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ABSTRACT

This study investigated the independent strategy development of 7- to 12-year-old children from the suburbs of Brisbane, Australia in solving a series of novel, two- and three-dimensional combinatorial problems. For each of the problem types, a sequence of five, increasingly complex, strategies was identified from the children's actions and explanations as they manipulated the problem materials. These strategies ranged from inefficient trial-and-error methods to efficient "odometer" procedures. Changes in the children's strategies as they progressed on the problems suggested modifications to their knowledge of the combinatorial domain. Children's knowledge growth was analyzed in terms of their acquisition of a number of domain-specific principles. Such principled knowledge is, in itself, insufficient for problem solution. Children's ability to monitor their actions, detect and correct errors, and recognize problem completion played a crucial role in goal attainment. With experience in problem solution, the older children, in particular, became more aware of the problems' underlying structure and how they could modify their inefficient strategies to produce more effective solution methods. Contains 51 references. (Author/MDH)

Development of children's strategic and metastrategic knowledge in a novel mathematical domain

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Abstract

This study investigated 7 to 12 year-olds' independent strategy development in solving a series of novel, two- and three-dimensional combinatorial problems. For each of the problem types a sequence of five, increasingly complex, strategies was identified from the children's actions and explanations as they manipulated the problem materials. These strategies ranged from inefficient trial-and-error methods to efficient "odometer" procedures. Changes in the children's strategy use as they progressed on the problems suggested modifications to their knowledge of the combinatorial domain. Children's knowledge growth is analyzed in terms of their acquisition of a number of domain-specific principles. Such principled knowledge is, in itself, insufficient for problem solution. Children's ability to monitor their actions, detect and correct errors, and recognize problem completion played a crucial role in goal attainment. With experience in problem solution, the older children, in particular, became more aware of the problems' underlying structure and of how they could modify their inefficient strategies to produce more effective solution methods. This metastrategic knowledge is considered to play a major role in children's acquisition and stabilization of efficient strategies and their discarding of less effective methods (Kuhn et al., 1988). A more sophisticated metastrategic knowledge base may be largely responsible for the older children's superior performance on the more complex combinatorial problems.

Development of children's strategic and metastrategic knowledge in a novel mathematical domain

Introduction

With changes in our societal needs and the recent research on learning and cognition there has emerged a new vision of education (Resnick & Resnick, 1990; Jones, 1991). This new perspective emphasises the development of students' problem-solving competence, higher-order thinking skills, and self-regulated learning (Halpern, 1992; Jones & Idol, 1990). Recent curriculum documents, such as the Curriculum and Evaluation Standards for School Mathematics (National Council of Teachers of Mathematics, 1989), reflect this trend with their emphasis on empowering children as thinkers and problem solvers. The Standards calls for the development of children's mathematical power, namely, their ability to "explore, conjecture, and reason logically," as well as their ability to "use a variety of mathematical methods effectively to solve nonroutine problems" (NCTM, p.5).

Despite the increased focus on problem solving and reasoning, studies on children's competence in solving novel problems have not been prolific, particularly in the mathematical domain. The bulk of the research in this field has examined children's skills in solving routine arithmetic problems (e.g. Bisanz & Lefevre, 1990; Carpenter, Moser, & Bebout, 1988; Hamann & Ashcraft, 1985; Siegler & Jenkins, 1989). These studies have shed considerable light on children's strategy discovery and generalization in numerical operations where accuracy, speed, and retrieval have been of prime concern (e.g. Siegler & Jenkins, 1989). Considerably less attention has been devoted to the strategies children apply in solving mathematical problems that do not involve written or oral computations. Problems where children can create, test, and modify their own solution strategies, while at the same time acquire important mathematical principles, play a significant role in the development of children's mathematical power. Such problems are particularly worthy of investigation.

The focus of the present study is on children's independent strategy development in solving a series of novel, "hands-on" combinatorial problems set within a meaningful context (dressing toy bears). It was hypothesised that, with no prior instruction and receiving feedback only through their interaction with the physical materials, children would apply their informal knowledge of the problem domain in their initial attempts at solution. By exercising their existing strategies in repeated encounters with these problems, at least some of the children would be likely to modify their beginning strategies (Anzai & Simon, 1979; Kuhn & Phelps, 1982). Children's adoption of more advanced strategies and their abandonment of old, inefficient methods would be indicative of their learning in the combinatorial domain. Their understanding of the problems and the problem domain would be inferred from their move sequences as they manipulated the problem materials (Ericsson & Oliver, 1988). Children's explanations of their strategies would provide some insight into their development of metastrategic knowledge (Garner & Alexander, 1989; Larkin, 1984) and would also highlight the role of the self-direction factor in children's cognitive growth (Gelman & Brown, 1986; Kuhn & Ho, 1980). This article adresses the findings of this study and has three main aims:

1. To describe the strategies that 7 to 12 year-olds apply to the solution of simple (X x Y) and more complex (X x Y x Z) combinatorial problems where no prior instruction has been given.
2. To delineate the changes in these strategies as children progress from the simple to the more complex problems.
3. To trace the development of the underlying principled knowledge (Gelman & Greeno, 1989) and metastrategic knowledge (e.g. Kuhn & Phelps, 1982) reflected in the children's strategies.

Background

The nature of strategies

While there are diverse opinions on what constitutes a strategy (Bisanz & LeFevre, 1990), there is nevertheless some agreement on its key features. It is usually accepted that strategies are "goal-directed operations employed to facilitate task performance" (Harnishfeger & Bjorklund, 1990, p.1). They are frequently seen as domain specific (Pressley, Borkowski, & Schneider, 1987) and designed to facilitate both knowledge acquisition and utilization (Prawatt, 1989). Some view strategies as necessarily involving a choice of procedures (Siegler & Jenkins, 1989), with the procedure being invoked in a "flexible, goal-directed manner ... that influences the selection and implementation of subsequent procedures" (Bisanz & LeFevre, 1990, p.236). Procedures which create new procedures or alter old ones in flexible ways are also considered strategic (Bisanz & LeFevre, 1990). There are others who emphasize the "potentially conscious and controllable" nature of strategies (Bjorklund, Muir-Broaddus, & Schneider, 1990; Pressley et al., 1987), as well as the "dynamic interaction" of strategies, one's knowledge of the strategies, and one's monitoring of their implementation (Pressley, Forrest-Pressley, Elliott-Faust, & Miller, 1985). Within the framework of the present study, strategies are viewed as goal-directed procedures which facilitate both problem solution and acquisition of domain-specific knowledge. They are also seen as potentially conscious and controllable procedures.

Children's strategy development

Research in the last decade has presented convincing evidence that children behave strategically, are able to direct their own learning, and acquire a knowledge of the domain in which they are working (e.g. English, 1991; Gelman & Brown, 1986; DeLoache, Sugarman, & Brown, 1985; Karmiloff-Smith, 1984; Gelman & Greeno, 1989). For children to behave strategically in solving problems, they must firstly realize that their actions influence their progress towards a goal and then keep the goal in mind as they solve the problem. As children become more aware of the outcomes of their actions in problem-solving situations, they give more attention to the behaviors they use to achieve the goal. This results in enhanced awarenesss of the connection between their actions and the goal. As this awareness improves, children will be more likely to monitor their progress towards the goal, resulting in heightened consciousness of their actions and increased effectiveness of their strategy (Bjorklund & Harnishfeger, 1990).

The work of DeLoache et al. (1985), involving children's free play with a set of nested cups, indicated that children progress from trial-and-error behavior to a careful consideration of the relationships among elements of the problem as a whole. Other studies have revealed this general progression from immature to mature activities, where children create and modify solutions, detect and correct their errors, and develop more mature strategies on their own own (e.g. Gelman & Brown, 1986; Karmiloff-Smith, 1979; Burton, 1992). This progression reflects "a general learning mechanism" that characterizes cross-age descriptions of children's initial attacks on a problem (Gelman & Brown, 1986, p.188). This general learning mechanism becomes increasingly effective with age, due mainly to underlying changes in children's knowledge base, their processing efficiency, and their self-monitoring skills (e.g. Bjorklund & Harnishfeger, 1990; Brown & Kane, 1988).

