steps required to represent the completion of one full stride.

Getting Nowhere

Geometry helps us describe a change in position. One method is to use a "cosmic golf game" of getting from here to there (describing transformations). We locate a point (A). We choose a direction ray and ... *fore*! blast off into the great unknown. Through the force of our shot (a fixed distance), we "find" another point. Consider how you drew triangles from three clues in Program 6.

There are other ways of describing transformations but unless the method has order the process can be nightmarish.

3. Let's explore a particular method. Write the numbers 1 to 25 on a piece of paper in a random manner. You have now assigned a location to each of your 25 points. But a plane has more than 25 points. How are you going to pinpoint all those blank spaces on your sheet? You could add fractions between the points numbering them 1.5 ... 1.25 ... etc.

You don't want to? Who can blame you? Because you will have to add more points between these ... and more points ... to infinity.

Pointless, But to the Point

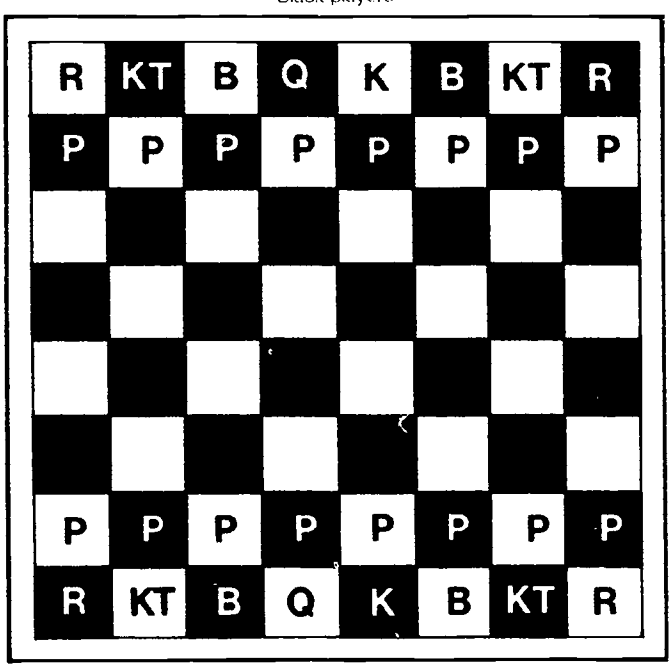
Mapping all of space with points is a useful way of describing shapes and transformations. To make it simple, the trick is to organize a way of describing most points by referring to a few others. The most convenient method is to imagine every point as the **intersection** of two lines at **right angles**. This is the grid.

4. The game of chess uses an old method of referring to rather large points (in fact, squares) on a board. Find the chess column in a newspaper. Can you unravel the puzzle of "chess notation" by using common sense and referring to the names of the pieces in Figure 8.1? Draw the chess grid with the names of the squares.

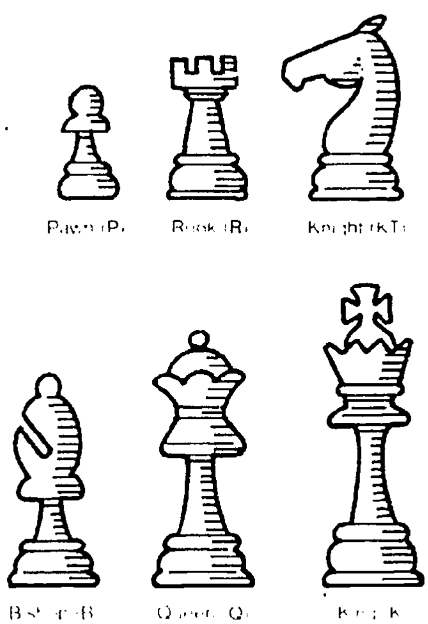
Battleships

5. Here's a tough homework assignment. Have you ever played "Battleships?" You could go out and buy the game — it sells for a

Black players



White players



few dollars. But if you understand grids, you can quickly make your own game using pencils and paper.

