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ABSTRACT

While the emphasis and content of school mathematics differ little between Japan and America, by the time students reach junior high school, performance as measured by international studies differs greatly. This report provides a profile of the attitudes toward, strategies used, and performance on mental computation of Japanese students in grades 2, 4, 6, and 8, in order to identify factors that may contribute to this difference. Students in two randomly selected classes in each of grades 2, 4, and 6, and six randomly selected classes in grade 8 were selected from three elementary and one junior high school in Tshuchiura, Japan. Data were collected using a preference survey of numerical computation methods used, an attitude survey, a mental computation test, and interviews of selected students in grades 4 and 8. The major findings were that: (1) Japanese students thought both mental computation and written computation were important; (2) teacher attitudes toward mental computation varied; (3) students across all grades preferred mental computations; (4) a wide range of performance on mental computation was found; (5) performance levels on items presented visually was significantly higher than those presented orally; (6) performance on common items increased as schooling increased; (7) the range of mental computation strategies was very narrow; and (8) few students in the interview were able to express alternative strategies for mentally computing an item. Six appendices contain copies of the instruments and the categorization of anticipated strategies. (MDH)

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Mental Computation Performance and Strategy Use of Japanese Students in Grades 2, 4, 6, and 8

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Preface

The American and Japanese people share a common language - the language of mathematics. The importance of mathematics in both countries is evident by the attention it receives in schooling beginning early in the primary grades. While the emphasis and content of mathematics throughout elementary school differ little between the countries, by the junior high years performance as measured by international comparisons differs markedly. The reasons for the chasm are complex and involve societal as well as pedagogical considerations. These issues are not the focus of this research effort. Instead, this research addresses one specific area of the mathematics curriculum, namely mental computation. It is hoped that the narrow focus will provide a better understanding of how Japanese students view this topic as well as how they think about and perform the process of mental computation. In addition, it is hoped that asking students to invent and/or apply mental thinking strategies to compute will provide a window through which it will be possible to view their developing number sense.

Why focus on mental computation tasks? Research of adult usage of mathematics shows that over 80 percent of all mathematical computations in daily life involve mental manipulation of numerical quantities rather than paper/pencil computation (Wandt & Brown, 1957; Edwards, 1984). Ironically, recent surveys show that much more time in elementary school mathematics in Japan and the United States is directed toward written computation than mental or mechanical (calculator) computation (Flanders, 1987; Nagasaki, 1987). The availability of calculators has placed an even greater premium on mental computation, estimation and sense making of numbers, where students and adults must be able to judge the reasonableness of calculated answers. In addition to the practical importance of mental computation in today's society, the process is viewed by some educational researchers as a way to study both an individual's reliance on learned procedures and their willingness and ability to assimilate their mathematical knowledge to construct and apply new procedures to accomplish new tasks.

In both Japan and the United States, the approach to teaching mathematics is shaped by its national character. In Japan, there is a standard national curriculum and although quality of instruction differs from classroom to classroom, direct instruction is the predominant method and whole class heterogeneous grouping is standard. In America, there are almost as many mathematics curricula as there are school districts and teaching approaches range from direct instruction to encouraging children to construct their own mathematical understandings. Ability grouping is common and small group discussions are currently being advocated . Worldwide, larger forces are molding the language of mathematics, including rapid change in schools, homes and workplaces. The importance of a mathematically literate society is well recognized and this implies a citizenry that can think flexibly with numbers, whether they are mentally calculating the best value at the grocery store, estimating the return of money market funds, or checking the reasonableness of a calculator result.

While swamped with comparative data which ranks countries according to student performance on general tests of mathematical ability, we know little about the curricular and/or instructional factors which contribute to these differences (Stigler, Lee, & Stevenson, 1991; Stevenson & Stigler, 1992). If we are to go beyond the "contest" mentality of such international comparisons, we must begin to share information and carefully study specific areas so that we can all learn ways of helping children worldwide be better educated and prepared to face a rapidly changing world.

This report provides a profile of mental computation in Japanese schools. It provides a comprehensive data base of Japanese student performance in grades 2, 4, 6 and 8 on mental computation. In addition, individual interviews with Japanese students reveal the thinking strategies and processes used when doing mental computation. Information gathered from Japanese students and their teachers provide a profile of their attitudes and preferences toward mental computation.

This project is the outgrowth of more than four years of planning and preparation. It began with a two month research leave to Japan in 1988 which resulted in a cooperative research project with mathematics educators at the University of Tsukuba (Reys, Reys, Nohda, Ishida, Yoshikawa and Shimizu, 1991). This cooperative effort led to a Japan-United States Joint Seminar, "Computation for the 21st Century: Cross Cultural Perspectives," supported jointly by the National Science Foundation and the Japan Society for the Promotion of Science (Reys & Nohda, in press). Discussions at the seminar contributed to the development of the current research project which was made possible by funding from the National Science Foundation in a special program designed to stimulate cooperative research between Japan and the United States.

We thank the National Science Foundation (Grant 9000203) for supporting this project, as well as acknowledge support from the University of Missouri and the Institute of Education at the University of Tsukuba. In addition we are grateful to all the students and teachers who participated in the fieldtesting as well as those who contributed to the final data collection. We thank the principals of the Japanese schools for their willingness to participate and the gracious hospitality shown. We thank our Japanese host, Professor Nobuhiko Nohda, University of Tsukuba, for his help and contributions throughout all stages of this project. We also thank Katsuhiko Shimizu, Japanese National Institute for Educational Research and Yasuhiro Sekiguchi, Research Assistant to the Dean and Professor Duwey, University of Tsukuba for their assistance. We give a special heartfelt thanks to Hideyo Emori for his help in all stages of the project--from translation of all the instruments to data analysis, and Tatsuya Mizoguchi for also helping on the data analysis. In addition, we thank two teachers Yutaka Todoroki and Masae Yamada, and the following doctoral course mathematics education students (Toshiyuki Kimura, Nanae Matuo, Mikio Miyazaki, and Kazuhiko Nunokawa) and master course students (Yuji Endo, Sengaku Enya, Fumi Ginshima, Nobuhisa Hinaji, Makiko Hujinaga, Aida Istono, Yasuharu Kagoshima, Kyoko Kakihana, Shinji Kurata, Eri Motoki, Yoko Nakabayashi, Hiroaki Nakajima, Kazunari Nakajima, Takumi Nishimura, Tetuya Nuruki, Yoshinori Suzuki, Hiromasa Ueda, Yasushi Yamamoto and Takeshi Yoshigaki) at the University of Tsukuba for their help. The researchers are grateful for this broad base of support, without which this research would not have been possible.

Many people helped us by reading and making editorial suggestions to earlier drafts of this report. We especially thank Alistair McIntosh, Edith Cowan University, Perth Australia, who read every page with a critical eye and made many valuable suggestions for improving the report.

Finally, we acknowledge that the content of this report is based on the opinion and interpretation of the principal investigators and does not necessarily reflect the opinion or position of the National Science Foundation, or any of the many people who contributed to this research effort.

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Introduction

Mental computation is the process of calculating an exact arithmetic result without the aid of an external calculating or recording device. It is recognized as both important and useful in everyday living as well as valuable in promoting higher level mathematical thinking. The benefits of developing mental computation and related thinking strategies have been discussed elsewhere (Josephina, 1960; B. Reys, 1985a; Cobb & Merkel, 1989; McIntosh, 1990; Sowder, 1990; and Reys & Barger, in press). Its significance is also highlighted in the Curriculum and Evaluation Standards for School Mathematics and other recent reports in mathematics education in the United States (Mathematical Sciences Education Board, 1990) and internationally (Cockroft, 1986; Japanese Ministry of Education, 1989 and Australian Education Council, 1991).

Although mental computation has long been valued and included in Japanese mathematics programs, data on the mental computation performance of Japanese students have either not been reported or are not widely available (Shibata, in press). Even when performance data have been reported, the procedures used in the research studies were often not clearly articulated (Shigematsu, Iwasaki and Koyama, in press). Previous research has not addressed the thinking processes used by Japanese students as they calculate mentally. For example, are Japanese students applying procedures they have been taught or, as some of their more proficient American counterparts, are they formulating and applying invented thinking strategies to efficiently mentally compute?

Studies documenting Japanese students outscoring American students on general mathematics achievement tests have been reported for more than 20 years (Husen, 1967; McKnight, et al., 1987). Although none of these international studies included assessments of mental computation, one significant comparative study of first and fifth grade Asian and American students has been reported (Stigler, Lee and Stevenson, 1991). Their research was designed to provide a global view of mathematical performance, so mental computation was only a small portion of the overall assessment. Students were given a fixed time (one minute) to do a series of straight computations (computations without a context) mentally. Only whole number computations were included and all of the items were presented visually. The mean number of problems correctly computed by students was reported and the research documented that both

Taiwanese and Japanese students did more computations correctly in the given time period than did American students in both first and fifth grades. Performance levels for individual items were not reported, nor did the study identify strategies students used in doing their mental computations.

Purpose of this Study

This study was conducted to obtain information from Japanese students and teachers in grades 2, 4, 6 and 8 related to mental computation. The research was designed to provide four different perspectives of mental computation:

1. A measure of attitude toward mental and written computation of Japanese students and teachers in grades 4, 6, and 8.
2. A survey of the kinds of computations which Japanese students in grades 2, 4, 6, and 8 prefer to do mentally.
3. An assessment of mental computation performance of Japanese students in grades 2, 4, 6, and 8.
4. Characterization of the kinds of strategies used by Japanese fourth and eighth graders as they mentally compute a range of arithmetic items.

These perspectives taken collectively should provide a useful data set for better understanding mental computation in Japanese schools. In addition, it is anticipated that this research will provide some valuable benchmarks and points of departure for future research in the areas of mental computation and number sense.

Procedures

This study includes data gathered from Japanese students and their teachers in the final third of the school year. The Japanese school year begins in early April and ends in March. All the data in this study were collected during February and March of 1992.

Three different survey instruments were constructed and utilized. In addition, an interview protocol was devised for selected students in grades 4 and 8. Construct validity for the instruments was provided through a series of reviews. The researchers developed drafts of the instruments which were reviewed by several other American, Australian and Japanese mathematics educators. The resulting instruments were then fieldtested with Japanese students and teachers, and revised further based on information gathered from the piloting.

The three survey instruments developed for and used in the study included a Preference Survey (PS), Attitude Survey (AS), and a survey of mental computation ability. All the surveys were administered (in the order listed) during one class period (50 minutes).

Parallel forms of the preference and attitude surveys were developed for teachers at each grade level. Teachers were asked to complete a preference survey and an attitude survey during the same time their students were completing their respective surveys.

The Mental Computation Test (MCT) contained two parts: an orally presented set of items (items read individually by the administrator) and a visually presented set of items (items presented individually using an overhead projector). Half of the classes in the sample took the first half of the test via an oral administration format followed by the second half of the test via a visual administration format. The administration format was reversed with the other half of the sample (visual administration on first half, oral administration on second half). Figure 1 describes the administration format for the grade 2 MCT. This plan provided an opportunity to examine any learning effect by form as well as any mode of presentation effect. Each of the other grade-level tests followed a similar pattern.

Figure 1.
Administration Pattern of MCT Forms A and B for Grade 2

Item Numbers	Administration Format	
	Form A	Form B
#1-15	oral	visual
#16-30	visual	oral

In an effort to document strategies used to mentally compute, individual student interviews in grades 4 and 8 were conducted. These interviews were constructed to learn about strategies and techniques students use to perform mental computation. Approximately one month following the administration of the three surveys, ten students in each of grades 4 and 8 were individually interviewed. These students were randomly selected from students scoring in the first (high) and third (middle) quintile on the MCT. Five students from each grade were selected from each quintile. All of the individual interviews were conducted during the regular school day by a team of Japanese mathematics educators and videotaped for review and analysis.

Subjects

Four different Japanese schools (3 elementary and 1 junior high) participated in the study. The schools were randomly selected from a "typical" city in Ibaraki prefecture. "Typical" is used here to mean a school district outside a major urban area with a balanced working class/professional population as well as a student population which is "typical" of the prefecture's population mix.

The city chosen for this study (Tsuchiura) has a population of 150,000 and is located about 50 km north of Tokyo. Within each elementary school, two classes were randomly selected at grade 2, 4 and 6. One class was randomly assigned to form A and the other to Form B of the MCT. At the junior high school, 6 classes of grade eight were randomly selected, three randomly assigned to Form A and three randomly assigned to Form B of the MCT. The class sizes showed a range of students in each grade (15 - 38, grade 2; 22 - 39, grade 4; 28 - 32, grade 6; and 29 - 36, grade 8). Students in all classes were heterogeneously grouped as is the custom in Japanese schools. Only students and teachers attending class during the test administration date were included in the sample of subjects. These procedures resulted in data for 176 second, 187 fourth, 186 sixth, and 206 eighth graders and 22 teachers (two teachers were absent the day their classes were tested).

Instruments

Preference Survey (PS)

Numerical computation can be done by various methods, including the use of mental computation, written computation or calculators. Among these alternatives, the Japanese school mathematics curriculum has placed greatest emphasis on written computation and least on calculators (Shibata, in press). The preference survey began with a reminder that different types of computational methods exist, and that each person must choose what method to use on a particular computation. The preference survey provided a series of numerical computations and asked participants if they would choose to do these computations mentally. For example, grade 4 students were asked to respond yes or no to whether they would mentally compute items such as: 500 + 300; 80 - 24; 14 x 83; and 100 x 45. Figure 2 shows an excerpt from the beginning of the preference survey for second graders.

Figure 2.
Excerpt from an English Translation of the Second Grade Preference Survey

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Computation is often involved in solving real-world problems. When solving problems, several computational methods exist:

- Sometimes people use a calculator.
- Sometimes people use paper and pencil.
- Sometimes people compute mentally without writing anything down.

We want to learn which problems you prefer to do mentally.
Please look at each problem below and decide if you prefer to do it mentally.
Circle YES or NO to indicate your response. It is not necessary for you to work the problems.

	Problem	I would do this problem mentally.	
1.	500 + 300	Yes	No
2.	Double 26	Yes	No
3.	58 + 34	Yes	No
4.	60 + 80	Yes	No

The participants were not asked to do the particular computation but only to decide if they would do the computation mentally if allowed a choice. Respondents indicated their answer by marking "yes" or "no." The same instructions were used for each grade level. The survey items at each grade level were selected to coincide with items commonly found in the mathematics curriculum at that particular grade level. Four items (14 x 83, 35 x 55, 4/7 + 2/5 and 0.35 x 567) that would be tedious to compute mentally were included to determine how discriminating the students were in their responses to the preferences. A few common items, such as 165 + 99, appeared in more than one grade level to provide a profile of preferences across grades. Some items in the PS were also included in the mental computation test, so that data reporting preferences could be compared with actual performance on the same item.

A parallel preference survey for teachers was also developed. The teachers were told, "We want to learn what calculations you feel students should do mentally. At the end of the grade level you teach, do you think students should do these calculations mentally? Please check the appropriate box indicating your response for each item." All items from the student survey were included in the preference survey for teachers. Administration time for the preference survey for both teachers and students ranged from 4 to 8 minutes. See Appendix A for a complete copy of both the fourth grade student and teacher preference survey in English and Japanese.

Attitude Survey (AS)

A series of statements designed to document a student's attitude toward mental computation was developed, field tested, refined and utilized in grades 4, 6 and 8. The final statements resulted from reviews of earlier versions of the attitude instruments by four mathematics educators as well as pilot information from Japanese teachers and students. The student survey included 28 statements clustered by five dimensions. The dimensions and an English translation of a sample of the statements are shown in Figure 3.