Changes in children's knowledge base

The development of children's conceptual and metastrategic knowledge during the course of novel problem solving is of fundamental importance to mathematics education (e.g. English, 1992; Schoenfeld, 1992; Garofalo & Lester, 1985). One promising avenue for exploring changes in children's mathematical knowledge lies in the competence model of Gelman and her associates (Gelman & Meck, 1986; Gelman & Greeno, 1989). They argue that many cognitive strategies reflect underlying principled knowledge of the problem domain and that children's initial understanding of the domain is principled, albeit in a "limited and implicit way." (Gelman and Greeno, 1989, p.126). As children gain experience with the problem domain, they acquire a knowledge of the principles of the domain and furthermore, demonstrate a more explicit or stateable understanding of these. Early principled knowledge structures attention towards domain-relevant inputs and guides the learning of new principles. This view is consistent with the well established notion that prior knowledge in a domain determines what and how other information is encoded and learned (Resnick, 1986; Siegler & Jenkins, 1989).

A model of principled learning also provides a means of determining whether children have an implicit theory about a domain. The more children's knowledge can be characterized in terms of the principles of the particular domain, the more it can be said they have a "theory" (Gelman & Greeno, 1989, p.130). Children's competence in the domain is viewed in terms of their ability to generate competent plans of action that meet the constraints of the knowledge principles in that domain.

While domain-specific competence is necessary for successful problem solving in a domain, it is not considered sufficient. Domain-general competencies are also required for the production of a competent plan of action (English, 1992; Gelman & Greeno, 1989; Greeno & Riley, 1987). Because the planning component of the Gelman and Greeno model has to determine whether a chosen strategy meets the requisites dictated by the principles, it can serve as a potential source of feedback to children solving a novel problem. If the requisite conditions are not met, the plan or its execution can be rejected or terminated. This means that the child can start again, without being explicitly told to do so.

The fact that children do modify their strategies during the course of repeated encounters with a problem, in the absence of instruction or experimenter input, highlights the contribution of metastrategic knowledge (Kuhn & Phelps, 1982; Kuhn et al., 1988; Pressley et al., 1987). Children's capacity to monitor their actions, noting the relationship between their outputs and the problem goal, promotes the growth of this knowledge. Through problem experience, children acquire not only knowledge about the particular problem, but also knowledge about their own strategies as they apply to the problem. That is, they come to realize how a particular strategy works, why it works, and why it is the most appropriate for the problem. Included in this metastrategic knowledge is knowledge about less efficient strategies, why these do not work or why they are inappropriate for the problem, and the errors that can result from their use (Kuhn et al., 1988).

Choosing an appropriate domain

As previously mentioned, non-routine problems which encourage children to create their own solution strategies rather than apply a learnt rule or algorithm play a significant role in mathematics education. In the absence of an expert solution procedure, children must rely on their general problem-solving strategies and hence will be more likely to display some metastrategic knowledge in solving these problems.

While the problems should be challenging to children they should nevertheless be solvable and hence their parameters need to be unfamiliar but their contexts familiar (Sternberg, 1985). The mathematical topic of combinatorics, involving the selection and arrangement of objects in a finite set, readily meets these criteria. Furthermore, the domain comprises a rich structure of significant mathematical principles which underlie several areas of the mathematics curriculum, including counting, computation, and probability. In simple mathematical terms, combinatorics may be viewed as the operation of <u>cross product</u>. The cross product of two sets, X and Y, is the set of combinations obtained by systematically pairing each member of X in turn with each member of Y, as shown in Figure 1 (v). In more complex examples involving combinations of three elements, each member of set X must be systematically matched with each member of set Y <u>and</u> set Z, as shown in Figure 2 (v). This tree diagram represents the most efficient way of forming X x Y x Z combinations. However there are several other, less efficient, methods of generating these combinations, as discussed later. Because the combinatorial domain offers meaningful problems which allow for varying levels of solution, it is an eminently suitable topic of investigation in the mathematics curriculum.

The combinatorial domain is also of significance from a developmental perspective. It is a major component of Piaget's theory where it plays a significant role in cognitive development (Piaget, 1957; Flavell, 1963). The combinatorial system is evident in a subject's ability to "link a set of base associations or correspondences with each other in all possible ways so as to draw from them the relationships of implication, disjunction, exclusion etc." (Inhelder & Piaget, 1958, p.107). The important cognitive strategies here are isolation or control of variables, and systematic combination. The appearance of a systematic method of generating combinations is said to occur at the onset of the formal operations stage (Piaget & Inhelder, 1975).

Overview of study

The next section describes the problems administered in the study and examines the domain knowledge expected of both the novice and expert problem solver. This is followed by a description of each of the strategies identified from the children's actions. An analysis of the responses of individual children across the problems with an emphasis on their strategy development is then examined. Finally, a theoretical analysis of the domain-specific and metastrategic knowledge underlying the children's strategy development is undertaken.

The combinatorial problems

A series of six problems was designed for the study. The problems required children to dress toy bears in all possible combinations of colored tops and pants (first three problems) or colored tops, pants, and tennis rackets (remaining three problems).

The bears were made of thin wood and and were placed on a stand. Once the bears had been dressed they were arranged in a line so that the completed outfits could be clearly seen. The clothing items were made of colored card and were backed with adhesive material to facilitate the dressing process.

Problems 1 to 3 were two-dimensional problems (tops X pants) which had been used in previous studies (English, 1988; 1992) while problems 4 to 6 were more complex three-dimensional examples (tops X pants X tennis rackets) designed specifically for this study. Problems 1 and 2 involved 6 combinations, namely, 2 sets of tops X 3 sets of pants (problem 1) and 3 sets of tops X 2 sets of pants (problem 2). Problem 3 extended the number of combinations to 9 (3 sets of tops X 3 sets of pants). Problems 4 and 5 incorporated 8 combinations (2 sets of tops X 2 sets of pants X 2 sets of tennis rackets). The final problem was the most complex, involving 12 combinations (2 sets of tops X 3 sets of pants X 2 sets of tennis rackets).

The successful completion of these problems requires children to make moves that are directed towards the satisfaction of goals determined by problem-solving constraints (Glaser & Pellegrino, 1982). The minimum set of constraints that children must meet in solving these problems is as follows:

1. A constraint on the types of items to be combined. That is, items of the same type cannot be combined, such as two tops or two pants. A combination must comprise one top and one pair of pants (and a tennis racket).

2. A constraint on similarity across combinations. That is, given the ordered pairs of items (a, b) and (c, d) where a and c represent any tops and b and d any pants (in the case of the two-dimensional

problems), different combinations will result if any of the following is adhered to:

i. a is different in color from c, and b is different in color from d;
ii. a is the same color as c, and b is different in color from d;
iii. a is different in color from c, and b is the same color as d.

A particular case of these constraints warrants citing:

iv. a is the same color as b, and c is the same color as d, but a is not the same color as c.

This fourth constraint allows items <u>within</u> a combination to be the same color (e.g. red top/red pants) while items <u>across</u> combinations must be different (e.g. red top/red pants and blue top/blue pants).

An awareness of the above constraints would be sufficient for a trial-and-error approach to problem solution where items would be generated in a random fashion, as indicated in Figure 1 (i), then selected and combined according to the the above rules. Since there would be no evidence of forward planning in such behavior (Rogoff, Gauvain, & Gardner, 1987, the children's self-monitoring processes would be particularly important. On the other hand, the most efficient strategy for solving the problems would reflect a clear plan of action with a focus on the overall goal of generating all possible combinations (Rogoff et al., 1988). In contrast to the novice strategies where an item is not selected more than once in succession, the expert strategy involves the repeated selection of an item (referred to here as "holding an item constant") and systematically matching it with each of the other, "varying" items. These latter items are varied in a cyclic fashion, as shown in Figure 1 (v) (Y_1, Y_2, Y_3, Y_1, Y_2, Y_3....). Because this method resembles the working of an odometer in a car, it has been labelled the "odometer" strategy (English, 1988; Scardamalia, 1977). In the case of the two-dimensional problems (X x Y) there is only one item (X) to be held constant at any one time, as indicated in Figure 1 (v). For the three-dimensional problems however, there are two items (X and Y) which are held constant at any one time. The item which is changed least often (X), that is, the slowest moving dimension, is referred to here as the <u>major constant item</u>; the item that is changed more frequently, the faster moving dimension, is termed the <u>minor constant item</u> (refer Figure 2 v).