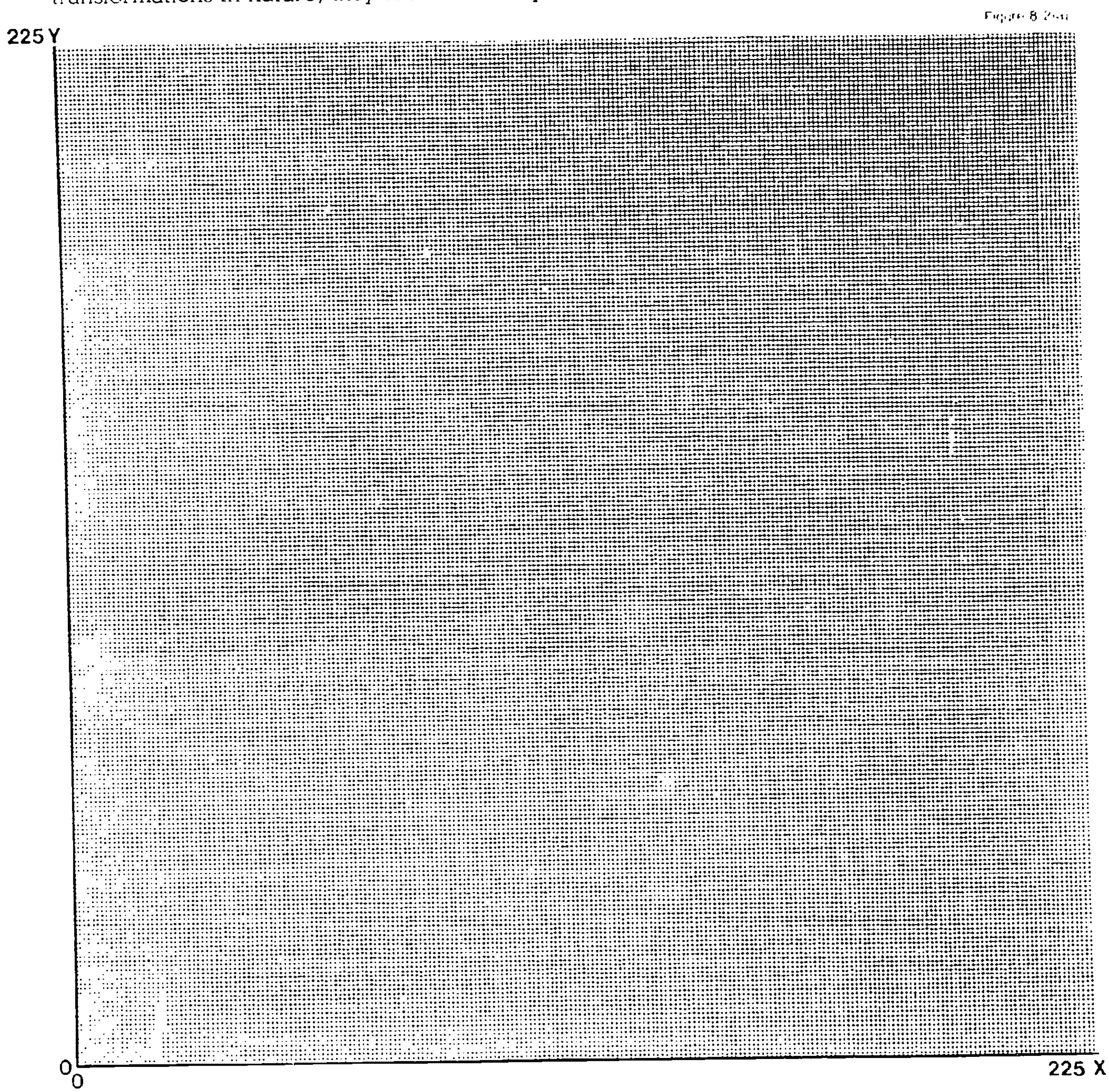
Find the rules of the game "Battleships" and get someone at home to help you to "play" your homework. Tell suspicious parents you are "developing a working familiarity with simple grid systems."

Computer Games

While it is difficult to clearly spot transformations in nature, they are abundantly evident in our man-made world. Today nearly all computers use a grid system to move shapes about on a video screen. (The very same process that entertains us with computer games is the one used to carry out serious tasks.)

6. Your objective is to program a rocketship flight. Although the instructions may seem a little complicated, writing the program is actually quite easy.

The screen has an X (horizontal) axis of 225 points and Y (vertical) axis of 225 points. 0,0 is at the lower left-hand corner of the screen.