Two types of statements are included within each dimension of the framework. One type provided a parallel mental computation statement to accompany each statement related to written computation. For example, the parallel statements, "It is important to be good at mental computation" and "It is important to be good at written computation" were both included on the survey. Another type of statement required responding to a judgmental statement such as, "I am better at written than mental computation." Each such judgmental statement was accompanied by a parallel statement, which in this case was, "I am better at mental than written computation." These pairings provide a further means of checking on consistency of student responses. (An

English and Japanese copy of the student attitude survey is provided in Appendix B.)

The teacher attitude survey consisted of a series of 24 statements reflecting three dimensions (beliefs, teaching practice, and evaluation). The organizing dimensions and a sample of statements are shown in Figure 4.

Most of the statements involved parallel statements of mental computation and written computation (for example, "I think mental computation is important" and "I think written computation is important.") Matched pairs of judgmental statements, such as "I think students

Figure 3.
Framework and Sample Items from Student Attitude Instrument

Interest and Enjoyment
I enjoy doing written computation.
I enjoy doing mental computation.
Written computation is more interesting than mental computation.
Mental computation is more interesting than written computation.

Perception of Competence
I am good at written computation.
I am good at mental computation.
I think written computation is more challenging than mental computation.
I think mental computation is more challenging than written computation.

Perception of Value
It is important to be good at written computation.
It is important to be good at mental computation.
It is more important to be good at written then mental computation.
It is more important to be good at mental than written computation.

Perception of Use
I think I will do written computation more than mental computation as an adult.
I think I will do mental computation more than written computation as an adult.
At school I do mental computation more than written computation.
At school I do written computation more than mental computation.

Perception of Source of Instruction
I learned to do mental computation at school.
I learned to do written computation at school.
I learned to do written computation by myself.
I learned to do mental computation by myself.

should learn some written computation before learning mental computation." and "I think students should do some computations mentally before learning paper/pencil computation." were also included in the survey.

The statements included in the attitude instruments were scrambled in the final forms given to students and teachers. In each case, the participants were asked to read each statement and respond by indicating, "Yes, No, or Not Sure." Administration time ranged from 6 to 10 minutes for both teachers and students (a copy of the teacher attitude instrument in both English and Japanese is in Appendix C).

Figure 4.
Framework for Teacher Attitude Instrument

Beliefs
I think paper/pencil computation is important.
I think mental computation is important.
I think students should learn some written computation before learning mental computation.
I think students should learn to do some computations mentally before learning paper/pencil computation.

Practice - Teaching
I encourage students to do easy computations mentally.
I encourage students to do easy computations with paper and pencil.
I teach some mental computation before paper/pencil computation.
I teach some paper/pencil computation before mental computation.
I try to help students learn ways to compute mentally.
I try to help students learn ways to compute with paper and pencil.

Practice - Evaluation
I test mental computation.
I test written computation.

Mental Computation Test (MCT)

The MCT was designed by the researchers for group administration. The grade 2 and 4 MCT contained 30 items, 15 administered orally and 15 administered visually. The grade 6 and 8 version contained 40 items, 20 administered orally, and 20 administered visually. Two different forms (A & B) were developed for each grade level. Each form contained the same set of items but differed in the presentation format as was illustrated in Figure 1. Prior research has documented the difficulty in obtaining valid and reliable measures of mental computation (Sachar, 1978; Reys, B., 1985b; Reys, R., 1986; Reys, Reys & Hope, in press; Shigematsu, Iwasaki and Koyama, in press). In an effort to provide an accurate assessment of mental computation, several steps were taken.

First, the mental computation test included only straight computational items (items devoid of a context). This allowed students to focus exclusively on the required computation thereby eliminating any decisions related to choice of operations.

Second, the mental computation test was composed of oral and visual items, with half of the items presented orally (read aloud by the administrator) and half presented visually (via an overhead projector). In an effort to investigate the order effect of the administration, half of the classes were given the oral portion of the test first followed by the visual portion of the test, and the other half experienced the visual portion of the test followed by the oral portion.

Third, all items on the mental computation test were given one at a time and the time for each item was carefully controlled. Prior research on group assessment had suggested that failure to control time would result in some students copying items and using written rather than mental computation techniques. Items were individually paced by the examiner with twenty second intervals between item presentation. The researchers felt 20 seconds provided ample time for students to think about the item presented and decide on a course of action. Pilot testing confirmed that 20 seconds was very generous for some students and yet adequate for nearly everyone to do the computations mentally. The visually presented items were individually displayed on an overhead screen for a period of 20 seconds. The orally administered items were read twice with a brief pause (2-3 seconds) between readings followed by a 20 second wait period between items. The test items were selected to be commensurate with the mathematics curriculum grade level. Several items used in earlier research studies (Reys, Reys & Hope, in press; Shigematsu, Iwasaki and Koyama, in press) were also included to provide some comparative benchmarks.

Fourth, a specially constructed answer sheet provided room only for a written answer, thereby discouraging copying the problem onto the paper.

Every response to the mental computation test was individually evaluated and coded for further analysis. Each student response was coded as correct, correct with signs of written computation, correct with evidence of several equivalent answers, incorrect, incorrect with signs of written computation, or no response.

A different form of the mental computation test was developed for each grade level. Figure 5 shows the distribution of items by operation and domain of numbers for each grade. In addition to providing a profile of student mental computation performance at each grade, the tests were designed to monitor some changes across grade level. A set of common items across grade levels was embedded within the test. Several sets of "nested" items (items related in mathematical structure) were also included (see Appendix D for a complete listing of items in each grade of the MCT).

Figure 5.
Mental Computation Test Item Distribution

Number Type	Operation	Grade 2	Grade 4	Grade 6	Grade 8
Whole Number	Addition	12	6	4	2
	Subtraction	12	6	4	2
	Multiplication	4	6	6	4
	Division	2	6	6	6
Fraction	Addition	-	2	4	4
	Subtraction	-	2	4	4
	Multiplication	-	-	2	4
	Division	-	-	-	2
Decimal	Addition	-	2	2	1
	Subtraction	-	-	2	2
	Multiplication	-	-	2	2
	Division	-	-	-	2
Percent	Multiplication	-	-	4	5
TOTAL		30	30	40	40

Interview Items and Protocols

Individual interviews were conducted in an effort to identify strategies and techniques students use in doing mental computation. In an effort to gain a range of information, interviews were conducted with students with high and average performance on the MCT. More specifically, individual interviews were conducted with 10 students in each of grades 4 and 8. At each grade, 5 students were randomly selected from the top quintile and 5 from the middle quintile on the MCT.

The interview items were drawn directly from the MCT. The interview for grade 4 included 8 items, 6 of which focused on whole number addition, subtraction, multiplication and division, one item on fraction addition, and one item on decimal addition. The grade 8 interview included 10 items, three whole number, 4 fraction, two decimal and one percent. The items, including the three common to both grade levels are shown in Figure 6.

The interview items were reviewed by the research team, other Japanese mathematics educators and Japanese classroom teachers. Each group was asked to examine the questions

Figure 6.
Mental Computation Interview Items for Grades 4 and 8

Item	Grade 4	Grade 8
A. 79 + 26	*	
B. 165 + 99	*	*
C. 105 - 97	*	
D. 100 - 68	*	
E. 7 x 25	*	
F. 7 x 49		*
G. 38 x 50	*	*
H. 1/2 + 3/4	*	*
I. 3 - 5/6		*
J. 4 x 3 1/2		*
K. 6 1/2 ÷ 2		*
L. 0.5 + 0.75	*	
M. 1.5 x 20		*
N. 90 ÷ 0.5		*
O. What is 75% of 48?		*

for clarity, grade level appropriateness, and likelihood that the item would elicit a range of strategies. All the mental computation items were judged to be commensurate with the grade 4 and 8 curriculum with the exception of one item at each grade. In grade 4, the fraction item (1/2 + 3/4) was placed on the interview even though paper/pencil computation of that problem type had not been addressed in the standard curriculum. The rationale for the inclusion of this item was that given conceptual understanding of fractions, these common fractions could be added by strictly conceptual knowledge (mentally model the fractions or decompose and recompose them) and therefore knowledge of a paper/pencil algorithm was not necessary. The researchers hoped that by placing this item on the interview, students would be encouraged to seek an invented method rather than a learned procedure. Likewise, a percent item (What is 75% of 48?) was included on the eighth grade protocol. Percent had been introduced to the eighth graders at the time of the interview although computation procedures had not been discussed.

The reviews resulted in items that were pilot tested via individual student interviews. Students were allowed as much time as needed to process each item and to describe their thinking, but the final set of questions for the interview was designed to be administered to students within a period of about 30 minutes.

To ensure consistency in the presentation of the interviews, specific probes were developed. The items and probes were then integrated into an interview package (see Appendix E for a complete listing of the interview protocols for grade 4 and 8). Training sessions using videotaped student interviews as well as discussions of interview techniques were held with the Japanese mathematics educators responsible for conducting the interviews. The interviewers also conducted several pilot interviews prior to conducting the interviews reported in this research. Discussion and further clarification with the Japanese interviewers followed these pilot interviews.

All interviews were conducted in Japanese and videotaped. The interviews were conducted by several interview teams, each consisting of two members - an interviewer and an observer who also operated a video camera. Each of the items in the interview was presented orally and the student was instructed to mentally compute then describe the thinking strategy used. Time was not a restraint as students were given as much time to both calculate and to describe their strategy as they desired. The members of the interview team did not know whether the student being interviewed was in the high or middle quintile, and were asked to treat all students in a similar manner.

Upon completion of an interview the researchers debriefed the interview team to learn of any unusual happenings during the interview, such as unscheduled interruptions or running short of time. After all interviews, videotapes were reviewed as strategies were identified and thinking reflected by the students was documented via script tapes (notes summarizing the interaction of interviewer and interviewee taken by the researchers as the videotape was reviewed). The script tapes included the response, approximate time needed to formulate the response, a written summary of the strategy described, the students' response to whether he/she could describe an alternative strategy, and any other comments about the problems and/or solution made by the student.

Prior to the interviews, a detailed categorization of anticipated strategies was formulated by the research team for each of the interview items. This categorization included a range of strategies, both standard (taught as part of regular school mathematics curriculum) and non-standard (not taught but rather invented by applying mathematics properties). For the first item on the grade 4 interview (79 + 26), Figure 7 shows the anticipated strategies and some grouping of them into like categories.

Figure 7
Categorization of Anticipated Strategies for Grade 4 Item A, 79 + 26

A. Group by tens and ones
    A1. L-R (Tens first): (70 + 20 = 90; 9 + 6 = 15; 90 + 15 = 105)
    A2. R-L (Ones first): (9 + 6 = 15; 70 + 20 = 90; 15 + 90 = 105)
    A3. Cumulating sums (70 + 20 = 90; 90 + 9 = 99); 99 + 6 = 105)
B. Hold one addend constant
    B1. First addend: (79 + 20 = 99; 99 + 6 = 105)
    B2. Second addend: (26 + 70 = 96; 96 + 9 = 105)
C. Round one or both addends to multiple of ten then adjust
    C1. First addend: (80 + 26 = 106; 106 - 1 = 105)
    C2. Second addend: (79 + 30 = 109; 109 - 4 = 105)
    C3. Both addends: (80 + 30 = 110; 110 - 1 - 4 = 105)
D. Round both addends to multiple of five then adjust
    D1. (75 + 25 = 100; 100 + 4 + 1 = 104)
E. Mental image of paper/pencil algorithm
F. Mental image of soroban

In the categorization of Item A, the first four strategies are those which have been found in previous research (Hope & Sherrill, 1987; Reys, B., 1985b) to be invented by individual students for the purpose of mentally computing. The last two strategies are ones which are normally utilized with a recording device (paper/pencil or soroban). A detailed listing of anticipated strategies for each interview item are in Appendix F.

Results and Discussion

Preference Survey

The Preference Survey (PS) focuses on computations which Japanese students prefer to do mentally and provides one perspective of mental computation. Most items in the PS were also included in the MCT, but four very difficult items (4/7 + 2/5, 14 x 83, 35 x 55 and 0.35 x 567) were included to provide a check on the validity of the PS data.

The results from the PS for each grade level are shown in Table 1. Of the 12 items in second grade, two-thirds or more of the students indicated on all but one item (165 + 99) that they would do the necessary computation mentally. In the fourth grade, one item involving complex computation (14 x 83) was included and as expected only a small portion (26 percent) of the students indicated they would attempt to do that computation mentally. On all other items, at least two-thirds and typically 80 percent or more of the fourth graders indicated they would do the computation mentally. Similar results were observed in grades six and eight.

One item in the PS (945 x 1000) was included as part of one of the earlier NAEP mathematics assessments. NAEP reported about 35 percent of the American 13 year-olds would do the computation mentally, with the remainder opting to use either paper/pencil or a calculator (National Assessment of Educational Progress, 1983). Over 80 percent of the Japanese sixth and eighth graders indicated they would do this computation mentally, which is markedly different from the American students.

Table 1 suggests Japanese students call upon mental computation to do appropriate computations and the MCT results reported later in Tables 11 - 14 confirm that on a majority of the PS items (which appeared in the MCT) the percent of students answering these items correctly, equals or exceeds the percent of students reporting they would do the computations mentally. This finding suggests that, in general, items which the Japanese students reported they would do mentally on the PS they were in fact able to do correctly on the MCT.

Table 1.
Computational Preferences Reported by Second, Fourth, Sixth and Eighth Grade Japanese Students

Item	Grade 2 (n=176)	Grade 4 (n=187)	Grade 6 (n=186)	Grade 8 (n=206)
6 + 8	94			
60 + 80	85	98		
36 + 9	85			
58 + 34	74	91	97	
47 + 54 + 23			74	68
265 + 100	86			
500 + 300	90	99		
165 + 99	56	64	75	74
74 - 30	67	93		
100 - 68	68			
73 - 23	76			
80 - 24	71	89		
264 - 99			67	57
6 - 4.5			63	87
1/2 + 3/4			81	74
4/7 + 2/5				27
1 - 1/3		67	80	
Double 26	63	83		
60 X 70		79	91	
14 X 83		26	24	12
35 X 55			25	
100 X 35		74		
945 X 1000			83	84
7 X 25		74	91	80
1/10 of 45			66	74
0.35 x 567				1
0.1 x 45			76	90
90 + 1/2				86
25% of 48				13

*Percent choosing to do the computation mentally is shown.

Table 2.
Computational Preferences Reported by Second, Fourth, Sixth and Eighth Grade Japanese Students in First, Third and Fifth Quintiles of Total Mental Computation Test*

Item	Grades and Quintiles											
	2			4			6			8		
	L	M	H	L	M	H	L	M	H	L	M	H
6 + 8	83	97	100									
60 + 80	71	81	94	95	100	100						
36 + 9	66	92	97									
58 + 34	49	72	97	84	95	97	95	97	100			
47 + 54 + 23							70	64	95	66	71	74
265 + 100	63	89	98									
500 + 300	77	94	94	95	100	100						
165 + 99	46	47	81	54	71	74	62	64	97	76	76	79
74 - 30	40	61	92	81	95	97						
100 - 68	37	61	97									
73 - 23	46	83	86									
80 - 24	46	64	89	86	95	89						
264 - 99							70	51	84	59	62	71
6 - 4.5							27	59	89	71	93	93
1/2 + 3/4							68	79	84	68	79	79
4/7 + 2/5										39	26	26
1 - 1/3				35	66	87	60	85	89			
Double 26	51	58	81	62	82	92						
60 X 70				62	82	87	81	87	97			
14 X 83				32	16	21	30	23	32	17	21	3
35 X 55							32	33	21			
100 X 35				51	79	89						
945 X 1000							49	90	95	49	93	98
7 X 25				68	76	84	92	85	97	66	86	90
1/10 of 45							32	64	92	44	81	95
0.35 x 567										3	3	0
0.1 x 45							46	79	92	73	95	100
90 ÷ 1/2										66	93	100
25% of 48										10	12	21

*Percent choosing to do the computation mentally is shown.
L, M, H designate low, middle and high quintile MCT groups.