It is worthwhile noting that, for young children in particular, the repeated selection of an item seemingly goes against the grain of the problem goal of <u>different</u> combinations. A previous study (English, 1988) had shown that some children are initially reluctant to select an item more than once in succession, perhaps because they interpret "different" to mean "different in all ways" and thus see the goal of "all different outfits" as an indication to make each new outfit completely different from the previous outfit(s). It could be that children avoid repeating the selection of an item because they see it as going against the problem goal. Such behavior reflects the difference-reduction method of problem solving (Anderson, 1985, p. 206) where problem solvers attempt to make the current state as similar as possible to the desired goal state. However a correct solution frequently involves going against the grain of similarity (Anderson, 1985). In the case of the present problems, selecting the <u>same</u> item in succession is a key feature of the most efficient combinatorial strategies.

The design of the study problems, from simple to complex, facilitated observation of the children's strategy development as they tried to accommodate the more complex three-dimensional examples. To assess the effects of the initial two-dimensional problems on the children's mastery of the harder examples, a control group of 11 and 12 year-old children was used. This group was admininstered the final two, three-dimensional problems only. Eleven and twelve year-olds were chosen because it was the older children who were showing the more sophisticated strategy development across the two problem types.

Method

Subjects

Ninety-six children participated in the study, 24 serving as a control group. None of the children had been subjects of previous studies involving combinatorial problems and none had been taught combinatorics in school. The children were randomly selected from one large state school and three small non-state schools located in middle class suburbs of Brisbane, Australia. There were 12 children in each of six age groups: 7 years 0 months to 7 years 6 months, 8 years 0 months to 8 years 6 months....12 years 0 months to 12 years 6 months. There were approximately equal numbers of males and females in each age group. The control group comprised 12 eleven year-olds and 12 twelve year-olds.

Procedure

The children were administered the problems on an individual basis by a research assistant who was a qualified teacher. Each child's responses were videotaped for subsequent analyses. At the commencement of each problem the child was instructed to dress the bears so that each had a different outfit (in terms of tops and pants only, or tops, pants, and tennis rackets). To ensure the child understood the problem, a familiarization task was administered first. The goal here was simply to dress the bears. The task was designed to test children's color recognition, as well as to establish an understanding of the terms, "outfit,"and "same/different outfits." The latter term was crucial in the interpretation of the problem goal, especially when a common item was present. For example, the outfits, red top/blue pants, and red top/yellow pants are different from each other even though they have a common item. During this familiarization period the children were not given any information that could bias their performance on the problems. For each of the remaining problems, the children were provided with more materials (both bears and items) than were needed. This was to ensure the children did not use item depletion as a signal that they had solved the problem. The children were expected to complete each problem without assistance and were asked to explain their procedure at the end of each of the last three (three-dimensional) problems. Oral reports were not sought for the two-dimensional problems as these had been obtained in previous studies (English, 1988, 1992).

Coding of protocols

The data obtained were in the form of videotaped responses of the children manipulating the task materials (that is, how they selected and combined items) and their verbal explanations of how they did this.

Each child's protocol on each problem was converted into a tree diagram. Since tree diagrams provide an effective visual tool for the generation of combinations (DeGuire, 1991; Graham, 1991), they were considered a suitable means of representing the children's strategies. The clothing items were represented on the tree diagrams by the labels X_{1-3} and Y_{1-3} for the two-dimensional problems and X_{1-3}, Y_{1-3}, and Z_{1-3} for the three-dimensional examples (refer Figures 1 and 2). Items labeled X were changed least frequently and were designated as the major constant items. Items labeled Y were changed more frequently and were the minor constant items in the case of the three-dimensional items and the varying items in the case of the two-dimensional problems. Items labeled Z were the varying items in the three-dimensional problems and were changed most frequently. An expert strategy would involve changing items X and Y the least number of times possible. That is, item X_1 would be held constant and would only be changed (and replaced by X_2) when it could no longer generate new combinations (refer Figures 1v and 2v). A novice strategy on the other hand, would change these items frequently, normally not using each X item more than once in succession.

The number of times children changed the major (X) and minor (Y) constant items was the main criterion used in distinguishing the more efficient strategies. This criterion however, did not clearly distinguish among the less efficient strategies where children changed the X and Y items frequently. In these cases, additional criteria were used. For the two-dimensional strategies, it was the presence of a pattern in children's item selection (usually in the selection of one item type only). For the three-dimensional examples, it was the extent to which the Y items were exhausted, that is, matched with both Z_1 and Z_2. More specifically, the criterion here was the ratio of the number of Y items exhausted to the number of Y items not exhausted. The next section describes the key features of each of the strategies and examines children's progress across the problems.

Results

Children's solution strategies

A series of ten, increasingly sophisticated, strategies became apparent after the children's responses were categorized using the foregoing criteria. The first five strategies were used in the solution of the two-dimensional problems and are labeled 1 through 5 for ease of reference. These strategies are illustrated by a series of tree diagrams for the 3 X 3 problem in this instance (refer Figure 1). The remaining five strategies, labeled 6 through 10, were applied to the solution of the three-dimensional problems and are represented by tree diagrams for the 2 X 3 X 2 problem (refer Figure 2). The categories used to classify the children's responses into strategies were subjected to a reliability check. The responses of 20 children were assessed by two independent raters; this yielded a 90 per cent level of inter-rater agreement. The discrepencies were mainly confined to the less efficient strategies and were readily rectified.

INSERT FIGURE 1 ABOUT HERE

Two-dimensional strategies

Children's strategies on the two-dimensional problems ranged from a random selection of items through to a pattern in item selection and finally, the complete odometer strategy.

Strategy 1.

This strategy is characterized by an absence of a pattern in item selection, as indicated in Figure 1 (i). Items are selected in a random manner reflecting a trial-and-error approach to problem solution. Children's scanning actions play an important checking role in these early strategies, with the effectiveness of their scanning largely determining goal attainment (English, 1988, 1992).

Strategies 2 and 3.

These strategies are distinguished from the first by the presence of a pattern in item selection. The pattern is of an alternating or cyclic nature and is usually confined to one item type, as indicated in Figure 1 (ii) and (iii) (X_1, X_2, X_3, X_1, X_2, X_3.....). In strategy 2 however, the pattern is not continued throughout problem execution and is replaced by random item selection. Strategy 3 on the other hand, features a uniform pattern of item selection throughout problem execution.

Strategies 4 and 5.

The final two strategies are clearly evidenced by the presence of a constant item, as indicated in Figure 1 (iv) and (v). Strategy 5 represents the most sophisticated procedure for solving the two-dimensional problems. It features only two new selections of item X for the 2 X 3 problem and three new selections for the 3 X 3 problem, these being the minimum number needed for the respective problems. This means that each constant item is exhausted, that is, it is used repeatedly until it can no longer generate unique combinations. Strategy 4 however, does not exhaust the possibilities for each constant item, thus necessitating an additional new selection of item X. That is, strategy 4 involves 3 new selections of item X for the 2 X 3 problem and 4 for the 3 X 3 problem. This renders it a less efficient procedure than strategy 5.

Three-dimensional strategies

As for the two-dimensional problems, children's responses on the three-dimensional examples were converted to tree diagrams to enable the various strategies to be distinguished.. As previously mentioned, the criterion used in identifying the more efficient strategies was the number of times children changed the major (X) and minor (Y) constant items. For the less efficient strategies, it was the extent to which the Y items were exhausted, that is, matched with both Z_1 and Z_2. The most efficient strategies feature the minimum number of new selections of the X and Y items. The least efficient strategies display the greatest number of new selections of the X and Y items because they do not exhaust complete sets of these items. That is, the X items are not matched systematically with <u>all</u> the Y items which, in turn, are not matched with <u>all</u> the Z items (refer Figure 2).