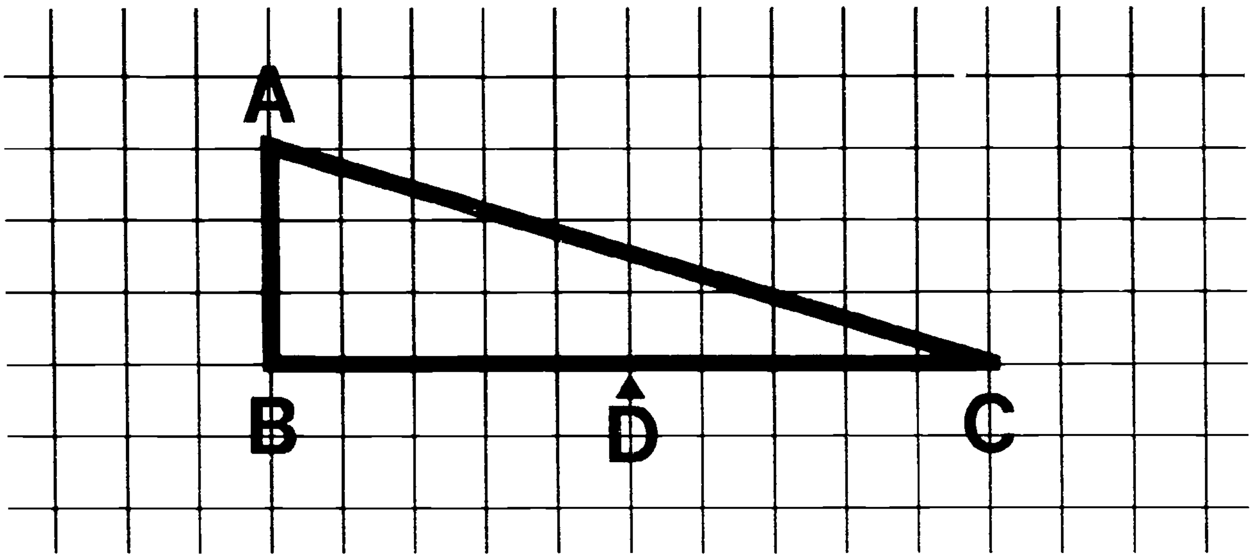


Figure 8.[illegible]

Your computer stores a rocketship image (with a schematic resemblance to the Columbia) under the name *rocket*.

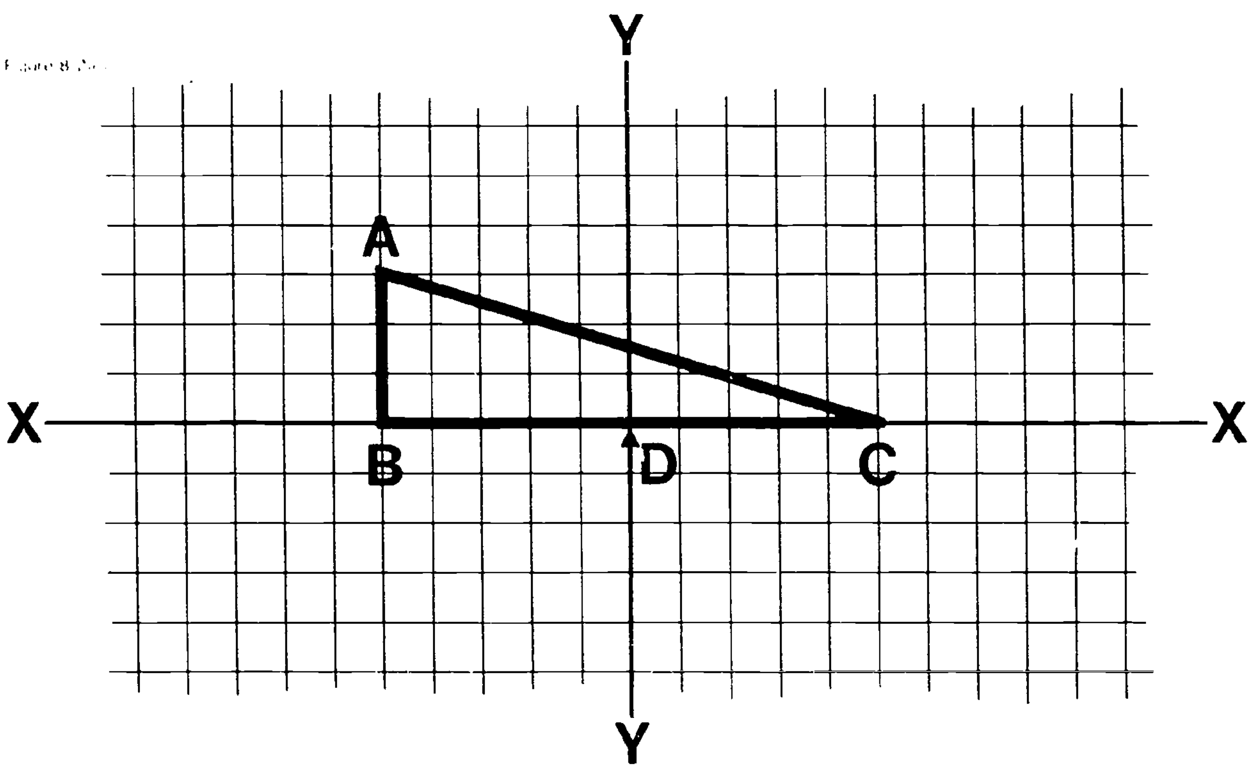
The rocket is ten points long and three points high. It is designated ABCD where D is the mid-point of BC.