Do students scoring high or low on the MCT differ in their selection of items to compute mentally? In an effort to answer this question, student responses were sorted by first, middle and fifth quintile according to their total score on the MCT. Then the PS was analyzed by the three quintile groups. The results are reported in Table 2. For example, at grade 2, whereas 86 percent of all second graders indicated they would compute 265 + 100 mentally, Table 2 shows that 63, 89 and 98 percent of the students in the bottom, middle and top quintile would choose to do that computation mentally.

An examination of Table 2 confirms that students in the three quintiles sometimes differ sharply in whether they choose to do a particular computation mentally. This pattern was observed across second, fourth and sixth grades. For example, 46, 64, and 89 percent of the second graders in the low, middle and high quintiles indicated they would prefer to do the computation 80 - 24 mentally. At the eighth grade, the difference between the low quintile and the middle quintile was often sizable, whereas the difference between the middle and high groups was usually negligible. For example, two-thirds of the eighth graders in the low quintile preferred to compute 90 ÷ 1/2 mentally, compared to 93 and 100 percent of the low eighth graders in the middle and high quintiles, respectively. Likewise, 49 percent of the lowest quintile sixth and eighth graders indicated a preference to compute 945 x 1000 mentally while over 90 percent of their middle and high quintile counterparts indicated the common expectation of doing this type of calculation mentally. In general, students more skilled in mental computation tended to choose mental computation as their preferred method.

In closing this discussion of the PS, it should be noted that examination of Tables 1 and 2 together with the actual MCT performance data reported in Tables 11 - 14 provides many opportunities to examine interactions and explore patterns. For example, Table 1 indicates that the percent of students preferring to do the computations mentally were generally very high, yet there were several items where few students indicated they would do the computation mentally. More specifically, only 13 percent of the eighth graders reported they would find 25% of 48 mentally. The item, 25% of 48, was chosen as a mental computation item by only 10, 12 and 21 percent of the students in the lowest, middle and highest quintile. Thus, the item was perceived as appropriate for mental computation by a very small percent of students across all mental computation ability performance levels. Considering the compatible numbers involved in this computation, it is surprising that so few students considered doing this computation mentally. Yet from Table 13 reporting results on the MCT, over half of the eighth graders correctly did the computation mentally. This suggests that although mental computation may not be the preferred mode of computation of 25% of 48, most of the eighth graders were capable of doing the computation mentally.

A preference survey was also given to teachers at each of the four grade levels. An examination of the teacher data raised some questions about the validity of the results. Follow-up discussions with several teachers revealed that some teachers completed their survey based on computations they themselves could do mentally rather than what their students should be able to do mentally. Since it was not possible to clarify how each teacher completed this survey, there was a cloud of uncertainty about the meaning of these data. Due to this ambiguity it seemed inappropriate to report or discuss these results, therefore the preference data from the Japanese teachers are not included in this report.

Attitude Survey

Student Attitude Survey. The statements in the Attitude Survey (AS) were scrambled and mixed as shown in Appendix B. In order to facilitate the review and analysis of the attitude data, these statements are grouped within clusters and shown in Table 3. For example, Table 3 reports that 32, 21 and 11 percent of the fourth, sixth and eighth grade students said 'yes' to statement #1, "I enjoy doing written computation." whereas 37, 26 and 12 percent of the students said 'yes' to statement #2, "I enjoy doing mental computation." An examination of Table 3 suggests many similar patterns of student responses across elementary and junior high school.

The statements relating to "Perception of Value" of mental computation illustrate the similarity of responses across elementary and junior high school. For example, about two-thirds of students at each grade level felt it was important to be good at mental computation and about the same percentages felt it was important to be good at written computation. Likewise very few (less that 15 percent) felt it was more important to be good at written computation than mental computation, and about half of the students agreed that it was more important to be good at

Table 3.
Percent of responses of "Yes" "No" and "Not Sure" by fourth, sixth and eighth grade Japanese students to statements reflecting attitudes

	Gr. 4 (n=187)			Gr. 6 (n=186)			Gr. 8 (n=206)		
	Y	N	NS	Y	N	NS	Y	N	NS
Interest and Enjoyment									
1. I enjoy doing written computation.	32	33	35	21	29	50	11	32	57
2. I enjoy doing mental computation.	37	40	19	26	38	35	12	43	43
3. I think written computation is interesting.	27	37	35	23	29	49	11	37	52
4. I think mental computation is interesting.	32	41	26	25	25	50	3	42	44
5. Written computation is more interesting than mental computation.	37	43	20	26	41	31	15	25	59
6. Mental computation is more interesting than written computation.	36	34	30	27	30	43	15	26	59
Perception of Competence									
7. Written computation is challenging to me.	33	41	24	28	40	31	32	18	49
8. Mental computation is challenging to me.	67	18	14	60	18	20	53	19	27
9. I am good at written computation.	49	23	26	44	19	37	18	18	64
10. I am good at mental computation.	33	45	19	18	54	26	11	57	32
11. I think written computation is more challenging than mental computation.	36	34	30	34	19	47	21	24	55
12. I think mental computation is more challenging than written computation.	44	40	16	31	42	27	34	31	35
13. I am better at written than mental computation.	64	20	15	68	16	16	69	9	22
14. I am better at mental than written computation.	29	49	22	18	57	25	16	57	27
Perception of Value									
15. It is important to be good at written computation.	60	18	22	67	11	22	69	5	26
16. It is important to be good at mental computation.	65	16	16	69	10	19	64	8	28
17. It is more important to be good at written then mental computation.	17	46	35	12	27	59	13	19	67
18. It is more important to be good at mental than written computation.	62	19	19	50	19	30	42	9	49
Perception of Use									
19. I think I will do written computation more than mental computation as an adult.	25	60	13	29	43	28	24	40	36
20. I think I will do mental computation more than written computation as an adult.	68	18	13	51	22	26	44	19	37
21. At school I do mental computation more than written computation.	29	54	17	18	55	26	17	58	24
22. At school I do written computation more than mental computation.	71	19	9	76	10	13	70	14	15
23. I do written computation more than mental computation away from school.	53	31	14	51	26	22	46	31	23
24. I do mental computation more than written computation away from school.	44	41	13	30	49	19	30	50	20

Perception of Source of Instruction									
25. I learned to do mental computation at school.	47	43	10	40	41	18	27	46	27
26. I learned to do written computation at school.	68	16	16	79	10	11	87	4	9
27. I learned to do written computation by myself.	31	51	14	19	63	17	11	68	21
28. I learned to do mental computation by myself.	42	49	9	26	61	13	24	70	6

* Percents totaling less than 100 for a statement at a given grade reflects "no response."

mental than written computation.

In regard to statements classified under the "Perception of Competence" heading about one-third of the students at each grade said written computation is challenging, while a majority at each grade said mental computation is challenging. About two-thirds of the students at each grade felt they were better at written than mental computation, whereas less than one third of the students at each grade level felt they were better at mental then written computation.

Do Japanese students attach equal importance to mental and written computation? Two thirds of the fourth graders, about half of the sixth graders and 44 percent of the junior high students felt they would do more mental computation than written computation as an adult, while only about one-fourth of the students at each grade said they would do more written computation than mental computation as an adult. About two-thirds of the students at each grade said they do more written than mental computation at school.

A majority of students (68, 79 and 87 percent of fourth, sixth and eighth grade respectively) reported learning written computation at school whereas less than half (47, 40 and 27 percent of fourth, sixth and eighth grade respectively) reported learning mental computation at school. At each grade level, more students reported learning mental than written computation by themselves.

Many messages are suggested in the attitude data but most would be decipherable only from case studies, careful observation and/or interviews with the students. Nevertheless there are some common themes which seem to cut across elementary and junior high. At the risk of overgeneralizing here is a possible characterization of the "typical" attitude of Japanese students:

> I learned to do written computation at school, and spend more time at school doing written computation than mental computation. I find mental computation challenging, but feel that I am better at doing written computation than mental

computation. I think it is important to be good at both mental and written computation, but mental computation will be used more as an adult and so it is more important than written computation. Although I learned to do some mental computation at school I learned to do much of it by myself.

Teacher Attitude. The summary of the teacher responses to the Attitude Survey is reported in Table 4. The limited number of teachers at each grade (at most six) make it foolhardy to draw conclusions or make general assertions regarding Japanese teachers. Nevertheless these data do provide a base for some interesting speculation.

An examination of the statements reflecting teacher "beliefs" (#1- 8) suggests that all the respondents at all four grade levels think both written and mental computation are important. An examination by grade level suggests that most instructional attention is given to mental computation in grade 2 (#9, 14, 15, 17, 19, 21 and 24); somewhat less in grade 4, and the attention to mental computation continues to decrease through grade 8. More teachers at each grade level felt their students were better at written computation than at mental computation. There was consensus in all grades that students should learn some written computation before learning mental computation.

The teacher beliefs coincide with teaching practice (#19 and 20) and evaluation (#21 and 24). For example, written computation (#13) was taught by all teachers in the four grades, whereas mental computation (#14) was taught by half the teachers in grade 2 and fewer than half the teachers in each of the other grades. More teachers reported helping students learn to do written computation (#19) than helping students to compute mentally (#20). Furthermore in all grades except grade 6, students practiced written computation (#17) more than mental computation (#18). Evaluation of computation paralleled what was taught, thus all the teachers responding in grades 2, 4 and 6, and half the eighth grade teachers test and monitor student progress in written computation; whereas a minority of teachers in grades 2 and 4, and none of the teachers in grades 6 and 8 reported testing or monitoring student progress in mental computation.

Table 4.
Responses of "Yes" "No" "Not Sure" by 6-second, 5-fourth, 5-sixth and 6 eighth grade Japanese teachers to statements reflecting attitudes

	Gr. 2			Gr. 4			Gr. 6			Gr. 8		
	Y	N	NS	Y	N	NS	Y	N	NS	Y	N	NS
Beliefs												
1. I think facility with written computation is important.	6	0	0	5	0	0	5	0	0	6	0	0
2. I think facility with mental computation is important.	6	0	0	5	0	0	5	0	0	5	0	1
3. I think it is important that students learn some written computation before learning mental computation.	5	1	0	3	1	1	5	0	0	5	0	1
4. I think it is important that students learn to do computations mentally before learning written computations.	1	2	3	3	1	1	1	0	4	0	6	0
5. I think my children are good at mental computation.	4	2	0	1	4	0	0	4	1	0	5	1
6. I think my children are good at written computation.	4	0	2	2	1	2	3	1	1	4	1	1
7. I think my children enjoy doing written computation.	3	1	2	2	1	2	0	3	2	1	3	2
8. I think my children enjoy doing mental computation.	2	1	3	1	0	4	1	1	3	3	0	3
Practice — Teaching												
9. I encourage students to do easy computations mentally.	4	0	2	5	0	0	4	1	0	2	3	1
10. I encourage students to do easy computations with paper and pencil.	4	2	0	4	1	0	2	2	1	3	0	3
11. I encourage students to do difficult computations with paper and pencil.	6	0	0	4	0	1	0	0	5	4	0	2
12. I encourage students to do difficult computations mentally.	0	6	0	0	5	0	0	4	1	0	6	0
13. I teach written computation.	6	0	0	5	0	0	5	0	0	4	2	0
14. I teach mental computation.	3	2	1	2	1	2	1	3	1	0	5	*
15. I teach mental computation prior to teaching written computation.	3	3	0	1	3	1	0	3	2	0	6	0
16. I teach written computation prior to teaching mental computation.	3	2	1	2	2	1	4	0	1	3	2	1
17. Children in my class practice mental computation.	3	2	1	4	1	0	1	3	1	0	6	0
18. Children in my class practice written computation.	6	0	0	3	2	0	0	0	5	2	4	0
19. I try to help students learn ways to compute mentally.	3	3	0	1	4	0	2	1	2	0	5	1
20. I try to help students learn ways to compute with paper and pencil.	6	0	0	2	3	0	5	0	0	4	2	0

Practice — Evaluation			
21. I test mental computation.	2 4 0	1 4 0	0 5 0	0 6 0
22. I test written computation.	6 0 0	5 0 0	4 0 *	3 3 0
23. I monitor student progress in written computation.	6 0 0	5 0 0	4 0 1	2 4 0
24. I monitor student progress in mental computation.	2 3 1	1 4 0	0 5 0	1 5 0

* denotes one "no response"

The biggest surprise from the teacher attitude data is their diversity. Although only data from 5 or 6 teachers at each grade level were reported in Table 1, on only one item (#1: I think facility with written computation is important) did all teachers in each grade level agree. In fact, out of the 24 items there was consensus at only 9 (#1, 2, 11, 12, 13, 18, 20, 22, 23), 6 (#1, 2, 9, 13, 22, 23), 9 (#1, 2, 3, 11, 13, 18, 19, 21, 24), and 6 (# 1, 4, 12, 15, 17, 21) items among second, fourth, sixth, and eighth grade teachers respectively. These responses (reflecting both belief and practices related to computational alternatives) torpedo the notion of conformity typically associated with Japanese teachers and suggest that considerable variability exists.

An examination of Japanese textbooks documents that these teaching practices mirror the attention given to the topics of mental computation and written computation. In textbooks although mental computation is often demonstrated and visually illustrated in the early development of computational techniques. Furthermore all addition and subtraction computations prior to the middle of grade two are presented horizontally. No written computational algorithms are presented until the last half of second grade. Nevertheless, most teachers reported (in both belief and practice) that written computation is introduced before mental computation (# 3 & 4 and #15 & 16). In general, more time is allotted in grades 2, 4, 6 and 8 to written computation than mental computation, however the amount of emphasis to computational facility decreases rapidly in grades 6 and 8.

Mental Computation Test

Mental computation has rarely been assessed and reported in Japan. Mental computation is not a visible part of national tests in Japan, nor is mental computation explicitly included in entrance examinations for junior or senior high schools. In addition, information gathered from the previously mentioned student and teacher attitude surveys confirm that mental computation is not regularly tested at the classroom level.

The purpose of this research was not to judge performance but rather to collect mental computation data and report them. Therefore the mental computation performance data are presented from several different perspectives in hopes that collectively these different views will be useful in discussing mental computation performance of Japanese students as well as providing comparative benchmarks for future research.

The MCT at each grade level was composed of items which the researchers felt were reasonable to mentally compute. Thus, it could be argued that students at each grade should answer all items correctly, and anything less than 100 percent would be judged as unsatisfactory performance. However, a range of performance levels was anticipated at each grade level, and indeed very wide ranges of performance among the Japanese students were observed at each grade level.

A summary of the MCT total scores is reported in Table 5. Each of the grade level tests was unique although some common items across grade levels were included. Therefore, grade level comparisons of group performance is inappropriate.