INSERT FIGURE 2 ABOUT HERE

To further define the three-dimensional strategies, the mean numbers of new X and Y item selections for each strategy were calculated from the responses of children across all problems. The results appear in Table 1.

INSERT TABLE 1 ABOUT HERE

<u>Strategy 6.</u>

This strategy involves the greatest number of new selections of items X and Y (refer Table1) and is thus the most inefficient for solving the three-dimensional problems. As can be seen in Figure 2 i, this strategy displays more Y items that have not been exhausted than have been exhausted.

Strategy 7.

This strategy is more efficient than the previous one in that, as many as, or more, Y items are exhausted than are not exhausted. However not all of the Y items are exhausted, as shown in Figure 2 (ii). The strategy is still an inefficient one, as can be seen by the frequency with which new X and Y items are selected (refer Table 1).

Strategy 8.

Strategy 8 is identified by the exhaustion of each Y item, that is, each Y item is matched systematically with both Z_1 and Z_2 (refer Figure 2 iii). While each Y item is exhausted, the X items are not. That is, strategy 8 does not exhaust a complete set of X and Y items. Nevertheless, this strategy is more efficient than the previous two as it does not make as many new selections of X and Y items (refer Table 1).

Strategies 9 and 10.

The distinguishing feature of these final two strategies is the complete exhaustion of a set of X and Y items and hence a considerable reduction in the number of new item selections. While both strategies feature the same number of new selections of item Y. (4 and 6 such selections in the 2 X 2 X 2 and 2 X 3 X 2 problems respectively), they differ in the number of new X items chosen. Strategy 10 displays the minimum number (2) of new X item selections while strategy 9 involves an additional selection, as shown in Figure 2 (iv) and (v).

The next section addresses a selection of findings from an analysis of children's strategy use across the problems. Following this, three case studies are presented to illustrate some of the ways in which children improved the efficiency of their strategies as they progressed on the problems. These case studies serve as the basis for analyzing children's strategic and metastrategic knowledge growth in the combinatorial domain.

Children's performance across the problems

Table 2 displays the frequencies with which children in each age group used the less efficient strategies (1 - 3 and 6 - 8) and the more efficient strategies (4 - 5 and 9 - 10) across the problems.

INSERT TABLE 2 ABOUT HERE

While there was no significant relationship between age and type of strategy used on the two-dimensional problems, the responses of the 11 and 12 year-olds on problem 3 are worth noting. In contrast to the 7 to 10 year-olds, the 11 and 12 year-olds mainly employed the more efficient strategies in solving this final, two-dimensional example, $r = 3.87$, df = 1, $p < 0.05$. On the three-dimensional problems, children in the 7 to 9 year age group did not perform as efficiently as their older counterparts. This was particularly the case on the fourth and fifth problems, where there was a significant correlation between strategy type and age group (7 to 9 years, 10 to 12 years), $r = 5.71$, df = 1, $p < 0.05$ (problem 4), $r = 4.96$, df = 1, $p < 0.05$ (problem 5). Children in both age groups displayed more efficient strategy use as they progressed to problem 5, suggesting the presence of a practice effect. The efficiency of the 11 and 12 year-olds on these two problems is particularly noticeable. They demonstrated the most efficient strategy use of all the children, $r = 9.26$, df = 1, $p < 0.005$ (problem 4); $r = 7.84$, df = 1, $p < 0.01$ (problem 5). The final problem however, proved more difficult for both the younger and older age groups. Both groups showed a decline in performance with the result that there was no significant difference in strategy use between them. It would have been interesting to compare the children's performance had an additional, 2 X 3 X 2 problem been presented.

A record of children's strategy changes across selected problems appears in Table 3.

INSERT TABLE 3 ABOUT HERE

Strategy changes within each problem set (problems 1 to 3 and 4 to 6) as well as between problem sets (problems 3 to 4 and 3 to 6) are shown in the table. With respect to the latter, half of the older children (10 to 12 years) changed from a sophisticated two-dimensional strategy (4 or 5) on problem 3 to a comparable three-dimensional strategy (9 or 10) on problem 4, suggesting they were aware, at least implicitly, of the odometer principle underlying their strategy. This point is revisited later. Significantly fewer of the younger children (7 to 9 years) demonstrated this type of change, $X^2 = 16.67$, df = 3, $p < 0.001$. They were more likely to use an inefficient three-dimensional strategy before progressing to an efficient one. Several children in both age groups demonstrated a change from the less efficient to the more efficient three-dimensional strategies (6/7 -> 9/10) in solving problems 4 to 6, although the differences between the groups were not significant.

The numbers of children who achieved success with each strategy type (less efficient/more efficient) on problems 1 to 3 and 4 to 6 are shown in Table 4. Successful performance on a set of problems was defined as solving all three problems in the set.

Children were classified as using a particular strategy type if they applied such strategies on at least two of the three problems.

INSERT TABLE 4 ABOUT HERE

On problems 1 to 3, children demonstrated success with both strategy types. Children using the less efficient strategies were successful largely because of their thorough checking actions. (The nature of these actions is discussed in English, 1992).

Children were not as successful on the more complex, three-dimensional problems. A significant relationship between success and strategy type was recorded for these problems, $r = 26.89$, df = 1, $p < 0.001$. Children using the less efficient strategies demonstrated a high failure rate, while those using the more efficient strategies were highly successful. No significant difference in the success of the younger and older age groups existed when the inefficient strategies were used. However for the more efficient strategies, there was a significant difference in the success of the older and younger age groups, with the older children achieving greater success than their younger counterparts, $r = 5.44$, df = 1, $p < 0.05$.

The progress of the older children on problems 4 to 6 raises the question of whether experience in solving the two-dimensional examples contributed to their successful application of sophisticated three-dimensional strategies. To address this issue, the responses of the 11 and 12 year-olds on problems 5 and 6 were compared with those in the control group who received these final two problems only. The results appear in Table 5.

INSERT TABLE 5 ABOUT HERE

The data of Table 5 clearly show that the experimental group outperformed the control group on problem 5, with few children in the control group applying advanced strategies to solve this problem. The results of a chi-square test on the children's use of strategies 9 and 10 (versus strategies 6 to 8) on problem 5 showed a significant difference between the two groups, $X^2\ (1) = 13.58$, $p < 0.001$. It is interesting that no significant difference between the groups occurred on problem 6. The control group demonstrated a significant change from the less efficient to the more efficient strategies between problems 5 and 6, $X^2 = 4.17$, df = 1, $p < 0.05$. This suggests that experience in solving just one of the three-dimensional problems was sufficient for the children in the control group to adopt efficient procedures.

Patterns of strategy development

Of interest to the present discussion are particular patterns of strategy development displayed by the children as they progressed across the problems. Three such patterns are examined:

i. an inefficient 2-D strategy (strategy 1 or 2) -> an efficient 2-D strategy (strategy 4 or 5) -> inefficient 3-D strategy (6 or 7) -> efficient 3-D strategy (9 or 10)

ii. an inefficient 2-D strategy -> an efficient 2-D strategy -> efficient 3-D strategy

iii. an inefficient 3-D strategy on problem 5 -> an efficient 3-D strategy on problem 6 (control group only)

Case studies of children demonstrating these changes are presented in this section.

Samuel, aged 8 years 2 months, third year of school

Samuel's performance illustrates the first type of strategy change. He commenced the problems with strategy 2 where he followed a pattern in his selection of tops but lost the pattern towards the end of the problem when he duplicated an outfit. This can be seen in his selection of items, reproduced below:

blue top and orange pants

green top and pink pants

blue top and blue pants

green top and orange pants

blue top and pink pants

At this stage, the child said he could not do any more. Upon being asked if he was sure of this, the child proceeded to make:

blue top and pink pants

The interviewer then asked if all the bears were dressed differently. Sam replied, "No," removed the sixth bear, undressed it, and made:

green top and blue pants

Sam displayed a similar strategy in solving the second problem. This time, he followed a pattern in his selection of pants which he commenced part way into the problem. On the third problem however, he improved his strategy and nearly generated the complete odometer strategy. His use of strategy 4 is illustrated below:

blue pants and blue top

orange pants and orange top

blue pants and green top

blue pants and orange top

orange pants and green top

orange pants and blue top

pink pants and green top

pink pants and orange top

pink pants and blue top

As Sam checked his results, he said aloud, "Three blues, threee greens, and three oranges." He then commented, "I've done all the colors so I know I can't do any more."