The computer constantly keeps track of points A, B, C, D and changes their address according to your instructions.

The computer language you use is called *transform* and it has the following commands:

- *Draw*(X,Y) Draws rocketship with its centre points at X,Y coordinates
- *Erase*(X,Y) Clears the screen
- *Slide*(X,Y) Slides rocketship to new location X,Y
- *Rotate*(?°, ∩ ∩) Rotates rocketship about D in specified degrees and either clockwise or counter-clockwise as specified
- *Flip*(AB, BC, AC) Flips rocketships about axis described by two points

Figure 8.[illegible]



Example: Rotate rocketship to get a better view of earth

a) *Draw* (50, 150) (We have to begin somewhere, so assume the rocketship is alligned as in Figure 8.2 (d).)

b) *Erase*

c) *Rotate* (180°, ↷)

Figure 8 2(d)

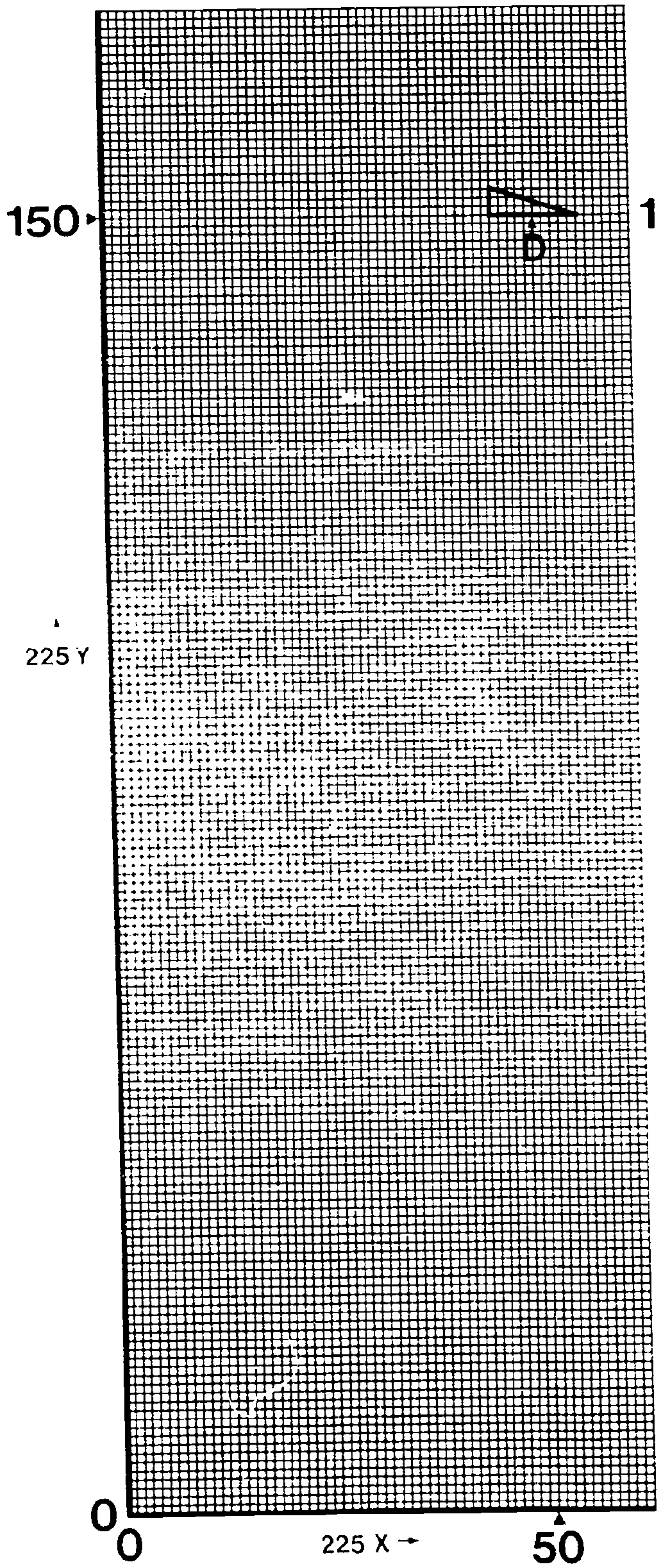
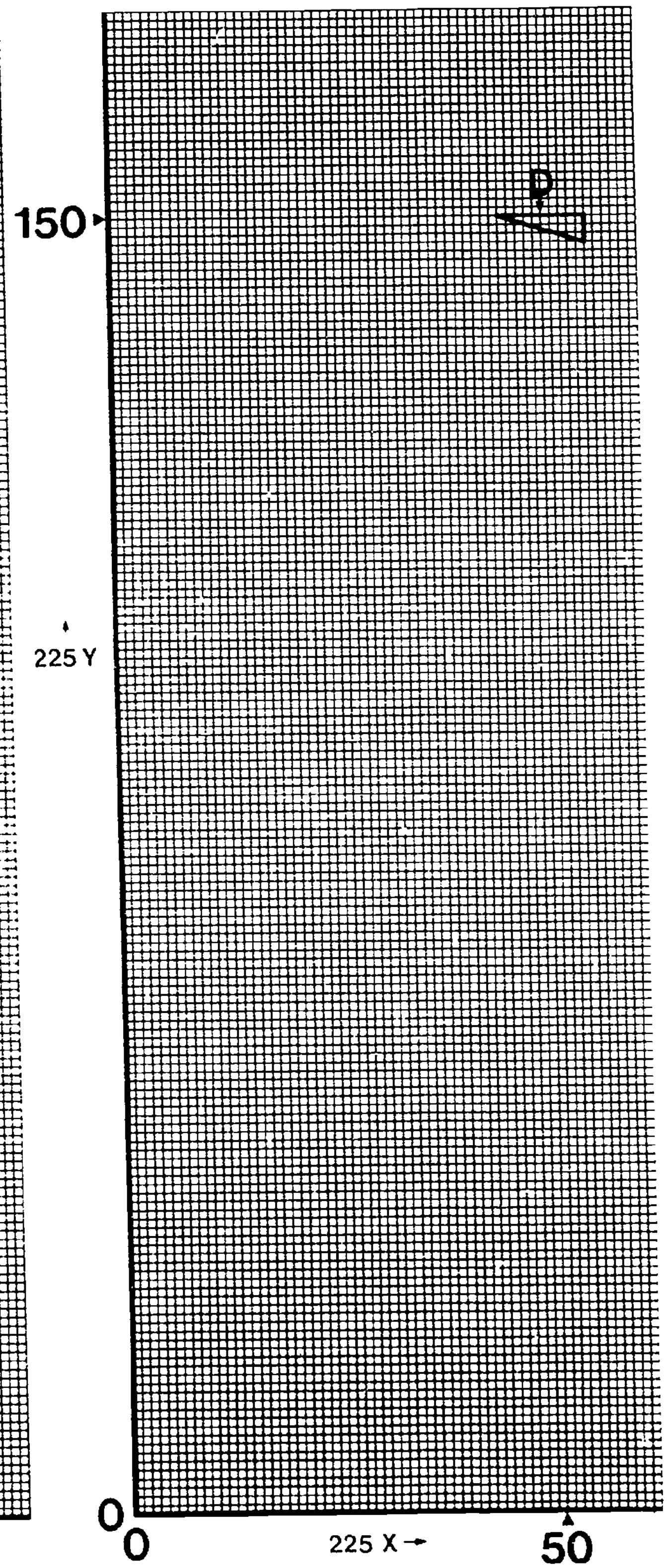


Figure 8 2(e)



d) *Erase*

Using these **transformation** commands, can you program the rocket to fly the path in Figure 8.2(f)? Draw the flight path on graph paper first to work out the slide coordinates.