Table 5.
Summary of Second, Fourth, Sixth, and Eighth Grade Japanese Student Performance on the MCT Total Score

	Grade 2	Grade 4	Grade 6	Grade 8
Number of Students	176	187	186	206
Minimum score achieved on MCT	0	0	4	3
Maximum score on MCT	30	30	40	40
Mean	19.45	18.09	28.39	28.85
Range	0 - 30	0 - 30	4 - 40	3 - 40
Standard Deviation	7.42	5.67	9.14	7.44
Error of Measurement	0.56	0.42	0.67	0.52

Figures 8 - 11 display frequency distributions of the performance at each grade level. These histograms provide visual evidence of the range of performance, with 5, 4, 2, and 2 of the second, fourth, sixth, and eighth graders respectively scoring less than 5, suggesting for these students the MCT was very difficult. On the other hand, at least 3 students at every grade and as many as 10 of the sixth graders answered all of the items correctly. The overall results confirm the researchers' belief that the items included in the MCT were reasonable to mentally compute at the specified grade level.

Figure 8. Histogram of MCT Total for Second Grade

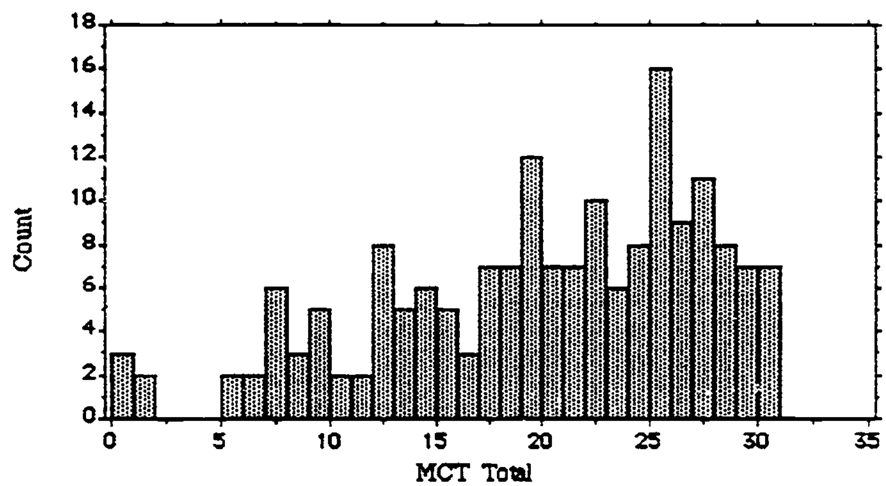


Figure 9. Histogram of MCT Total for Fourth Grade

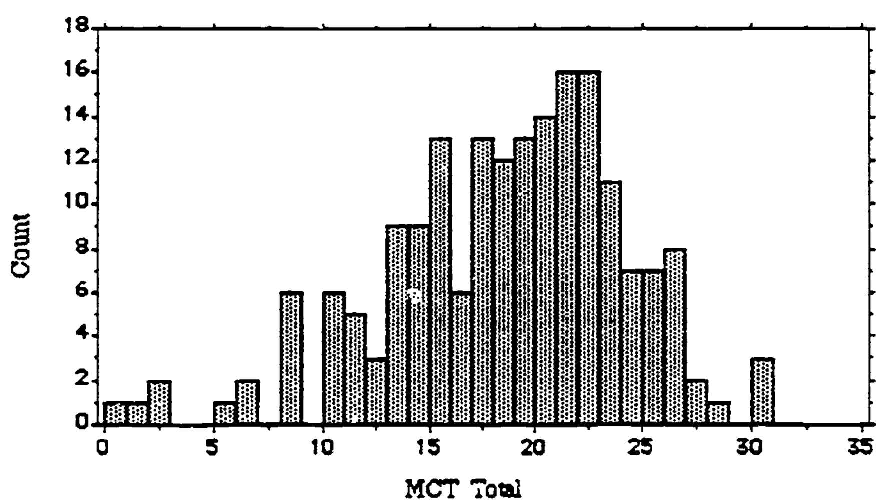


Figure 10. Historgam of MCT Total for Sixth Grade

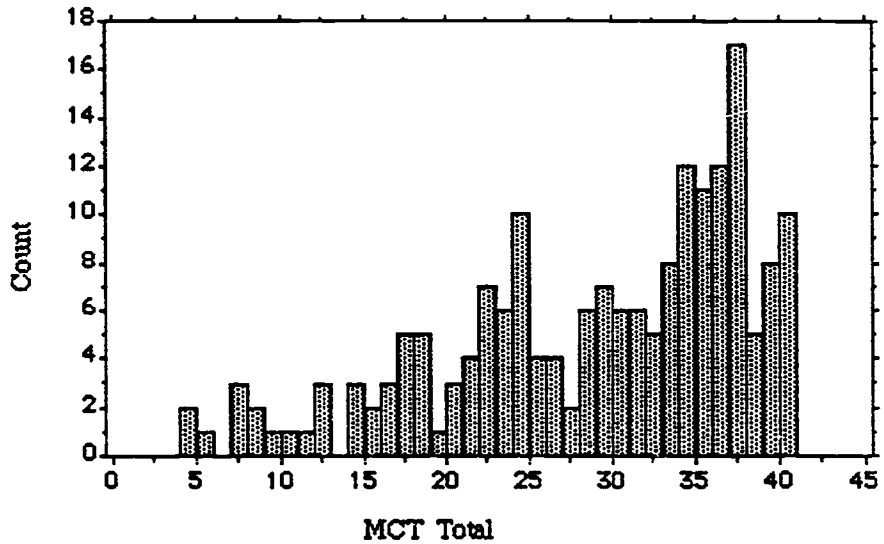
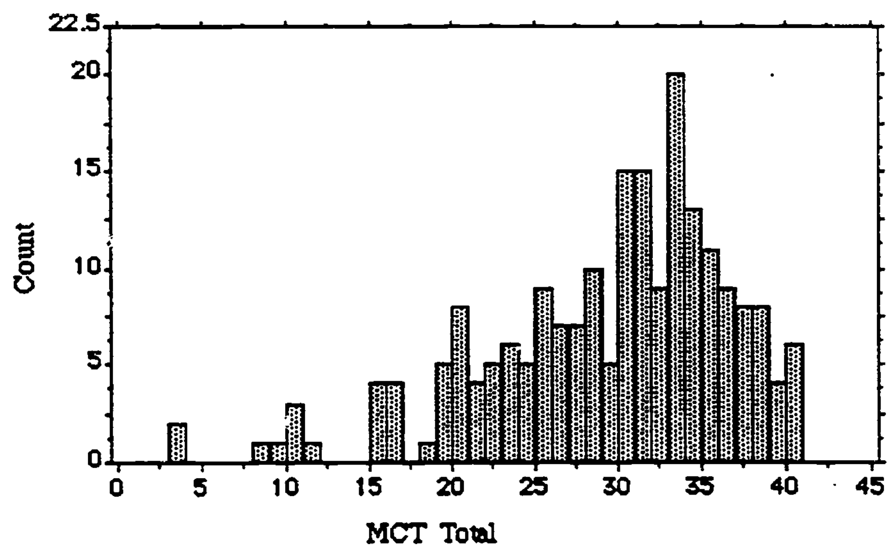


Figure 11. Histogram of MCT Total for Eighth Grade



The research design included six classes at each grade level. A summary of the MCT results by class is reported in Table 6 and reveals variation across classes in number of students (for example, ranging from 15 to 38 in second grade) as well as performance on the MCT at every grade level. For example, at sixth grade the range of scores for a particular class were as small as 25 and as large as 36. A review of Table 6 reveals similar ranges of extreme scores at each grade level, which produced means which had a minimum range of 2.46 (18.96 - 16.50) at grade four to a maximum of 6.79 (32.53 - 25.74) at sixth grade.

Table 6.
Summary of MCT Total Scores by Class

Grade	Class	N	Mean	SD	Range
2	1	31	22.26	5.00	11 - 30
	2	33	20.42	8.37	0 - 29
	3	38	17.87	7.73	1 - 30
	4	38	18.74	7.53	1 - 30
	5	21	17.71	6.18	6 - 30
	6	15	19.73	8.97	7 - 30
	Total	176	19.45	7.42	0 - 30
4	1	22	16.50	6.43	1 - 27
	2	32	18.19	4.30	11 - 28
	3	33	17.94	6.40	2 - 30
	4	39	18.03	5.99	2 - 30
	5	39	18.62	5.29	0 - 26
	6	22	18.96	5.88	8 - 30
	Total	187	18.09	5.67	0 - 30
6	1	31	25.74	9.64	5 - 39
	2	28	28.00	10.14	4 - 39
	3	31	27.71	9.41	8 - 40
	4	32	27.47	9.60	4 - 40
	5	32	28.75	8.27	12 - 40
	6	32	32.53	6.83	16 - 40
	Total	186	28.39	9.14	4 - 40
8	1	36	29.81	6.59	15 - 40
	2	35	27.60	8.58	3 - 40
	3	33	29.21	6.84	10 - 40
	4	32	28.03	8.45	3 - 39
	5	35	27.86	7.89	9 - 39
	6	35	30.54	6.03	11 - 40
	Total	206	28.85	7.44	3 - 40

In an effort to test if the classes were significantly different in their performance on the MCT, a one way ANOV was done at each grade level on the MCT Total Score. These results are reported in Table 7. The analysis confirms that the classes did not differ significantly ($p > 0.05$) from each other. Thus it is appropriate to discuss the MCT results at each grade level and not focus on minor variation across either classes or schools.

Table 7
ANOVA of MCT Total Scores by Class

Grade	Source	Df	SS	MS	F-test
2	Between	5	454.614	90.923	1.682
	Within	170	9188.926	54.053	p = .14
	Total	175	9643.540		
4	Between	5	84.041	16.808	0.516
	Within	181	5901.413	32.604	p = .76
	Total	186	5985.455		
6	Between	5	816.089	163.218	2.007
	Within	180	14638.260	81.324	p = .08
	Total	185	15454.349		
8	Between	5	248.137	49.627	0.895
	Within	200	11087.494	55.437	p = .49
	Total	205	11335.631		

The MCT consisted of two parts, one set of items administered visually and one set administered orally. In order to examine the order effect of the different modes of presentation two different forms of the MCT were used at each grade level. In Form A the first half of the items were presented orally, the second half presented visually. For Form B the first half of the items were presented visually, the second half orally. Table 8 reports the means of the two forms of the MCT for each grade and shows that the MCT total scores on the two forms were not significantly different ($p > 0.05$).

Table 8
t-test of MCT Total Scores on Forms A and B

Grade	Form	N	Mean	SD	Df	t-ratio	P
2	A	84	19.89	7.29	174	0.64	0.53
	B	92	19.11	7.56			
4	A	94	17.64	6.20	185	-1.10	0.27
	B	93	18.55	5.07			
6	A	94	28.70	9.08	184	0.46	0.64
	B	92	28.08	9.24			
8	A	104	28.96	7.11	204	0.21	0.84
	B	102	28.74	7.79			

An examination of subscores on each half of the test revealed that for each form the mean of the visually presented items was higher than the mean for the orally presented items. For example, Table 9 shows at the second grade the mean on the oral items of Form A was 8.81, while the mean on the same items taken visually on Form B was 10.05. In order to investigate the mode of presentation effect on the MCT, a t-test was run between the first half of Form A and the first half of Form B (like items which were presented orally to the Form A classes and visually to the Form B classes) and between the second half of Form A and the second half of Form B (like items which were presented visually to Form A classes and orally to Form B classes). The t-test for grade two showed a significant difference ($p < 0.05$) between the two means (8.81 oral and 10.05

visual). The results of the complete analysis are reported in Table 9 and show that at every grade level, the mean for the visually presented items was significantly higher (at least beyond the 0.05 level of significance) than the same items taken orally.

Table 9.
t-tests for halves of MCT by form (comparing effect of presentation mode)

Grade	Half	Form	Mode	N	Mean	SD	Df	t-ratio
2	1st	A	O	84	8.81	4.15	174	-2.014
	1st	B	V	92	10.05	4.05		p = .046
	2nd	A	V	84	11.01	3.50	174	3.534
	2nd	B	O	92	9.07	3.79		p = .0005
4	1st	A	O	94	7.90	3.68	185	-4.571
	1st	B	V	93	10.07	2.70		p = .0001
	2nd	A	V	94	9.73	2.95	185	2.942
	2nd	B	O	93	8.48	2.87		p = .0037
6	1st	A	O	94	13.04	4.93	184	-3.03
	1st	B	V	92	15.16	4.61		p = .0028
	2nd	A	V	94	15.66	4.65	184	4.09
	2nd	B	O	92	12.80	5.06		p = .0001
8	1st	A	O	104	13.70	4.09	204	-2.69
	1st	B	V	102	15.24	4.08		p = .004
	2nd	A	V	104	15.26	3.47	204	3.3230
	2nd	B	O	102	13.51	4.11		p = .0005

These results provide strong evidence that mode of presentation has a significant effect on mental computation performance levels achieved by Japanese students in elementary and junior high school. Thus any meaningful report and discussion of student performance on individual mental computation items must also describe the mode of presentation.

Do Japanese boys and girls differ in their ability to do mental computation? An examination of the means on the MCT total score showed that the mean of the boys were higher than the mean of the girls at each grade. Table 10 reports a summary of t-tests by sex and shows that only at the second grade were the means significantly different beyond the 0.05 level.

Table 10
t-test of MCT Total Scores for Males and Females

Grade	Sex	N	Mean	SD	Df	t-ratio
2	Males	91	20.65	7.51	174	2.243
	Females	85	18.17	7.15		p = .03
4	Males	98	18.85	5.77	185	1.93
	Females	89	17.26	5.48		p = .06
6	Males	99	28.80	9.96	184	0.64
	Females	87	27.94	8.13		p = .52
8	Males	103	29.79	7.57	204	1.81
	Females	103	27.92	7.22		p = .07

Tables 11 - 14 summarize the mental computation performance by item and mode of presentation for all grades for each of the four operations. Performance levels on individual items confirm that for all but a few items, performance was higher when the item was presented visually. Although for a few items in each grade mode of presentation did not influence performance (e.g., see p-values for 60 + 80 in grades 2 and 4, and 0.1 x 45 in grades 6 and 8) the mode of presentation more often produced dramatically different results (e.g., 165 + 99 was answered correctly by 45 percent of the fourth graders when presented orally and 88 percent when presented visually). Format differences on the same items were found to be consistent across grade levels. For example, 105 - 26 resulted in higher performance across all four grades when given visually. More specifically, this item p~oduced differences of 26, 31, 8 and 8 percent across grades 2, 4, 6, and 8, respectively.