Sam displayed an inefficient strategy (7) on the fourth problem, selecting the following order of items:

orange top, pink pants, and yellow tennis racket
orange top, yellow pants, and yellow tennis racket
pink pants, green top, and blue tennis racket
green top, yellow pants, and blue tennis racket
orange top, pink pants, and blue tennis racket
green top, yellow pants, and yellow tennis racket

At this point, the child stated, "I'll just check what I've done. He then proceeded to make:

green top, pink pants, and yellow tennis racket
orange top, yellow pants, and blue tennis racket

The interviewer asked the child what he was saying to himself as he was checking. He responded, "Nothing much this time; just saying yellow (tennis rackets) and two different pants, green (tops) and two different pants, orange (top) and the same racket but different colored pants and this one (pointing to the green top/yellow pants/ yellow tennis racket) is just like this one (pointing to green top/yellow pants/blue tennis racket) but it has a yellow tennis racket. This one (pointing to orange top/yellow pants/blue tennis racket) is just like this one (pointing to orange top/yellow pants/yellow tennis racket) but it has a blue racket.

On both the fifth and sixth problems, Sam adopted the most efficient strategy (10), as can be seen in his sequence of item selection on the final problem:

orange top, blue tennis racket, and orange pants
orange top, blue tennis racket, and blue pants
orange top, yellow tennis racket, and orange pants
orange top, yellow tennis racket, and blue pants
blue top, yellow tennis racket, and orange pants
blue top, yellow tennis racket, and blue pants
blue top, blue tennis racket and blue pants
blue top, blue tennis racket, and orange pants
yellow top, blue tennis racket, and blue pants
yellow top, blue tennis racket, and orange pants
yellow top, yellow tennis racket, and blue pants
yellow top, yellow tennis racket, and orange pants

When Sam had finished each problem, he stated that he could not do any more. When asked how he knew this, Sam replied, "Four greens and four yellows" (problem 5). When asked what he did with these, he responded, "I did green and blue with an orange racket and then green and orange with an orange racket. Then I did the same ones with a blue racket." When the interviewer queried the child on his method for the final problem, he replied, "I used two orange tops with blue rackets and then I copied them down there using yellow rackets. I did the same thing each time. I made two outfits and swapped them around with the rackets."

Mark, aged 10 years 11 months, fifth year of school

Mark's performance demonstrates the second type of strategy change. On problem 1, Mark followed strategy 2. He progressed to strategy 3 on the second problem, following a cyclic pattern in his selection of pants:

blue top and yellow pants

orange top and pink pants
blue top and yellow pants
orange top and pink pants
green top and yellow pants
green top and pink pants

It was only at this point that Mark realized he had dressed the third bear the same as the first bear. He proceeded to remove the yellow pants of the third bear and replaced it with pink pants. He then continued to form a further combination:

orange top and yellow pants

When asked if all the bears were dressed in different outfits, Mark replied, "No. The fourth is the same as the second," and proceeded to remove the clothes of the fourth bear. While he tried a few times to change this outfit to the correct one, he was unable to do so.

Despite using a cyclic pattern, Mark made two errors because of his failure to carefully monitor his actions. On the third problem however, Mark demonstrated a complete odometer strategy (5) which he used confidently and efficiently. When presented with the three-dimensional problems, he used the most sophisticated strategy (10) and could clearly explain his method. He was consistent in his explanations for each of the three-dimensional problems. For example, he completed problem 4 as follows:

orange top, yellow pants, and blue tennis racket
orange top, yellow pants, and yellow tennis racket
green top, yellow pants, and blue tennis racket
green top, yellow pants, and yellow tennis racket
orange top, pink pants, and blue tennis racket
orange top, pink pants, and yellow tennis racket
green top, pink pants, and blue tennis racket

green top, pink pants, and yellow tennis racket

When asked to describe his procedure, Mark explained, "On two I did orange top/yellow pants with different rackets. I did green top/yellow pants and changed the rackets. I did pink pants/orange tops and changed the racket and pink pants/green tops and changed the rackets." A similar explanation was given for his strategy on the fifth and sixth problems. On the latter, he stated, "I did it the same as before. I went through all the tops with yellow pants and changed the rackets. Then I went through all the tops with pink pants and changed the rackets."

Similar explanations were offered by other children who used strategy 10, including Samuel as cited previously. Ann, a 9 year-old fourth-grade student stated, "I dressed two in the same colors and changed the colors of the tennis racket and I did that with all of them." Alicia (10 years) explained, "I did all the same outfits with different tennis rackets. I did two the same and changed the rackets." Some children referred to the number of outfits they could make before changing the major constant item. For example, Quentin (7 years) said, "I did six with orange and then I knew how many I could do with purple pants and I changed the tennis rackets." Twelve year-old Andrew claimed, "For the first four I used all yellow pants, two of each colored top and changed the colour of the tennis racket and I did the same with all of them with pink pants."

<u>Lauretta, aged 11 years, sixth year of school</u>

Lauretta belonged to the control group who completed problems 5 and 6 only. She applied strategy 7 to problem 5 and made an error in doing so. On problem 6 however, she used strategy 10, without error. On problem 5, she displayed the following order of combinations:

blue pants, yellow top, and blue tennis racket
green top, orange pants, and orange tennis racket
blue pants, green top, and blue tennis racket
orange pants, yellow top, and orange tennis racket
yellow top, blue pants, and orange tennis racket
orange tennis racket, green top, and blue pants
orange pants, yellow top, and blue tennis racket
blue pants, yellow top, and blue tennis racket
orange pants, green top, and blue tennis racket

On completion of these, Lauretta stated, "That's all I can do and I think I've made two the same. The first and the eighth are the same." She then removed the eighth outfit and attempted to dress it differently but realized it was again the same as another bear. She tried to dress .e bear but unsuccessfully, commenting that she "can't fix it up." When asked to describe the procedure she followed, Lauretta explained, "First of all I just made an outfit and later tried to make one the same but with a different racket and I looked along and I tried to add something different that the one that was similar did not have."

On the next (final) problem however, Lauretta applied strategy 10 confidently and efficiently. She explained her procedure thus: "Well, first of all I used pink pants. I put pink pants, yellow shirt, and blue tennis racket. I did two with yellow shirts (and pink pants) but changed the tennis racket. Then I did the same with all the rest (referring to the remaining shirts) and then changed to orange pants.

Then I did two yellow shirts with orange pants and changed the tennis rackets and I just did that all the way along."

Given these changes in the children's strategies, it is worthwhile considering just what constitutes children's thinking as they generate solutions to the combinatorial problems. More specifically, the issue in question is the nature of the principled knowledge underlying children's solution strategies and the way in which this knowledge changes as children adopt more sophisticated procedures. In addition, the role of metastrategic knowledge in the solution process is examined. In addressing these issues, the cases studies of the previous section are revisited.

Changes in children's knowledge on the two-dimensional problems

It was noted earlier that children must meet a minimum set of constraints in solving the given problems. These constraints pertain to the types of items to be combined and to the uniqueness of the resulting combinations. To identify the minimum principled knowledge children need to bring to these problems, consideration is given to Samuel's performance on the first problem. Sam chose an item of each type to combine and continued to form combinations until he believed he had finished. When prompted to look for further possibilities, he made another combination. However he made an error in doing so and did not detect the error until it was pointed out to him, indicating a failure to effectively monitor his actions. This may have been due to Sam's preoccupation with the pattern he was following in his selection of tops. Nevertheless, Sam realized he could repair the incorrect combination and proceeded to do so. Sam's actions on this first problem suggest that he had an implicit knowledge of the principles of combination and difference (English, 1988), as they pertain to the formation of two-dimensional combinations. These principles may be defined thus:

> The principle of two-dimensional combination asserts that an item from each of two sets of items must be combined.
>
> The principle of difference asserts that two or more combinations of items will be different from each other if they differ in at least one item.