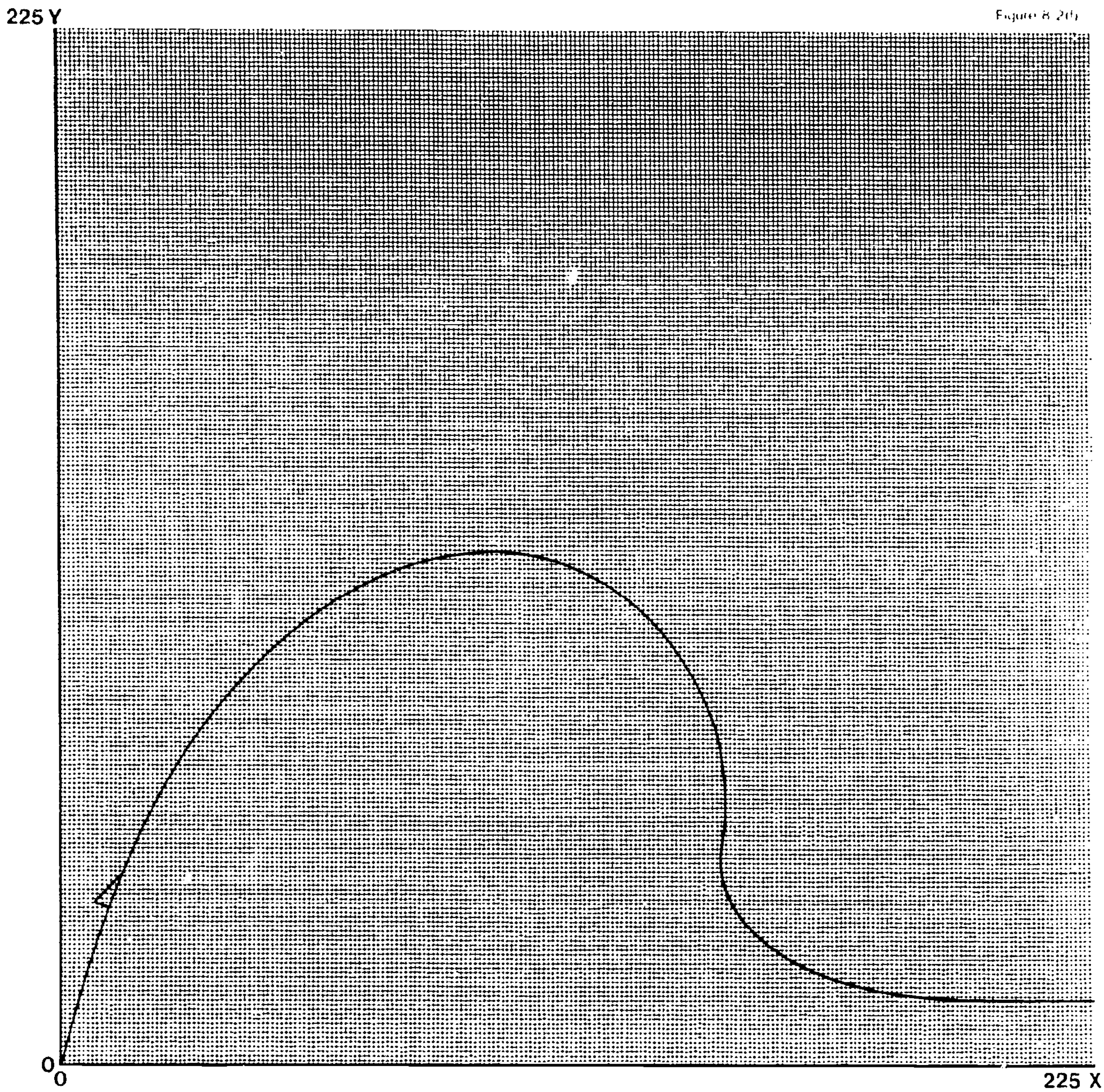


Figure 8.2(f)

Here, There, and Anywhere

Just to point out there's an exception to every rule, botanists use a simple grid to help them *not know* where things are.

7. Make a sampling grid. It's nothing more than four meter-long sticks, braced at the corners, and subdivided by a grid of strings (Figure 8.3) every 25 cm.

Stand in any grassy or weedy field. Toss the grid backward over your shoulder. Press the grid to the ground where it falls. In each small square examine the plants. Count and describe each different kind.