Table 11.
Performance of Second, Fourth, Sixth and Eighth Grade Japanese Students on Mental Computation Addition Items Administered Orally and Visually

Item	Grade 2		Grade 4		Grade 6		Grade 8	
	O	V	O	V	O	V	O	V
6+8	91	95						
16+9	84	94						
36+9	80	86						
20+70	99	92						
36+20	81	91						
60+80	73	77	95	97				
68+32	60	78	83	98				
25+27	67	82						
79+26	27	70	67	86	78	90	90	93
25+99	38	48						
58 + 34	41	80	83	95	92	97		
182+97	14	60	46	88	51	91		
165+99			45	88	65	74	70	85
1/2+1/4			5	3	73	70	88	93
1/2+3/4			3	4	69	84	77	89
2 1/2+3 1/2					51	66	61	80
2 1/2+3 3/4					32	76	38	73
0.5+0.75			58	64	69	75		
6.2+4.9			48	72	64	85	81	90

Table 12.
Performance of Second, Fourth, Sixth and Eighth Grade Japanese Students on Mental Computation Subtraction Items Administered Orally and Visually

Item	Grade 2		Grade 4		Grade 6		Grade 8	
	O	V	O	V	O	V	O	V
14 - 6	82	88						
36 - 9	56	69						
36 - 10	82	85						
90 - 70	90	92						
74 - 30	61	79	84	82				
73 - 23	64	79						
140 - 60	66	75	95	93				
80 - 24	51	52	77	89	83	95		
100 - 25	62	67						
100 - 68	48	54	77	92	90	94		
105 - 26	24	50	54	85	70	78	79	87
105 - 97	34	58	66	79				
264 - 99					36	71	55	75
1 - 1/3			60	54	80	86	83	92
3/4 - 1/2			4	3	76	79		
4 1/2 - 3					69	71		
6 - 4 1/2					45	54	65	74
3 - 5/6							65	76
5 1/4 - 2 3/4							28	68
6 - 4.5					76	77	90	95
4.5 - 3					80	82	94	95

Table 13.
Performance of Second, Fourth, Sixth and Eighth Grade Japanese Students on Mental Computation Multiplication Items Administered Orally and Visually

Item	Grade 2		Grade 4		Grade 6		Grade 8	
	O	V	O	V	O	V	O	V
Double 8	67	62						
Double 50	63	60						
Double 16	46	47						
Double 26	39	39	69	73	92	94		
60 x 70			49	35	86	86	91	93
100 x 35			62	66				
300 x 40			41	58	86	89		
7 x 25			60	75	65	89	78	94
38 x 50			28	65	47	81	71	81
7 x 49					45	71	61	78
1/10 of 45					78	77	85	84
4 x 3 1/2					19	41	44	51
2/3 of 90							74	84
1/2 x 6 1/2							30	37
0.5 x 48					45	63		
0.1 x 45					79	79	92	89
1.5 x 20							77	74
50% of 48					51	63	64	71
100% of 48					55	57	55	66
25% of 48					42	50	53	59
10% of 45					58	59	78	67
75% of 48							34	30

Table 14.
Performance of Second, Fourth, Sixth and Eighth Grade Japanese Students on Mental Computation Division Items Administered Orally and Visually

Item	Grade 2		Grade 4		Grade 6		Grade 8	
	O	V	O	V	O	V	O	V
Half of 16	50	48						
Half of 30	52	54						
Half of 52			65	65				
300 ÷ 5			78	81				
3500 ÷ 35			51	72	80	94	82	96
4200 ÷ 60			33	43	60	75	84	89
450 ÷ 15			29	39	66	76	88	89
150 ÷ 25			25	38	48	75	68	81
440 ÷ 8					77	87	81	91
12,000 ÷ 40					57	80	77	86
90 ÷ 1/2							46	35
6 1/2 ÷ 2							28	34
3.5 ÷ 0.5							83	75
90 ÷ 0.5							57	67

Table 15 highlights the range of differences on individual items by mode of presentation. It summarizes all of the MCT items according to the magnitude of difference on performance produced by the mode of presentation. Specifically, a tally of every item by operation and magnitude of mode of presentation difference was compiled at each grade. For example, the entry "1-S" in grade 6 under 21^+ means that one subtraction item (namely 264 - 99, see Table 12) in sixth grade had differences in performance by mode of presentation exceeding 20 percent and the visual presentation resulted in the higher performance. At the fourth grade, the entry 1-M under 10 - 20 % on the right side of Table 15 means one multiplication item (namely 60 x 70, see Table 13) had differences in performance between 10 - 20% and the oral presentation resulted in higher performance. Table 15 documents that when differences of 10 percent or greater do exist, the visual mode of presentation typically produced higher performance. More specifically, while there were 65 items where the p-value was over 10 percent higher for the visual mode, there were only 3 items where the p-value was over 10 percent higher for the oral mode (and none greater than 20 percent higher).

Table 15.
MCT Items by Operation According to Magnitude of Difference of Visual and Oral Presentation

Grade	Number of items where p-value was higher for visual mode: 21⁺%	Number of items where p-value was higher for visual mode: 10 - 20%	Number of items where visual and oral performance differed by less than 10 percent.	Number of items where p-value was higher for oral mode: 10 - 20%	Number of items where p-value was higher for oral mode: 21⁺%
2	3-A 2-S	5-A 3-S	4-A 7-S 4-M 2-D		
4	3-A 1-S 1-M	3-A 3-S 2-M 4-D	4-A 4-S 2-M 2-D	1-M	
6	3-A 1-S 4-M 2-D	3-A 1-S 2-M 4-D	4-A 8-S 8-M		
8	1-A 1-S	3-A 2-S 5-M 4-D	3-A 5-S 9-M 5-D	1-M 1-D	

Summary. The amount of data resulting from about 800 students on the MCT makes perfect patterns impossible. There are exceptions and "statistical outliers" for almost any conclusion that might be reported. Nevertheless some trends were strong and provide a bases for the following observations.

Across all grades:

1. A wide range of performance on mental computation was found in every grade level, dispelling any notion that all Japanese students are uniform in their ability to mentally compute.

2. The mode of presentation significantly effects performance levels on individual items, and when there are significant differences the visual mode of presentation typically produces the highest performance.

3. As expected, the performance across grade levels on common items increased as the years of schooling increased. Whereas significant levels of improvement typically appeared from grade two to four and from grade four to six, the improvement from grades 6 to 8 differed much less.

4. The mental computation test produced a sharp difference between performance on fractions and decimals. At all grade levels performance on decimals (regardless of mode of presentation) was higher than on fractions.

5. The level of conceptual understanding is not revealed directly from the test results but some items suggest direction for additional exploration. For example, about 2/3 of the fourth graders correctly mentally computed "half of 52" which was far greater than the portion of sixth graders who successfully mentally computed "0.5 x 48." The lower performance on the decimal item suggests the need to learn how the students conceptualize these computations and determine what connections between fractions and decimals are used. The higher performance on the visual form of 0.5 x 48 suggests that students probably approached the computation algorithmically, probably applying a written algorithm mentally.

Percent suggests yet another area for exploration of conceptual understanding. For example, the performance of sixth and eighth graders on the item "100% of 48" was generally lower than on the item "50% of 48." Perhaps students viewed the first item as too simple. Yet an examination of the incorrect answers provided for this response (4800 and 0.48 being the two most common errors) provides evidence of fundamental conceptual errors involving percent.

Grade 2

1. Second graders demonstrated a high level (80% or higher correct) of mastery of addition and subtraction basic facts (such as 6 + 8) and extended basic facts (90 - 70).

2. Second graders demonstrated a slightly lower (60% - 80%) mastery on addition and subtraction of two digit numbers without regrouping.

3. As expected, second graders demonstrated lowest mastery on two digit numbers involving regrouping.

4. Second graders produced higher performance on addition items than parallel subtraction items. For example, the performance on 36 + 9 was about 30 percent higher than for 36-9.

5. Second grade performance level on computations involving near compatible numbers (25 + 99) was often below fifty percent.

6. About half the second graders were able to compute the double and half of two digit numbers, some of which required regrouping.

Grade 4

1. Fourth graders demonstrated a very high level of mastery (90% or higher correct) of extended basic facts in addition(60 + 80), but less than half were successful with extended multiplication basic facts (60 x 70) and extended division basic facts (4200 ÷ 60).

2. Fourth graders computed compatible numbers (68 + 32) and near compatible numbers (105 - 97) at a much higher performance level than second graders.

3. Fourth graders found computation with common fractions (1/2 + 1/4) very difficult (5% or less correct), although slightly over half correctly answered 1 - 1/3.

4. Fourth graders were more successful mentally computing decimals than the equivalent fraction items. For example, whereas over half correctly computed 0.5 + 0.75, less than 5 percent correctly computed 1/2 + 3/4.

5. Fourth graders performed about equally well on families involving the same numbers. For example, performance on 79 + 26 was about the same as on 105 - 26. Likewise for multiplication and division, performance on 60 x 70 was similar to 4200 ÷ 60.

Grade 6

1. Sixth graders demonstrated a high level of mastery (70% or higher correct) on whole number addition subtraction, multiplication and division.

2. Sixth graders performed higher on multiplication than division on families involving the same numbers. For example, 12000 ÷ 40 was more difficult than 300 x 40.

3. Sixth graders showed increased performance on addition and subtraction of fractions over their fourth grade counterparts. For example, whereas less than 5 percent of the fourth graders answered "1/2 + 3/4" correctly, about 70 percent of the sixth graders gave a correct response. Nevertheless, on only one item (1 - 1/3) did both visual and oral performance exceed 80 percent correct.

4. Sixth graders performed consistently around the 50 percent correct level on all items involving percents. In fact, about the same percent correctly reported 100% of 48 as did 50% of 48.

5. Sixth graders performance on percent (50% of 48) and equivalent decimal computations (0.5 x 48) were very similar. Likewise was the performance on multiplication of decimals (0.1 x 45) and fractions (1/10 of 45).

Grade 8

1. Eighth graders demonstrated a very high level of mastery (80% or higher correct) on whole number addition subtraction, multiplication and division.

2. Eighth graders showed marked improvement from sixth graders on addition and subtraction with fractions and decimals, and demonstrated a high level of mastery (70% or higher) on these operations. Performance with decimals was generally higher than with similar types of computation involving fractions.

3. Eighth graders showed higher performance with multiplication of decimals than with multiplication of fractions.

4. Eighth graders demonstrated improved performance on percentages from sixth grade but performance with percents (10% of 45) was below equivalent computations with decimals (0.1 x 45). More than one third were unable to correctly solve 100% of 48.

Interview Results

Upon completion of all interviews, the script tapes were reviewed item by item by the research team. Using the categorization of anticipated strategies (see Appendix F), each strategy was coded. The student responses were categorized and the resulting data were compiled for each interview. Once all coding was complete it was revealed in what quintile (middle or high) each student scored on the MCT. The resulting interview data are summarized in Tables 16-30. For example, on item A (79 + 26) administered to grade 4 students, Table 16 shows that 4 students (2 high quintile and 2 middle quintile) used a group by tens and ones strategy, 1 student from the high quintile group held one addend constant (79 + 20 ... + 6), 2 students (1 high quintile, 1 middle quintile) mentally visualized the paper/pencil algorithm, 2 students (1 high quintile, 1 middle quintile) visualized the soroban, and 1 middle quintile student failed to explain the strategy used. Although the sample size was limited to 20 students and the number of items included in the interview was small, the interviews provide insight into the strategies and techniques Japanese students use in doing mental computation and their willingness and ability to describe alternative strategies.

Students at both fourth and eighth grade were generally successful in mentally computing the interview items, with students from the upper quintile on the MCT having the greatest success in each grade, and students in grade 8 doing better than their counterparts in grade 4 on common items. Scores (number of initial correct responses) of the fourth grade sample ranged from 3 to 6 (out of a possible 8) for the middle quintile students and from 5 to 8 for the high quintile students. For grade 8 scores ranged from 5 to 8 (out of a possible 10) for the middle quintile students and from 7 to 10 for the high quintile students. Table 31 summarizes the number of initial correct responses by interview item.

Table 16.
Initial strategy used on mental computation interview item A : 79 + 26

Grade 4, item 1

Strategy	Frequency
A. Group by Ten and Ones	
A1. L-R (Tens first): (70+20=90; 9+6=15; 90+15=105)	H*
A2. R-L (Ones first): (9+6=15; 70+20=90; 15+90=105)	H
A3. Cumulating Sum (70+20...+9...+6 or 70+20...+10...+5)	MM
B. Hold one addend constant	
B1. First addend: (79+20=99; 99+6=105)	H
B2. Second addend: (26+70=96; 96+9=105)	
C. Round one or both addends to multiple of ten then adjust	
C1. First addend: (80+26=106; 106-1=105)	
C2. Second addend: (79+30=109; 109-4=105)	
C3. Both addends: (80+30=110; 110-1-4=105)	
D. Round both addends to multiple of five then adjust (75+25=100; 100+4+1=105)	
E. Mental image of paper/pencil algorithm	HM
F. Mental image of soroban	HM
G. Can't explain	M

Note: "H" indicates one high quintile student, "M" indicates one middle quintile student.

Table 17.
Initial strategy used on mental computation interview item B: 165 + 99

Grade 4, item 2; Grade 8, item 1

Strategy	Frequency Gr. 4	Frequency Gr. 8
A. Group by Ten and Ones		
A1. L-R (Tens first): (60+90=150; 5+9=14; 100+150+14=264)	HMM	HH
A2. R-L (Ones first): (5+9=14; 60+90=150; 14+150+100=264)	H	
A3. Cumulating Sum (100+60 ... + 90 ... + 5 ... + 9)	M	
B. Hold one addend constant		
B1. First addend: (165+90=255; 255+9=264)	H	
B2. Second addend: (99+100=199; 199+60=259; 259+5=264)		
C. Round one or both addends to multiple of ten then adjust		
C1. First addend: (170+99) -5 or 200+99-35		
C2. Second addend: (165+100) -1		HHM
C3. Both addends: (170+100=270; 270-5-1=264)		
E. Mental image of paper/pencil algorithm	HMM	HMMMM
F. Mental image of soroban	H	
G. Can't explain		

Table 18.
Initial strategy used on mental computation interview item C: 105-97

Grade 4, item 3

Strategy	Frequency
A. Related Addition Problem, 97+?=105	
A1. Count up (by ones to 105)	
A2. Count up (by ones to 100 then add and five)	
A3. Known fact	
A4. Guess and Check	H
B. Round numbers then adjust	
B1. First (107 - 97=10; 10-2=8) OR (100 - 97=3; 3+5=8)	HMM
B2 Second (105-95=10; 10-2=8) OR (105-100=5; 5+3=8)	
B3. Both (100-90=10; 15-7=8)	M
E. Mental image of paper/pencil algorithm	HHHMM
F. Mental image of soroban	
G. Can't explain	

Table 19.
Initial strategy used on mental computation interview item D: 100-68

Grade 4, item 4

Strategy	Frequency
A. Tens and Ones	
A1. Partial (100-60=40; 40-8=32) OR (100-8=92; 92-60=32)	HHMMM
B. Related Addition Problem, 68+?=100	
B1. Count up (by ones)	
B2. Count up (by ones to 70 then tens to 100)	
B3. Known fact	
C. Round numbers then adjust	
C1. First --- not applicable---	
C2. Second (100-70=30; 30+2=32) OR (10-65=35; 35-3=32)	
E. Mental image of paper/pencil algorithm	HHHMM
F. Mental image of soroban	
G. Can't explain	

Table 20.
Initial strategy used on mental computation interview item E: 7x25

Grade 4, item 6

Strategy	Frequency
A. Partial Products (Distributivity)	
A1. Addition (7x20 + 7x5 = 175)	HM
A2. Subtraction (7x30 - 7x5= 175)	
B. Round factors to powers of ten, then adjust	
B1. First factor (10x25=250; 2=3x25=75; 250-75=175)	
B2. Second factor (7x100=700; 700+4=175)	
E. Mental image of paper/pencil algorithm	HHH MMMM
F. Mental image of soroban	H
G. Can't explain	

Table 21.
Initial strategy used on mental computation interview item F: 7x49

Grade 8, item 2

Strategy	Frequency
A. Partial Products (Distributivity)	
A1. Addition (7x40 + 7x9 = 343)	HM
A2. Subtraction (7x50 - 7x1= 343)	
E. Mental image of paper/pencil algorithm	HHHMMMM
F. Mental image of soroban	H
G. Can't explain	