A knowledge of two other fundamental principles is also assumed in Sam's actions. These principles are derived from the counting domain and refer firstly, to the order irrelevance of item selection (Gelman and Gallistel, 1978, p.82) that is, it does not matter whether a top, pair of pants, or tennis racket is selected first. The second principle pertains to one-to-one matching (Gelman and Gallistel, 1978, p.77) where each item is associated with its corresponding position on each bear. This latter principle was violated by a few preschoolers in an earlier study (English, 1988) where children attempted to create further combinations by reversing the positions of the items on the bears. It is interesting to note that only preschool children made this error and furthermore, knew that they were making an error. To them, it was a playful attempt at creating something "different." The school children on the other hand, preferred to adhere strictly to the "rules of the problem."

Mark's performance on the second problem suggests that he was attempting to improve solution efficiency by following a complete pattern in item selection (yellow pants, pink pants, yellow pants, pink pants......). However Mark failed to monitor his progress and consequently made errors which he was unable to repair. He retained his cyclic pattern however, and incorporated this with the use of a constant item on the third problem to produce a perfect odometer strategy. Sam had also developed an (almost) odometer strategy by the third problem. While he did not apply it perfectly, his comments as he checked his results indicated an understanding of this procedure ("Three blues, three greens, and three oranges. I've done all the colours.")

There would appear to be four key components of the children's progress on the two-dimensional problems. Firstly, there was the appearance of a cyclic pattern in item selection. Both children commenced the problems with a cyclic pattern which they did not carry through to completion. They perfected this however, as they progressed to the second and third problems. This would suggest that these children commenced the problems with a partial knowledge of systematic variation (English, 1988) which became more complete with experience. For the two-dimensional problems, this principle may be defined thus:

> The principle of systematic variation asserts that, within the one cycle of selection from two discrete sets of items, different combinations will be generated if items of the one type are varied systematically. This is irrespective of the second item chosen.

The second component of the children's strategy development on the two-dimensional problems is the emergence of a constant item together with the retention of the cyclic pattern of item selection. This would suggest a knowledge of the principle of constancy for two-dimensional combinations (English, 1988):

> The principle of constancy asserts that, within the one cycle of

selection from two discrete sets of items, different combinations will be generated if an item of the one type remains constant while items of the other type vary systematically.

The third and fourth components pertain to the exhaustion of the constant items and recognition of problem completion. Mark held items constant until they could no longer generate unique combinations; that is, he did not change his constant items until he had exhausted them. Furthermore, he knew when he had solved the problem and did not attempt to generate more than the maximum number of possible combinations. Sam, on the other hand, switched from blue pants to orange pants in the early stages of solving problem 3 and hence failed to exhaust these items. He nevertheless held the remaining blue and orange pants constant and also exhausted the pink pants. Furthermore, he recognized problem completion. The actions of these children suggest that both had a knowledge of the odometer strategy, however Mark had a more thorough understanding than Sam. More specifically, it is hypothesized that Mark had an implicit understanding of two additional principles, namely, exhaustion and completion (English, 1988), while Sam appeared not to have acquired a complete understanding of the former principle. These principles may be defined for two-dimensional combinations, as follows:

The principle of exhaustion asserts that a given constant item is exhausted when it can no longer generate combinations which are different from existing combinations.

The principle of completion asserts that all possible combinations have been generated when all constant items have been exhausted.

While this principled knowledge is considered essential to problem success, it does not guarantee it. Other factors such as interpretative errors, difficulties in executing a plan correctly, and a failure to carefully monitor performance, can prevent problem solution (English, 1991, 1992; Gelman & Greeno, 1989). In the present study, children's skill in monitoring their actions and detecting errors is particularly important. However unless children can repair these errors, they will not achieve the problem goal of all different combinations as was evident in Mark's actions on the first problem. Initial problem success thus requires not only a knowledge of the aforementioned principles but also a metastrategic knowledge of the requirements of the problem, the means by which strategies can be monitored, the point at which repairs must be made and how these repairs can best be implemented.

Changes in children's knowledge on the three-dimensional problems

The responses of children displaying strategy 6, the most inefficient of the 3-D strategies, indicate that these children were adopting a trial-and-error approach. Even if they did have an implicit knowledge of the foregoing combinatorial principles, their actions and explanations suggest that they were unable to apply this to the solution of the three-dimensional problems. The responses of the following children support this hypothesis:

> "What I did, well, first I did any outfits I wanted to. Then I looked at them and then I changed the colour of the pants and I just kept changing them." (Rachel, 8 years)

"I dressed them up in any colours and I put a tennis racket on and saw if any others were different and I tried to change it if it wasn't different." (Christine, 10 years)

"I looked at the tops and the rackets, then I looked at the pants and then I looked at the one I was dressing to make sure it wasn't the same." (Brett, 10 years)

One 7 year-old child realized at the end of the fourth problem that she could improve on her inefficient method and did so on the next problem. She explained:

"Well, I did one (outfit) and then I looked up at it and then I did another one and as I went, I kept looking at all the other clothes that were different and in case it was the same, I'd look at it and change it. Oh, now I know what I could have done. I could have put all the blues in a line and all the yellows in a line."

Children who followed strategies 7 and 8 however, demonstrated an awareness of the need to hold two items constant while varying the third. These children frequently referred to the formation of pairs of outfits where two items are held constant and a third is varied, for example:

"First of all I just made an outfit and later tried to make one the same but with a different racket." (Lauretta, 11 years, strategy 7)

"What I do is, say I have a pink top and blue pair of pants and a blue racket, well I then do another one only with a yellow racket." (Peter 7 years, strategy 7)

"I just put the same colors on both bears and just put different tennis rackets on and I did that with all of them." (Julie, 11 years, strategy 8).

The responses of these children suggest that they were able to apply a knowledge of constancy and systematic variation to the solution of the three-dimensional problems. However they did not always exhaust their constant items, as was evident in the responses of Samuel and Lauretta. It would seem that the complexity of these problems made it difficult for them to coordinate all components of the knowledge required for an efficient solution.

While children displaying strategies 7 and 8 mentioned using two items twice and varying the third, they did not comment on exhausting an item. This was in contrast to children who displayed the more sophisticated strategies, 9 and 10. These children usually referred to using an item until it could no longer generate new combinations, with some also stating the number of combinations they could form with a particular constant item. Children's explanations of these advanced strategies would appear to reflect a more comprehensive understanding of the combinatorial domain. To support this hypothesis, some of the children's responses are reproduced below:

"I did six with orange and then I knew how many I could do with pink pants and I changed the tennis rackets." (Quentin, 7 years).

"I used the same method as before. All the pink pants with different colored tops and different colored tennis rackets and all the orange pants with different colored tops and different colored rackets." (Danielle, 12 years).

"I used all the pink pants with different colored tops and I put an orange tennis racket on three of them and then a blue racket on three of them. Then I used all the orange pants with different colored tops and three with orange rackets and three with blue rackets." (Nicole, 10 years).

"I had the same pants and the same top and two different rackets. Because there were two different colored rackets, I used two tops. I changed the tops again and put the two different rackets ...." (Kevin, 12 years).