When you have totals for each square, add them all together for the meter square. You now have a figure for plant density per square meter. In this way you can compare vegetation in different areas.

Why do you think the grid is thrown over the shoulder? Considering the way you use them, how important are each of the individual squares on the grid?

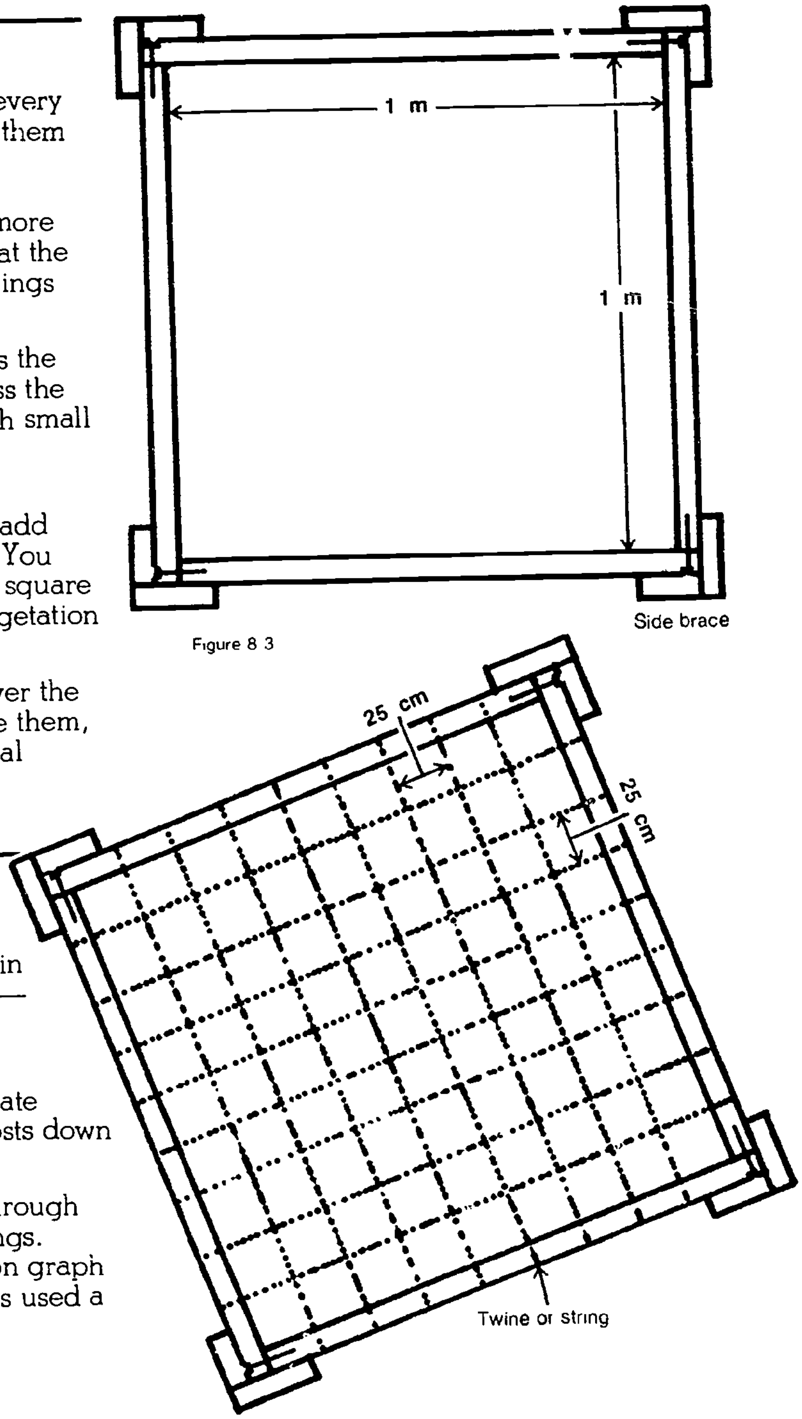


Figure 8 3

Moving House

8. There are three simple ways of transforming shapes: The slide — in geometry called **translation**; the turn — **rotation**; the flip — **reflection**.

Architects make endless use of these transformations in their work — to create pleasing patterns, and to help keep costs down by standardizing parts.

Look about your neighborhood and through magazines at houses and other buildings. Draw a simplified view of a building on graph paper. Identify where the architect has used a slide, a turn, and a flip.

ORDERING INFORMATION

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