Table 22.
Initial strategy used on mental computation interview item G: 38x50

Grade 4, item 5; Grade 8, item 3

Strategy	Frequency Gr. 4	Gr. 8
A. Round factors then adjust		
A1. First factor (40x50=2000; 2x50=100; 2000-100=1900)		
A2. Second factor (38x100=3800; 3800÷2=1900)		H
B. Partial Products		
B1. Distributivity (30x50)+(8x50) OR [(30x5)+(8x5)] x10	HM	H
E. Mental image of paper/pencil algorithm	HHH MMMM	HH MMMMM
F. Mental image of soroban	H	H
G. Can't explain		

Table 23.
Initial strategy used on mental computation interview item H: 1/2 + 3/4

Grade 4, item 7; Grade 8, item 4

Strategy	Frequency Gr. 4	Gr. 8
A. Decomposition		
A1. 1/2 + 1/2 + 1/4		
A2. 1/2 + 1 - 1/4		
B. Convert to decimals and compute		
B1. 0.5 +0.75 = 1.25 (1 1/4)		
C/E. Convert to common denominator and add (2/4 + 3/4)	HHH	HHHHH MMMMM
D. Misconception -- Add numerator and denominator (4/6)	HM	
F. Mental image of soroban		
G. Can't explain		
H. Can't do	HMMMM	

Table 24.
Initial strategy used on mental computation interview item I: 3 - 5/6

Grade 8, item 5

Strategy	Frequency
A. Related addition problem (5/6 + ? = 3)	
A1. Count up to 1 then to 3	
B. Round one factor then adjust	
B1. 2 5/6 - 5/6 = 2; 2 + 1/6 = 2 1/6	
B2. 3-1=2; 2 + 1/6 = 2 1/6)	H
C. Common denominator (3 = 18/6; 18/6 - 5/6 = 13/6)	HHHMMMM
D. Misconception	M
E. Mental image of paper/pencil algorithm	H
F. Mental image of soroban	
G. Can't explain	

Table 25.
Initial strategy used on mental computation interview item J: 4 x 3 1/2

Grade 8, item 6

Strategy	Frequency
A. Distributivity	
A1. Addition (4x3 + 4x1/2)	HH
A2. Subtraction (4x4 - 4x1/2= 175)	
B. Convert to decimals (4x3.5)	
D. Misconception (4/1 x 3 1/2, cancel 4 and 2, 2x3x1=6)	M
E. Mental image of paper/pencil algorithm (3 1/2 = 7/2; 4/1 x 7/2 = 14)	HHHMMMM
F. Mental image of soroban	

Table 26.
Initial strategy used on mental computation interview item K: 6 1/2 ÷ 2

Grade 8, item 7

Strategy	Frequency
A. Related multiplication problem (2 x ? = 6 1/2)	
A1. Guess and check	
A2. Take half of 6 1/2	
B. Convert to decimal form	
B1. 6.5 x 2	
C. Distributivity	
C1. 6÷2 + 1/2÷2	H
D. Misconception (inverted dividend rather than divisor in p/p algorithm)	MM
E. Mental image of paper/pencil algorithm (6 1/2 ÷ 2 = 13/2 x 1/2)	HHHHMMM
F. Mental image of soroban	
G. Can't explain	

Table 27.
Initial strategy used on mental computation interview item L: 0.5+0.75

Grade 4, item 8

Strategy	Frequency
A. Tenths and Hundredths (.5+.7 = 1.2; 1.2 + 0.05 = 1.25)	HHHMM
B. Decomposition	
B1. 0.5 + 0.5 + 0.25	
B2. 0.5 + 1 - 0.25	
C. Convert to fractions and compute	
C1. 1/2 + 3/4 = 1 1/4	
D. Misconception (ignore pv, add 5 and 75 . . . 0.8)	M
E. Mental image of paper/pencil algorithm	HMM
F. Mental image of soroban	H
G. Can't explain	

Table 28.
Initial strategy used on mental computation interview item M: 1.5 x 20

Grade 8, item 8

Strategy	Frequency
A. Partial Products (Distributivity)	
A1. Addition (1x20 + 1/2 x 20)	
A2. Subtraction (2x20 - 1/2 x 20)	
B. Convert to fraction (1 1/2 x 20 or 3/2 x 20)	
C. Shift decimal place (1.5x2x10) or 15x20=300 ... 30	HHHHMM
D. Equivalent form (1.5x 20 = 15 x 2)	HMM
E. Mental image of paper/pencil algorithm	M
F. Mental image of soroban algorithm	
G. Can't explain	

Table 29.
Initial strategy used on mental computation interview item N: 90 ÷ 0.5

Grade 8, item 9

Strategy	Frequency*
A. Related multiplication (dividing by 0.5 equivalent to multiplying by 2)	HH
B. Convert to fraction (90 ÷ 1/2)	H
C. Equivalent form (90÷0.5 => 900÷5)	MM
E. Mental image of paper/pencil algorithm	HHMM
F. Mental image of soroban algorithm	
G. Can't explain	

Note: one middle quintile student did not respond to this item.

Table 30.
Initial strategy used on mental computation interview item O: 75% of 48

Grade 8, item 10

Strategy	Frequency
B. Convert to fraction	
B1. 3/4 x 48	HHH
B2. 48 x 75/100; (48 x 75) + 100	H
B3. 75% = 3/4; (48 +4) x 3	H
C. Convert to decimal (48 x 0.75)	
E. Mental image of paper/pencil algorithm	
F. Mental image of soroban algorithm	
G. Can't explain	
H. Can't do	MMMMM

Table 31.
Frequency of Initial Correct Responses on MCT Interview Items

Item	Frequency of Correct Responses*			
	Grade 4		Grade 8	
	Middle	High	Middle	High
A. 79+26	5	5		
B. 165+99	3	3	4	5
C. 105-97	4	5		
D. 100-68	5	5		
E. 7x25	2	4		
F. 7x49			4	5
G. 38x50	1	3	5	5
H. 1/2 + 3/4	0	3	5	5
I. 3 - 5/6			4	4
J. 4 x 3 1/2			2	4
K. 6 1/2 + 2			1	4
L. 0.5 + 0.75	4	4		
M. 1.5 x 20			2	4
N. 90 + 0.5			4	3
O. 75% of 48			0	4

*Note: Five students in each category were interviewed. Therefore, an entry of "5" indicates all students in the category responded correctly to the item.

The most difficult interview items for the fourth graders were: 165 + 99 (missed by 2 high and 2 middle quintile students); 7 x 25 (missed by 1 high and 3 middle quintile students); 38 x 50 (missed by 2 high and 4 middle quintile students); and 1/2 + 3/4 (unattempted by 5 students, missed by 2 students). The most difficult items for the eighth graders were: 4 x 3 1/2 (missed by 3 middle and 1 high quintile student); 6 1/2 + 2 (missed by 4 middle and 1 high quintile student); 1.5 x 20 (missed by 3 middle and 1 high quintile students) and 75% of 48 (unattempted by 5 students, missed by 1 student).

As anticipated, the most difficult item at each grade level (1/2 + 3/4 for grade 4 and 75% of 48 for grade 8) was the item reviewers had acknowledged would be least familiar to the students. That is, at the time of the interview, students would not have been taught a procedure to compute the item, although the students had been introduced to the concepts involved in each problem (fractions in grade 4, percents at grade 8). The two items were not solved by any of the middle quintile students at either grade level. Five of the grade 4 students (4 middle, 1 high) said they couldn't do this type of problem. Two other grade 4 students (1 middle, 1 high) produced an answer of 4/6 by adding numerators and denominators. The three fourth grade students (all from the highest quintile) who successfully solved the problem all used the same strategy, namely converting to a common denominator then adding numerators.

On the parallel decimal item (0.5 + 0.75) the fourth graders had less difficulty. In fact, all attempted the item, and all but two of them were successful. It was also observed that none of the fourth graders verbalized any connections or relationships between these two parallel computational items, and none of the students talked about using the equivalent fraction/decimal form for solving either item. These students were more comfortable with mental computation of decimals than fractions. This reflects not only the lack of opportunity to handle these computations within the regular Japanese school curriculum but also within the Japanese culture, which provides very few opportunities to experience fractions compared to the United States. For example, use of the metric system means that measurement is generally recorded in decimal form (meters, centimeters, etc.) and even common consumer applications are measured by tens (e.g., eggs are sold in a carton of 10).

For the eighth graders, 5 students (all from the middle quintile) did not attempt the unfamiliar item (75% of 48), indicating that they didn't know how. One middle quintile student after saying he couldn't solve the problem went on to say, "I remember I have studied about percent. Even when I studied about percent, I couldn't understand that." The four students who did attempt to solve the problem (all from the high quintile) converted 75% to a fraction (3/4 or 75/100) then multiplied using a fraction computation strategy, one unsuccessfully.

A review of Tables 16 - 30 reveals a reliance on mental images of either a paper/pencil or soroban algorithm. For example, for item B (165 + 99) which was common to both grade levels, Table 17 indicates that 8 of the 20 students used a mental image of a learned paper/pencil algorithm. These learned strategies heavily influenced students thinking as evidenced by two eighth graders, one of whom said, "Because I learned the abacus, the answer came out in a flash." Another student said, "I added the numbers in my head just as I do written calculations." Only 3 students (all eighth graders) used a compensation strategy (add 100 to 165 then subtract 1). Efficiency did not seem to be an important factor to the students on item B who used the paper/pencil strategy as they took considerably longer (20-30 seconds as opposed to 3-8 seconds) to arrive at an answer than students using other strategies. On the other hand, students who used a mental image of a soroban were generally very quick.

Further evidence of the influence of paper/pencil algorithms is illustrated by item F: 7x49 (see Table 21). The numbers in this computation lend themselves to use of distributivity and it was anticipated that some students would use this mathematical property. Table 21 shows that only two of the ten eighth graders used the distributive property with the majority using a mental image of a paper/pencil algorithm. Since 9 of the students produced correct answers, the students were able to successfully apply their chosen strategy. Although there were some items, such as item F, where correct answers were produced regardless of strategy, accuracy was generally higher when strategies other than mental image of a paper/pencil algorithm were employed. For example, 2 of the 3 fourth graders and 1 of the eighth graders who used this strategy made an error. On the other hand, all three eighth graders who used a compensation strategy did so successfully. Across all items, 63 percent of the errors made by the fourth graders were the result of an error in applying the paper/pencil algorithm mentally. Likewise, 56 percent of the errors made by the eighth graders were the result of applying the paper/pencil algorithm.

Item G: 38 x 50 (see Table 22) provides further evidence of the tendency to use learned algorithms rather than to invent an efficient method for mentally computing. Fourteen of the 20 students (7 fourth graders, 7 eighth graders) utilized a paper/pencil strategy (mentally arranging factors in a horizontal format then multiplying from the right), although 7 of these students did temporarily drop the 0 in 50 then annex it later. As mentioned earlier, for the fourth graders this strategy lead to frequent errors.

The limited use of invented strategies and the lack of a variety of strategies across subjects is apparent in the responses of students when asked if they could describe a second (alternative) method for mentally computing each item. After describing the initial strategy used on an item the student was asked by the interviewer if they could think of another way to mentally compute the answer. For example, on item G (38 x 50) one eighth grader (from the high quintile) initially rounded 50 to 100, found the product of 38 and 100, then adjusted by dividing the result by 2. The alternative method described was to multiply 38 by 5 (using paper/pencil algorithm) then annex the 0. It is surprising that no student described a distributive strategy for either item G (38 x 50) or F(7 x 25) even as an alternative strategy. Students at both grade and quintile levels were generally reluctant to offer alternative strategies, frequently saying there wasn't one. In fact, in 62 out of 81 instances (77%) where the question was asked to the fourth graders, students responded in one of three ways: there is no other way, repeated same procedure, or said they didn't know of another way. For eighth graders these three responses occurred in 71 of the 94 instances (76%) where the question was posed. Perhaps the effective use of an initial strategy discourages students from considering other ways of computing. This conjecture is supported by an eighth grader from the top quintile. When asked for another way to compute 38x50 the response was, "Because I am good at using an abacus, I don't think about another way very much."

When asked if they could describe an alternative method for mentally computing item B (165+99), four of the fourth graders and six of the eighth graders were unable (or unwilling) to describe an alternative strategy. In fact, one middle ability eighth grader, after initially utilizing the paper/pencil strategy to produce an incorrect solution said, "There is no other way." Those students who did produce an alternative method most often either restated their original method or described a paper/pencil strategy. A summary of alternative strategies described for the interview item pool can be found in Table 32. The students' general reluctance and/or inability to formulate alternative methods may have been affected by the interview situation and/or unfamiliarity with such a request, however it is an area worthy of further investigation.

Table 32.
Frequency of Alternative Strategies Described for Interview Items

Strategy	Grade 4		Grade 8	
	Middle	High	Middle	High
Group by Tens and Ones	1			
Hold One Addend Constant		1		
Round then Adjust				
Partial Products		1		1
Convert to Common Denominator		1		2
Distributivity				1
Shift Decimal Place				2
Equivalent Form				2
Related Multiplication			1	
Convert to fraction/decimal				3
Mental Image of Paper/Pencil Algorithm	7	7	2	9
Mental Image of Soroban	1	2		
Repeat Same Strategy	6	6	2	1
"There is none" or "I don't know another way"	25	25	42	26

In summary, the students interviewed tended to utilize learned strategies for mentally computing, particularly a mental image of a paper/pencil algorithm. This tendency was more pronounced with the middle quintile students than the high quintile students. While more errors were made when utilizing paper/pencil algorithms than with other non-standard methods, students often caught these errors when describing the process. The students interviewed used a very narrow range of strategies as evidenced by the list of initial strategies used and the reluctance to offer an alternative strategy. This was particularly apparent as items specifically designed to offer multiple approaches (e.g. 38 x 50, 7 x 49, or 165 + 99) were treated with standard algorithms at the same level as other items in the interview pool. Students from the upper quintile group were more likely to use non-standard approaches than their middle quintile counterparts and were more successful in whatever approach they chose.

Summary

This research was designed to establish a data base among Japanese students on mental computation. Several different instruments were developed and used to provide a collect the data. More specifically, data were collected to document student attitude toward mental computation, preference for doing mental computation, and performance on mental computation items. Teacher attitudes toward mental computation were also examined.

An Attitude Survey, Preference Survey and Mental Computation Test were developed for each grade. These instruments were group-administered to a sample of 24 classes (6 at each grade level) of students randomly selected from schools in one "typical" Japanese city. A sample of nearly 800 Japanese students in grades 2, 4, 6, and 8 from four different schools participated.

In addition to group information on mental computation, it was deemed essential to learn how Japanese students "do" mental computation. That is, what mental models are employed and what strategies and techniques are used when mental computation is performed. Thus, 20 students were selected from the grade 4 and grade 8 classes to participate in an individual interview where they were encouraged to describe their thinking as they solved a variety of mental computation items. This interview data provide the basis for identifying and characterizing the strategies used by these Japanese students when doing computation mentally.

The major findings of this study include:

1. The Japanese students sampled thought both mental computation and written computation were important. Most students thought they would do more mental computation than written computation as an adult. However, these students reported that they spend more time in school learning written computation than mental computation. In fact, less than half of the students reported learning mental computation at school.

2. Teacher attitudes and belief toward mental computation were varied. The range of responses at each grade level suggests considerable diversity among the Japanese teachers regarding their perception and treatment of mental computation within their

mathematics classes.