It would seem that these children were able to apply an understanding of all of the combinatorial principles to the solution of the three-dimensional problems. However this requires more than a mere application of the principled knowledge used in the solution of the two-dimensional examples. Children's knowledge of holding constant, throughout a single cycle, an item from one set (X), must now be extended to include a second (minor) item (Y) which remains constant for only a part of the overall cycle (refer Figure 2). The frequency with which this minor constant item is changed is governed by the number of different items in the remaining, third set (Z), as 12 year-old Kevin noted. After each systematic variation of the items in the third set, a new minor item is selected. Knowing whether or not a major constant item has been exhausted involves consideration of whether each of the minor constant items has been exhausted. It was clear in the children's strategies that less competent problem solvers did not consider the exhaustion of either constant item, while more competent children focussed on the exhaustion of one constant item, and the most competent addressed both. As Fiona (12 years) mentioned while explaining her use of strategy 10 on problem 6, "I kept the same pink pants, two yellow shirts, and different rackets. Then I had to change to orange pants because there were no other colored shirts to use." It is argued that this developing ability to monitor and coordinate the exhaustion of two constant items represents a growth in children's metastrategic knowledge. With experience in solving these problems, children become more aware of the problems' underlying structure and of how they can modify their inefficient strategies to produce more effective solution methods. This, in turn, enhances their understanding of the strategy (Gelman & Brown, 1986; Gelman & Greeno, 1989; Kuhn & Phelps, 1982; Kuhn, Amsel, & O'Loughlin, 1988; Pressley et al., 1985; Pressley et al., 1987). Metastrategic knowledge is thus considered to play a major role in children's acquisition and stabilization of efficient strategies and their discarding of less effective strategies (Kuhn et al., 1988).

It may be that the superior performance of the older children was due to their better developed metastrategic knowledge. It has been argued that older children are more capable of reflecting on the problem situation and that they "look for the rules of the game, and expect to extract a general rule" (Brown & Kane, 1988, p.519). Furthermore, the more sophisticated planning skills of older children would assist them in generating efficient solution strategies. As Rogoff et al. (1987, p.305) contend, planning involves both the context of the task and the problem constraints, together with the skills and knowledge the problem solver brings to the situation. Older children are better able to integrate existing knowledge, are more likely to adjust their planning to the problem conditions, and are more able to coordinate multiple considerations in solving problems. The less effective planning of younger children may be due in part to their difficulty in coordinating the means and the goal in more complex problem solving.

Conclusion

The present study investigated children's independent strategy development in solving a series of novel, two- and three-dimensional combinatorial problems. For each problem type, a sequence of five, increasingly complex, strategies was identified from the children's actions and explanations as they manipulated the problem materials. These strategies ranged from inefficient trial-and-error methods to efficient odometer procedures. Changes in children's strategy use as they progressed on the problems suggested modifications to their knowledge of the combinatorial domain. It was argued that children brought with them a body of principled knowledge that was requisite to problem solution. This included an implicit understanding of the principles of difference, combination, order-irrelevance, and one-to-one matching. However this principled knowledg alone, was not sufficient for problem solution. Children's ability to monitor their actions, detect and correct errors, and recognize problem completion played a crucial role in goal attainment.

Previous studies (e.g. English, 1992) have shown that the nature and extent of children's monitoring has a significant bearing on their solution of novel combinatorial problems. However the major determinant of children's success in solving these problems is the interaction between their domain-specific knowledge and their general problem-solving processes (English, 1992). Case studies of children's responses in the present study have illustrated the importance of this interaction.

The trends observed in this study reflect those of other studies of children's problem solving (e.g. DeLoache et al., 1985, Gelman & Brown, 1986). The "general learning mechanism" which, according to Gelman and Brown (1986, p.188), characterizes children's problem-solving behavior across age groups was evident in the present study. Children progressed from immature, trial-and-error behavior to efficient "expert" strategies, reflecting an awareness of the relationships among the components of each problem as a whole. The older children in particular, demonstrated this understanding with their ability to operate effectively with two constant items in solving the 3-D problems. The fact these children did switch, very rapidly, to more efficient strategies and abandoned their old inefficient ones suggests that they were becoming increasingly aware of the structure of the problems and were looking for ways to streamline their progress towards goal attainment. As children developed more advanced strategies they offered more sophisticated accounts of their actions. It is concluded that their experience in solving these problems contributed to a growth in both their strategic and metastrategic knowledge.

Other studies have shown that children can invent procedures for solving arithmetical tasks without instruction (e.g. Carpenter, 1985; Silver & Marshall, 1990). Analyses of children's solution strategies in solving such tasks has indicated that they attend to the semantics of the problem situation, that is, they construct a representation that is based on an adequate understanding of this situation (Silver & Marshall, 1990). With respect to the present study, problems were selected from a novel domain and presented in a meaningful context. Problems of this nature enable children to connect new information to their existing knowledge structures and construct new relationships among those structures (Silver & Marshall, 1990). The domain of combinatorics provides a fertile source of problem-solving activities in which children can discover for themselves important mathematical concepts and principles. Problems from this domain should form a part of the regular mathematics curriculum.

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Table 1

Mean Numbers of New X and Y Item Selections for the Three-dimensional Strategies

	New selections of item X on the 2 X 2 X 2 tasks			New selections of item Y on the 2 X 2 X 2 tasks		
Strategy	Mean	Range	SD	Mean	Range	SD
6	5.2	3	0.8	6.5	4	1.0
7	4.6	2	0.7	5.6	3	0.9
8	4	0	0	4	0	0
9	3	0	0	4	0	0
10	2	0	0	4	0	0

	New selections of item X on the 2 X 3 X 2 task			New selections of item Y on the 2 X 3 X 2 task		
Strategy	Mean	Range	SD	Mean	Range	SD
6	7.3	3	1.1	9.2	4	1.4
7	5.8	3	1.6	7.4	3	1.1
8	4.6	2	0.7	5.4	2	0.7
9	3	0	0	6	0	0
10	2	0	0	6	0	0

Table 2

Frequency of Strategy Use (less efficient / more efficient) by Age on Problems 1 to 6

Age (Yrs)	Problem 1 Strategies* 1-3	Problem 1 Strategies* 4-5	Problem 2 Strategies* 1-3	Problem 2 Strategies* 4-5	Problem 3 Strategies* 1-3	Problem 3 Strategies* 4-5
7	6	6	4	8	4	8
8	8	4	6	6	6	6
9	5	7	1	11	5	7
10	3	9	4	8	4	8
11	8	4	4	8	3	9
12	3	9	1	11	1	11

Age (Yrs)	Problem 4 Strategies* 6-8	Problem 4 Strategies* 9-10	Problem 5 Strategies* 6-8	Problem 5 Strategies* 9-10	Problem 6 Strategies* 6-8	Problem 6 Strategies* 9-10
7	11	1	7	5	8	4
8	7	5	4	8	7	5
9	8	4	6	6	5	7
10	8	4	5	7	6	6
11	5	7	1	11	4	8
12	3	9	2	10	4	8

* Note. Inefficient strategies: 1-3 (2-D) and 6-8 (3-D)
Efficient strategies: 4-5 (2-D) and 9-10 (3-D)

Table 3

Specific Strategy Changes Across Problems

Type of strategy change	Problems 1 -> 3		Problems 3 -> 4		Problems 3 -> 6		Problems 4 -> 5		Problems 5 -> 6		Problems 4 -> 6	
	7-9 yrs	10-12 yrs	7-9 yrs	10-12 yrs	7-9 yrs	10-12 yrs	7-9 yrs	10-12 yrs	7-9 yrs	10-12 yrs	7-9 yrs	10-12 yr.
1/2 -> 4/5	6	7										
1/2 -> 9/10			2	2	4	5						
4/5 -> 9/10			8	18	12	17						
6/7 -> 9/10							11	7	2	2	10	6
6/7 -> 8							1	0	0	3	1	4

Note. N (7-9) yrs) = 36
N (10-12 yrs) = 36

Empty cells appear because strategies 1 to 5 refer to problems 1 to 3 only and strategies 6 to 10 refer to problems 4 to 6 only.