3. Students across all grades were consistent (typically two-thirds or more) in selecting computations preferred to do mentally. For example, over 80 percent of students in grades 6 and 8 chose mental computation as the preferred mode for computing 945 x 1000. In contrast, when a similar question was posed to American students as part of the National Assessment of Educational Progress, about one-third chose mental computation as their preferred mode of computation for this item. Furthermore, the more skilled Japanese students were in mental computation the more they tended to choose mental computation as their preferred method of computation.

4. A wide range of performance on mental computation was found with respect to all types of numbers (whole numbers, decimals and fractions) and all operations in every grade level. There were also wide ranges of performance within every class at every grade level. For example, scores ranged from 0 to 30 (with a 30 maximum) in grade 2, and from 3 to 40 (with a 40 maximum) in grade 8. These results dispel any notion that all Japanese students are uniform in their ability to mentally compute.

5. The mode of presentation (visual or oral) significantly effects performance levels on individual items with items being presented visually generally producing higher performance levels. The mode of presentation effect phenomenon was observed at each grade level.

6. As expected, the performance across grade levels on common items increased as the years of schooling increased. However, the improvement on common items across grade levels was greatest from grades 2 to 4 and 4 to 6, with the least change found from grades 6 to 8. This performance pattern reflects the curricular emphasis on computation which is greatest in elementary school, and given little direct attention in junior high school.

7. The range of strategies for mentally computing described in the interviews was very narrow. For students in grades 4 and 8, the most popular mental computation strategy described reflected a learned "paper/pencil" strategy. For example, when computing 165 + 99, a majority of the students at each grade either grouped by tens and ones (16 tens plus 9 tens is 25 tens; 5 ones and 9 ones is 14; 250 plus 14 is 264), or

described a mental image of a paper/pencil algorithm to "mentally" compute this sum. Only three (out of 20) students used a strategy that seems particularly effective when mentally computing, i. e., rounding 99 to 100, adding 100 + 65, then subtracting 1 to adjust for initially rounding. The use of non-standard (not taught) strategies was rarely observed. Students from the upper quintile (based on the mental computation test) were more likely to use non-standard approaches than their middle quintile counterparts and were more successful in whatever approach they chose.

8). Few students in the interview were able to express alternative strategies for mentally computing an item. Once students described their initial choice of a mental computation strategy and utilized it to produce an answer, they were asked if they could think of another way of doing the computation mentally. This search for alternate strategies was generally unproductive. Most students were unable to describe any other approach to the problem and seemed surprised that they would be asked for another method.

In addition to the above findings, this research suggests the need for additional exploration along several directions. For example:

Does the wide range of performance levels on mental computation at each grade suggest similar ranges of performance on other mathematical topics? The focus on class effort and group success suggests that certain thresholds of performance would be obtained by all Japanese students, yet consistency within grades was not reflected by the wide range of performance on mental computation.

Do the strategies identified in this research accurately reflect the range of strategies possessed by Japanese students? An examination of reports on Japanese education describing the emphasis on student contributions and sharing of strategies (Stevenson & Stigler, 1992) led us to believe that students would have developed a wide range of appropriate strategies for mental computation. Perhaps different items or a different interview approach would have been more productive in stimulating different strategies and approaches among the Japanese students.

Does this examination of mental computation reveal any conceptual misunderstanding of fundamental concepts? Percent and the connection between fractions and decimals provide two examples for consideration. For example, the low percent of correct answers in grades 6 and 8 to the question "What is 100% of 48?" suggests the need for additional research to learn if this was an anomaly or is representative of the Japanese students' knowledge of percent. The lack of connection between fractions and decimals is reflected in their performance on mental computation involving fractions and decimals. For example, less than 5 percent of the fourth graders answered 1/2 + 3/4 correctly, while about 60 percent answered 0.5 + .75 correctly. The lack of connection was further evident in the interviews where both problems were given to fourth and eighth grade students, and the students were asked to describe their strategy. None of these students made any comments to suggest any conceptual links, connections or similarities between the fraction and decimal computations.

Do Japanese teachers value and emphasize mental computation? The data from students, over half of which said they learned mental computation on their own, and teachers, many of whom reported different beliefs and practices toward mental computation, suggests that the attention to mental computation varies greatly. The number of classroom teachers (22) involved in this research was too limited to make any generalizations. However, given the strict guidelines for curriculum provided by the Japanese Ministry of Education, it is surprising that so much variability would exist from either the student or teacher perspective.

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Name: ______________________ Grade: 4 School: ______________________
(last) (first)

Computation is often involved in solving real-world problems. When solving problems, several computational methods exist:

- Sometimes people use a calculator.
- Sometimes people use paper and pencil.
- Sometimes people compute mentally without writing anything down.

We want to learn which problems you prefer to do mentally. Please look at each problem below and decide if you prefer to do it mentally. Circle YES or NO to indicate your response. It is **not** necessary for you to work the problems.

	Problem	**I would do this problem mentally.**	
1.	500 + 300	Yes	No
2.	Double 26	Yes	No
3.	58 + 34	Yes	No
4.	60 + 80	Yes	No
5.	74 - 30	Yes	No
6.	80 - 24	Yes	No
7.	60 x 70	Yes	No
8.	14 x 83	Yes	No
9.	100 x 35	Yes	No
10.	1 - 1/3	Yes	No
11.	165 + 99	Yes	No
12.	7 x 25	Yes	No

Name: ______________________ Grade: 4 School: ______________________
(last) (first)

Computation is often involved in solving real-world problems. When solving problems, several computational methods exist:

- Sometimes people use a calculator.
- Sometimes people use paper and pencil.
- Sometimes people do computation mentally without writing anything down.

We want to learn what calculations you feel students should do mentally. **At the end of fourth grade, do you think students should do these calculations mentally?** Circle YES or NO to indicate your response.

Item	Response	
1. 500 + 300	Yes	No
2. Double 26	Yes	No
3. 58 + 34	Yes	No
4. 60 + 80	Yes	No
5. 70 - 30	Yes	No
6. 80 - 24	Yes	No
7. 60 x 70	Yes	No
8. 14 x 83	Yes	No
9. 100 x 35	Yes	No
10. 1 - 1/3	Yes	No
11. 165 + 99	Yes	No
12. 7 x 25	Yes	No
13. 47 + 54 + 23	Yes	No
14. 123 x 12	Yes	No
15. 648 + 286	Yes	No
16. 1/2 + 3/4	Yes	No
17. 0.5 + 0.75	Yes	No
18. 945 x 1000	Yes	No

Please describe your thinking in making the above choices. Use the reverse side of this page if necessary.

小学校　4年　組　番　氏名

毎日の生活の中で、私たちはよく計算をします。そのとき、次のような方法で計算します。

・私たちは電卓を使って計算します。

・私たちは筆算で計算します。

・私たちは何も書かないで頭の中で計算します。

ここでは、あなたがどの問題を頭の中で計算するかを知りたいと思います。下の問題を頭の中で計算しますか、しませんか。はい、いいえのあてはまる方に○をつけてください。計算はしなくてよいです。

問　題	この問題を頭の中で計算しますか。	
1.　500＋300	はい	いいえ
2.　26の2倍	はい	いいえ
3.　58＋34	はい	いいえ
4.　60＋80	はい	いいえ
5.　74－30	はい	いいえ
6.　80－24	はい	いいえ
7.　60×70	はい	いいえ
8.　14×83	はい	いいえ
9.　100×35	はい	いいえ
10.　$1-\frac{1}{3}$	はい	いいえ
11.　165＋99	はい	いいえ
12.　7×25	はい	いいえ

小学校　4年　　組担当　氏名

（姓）　　（名）

日常生活で、私たちは問題を解くとき、よく計算をします。問題を解くときに、次のような方法で計算します。

・私たちは電卓を使って計算します。

・私たちは筆算で計算します。

・私たちは何も書かないで暗算で計算します。

この調査では、児童がどの問題を暗算で計算すると考えるかについて知りたいと思います。４年生の終わりに、あなたは児童が次の問題を暗算で計算すると考えますか。それぞれの問題に対する該当すると思われるものを○印で囲んでください。

問　題		この問題を暗算で計算しますか。	
1.	５００＋３００	はい	いいえ
2.	２６の２倍	はい	いいえ
3.	５８＋３４	はい	いいえ
4.	６０＋８０	はい	いいえ
5.	７０－３０	はい	いいえ
6.	８０－２４	はい	いいえ
7.	６０×７０	はい	いいえ
8.	１４×８３	はい	いいえ
9.	１００×３５	はい	いいえ
10.	１－１／３	はい	いいえ
11.	１６５＋９９	はい	いいえ
12.	７×２５	はい	いいえ
13.	４７＋５４＋２３	はい	いいえ
14.	１２３×１２	はい	いいえ
15.	６４８＋２８６	はい	いいえ
16.	１／２＋３／４	はい	いいえ
17.	０.５＋０.７５	はい	いいえ
18.	９４５×１０００	はい	いいえ

あなたが上の問題について、その答えを選んだ理由について、あなたの考えを書いてください。必要があれば、裏を使ってもかまいません。

Name: ______________________ Grade: ___ Class: ___ School: ________________________
(last) (first)

Here are some statements about written and mental computation. Mark the space which best describes your feeling about each statement.

	Yes	No	Not Sure
1. I enjoy doing written computation.	|___|	|___|	|___|
2. I think written computation is interesting.	|___|	|___|	|___|
3. I think mental computation is more challenging than written computation.	|___|	|___|	|___|
4. I am better at written than mental computation.	|___|	|___|	|___|
5. At school I do written computation more than mental computation.	|___|	|___|	|___|
6. I do mental computation more than written computation away from school.	|___|	|___|	|___|
7. I learned to do mental computation by myself.	|___|	|___|	|___|
8. It is more important to be good at mental than written computation.	|___|	|___|	|___|
9. I think I will do written computation more than mental computation as an adult.	|___|	|___|	|___|
10. I learned to do written computation by myself.	|___|	|___|	|___|
11. I do written computation more than mental computation away from school.	|___|	|___|	|___|
12. I am good at written computation.	|___|	|___|	|___|
13. I think written computation is more challenging than mental computation.	|___|	|___|	|___|
14. Written computation is more interesting than mental computation.	|___|	|___|	|___|
15. I enjoy doing mental computation.	|___|	|___|	|___|
16. Mental computation is challening to me.	|___|	|___|	|___|
17. I am good at mental computation.	|___|	|___|	|___|
18. At school I do mental computation more than written computation.	|___|	|___|	|___|
19. It is important to be good at written computation.	|___|	|___|	|___|
20. I learned to do mental computation at school.	|___|	|___|	|___|
21. Written computation is challenging to me.	|___|	|___|	|___|
22. I am better at mental than written computation.	|___|	|___|	|___|
23. I think mental computation is interesting.	|___|	|___|	|___|
24. I think I will do mental computation more than written computation as an adult.	|___|	|___|	|___|
25. It is important to be good at mental computation.	|___|	|___|	|___|
26. It is more important to be good at written then mental computation.	|___|	|___|	|___|
27. Mental computation is more interesting than written computation.	|___|	|___|	|___|
28. I learned to do written computation at school.	|___|	|___|	|___|

中学校　2年　組　番　氏名

筆算や暗算に関する文章があります。それぞれの項目について、あなたの気持ちにあっているものを○印で囲んでください。

質問項目			
1．筆算をすることは、楽しい。	はい	いいえ	どちらでもない
2．筆算はおもしろい。	はい	いいえ	どちらでもない
3．暗算は難しいけれど、筆算よりやりがいがあると思う。	はい	いいえ	どちらでもない
4．暗算より筆算の方が得意だ。	はい	いいえ	どちらでもない
5．学校では、暗算より筆算で計算することの方が多い。	はい	いいえ	どちらでもない
6．学校から帰った後では、筆算より暗算の方をよく使う。	はい	いいえ	どちらでもない
7．自分ひとりで、暗算の練習をしたことがある。	はい	いいえ	どちらでもない
8．筆算ができるより暗算ができる方が大切だ。	はい	いいえ	どちらでもない
9．大人になれば、暗算より筆算で計算することの方が多くなるだろう。	はい	いいえ	どちらでもない
10．自分自身で、筆算の勉強をしている。	はい	いいえ	どちらでもない
11．学校から帰った後では、暗算より筆算の方をよく使う。	はい	いいえ	どちらでもない
12．筆算が得意だ。	はい	いいえ	どちらでもない
13．筆算は、暗算よりやりがいがあると思う。	はい	いいえ	どちらでもない
14．筆算は、暗算よりもおもしろいと思う。	はい	いいえ	どちらでもない
15．暗算をすることは楽しい。	はい	いいえ	どちらでもない
16．暗算をすることは、自分の力が試されているような気がする。	はい	いいえ	どちらでもない
17．暗算が得意だ。	はい	いいえ	どちらでもない
18．学校では、筆算より暗算で計算することの方が多い。	はい	いいえ	どちらでもない
19．筆算がじょうずにできることは大切だ。	はい	いいえ	どちらでもない
20．学校で暗算を習ったことがある。	はい	いいえ	どちらでもない
21．筆算をすることによって、自分の力が伸びるような気がする。	はい	いいえ	どちらでもない
22．筆算より暗算の方が得意だ。	はい	いいえ	どちらでもない
23．暗算はおもしろい。	はい	いいえ	どちらでもない
24．大人になれば、筆算より暗算で計算することの方が多くなるだろう。	はい	いいえ	どちらでもない
25．暗算がじょうずにできることは大切だ。	はい	いいえ	どちらでもない
26．暗算ができるより筆算ができる方が大切だ。	はい	いいえ	どちらでもない
27．暗算は、筆算よりもおもしろいと思う。	はい	いいえ	どちらでもない
28．学校で筆算を習ったことがある。	はい	いいえ	どちらでもない

Name: ______________________ Grade: ___ Class: ___ School: __________________
(last) (first)

Please respond to the following statements by placing a check in the appropriate column.