Table 4

Frequency of Problem Success by Strategy Type and Age

	Problems 1 to 3				Problems 4 to 6			
	Strategies 1-3		Strategies 4-6		Strategies 6-8		Strategies 9-10	
Age (Yrs)	Succeed	Fail	Succeed	Fail	Succeed	Fail	Succeed	Fail
7	4	2	4	2	3	6	2	1
8	5	2	5	0	2	5	4	1
9	2	1	8	1	1	4	4	3
10	2	2	6	2	0	6	6	0
11	3	0	8	1	1	4	6	1
12	1	0	10	1	1	2	9	0

Table 5

Strategy use on Problems 5 and 6 by the Control and Experimental Groups

	Problem			
	5		6	
Strategy type	Control	Experimental	Control	Experimental
6	10	1	3	1
7	3	2	2	1
8	2	0	5	6
9	4	7	1	5
10	5	14	13	11

Author Notes

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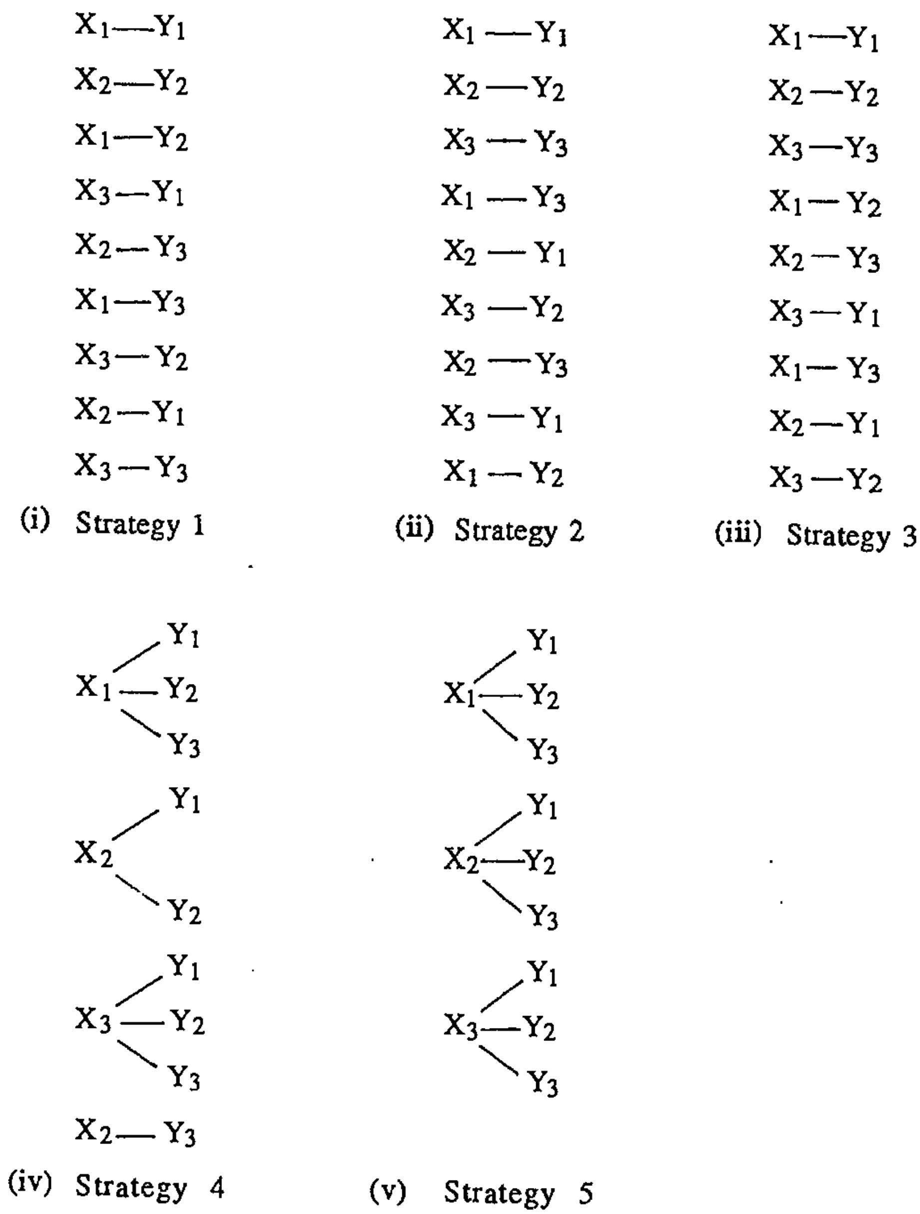
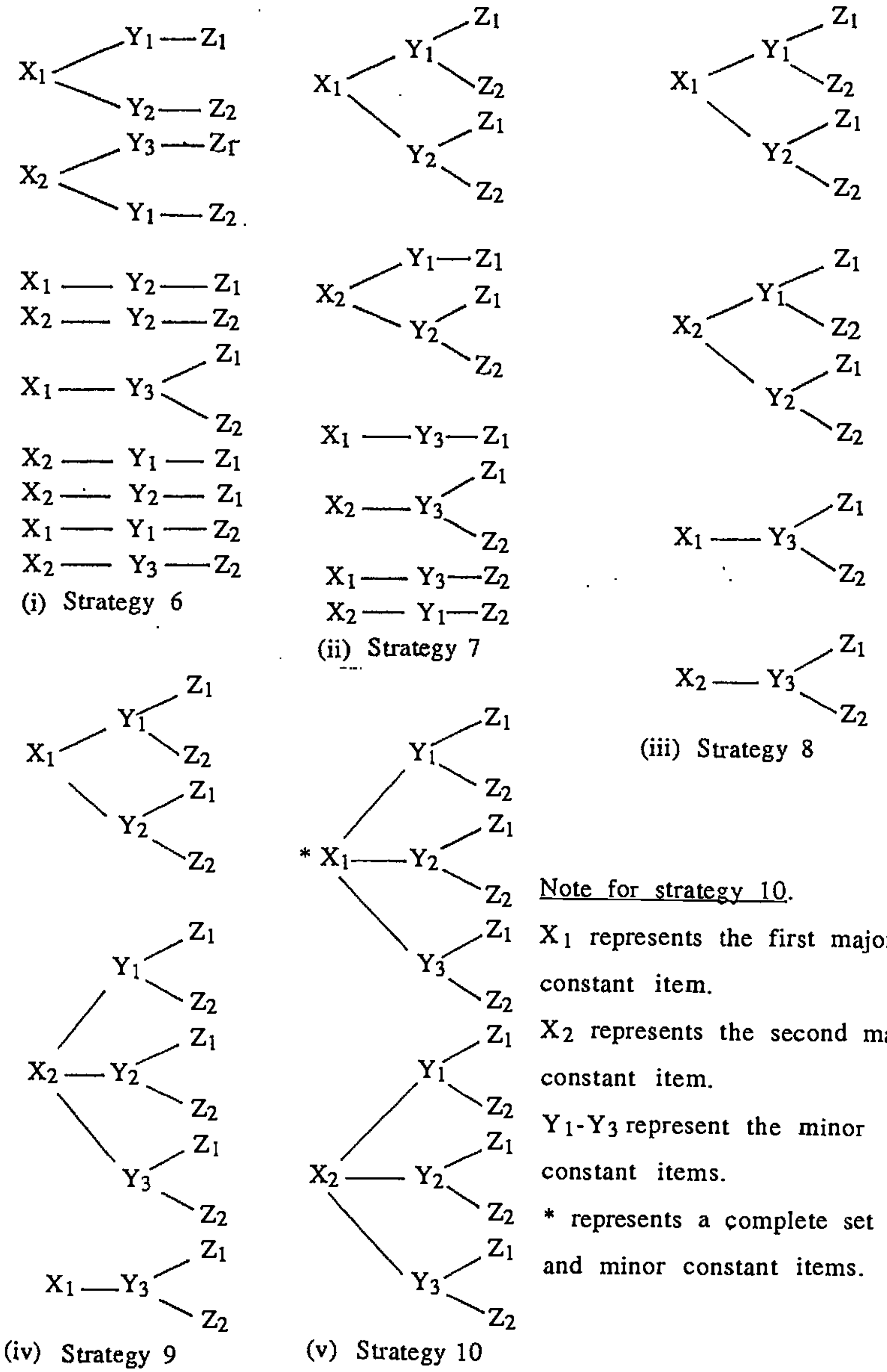


Figure 1. Tree diagrams for the two-dimensional strategies



Note for strategy 10.

X_1 represents the first major constant item.

X_2 represents the second major constant item.

Y_1-Y_3 represent the minor constant items.

* represents a complete set of major and minor constant items.

Figure 2. Tree diagrams for the three-dimensional strategies