Statement:	Yes	No	Not Sure
1. I think facility with written computation is important.	|___|	|___|	|___|
2. I think facility with mental computation is important.	|___|	|___|	|___|
3. I think it is important that students learn some written computation before learning mental computation.	|___|	|___|	|___|
4. I think it is important that students learn to do some computations mentally before learning written computation.	|___|	|___|	|___|
5. I think my children are good at mental computation.	|___|	|___|	|___|
6. I think my children are good at paper/pencil computation.	|___|	|___|	|___|
7. I think my children enjoy doing written computation.	|___|	|___|	|___|
8. I think my children enjoy doing mental computation.	|___|	|___|	|___|
9. I monitor student progress in mental computation.	|___|	|___|	|___|
10. I monitor student progress on written computation.	|___|	|___|	|___|
11. I encourage students to do easy computations mentally.	|___|	|___|	|___|
12. I encourage students to do easy computations with paper and pencil.	|___|	|___|	|___|
13. I encourage students to do difficult computations with paper and pencil.	|___|	|___|	|___|
14. I encourage students to do difficult computations mentally.	|___|	|___|	|___|
15. I teach mental computation.	|___|	|___|	|___|
16. I teach written computation.	|___|	|___|	|___|
17. I teach mental computation prior to teaching written computation.	|___|	|___|	|___|
18. I teach written computation prior to teaching mental computation.	|___|	|___|	|___|
19. Children in my class practice mental computation.	|___|	|___|	|___|
20. Children in my class practice written computation.	|___|	|___|	|___|
21. I try to help students learn ways to compute mentally.	|___|	|___|	|___|
22. I try to help students learn ways to compute with paper and pencil.	|___|	|___|	|___|
23. I test mental computation.	|___|	|___|	|___|
24. I test written computation.	|___|	|___|	|___|

________学校　___年　__組担当　氏名__________

計算に関する次の質問項目について、適切であると思われるものを○印で囲んで下さい。

質　問　項　目			
1．筆算ができることは大切だと思いますか。	はい	いいえ	どちらでもない
2．暗算ができることは大切だと思いますか。	はい	いいえ	どちらでもない
3．暗算を学習する前に、筆算を学んでおくことは大切なことだと思いますか。	はい	いいえ	どちらでもない
4．筆算を学習する前に、暗算を学んでおくことは大切なことだと思いますか。	はい	いいえ	どちらでもない
5．あなたが教えている多くの子供たちは、暗算が得意ですか。	はい	いいえ	どちらでもない
6．あなたが教えている多くの子供たちは、筆算が得意ですか。	はい	いいえ	どちらでもない
7．子供たちは、筆算することを楽しんでいると思いますか。	はい	いいえ	どちらでもない
8．子供たちは、暗算することを楽しんでいると思いますか。	はい	いいえ	どちらでもない
9．あなたは、子供たちの暗算能力が向上しているかどうかを授業でチェックしていますか。	はい	いいえ	どちらでもない
10．あなたは、子供たちの筆算能力が向上しているかどうかを授業でチェックしていますか。	はい	いいえ	どちらでもない
11．簡単な計算は暗算でするように指導していますか。	はい	いいえ	どちらでもない
12．簡単な計算でも、筆算で計算するように指導していますか。	はい	いいえ	どちらでもない
13．難しい計算は、筆算でするように指導していますか。	はい	いいえ	どちらでもない
14．難しい計算でも暗算で行うように指導していますか。	はい	いいえ	どちらでもない
15．暗算を教えていますか。	はい	いいえ	どちらでもない
16．筆算を教えていますか。	はい	いいえ	どちらでもない
17．筆算を教える前に、暗算を教えていますか。	はい	いいえ	どちらでもない
18．暗算を教える前に、筆算を教えていますか。	はい	いいえ	どちらでもない
19．授業で、子供たちは暗算の練習をしますか。	はい	いいえ	どちらでもない
20．授業で、子供たちは筆算の練習をしますか。	はい	いいえ	どちらでもない
21．あなたは、子供たちが暗算で計算するのを手助けしていますか。	はい	いいえ	どちらでもない
22．あなたは、子供たちが筆算で計算するのを手助けしていますか。	はい	いいえ	どちらでもない
23．あなたは、暗算の試験を行っていますか。	はい	いいえ	どちらでもない
24．あなたは、筆算の試験を行っていますか。	はい	いいえ	どちらでもない

MENTAL COMPUTATION TEST
GRADE 2

1. 36 + 9
2. 20 + 70
3. 36 + 20
4. 68 + 32
5. 25 + 27
6. 25 + 99
7. 36 - 9
8. 36 - 10
9. 73 - 23
10. 80 - 24
11. 100 - 68
12. 105 - 26
13. Double 15
14. Double 26
15. Take half of 30

16. 6 + 8
17. 16 + 9
18. 60 + 80
19. 79 + 26
20. 58 + 34
21. 182 + 97
22. 14 - 6
23. 90 - 70
24. 74 - 30
25. 140 - 60
26. 100 - 25
27. 105 - 97
28. Double 8
29. Double 50
30. Take half of 16

MENTAL COMPUTATION TEST
GRADE 4

1. 58 + 34
2. 68 + 32
3. 165 + 99
4. 80 - 24
5. 100 - 68
6. 105 - 26
7. Double 26
8. 300 x 40
9. 7 x 25
10. Take half of 52
11. 3500 + 35
12. 450 + 15
13. 1/2 + 1/4
14. 3/4 - 1/2
15. 6.2 + 4.9

16. 60 + 80
17. 79 + 26
18. 182 + 97
19. 74 - 30
20. 140 - 60
21. 105 - 97
22. 60 x 70
23. 100 x 35
24. 38 x 50
25. 300 + 5
26. 4200 + 60
27. 150 + 25
28. 1/2 + 3/4
29. 1 - 1/3
30. 0.5 + 0.75

MENTAL COMPUTATION TEST
GRADE 6

1. 58 + 34
2. 165 + 99
3. 100 - 68
4. 105 - 26
5. Double 26
6. 300 x 40
7. 7 x 25
8. 3500 + 35
9. 450 + 15
10. 12,000 + 40
11. 1/2 + 1/4
12. 2 1/2 + 3 1/2
13. 3/4 - 1/2
14. 6 - 4 1/2
15. 4 x 3 1/2
16. 6.2 + 4.9
17. 6 - 4.5
18. 0.5 x 48
19. What is 50% of 48?
20. What is 25% of 48

21. 79 + 26
22. 182 + 97
23. 80 - 24
24. 264 - 99
25. 60 x 70
26. 7 x 49
27. 38 x 50
28. 150 + 25
29. 4200 + 60
30. 440 + 8
31. 1/2 + 3/4
32. 2 1/2 + 3 3/4
33. 1 - 1/3
34. 4 1/2 - 3
35. What is 1/10 of 45?
36. 0.5 + 0.75
37. 4.5 - 3
38. 0.1 x 45
39. What is 100% of 48?
40. What is 10% of 45?

MENTAL COMPUTATION TEST
GRADE 8

1. 165 + 99
2. 105 - 26
3. 7 x 25
4. 7 x 49
5. 3500 + 35
6. 450 + 15
7. 12,000 + 40
8. 1/2 + 1/4
9. 2 1/2 + 3 1/2
10. 3 - 5/6
11. 6 - 4 1/2
12. What is 2/3 of 90?
13. 4 x 3 1/2
14. 90 + 1/2
15. 6 - 4.5
16. 1.5 x 20
17. 90 + 0.5
18. What is 100% of 48?
19. What is 50% of 48?
20. What is 25% of 48?

21. 79 + 26
22. 264 - 99
23. 60 x 70
24. 38 x 50
25. 150 + 25
26. 4200 + 60
27. 440 + 8
28. 1/2 + 3/4
29. 2 1/2 + 3 3/4
30. 1 - 1/3
31. 5 1/4 - 2 3/4
32. What is 1/10 of 45?
33. 1/2 x 6 1/2
34. 6 1/2 + 2
35. 6.2 + 4.9
36. 4.5 - 3
37. 0.1 x 45
38. 3.5 + 0.5
39. What is 10% of 45?
40. What is 75% of 48?

Mental Computation Interview Items and Protocols

Grade 4

1. 79 + 26
2. 165 + 99
3. 105 - 97
4. 100 - 68
5. 38 x 50
6. 7 x 25
7. 1/2 + 3/4
8. 0.5 + 0.75

Grade 8

1. 165 + 99*
2. 7 x 49
3. 38 x 50*
4. 1/2 + 3/4*
5. 3 - 5/6
6. 4 x 3 1/2
7. 6 1/2 ÷ 2
8. 1.5 x 20
9. 90 ÷ 0.5
10. 75% of 48

* Denotes items on gr. 4 interview.

General guidelines for mental computation interview:

1. Present each item orally, one at a time. Repeat the item if necessary.
2. Make it clear that you are interested in both the answer and how the answer was generated.
3. Ask the student to compute the answer mentally and tell how they got their answer. They could think aloud as they go along or tell what they did after they produced an answer.
4. Give the student as much time as they need to compute an answer and explain their process.
5. Do not give feeback regarding the correctness or incorrectness of the response or the strategy described.
6. After an answer and explanation have been given ask the student, "Can you think of another way to do this problem mentally?"
7. Discourage students from writing or recording anything.
8. Use probes to encourage elaboration of thought process. These may include, but are not limited to:
   How did you say you did that?
   How would you explain to a friend what you did?
   Tell me again exactly what you did.
   As a last resort to clarifying the process, interviewer might say: "Let me see if I understand this..." (the interviewer repeats what they think the student said).

Mental Computation Interview
Categorization of Anticipated Strategies

Item A: 79 + 26

Strategies:

A. Group by Ten and Ones
  A1. L-R (Tens first): (70+20=90; 9+6=15; 90+15=105)
  A2. R-L (Ones first): (9+6=15; 70+20=90; 15+90=105)
  A3. Cumulating Sum (70+20 ...+9 ...+6 or 70+20 ...+10 ...+5))
B. Hold one addend constant
  B1. First addend: (79+20=99; 99+6=105)
  B2. Second addend: (26+70=96; 96+9=105)
C. Round one or both addends to multiple of ten then adjust
  C1. First addend: (80+26=106; 106-1=105)
  C2. Second addend: (79+30=109; 109-4=105)
  C3. Both addends: (80+30=110; 110-1-4=105)
D. Round both addends to multiple of five then adjust
  (75+25=100; 100+4+1=105)
E. Mental image of paper/pencil algorithm
F. Mental image of soroban
G. Can't explain

Item B: 165 + 99

Strategies:

A. Group by Ten and Ones
  A1. L-R (Tens first): (60+90=150; 5+9=14; 100+150+14=264)
  A2. R-L (Ones first): (5+9=14; 60+90=150; 14+150+100=264)
  A3. Cumulating Sum (100+60 ... + 90 ... + 5 ... + 9)
B. Hold one addend constant
  B1. First addend: (165+90=255; 255+9=264)
  B2. Second addend: (99+100=199; 199+60=259; 259+5=264)
C. Round one or both addends to multiple of ten then adjust
  C1. First addend: (170+99=269; 269-5=264)
  C2. Second addend: (165+100=265; 265-1=264)
  C3. Both addends: (170+100=270; 270-5-1=264)
D. Compensate
  D1. 165 + 99 = 165 + 100 - 1
  D2. 165 + 99 = 200 + (99 - 35)
E. Mental image of paper/pencil algorithm
F. Mental image of soroban
G. Can't explain

Item C: 105-97

Strategies:

A. Related Addition Problem, 97+?=105
  - A1. Count up (by ones to 105)
  - A2. Count up (by ones to 100 then add and five)
  - A3. Known fact
  - A4. Guess and Check

B. Round numbers then adjust
  - B1. First (107 - 97=10; 10-2=8) OR (100 - 97=3; 3+5=8)
  - B2 Second (105-95=10; 10-2=8) OR (105-100=5; 5+3=8)
  - B3. Both (100-90=10; 15-7=8)

E. Mental image of paper/pencil algorithm

F. Mental image of soroban

G. Can't explain

Item D: 100-68

Strategies:

A. Tens and Ones
  - A1. Partial (100-60=40; 40-8=32) OR (100-8=92; 92-60=32)

B. Related Addition Problem, 68+?=100
  - B1. Count up (by ones)
  - B2. Count up (by ones to 70 then tens to 100)
  - B3. Known fact

C. Round numbers then adjust
  - C1. First --- not applicable---
  - C2. Second (100-70=30; 30+2=32) OR (10-65=35; 35-3=32)

E. Mental image of paper/pencil algorithm

F. Mental image of soroban

G. Can't explain

Item E: 7x25

Strategies:

A. Partial Products (Distributivity)
  - A1. Addition (7x20 + 7x5 = 175)
  - A2. Subtraction (7x30 - 7x5= 175)

B. Round factors to powers of ten, then adjust
  - B1. First factor (10x25=250; 2=3x25=75; 250-75=175)
  - B2. Second factor (7x100=700; 700+4=175)

E. Mental image of paper/pencil algorithm

F. Mental image of soroban

G. Can't explain

Grade 8, item 2

Item F: 7x49

Strategies:

A. Partial Products (Distributivity)
    A1. Addition (7x40 + 7x9 = 343)
    A2. Subtraction (7x50 - 7x1= 343)
E. Mental image of paper/pencil algorithm
F. Mental image of soroban
G. Can't explain

Item G: 38x50

Strategies:

A. Round factors then adjust
    A1. First factor (40x50=2000; 2x50=100; 2000-100=1900)
    A2. Second factor (38x100=3800; 3800÷2=1900)
B. Partial Products
    B1. Distributivity (30x50) + (8x50) OR [(30x5) + (8x5)] x 10
E. Mental image of paper/pencil algorithm
F. Mental image of soroban
G. Can't explain

Item H: 1/2 + 3/4

Strategies:

A. Decomposition
    A1. 1/2 + 1/2 + 1/4
    A2. 1/2 + 1 - 1/4
B. Convert to decimals and compute
    B1. 0.5 +0.75 = 1.25 (1 1/4)
C/E. Convert to common denominator and add (2/4 + 3/4)
D. Misconception -- Add numerator and denominator (4/6)
F. Mental image of soroban
G. Can't explain

Item I: 3 - 5/6

Strategies:

A. Related addition problem (5/6 + ? = 3)
    A1. Count up to 1 then to 3
B. Round one factor then adjust
    B1. 2 5/6 - 5/6 = 2; 2 + 1/6 = 2 1/6
C. Common denominator (3 = 18/6; 18/6 - 5/6 = 13/6)
D. Misconception (---)
E. Mental image of paper/pencil algorithm
F. Mental image of soroban
G. Can't explain

Item J: 4 x 3 1/2

Strategies:

A. Distributivity
    A1. Addition (4x3 + 4x1/2)
    A2. Subtraction (4x4 - 4x1/2= 175)
B. Convert to decimals (4x3.5)
D. Misconception (4/1 x 3 1/2, cancel 4 and 2, 2x3x1=6)
E. Mental image of paper/pencil algorithm (3 1/2 = 7/2; 7/2 x 4 = 14)
F. Mental image of soroban

Item K: 6 1/2 ÷ 2

Strategies:

A. Related multiplication problem (2 x ? = 6 1/2)
    A1. Guess and check
    A2. Take half of 6 1/2
B. Convert to decimal form
    B1. 6.5 x 2
C. Distributivity
    C1. 6÷2 + 1/2÷2
D. Misconception (inverted dividend rather than divsor in p/p strategy)
E. Mental image of paper/pencil algorithm (6 1/2 ÷ 2 = 13/2 x 1/2)
F. Mental image of soroban
G. Can't explain

Item L: 0.5+0.75

Strategies:

A. Tens and ones
   Ai. .5+.7 = 1.2; 1.2 + 0.05 = 1.25
B. Decomposition
   B1. 0.5 + 0.5 + 0.25
   B2. 0.5 + 1 - 0.25
C. Convert to fractions and compute
   C1. 1/2 + 3/4 = 1 1/4
D. Misconception (ignore pv, add 5 and 75 . . . 0.8)
E. Mental image of paper/pencil algorithm
F. Mental image of soroban
G. Can't explain

Item M: 1.5 x 20

Strategies:

A. Partial Products (Distributivity)
   A1. Addition (1x20 + 1/2 x 20)
   A2. Subtraction (2x20 - 1/2 x 20)
B. Convert to fraction (1 1/2 x 20 or 3/2 x 20)
C. Shift decimal place (1.5x2x10) or 15x20=300 . . . 30
D. Equivalent form (1.5x 20 = 15 x 2)
E. Mental image of paper/pencil algorithm
F. Mental image of soroban algorithm
G. Can't explain

Item N: 90 ÷ 0.5

Strategies:

A. Related multiplication (dividing by 0.5 equivalent to multiplying by 2)
B. Convert to fraction (90 ÷ 1/2)
E. Mental image of paper/pencil algorithm
F. Mental image of soroban algorithm
G. Can't explain

Item O: 75% of 48

Strategies:

B. Convert to fraction
    B1. 3/4 x 48
    B2. 48 x 75/100
    B3. 75% = 3/4; (48 +4) x 3
C. Convert to decimal (48 x 0.75)
E. Mental image of paper/pencil algorithm
F. Mental image of soroban algorithm
G. Can't explain